Sun Performance Library Reference Manual

Sun™ ONE Studio 8

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This reference manual is the Sun Performance Library section 3P man pages, available in HTML and PDF formats. For additional information, see the *Sun Performance Library User's Guide*, available on `docs.sun.com`, or the *LAPACK Users' Guide*, available from the Society for Industrial and Applied Mathematics (SIAM).

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cbdimm - cbdimm - block diagonal format matrix-matrix multiply

cbdism - cbdism - block diagonal format triangular solve

cbdsqr - cbdsqr - compute the singular value decomposition (SVD) of a real N-by-N (upper or lower) bidiagonal matrix B.

cbelmm - cbelmm - block Ellpack format matrix-matrix multiply

cbelsm - cbelsm - block Ellpack format triangular solve

cbscmm - cbscmm - block sparse column matrix-matrix multiply

cbscsm - cbscsm - block sparse column format triangular solve

cbsrmm - cbsrmm - block sparse row format matrix-matrix multiply

cbsrsm - cbsrsm - block sparse row format triangular solve

ccnvcor - ccnvcor - compute the convolution or correlation of complex vectors

ccnvcor2 - ccnvcor2 - compute the convolution or correlation of complex matrices

ccoomm - ccoomm - coordinate matrix-matrix multiply

ccopy - ccopy - Copy x to y

ccscmm - ccscmm - compressed sparse column format matrix-matrix multiply

ccscsm - ccscsm - compressed sparse column format triangular solve

ccsrmm - ccsrmm - compressed sparse row format matrix-matrix multiply

ccsrsm - ccsrsm - compressed sparse row format triangular solve

cdiamm - cdiamm - diagonal format matrix-matrix multiply

cdiasm - cdiasm - diagonal format triangular solve

cdotc - cdotc - compute the dot product of two vectors conjg(x) and y.

cdotci - cdotci - Compute the complex conjugated indexed dot product.

cdotu - cdotu - compute the dot product of two vectors x and y.

cdotui - cdotci - Compute the complex conjugated indexed dot product.

cellmm - cellmm - Ellpack format matrix-matrix multiply

cellsm - cellsm - Ellpack format triangular solve

cfft2b - cfft2b - compute a periodic sequence from its Fourier coefficients. The xFFT operations are unnormalized, so a call of xFFT2F followed by a call of xFFT2B will multiply the input sequence by M*N.

cfft2f - cfft2f - compute the Fourier coefficients of a periodic sequence. The xFFT operations are unnormalized, so a call of xFFT2F followed by a call of xFFT2B will multiply the input sequence by M*N.

cfft2i - cfft2i - initialize the array WSAVE, which is used in both the forward and backward transforms.

cfft3b - cfft3b - compute a periodic sequence from its Fourier coefficients. The FFT operations are unnormalized, so a call of CFFT3F followed by a call of CFFT3B will multiply the input sequence by M*N*K.

cfft3f - cfft3f - compute the Fourier coefficients of a periodic sequence. The FFT operations are unnormalized, so a call of CFFT3F followed by a call of CFFT3B will multiply the input sequence by M*N*K.

cfft3i - cfft3i - initialize the array WSAVE, which is used in both CFFT3F and CFFT3B.

cfftb - cfftb - compute a periodic sequence from its Fourier coefficients. The FFT operations are unnormalized, so a call of CFFTF followed by a call of CFFTB will multiply the input sequence by N.

cfftc - cfftc - initialize the trigonometric weight and factor tables or compute the Fast Fourier transform (forward or inverse) of a complex sequence.

cfftc2 - cfftc2 - initialize the trigonometric weight and factor tables or compute the two-dimensional Fast Fourier Transform (forward or inverse) of a two-dimensional complex array.

cfftc3 - cfftc3 - initialize the trigonometric weight and factor tables or compute the three-dimensional Fast Fourier Transform (forward or inverse) of a three-dimensional complex array.

cfftcm - cfftcm - initialize the trigonometric weight and factor tables or compute the one-dimensional Fast Fourier Transform (forward or inverse) of a set of data sequences stored in a two-dimensional complex array.

cfftf - cfftf - compute the Fourier coefficients of a periodic sequence. The FFT operations are unnormalized, so a call of CFFTF followed by a call of CFFTB will multiply the input sequence by N.

cffti - cffti - initialize the array WSAVE, which is used in both CFFTF and CFFTB.

cfftopt - cfftopt - compute the length of the closest fast FFT

cffts - cffts - initialize the trigonometric weight and factor tables or compute the inverse Fast Fourier Transform of a complex sequence as follows.

cffts2 - cffts2 - initialize the trigonometric weight and factor tables or compute the two-dimensional inverse Fast Fourier Transform of a two-dimensional complex array.

cffts3 - cffts3 - initialize the trigonometric weight and factor tables or compute the three-dimensional inverse Fast Fourier Transform of a three-dimensional complex array.

cfftsm - cfftsm - initialize the trigonometric weight and factor tables or compute the one-dimensional inverse Fast Fourier Transform of a set of complex data sequences stored in a two-dimensional array.

cgbbrd - cgbbrd - reduce a complex general m-by-n band matrix A to real upper bidiagonal form B by a unitary transformation

cgbcon - cgbcon - estimate the reciprocal of the condition number of a complex general band matrix A, in either the 1-norm or the infinity-norm,

cgbequ - cgbequ - compute row and column scalings intended to equilibrate an M-by-N band matrix A and reduce its condition number

cgbmv - cgbmv - perform one of the matrix-vector operations y := alpha*A*x + beta*y, or y := alpha*A'*x + beta*y, or y := alpha*conjg(A')*x + beta*y

cgbrfs - cgbrfs - improve the computed solution to a system of linear equations when the coefficient matrix is banded, and provides error bounds and backward error estimates for the solution

cgbsv - cgbsv - compute the solution to a complex system of linear equations A * X = B, where A is a band matrix of order N with KL subdiagonals and KU superdiagonals, and X and B are N-by-NRHS matrices

cgbsvx - cgbsvx - use the LU factorization to compute the solution to a complex system of linear equations A * X = B, A**T * X = B, or A**H * X = B,

cgbtf2 - cgbtf2 - compute an LU factorization of a complex m-by-n band matrix A using partial pivoting with row interchanges

cgbtrf - cgbtrf - compute an LU factorization of a complex m-by-n band matrix A using partial pivoting with row interchanges cgbtrs - cgbtrs - solve a system of linear equations A * X = B, A**T * X = B, or A**H * X = B with a general band matrix A using the LU factorization computed by CGBTRF

cgebak - cgebak - form the right or left eigenvectors of a complex general matrix by backward transformation on the computed eigenvectors of the balanced matrix output by CGEBAL

cgebal - cgebal - balance a general complex matrix A

cgebrd - cgebrd - reduce a general complex M-by-N matrix A to upper or lower bidiagonal form B by a unitary transformation

cgecon - cgecon - estimate the reciprocal of the condition number of a general complex matrix A, in either the 1-norm or the infinity-norm, using the LU factorization computed by CGETRF

cgeequ - cgeequ - compute row and column scalings intended to equilibrate an M-by-N matrix A and reduce its condition number

cgees - cgees - compute for an N-by-N complex nonsymmetric matrix A, the eigenvalues, the Schur form T, and, optionally, the matrix of Schur vectors Z

cgeesx - cgeesx - compute for an N-by-N complex nonsymmetric matrix A, the eigenvalues, the Schur form T, and, optionally, the matrix of Schur vectors Z

cgeev - cgeev - compute for an N-by-N complex nonsymmetric matrix A, the eigenvalues and, optionally, the left and/or right eigenvectors

cgeevx - cgeevx - compute for an N-by-N complex nonsymmetric matrix A, the eigenvalues and, optionally, the left and/or right eigenvectors

cgegs - cgegs - routine is deprecated and has been replaced by routine CGGES

cgegv - cgegv - routine is deprecated and has been replaced by routine CGGEV

cgehrd - cgehrd - reduce a complex general matrix A to upper Hessenberg form H by a unitary similarity transformation

cgelqf - cgelqf - compute an LQ factorization of a complex M-by-N matrix A

cgels - cgels - solve overdetermined or underdetermined complex linear systems involving an M-by-N matrix A, or its conjugate-transpose, using a QR or LQ factorization of A

cgelsd - cgelsd - compute the minimum-norm solution to a real linear least squares problem cgelss - cgelss - compute the minimum norm solution to a complex linear least squares problem

cgelsx - cgelsx - routine is deprecated and has been replaced by routine CGELSY

cgelsy - cgelsy - compute the minimum-norm solution to a complex linear least squares problem

cgemm - cgemm - perform one of the matrix-matrix operations C := alpha*op(A)*op(B) + beta*C

cgemv - cgemv - perform one of the matrix-vector operations y := alpha*A*x + beta*y, or y := alpha*A'*x + beta*y, or y := alpha*conjg(A')*x + beta*y

cgeqlf - cgeqlf - compute a QL factorization of a complex M-by-N matrix A

cgeqp3 - cgeqp3 - compute a QR factorization with column pivoting of a matrix A

cgeqpf - cgeqpf - routine is deprecated and has been replaced by routine CGEQP3

cgeqrf - cgeqrf - compute a QR factorization of a complex M-by-N matrix A

cgerc - cgerc - perform the rank 1 operation A := alpha*x*conjg(y') + A

cgerfs - cgerfs - improve the computed solution to a system of linear equations and provides error bounds and backward error estimates for the solution

cgerqf - cgerqf - compute an RQ factorization of a complex M-by-N matrix A

cgeru - cgeru - perform the rank 1 operation A := alpha*x*y' + A

cgesdd - cgesdd - compute the singular value decomposition (SVD) of a complex M-by-N matrix A, optionally computing the left and/or right singular vectors, by using divide-and-conquer method

cgesv - cgesv - compute the solution to a complex system of linear equations A * X = B,

cgesvd - cgesvd - compute the singular value decomposition (SVD) of a complex M-by-N matrix A, optionally computing the left and/or right singular vectors

cgesvx - cgesvx - use the LU factorization to compute the solution to a complex system of linear equations A * X = B,

cgetf2 - cgetf2 - compute an LU factorization of a general m-by-n matrix A using partial pivoting with row interchanges cgetrf - cgetrf - compute an LU factorization of a general M-by-N matrix A using partial pivoting with row interchanges

cgetri - cgetri - compute the inverse of a matrix using the LU factorization computed by CGETRF

cgetrs - cgetrs - solve a system of linear equations A * X = B, A**T * X = B, or A**H * X = B with a general N-by-N matrix A using the LU factorization computed by CGETRF

cggbak - cggbak - form the right or left eigenvectors of a complex generalized eigenvalue problem A*x = lambda*B*x, by backward transformation on the computed eigenvectors of the balanced pair of matrices output by CGGBAL

cggbal - cggbal - balance a pair of general complex matrices (A,B)

cgges - cgges - compute for a pair of N-by-N complex nonsymmetric matrices (A,B), the generalized eigenvalues, the generalized complex Schur form (S, T), and optionally left and/or right Schur vectors (VSL and VSR)

cggesx - cggesx - compute for a pair of N-by-N complex nonsymmetric matrices (A,B), the generalized eigenvalues, the complex Schur form (S,T),

cggev - cggev - compute for a pair of N-by-N complex nonsymmetric matrices (A,B), the generalized eigenvalues, and optionally, the left and/or right generalized eigenvectors

cggevx - cggevx - compute for a pair of N-by-N complex nonsymmetric matrices (A,B) the generalized eigenvalues, and optionally, the left and/or right generalized eigenvectors

cggglm - cggglm - solve a general Gauss-Markov linear model (GLM) problem

cgghrd - cgghrd - reduce a pair of complex matrices (A,B) to generalized upper Hessenberg form using unitary transformations, where A is a general matrix and B is upper triangular

cgglse - cgglse - solve the linear equality-constrained least squares (LSE) problem

cggqrf - cggqrf - compute a generalized QR factorization of an N-by-M matrix A and an N-by-P matrix B.

cggrqf - cggrqf - compute a generalized RQ factorization of an M-by-N matrix A and a P-by-N matrix B

cggsvd - cggsvd - compute the generalized singular value decomposition (GSVD) of an M-by-N complex matrix A and P-by-N complex matrix B

cggsvp - cggsvp - compute unitary matrices U, V and Q such that N-K-L K L U'*A*Q = K (0 A12 A13) if M-K-L >= 0 cgssco - cgssco - General sparse solver condition number estimate.

cgssda - cgssda - Deallocate working storage for the general sparse solver.

cgssfa - cgssfa - General sparse solver numeric factorization.

cgssfs - cgssfs - General sparse solver one call interface.

cgssin - cgssin - Initialize the general sparse solver.

cgssor - cgssor - General sparse solver ordering and symbolic factorization.

cgssps - cgssps - Print general sparse solver statics.

cgssrp - cgssrp - Return permutation used by the general sparse solver.

cgsssl - cgsssl - Solve routine for the general sparse solver.

cgssuo - cgssuo - User supplied permutation for ordering used in the general sparse solver.

cgtcon - cgtcon - estimate the reciprocal of the condition number of a complex tridiagonal matrix A using the LU factorization as computed by CGTTRF

cgthr - cgthr - Gathers specified elements from y into x.

cgthrz - cgthrz - Gather and zero.

cgtrfs - cgtrfs - improve the computed solution to a system of linear equations when the coefficient matrix is tridiagonal, and provides error bounds and backward error estimates for the solution

cgtsv - cgtsv - solve the equation A*X = B,

cgtsvx - cgtsvx - use the LU factorization to compute the solution to a complex system of linear equations A * X = B, A**T * X = B, or A**H * X = B,

cgttrf - cgttrf - compute an LU factorization of a complex tridiagonal matrix A using elimination with partial pivoting and row interchanges

cgttrs - cgttrs - solve one of the systems of equations A * X = B, A**T * X = B, or A**H * X = B,

chbev - chbev - compute all the eigenvalues and, optionally, eigenvectors of a complex Hermitian band matrix A chbevd - chbevd - compute all the eigenvalues and, optionally, eigenvectors of a complex Hermitian band matrix A

chbevx - chbevx - compute selected eigenvalues and, optionally, eigenvectors of a complex Hermitian band matrix A

chbgst - chbgst - reduce a complex Hermitian-definite banded generalized eigenproblem A*x = lambda*B*x to standard form C*y = lambda*y,

chbgv - chbgv - compute all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite banded eigenproblem, of the form A*x=(lambda)*B*x

chbgvd - chbgvd - compute all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite banded eigenproblem, of the form A*x=(lambda)*B*x

chbgvx - chbgvx - compute all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite banded eigenproblem, of the form A*x=(lambda)*B*x

chbmv - chbmv - perform the matrix-vector operation y := alpha*A*x + beta*y

chbtrd - chbtrd - reduce a complex Hermitian band matrix A to real symmetric tridiagonal form T by a unitary similarity transformation

checon - checon - estimate the reciprocal of the condition number of a complex Hermitian matrix A using the factorization A = U*D*U**H or A = L*D*L**H computed by CHETRF

cheev - cheev - compute all eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A

cheevd - cheevd - compute all eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A

cheevr - cheevr - compute selected eigenvalues and, optionally, eigenvectors of a complex Hermitian tridiagonal matrix T

cheevx - cheevx - compute selected eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A

chegs2 - chegs2 - reduce a complex Hermitian-definite generalized eigenproblem to standard form

chegst - chegst - reduce a complex Hermitian-definite generalized eigenproblem to standard form

chegv - chegv - compute all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

chegvd - chegvd - compute all the eigenvalues, and optionally, the eigenvectors of a complex generalized

Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

chegvx - chegvx - compute selected eigenvalues, and optionally, eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

chemm - chemm - perform one of the matrix-matrix operations C := alpha*A*B + beta*C or C := alpha*B*A + beta*C

chemv - chemv - perform the matrix-vector operation y := alpha*A*x + beta*y

cher - cher - perform the hermitian rank 1 operation A := alpha*x*conjg(x') + A

cher2 - cher2 - perform the hermitian rank 2 operation A := alpha*x*conjg(y') + conjg(alpha)*y*conjg(x') + A

cher2k - cher2k - perform one of the Hermitian rank 2k operations C := alpha*A*conjg(B') + conjg(alpha) *B*conjg(A') + beta*C or C := alpha*conjg(A')*B + conjg(alpha)*conjg(B')*A + beta*C

cherfs - cherfs - improve the computed solution to a system of linear equations when the coefficient matrix is Hermitian indefinite, and provides error bounds and backward error estimates for the solution

cherk - cherk - perform one of the Hermitian rank k operations C := alpha*A*conjg(A') + beta*C or C := alpha*conjg(A')*A + beta*C

chesv - chesv - compute the solution to a complex system of linear equations A * X = B,

chesvx - chesvx - use the diagonal pivoting factorization to compute the solution to a complex system of linear equations A * X = B,

chetf2 - chetf2 - compute the factorization of a complex Hermitian matrix A using the Bunch-Kaufman diagonal pivoting method

chetrd - chetrd - reduce a complex Hermitian matrix A to real symmetric tridiagonal form T by a unitary similarity transformation

chetrf - chetrf - compute the factorization of a complex Hermitian matrix A using the Bunch-Kaufman diagonal pivoting method

chetri - chetri - compute the inverse of a complex Hermitian indefinite matrix A using the factorization A = U*D*U**H or A = L*D*L**H computed by CHETRF

chetrs - chetrs - solve a system of linear equations A*X = B with a complex Hermitian matrix A using the factorization A = U*D*U**H or A = L*D*L**H computed by CHETRF chgeqz - chgeqz - implement a single-shift version of the QZ method for finding the generalized eigenvalues w(i) =ALPHA(i)/BETA(i) of the equation det(A-w(i) B) = 0 If JOB='S', then the pair (A,B) is simultaneously reduced to Schur form (i.e., A and B are both upper triangular) by applying one unitary tranformation (usually called Q) on the left and another (usually called Z) on the right

chpcon - chpcon - estimate the reciprocal of the condition number of a complex Hermitian packed matrix A using the factorization A = U*D*U**H or A = L*D*L**H computed by CHPTRF

chpev - chpev - compute all the eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix in packed storage

chpevd - chpevd - compute all the eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A in packed storage

chpevx - chpevx - compute selected eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A in packed storage

chpgst - chpgst - reduce a complex Hermitian-definite generalized eigenproblem to standard form, using packed storage

chpgv - chpgv - compute all the eigenvalues and, optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

chpgvd - chpgvd - compute all the eigenvalues and, optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

chpgvx - chpgvx - compute selected eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

chpmv - chpmv - perform the matrix-vector operation y := alpha*A*x + beta*y

chpr - chpr - perform the hermitian rank 1 operation A := alpha*x*conjg(x') + A

chpr2 - chpr2 - perform the Hermitian rank 2 operation A := alpha*x*conjg(y') + conjg(alpha)*y*conjg(x') + A

chprfs - chprfs - improve the computed solution to a system of linear equations when the coefficient matrix is Hermitian indefinite and packed, and provides error bounds and backward error estimates for the solution

chpsv - chpsv - compute the solution to a complex system of linear equations A * X = B,

chpsvx - chpsvx - use the diagonal pivoting factorization A = U*D*U**H or A = L*D*L**H to compute the solution to a complex system of linear equations A * X = B, where A is an N-by-N Hermitian matrix stored in packed format and X and B are N-by-NRHS matrices chptrd - chptrd - reduce a complex Hermitian matrix A stored in packed form to real symmetric tridiagonal form T by a unitary similarity transformation

chptrf - chptrf - compute the factorization of a complex Hermitian packed matrix A using the Bunch-Kaufman diagonal pivoting method

chptri - chptri - compute the inverse of a complex Hermitian indefinite matrix A in packed storage using the factorization A = U*D*U**H or A = L*D*L**H computed by CHPTRF

chptrs - chptrs - solve a system of linear equations A*X = B with a complex Hermitian matrix A stored in packed format using the factorization A = U*D*U**H or A = L*D*L**H computed by CHPTRF

chsein - chsein - use inverse iteration to find specified right and/or left eigenvectors of a complex upper Hessenberg matrix H

chseqr - chseqr - compute the eigenvalues of a complex upper Hessenberg matrix H, and, optionally, the matrices T and Z from the Schur decomposition H = Z T Z**H, where T is an upper triangular matrix (the Schur form), and Z is the unitary matrix of Schur vectors

cjadmm - cjadmm - Jagged diagonal matrix-matrix multiply (modified Ellpack)

cjadrp - cjadrp - right permutation of a jagged diagonal matrix

cjadsm - cjadsm - Jagged-diagonal format triangular solve

clarz - clarz - applie a complex elementary reflector H to a complex M-by-N matrix C, from either the left or the right

clarzb - clarzb - applie a complex block reflector H or its transpose H**H to a complex distributed M-by-N C from the left or the right

clarzt - clarzt - form the triangular factor T of a complex block reflector H of order > n, which is defined as a product of k elementary reflectors

clatzm - clatzm - routine is deprecated and has been replaced by routine CUNMRZ

cosqb - cosqb - synthesize a Fourier sequence from its representation in terms of a cosine series with odd wave numbers. The COSQ operations are unnormalized inverses of themselves, so a call to COSQF followed by a call to COSQB will multiply the input sequence by 4 * N.

cosqf - cosqf - compute the Fourier coefficients in a cosine series representation with only odd wave numbers. The COSQ operations are unnormalized inverses of themselves, so a call to COSQF followed by a call to COSQB will multiply the input sequence by 4 * N.

cosqi - cosqi - initialize the array WSAVE, which is used in both COSQF and COSQB.

cost - cost - compute the discrete Fourier cosine transform of an even sequence. The COST transforms are unnormalized inverses of themselves, so a call of COST followed by another call of COST will multiply the input sequence by 2 * (N-1).

costi - costi - initialize the array WSAVE, which is used in COST.

cpbcon - cpbcon - estimate the reciprocal of the condition number (in the 1-norm) of a complex Hermitian positive definite band matrix using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPBTRF

cpbequ - cpbequ - compute row and column scalings intended to equilibrate a Hermitian positive definite band matrix A and reduce its condition number (with respect to the two-norm)

cpbrfs - cpbrfs - improve the computed solution to a system of linear equations when the coefficient matrix is Hermitian positive definite and banded, and provides error bounds and backward error estimates for the solution

cpbstf - cpbstf - compute a split Cholesky factorization of a complex Hermitian positive definite band matrix A

cpbsv - cpbsv - compute the solution to a complex system of linear equations A * X = B,

cpbsvx - cpbsvx - use the Cholesky factorization A = U**H*U or A = L*L**H to compute the solution to a complex system of linear equations A * X = B,

cpbtf2 - cpbtf2 - compute the Cholesky factorization of a complex Hermitian positive definite band matrix A

cpbtrf - cpbtrf - compute the Cholesky factorization of a complex Hermitian positive definite band matrix A

cpbtrs - cpbtrs - solve a system of linear equations A*X = B with a Hermitian positive definite band matrix A using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPBTRF

cpocon - cpocon - estimate the reciprocal of the condition number (in the 1-norm) of a complex Hermitian positive definite matrix using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPOTRF

cpoequ - cpoequ - compute row and column scalings intended to equilibrate a Hermitian positive definite matrix A and reduce its condition number (with respect to the two-norm)

cporfs - cporfs - improve the computed solution to a system of linear equations when the coefficient matrix is Hermitian positive definite,

cposv - cposv - compute the solution to a complex system of linear equations A * X = B, cposvx - cposvx - use the Cholesky factorization A = U**H*U or A = L*L**H to compute the solution to a complex system of linear equations A * X = B,

cpotf2 - cpotf2 - compute the Cholesky factorization of a complex Hermitian positive definite matrix A

cpotrf - cpotrf - compute the Cholesky factorization of a complex Hermitian positive definite matrix A

cpotri - cpotri - compute the inverse of a complex Hermitian positive definite matrix A using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPOTRF

cpotrs - cpotrs - solve a system of linear equations A*X = B with a Hermitian positive definite matrix A using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPOTRF

cppcon - cppcon - estimate the reciprocal of the condition number (in the 1-norm) of a complex Hermitian positive definite packed matrix using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPPTRF

cppequ - cppequ - compute row and column scalings intended to equilibrate a Hermitian positive definite matrix A in packed storage and reduce its condition number (with respect to the two-norm)

cpprfs - cpprfs - improve the computed solution to a system of linear equations when the coefficient matrix is Hermitian positive definite and packed, and provides error bounds and backward error estimates for the solution

cppsv - cppsv - compute the solution to a complex system of linear equations A * X = B,

cppsvx - cppsvx - use the Cholesky factorization A = U**H*U or A = L*L**H to compute the solution to a complex system of linear equations A * X = B,

cpptrf - cpptrf - compute the Cholesky factorization of a complex Hermitian positive definite matrix A stored in packed format

cpptri - cpptri - compute the inverse of a complex Hermitian positive definite matrix A using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPPTRF

cpptrs - cpptrs - solve a system of linear equations A*X = B with a Hermitian positive definite matrix A in packed storage using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPPTRF

cptcon - cptcon - compute the reciprocal of the condition number (in the 1-norm) of a complex Hermitian positive definite tridiagonal matrix using the factorization A = L*D*L**H or A = U**H*D*U computed by CPTTRF

cpteqr - cpteqr - compute all eigenvalues and, optionally, eigenvectors of a symmetric positive definite tridiagonal matrix by first factoring the matrix using SPTTRF and then calling CBDSQR to compute the singular values of the bidiagonal factor cptrfs - cptrfs - improve the computed solution to a system of linear equations when the coefficient matrix is Hermitian positive definite and tridiagonal, and provides error bounds and backward error estimates for the solution

cptsv - cptsv - compute the solution to a complex system of linear equations A*X = B, where A is an N-by-N Hermitian positive definite tridiagonal matrix, and X and B are N-by-NRHS matrices.

cptsvx - cptsvx - use the factorization A = L*D*L**H to compute the solution to a complex system of linear equations A*X = B, where A is an N-by-N Hermitian positive definite tridiagonal matrix and X and B are N-by-NRHS matrices

cpttrf - cpttrf - compute the L*D*L' factorization of a complex Hermitian positive definite tridiagonal matrix A

cpttrs - cpttrs - solve a tridiagonal system of the form A * X = B using the factorization A = U'*D*U or A = L*D*L' computed by CPTTRF

cptts2 - cptts2 - solve a tridiagonal system of the form A * X = B using the factorization A = U'*D*U or A = L*D*L' computed by CPTTRF

crot - crot - apply a plane rotation, where the cos (C) is real and the sin (S) is complex, and the vectors X and Y are complex

crotg - crotg - Construct a Given's plane rotation

cscal - cscal - Compute y := alpha * y

csctr - csctr - Scatters elements from x into y.

cskymm - cskymm - Skyline format matrix-matrix multiply

cskysm - cskysm - Skyline format triangular solve

cspcon - cspcon - estimate the reciprocal of the condition number (in the 1-norm) of a complex symmetric packed matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by CSPTRF

csprfs - csprfs - improve the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite and packed, and provides error bounds and backward error estimates for the solution

cspsv - cspsv - compute the solution to a complex system of linear equations A * X = B,

cspsvx - cspsvx - use the diagonal pivoting factorization A = U*D*U**T or A = L*D*L**T to compute the solution to a complex system of linear equations A * X = B, where A is an N-by-N symmetric matrix stored in packed format and X and B are N-by-NRHS matrices

csptrf - csptrf - compute the factorization of a complex symmetric matrix A stored in packed format using the Bunch-Kaufman diagonal pivoting method

csptri - csptri - compute the inverse of a complex symmetric indefinite matrix A in packed storage using the factorization A = U*D*U**T or A = L*D*L**T computed by CSPTRF

csptrs - csptrs - solve a system of linear equations A*X = B with a complex symmetric matrix A stored in packed format using the factorization A = U*D*U**T or A = L*D*L**T computed by CSPTRF

csrot - csrot - Apply a plane rotation.

csscal - csscal - Compute y := alpha * y

cstedc - cstedc - compute all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the divide and conquer method

cstegr - cstegr - Compute T-sigma_i = L_i D_i L_i^T, such that L_i D_i L_i^T is a relatively robust representation

cstein - cstein - compute the eigenvectors of a real symmetric tridiagonal matrix T corresponding to specified eigenvalues, using inverse iteration

csteqr - csteqr - compute all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the implicit QL or QR method

cstsv - cstsv - compute the solution to a complex system of linear equations A * X = B where A is a Hermitian tridiagonal matrix

csttrf - csttrf - compute the factorization of a complex Hermitian tridiagonal matrix A

csttrs - csttrs - computes the solution to a complex system of linear equations A * X = B

cswap - cswap - Exchange vectors x and y.

csycon - csycon - estimate the reciprocal of the condition number (in the 1-norm) of a complex symmetric matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by CSYTRF

csymm - csymm - perform one of the matrix-matrix operations C := alpha*A*B + beta*C or C := alpha*B*A + beta*C

csyr2k - csyr2k - perform one of the symmetric rank 2k operations C := alpha*A*B' + alpha*B*A' + beta*C or C :

= alpha*A'*B + alpha*B'*A + beta*C

csyrfs - csyrfs - improve the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite, and provides error bounds and backward error estimates for the solution

csyrk - csyrk - perform one of the symmetric rank k operations C := alpha*A*A' + beta*C or C := alpha*A'*A + beta*C

csysv - csysv - compute the solution to a complex system of linear equations A * X = B,

csysvx - csysvx - use the diagonal pivoting factorization to compute the solution to a complex system of linear equations A * X = B,

csytf2 - csytf2 - compute the factorization of a complex symmetric matrix A using the Bunch-Kaufman diagonal pivoting method

csytrf - csytrf - compute the factorization of a complex symmetric matrix A using the Bunch-Kaufman diagonal pivoting method

csytri - csytri - compute the inverse of a complex symmetric indefinite matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by CSYTRF

csytrs - csytrs - solve a system of linear equations A*X = B with a complex symmetric matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by CSYTRF

ctbcon - ctbcon - estimate the reciprocal of the condition number of a triangular band matrix A, in either the 1-norm or the infinity-norm

ctbmv - ctbmv - perform one of the matrix-vector operations x := A*x, or x := A'*x, or x := conjg(A')*x

ctbrfs - ctbrfs - provide error bounds and backward error estimates for the solution to a system of linear equations with a triangular band coefficient matrix

ctbsv - ctbsv - solve one of the systems of equations A*x = b, or A'*x = b, or conjg(A')*x = b

ctbtrs - ctbtrs - solve a triangular system of the form A * X = B, A**T * X = B, or A**H * X = B,

ctgevc - ctgevc - compute some or all of the right and/or left generalized eigenvectors of a pair of complex upper triangular matrices (A,B)

ctgexc - ctgexc - reorder the generalized Schur decomposition of a complex matrix pair (A,B), using an unitary equivalence transformation (A, B) := Q * (A, B) * Z', so that the diagonal block of (A, B) with row index IFST is moved to row ILST ctgsen - ctgsen - reorder the generalized Schur decomposition of a complex matrix pair (A, B) (in terms of an unitary equivalence trans- formation Q' * (A, B) * Z), so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the pair (A,B)

ctgsja - ctgsja - compute the generalized singular value decomposition (GSVD) of two complex upper triangular (or trapezoidal) matrices A and B

ctgsna - ctgsna - estimate reciprocal condition numbers for specified eigenvalues and/or eigenvectors of a matrix pair (A, B)

ctgsyl - ctgsyl - solve the generalized Sylvester equation

ctpcon - ctpcon - estimate the reciprocal of the condition number of a packed triangular matrix A, in either the 1-norm or the infinity-norm

ctpmv - ctpmv - perform one of the matrix-vector operations x := A*x, or x := A'*x, or x := conjg(A')*x

ctprfs - ctprfs - provide error bounds and backward error estimates for the solution to a system of linear equations with a triangular packed coefficient matrix

ctpsv - ctpsv - solve one of the systems of equations A*x = b, or A'*x = b, or conjg(A')*x = b

ctptri - ctptri - compute the inverse of a complex upper or lower triangular matrix A stored in packed format

ctptrs - ctptrs - solve a triangular system of the form A * X = B, A**T * X = B, or A**H * X = B,

ctrans - ctrans - transpose and scale source matrix

ctrcon - ctrcon - estimate the reciprocal of the condition number of a triangular matrix A, in either the 1-norm or the infinity-norm

ctrevc - ctrevc - compute some or all of the right and/or left eigenvectors of a complex upper triangular matrix T

ctrexc - ctrexc - reorder the Schur factorization of a complex matrix A = Q*T*Q**H, so that the diagonal element of T with row index IFST is moved to row ILST

ctrmm - ctrmm - perform one of the matrix-matrix operations B := alpha*op(A)*B, or B := alpha*B*op(A) where alpha is a scalar, B is an m by n matrix, A is a unit, or non-unit, upper or lower triangular matrix and op (A) is one of op(A) = A or op(A) = A' or op(A) = conjg(A')

ctrmv - ctrmv - perform one of the matrix-vector operations x := A*x, or x := A'*x, or x := conjg(A')*x ctrrfs - ctrrfs - provide error bounds and backward error estimates for the solution to a system of linear equations with a triangular coefficient matrix

ctrsen - ctrsen - reorder the Schur factorization of a complex matrix A = Q*T*Q**H, so that a selected cluster of eigenvalues appears in the leading positions on the diagonal of the upper triangular matrix T, and the leading columns of Q form an orthonormal basis of the corresponding right invariant subspace

ctrsm - ctrsm - solve one of the matrix equations op(A)*X = alpha*B, or X*op(A) = alpha*B

ctrsna - ctrsna - estimate reciprocal condition numbers for specified eigenvalues and/or right eigenvectors of a complex upper triangular matrix T (or of any matrix Q*T*Q**H with Q unitary)

ctrsv - ctrsv - solve one of the systems of equations A*x = b, or A'*x = b, or conjg(A')*x = b

ctrsyl - ctrsyl - solve the complex Sylvester matrix equation

ctrti2 - ctrti2 - compute the inverse of a complex upper or lower triangular matrix

ctrtri - ctrtri - compute the inverse of a complex upper or lower triangular matrix A

ctrtrs - ctrtrs - solve a triangular system of the form A * X = B, A**T * X = B, or A**H * X = B,

ctzrqf - ctzrqf - routine is deprecated and has been replaced by routine CTZRZF

ctzrzf - ctzrzf - reduce the M-by-N (M<=N) complex upper trapezoidal matrix A to upper triangular form by means of unitary transformations

cung2l - cung2l - generate an m by n complex matrix Q with orthonormal columns,

cung2r - cung2r - generate an m by n complex matrix Q with orthonormal columns,

cungbr - cungbr - generate one of the complex unitary matrices Q or P**H determined by CGEBRD when reducing a complex matrix A to bidiagonal form

cunghr - cunghr - generate a complex unitary matrix Q which is defined as the product of IHI-ILO elementary reflectors of order N, as returned by CGEHRD

cungl2 - cungl2 - generate an m-by-n complex matrix Q with orthonormal rows,

cunglq - cunglq - generate an M-by-N complex matrix Q with orthonormal rows,

cungql - cungql - generate an M-by-N complex matrix Q with orthonormal columns, cungqr - cungqr - generate an M-by-N complex matrix Q with orthonormal columns,

cungr2 - cungr2 - generate an m by n complex matrix Q with orthonormal rows,

cungrq - cungrq - generate an M-by-N complex matrix Q with orthonormal rows,

cungtr - cungtr - generate a complex unitary matrix Q which is defined as the product of n-1 elementary reflectors of order N, as returned by CHETRD

cunmbr - cunmbr - VECT = 'Q', CUNMBR overwrites the general complex M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

cunmhr - cunmhr - overwrite the general complex M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

cunml2 - cunml2 - overwrite the general complex m-by-n matrix C with Q * C if SIDE = 'L' and TRANS = 'N', or Q'* C if SIDE = 'L' and TRANS = 'C', or C * Q if SIDE = 'R' and TRANS = 'N', or C * Q' if SIDE = 'R' and TRANS = 'C',

cunmlq - cunmlq - overwrite the general complex M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

cunmql - cunmql - overwrite the general complex M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

cunmqr - cunmqr - overwrite the general complex M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

cunmr2 - cunmr2 - overwrite the general complex m-by-n matrix C with Q * C if SIDE = 'L' and TRANS = 'N', or Q'* C if SIDE = 'L' and TRANS = 'C', or C * Q if SIDE = 'R' and TRANS = 'N', or C * Q' if SIDE = 'R' and TRANS = 'C',

cunmrq - cunmrq - overwrite the general complex M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

cunmrz - cunmrz - overwrite the general complex M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

cunmtr - cunmtr - overwrite the general complex M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

cupgtr - cupgtr - generate a complex unitary matrix Q which is defined as the product of n-1 elementary reflectors H(i) of order n, as returned by CHPTRD using packed storage

cupmtr - cupmtr - overwrite the general complex M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

cvbrmm - cvbrmm - variable block sparse row format matrix-matrix multiply

cvbrsm - cvbrsm - variable block sparse row format triangular solve cvmul - cvmul - compute the scaled product of complex vectors

dasum - dasum - Return the sum of the absolute values of a vector x.

daxpy - daxpy - compute y := alpha * x + y

daxpyi - daxpyi - Compute y := alpha * x + y

dbcomm - dbcomm - block coordinate matrix-matrix multiply

dbdimm - dbdimm - block diagonal format matrix-matrix multiply

dbdism - dbdism - block diagonal format triangular solve

dbdsdc - dbdsdc - compute the singular value decomposition (SVD) of a real N-by-N (upper or lower) bidiagonal matrix B

dbdsqr - dbdsqr - compute the singular value decomposition (SVD) of a real N-by-N (upper or lower) bidiagonal matrix B.

dbelmm - dbelmm - block Ellpack format matrix-matrix multiply

dbelsm - dbelsm - block Ellpack format triangular solve

dbscmm - dbscmm - block sparse column matrix-matrix multiply

dbscsm - dbscsm - block sparse column format triangular solve

dbsrmm - dbsrmm - block sparse row format matrix-matrix multiply

dbsrsm - dbsrsm - block sparse row format triangular solve

dcnvcor - dcnvcor - compute the convolution or correlation of real vectors

dcnvcor2 - dcnvcor2 - compute the convolution or correlation of real matrices

dcoomm - dcoomm - coordinate matrix-matrix multiply

dcopy - dcopy - Copy x to y

dcosqb - dcosqb - synthesize a Fourier sequence from its representation in terms of a cosine series with odd wave numbers. The COSQ operations are unnormalized inverses of themselves, so a call to COSQF followed by a call to COSQB will multiply the input sequence by 4 * N.

dcosqf - dcosqf - compute the Fourier coefficients in a cosine series representation with only odd wave numbers. The COSQ operations are unnormalized inverses of themselves, so a call to COSQF followed by a call to COSQB will multiply the input sequence by 4 * N.

dcosqi - dcosqi - initialize the array WSAVE, which is used in both COSQF and COSQB.

dcost - dcost - compute the discrete Fourier cosine transform of an even sequence. The COST transforms are unnormalized inverses of themselves, so a call of COST followed by another call of COST will multiply the input sequence by 2 * (N-1).

dcosti - dcosti - initialize the array WSAVE, which is used in COST.

dcscmm - dcscmm - compressed sparse column format matrix-matrix multiply

dcscsm - dcscsm - compressed sparse column format triangular solve

dcsrmm - dcsrmm - compressed sparse row format matrix-matrix multiply

dcsrsm - dcsrsm - compressed sparse row format triangular solve

ddiamm - ddiamm - diagonal format matrix-matrix multiply

ddiasm - ddiasm - diagonal format triangular solve

ddisna - ddisna - compute the reciprocal condition numbers for the eigenvectors of a real symmetric or complex Hermitian matrix or for the left or right singular vectors of a general m-by-n matrix

ddot - ddot - compute the dot product of two vectors x and y.

ddoti - ddoti - Compute the indexed dot product.

dellmm - dellmm - Ellpack format matrix-matrix multiply

dellsm - dellsm - Ellpack format triangular solve

dezftb - dezftb - computes a periodic sequence from its Fourier coefficients. DEZFTB is a simplified but slower version of DFFTB.

dezftf - dezftf - computes the Fourier coefficients of a periodic sequence. DEZFTF is a simplified but slower version of DFFTF.

dezfti - dezfti - initializes the array WSAVE, which is used in both DEZFTF and DEZFTB.

dfft2b - dfft2b - compute a periodic sequence from its Fourier coefficients. The DFFT operations are unnormalized, so a call of DFFT2F followed by a call of DFFT2B will multiply the input sequence by M*N.

dfft2f - dfft2f - compute the Fourier coefficients of a periodic sequence. The DFFT operations are unnormalized, so a call of DFFT2F followed by a call of DFFT2B will multiply the input sequence by M*N.

dfft2i - dfft2i - initialize the array WSAVE, which is used in both the forward and backward transforms.

dfft3b - dfft3b - compute a periodic sequence from its Fourier coefficients. The DFFT operations are unnormalized, so a call of DFFT3F followed by a call of DFFT3B will multiply the input sequence by M*N*K.

dfft3f - dfft3f - compute the Fourier coefficients of a real periodic sequence. The DFFT operations are unnormalized, so a call of DFFT3F followed by a call of DFFT3B will multiply the input sequence by M*N*K.

dfft3i - dfft3i - initialize the array WSAVE, which is used in both DFFT3F and DFFT3B.

dfftb - dfftb - compute a periodic sequence from its Fourier coefficients. The DFFT operations are unnormalized, so a call of DFFTF followed by a call of DFFTB will multiply the input sequence by N.

dfftf - dfftf - compute the Fourier coefficients of a periodic sequence. The FFT operations are unnormalized, so a call of DFFTF followed by a call of DFFTB will multiply the input sequence by N.

dffti - dffti - initialize the array WSAVE, which is used in both DFFTF and DFFTB.

dfftopt - dfftopt - compute the length of the closest fast FFT

dfftz - dfftz - initialize the trigonometric weight and factor tables or compute the forward Fast Fourier Transform of a double precision sequence.

dfftz2 - dfftz2 - initialize the trigonometric weight and factor tables or compute the two-dimensional forward Fast Fourier Transform of a two-dimensional double precision array.

dfftz3 - dfftz3 - initialize the trigonometric weight and factor tables or compute the three-dimensional forward Fast Fourier Transform of a three-dimensional double complex array.

dfftzm - dfftzm - initialize the trigonometric weight and factor tables or compute the one-dimensional forward Fast Fourier Transform of a set of double precision data sequences stored in a two-dimensional array.

dgbbrd - dgbbrd - reduce a real general m-by-n band matrix A to upper bidiagonal form B by an orthogonal transformation

dgbcon - dgbcon - estimate the reciprocal of the condition number of a real general band matrix A, in either the 1-norm or the infinity-norm,

dgbequ - dgbequ - compute row and column scalings intended to equilibrate an M-by-N band matrix A and reduce its condition number

dgbmv - dgbmv - perform one of the matrix-vector operations y := alpha*A*x + beta*y or y := alpha*A'*x + beta*y

dgbrfs - dgbrfs - improve the computed solution to a system of linear equations when the coefficient matrix is banded, and provides error bounds and backward error estimates for the solution

dgbsv - dgbsv - compute the solution to a real system of linear equations A * X = B, where A is a band matrix of order N with KL subdiagonals and KU superdiagonals, and X and B are N-by-NRHS matrices

dgbsvx - dgbsvx - use the LU factorization to compute the solution to a real system of linear equations A * X = B, A**T * X = B, or A**H * X = B,

dgbtf2 - dgbtf2 - compute an LU factorization of a real m-by-n band matrix A using partial pivoting with row interchanges

dgbtrf - dgbtrf - compute an LU factorization of a real m-by-n band matrix A using partial pivoting with row interchanges

dgbtrs - dgbtrs - solve a system of linear equations A * X = B or A' * X = B with a general band matrix A using the LU factorization computed by SGBTRF

dgebak - dgebak - form the right or left eigenvectors of a real general matrix by backward transformation on the computed eigenvectors of the balanced matrix output by SGEBAL

dgebal - dgebal - balance a general real matrix A

dgebrd - dgebrd - reduce a general real M-by-N matrix A to upper or lower bidiagonal form B by an orthogonal transformation

dgecon - dgecon - estimate the reciprocal of the condition number of a general real matrix A, in either the 1-norm or the infinity-norm, using the LU factorization computed by SGETRF

dgeequ - dgeequ - compute row and column scalings intended to equilibrate an M-by-N matrix A and reduce its condition number dgees - dgees - compute for an N-by-N real nonsymmetric matrix A, the eigenvalues, the real Schur form T, and, optionally, the matrix of Schur vectors Z

dgeesx - dgeesx - compute for an N-by-N real nonsymmetric matrix A, the eigenvalues, the real Schur form T, and, optionally, the matrix of Schur vectors Z

dgeev - dgeev - compute for an N-by-N real nonsymmetric matrix A, the eigenvalues and, optionally, the left and/or right eigenvectors

dgeevx - dgeevx - compute for an N-by-N real nonsymmetric matrix A, the eigenvalues and, optionally, the left and/or right eigenvectors

dgegs - dgegs - routine is deprecated and has been replaced by routine SGGES

dgegv - dgegv - routine is deprecated and has been replaced by routine SGGEV

dgehrd - dgehrd - reduce a real general matrix A to upper Hessenberg form H by an orthogonal similarity transformation

dgelqf - dgelqf - compute an LQ factorization of a real M-by-N matrix A

dgels - dgels - solve overdetermined or underdetermined real linear systems involving an M-by-N matrix A, or its transpose, using a QR or LQ factorization of A

dgelsd - dgelsd - compute the minimum-norm solution to a real linear least squares problem

dgelss - dgelss - compute the minimum norm solution to a real linear least squares problem

dgelsx - dgelsx - routine is deprecated and has been replaced by routine SGELSY

dgelsy - dgelsy - compute the minimum-norm solution to a real linear least squares problem

dgemm - dgemm - perform one of the matrix-matrix operations C := alpha*op(A)*op(B) + beta*C

dgemv - dgemv - perform one of the matrix-vector operations y := alpha*A*x + beta*y or y := alpha*A'*x + beta*y

dgeqlf - dgeqlf - compute a QL factorization of a real M-by-N matrix A

dgeqp3 - dgeqp3 - compute a QR factorization with column pivoting of a matrix A

dgeqpf - dgeqpf - routine is deprecated and has been replaced by routine SGEQP3 dgeqrf - dgeqrf - compute a QR factorization of a real M-by-N matrix A

dger - dger - perform the rank 1 operation A := alpha*x*y' + A

dgerfs - dgerfs - improve the computed solution to a system of linear equations and provides error bounds and backward error estimates for the solution

dgerqf - dgerqf - compute an RQ factorization of a real M-by-N matrix A

dgesdd - dgesdd - compute the singular value decomposition (SVD) of a real M-by-N matrix A, optionally computing the left and right singular vectors

dgesv - dgesv - compute the solution to a real system of linear equations A * X = B,

dgesvd - dgesvd - compute the singular value decomposition (SVD) of a real M-by-N matrix A, optionally computing the left and/or right singular vectors

dgesvx - dgesvx - use the LU factorization to compute the solution to a real system of linear equations A * X = B,

dgetf2 - dgetf2 - compute an LU factorization of a general m-by-n matrix A using partial pivoting with row interchanges

dgetrf - dgetrf - compute an LU factorization of a general M-by-N matrix A using partial pivoting with row interchanges

dgetri - dgetri - compute the inverse of a matrix using the LU factorization computed by SGETRF

dgetrs - dgetrs - solve a system of linear equations A * X = B or A' * X = B with a general N-by-N matrix A using the LU factorization computed by SGETRF

dggbak - dggbak - form the right or left eigenvectors of a real generalized eigenvalue problem A*x = lambda*B*x, by backward transformation on the computed eigenvectors of the balanced pair of matrices output by SGGBAL

dggbal - dggbal - balance a pair of general real matrices (A,B)

dgges - dgges - compute for a pair of N-by-N real nonsymmetric matrices (A,B),

dggesx - dggesx - compute for a pair of N-by-N real nonsymmetric matrices (A,B), the generalized eigenvalues, the real Schur form (S,T), and,

dggev - dggev - compute for a pair of N-by-N real nonsymmetric matrices (A,B) dggevx - dggevx - compute for a pair of N-by-N real nonsymmetric matrices (A,B)

dggglm - dggglm - solve a general Gauss-Markov linear model (GLM) problem

dgghrd - dgghrd - reduce a pair of real matrices (A,B) to generalized upper Hessenberg form using orthogonal transformations, where A is a general matrix and B is upper triangular

dgglse - dgglse - solve the linear equality-constrained least squares (LSE) problem

dggqrf - dggqrf - compute a generalized QR factorization of an N-by-M matrix A and an N-by-P matrix B.

dggrqf - dggrqf - compute a generalized RQ factorization of an M-by-N matrix A and a P-by-N matrix B

dggsvd - dggsvd - compute the generalized singular value decomposition (GSVD) of an M-by-N real matrix A and P-by-N real matrix B

dggsvp - dggsvp - compute orthogonal matrices U, V and Q such that N-K-L K L U'*A*Q = K (0 A12 A13) if M-K-L >= 0

dgssco - dgssco - General sparse solver condition number estimate.

dgssda - dgssda - Deallocate working storage for the general sparse solver.

dgssfa - dgssfa - General sparse solver numeric factorization.

dgssfs - dgssfs - General sparse solver one call interface.

dgssin - dgssin - Initialize the general sparse solver.

dgssor - dgssor - General sparse solver ordering and symbolic factorization.

dgssps - dgssps - Print general sparse solver statics.

dgssrp - dgssrp - Return permutation used by the general sparse solver.

dgsssl - dgsssl - Solve routine for the general sparse solver.

dgssuo - dgssuo - User supplied permutation for ordering used in the general sparse solver.

dgtcon - dgtcon - estimate the reciprocal of the condition number of a real tridiagonal matrix A using the LU factorization as computed by SGTTRF dgthr - dgthr - Gathers specified elements from y into x.

dgthrz - dgthrz - Gather and zero.

dgtrfs - dgtrfs - improve the computed solution to a system of linear equations when the coefficient matrix is tridiagonal, and provides error bounds and backward error estimates for the solution

dgtsv - dgtsv - solve the equation A*X = B,

dgtsvx - dgtsvx - use the LU factorization to compute the solution to a real system of linear equations A * X = B or A**T * X = B,

dgttrf - dgttrf - compute an LU factorization of a real tridiagonal matrix A using elimination with partial pivoting and row interchanges

dgttrs - dgttrs - solve one of the systems of equations A*X = B or A'*X = B,

dhgeqz - dhgeqz - implement a single-/double-shift version of the QZ method for finding the generalized eigenvalues w(j)=(ALPHAR(j) + i*ALPHAI(j))/BETAR(j) of the equation det(A-w(i) B) = 0 In addition, the pair A,B may be reduced to generalized Schur form

dhsein - dhsein - use inverse iteration to find specified right and/or left eigenvectors of a real upper Hessenberg matrix H

dhseqr - dhseqr - compute the eigenvalues of a real upper Hessenberg matrix H and, optionally, the matrices T and Z from the Schur decomposition H = Z T Z**T, where T is an upper quasi-triangular matrix (the Schur form), and Z is the orthogonal matrix of Schur vectors

djadmm - djadmm - Jagged diagonal matrix-matrix multiply (modified Ellpack)

djadrp - djadrp - right permutation of a jagged diagonal matrix

djadsm - djadsm - Jagged-diagonal format triangular solve

dlagtf - dlagtf - factorize the matrix (T-lambda*I), where T is an n by n tridiagonal matrix and lambda is a scalar, as T-lambda*I = PLU

dlamrg - dlamrg - will create a permutation list which will merge the elements of A (which is composed of two independently sorted sets) into a single set which is sorted in ascending order

dlarz - dlarz - applies a real elementary reflector H to a real M-by-N matrix C, from either the left or the right dlarzb - dlarzb - applies a real block reflector H or its transpose H**T to a real distributed M-by-N C from the left or the right

dlarzt - dlarzt - form the triangular factor T of a real block reflector H of order > n, which is defined as a product of k elementary reflectors

dlasrt - dlasrt - the numbers in D in increasing order (if ID = 'I') or in decreasing order (if ID = 'D')

dlatzm - dlatzm - routine is deprecated and has been replaced by routine SORMRZ

dnrm2 - dnrm2 - Return the Euclidian norm of a vector.

dopgtr - dopgtr - generate a real orthogonal matrix Q which is defined as the product of n-1 elementary reflectors H(i) of order n, as returned by SSPTRD using packed storage

dopmtr - dopmtr - overwrite the general real M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

dorg2l - dorg2l - generate an m by n real matrix Q with orthonormal columns,

dorg2r - dorg2r - generate an m by n real matrix Q with orthonormal columns,

dorgbr - dorgbr - generate one of the real orthogonal matrices Q or P**T determined by SGEBRD when reducing a real matrix A to bidiagonal form

dorghr - dorghr - generate a real orthogonal matrix Q which is defined as the product of IHI-ILO elementary reflectors of order N, as returned by SGEHRD

dorgl2 - dorgl2 - generate an m by n real matrix Q with orthonormal rows,

dorglq - dorglq - generate an M-by-N real matrix Q with orthonormal rows,

dorgql - dorgql - generate an M-by-N real matrix Q with orthonormal columns,

dorgqr - dorgqr - generate an M-by-N real matrix Q with orthonormal columns,

dorgr2 - dorgr2 - generate an m by n real matrix Q with orthonormal rows,

dorgrq - dorgrq - generate an M-by-N real matrix Q with orthonormal rows,

dorgtr - dorgtr - generate a real orthogonal matrix Q which is defined as the product of n-1 elementary reflectors of order N, as returned by SSYTRD dormbr - dormbr - VECT = 'Q', SORMBR overwrites the general real M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

dormhr - dormhr - overwrite the general real M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

dormlq - dormlq - overwrite the general real M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

dormql - dormql - overwrite the general real M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

dormqr - dormqr - overwrite the general real M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

dormrq - dormrq - overwrite the general real M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

dormrz - dormrz - overwrite the general real M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

dormtr - dormtr - overwrite the general real M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

dpbcon - dpbcon - estimate the reciprocal of the condition number (in the 1-norm) of a real symmetric positive definite band matrix using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPBTRF

dpbequ - dpbequ - compute row and column scalings intended to equilibrate a symmetric positive definite band matrix A and reduce its condition number (with respect to the two-norm)

dpbrfs - dpbrfs - improve the computed solution to a system of linear equations when the coefficient matrix is symmetric positive definite and banded, and provides error bounds and backward error estimates for the solution

dpbstf - dpbstf - compute a split Cholesky factorization of a real symmetric positive definite band matrix A

dpbsv - dpbsv - compute the solution to a real system of linear equations A * X = B,

dpbsvx - dpbsvx - use the Cholesky factorization A = U**T*U or A = L*L**T to compute the solution to a real system of linear equations A * X = B,

dpbtf2 - dpbtf2 - compute the Cholesky factorization of a real symmetric positive definite band matrix A

dpbtrf - dpbtrf - compute the Cholesky factorization of a real symmetric positive definite band matrix A

dpbtrs - dpbtrs - solve a system of linear equations A*X = B with a symmetric positive definite band matrix A using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPBTRF

dpocon - dpocon - estimate the reciprocal of the condition number (in the 1-norm) of a real symmetric positive definite matrix using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPOTRF dpoequ - dpoequ - compute row and column scalings intended to equilibrate a symmetric positive definite matrix A and reduce its condition number (with respect to the two-norm)

dporfs - dporfs - improve the computed solution to a system of linear equations when the coefficient matrix is symmetric positive definite,

dposv - dposv - compute the solution to a real system of linear equations A * X = B,

dposvx - dposvx - use the Cholesky factorization A = U**T*U or A = L*L**T to compute the solution to a real system of linear equations A * X = B,

dpotf2 - dpotf2 - compute the Cholesky factorization of a real symmetric positive definite matrix A

dpotrf - dpotrf - compute the Cholesky factorization of a real symmetric positive definite matrix A

dpotri - dpotri - compute the inverse of a real symmetric positive definite matrix A using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPOTRF

dpotrs - dpotrs - solve a system of linear equations A*X = B with a symmetric positive definite matrix A using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPOTRF

dppcon - dppcon - estimate the reciprocal of the condition number (in the 1-norm) of a real symmetric positive definite packed matrix using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPPTRF

dppequ - dppequ - compute row and column scalings intended to equilibrate a symmetric positive definite matrix A in packed storage and reduce its condition number (with respect to the two-norm)

dpprfs - dpprfs - improve the computed solution to a system of linear equations when the coefficient matrix is symmetric positive definite and packed, and provides error bounds and backward error estimates for the solution

dppsv - dppsv - compute the solution to a real system of linear equations A * X = B,

dppsvx - dppsvx - use the Cholesky factorization A = U**T*U or A = L*L**T to compute the solution to a real system of linear equations A * X = B,

dpptrf - dpptrf - compute the Cholesky factorization of a real symmetric positive definite matrix A stored in packed format

dpptri - dpptri - compute the inverse of a real symmetric positive definite matrix A using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPPTRF

dpptrs - dpptrs - solve a system of linear equations A*X = B with a symmetric positive definite matrix A in packed storage using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPPTRF dptcon - dptcon - compute the reciprocal of the condition number (in the 1-norm) of a real symmetric positive definite tridiagonal matrix using the factorization A = L*D*L**T or A = U**T*D*U computed by SPTTRF

dpteqr - dpteqr - compute all eigenvalues and, optionally, eigenvectors of a symmetric positive definite tridiagonal matrix by first factoring the matrix using SPTTRF, and then calling SBDSQR to compute the singular values of the bidiagonal factor

dptrfs - dptrfs - improve the computed solution to a system of linear equations when the coefficient matrix is symmetric positive definite and tridiagonal, and provides error bounds and backward error estimates for the solution

dptsv - dptsv - compute the solution to a real system of linear equations A*X = B, where A is an N-by-N symmetric positive definite tridiagonal matrix, and X and B are N-by-NRHS matrices.

dptsvx - dptsvx - use the factorization A = L*D*L**T to compute the solution to a real system of linear equations A*X = B, where A is an N-by-N symmetric positive definite tridiagonal matrix and X and B are N-by-NRHS matrices

dpttrf - dpttrf - compute the L*D*L' factorization of a real symmetric positive definite tridiagonal matrix A

dpttrs - dpttrs - solve a tridiagonal system of the form A * X = B using the L*D*L' factorization of A computed by SPTTRF

dptts2 - dptts2 - solve a tridiagonal system of the form A * X = B using the L*D*L' factorization of A computed by SPTTRF

dqdota - dqdota - compute a double precision constant plus an extended precision constant plus the extended precision dot product of two double precision vectors x and y.

dqdoti - dqdoti - compute a constant plus the extended precision dot product of two double precision vectors x and y.

drot - drot - Apply a Given's rotation constructed by SROTG.

drotg - drotg - Construct a Given's plane rotation

droti - droti - Apply an indexed Givens rotation.

drotm - drotm - Apply a Gentleman's modified Given's rotation constructed by SROTMG.

drotmg - drotmg - Construct a Gentleman's modified Given's plane rotation dsbev - dsbev - compute all the eigenvalues and, optionally, eigenvectors of a real symmetric band matrix A

dsbevd - dsbevd - compute all the eigenvalues and, optionally, eigenvectors of a real symmetric band matrix A

dsbevx - dsbevx - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric band matrix A

dsbgst - dsbgst - reduce a real symmetric-definite banded generalized eigenproblem A*x = lambda*B*x to standard form C*y = lambda*y,

dsbgv - dsbgv - compute all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form A*x=(lambda)*B*x

dsbgvd - dsbgvd - compute all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form A*x=(lambda)*B*x

dsbgvx - dsbgvx - compute selected eigenvalues, and optionally, eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form A*x=(lambda)*B*x

dsbmv - dsbmv - perform the matrix-vector operation y := alpha*A*x + beta*y

dsbtrd - dsbtrd - reduce a real symmetric band matrix A to symmetric tridiagonal form T by an orthogonal similarity transformation

dscal - dscal - Compute y := alpha * y

dsctr - dsctr - Scatters elements from x into y.

dsdot - dsdot - compute the double precision dot product of two single precision vectors x and y.

dsecnd - dsecnd - return the user time for a process in seconds

dsinqb - dsinqb - synthesize a Fourier sequence from its representation in terms of a sine series with odd wave numbers. The SINQ operations are unnormalized inverses of themselves, so a call to SINQF followed by a call to SINQB will multiply the input sequence by 4 * N.

dsinqf - dsinqf - compute the Fourier coefficients in a sine series representation with only odd wave numbers. The SINQ operations are unnormalized inverses of themselves, so a call to SINQF followed by a call to SINQB will multiply the input sequence by 4 * N.

dsinqi - dsinqi - initialize the array xWSAVE, which is used in both SINQF and SINQB.

dsint - dsint - compute the discrete Fourier sine transform of an odd sequence. The SINT transforms are unnormalized inverses of themselves, so a call of SINT followed by another call of SINT will multiply the input sequence by 2 * (N+1).

dsinti - dsinti - initialize the array WSAVE, which is used in subroutine SINT.

dskymm - dskymm - Skyline format matrix-matrix multiply

dskysm - dskysm - Skyline format triangular solve

dspcon - dspcon - estimate the reciprocal of the condition number (in the 1-norm) of a real symmetric packed matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by SSPTRF

dspev - dspev - compute all the eigenvalues and, optionally, eigenvectors of a real symmetric matrix A in packed storage

dspevd - dspevd - compute all the eigenvalues and, optionally, eigenvectors of a real symmetric matrix A in packed storage

dspevx - dspevx - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix A in packed storage

dspgst - dspgst - reduce a real symmetric-definite generalized eigenproblem to standard form, using packed storage

dspgv - dspgv - compute all the eigenvalues and, optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

dspgvd - dspgvd - compute all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

dspgvx - dspgvx - compute selected eigenvalues, and optionally, eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

dspmv - dspmv - perform the matrix-vector operation y := alpha*A*x + beta*y

dspr - dspr - perform the symmetric rank 1 operation A := alpha*x*x' + A

dspr2 - dspr2 - perform the symmetric rank 2 operation A := alpha*x*y' + alpha*y*x' + A

dsprfs - dsprfs - improve the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite and packed, and provides error bounds and backward error estimates for the solution

dspsv - dspsv - compute the solution to a real system of linear equations A * X = B, dspsvx - dspsvx - use the diagonal pivoting factorization A = U*D*U**T or A = L*D*L**T to compute the solution to a real system of linear equations A * X = B, where A is an N-by-N symmetric matrix stored in packed format and X and B are N-by-NRHS matrices

dsptrd - dsptrd - reduce a real symmetric matrix A stored in packed form to symmetric tridiagonal form T by an orthogonal similarity transformation

dsptrf - dsptrf - compute the factorization of a real symmetric matrix A stored in packed format using the Bunch-Kaufman diagonal pivoting method

dsptri - dsptri - compute the inverse of a real symmetric indefinite matrix A in packed storage using the factorization A = U*D*U**T or A = L*D*L**T computed by SSPTRF

dsptrs - dsptrs - solve a system of linear equations A*X = B with a real symmetric matrix A stored in packed format using the factorization A = U*D*U**T or A = L*D*L**T computed by SSPTRF

dstebz - dstebz - compute the eigenvalues of a symmetric tridiagonal matrix T

dstedc - dstedc - compute all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the divide and conquer method

dstegr - dstegr - (a) Compute T-sigma_i = L_i D_i L_i^T, such that L_i D_i L_i^T is a relatively robust representation

dstein - dstein - compute the eigenvectors of a real symmetric tridiagonal matrix T corresponding to specified eigenvalues, using inverse iteration

dsteqr - dsteqr - compute all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the implicit QL or QR method

dsterf - dsterf - compute all eigenvalues of a symmetric tridiagonal matrix using the Pal-Walker-Kahan variant of the QL or QR algorithm

dstev - dstev - compute all eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix A

dstevd - dstevd - compute all eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix

dstevr - dstevr - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix T

dstevx - dstevx - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix A dstsv - dstsv - compute the solution to a system of linear equations A * X = B where A is a symmetric tridiagonal matrix

dsttrf - dsttrf - compute the factorization of a symmetric tridiagonal matrix A

dsttrs - dsttrs - computes the solution to a real system of linear equations A * X = B

dswap - dswap - Exchange vectors x and y.

dsycon - dsycon - estimate the reciprocal of the condition number (in the 1-norm) of a real symmetric matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by SSYTRF

dsyev - dsyev - compute all eigenvalues and, optionally, eigenvectors of a real symmetric matrix A

dsyevd - dsyevd - compute all eigenvalues and, optionally, eigenvectors of a real symmetric matrix A

dsyevr - dsyevr - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix T

dsyevx - dsyevx - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix A

dsygs2 - dsygs2 - reduce a real symmetric-definite generalized eigenproblem to standard form

dsygst - dsygst - reduce a real symmetric-definite generalized eigenproblem to standard form

dsygv - dsygv - compute all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

dsygvd - dsygvd - compute all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

dsygvx - dsygvx - compute selected eigenvalues, and optionally, eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

dsymm - dsymm - perform one of the matrix-matrix operations C := alpha*A*B + beta*C or C := alpha*B*A + beta*C

dsymv - dsymv - perform the matrix-vector operation y := alpha*A*x + beta*y

dsyr - dsyr - perform the symmetric rank 1 operation A := alpha*x*x' + A

dsyr2 - dsyr2 - perform the symmetric rank 2 operation A := alpha*x*y' + alpha*y*x' + A dsyr2k - dsyr2k - perform one of the symmetric rank 2k operations C := alpha*A*B' + alpha*B*A' + beta*C or C := alpha*A'*B + alpha*B'*A + beta*C

dsyrfs - dsyrfs - improve the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite, and provides error bounds and backward error estimates for the solution

dsyrk - dsyrk - perform one of the symmetric rank k operations C := alpha*A*A' + beta*C or C := alpha*A'*A + beta*C

dsysv - dsysv - compute the solution to a real system of linear equations A * X = B,

dsysvx - dsysvx - use the diagonal pivoting factorization to compute the solution to a real system of linear equations A * X = B,

dsytd2 - dsytd2 - reduce a real symmetric matrix A to symmetric tridiagonal form T by an orthogonal similarity transformation

dsytf2 - dsytf2 - compute the factorization of a real symmetric matrix A using the Bunch-Kaufman diagonal pivoting method

dsytrd - dsytrd - reduce a real symmetric matrix A to real symmetric tridiagonal form T by an orthogonal similarity transformation

dsytrf - dsytrf - compute the factorization of a real symmetric matrix A using the Bunch-Kaufman diagonal pivoting method

dsytri - dsytri - compute the inverse of a real symmetric indefinite matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by SSYTRF

dsytrs - dsytrs - solve a system of linear equations A*X = B with a real symmetric matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by SSYTRF

dtbcon - dtbcon - estimate the reciprocal of the condition number of a triangular band matrix A, in either the 1-norm or the infinity-norm

dtbmv - dtbmv - perform one of the matrix-vector operations x := A*x, or x := A'*x

dtbrfs - dtbrfs - provide error bounds and backward error estimates for the solution to a system of linear equations with a triangular band coefficient matrix

dtbsv - dtbsv - solve one of the systems of equations A*x = b, or A'*x = b

dtbtrs - dtbtrs - solve a triangular system of the form A * X = B or A**T * X = B, dtgevc - dtgevc - compute some or all of the right and/or left generalized eigenvectors of a pair of real upper triangular matrices (A,B)

dtgexc - dtgexc - reorder the generalized real Schur decomposition of a real matrix pair (A,B) using an orthogonal equivalence transformation (A, B) = Q * (A, B) * Z',

dtgsen - dtgsen - reorder the generalized real Schur decomposition of a real matrix pair (A, B) (in terms of an orthonormal equivalence trans- formation Q' * (A, B) * Z), so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper quasi-triangular matrix A and the upper triangular B

dtgsja - dtgsja - compute the generalized singular value decomposition (GSVD) of two real upper triangular (or trapezoidal) matrices A and B

dtgsna - dtgsna - estimate reciprocal condition numbers for specified eigenvalues and/or eigenvectors of a matrix pair (A, B) in generalized real Schur canonical form (or of any matrix pair (Q*A*Z', Q*B*Z') with orthogonal matrices Q and Z, where Z' denotes the transpose of Z

dtgsyl - dtgsyl - solve the generalized Sylvester equation

dtpcon - dtpcon - estimate the reciprocal of the condition number of a packed triangular matrix A, in either the 1-norm or the infinity-norm

dtpmv - dtpmv - perform one of the matrix-vector operations x := A*x, or x := A'*x

dtprfs - dtprfs - provide error bounds and backward error estimates for the solution to a system of linear equations with a triangular packed coefficient matrix

dtpsv - dtpsv - solve one of the systems of equations A*x = b, or A'*x = b

dtptri - dtptri - compute the inverse of a real upper or lower triangular matrix A stored in packed format

dtptrs - dtptrs - solve a triangular system of the form A * X = B or A**T * X = B,

dtrans - dtrans - transpose and scale source matrix

dtrcon - dtrcon - estimate the reciprocal of the condition number of a triangular matrix A, in either the 1-norm or the infinity-norm

dtrevc - dtrevc - compute some or all of the right and/or left eigenvectors of a real upper quasi-triangular matrix T

dtrexc - dtrexc - reorder the real Schur factorization of a real matrix A = Q*T*Q**T, so that the diagonal block of T with row index IFST is moved to row ILST dtrmm - dtrmm - perform one of the matrix-matrix operations B := alpha*op(A)*B, or B := alpha*B*op(A)

dtrmv - dtrmv - perform one of the matrix-vector operations x := A*x, or x := A'*x

dtrrfs - dtrrfs - provide error bounds and backward error estimates for the solution to a system of linear equations with a triangular coefficient matrix

dtrsen - dtrsen - reorder the real Schur factorization of a real matrix A = Q*T*Q**T, so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper quasi-triangular matrix T,

dtrsm - dtrsm - solve one of the matrix equations op(A)*X = alpha*B, or X*op(A) = alpha*B

dtrsna - dtrsna - estimate reciprocal condition numbers for specified eigenvalues and/or right eigenvectors of a real upper quasi-triangular matrix T (or of any matrix Q*T*Q**T with Q orthogonal)

dtrsv - dtrsv - solve one of the systems of equations A*x = b, or A'*x = b

dtrsyl - dtrsyl - solve the real Sylvester matrix equation

dtrti2 - dtrti2 - compute the inverse of a real upper or lower triangular matrix

dtrtri - dtrtri - compute the inverse of a real upper or lower triangular matrix A

dtrtrs - dtrtrs - solve a triangular system of the form A * X = B or A**T * X = B,

dtzrqf - dtzrqf - routine is deprecated and has been replaced by routine STZRZF

dtzrzf - dtzrzf - reduce the M-by-N (M<=N) real upper trapezoidal matrix A to upper triangular form by means of orthogonal transformations

dvbrmm - dvbrmm - variable block sparse row format matrix-matrix multiply

dvbrsm - dvbrsm - variable block sparse row format triangular solve

dwiener - dwiener - perform Wiener deconvolution of two signals

dzasum - dzasum - Return the sum of the absolute values of a vector x.

dznrm2 - dznrm2 - Return the Euclidian norm of a vector.

ezfftb - ezfftb - computes a periodic sequence from its Fourier coefficients. EZFFTB is a simplified but slower version of RFFTB.

ezfftf - ezfftf - computes the Fourier coefficients of a periodic sequence. EZFFTF is a simplified but slower version of RFFTF.

ezffti - ezffti - initializes the array WSAVE, which is used in both EZFFTF and EZFFTB.

fft - Overview of Fast Fourier Transform subroutines

icamax - icamax - return the index of the element with largest absolute value.

idamax - idamax - return the index of the element with largest absolute value.

ilaenv - The name of the calling subroutine, in either upper case or lower case.

isamax - isamax - return the index of the element with largest absolute value.

izamax - izamax - return the index of the element with largest absolute value.

lsame - lsame - returns .TRUE. if CA is the same letter as CB regardless of case

rfft2b - rfft2b - compute a periodic sequence from its Fourier coefficients. The RFFT operations are unnormalized, so a call of RFFT2F followed by a call of RFFT2B will multiply the input sequence by M*N.

rfft2f - rfft2f - compute the Fourier coefficients of a periodic sequence. The RFFT operations are unnormalized, so a call of RFFT2F followed by a call of RFFT2B will multiply the input sequence by M*N.

rfft2i - rfft2i - initialize the array WSAVE, which is used in both the forward and backward transforms.

rfft3b - rfft3b - compute a periodic sequence from its Fourier coefficients. The RFFT operations are unnormalized, so a call of RFFT3F followed by a call of RFFT3B will multiply the input sequence by M*N*K.

rfft3f - rfft3f - compute the Fourier coefficients of a real periodic sequence. The RFFT operations are unnormalized, so a call of RFFT3F followed by a call of RFFT3B will multiply the input sequence by M*N*K.

rfft3i - rfft3i - initialize the array WSAVE, which is used in both RFFT3F and RFFT3B.

rfftb - rfftb - compute a periodic sequence from its Fourier coefficients. The RFFT operations are unnormalized, so a call of RFFTF followed by a call of RFFTB will multiply the input sequence by N.

rfftf - rfftf - compute the Fourier coefficients of a periodic sequence. The FFT operations are unnormalized, so a call of RFFTF followed by a call of RFFTB will multiply the input sequence by N.

rffti - rffti - initialize the array WSAVE, which is used in both RFFTF and RFFTB.

rfftopt - rfftopt - compute the length of the closest fast FFT

sasum - sasum - Return the sum of the absolute values of a vector x.

saxpy - saxpy - compute y := alpha * x + y

saxpyi - saxpyi - Compute y := alpha * x + y

sbcomm - sbcomm - block coordinate matrix-matrix multiply

sbdimm - sbdimm - block diagonal format matrix-matrix multiply

sbdism - sbdism - block diagonal format triangular solve

sbdsdc - sbdsdc - compute the singular value decomposition (SVD) of a real N-by-N (upper or lower) bidiagonal matrix B

sbdsqr - sbdsqr - compute the singular value decomposition (SVD) of a real N-by-N (upper or lower) bidiagonal matrix B.

sbelmm - sbelmm - block Ellpack format matrix-matrix multiply

sbelsm - sbelsm - block Ellpack format triangular solve

sbscmm - sbscmm - block sparse column matrix-matrix multiply

sbscsm - sbscsm - block sparse column format triangular solve

sbsrmm - sbsrmm - block sparse row format matrix-matrix multiply

sbsrsm - sbsrsm - block sparse row format triangular solve

scasum - scasum - Return the sum of the absolute values of a vector x.

scnrm2 - scnrm2 - Return the Euclidian norm of a vector.

scnvcor - scnvcor - compute the convolution or correlation of real vectors

scnvcor2 - scnvcor2 - compute the convolution or correlation of real matrices scoomm - scoomm - coordinate matrix-matrix multiply

scopy - scopy - Copy x to y

scscmm - scscmm - compressed sparse column format matrix-matrix multiply

scscsm - scscsm - compressed sparse column format triangular solve

scsrmm - scsrmm - compressed sparse row format matrix-matrix multiply

scsrsm - scsrsm - compressed sparse row format triangular solve

sdiamm - sdiamm - diagonal format matrix-matrix multiply

sdiasm - sdiasm - diagonal format triangular solve

sdisna - sdisna - compute the reciprocal condition numbers for the eigenvectors of a real symmetric or complex Hermitian matrix or for the left or right singular vectors of a general m-by-n matrix

sdot - sdot - compute the dot product of two vectors x and y.

sdoti - sdoti - Compute the indexed dot product.

sdsdot - sdsdot - compute a constant plus the double precision dot product of two single precision vectors x and y

second - second - return the user time for a process in seconds

sellmm - sellmm - Ellpack format matrix-matrix multiply

sellsm - sellsm - Ellpack format triangular solve

sfftc - sfftc - initialize the trigonometric weight and factor tables or compute the forward Fast Fourier Transform of a real sequence.

sfftc2 - sfftc2 - initialize the trigonometric weight and factor tables or compute the two-dimensional forward Fast Fourier Transform of a two-dimensional real array.

sfftc3 - sfftc3 - initialize the trigonometric weight and factor tables or compute the three-dimensional forward Fast Fourier Transform of a three-dimensional complex array.

sfftcm - sfftcm - initialize the trigonometric weight and factor tables or compute the one-dimensional forward

Fast Fourier Transform of a set of real data sequences stored in a two-dimensional array.

sgbbrd - sgbbrd - reduce a real general m-by-n band matrix A to upper bidiagonal form B by an orthogonal transformation

sgbcon - sgbcon - estimate the reciprocal of the condition number of a real general band matrix A, in either the 1-norm or the infinity-norm,

sgbequ - sgbequ - compute row and column scalings intended to equilibrate an M-by-N band matrix A and reduce its condition number

sgbmv - sgbmv - perform one of the matrix-vector operations y := alpha*A*x + beta*y or y := alpha*A'*x + beta*y

sgbrfs - sgbrfs - improve the computed solution to a system of linear equations when the coefficient matrix is banded, and provides error bounds and backward error estimates for the solution

sgbsv - sgbsv - compute the solution to a real system of linear equations A * X = B, where A is a band matrix of order N with KL subdiagonals and KU superdiagonals, and X and B are N-by-NRHS matrices

sgbsvx - sgbsvx - use the LU factorization to compute the solution to a real system of linear equations A * X = B, A**T * X = B, or A**H * X = B,

sgbtf2 - sgbtf2 - compute an LU factorization of a real m-by-n band matrix A using partial pivoting with row interchanges

sgbtrf - sgbtrf - compute an LU factorization of a real m-by-n band matrix A using partial pivoting with row interchanges

sgbtrs - sgbtrs - solve a system of linear equations A * X = B or A' * X = B with a general band matrix A using the LU factorization computed by SGBTRF

sgebak - sgebak - form the right or left eigenvectors of a real general matrix by backward transformation on the computed eigenvectors of the balanced matrix output by SGEBAL

sgebal - sgebal - balance a general real matrix A

sgebrd - sgebrd - reduce a general real M-by-N matrix A to upper or lower bidiagonal form B by an orthogonal transformation

sgecon - sgecon - estimate the reciprocal of the condition number of a general real matrix A, in either the 1-norm or the infinity-norm, using the LU factorization computed by SGETRF sgeequ - sgeequ - compute row and column scalings intended to equilibrate an M-by-N matrix A and reduce its condition number

sgees - sgees - compute for an N-by-N real nonsymmetric matrix A, the eigenvalues, the real Schur form T, and, optionally, the matrix of Schur vectors Z

sgeesx - sgeesx - compute for an N-by-N real nonsymmetric matrix A, the eigenvalues, the real Schur form T, and, optionally, the matrix of Schur vectors Z

sgeev - sgeev - compute for an N-by-N real nonsymmetric matrix A, the eigenvalues and, optionally, the left and/or right eigenvectors

sgeevx - sgeevx - compute for an N-by-N real nonsymmetric matrix A, the eigenvalues and, optionally, the left and/or right eigenvectors

sgegs - sgegs - routine is deprecated and has been replaced by routine SGGES

sgegv - sgegv - routine is deprecated and has been replaced by routine SGGEV

sgehrd - sgehrd - reduce a real general matrix A to upper Hessenberg form H by an orthogonal similarity transformation

sgelqf - sgelqf - compute an LQ factorization of a real M-by-N matrix A

sgels - sgels - solve overdetermined or underdetermined real linear systems involving an M-by-N matrix A, or its transpose, using a QR or LQ factorization of A

sgelsd - sgelsd - compute the minimum-norm solution to a real linear least squares problem

sgelss - sgelss - compute the minimum norm solution to a real linear least squares problem

sgelsx - sgelsx - routine is deprecated and has been replaced by routine SGELSY

sgelsy - sgelsy - compute the minimum-norm solution to a real linear least squares problem

sgemm - sgemm - perform one of the matrix-matrix operations C := alpha*op(A)*op(B) + beta*C

sgemv - sgemv - perform one of the matrix-vector operations y := alpha*A*x + beta*y or y := alpha*A'*x + beta*y

sgeqlf - sgeqlf - compute a QL factorization of a real M-by-N matrix A sgeqp3 - sgeqp3 - compute a QR factorization with column pivoting of a matrix A

sgeqpf - sgeqpf - routine is deprecated and has been replaced by routine SGEQP3

sgeqrf - sgeqrf - compute a QR factorization of a real M-by-N matrix A

sger - sger - perform the rank 1 operation A := alpha*x*y' + A

sgerfs - sgerfs - improve the computed solution to a system of linear equations and provides error bounds and backward error estimates for the solution

sgerqf - sgerqf - compute an RQ factorization of a real M-by-N matrix A

sgesdd - sgesdd - compute the singular value decomposition (SVD) of a real M-by-N matrix A, optionally computing the left and right singular vectors

sgesv - sgesv - compute the solution to a real system of linear equations A * X = B,

sgesvd - sgesvd - compute the singular value decomposition (SVD) of a real M-by-N matrix A, optionally computing the left and/or right singular vectors

sgesvx - sgesvx - use the LU factorization to compute the solution to a real system of linear equations A * X = B,

sgetf2 - sgetf2 - compute an LU factorization of a general m-by-n matrix A using partial pivoting with row interchanges

sgetrf - sgetrf - compute an LU factorization of a general M-by-N matrix A using partial pivoting with row interchanges

sgetri - sgetri - compute the inverse of a matrix using the LU factorization computed by SGETRF

sgetrs - sgetrs - solve a system of linear equations A * X = B or A' * X = B with a general N-by-N matrix A using the LU factorization computed by SGETRF

sggbak - sggbak - form the right or left eigenvectors of a real generalized eigenvalue problem A*x = lambda*B*x, by backward transformation on the computed eigenvectors of the balanced pair of matrices output by SGGBAL

sggbal - sggbal - balance a pair of general real matrices (A,B)

sgges - sgges - compute for a pair of N-by-N real nonsymmetric matrices (A,B), sggesx - sggesx - compute for a pair of N-by-N real nonsymmetric matrices (A,B), the generalized eigenvalues, the real Schur form (S,T), and,

sggev - sggev - compute for a pair of N-by-N real nonsymmetric matrices (A,B)

sggevx - sggevx - compute for a pair of N-by-N real nonsymmetric matrices (A,B)

sggglm - sggglm - solve a general Gauss-Markov linear model (GLM) problem

sgghrd - sgghrd - reduce a pair of real matrices (A,B) to generalized upper Hessenberg form using orthogonal transformations, where A is a general matrix and B is upper triangular

sgglse - sgglse - solve the linear equality-constrained least squares (LSE) problem

sggqrf - sggqrf - compute a generalized QR factorization of an N-by-M matrix A and an N-by-P matrix B.

sggrqf - sggrqf - compute a generalized RQ factorization of an M-by-N matrix A and a P-by-N matrix B

sggsvd - sggsvd - compute the generalized singular value decomposition (GSVD) of an M-by-N real matrix A and P-by-N real matrix B

sggsvp - sggsvp - compute orthogonal matrices U, V and Q such that N-K-L K L U'*A*Q = K (0 A12 A13) if M-K-L >= 0

sgssco - sgssco - General sparse solver condition number estimate.

sgssda - sgssda - Deallocate working storage for the general sparse solver.

sgssfa - sgssfa - General sparse solver numeric factorization.

sgssfs - sgssfs - General sparse solver one call interface.

sgssin - sgssin - Initialize the general sparse solver.

sgssor - sgssor - General sparse solver ordering and symbolic factorization.

sgssps - sgssps - Print general sparse solver statics.

sgssrp - sgssrp - Return permutation used by the general sparse solver.

sgsssl - sgsssl - Solve routine for the general sparse solver.

sgssuo - sgssuo - User supplied permutation for ordering used in the general sparse solver.

sgtcon - sgtcon - estimate the reciprocal of the condition number of a real tridiagonal matrix A using the LU factorization as computed by SGTTRF

sgthr - sgthr - Gathers specified elements from y into x.

sgthrz - sgthrz - Gather and zero.

sgtrfs - sgtrfs - improve the computed solution to a system of linear equations when the coefficient matrix is tridiagonal, and provides error bounds and backward error estimates for the solution

sgtsv - sgtsv - solve the equation A*X = B,

sgtsvx - sgtsvx - use the LU factorization to compute the solution to a real system of linear equations A * X = B or A**T * X = B,

sgttrf - sgttrf - compute an LU factorization of a real tridiagonal matrix A using elimination with partial pivoting and row interchanges

sgttrs - sgttrs - solve one of the systems of equations A*X = B or A'*X = B,

shgeqz - shgeqz - implement a single-/double-shift version of the QZ method for finding the generalized eigenvalues w(j)=(ALPHAR(j) + i*ALPHAI(j))/BETAR(j) of the equation det(A-w(i) B) = 0 In addition, the pair A,B may be reduced to generalized Schur form

shsein - shsein - use inverse iteration to find specified right and/or left eigenvectors of a real upper Hessenberg matrix H

shseqr - shseqr - compute the eigenvalues of a real upper Hessenberg matrix H and, optionally, the matrices T and Z from the Schur decomposition H = Z T Z**T, where T is an upper quasi-triangular matrix (the Schur form), and Z is the orthogonal matrix of Schur vectors

sinqb - sinqb - synthesize a Fourier sequence from its representation in terms of a sine series with odd wave numbers. The SINQ operations are unnormalized inverses of themselves, so a call to SINQF followed by a call to SINQB will multiply the input sequence by 4 * N.

sinqf - sinqf - compute the Fourier coefficients in a sine series representation with only odd wave numbers. The SINQ operations are unnormalized inverses of themselves, so a call to SINQF followed by a call to SINQB will multiply the input sequence by 4 * N.

sinqi - sinqi - initialize the array xWSAVE, which is used in both SINQF and SINQB.

sint - sint - compute the discrete Fourier sine transform of an odd sequence. The SINT transforms are unnormalized inverses of themselves, so a call of SINT followed by another call of SINT will multiply the input sequence by 2 * (N+1).

sinti - sinti - initialize the array WSAVE, which is used in subroutine SINT.

sjadmm - sjadmm - Jagged diagonal matrix-matrix multiply (modified Ellpack)

sjadrp - sjadrp - right permutation of a jagged diagonal matrix

sjadsm - sjadsm - Jagged-diagonal format triangular solve

slagtf - slagtf - factorize the matrix (T-lambda*I), where T is an n by n tridiagonal matrix and lambda is a scalar, as T-lambda*I = PLU

slamrg - slamrg - will create a permutation list which will merge the elements of A (which is composed of two independently sorted sets) into a single set which is sorted in ascending order

slarz - slarz - applies a real elementary reflector H to a real M-by-N matrix C, from either the left or the right

slarzb - slarzb - applies a real block reflector H or its transpose H**T to a real distributed M-by-N C from the left or the right

slarzt - slarzt - form the triangular factor T of a real block reflector H of order > n, which is defined as a product of k elementary reflectors

slasrt - slasrt - the numbers in D in increasing order (if ID = 'I') or in decreasing order (if ID = 'D')

slatzm - slatzm - routine is deprecated and has been replaced by routine SORMRZ

snrm2 - snrm2 - Return the Euclidian norm of a vector.

sopgtr - sopgtr - generate a real orthogonal matrix Q which is defined as the product of n-1 elementary reflectors H (i) of order n, as returned by SSPTRD using packed storage

sopmtr - sopmtr - overwrite the general real M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

sorg2l - sorg2l - generate an m by n real matrix Q with orthonormal columns,

sorg2r - sorg2r - generate an m by n real matrix Q with orthonormal columns,

sorgbr - sorgbr - generate one of the real orthogonal matrices Q or P**T determined by SGEBRD when reducing a real matrix A to bidiagonal form

sorghr - sorghr - generate a real orthogonal matrix Q which is defined as the product of IHI-ILO elementary reflectors of order N, as returned by SGEHRD

sorgl2 - sorgl2 - generate an m by n real matrix Q with orthonormal rows,

sorglq - sorglq - generate an M-by-N real matrix Q with orthonormal rows,

sorgql - sorgql - generate an M-by-N real matrix Q with orthonormal columns,

sorgqr - sorgqr - generate an M-by-N real matrix Q with orthonormal columns,

sorgr2 - sorgr2 - generate an m by n real matrix Q with orthonormal rows,

sorgrq - sorgrq - generate an M-by-N real matrix Q with orthonormal rows,

sorgtr - sorgtr - generate a real orthogonal matrix Q which is defined as the product of n-1 elementary reflectors of order N, as returned by SSYTRD

sormbr - sormbr - VECT = 'Q', SORMBR overwrites the general real M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

sormhr - sormhr - overwrite the general real M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

sormlq - sormlq - overwrite the general real M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

sormql - sormql - overwrite the general real M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

sormqr - sormqr - overwrite the general real M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

sormrq - sormrq - overwrite the general real M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

sormrz - sormrz - overwrite the general real M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

sormtr - sormtr - overwrite the general real M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

spbcon - spbcon - estimate the reciprocal of the condition number (in the 1-norm) of a real symmetric positive definite band matrix using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPBTRF

spbequ - spbequ - compute row and column scalings intended to equilibrate a symmetric positive definite band matrix A and reduce its condition number (with respect to the two-norm) spbrfs - spbrfs - improve the computed solution to a system of linear equations when the coefficient matrix is symmetric positive definite and banded, and provides error bounds and backward error estimates for the solution

spbstf - spbstf - compute a split Cholesky factorization of a real symmetric positive definite band matrix A

spbsv - spbsv - compute the solution to a real system of linear equations A * X = B,

spbsvx - spbsvx - use the Cholesky factorization A = U**T*U or A = L*L**T to compute the solution to a real system of linear equations A * X = B,

spbtf2 - spbtf2 - compute the Cholesky factorization of a real symmetric positive definite band matrix A

spbtrf - spbtrf - compute the Cholesky factorization of a real symmetric positive definite band matrix A

spbtrs - spbtrs - solve a system of linear equations A*X = B with a symmetric positive definite band matrix A using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPBTRF

spocon - spocon - estimate the reciprocal of the condition number (in the 1-norm) of a real symmetric positive definite matrix using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPOTRF

spoequ - spoequ - compute row and column scalings intended to equilibrate a symmetric positive definite matrix A and reduce its condition number (with respect to the two-norm)

sporfs - sporfs - improve the computed solution to a system of linear equations when the coefficient matrix is symmetric positive definite,

sposv - sposv - compute the solution to a real system of linear equations A * X = B,

sposvx - sposvx - use the Cholesky factorization A = U**T*U or A = L*L**T to compute the solution to a real system of linear equations A * X = B,

spotf2 - spotf2 - compute the Cholesky factorization of a real symmetric positive definite matrix A

spotrf - spotrf - compute the Cholesky factorization of a real symmetric positive definite matrix A

spotri - spotri - compute the inverse of a real symmetric positive definite matrix A using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPOTRF

spotrs - spotrs - solve a system of linear equations A*X = B with a symmetric positive definite matrix A using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPOTRF

sppcon - sppcon - estimate the reciprocal of the condition number (in the 1-norm) of a real symmetric positive definite packed matrix using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPPTRF

sppequ - sppequ - compute row and column scalings intended to equilibrate a symmetric positive definite matrix A in packed storage and reduce its condition number (with respect to the two-norm)

spprfs - spprfs - improve the computed solution to a system of linear equations when the coefficient matrix is symmetric positive definite and packed, and provides error bounds and backward error estimates for the solution

sppsv - sppsv - compute the solution to a real system of linear equations A * X = B,

sppsvx - sppsvx - use the Cholesky factorization A = U**T*U or A = L*L**T to compute the solution to a real system of linear equations A * X = B,

spptrf - spptrf - compute the Cholesky factorization of a real symmetric positive definite matrix A stored in packed format

spptri - spptri - compute the inverse of a real symmetric positive definite matrix A using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPPTRF

spptrs - spptrs - solve a system of linear equations A*X = B with a symmetric positive definite matrix A in packed storage using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPPTRF

sptcon - sptcon - compute the reciprocal of the condition number (in the 1-norm) of a real symmetric positive definite tridiagonal matrix using the factorization A = L*D*L**T or A = U**T*D*U computed by SPTTRF

spteqr - spteqr - compute all eigenvalues and, optionally, eigenvectors of a symmetric positive definite tridiagonal matrix by first factoring the matrix using SPTTRF, and then calling SBDSQR to compute the singular values of the bidiagonal factor

sptrfs - sptrfs - improve the computed solution to a system of linear equations when the coefficient matrix is symmetric positive definite and tridiagonal, and provides error bounds and backward error estimates for the solution

sptsv - sptsv - compute the solution to a real system of linear equations A*X = B, where A is an N-by-N symmetric positive definite tridiagonal matrix, and X and B are N-by-NRHS matrices.

sptsvx - sptsvx - use the factorization A = L*D*L**T to compute the solution to a real system of linear equations A*X = B, where A is an N-by-N symmetric positive definite tridiagonal matrix and X and B are N-by-NRHS matrices

spttrf - spttrf - compute the L*D*L' factorization of a real symmetric positive definite tridiagonal matrix A

spttrs - spttrs - solve a tridiagonal system of the form A * X = B using the L*D*L' factorization of A computed by SPTTRF sptts2 - sptts2 - solve a tridiagonal system of the form A * X = B using the L*D*L' factorization of A computed by SPTTRF

srot - srot - Apply a Given's rotation constructed by SROTG.

srotg - srotg - Construct a Given's plane rotation

sroti - sroti - Apply an indexed Givens rotation.

srotm - srotm - Apply a Gentleman's modified Given's rotation constructed by SROTMG.

srotmg - srotmg - Construct a Gentleman's modified Given's plane rotation

ssbev - ssbev - compute all the eigenvalues and, optionally, eigenvectors of a real symmetric band matrix A

ssbevd - ssbevd - compute all the eigenvalues and, optionally, eigenvectors of a real symmetric band matrix A

ssbevx - ssbevx - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric band matrix A

ssbgst - ssbgst - reduce a real symmetric-definite banded generalized eigenproblem A*x = lambda*B*x to standard form C*y = lambda*y,

ssbgv - ssbgv - compute all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form A*x=(lambda)*B*x

ssbgvd - ssbgvd - compute all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form A*x=(lambda)*B*x

ssbgvx - ssbgvx - compute selected eigenvalues, and optionally, eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form A*x=(lambda)*B*x

ssbmv - ssbmv - perform the matrix-vector operation y := alpha*A*x + beta*y

ssbtrd - ssbtrd - reduce a real symmetric band matrix A to symmetric tridiagonal form T by an orthogonal similarity transformation

sscal - sscal - Compute y := alpha * y

ssctr - ssctr - Scatters elements from x into y.

sskymm - sskymm - Skyline format matrix-matrix multiply sskysm - sskysm - Skyline format triangular solve

sspcon - sspcon - estimate the reciprocal of the condition number (in the 1-norm) of a real symmetric packed matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by SSPTRF

sspev - sspev - compute all the eigenvalues and, optionally, eigenvectors of a real symmetric matrix A in packed storage

sspevd - sspevd - compute all the eigenvalues and, optionally, eigenvectors of a real symmetric matrix A in packed storage

sspevx - sspevx - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix A in packed storage

sspgst - sspgst - reduce a real symmetric-definite generalized eigenproblem to standard form, using packed storage

sspgv - sspgv - compute all the eigenvalues and, optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

sspgvd - sspgvd - compute all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

sspgvx - sspgvx - compute selected eigenvalues, and optionally, eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

sspmv - sspmv - perform the matrix-vector operation y := alpha*A*x + beta*y

sspr - sspr - perform the symmetric rank 1 operation A := alpha*x*x' + A

sspr2 - sspr2 - perform the symmetric rank 2 operation A := alpha*x*y' + alpha*y*x' + A

ssprfs - ssprfs - improve the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite and packed, and provides error bounds and backward error estimates for the solution

sspsv - sspsv - compute the solution to a real system of linear equations A * X = B,

sspsvx - sspsvx - use the diagonal pivoting factorization A = U*D*U**T or A = L*D*L**T to compute the solution to a real system of linear equations A * X = B, where A is an N-by-N symmetric matrix stored in packed format and X and B are N-by-NRHS matrices

ssptrd - ssptrd - reduce a real symmetric matrix A stored in packed form to symmetric tridiagonal form T by an orthogonal similarity transformation ssptrf - ssptrf - compute the factorization of a real symmetric matrix A stored in packed format using the Bunch-Kaufman diagonal pivoting method

ssptri - ssptri - compute the inverse of a real symmetric indefinite matrix A in packed storage using the factorization A = U*D*U**T or A = L*D*L**T computed by SSPTRF

ssptrs - ssptrs - solve a system of linear equations A*X = B with a real symmetric matrix A stored in packed format using the factorization A = U*D*U**T or A = L*D*L**T computed by SSPTRF

sstebz - sstebz - compute the eigenvalues of a symmetric tridiagonal matrix T

sstedc - sstedc - compute all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the divide and conquer method

sstegr - sstegr - (a) Compute T-sigma_i = L_i D_i L_i^T, such that L_i D_i L_i^T is a relatively robust representation

sstein - sstein - compute the eigenvectors of a real symmetric tridiagonal matrix T corresponding to specified eigenvalues, using inverse iteration

ssteqr - ssteqr - compute all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the implicit QL or QR method

ssterf - ssterf - compute all eigenvalues of a symmetric tridiagonal matrix using the Pal-Walker-Kahan variant of the QL or QR algorithm

sstev - sstev - compute all eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix A

sstevd - sstevd - compute all eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix

sstevr - sstevr - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix T

sstevx - sstevx - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix A

sstsv - sstsv - compute the solution to a system of linear equations A * X = B where A is a symmetric tridiagonal matrix

ssttrf - ssttrf - compute the factorization of a symmetric tridiagonal matrix A

ssttrs - ssttrs - computes the solution to a real system of linear equations A * X = B sswap - sswap - Exchange vectors x and y.

ssycon - ssycon - estimate the reciprocal of the condition number (in the 1-norm) of a real symmetric matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by SSYTRF

ssyev - ssyev - compute all eigenvalues and, optionally, eigenvectors of a real symmetric matrix A

ssyevd - ssyevd - compute all eigenvalues and, optionally, eigenvectors of a real symmetric matrix A

ssyevr - ssyevr - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix T

ssyevx - ssyevx - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix A

ssygs2 - ssygs2 - reduce a real symmetric-definite generalized eigenproblem to standard form

ssygst - ssygst - reduce a real symmetric-definite generalized eigenproblem to standard form

ssygv - ssygv - compute all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

ssygvd - ssygvd - compute all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

ssygvx - ssygvx - compute selected eigenvalues, and optionally, eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

ssymm - ssymm - perform one of the matrix-matrix operations C := alpha*A*B + beta*C or C := alpha*B*A + beta*C

ssymv - ssymv - perform the matrix-vector operation y := alpha*A*x + beta*y

ssyr - ssyr - perform the symmetric rank 1 operation A := alpha*x*x' + A

ssyr2 - ssyr2 - perform the symmetric rank 2 operation A := alpha*x*y' + alpha*y*x' + A

ssyr2k - ssyr2k - perform one of the symmetric rank 2k operations C := alpha*A*B' + alpha*B*A' + beta*C or C := alpha*A'*B + alpha*B'*A + beta*C

ssyrfs - ssyrfs - improve the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite, and provides error bounds and backward error estimates for the solution ssyrk - ssyrk - perform one of the symmetric rank k operations C := alpha*A*A' + beta*C or C := alpha*A'*A + beta*C

ssysv - ssysv - compute the solution to a real system of linear equations A * X = B,

ssysvx - ssysvx - use the diagonal pivoting factorization to compute the solution to a real system of linear equations A * X = B,

ssytd2 - ssytd2 - reduce a real symmetric matrix A to symmetric tridiagonal form T by an orthogonal similarity transformation

ssytf2 - ssytf2 - compute the factorization of a real symmetric matrix A using the Bunch-Kaufman diagonal pivoting method

ssytrd - ssytrd - reduce a real symmetric matrix A to real symmetric tridiagonal form T by an orthogonal similarity transformation

ssytrf - ssytrf - compute the factorization of a real symmetric matrix A using the Bunch-Kaufman diagonal pivoting method

ssytri - ssytri - compute the inverse of a real symmetric indefinite matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by SSYTRF

ssytrs - ssytrs - solve a system of linear equations A*X = B with a real symmetric matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by SSYTRF

stbcon - stbcon - estimate the reciprocal of the condition number of a triangular band matrix A, in either the 1-norm or the infinity-norm

stbmv - stbmv - perform one of the matrix-vector operations x := A*x, or x := A'*x

stbrfs - stbrfs - provide error bounds and backward error estimates for the solution to a system of linear equations with a triangular band coefficient matrix

stbsv - stbsv - solve one of the systems of equations A*x = b, or A'*x = b

stbtrs - stbtrs - solve a triangular system of the form A * X = B or A**T * X = B,

stgevc - stgevc - compute some or all of the right and/or left generalized eigenvectors of a pair of real upper triangular matrices (A,B)

stgexc - stgexc - reorder the generalized real Schur decomposition of a real matrix pair (A,B) using an orthogonal equivalence transformation (A, B) = Q * (A, B) * Z', stgsen - stgsen - reorder the generalized real Schur decomposition of a real matrix pair (A, B) (in terms of an orthonormal equivalence trans- formation Q' * (A, B) * Z), so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper quasi-triangular matrix A and the upper triangular B

stgsja - stgsja - compute the generalized singular value decomposition (GSVD) of two real upper triangular (or trapezoidal) matrices A and B

stgsna - stgsna - estimate reciprocal condition numbers for specified eigenvalues and/or eigenvectors of a matrix pair (A, B) in generalized real Schur canonical form (or of any matrix pair (Q*A*Z', Q*B*Z') with orthogonal matrices Q and Z, where Z' denotes the transpose of Z

stgsyl - stgsyl - solve the generalized Sylvester equation

stpcon - stpcon - estimate the reciprocal of the condition number of a packed triangular matrix A, in either the 1-norm or the infinity-norm

stpmv - stpmv - perform one of the matrix-vector operations x := A*x, or x := A'*x

stprfs - stprfs - provide error bounds and backward error estimates for the solution to a system of linear equations with a triangular packed coefficient matrix

stpsv - stpsv - solve one of the systems of equations A*x = b, or A'*x = b

stptri - stptri - compute the inverse of a real upper or lower triangular matrix A stored in packed format

stptrs - stptrs - solve a triangular system of the form A * X = B or A**T * X = B,

strans - strans - transpose and scale source matrix

strcon - strcon - estimate the reciprocal of the condition number of a triangular matrix A, in either the 1-norm or the infinity-norm

strevc - strevc - compute some or all of the right and/or left eigenvectors of a real upper quasi-triangular matrix T

strexc - strexc - reorder the real Schur factorization of a real matrix A = Q*T*Q**T, so that the diagonal block of T with row index IFST is moved to row ILST

strmm - strmm - perform one of the matrix-matrix operations B := alpha*op(A)*B, or B := alpha*B*op(A)

strmv - strmv - perform one of the matrix-vector operations x := A*x, or x := A'*x

strrfs - strrfs - provide error bounds and backward error estimates for the solution to a system of linear equations with a triangular coefficient matrix

strsen - strsen - reorder the real Schur factorization of a real matrix A = Q*T*Q**T, so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper quasi-triangular matrix T,

strsm - strsm - solve one of the matrix equations op(A)*X = alpha*B, or X*op(A) = alpha*B

strsna - strsna - estimate reciprocal condition numbers for specified eigenvalues and/or right eigenvectors of a real upper quasi-triangular matrix T (or of any matrix Q*T*Q**T with Q orthogonal)

strsv - strsv - solve one of the systems of equations A*x = b, or A'*x = b

strsyl - strsyl - solve the real Sylvester matrix equation

strti2 - strti2 - compute the inverse of a real upper or lower triangular matrix

strtri - strtri - compute the inverse of a real upper or lower triangular matrix A

strtrs - strtrs - solve a triangular system of the form A * X = B or A**T * X = B,

stzrqf - stzrqf - routine is deprecated and has been replaced by routine STZRZF

stzrzf - stzrzf - reduce the M-by-N (M<=N) real upper trapezoidal matrix A to upper triangular form by means of orthogonal transformations

sunperf_version - sunperf_version - gets library information .HP 1i SUBROUTINE SUNPERF_VERSION (VERSION, PATCH, UPDATE) .HP 1i INTEGER VERSION, PATCH, UPDATE .HP 1i

svbrmm - svbrmm - variable block sparse row format matrix-matrix multiply

svbrsm - svbrsm - variable block sparse row format triangular solve

swiener - swiener - perform Wiener deconvolution of two signals

use_threads - use_threads - set the upper bound on the number of threads that the calling thread wants used

using_threads - using_threads - returns the current Use number set by the USE_THREADS subroutine

vcfftb - vcfftb - compute a periodic sequence from its Fourier coefficients. The VCFFT operations are normalized, so a call of VCFFTF followed by a call of VCFFTB will return the original sequence.

vcfftf - vcfftf - compute the Fourier coefficients of a periodic sequence. The VCFFT operations are normalized, so a call of VCFFTF followed by a call of VCFFTB will return the original sequence.

vcffti - vcffti - initialize the array WSAVE, which is used in both VCFFTF and VCFFTB.

vcosqb - vcosqb - synthesize a Fourier sequence from its representation in terms of a cosine series with odd wave numbers. The VCOSQ operations are normalized, so a call of VCOSQF followed by a call of VCOSQB will return the original sequence.

vcosqf - vcosqf - compute the Fourier coefficients in a cosine series representation with only odd wave numbers. The VCOSQ operations are normalized, so a call of VCOSQF followed by a call of VCOSQB will return the original sequence.

vcosqi - vcosqi - initialize the array WSAVE, which is used in both VCOSQF and VCOSQB.

vcost - vcost - compute the discrete Fourier cosine transform of an even sequence. The VCOST transform is normalized, so a call of VCOST followed by a call of VCOST will return the original sequence.

vcosti - vcosti - initialize the array WSAVE, which is used in VCOST.

vdcosqb - vdcosqb - synthesize a Fourier sequence from its representation in terms of a cosine series with odd wave numbers. The VCOSQ operations are normalized, so a call of VCOSQF followed by a call of VCOSQB will return the original sequence.

vdcosqf - vdcosqf - compute the Fourier coefficients in a cosine series representation with only odd wave numbers. The VCOSQ operations are normalized, so a call of VCOSQF followed by a call of VCOSQB will return the original sequence.

vdcosqi - vdcosqi - initialize the array WSAVE, which is used in both VCOSQF and VCOSQB.

vdcost - vdcost - compute the discrete Fourier cosine transform of an even sequence. The VCOST transform is normalized, so a call of VCOST followed by a call of VCOST will return the original sequence.

vdcosti - vdcosti - initialize the array WSAVE, which is used in VCOST.

vdfftb - vdfftb - compute a periodic sequence from its Fourier coefficients. The VRFFT operations are normalized, so a call of VRFFTF followed by a call of VRFFTB will return the original sequence.

vdfftf - vdfftf - compute the Fourier coefficients of a periodic sequence. The VRFFT operations are normalized, so a call of VRFFTF followed by a call of VRFFTB will return the original sequence.

vdffti - vdffti - initialize the array WSAVE, which is used in both VRFFTF and VRFFTB.

vdsinqb - vdsinqb - synthesize a Fourier sequence from its representation in terms of a sine series with odd wave numbers. The VSINQ operations are normalized, so a call of VSINQF followed by a call of VSINQB will return the original sequence.

vdsinqf - vdsinqf - compute the Fourier coefficients in a sine series representation with only odd wave numbers. The VSINQ operations are normalized, so a call of VSINQF followed by a call of VSINQB will return the original sequence.

vdsinqi - vdsinqi - initialize the array WSAVE, which is used in both VSINQF and VSINQB.

vdsint - vdsint - compute the discrete Fourier sine transform of an odd sequence. The VSINT transforms are unnormalized inverses of themselves, so a call of VSINT followed by another call of VSINT will multiply the input sequence by 2 * (N+1). The VSINT transforms are normalized, so a call of VSINT followed by a call of VSINT will return the original sequence.

vdsinti - vdsinti - initialize the array WSAVE, which is used in subroutine VSINT.

vrfftb - vrfftb - compute a periodic sequence from its Fourier coefficients. The VRFFT operations are normalized, so a call of VRFFTF followed by a call of VRFFTB will return the original sequence.

vrfftf - vrfftf - compute the Fourier coefficients of a periodic sequence. The VRFFT operations are normalized, so a call of VRFFTF followed by a call of VRFFTB will return the original sequence.

vrffti - vrffti - initialize the array WSAVE, which is used in both VRFFTF and VRFFTB.

vsinqb - vsinqb - synthesize a Fourier sequence from its representation in terms of a sine series with odd wave numbers. The VSINQ operations are normalized, so a call of VSINQF followed by a call of VSINQB will return the original sequence.

vsinqf - vsinqf - compute the Fourier coefficients in a sine series representation with only odd wave numbers. The VSINQ operations are normalized, so a call of VSINQF followed by a call of VSINQB will return the original sequence.

vsinqi - vsinqi - initialize the array WSAVE, which is used in both VSINQF and VSINQB.

vsint - vsint - compute the discrete Fourier sine transform of an odd sequence. The VSINT transforms are unnormalized inverses of themselves, so a call of VSINT followed by another call of VSINT will multiply the input sequence by 2 * (N+1). The VSINT transforms are normalized, so a call of VSINT followed by a call of VSINT will return the original sequence.

vsinti - vsinti - initialize the array WSAVE, which is used in subroutine VSINT.

vzfftb - vzfftb - compute a periodic sequence from its Fourier coefficients. The VZFFT operations are normalized, so a call of VZFFTF followed by a call of VZFFTB will return the original sequence.

vzfftf - vzfftf - compute the Fourier coefficients of a periodic sequence. The VZFFT operations are normalized, so a call of VZFFTF followed by a call of VZFFTB will return the original sequence.

vzffti - vzffti - initialize the array WSAVE, which is used in both VZFFTF and VZFFTB.

zaxpy - zaxpy - compute y := alpha * x + y

zaxpyi - zaxpyi - Compute y := alpha * x + y

zbcomm - zbcomm - block coordinate matrix-matrix multiply

zbdimm - zbdimm - block diagonal format matrix-matrix multiply

zbdism - zbdism - block diagonal format triangular solve

zbdsqr - zbdsqr - compute the singular value decomposition (SVD) of a real N-by-N (upper or lower) bidiagonal matrix B.

zbelmm - zbelmm - block Ellpack format matrix-matrix multiply

zbelsm - zbelsm - block Ellpack format triangular solve

zbscmm - zbscmm - block sparse column matrix-matrix multiply

zbscsm - zbscsm - block sparse column format triangular solve

zbsrmm - zbsrmm - block sparse row format matrix-matrix multiply

zbsrsm - zbsrsm - block sparse row format triangular solve

zcnvcor - zcnvcor - compute the convolution or correlation of complex vectors

zcnvcor2 - zcnvcor2 - compute the convolution or correlation of complex matrices

zcoomm - zcoomm - coordinate matrix-matrix multiply

zcopy - zcopy - Copy x to y

zcscmm - zcscmm - compressed sparse column format matrix-matrix multiply

zcscsm - zcscsm - compressed sparse column format triangular solve zcsrmm - zcsrmm - compressed sparse row format matrix-matrix multiply

zcsrsm - zcsrsm - compressed sparse row format triangular solve

zdiamm - zdiamm - diagonal format matrix-matrix multiply

zdiasm - zdiasm - diagonal format triangular solve

zdotc - zdotc - compute the dot product of two vectors conjg(x) and y.

zdotci - zdotci - Compute the complex conjugated indexed dot product.

zdotu - zdotu - compute the dot product of two vectors x and y.

zdotui - zdotui - Compute the complex unconjugated indexed dot product.

zdrot - zdrot - Apply a plane rotation.

zdscal - zdscal - Compute y := alpha * y

zellmm - zellmm - Ellpack format matrix-matrix multiply

zellsm - zellsm - Ellpack format triangular solve

zfft2b - zfft2b - compute a periodic sequence from its Fourier coefficients. The FFT operations are unnormalized, so a call of ZFFT2F followed by a call of ZFFT2B will multiply the input sequence by M*N.

zfft2f - zfft2f - compute the Fourier coefficients of a periodic sequence. The FFT operations are unnormalized, so a call of ZFFT2F followed by a call of ZFFT2B will multiply the input sequence by M*N.

zfft2i - zfft2i - initialize the array WSAVE, which is used in both the forward and backward transforms.

zfft3b - zfft3b - compute a periodic sequence from its Fourier coefficients. The FFT operations are unnormalized, so a call of ZFFT3F followed by a call of ZFFT3B will multiply the input sequence by M*N*K.

zfft3f - zfft3f - compute the Fourier coefficients of a periodic sequence. The FFT operations are unnormalized, so a call of ZFFT3F followed by a call of ZFFT3B will multiply the input sequence by M*N*K.

zfft3i - zfft3i - initialize the array WSAVE, which is used in both ZFFT3F and ZFFT3B.

zfftb - zfftb - compute a periodic sequence from its Fourier coefficients. The FFT operations are unnormalized, so a call of ZFFTF followed by a call of ZFFTB will multiply the input sequence by N.

zfftd - zfftd - initialize the trigonometric weight and factor tables or compute the inverse Fast Fourier Transform of a double complex sequence.

zfftd2 - zfftd2 - initialize the trigonometric weight and factor tables or compute the two-dimensional inverse Fast Fourier Transform of a two-dimensional double complex array.

zfftd3 - zfftd3 - initialize the trigonometric weight and factor tables or compute the three-dimensional inverse Fast Fourier Transform of a three-dimensional double complex array.

zfftdm - zfftdm - initialize the trigonometric weight and factor tables or compute the one-dimensional inverse Fast Fourier Transform of a set of double complex data sequences stored in a two-dimensional array.

zfftf - zfftf - compute the Fourier coefficients of a periodic sequence. The FFT operations are unnormalized, so a call of ZFFTF followed by a call of ZFFTB will multiply the input sequence by N.

zffti - zffti - initialize the array WSAVE, which is used in both ZFFTF and ZFFTB.

zfftopt - zfftopt - compute the length of the closest fast FFT

zfftz - zfftz - initialize the trigonometric weight and factor tables or compute the Fast Fourier transform (forward or inverse) of a double complex sequence.

zfftz2 - zfftz2 - initialize the trigonometric weight and factor tables or compute the two-dimensional Fast Fourier Transform (forward or inverse) of a two-dimensional double complex array.

zfftz3 - zfftz3 - initialize the trigonometric weight and factor tables or compute the three-dimensional Fast Fourier Transform (forward or inverse) of a three-dimensional double complex array.

zfftzm - zfftzm - initialize the trigonometric weight and factor tables or compute the one-dimensional Fast Fourier Transform (forward or inverse) of a set of data sequences stored in a two-dimensional double complex array.

zgbbrd - zgbbrd - reduce a complex general m-by-n band matrix A to real upper bidiagonal form B by a unitary transformation

zgbcon - zgbcon - estimate the reciprocal of the condition number of a complex general band matrix A, in either the 1-norm or the infinity-norm,

zgbequ - zgbequ - compute row and column scalings intended to equilibrate an M-by-N band matrix A and reduce its condition number

zgbmv - zgbmv - perform one of the matrix-vector operations y := alpha*A*x + beta*y, or y := alpha*A'*x + beta*y, or y := alpha*conjg(A')*x + beta*y zgbrfs - zgbrfs - improve the computed solution to a system of linear equations when the coefficient matrix is banded, and provides error bounds and backward error estimates for the solution

zgbsv - zgbsv - compute the solution to a complex system of linear equations A * X = B, where A is a band matrix of order N with KL subdiagonals and KU superdiagonals, and X and B are N-by-NRHS matrices

zgbsvx - zgbsvx - use the LU factorization to compute the solution to a complex system of linear equations A * X = B, A**T * X = B, or A**H * X = B,

zgbtf2 - zgbtf2 - compute an LU factorization of a complex m-by-n band matrix A using partial pivoting with row interchanges

zgbtrf - zgbtrf - compute an LU factorization of a complex m-by-n band matrix A using partial pivoting with row interchanges

zgbtrs - zgbtrs - solve a system of linear equations A * X = B, A**T * X = B, or A**H * X = B with a general band matrix A using the LU factorization computed by CGBTRF

zgebak - zgebak - form the right or left eigenvectors of a complex general matrix by backward transformation on the computed eigenvectors of the balanced matrix output by CGEBAL

zgebal - zgebal - balance a general complex matrix A

zgebrd - zgebrd - reduce a general complex M-by-N matrix A to upper or lower bidiagonal form B by a unitary transformation

zgecon - zgecon - estimate the reciprocal of the condition number of a general complex matrix A, in either the 1-norm or the infinity-norm, using the LU factorization computed by CGETRF

zgeequ - zgeequ - compute row and column scalings intended to equilibrate an M-by-N matrix A and reduce its condition number

zgees - zgees - compute for an N-by-N complex nonsymmetric matrix A, the eigenvalues, the Schur form T, and, optionally, the matrix of Schur vectors Z

zgeesx - zgeesx - compute for an N-by-N complex nonsymmetric matrix A, the eigenvalues, the Schur form T, and, optionally, the matrix of Schur vectors Z

zgeev - zgeev - compute for an N-by-N complex nonsymmetric matrix A, the eigenvalues and, optionally, the left and/or right eigenvectors

zgeevx - zgeevx - compute for an N-by-N complex nonsymmetric matrix A, the eigenvalues and, optionally, the left and/or right eigenvectors zgegs - zgegs - routine is deprecated and has been replaced by routine CGGES

zgegv - zgegv - routine is deprecated and has been replaced by routine CGGEV

zgehrd - zgehrd - reduce a complex general matrix A to upper Hessenberg form H by a unitary similarity transformation

zgelqf - zgelqf - compute an LQ factorization of a complex M-by-N matrix A

zgels - zgels - solve overdetermined or underdetermined complex linear systems involving an M-by-N matrix A, or its conjugate-transpose, using a QR or LQ factorization of A

zgelsd - zgelsd - compute the minimum-norm solution to a real linear least squares problem

zgelss - zgelss - compute the minimum norm solution to a complex linear least squares problem

zgelsx - zgelsx - routine is deprecated and has been replaced by routine CGELSY

zgelsy - zgelsy - compute the minimum-norm solution to a complex linear least squares problem

zgemm - zgemm - perform one of the matrix-matrix operations C := alpha*op(A)*op(B) + beta*C

zgemv - zgemv - perform one of the matrix-vector operations y := alpha*A*x + beta*y, or y := alpha*A'*x + beta*y, or y := alpha*conjg(A')*x + beta*y

zgeqlf - zgeqlf - compute a QL factorization of a complex M-by-N matrix A

zgeqp3 - zgeqp3 - compute a QR factorization with column pivoting of a matrix A

zgeqpf - zgeqpf - routine is deprecated and has been replaced by routine CGEQP3

zgeqrf - zgeqrf - compute a QR factorization of a complex M-by-N matrix A

zgerc - zgerc - perform the rank 1 operation A := alpha*x*conjg(y') + A

zgerfs - zgerfs - improve the computed solution to a system of linear equations and provides error bounds and backward error estimates for the solution

zgerqf - zgerqf - compute an RQ factorization of a complex M-by-N matrix A

zgeru - zgeru - perform the rank 1 operation A := alpha*x*y' + A zgesdd - zgesdd - compute the singular value decomposition (SVD) of a complex M-by-N matrix A, optionally computing the left and/or right singular vectors, by using divide-and-conquer method

zgesv - zgesv - compute the solution to a complex system of linear equations A * X = B,

zgesvd - zgesvd - compute the singular value decomposition (SVD) of a complex M-by-N matrix A, optionally computing the left and/or right singular vectors

zgesvx - zgesvx - use the LU factorization to compute the solution to a complex system of linear equations A * X = B,

zgetf2 - zgetf2 - compute an LU factorization of a general m-by-n matrix A using partial pivoting with row interchanges

zgetrf - zgetrf - compute an LU factorization of a general M-by-N matrix A using partial pivoting with row interchanges

zgetri - zgetri - compute the inverse of a matrix using the LU factorization computed by CGETRF

zgetrs - zgetrs - solve a system of linear equations A * X = B, A**T * X = B, or A**H * X = B with a general N-by-N matrix A using the LU factorization computed by CGETRF

zggbak - zggbak - form the right or left eigenvectors of a complex generalized eigenvalue problem A*x = lambda*B*x, by backward transformation on the computed eigenvectors of the balanced pair of matrices output by CGGBAL

zggbal - zggbal - balance a pair of general complex matrices (A,B)

zgges - zgges - compute for a pair of N-by-N complex nonsymmetric matrices (A,B), the generalized eigenvalues, the generalized complex Schur form (S, T), and optionally left and/or right Schur vectors (VSL and VSR)

zggesx - zggesx - compute for a pair of N-by-N complex nonsymmetric matrices (A,B), the generalized eigenvalues, the complex Schur form (S,T),

zggev - zggev - compute for a pair of N-by-N complex nonsymmetric matrices (A,B), the generalized eigenvalues, and optionally, the left and/or right generalized eigenvectors

zggevx - zggevx - compute for a pair of N-by-N complex nonsymmetric matrices (A,B) the generalized eigenvalues, and optionally, the left and/or right generalized eigenvectors

zggglm - zggglm - solve a general Gauss-Markov linear model (GLM) problem zgghrd - zgghrd - reduce a pair of complex matrices (A,B) to generalized upper Hessenberg form using unitary transformations, where A is a general matrix and B is upper triangular

zgglse - zgglse - solve the linear equality-constrained least squares (LSE) problem

zggqrf - zggqrf - compute a generalized QR factorization of an N-by-M matrix A and an N-by-P matrix B.

zggrqf - zggrqf - compute a generalized RQ factorization of an M-by-N matrix A and a P-by-N matrix B

zggsvd - zggsvd - compute the generalized singular value decomposition (GSVD) of an M-by-N complex matrix A and P-by-N complex matrix B

zggsvp - zggsvp - compute unitary matrices U, V and Q such that N-K-L K L U'*A*Q = K (0 A12 A13) if M-K-L >= 0

zgssco - zgssco - General sparse solver condition number estimate.

zgssda - zgssda - Deallocate working storage for the general sparse solver.

zgssfa - zgssfa - General sparse solver numeric factorization.

zgssfs - zgssfs - General sparse solver one call interface.

zgssin - zgssin - Initialize the general sparse solver.

zgssor - zgssor - General sparse solver ordering and symbolic factorization.

zgssps - zgssps - Print general sparse solver statics.

zgssrp - zgssrp - Return permutation used by the general sparse solver.

zgsssl - zgsssl - Solve routine for the general sparse solver.

zgssuo - zgssuo - User supplied permutation for ordering used in the general sparse solver.

zgtcon - zgtcon - estimate the reciprocal of the condition number of a complex tridiagonal matrix A using the LU factorization as computed by CGTTRF

zgthr - zgthr - Gathers specified elements from y into x.

zgthrz - zgthrz - Gather and zero.

zgtrfs - zgtrfs - improve the computed solution to a system of linear equations when the coefficient matrix is tridiagonal, and provides error bounds and backward error estimates for the solution

zgtsv - zgtsv - solve the equation A*X = B,

zgtsvx - zgtsvx - use the LU factorization to compute the solution to a complex system of linear equations A * X = B, A**T * X = B, or A**H * X = B,

zgttrf - zgttrf - compute an LU factorization of a complex tridiagonal matrix A using elimination with partial pivoting and row interchanges

zgttrs - zgttrs - solve one of the systems of equations A * X = B, A**T * X = B, or A**H * X = B,

zhbev - zhbev - compute all the eigenvalues and, optionally, eigenvectors of a complex Hermitian band matrix A

zhbevd - zhbevd - compute all the eigenvalues and, optionally, eigenvectors of a complex Hermitian band matrix A

zhbevx - zhbevx - compute selected eigenvalues and, optionally, eigenvectors of a complex Hermitian band matrix A

zhbgst - zhbgst - reduce a complex Hermitian-definite banded generalized eigenproblem A*x = lambda*B*x to standard form C*y = lambda*y,

zhbgv - zhbgv - compute all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite banded eigenproblem, of the form A*x=(lambda)*B*x

zhbgvd - zhbgvd - compute all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite banded eigenproblem, of the form A*x=(lambda)*B*x

zhbgvx - zhbgvx - compute all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite banded eigenproblem, of the form A*x=(lambda)*B*x

zhbmv - zhbmv - perform the matrix-vector operation y := alpha*A*x + beta*y

zhbtrd - zhbtrd - reduce a complex Hermitian band matrix A to real symmetric tridiagonal form T by a unitary similarity transformation

zhecon - zhecon - estimate the reciprocal of the condition number of a complex Hermitian matrix A using the factorization A = U*D*U**H or A = L*D*L**H computed by CHETRF

zheev - zheev - compute all eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A zheevd - zheevd - compute all eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A

zheevr - zheevr - compute selected eigenvalues and, optionally, eigenvectors of a complex Hermitian tridiagonal matrix T

zheevx - zheevx - compute selected eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A

zhegs2 - zhegs2 - reduce a complex Hermitian-definite generalized eigenproblem to standard form

zhegst - zhegst - reduce a complex Hermitian-definite generalized eigenproblem to standard form

zhegv - zhegv - compute all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

zhegvd - zhegvd - compute all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

zhegvx - zhegvx - compute selected eigenvalues, and optionally, eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

zhemm - zhemm - perform one of the matrix-matrix operations C := alpha*A*B + beta*C or C := alpha*B*A + beta*C

zhemv - zhemv - perform the matrix-vector operation y := alpha*A*x + beta*y

zher - zher - perform the hermitian rank 1 operation A := alpha*x*conjg(x') + A

zher2 - zher2 - perform the hermitian rank 2 operation A := alpha*x*conjg(y') + conjg(alpha)*y*conjg(x') + A

zher2k - zher2k - perform one of the Hermitian rank 2k operations C := alpha*A*conjg(B') + conjg(alpha) *B*conjg(A') + beta*C or C := alpha*conjg(A')*B + conjg(alpha)*conjg(B')*A + beta*C

zherfs - zherfs - improve the computed solution to a system of linear equations when the coefficient matrix is Hermitian indefinite, and provides error bounds and backward error estimates for the solution

zherk - zherk - perform one of the Hermitian rank k operations C := alpha*A*conjg(A') + beta*C or C := alpha*conjg(A')*A + beta*C

zhesv - zhesv - compute the solution to a complex system of linear equations A * X = B,

zhesvx - zhesvx - use the diagonal pivoting factorization to compute the solution to a complex system of linear equations A * X = B, zhetf2 - zhetf2 - compute the factorization of a complex Hermitian matrix A using the Bunch-Kaufman diagonal pivoting method

zhetrd - zhetrd - reduce a complex Hermitian matrix A to real symmetric tridiagonal form T by a unitary similarity transformation

zhetrf - zhetrf - compute the factorization of a complex Hermitian matrix A using the Bunch-Kaufman diagonal pivoting method

zhetri - zhetri - compute the inverse of a complex Hermitian indefinite matrix A using the factorization A = U*D*U**H or A = L*D*L**H computed by CHETRF

zhetrs - zhetrs - solve a system of linear equations A*X = B with a complex Hermitian matrix A using the factorization A = U*D*U**H or A = L*D*L**H computed by CHETRF

zhgeqz - zhgeqz - implement a single-shift version of the QZ method for finding the generalized eigenvalues w(i) =ALPHA(i)/BETA(i) of the equation det(A-w(i) B) = 0 If JOB='S', then the pair (A,B) is simultaneously reduced to Schur form (i.e., A and B are both upper triangular) by applying one unitary tranformation (usually called Q) on the left and another (usually called Z) on the right

zhpcon - zhpcon - estimate the reciprocal of the condition number of a complex Hermitian packed matrix A using the factorization A = U*D*U**H or A = L*D*L**H computed by CHPTRF

zhpev - zhpev - compute all the eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix in packed storage

zhpevd - zhpevd - compute all the eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A in packed storage

zhpevx - zhpevx - compute selected eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A in packed storage

zhpgst - zhpgst - reduce a complex Hermitian-definite generalized eigenproblem to standard form, using packed storage

zhpgv - zhpgv - compute all the eigenvalues and, optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

zhpgvd - zhpgvd - compute all the eigenvalues and, optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

zhpgvx - zhpgvx - compute selected eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x zhpmv - zhpmv - perform the matrix-vector operation y := alpha*A*x + beta*y

zhpr - zhpr - perform the hermitian rank 1 operation A := alpha*x*conjg(x') + A

zhpr2 - zhpr2 - perform the Hermitian rank 2 operation A := alpha*x*conjg(y') + conjg(alpha)*y*conjg(x') + A

zhprfs - zhprfs - improve the computed solution to a system of linear equations when the coefficient matrix is Hermitian indefinite and packed, and provides error bounds and backward error estimates for the solution

zhpsv - zhpsv - compute the solution to a complex system of linear equations A * X = B,

zhpsvx - zhpsvx - use the diagonal pivoting factorization A = U*D*U**H or A = L*D*L**H to compute the solution to a complex system of linear equations A * X = B, where A is an N-by-N Hermitian matrix stored in packed format and X and B are N-by-NRHS matrices

zhptrd - zhptrd - reduce a complex Hermitian matrix A stored in packed form to real symmetric tridiagonal form T by a unitary similarity transformation

zhptrf - zhptrf - compute the factorization of a complex Hermitian packed matrix A using the Bunch-Kaufman diagonal pivoting method

zhptri - zhptri - compute the inverse of a complex Hermitian indefinite matrix A in packed storage using the factorization A = U*D*U**H or A = L*D*L**H computed by CHPTRF

zhptrs - zhptrs - solve a system of linear equations A*X = B with a complex Hermitian matrix A stored in packed format using the factorization A = U*D*U**H or A = L*D*L**H computed by CHPTRF

zhsein - zhsein - use inverse iteration to find specified right and/or left eigenvectors of a complex upper Hessenberg matrix H

zhseqr - zhseqr - compute the eigenvalues of a complex upper Hessenberg matrix H, and, optionally, the matrices T and Z from the Schur decomposition H = Z T Z**H, where T is an upper triangular matrix (the Schur form), and Z is the unitary matrix of Schur vectors

zjadmm - zjadmm - Jagged diagonal matrix-matrix multiply (modified Ellpack)

zjadrp - zjadrp - right permutation of a jagged diagonal matrix

zjadsm - zjadsm - Jagged-diagonal format triangular solve

zlarz - zlarz - applie a complex elementary reflector H to a complex M-by-N matrix C, from either the left or the right zlarzb - zlarzb - applie a complex block reflector H or its transpose H**H to a complex distributed M-by-N C from the left or the right

zlarzt - zlarzt - form the triangular factor T of a complex block reflector H of order > n, which is defined as a product of k elementary reflectors

zlatzm - zlatzm - routine is deprecated and has been replaced by routine CUNMRZ

zpbcon - zpbcon - estimate the reciprocal of the condition number (in the 1-norm) of a complex Hermitian positive definite band matrix using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPBTRF

zpbequ - zpbequ - compute row and column scalings intended to equilibrate a Hermitian positive definite band matrix A and reduce its condition number (with respect to the two-norm)

zpbrfs - zpbrfs - improve the computed solution to a system of linear equations when the coefficient matrix is Hermitian positive definite and banded, and provides error bounds and backward error estimates for the solution

zpbstf - zpbstf - compute a split Cholesky factorization of a complex Hermitian positive definite band matrix A

zpbsv - zpbsv - compute the solution to a complex system of linear equations A * X = B,

zpbsvx - zpbsvx - use the Cholesky factorization A = U**H*U or A = L*L**H to compute the solution to a complex system of linear equations A * X = B,

zpbtf2 - zpbtf2 - compute the Cholesky factorization of a complex Hermitian positive definite band matrix A

zpbtrf - zpbtrf - compute the Cholesky factorization of a complex Hermitian positive definite band matrix A

zpbtrs - zpbtrs - solve a system of linear equations A*X = B with a Hermitian positive definite band matrix A using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPBTRF

zpocon - zpocon - estimate the reciprocal of the condition number (in the 1-norm) of a complex Hermitian positive definite matrix using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPOTRF

zpoequ - zpoequ - compute row and column scalings intended to equilibrate a Hermitian positive definite matrix A and reduce its condition number (with respect to the two-norm)

zporfs - zporfs - improve the computed solution to a system of linear equations when the coefficient matrix is Hermitian positive definite,

zposv - zposv - compute the solution to a complex system of linear equations A * X = B, zposvx - zposvx - use the Cholesky factorization A = U**H*U or A = L*L**H to compute the solution to a complex system of linear equations A * X = B,

zpotf2 - zpotf2 - compute the Cholesky factorization of a complex Hermitian positive definite matrix A

zpotrf - zpotrf - compute the Cholesky factorization of a complex Hermitian positive definite matrix A

zpotri - zpotri - compute the inverse of a complex Hermitian positive definite matrix A using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPOTRF

zpotrs - zpotrs - solve a system of linear equations A*X = B with a Hermitian positive definite matrix A using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPOTRF

zppcon - zppcon - estimate the reciprocal of the condition number (in the 1-norm) of a complex Hermitian positive definite packed matrix using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPPTRF

zppequ - zppequ - compute row and column scalings intended to equilibrate a Hermitian positive definite matrix A in packed storage and reduce its condition number (with respect to the two-norm)

zpprfs - zpprfs - improve the computed solution to a system of linear equations when the coefficient matrix is Hermitian positive definite and packed, and provides error bounds and backward error estimates for the solution

zppsv - zppsv - compute the solution to a complex system of linear equations A * X = B,

zppsvx - zppsvx - use the Cholesky factorization A = U**H*U or A = L*L**H to compute the solution to a complex system of linear equations A * X = B,

zpptrf - zpptrf - compute the Cholesky factorization of a complex Hermitian positive definite matrix A stored in packed format

zpptri - zpptri - compute the inverse of a complex Hermitian positive definite matrix A using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPPTRF

zpptrs - zpptrs - solve a system of linear equations A*X = B with a Hermitian positive definite matrix A in packed storage using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPPTRF

zptcon - zptcon - compute the reciprocal of the condition number (in the 1-norm) of a complex Hermitian positive definite tridiagonal matrix using the factorization A = L*D*L**H or A = U**H*D*U computed by CPTTRF

zpteqr - zpteqr - compute all eigenvalues and, optionally, eigenvectors of a symmetric positive definite tridiagonal matrix by first factoring the matrix using SPTTRF and then calling CBDSQR to compute the singular values of the bidiagonal factor zptrfs - zptrfs - improve the computed solution to a system of linear equations when the coefficient matrix is Hermitian positive definite and tridiagonal, and provides error bounds and backward error estimates for the solution

zptsv - zptsv - compute the solution to a complex system of linear equations A*X = B, where A is an N-by-N Hermitian positive definite tridiagonal matrix, and X and B are N-by-NRHS matrices.

zptsvx - zptsvx - use the factorization A = L*D*L**H to compute the solution to a complex system of linear equations A*X = B, where A is an N-by-N Hermitian positive definite tridiagonal matrix and X and B are N-by-NRHS matrices

zpttrf - zpttrf - compute the L*D*L' factorization of a complex Hermitian positive definite tridiagonal matrix A

zpttrs - zpttrs - solve a tridiagonal system of the form A * X = B using the factorization A = U'*D*U or A = L*D*L' computed by CPTTRF

zptts2 - zptts2 - solve a tridiagonal system of the form A * X = B using the factorization A = U'*D*U or A = L*D*L' computed by CPTTRF

zrot - zrot - apply a plane rotation, where the cos (C) is real and the sin (S) is complex, and the vectors X and Y are complex

zrotg - zrotg - Construct a Given's plane rotation

zscal - zscal - Compute y := alpha * y

zsctr - zsctr - Scatters elements from x into y.

zskymm - zskymm - Skyline format matrix-matrix multiply

zskysm - zskysm - Skyline format triangular solve

zspcon - zspcon - estimate the reciprocal of the condition number (in the 1-norm) of a complex symmetric packed matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by CSPTRF

zsprfs - zsprfs - improve the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite and packed, and provides error bounds and backward error estimates for the solution

zspsv - zspsv - compute the solution to a complex system of linear equations A * X = B,

zspsvx - zspsvx - use the diagonal pivoting factorization A = U*D*U**T or A = L*D*L**T to compute the solution to a complex system of linear equations A * X = B, where A is an N-by-N symmetric matrix stored in packed format and X and B are N-by-NRHS matrices zsptrf - zsptrf - compute the factorization of a complex symmetric matrix A stored in packed format using the Bunch-Kaufman diagonal pivoting method

zsptri - zsptri - compute the inverse of a complex symmetric indefinite matrix A in packed storage using the factorization A = U*D*U**T or A = L*D*L**T computed by CSPTRF

zsptrs - zsptrs - solve a system of linear equations A*X = B with a complex symmetric matrix A stored in packed format using the factorization A = U*D*U**T or A = L*D*L**T computed by CSPTRF

zstedc - zstedc - compute all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the divide and conquer method

zstegr - zstegr - Compute T-sigma_i = L_i D_i L_i^T, such that L_i D_i L_i^T is a relatively robust representation

zstein - zstein - compute the eigenvectors of a real symmetric tridiagonal matrix T corresponding to specified eigenvalues, using inverse iteration

zsteqr - zsteqr - compute all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the implicit QL or QR method

zstsv - zstsv - compute the solution to a complex system of linear equations A * X = B where A is a Hermitian tridiagonal matrix

zsttrf - zsttrf - compute the factorization of a complex Hermitian tridiagonal matrix A

zsttrs - zsttrs - computes the solution to a complex system of linear equations A * X = B

zswap - zswap - Exchange vectors x and y.

zsycon - zsycon - estimate the reciprocal of the condition number (in the 1-norm) of a complex symmetric matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by CSYTRF

zsymm - zsymm - perform one of the matrix-matrix operations C := alpha*A*B + beta*C or C := alpha*B*A + beta*C

zsyr2k - zsyr2k - perform one of the symmetric rank 2k operations C := alpha*A*B' + alpha*B*A' + beta*C or C := alpha*A'*B + alpha*B'*A + beta*C

zsyrfs - zsyrfs - improve the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite, and provides error bounds and backward error estimates for the solution zsyrk - zsyrk - perform one of the symmetric rank k operations C := alpha*A*A' + beta*C or C := alpha*A'*A + beta*C

zsysv - zsysv - compute the solution to a complex system of linear equations A * X = B,

zsysvx - zsysvx - use the diagonal pivoting factorization to compute the solution to a complex system of linear equations A * X = B,

zsytf2 - zsytf2 - compute the factorization of a complex symmetric matrix A using the Bunch-Kaufman diagonal pivoting method

zsytrf - zsytrf - compute the factorization of a complex symmetric matrix A using the Bunch-Kaufman diagonal pivoting method

zsytri - zsytri - compute the inverse of a complex symmetric indefinite matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by CSYTRF

zsytrs - zsytrs - solve a system of linear equations A*X = B with a complex symmetric matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by CSYTRF

ztbcon - ztbcon - estimate the reciprocal of the condition number of a triangular band matrix A, in either the 1-norm or the infinity-norm

ztbmv - ztbmv - perform one of the matrix-vector operations x := A*x, or x := A'*x, or x := conjg(A')*x

ztbrfs - ztbrfs - provide error bounds and backward error estimates for the solution to a system of linear equations with a triangular band coefficient matrix

ztbsv - ztbsv - solve one of the systems of equations A*x = b, or A'*x = b, or conjg(A')*x = b

ztbtrs - ztbtrs - solve a triangular system of the form A * X = B, A**T * X = B, or A**H * X = B,

ztgevc - ztgevc - compute some or all of the right and/or left generalized eigenvectors of a pair of complex upper triangular matrices (A,B)

ztgexc - ztgexc - reorder the generalized Schur decomposition of a complex matrix pair (A,B), using an unitary equivalence transformation (A, B) := Q * (A, B) * Z', so that the diagonal block of (A, B) with row index IFST is moved to row ILST

ztgsen - ztgsen - reorder the generalized Schur decomposition of a complex matrix pair (A, B) (in terms of an unitary equivalence trans- formation Q' * (A, B) * Z), so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the pair (A,B) ztgsja - ztgsja - compute the generalized singular value decomposition (GSVD) of two complex upper triangular (or trapezoidal) matrices A and B

ztgsna - ztgsna - estimate reciprocal condition numbers for specified eigenvalues and/or eigenvectors of a matrix pair (A, B)

ztgsyl - ztgsyl - solve the generalized Sylvester equation

ztpcon - ztpcon - estimate the reciprocal of the condition number of a packed triangular matrix A, in either the 1-norm or the infinity-norm

ztpmv - ztpmv - perform one of the matrix-vector operations x := A*x, or x := A'*x, or x := conjg(A')*x

ztprfs - ztprfs - provide error bounds and backward error estimates for the solution to a system of linear equations with a triangular packed coefficient matrix

ztpsv - ztpsv - solve one of the systems of equations A*x = b, or A'*x = b, or conjg(A')*x = b

ztptri - ztptri - compute the inverse of a complex upper or lower triangular matrix A stored in packed format

ztptrs - ztptrs - solve a triangular system of the form A * X = B, A**T * X = B, or A**H * X = B,

ztrans - ztrans - transpose and scale source matrix

ztrcon - ztrcon - estimate the reciprocal of the condition number of a triangular matrix A, in either the 1-norm or the infinity-norm

ztrevc - ztrevc - compute some or all of the right and/or left eigenvectors of a complex upper triangular matrix T

ztrexc - ztrexc - reorder the Schur factorization of a complex matrix A = Q*T*Q**H, so that the diagonal element of T with row index IFST is moved to row ILST

ztrmm - ztrmm - perform one of the matrix-matrix operations B := alpha*op(A)*B, or B := alpha*B*op(A) where alpha is a scalar, B is an m by n matrix, A is a unit, or non-unit, upper or lower triangular matrix and op (A) is one of op(A) = A or op(A) = A' or op(A) = conjg(A')

ztrmv - ztrmv - perform one of the matrix-vector operations x := A*x, or x := A'*x, or x := conjg(A')*x

ztrrfs - ztrrfs - provide error bounds and backward error estimates for the solution to a system of linear equations with a triangular coefficient matrix

ztrsen - ztrsen - reorder the Schur factorization of a complex matrix A = Q*T*Q**H, so that a selected cluster of eigenvalues appears in the leading positions on the diagonal of the upper triangular matrix T, and the leading columns of Q form an orthonormal basis of the corresponding right invariant subspace

ztrsm - ztrsm - solve one of the matrix equations op(A)*X = alpha*B, or X*op(A) = alpha*B

ztrsna - ztrsna - estimate reciprocal condition numbers for specified eigenvalues and/or right eigenvectors of a complex upper triangular matrix T (or of any matrix Q*T*Q**H with Q unitary)

ztrsv - ztrsv - solve one of the systems of equations A*x = b, or A'*x = b, or conjg(A')*x = b

ztrsyl - ztrsyl - solve the complex Sylvester matrix equation

ztrti2 - ztrti2 - compute the inverse of a complex upper or lower triangular matrix

ztrtri - ztrtri - compute the inverse of a complex upper or lower triangular matrix A

ztrtrs - ztrtrs - solve a triangular system of the form A * X = B, A**T * X = B, or A**H * X = B,

ztzrqf - ztzrqf - routine is deprecated and has been replaced by routine CTZRZF

ztzrzf - ztzrzf - reduce the M-by-N (M<=N) complex upper trapezoidal matrix A to upper triangular form by means of unitary transformations

zung2l - zung2l - generate an m by n complex matrix Q with orthonormal columns,

zung2r - zung2r - generate an m by n complex matrix Q with orthonormal columns,

zungbr - zungbr - generate one of the complex unitary matrices Q or P**H determined by CGEBRD when reducing a complex matrix A to bidiagonal form

zunghr - zunghr - generate a complex unitary matrix Q which is defined as the product of IHI-ILO elementary reflectors of order N, as returned by CGEHRD

zungl2 - zungl2 - generate an m-by-n complex matrix Q with orthonormal rows,

zunglq - zunglq - generate an M-by-N complex matrix Q with orthonormal rows,

zungql - zungql - generate an M-by-N complex matrix Q with orthonormal columns,

zungqr - zungqr - generate an M-by-N complex matrix Q with orthonormal columns,

zungr2 - zungr2 - generate an m by n complex matrix Q with orthonormal rows, zungrq - zungrq - generate an M-by-N complex matrix Q with orthonormal rows,

zungtr - zungtr - generate a complex unitary matrix Q which is defined as the product of n-1 elementary reflectors of order N, as returned by CHETRD

zunmbr - zunmbr - VECT = 'Q', CUNMBR overwrites the general complex M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

zunmhr - zunmhr - overwrite the general complex M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

zunml2 - zunml2 - overwrite the general complex m-by-n matrix C with Q * C if SIDE = 'L' and TRANS = 'N', or Q'* C if SIDE = 'L' and TRANS = 'C', or C * Q if SIDE = 'R' and TRANS = 'N', or C * Q' if SIDE = 'R' and TRANS = 'C',

zunmlq - zunmlq - overwrite the general complex M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

zunmql - zunmql - overwrite the general complex M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

zunmqr - zunmqr - overwrite the general complex M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

zunmr2 - zunmr2 - overwrite the general complex m-by-n matrix C with Q * C if SIDE = 'L' and TRANS = 'N', or Q'* C if SIDE = 'L' and TRANS = 'C', or C * Q if SIDE = 'R' and TRANS = 'N', or C * Q' if SIDE = 'R' and TRANS = 'C',

zunmrq - zunmrq - overwrite the general complex M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

zunmrz - zunmrz - overwrite the general complex M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

zunmtr - zunmtr - overwrite the general complex M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

zupgtr - zupgtr - generate a complex unitary matrix Q which is defined as the product of n-1 elementary reflectors H(i) of order n, as returned by CHPTRD using packed storage

zupmtr - zupmtr - overwrite the general complex M-by-N matrix C with SIDE = 'L' SIDE = 'R' TRANS = 'N'

zvbrmm - zvbrmm - variable block sparse row format matrix-matrix multiply

zvbrsm - zvbrsm - variable block sparse row format triangular solve

zvmul - zvmul - compute the scaled product of complex vectors

Contents

NAME

```
available_threads - returns information about current thread
usage
```

SYNOPSIS

```
SUBROUTINE AVAILABLE_THREADS(NTOTAL, NUSING)

INTEGER NTOTAL, NUSING

SUBROUTINE AVAILABLE_THREADS_64(NTOTAL, NUSING)

INTEGER*8 NTOTAL, NUSING
```

F95 INTERFACE

```
SUBROUTINE AVAILABLE_THREADS(NTOTAL, NUSING)

INTEGER :: NTOTAL, NUSING

SUBROUTINE AVAILABLE_THREADS_64(NTOTAL, NUSING)

INTEGER(8) :: NTOTAL, NUSING
```

C INTERFACE

```
#include <sunperf.h>

void available_threads(int *ntotal, int *nusing);

void available_threads_64(long *ntotal, long *nusing);
```

PURPOSE

available_threads threads returns NTOTAL, which is the total number of CPUs available to the job (generally the number of CPUs presently on-line in the partition), and NUSING, which is the sum of the current Use numbers for all threads specified in USE_THREADS. If NTOTAL < NUSING then the system is potentially overcommitted.

ARGUMENTS

NTOTAL (output)
Total number of CPUs available.

NUSING (output)
Sum of current Use numbers for all threads specified in USE_THREADS.

Contents

NAME

```
blas_dpermute - permutes a real (double precision) array  in
terms of the permutation vector P, output by dsortv
```

SYNOPSIS

```
SUBROUTINE BLAS_DPERMUTE (N, P, INCP, X, INCX)

INTEGER N
INTEGER P(*)
INTEGER INCP
REAL*8 X(*)
INTEGER INCX

SUBROUTINE BLAS_DPERMUTE_64 (N, P, INCP, X, INCX)

INTEGER*8 N
INTEGER*8 P(*)
INTEGER*8 INCP
REAL*8 X(*)
INTEGER*8  INCX
```

F95 INTERFACE

```
SUBROUTINE PERMUTE (X, P)

USE SUNPERF

SUBROUTINE PERMUTE_64 (X, P)

USE SUNPERF
```

ARGUMENTS

```
N (input) INTEGER, the number of elements to be permuted in X
          If  N  <= 1, the subroutine returns without trying
          to permute X.

P (input) INTEGER((N-1)*|INCP|+1), the permutation (index)
          vector defined follows  the  same  conventions  as
          that  for  DTYPE  SORTV. It records the details of
          the interchanges of the elements of X during sort-
          ing. That is X = P*X. In current implementation, P
          contains the index of sorted X.

INCP  (input) INTEGER, increment for P
          INCP must not be zero. INCP could be negative.  If
          INCP  < 0, the permutation is applied in the oppo-
          site direction.  That is
          If INCP > 0,
            if INCX > 0,
              sorted X((i-1)*INCX+1) = X(P((i-1)*INCP+1)),
            if INCX < 0,
              sorted X((N-i)*|INCX|+1) = X(P((i-1)*INCP+1));
          If INCP < 0,
            if INCX > 0,
              sorted X((i-1)*INCX+1) = X(P((N-i)*|INCP|+1)).
            if INCX < 0,
              sorted X((N-i)*|INCX|+1)
                   = X(P((N-i)*|INCP|+1)).

X (input/output) REAL*8(KIND)((N-1)*|INCX|+1), the array to
          be  permuted.  Minimum  size   (N-1)*|INCX|+1   is
          required

INCX  (input) INTEGER, increment for X
          INCX must not be zero. INCX could be negative.  If
          INCX < 0, X will be permuted in a reverse way (see
          the description for INCP above).
```

SEE ALSO

blas_dsortv(3P), blas_dsort(3P)

Contents

NAME

```
blas_dsort - sorts a real (double  precision)  vector  X  in
increasing or decreasing order using quick sort algorithm
```

SYNOPSIS

```
SUBROUTINE BLAS_DSORT (SORT, N, X, INCX)

INTEGER SORT
INTEGER N
REAL*8 X(*)
INTEGER INCX

SUBROUTINE BLAS_DSORT_64 (SORT, N, X, INCX)

INTEGER*8 SORT
INTEGER*8  N
REAL*8 X(*)
INTEGER*8  INCX
```

F95 INTERFACE

```
SUBROUTINE SORT (X [, SORT])

USE SUNPERF

SUBROUTINE SORT_64 (X [, SORT])

USE SUNPERF

The functionality of SORT is covered by SORTV
```

ARGUMENTS

```
SORT (input) INTEGER, indicating sort directions
          SORT = 0, descending
          SORT = 1, ascending
          SORT = other value, error
          SORT is default to 1 for F95 INTERFACE

N (input) INTEGER, the number of elements to be sorted in X
          If N <= 1, the subroutine returns  without  trying
          to sort X.

X (input/output) REAL*8((N-1)*|INCX|+1), the array to be
          sorted
          Minimum size (N-1)*|INCX|+1 is required

INCX  (input) INTEGER, increment for X
          INCX must not be zero. INCX could be negative.  If
          INCX  < 0, change the sorting direction defined by
          SORT. That is
          If SORT = 0, let SORT = 1, INCX = |INCX|;
          If SORT = 1, let SORT = 0, INCX = |INCX|.
```

SEE ALSO

blas_dsortv(3P), blas_dpermute(3P)

Contents

NAME

```
blas_dsortv - sorts a real (double precision)  vector  X  in
increasing  or  decreasing  order using quick sort algorithm
and overwrite P with the permutation vector
```

SYNOPSIS

```
SUBROUTINE BLAS_DSORTV (SORT, N, X, INCX, P, INCP)

INTEGER SORT
INTEGER N
REAL*8 X(*)
INTEGER INCX
INTEGER P(*)
INTEGER INCP

SUBROUTINE BLAS_DSORTV_64 (SORT, N, X, INCX, P, INCP)

INTEGER*8 SORT
INTEGER*8  N
REAL*8 X(*)
INTEGER*8  INCX
INTEGER*8 P(*)
INTEGER*8  INCP
```

F95 INTERFACE

```
SUBROUTINE SORTV (X [, SORT] [, P])

USE SUNPERF

SUBROUTINE SORTV_64 (X [, SORT] [, P])

USE SUNPERF

SORTV covers the functionality of SORT
```

ARGUMENTS

```
SORT (input) INTEGER, indicating sort directions
          SORT = 0, descending
          SORT = 1, ascending
          SORT = other value, error
          SORT is default to 1 for F95 INTERFACE

N (input) INTEGER, the number of elements to be sorted in X
          If N <= 1, the subroutine returns  without  trying
          to sort X.

X (input/output) REAL*8((N-1)*|INCX|+1), the array to be
          sorted
          Minimum size (N-1)*|INCX|+1 is required

INCX  (input) INTEGER, increment for X
          INCX must not be zero. INCX could be negative.  If
          INCX  < 0, change the sorting direction defined by
          SORT. That is
          If SORT = 0, let SORT = 1, INCX = |INCX|;
          If SORT = 1, let SORT = 0, INCX = |INCX|.

P (output) INTEGER((N-1)*|INCP|+1), the permutation (index)
          vector recording the details of  the  interchanges
          of  the  elements of X during sorting. That is X =
          P*X. In this implementation, P contains the  index
          of sorted X.

INCP (input) INTEGER, increment fpr P
          INCP must not be zero. INCP could be negative.  If
          INCP < 0, store P(i) in reverse order. That is
          If INCP > 0,
            if INCX > 0,
              sorted X((i-1)*INCX+1) = X(P((i-1)*INCP+1)),
            if INCX < 0,
              sorted X((N-i)*|INCX|+1) = X(P((i-1)*INCP+1));
          If INCP < 0,
            if INCX > 0,
              sorted X((i-1)*INCX+1) = X(P((N-i)*|INCP|+1)),
            if INCX < 0,
              sorted X((N-i)*|INCX|+1)
                   = X(P((N-i)*|INCP|+1)).
```

SEE ALSO

blas_dsort(3P), blas_dpermute(3P)

Contents

NAME

```
blas_ipermute - permutes an integer array in  terms  of  the
permutation vector P, output by dsortv
```

SYNOPSIS

```
SUBROUTINE BLAS_IPERMUTE (N, P, INCP, X, INCX)

INTEGER N
INTEGER P(*)
INTEGER INCP
INTEGER X(*)
INTEGER INCX

SUBROUTINE BLAS_IPERMUTE_64 (N, P, INCP, X, INCX)

INTEGER*8 N
INTEGER*8 P(*)
INTEGER*8 INCP
INTEGER*8 X(*)
INTEGER*8  INCX
```

F95 INTERFACE

```
SUBROUTINE PERMUTE (X, P)

USE SUNPERF

SUBROUTINE PERMUTE_64 (X, P)

USE SUNPERF
```

ARGUMENTS

```
N (input) INTEGER, the number of elements to be permuted in X
          If  N  <= 1, the subroutine returns without trying
          to permute X.

P (input) INTEGER((N-1)*|INCP|+1), the permutation (index)
          vector defined follows  the  same  conventions  as
          that  for  DTYPE  SORTV. It records the details of
          the interchanges of the elements of X during sort-
          ing. That is X = P*X. In current implementation, P
          contains the index of sorted X.

INCP  (input) INTEGER, increment for P
          INCP must not be zero. INCP could be negative.  If
          INCP  < 0, the permutation is applied in the oppo-
          site direction.  That is
          If INCP > 0,
            if INCX > 0,
              sorted X((i-1)*INCX+1) = X(P((i-1)*INCP+1)),
            if INCX < 0,
              sorted X((N-i)*|INCX|+1) = X(P((i-1)*INCP+1));
          If INCP < 0,
            if INCX > 0,
              sorted X((i-1)*INCX+1) = X(P((N-i)*|INCP|+1)).
            if INCX < 0,
              sorted X((N-i)*|INCX|+1)
                   = X(P((N-i)*|INCP|+1)).

X (input/output) INTEGER(KIND)((N-1)*|INCX|+1), the array
          to be permuted.  Minimum  size  (N-1)*|INCX|+1  is
          required

INCX  (input) INTEGER, increment for X
          INCX must not be zero. INCX could be negative.  If
          INCX < 0, X will be permuted in a reverse way (see
          the description for INCP above).
```

SEE ALSO

blas_isortv(3P), blas_isort(3P)

Contents

NAME

```
blas_isort - sorts an integer  vector  X  in  increasing  or
decreasing order using quick sort algorithm
```

SYNOPSIS

```
SUBROUTINE BLAS_ISORT (SORT, N, X, INCX)

INTEGER SORT
INTEGER N
INTEGER X(*)
INTEGER INCX

SUBROUTINE BLAS_ISORT_64 (SORT, N, X, INCX)

INTEGER*8 SORT
INTEGER*8  N
INTEGER*8 X(*)
INTEGER*8  INCX
```

F95 INTERFACE

```
SUBROUTINE SORT (X [, SORT])

USE SUNPERF

SUBROUTINE SORT_64 (X [, SORT])

USE SUNPERF

The functionality of SORT is covered by SORTV
```

ARGUMENTS

```
SORT (input) INTEGER, indicating sort directions
          SORT = 0, descending
          SORT = 1, ascending
          SORT = other value, error
          SORT is default to 1 for F95 INTERFACE

N (input) INTEGER, the number of elements to be sorted in X
          If N <= 1, the subroutine returns  without  trying
          to sort X.

X (input/output) INTEGER((N-1)*|INCX|+1), the array to be
          sorted
          Minimum size (N-1)*|INCX|+1 is required

INCX  (input) INTEGER, increment for X
          INCX must not be zero. INCX could be negative.  If
          INCX  < 0, change the sorting direction defined by
          SORT. That is
          If SORT = 0, let SORT = 1, INCX = |INCX|;
          If SORT = 1, let SORT = 0, INCX = |INCX|.
```

SEE ALSO

blas_isortv(3P), blas_ipermute(3P)

Contents

NAME

```
blas_isortv -  sorts  a  real  vector  X  in  increasing  or
decreasing  order using quick sort algorithm and overwrite P
with the permutation vector
```

SYNOPSIS

```
SUBROUTINE BLAS_ISORTV (SORT, N, X, INCX, P, INCP)

INTEGER SORT
INTEGER N
INTEGER X(*)
INTEGER INCX
INTEGER P(*)
INTEGER INCP

SUBROUTINE BLAS_ISORTV_64 (SORT, N, X, INCX, P, INCP)

INTEGER*8 SORT
INTEGER*8 N
INTEGER*8 X(*)
INTEGER*8 INCX
INTEGER*8 P(*)
INTEGER*8 INCP
```

F95 INTERFACE

```
SUBROUTINE SORTV (X [, SORT] [, P])

USE SUNPERF

SUBROUTINE SORTV_64 (X [, SORT] [, P])

USE SUNPERF

SORTV covers the functionality of SORT
```

ARGUMENTS

```
SORT (input) INTEGER, indicating sort directions
          SORT = 0, descending
          SORT = 1, ascending
          SORT = other value, error
          SORT is default to 1 for F95 INTERFACE

N (input) INTEGER, the number of elements to be sorted in X
          If N <= 1, the subroutine returns  without  trying
          to sort X.

X (input/output) INTEGER((N-1)*|INCX|+1), the array to be
          sorted
          Minimum size (N-1)*|INCX|+1 is required

INCX  (input) INTEGER, increment for X
          INCX must not be zero. INCX could be negative.  If
          INCX  < 0, change the sorting direction defined by
          SORT. That is
          If SORT = 0, let SORT = 1, INCX = |INCX|;
          If SORT = 1, let SORT = 0, INCX = |INCX|.

P (output) INTEGER((N-1)*|INCP|+1), the permutation (index)
          vector recording the details of  the  interchanges
          of  the  elements of X during sorting. That is X =
          P*X. In this implementation, P contains the  index
          of sorted X.

INCP (input) INTEGER, increment fpr P
          INCP must not be zero. INCP could be negative.  If
          INCP < 0, store P(i) in reverse order. That is
          If INCP > 0,
            if INCX > 0,
              sorted X((i-1)*INCX+1) = X(P((i-1)*INCP+1)),
            if INCX < 0,
              sorted X((N-i)*|INCX|+1) = X(P((i-1)*INCP+1));
          If INCP < 0,
            if INCX > 0,
              sorted X((i-1)*INCX+1) = X(P((N-i)*|INCP|+1)),
            if INCX < 0,
              sorted X((N-i)*|INCX|+1)
                   = X(P((N-i)*|INCP|+1)).
```

SEE ALSO

blas_isort(3P), blas_ipermute(3P)

Contents

NAME

blas_spermute - permutes a real array in terms of the permutation vector P, output by dsortv

SYNOPSIS

```
SUBROUTINE BLAS_SPERMUTE (N, P, INCP, X, INCX)

INTEGER N
INTEGER P(*)
INTEGER INCP
REAL X(*)
INTEGER INCX

SUBROUTINE BLAS_SPERMUTE_64 (N, P, INCP, X, INCX)

INTEGER*8 N
INTEGER*8 P(*)
INTEGER*8 INCP
REAL X(*)
INTEGER*8  INCX
```

F95 INTERFACE

```
SUBROUTINE PERMUTE (X, P)

USE SUNPERF

SUBROUTINE PERMUTE_64 (X, P)

USE SUNPERF
```

ARGUMENTS

```
N (input) INTEGER, the number of elements to be permuted in X
          If  N  <= 1, the subroutine returns without trying
          to permute X.

P (input) INTEGER((N-1)*|INCP|+1), the permutation (index)
          vector defined follows  the  same  conventions  as
          that  for  DTYPE  SORTV. It records the details of
          the interchanges of the elements of X during sort-
          ing. That is X = P*X. In current implementation, P
          contains the index of sorted X.

INCP  (input) INTEGER, increment for P
          INCP must not be zero. INCP could be negative.  If
          INCP  < 0, the permutation is applied in the oppo-
          site direction.  That is
          If INCP > 0,
            if INCX > 0,
              sorted X((i-1)*INCX+1) = X(P((i-1)*INCP+1)),
            if INCX < 0,
              sorted X((N-i)*|INCX|+1) = X(P((i-1)*INCP+1));
          If INCP < 0,
            if INCX > 0,
              sorted X((i-1)*INCX+1) = X(P((N-i)*|INCP|+1)).
            if INCX < 0,
              sorted X((N-i)*|INCX|+1)
                   = X(P((N-i)*|INCP|+1)).

X (input/output) REAL(KIND)((N-1)*|INCX|+1), the array to be
          permuted. Minimum size (N-1)*|INCX|+1 is required

INCX  (input) INTEGER, increment for X
          INCX must not be zero. INCX could be negative.  If
          INCX < 0, X will be permuted in a reverse way (see
          the description for INCP above).
```

SEE ALSO

blas_ssortv(3P), blas_ssort(3P)

Contents

NAME

```
blas_ssort - sorts a real vector X in increasing or decreas-
ing order using quick sort algorithm
```

SYNOPSIS

```
SUBROUTINE BLAS_SSORT (SORT, N, X, INCX)

INTEGER SORT
INTEGER N
REAL X(*)
INTEGER INCX

SUBROUTINE BLAS_SSORT_64 (SORT, N, X, INCX)

INTEGER*8 SORT
INTEGER*8  N
REAL X(*)
INTEGER*8  INCX
```

F95 INTERFACE

```
SUBROUTINE SORT (X [, SORT])

USE SUNPERF

SUBROUTINE SORT_64 (X [, SORT])

USE SUNPERF

The functionality of SORT is covered by SORTV
```

ARGUMENTS

```
SORT (input) INTEGER, indicating sort directions
          SORT = 0, descending
          SORT = 1, ascending
          SORT = other value, error
          SORT is default to 1 for F95 INTERFACE

N (input) INTEGER, the number of elements to be sorted in X
          If N <= 1, the subroutine returns  without  trying
          to sort X.

X (input/output) REAL((N-1)*|INCX|+1), the array to be
          sorted
          Minimum size (N-1)*|INCX|+1 is required

INCX  (input) INTEGER, increment for X
          INCX must not be zero. INCX could be negative.  If
          INCX  < 0, change the sorting direction defined by
          SORT. That is
          If SORT = 0, let SORT = 1, INCX = |INCX|;
          If SORT = 1, let SORT = 0, INCX = |INCX|.
```

SEE ALSO

blas_ssortv(3P), blas_spermute(3P)

Contents

NAME

```
blas_ssortv -  sorts  a  real  vector  X  in  increasing  or
decreasing  order using quick sort algorithm and overwrite P
with the permutation vector
```

SYNOPSIS

```
SUBROUTINE BLAS_SSORTV (SORT, N, X, INCX, P, INCP)

INTEGER SORT
INTEGER N
REAL X(*)
INTEGER INCX
INTEGER P(*)
INTEGER INCP

SUBROUTINE BLAS_SSORTV_64 (SORT, N, X, INCX, P, INCP)

INTEGER*8 SORT
INTEGER*8  N
REAL X(*)
INTEGER*8  INCX
INTEGER*8 P(*)
INTEGER*8  INCP
```

F95 INTERFACE

```
SUBROUTINE SORTV (X [, SORT] [, P])

USE SUNPERF

SUBROUTINE SORTV_64 (X [, SORT] [, P])

USE SUNPERF

SORTV covers the functionality of SORT
```

ARGUMENTS

```
SORT (input) INTEGER, indicating sort directions
          SORT = 0, descending
          SORT = 1, ascending
          SORT = other value, error
          SORT is default to 1 for F95 INTERFACE

N (input) INTEGER, the number of elements to be sorted in X
          If N <= 1, the subroutine returns  without  trying
          to sort X.

X (input/output) REAL((N-1)*|INCX|+1), the array to be
          sorted
          Minimum size (N-1)*|INCX|+1 is required

INCX  (input) INTEGER, increment for X
          INCX must not be zero. INCX could be negative.  If
          INCX  < 0, change the sorting direction defined by
          SORT. That is
          If SORT = 0, let SORT = 1, INCX = |INCX|;
          If SORT = 1, let SORT = 0, INCX = |INCX|.

P (output) INTEGER((N-1)*|INCP|+1), the permutation (index)
          vector recording the details of  the  interchanges
          of  the  elements of X during sorting. That is X =
          P*X. In this implementation, P contains the  index
          of sorted X.

INCP (input) INTEGER, increment fpr P
          INCP must not be zero. INCP could be negative.  If
          INCP < 0, store P(i) in reverse order. That is
          If INCP > 0,
            if INCX > 0,
              sorted X((i-1)*INCX+1) = X(P((i-1)*INCP+1)),
            if INCX < 0,
              sorted X((N-i)*|INCX|+1) = X(P((i-1)*INCP+1));
          If INCP < 0,
            if INCX > 0,
              sorted X((i-1)*INCX+1) = X(P((N-i)*|INCP|+1)),
            if INCX < 0,
              sorted X((N-i)*|INCX|+1)
                   = X(P((N-i)*|INCP|+1)).
```

SEE ALSO

blas_ssort(3P), blas_spermute(3P)

Contents

NAME

```
caxpy - compute y := alpha * x + y
```

SYNOPSIS

```
SUBROUTINE CAXPY(N, ALPHA, X, INCX, Y, INCY)

COMPLEX ALPHA
COMPLEX X(*), Y(*)
INTEGER N, INCX, INCY

SUBROUTINE CAXPY_64(N, ALPHA, X, INCX, Y, INCY)

COMPLEX ALPHA
COMPLEX X(*), Y(*)
INTEGER*8 N, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE AXPY([N], ALPHA, X, [INCX], Y, [INCY])

COMPLEX :: ALPHA
COMPLEX, DIMENSION(:) :: X, Y
INTEGER :: N, INCX, INCY

SUBROUTINE AXPY_64([N], ALPHA, X, [INCX], Y, [INCY])

COMPLEX :: ALPHA
COMPLEX, DIMENSION(:) :: X, Y
INTEGER(8) :: N, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>
```

```
void caxpy(int n, complex *alpha, complex *x, int incx, com-
          plex *y, int incy);

void caxpy_64(long n, complex *alpha, complex *x, long incx,
          complex *y, long incy);
```

PURPOSE

caxpy compute y := alpha * x + y where alpha is a scalar and x and y are n-vectors.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
array of DIMENSION at least (1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input/output)
array of DIMENSION at least (1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
caxpyi - Compute y := alpha * x + y
```

SYNOPSIS

```
 SUBROUTINE CAXPYI(NZ, A, X, INDX, Y)

 COMPLEX A
 COMPLEX X(*), Y(*)
 INTEGER NZ
 INTEGER INDX(*)

 SUBROUTINE CAXPYI_64(NZ, A, X, INDX, Y)

 COMPLEX A
 COMPLEX X(*), Y(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 SUBROUTINE AXPYI([NZ], [A], X, INDX, Y)

 COMPLEX :: A
 COMPLEX, DIMENSION(:) :: X, Y
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 SUBROUTINE AXPYI_64([NZ], [A], X, INDX, Y)

 COMPLEX :: A
 COMPLEX, DIMENSION(:) :: X, Y
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
CAXPYI Compute y := alpha * x + y where alpha is a scalar, x
is a sparse vector, and y is a vector in full storage form

 do i = 1, n
   y(indx(i)) = alpha * x(i) + y(indx(i))
 enddo
```

ARGUMENTS

```
NZ (input) - INTEGER
        Number of elements in the compressed form.
        Unchanged on exit.

A (input)
        On entry, A(LPHA) specifies the scaling value.
        Unchanged on exit. A is defaulted to (1.0E0, 0.0E0)
        for F95 INTERFACE.
X (input)
        Vector containing the values of the compressed form.
        Unchanged on exit.

INDX (input) - INTEGER
        Vector containing the indices of the compressed
        form.  It is assumed that the elements in INDX are
        distinct and greater than zero.  Unchanged on exit.

Y (output)
        Vector on input which contains the vector Y in full
        storage form.  On exit, only the elements
        corresponding to the indices in INDX have been
        modified.
```

Contents

NAME

cbcomm - block coordinate matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE CBCOMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
*            VAL, BINDX, BJNDX, BNNZ, LB,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, KB, DESCRA(5), BNNZ, LB,
*            LDB, LDC, LWORK
 INTEGER    BINDX(BNNZ), BJNDX(BNNZ)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CBCOMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
*            VAL, BINDX, BJNDX, BNNZ, LB,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), BNNZ, LB,
*            LDB, LDC, LWORK
 INTEGER*8  BINDX(BNNZ), BJNDX(BNNZ)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
  SUBROUTINE BCOMM(TRANSA,MB,N,KB,ALPHA,DESCRA,VAL,BINDX, BJNDX,
*   BNNZ, LB, B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, N, KB, BNNZ, LB
 INTEGER, DIMENSION(:) ::    DESCRA, BINDX, BJNDX
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL
 COMPLEX, DIMENSION(:, :) ::  B, C

  SUBROUTINE BCOMM_64(TRANSA,MB,N,KB,ALPHA,DESCRA,VAL,BINDX, BJNDX,
```

```
*   BNNZ, LB, B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, N, KB,  BNNZ, LB
 INTEGER*8, DIMENSION(:) ::   DESCRA, BINDX, BJNDX
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL
 COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
         C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block coordinate format and
op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if the matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
```

```
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices

VAL()          scalar array of length LB*LB*BNNZ consisting of
               the non-zero block entries of A, in any order.
               Each block is stored in standard column-major form.

BINDX()        integer array of length BNNZ consisting of the
               block row indices of the block entries of A.

BJNDX()        integer array of length BNNZ consisting of the
               block column indices of the block entries of A.

BNNZ           number of block entries

LB             dimension of dense blocks composing A.

B()            rectangular array with first dimension LDB.
LDB            leading dimension of B

BETA           Scalar parameter

C()            rectangular array with first dimension LDC.

LDC            leading dimension of C

WORK()         scratch array of length LWORK. WORK is not
               referenced in the current version.

LWORK          length of WORK array. LWORK is not referenced
               in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

cbdimm - block diagonal format matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE CBDIMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BLDA, IBDIAG, NBDIAG, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, KB, DESCRA(5), BLDA, NBDIAG, LB,
*           LDB, LDC, LWORK
 INTEGER    IBDIAG(NBDIAG)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(LB*LB*BLDA*NBDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CBDIMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BLDA, IBDIAG, NBDIAG, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), BLDA, NBDIAG, LB,
*           LDB, LDC, LWORK
 INTEGER*8  IBDIAG(NBDIAG)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(LB*LB*BLDA*NBDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
  SUBROUTINE BDIMM(TRANSA,MB, [N], KB, ALPHA, DESCRA, VAL, BLDA,
*    IBDIAG, NBDIAG, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, KB, BLDA, NBDIAG, LB
 INTEGER, DIMENSION(:) ::    DESCRA, IBDIAG
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL
 COMPLEX, DIMENSION(:, :) ::  B, C

  SUBROUTINE BDIMM_64(TRANSA,MB, [N], KB, ALPHA, DESCRA, VAL, BLDA,
```

```
*    IBDIAG, NBDIAG, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, KB, BLDA, NBDIAG, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, IBDIAG
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL
 COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
         C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block diagonal format and
op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                     ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix  is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
```

```
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

 VAL()          two-dimensional LB*LB*BLDA-by-NBDIAG scalar array
                consisting of the NBDIAG nonzero block diagonal in
                any order. Each dense block is stored in standard
                column-major form.

 BLDA           leading block dimension of VAL().

 IBDIAG()       integer array of length NBDIAG consisting of the
                corresponding diagonal offsets of the non-zero
                block diagonals of A in VAL.  Lower triangular
                block diagonals have negative offsets, the main
                block diagonal has offset 0, and upper triangular
                block diagonals have positive offset.

 NBDIAG         the number of non-zero block diagonals in A.
 LB             dimension of dense blocks composing A.

 B()            rectangular array with first dimension LDB.

 LDB            leading dimension of B

 BETA           Scalar parameter

 C()            rectangular array with first dimension LDC.

 LDC            leading dimension of C

 WORK()         scratch array of length LWORK. WORK is not
                referenced in the current version.

 LWORK          length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

```
cbdism -  block diagonal format triangular solve
```

SYNOPSIS

```
 SUBROUTINE CBDISM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, BLDA, IBDIAG, NBDIAG, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), BLDA, NBDIAG, LB,
*           LDB, LDC, LWORK
 INTEGER    IBDIAG(NBDIAG)
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(MB*LB*LB), VAL(LB*LB*BLDA, NBDIAG), B(LDB,*), C(LDC,*),
*           WORK(LWORK)

 SUBROUTINE CBDISM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, BLDA, IBDIAG, NBDIAG, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), BLDA, NBDIAG, LB,
*           LDB, LDC, LWORK
 INTEGER*8  IBDIAG(NBDIAG)
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(MB*LB*LB), VAL(LB*LB*BLDA, NBDIAG), B(LDB,*), C(LDC,*),
*           WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE BDISM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BLDA,
*   IBDIAG, NBDIAG, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, N, UNITD, BLDA, NBDIAG, LB
 INTEGER, DIMENSION(:) ::    DESCRA, IBDIAG
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL, DV
 COMPLEX, DIMENSION(:, :) ::   B, C
```

```
 SUBROUTINE BDISM_64(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BLDA,
*   IBDIAG, NBDIAG, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, N, UNITD, BLDA, NBDIAG, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, IBDIAG
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL, DV
 COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in block diagonal format
and  op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)

DV()          Array of length MB*LB*LB containing the elements of
              the diagonal blocks of the matrix D.  The size of each
              square block is LB-by-LB and each block
              is stored in standard column-major form.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                 0 : general
                 1 : symmetric (A=A')
                 2 : Hermitian (A= CONJG(A'))
                 3 : Triangular
                 4 : Skew(Anti)-Symmetric (A=-A')
                 5 : Diagonal
                 6 : Skew-Hermitian (A= -CONJG(A'))
               Note: For the routine, DESCRA(1)=3 is only supported.

               DESCRA(2) upper/lower triangular indicator
                 1 : lower
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-identity blocks on the main diagonal
                 1 : identity diagonal blocks
                 2 : diagonal blocks are dense matrices
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices
VAL()          Two-dimensional LB*LB*BLDA-by-NBDIAG scalar array
               consisting of the NBDIAG non-zero block diagonal.
               Each dense block is stored in standard column-major form.

BLDA           Leading block dimension of VAL().  Should be greater
               than or equal to MB.

IBDIAG()       integer array of length NBDIAG consisting of the
               corresponding diagonal offsets of the non-zero block
               diagonals of A in VAL.  Lower triangular block diagonals
               have negative offsets, the main block diagonal has offset
               0, and upper triangular block diagonals have positive offset.
               Elements of IBDIAG  MUST be sorted in  increasing order.

NBDIAG         The number of non-zero block diagonals in A.

LB             Dimension of dense blocks composing A.

B()            Rectangular array with first dimension LDB.

LDB            Leading dimension of B.

BETA           Scalar parameter.

C()            Rectangular array with first dimension LDC.

LDC            Leading dimension of C.
```

```
WORK()         scratch array of length LWORK.
               On exit, if LWORK= -1, WORK(1) returns the optimum  size
               of LWORK.

LWORK         length of WORK array. LWORK should be at least
              MB*LB.

              For good performance, LWORK should generally be larger.
              For optimum performance on multiple processors, LWORK
              >=MB*LB*N_CPUS where N_CPUS is the maximum number of
              processors available to the program.

              If LWORK=0, the routine is to allocate workspace needed.

              If LWORK = -1, then a workspace query is assumed; the
              routine only calculates the optimum size of the WORK array,
              returns this value as the first entry of the WORK array,
              and no error message related to LWORK is issued by XERBLA.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If *DESCRA*(3)=0,the lower or upper triangular part of each diagonal block is used by the routine depending on *DESCRA*(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might not be referenced in the BDI representation of a sparse matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense matrices and the LU factorization with partial pivoting is used by the routine.

WORK(1)=0 on return if the factorization for all diagonal blocks has been completed successfully, otherwise *WORK*(1) = -i where i is the block number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

```
cbdsqr - compute the singular value decomposition (SVD) of a
real N-by-N (upper or lower) bidiagonal matrix B.
```

SYNOPSIS

```
SUBROUTINE CBDSQR(UPLO, N, NCVT, NRU, NCC, D, E, VT, LDVT, U, LDU, C,
      LDC, WORK, INFO)

CHARACTER * 1 UPLO
COMPLEX VT(LDVT,*), U(LDU,*), C(LDC,*)
INTEGER N, NCVT, NRU, NCC, LDVT, LDU, LDC, INFO
REAL D(*), E(*), WORK(*)

SUBROUTINE CBDSQR_64(UPLO, N, NCVT, NRU, NCC, D, E, VT, LDVT, U, LDU,
      C, LDC, WORK, INFO)

CHARACTER * 1 UPLO
COMPLEX VT(LDVT,*), U(LDU,*), C(LDC,*)
INTEGER*8 N, NCVT, NRU, NCC, LDVT, LDU, LDC, INFO
REAL D(*), E(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE BDSQR(UPLO, [N], [NCVT], [NRU], [NCC], D, E, VT, [LDVT],
       U, [LDU], C, [LDC], [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: VT, U, C
INTEGER :: N, NCVT, NRU, NCC, LDVT, LDU, LDC, INFO
REAL, DIMENSION(:) :: D, E, WORK

SUBROUTINE BDSQR_64(UPLO, [N], [NCVT], [NRU], [NCC], D, E, VT, [LDVT],
       U, [LDU], C, [LDC], [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: VT, U, C
INTEGER(8) :: N, NCVT, NRU, NCC, LDVT, LDU, LDC, INFO
REAL, DIMENSION(:) :: D, E, WORK
```

C INTERFACE

```
#include <sunperf.h>

void cbdsqr(char uplo, int n, int ncvt, int  nru,  int  ncc,
          float *d, float *e, complex *vt, int ldvt, complex
          *u, int ldu, complex *c, int ldc, int *info);

void cbdsqr_64(char uplo, long n, long ncvt, long nru,  long
          ncc,  float  *d, float *e, complex *vt, long ldvt,
          complex *u, long ldu, complex *c, long  ldc,  long
          *info);
```

PURPOSE

cbdsqr computes the singular value decomposition (SVD) of a real N-by-N (upper or lower) bidiagonal matrix B: B = Q * S * P' (P' denotes the transpose of P), where S is a diagonal matrix with non-negative diagonal elements (the singular values of B), and Q and P are orthogonal matrices.

The routine computes S, and optionally computes U * Q, P' * VT, or Q' * C, for given complex input matrices U, VT, and C.

See "Computing Small Singular Values of Bidiagonal Matrices With Guaranteed High Relative Accuracy," by J. Demmel and W. Kahan, LAPACK Working Note #3 (or SIAM J. Sci. Statist. Comput. vol. 11, no. 5, pp. 873-912, Sept 1990) and "Accurate singular values and differential qd algorithms," by B. Parlett and V. Fernando, Technical Report CPAM-554, Mathematics Department, University of California at Berkeley, July 1992 for a detailed description of the algorithm.

ARGUMENTS

UPLO (input)
= 'U': B is upper bidiagonal;
= 'L': B is lower bidiagonal.

N (input) The order of the matrix B. N >= 0.

```
NCVT (input)
          The number of columns of the matrix VT. NCVT >= 0.

NRU (input)
          The number of rows of the matrix U. NRU >= 0.

NCC (input)
          The number of columns of the matrix C. NCC >= 0.

D (input/output)
          On entry, the n diagonal elements of the  bidiago-
          nal  matrix  B.   On exit, if INFO=0, the singular
          values of B in decreasing order.
E (input/output)
          On entry, the elements of E contain the  offdiago-
          nal elements of of the bidiagonal matrix whose SVD
          is desired. On normal exit (INFO = 0), E  is  des-
          troyed.   If the algorithm does not converge (INFO
          > 0), D and E will contain the diagonal and super-
          diagonal  elements of a bidiagonal matrix orthogo-
          nally equivalent to the one given as  input.  E(N)
          is used for workspace.

VT (input/output)
          On entry, an N-by-NCVT matrix VT.  On exit, VT  is
          overwritten  by  P' * VT.  VT is not referenced if
          NCVT = 0.

LDVT (input)
          The leading dimension of the array  VT.   LDVT  >=
          max(1,N) if NCVT > 0; LDVT >= 1 if NCVT = 0.

U (input/output)
          On entry, an NRU-by-N matrix U.   On  exit,  U  is
          overwritten  by U * Q.  U is not referenced if NRU
          = 0.

LDU (input)
          The leading dimension of  the  array  U.   LDU  >=
          max(1,NRU).

C (input/output)
          On entry, an N-by-NCC matrix C.   On  exit,  C  is
          overwritten by Q' * C.  C is not referenced if NCC
          = 0.

LDC (input)
          The leading dimension of  the  array  C.   LDC  >=
          max(1,N) if NCC > 0; LDC >=1 if NCC = 0.
```

WORK (workspace)
dimension (4*N)

INFO (output)
= 0: successful exit
< 0: If INFO = -i, the i-th argument had an illegal value
> 0: the algorithm did not converge; D and E contain the elements of a bidiagonal matrix which is orthogonally similar to the input matrix B; if INFO = i, i elements of E have not converged to zero.

Contents

NAME

cbelmm - block Ellpack format matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE CBELMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
*            VAL, BINDX, BLDA, MAXBNZ, LB,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, KB, DESCRA(5), BLDA, MAXBNZ, LB,
*            LDB, LDC, LWORK
 INTEGER    BINDX(BLDA,MAXBNZ)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(LB*LB*BLDA*MAXBNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CBELMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
*            VAL, BINDX, BLDA, MAXBNZ, LB,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), BLDA, MAXBNZ, LB,
*            LDB, LDC, LWORK
 INTEGER*8  BINDX(BLDA,MAXBNZ)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(LB*LB*BLDA*MAXBNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE BELMM( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
*            BLDA, MAXBNZ, LB, B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, KB, BLDA, MAXBNZ, LB
 INTEGER, DIMENSION(:) ::    DESCRA, BINDX
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL
 COMPLEX, DIMENSION(:, :) ::  B, C

 SUBROUTINE BELMM_64( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
```

```
*            BLDA, MAXBNZ, LB, B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, KB, BLDA, MAXBNZ, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL
 COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block Ellpack format and
op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
```

```
                   0 : non-unit
                   1 : unit
                 DESCRA(4) Array base  (NOT IMPLEMENTED)
                   0 : C/C++ compatible
                   1 : Fortran compatible
                 DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                   0 : unknown
                   1 : no repeated indices

 VAL()           scalar array of length LB*LB*BLDA*MAXBNZ containing
                 matrix entries, stored column-major within each dense
                 block.

 BINDX()         two-dimensional integer BLDA-by-MAXBNZ array such
                 BINDX(i,:) consists of the block column indices of the
                 nonzero blocks in block row i, padded by the integer
                 value i if the number of nonzero blocks is less than
                 MAXBNZ.

 BLDA            leading dimension of BINDX(:,:).

 MAXBNZ          max number of nonzeros blocks per row.
 LB              row and column dimension of the dense blocks composing
                 VAL.

 B()             rectangular array with first dimension LDB.

 LDB             leading dimension of B

 BETA            Scalar parameter

 C()             rectangular array with first dimension LDC.

 LDC             leading dimension of C

 WORK()          scratch array of length LWORK. WORK is not
                 referenced in the current version.

 LWORK           length of WORK array. LWORK is not referenced
                 in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS)

Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

cbelsm - block Ellpack format triangular solve

SYNOPSIS

```
 SUBROUTINE CBELSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, BINDX, BLDA, MAXBNZ, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), BLDA, MAXBNZ, LB,
*           LDB, LDC, LWORK
 INTEGER    BINDX(BLDA,MAXBNZ)
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(MB*LB*LB), VAL(LB*LB*BLDA*MAXBNZ), B(LDB,*), C(LDC,*),
*           WORK(LWORK)

 SUBROUTINE CBELSM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, BINDX, BLDA, MAXBNZ, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), BLDA, MAXBNZ, LB,
*           LDB, LDC, LWORK
 INTEGER*8  BINDX(BLDA,MAXBNZ)
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(MB*LB*LB), VAL(LB*LB*BLDA*MAXBNZ), B(LDB,*), C(LDC,*),
*           WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE BELSM( TRANSA, MB, [N], UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*   BLDA, MAXBNZ, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, UNITD,  BLDA, MAXBNZ, LB
 INTEGER, DIMENSION(:) ::    DESCRA,  BINDX
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL, DV
 COMPLEX, DIMENSION(:, :) ::   B, C
```

```
 SUBROUTINE BELSM_64( TRANSA, MB, [N], UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*   BLDA, MAXBNZ, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8  TRANSA, MB, UNITD,  BLDA, MAXBNZ, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA,  BINDX
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL, DV
 COMPLEX, DIMENSION(:, :) ::   B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in block Ellpack format and
op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)

DV()          Array of the length MB*LB*LB consisting of the block
              entries of block diagonal matrix D where each
              block is stored in standard column-major form.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
```

```
                   1 : symmetric (A=A')
                   2 : Hermitian (A= CONJG(A'))
                   3 : Triangular
                   4 : Skew(Anti)-Symmetric (A=-A')
                   5 : Diagonal
                   6 : Skew-Hermitian (A= -CONJG(A'))
                 Note: For the routine, DESCRA(1)=3 is only supported.

                 DESCRA(2) upper/lower triangular indicator
                   1 : lower
                   2 : upper
                 DESCRA(3) main diagonal type
                   0 : non-identity blocks on the main diagonal
                   1 : identity diagonal blocks
                   2 : diagonal blocks are dense matrices
                 DESCRA(4) Array base  (NOT IMPLEMENTED)
                   0 : C/C++ compatible
                   1 : Fortran compatible
                 DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                   0 : unknown
                   1 : no repeated indices
  VAL()          scalar array of length LB*LB*BLDA*MAXBNZ containing
                 matrix entries, stored column-major within each dense
                 block.

  BINDX()        two-dimensional integer BLDA-by-MAXBNZ array such
                 BINDX(i,:) consists of the block column indices of the
                 nonzero blocks in block row i, padded by the integer
                 value i if the number of nonzero blocks is less than
                 MAXBNZ.  The block column indices MUST be sorted
                 in increasing order for each block row.

  BLDA           leading dimension of BINDX(:,:).

  MAXBNZ         max number of nonzeros blocks per row.

  LB             row and column dimension of the dense blocks composing A.

  B()            rectangular array with first dimension LDB.

  LDB            leading dimension of B

  BETA           Scalar parameter

  C()            rectangular array with first dimension LDC.

  LDC            leading dimension of C

  WORK()         scratch array of length LWORK.
                 On exit, if LWORK= -1, WORK(1) returns the minimum
```

size of LWORK.

LWORK — length of WORK array. LWORK should be at least MB*LB.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=MB*LB*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If *DESCRA*(3)=0,the lower or upper triangular part of each diagonal block is used by the routine depending on *DESCRA*(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might not be referenced in the BEL representation of a sparse matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense matrices and the LU factorization with partial pivoting is used by the routine. *WORK*(1)=0 on return if the factorization for all diagonal blocks has been completed successfully, otherwise *WORK*(1) = -i where i is the block number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

cbscmm - block sparse column matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE CBSCMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BPNTRB, BPNTRE, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, KB, DESCRA(5), LB,
*           LDB, LDC, LWORK
 INTEGER    BINDX(BNNZ), BPNTRB(KB), BPNTRE(KB)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CBSCMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BPNTRB, BPNTRE, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), LB,
*           LDB, LDC, LWORK
 INTEGER*8  BINDX(BNNZ), BPNTRB(KB), BPNTRE(KB)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where: BNNZ = BPNTRE(KB)-BPNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE BSCMM( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB,  KB, LB
 INTEGER, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL
 COMPLEX, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE BSCMM_64( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, KB, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL
 COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block sparse column format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
```

```
               DESCRA(3) main diagonal type
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices

VAL()          scalar array of length LB*LB*BNNZ consisting of
               the block entries stored column-major within each
               dense block.

BINDX()        integer array of length BNNZ consisting of the
               block row indices of the block entries of A.

BPNTRB()       integer array of length KB such that
               BPNTRB(J)-BPNTRB(1)+1 points to location in BINDX
               of the first block entry of the J-th block column
               of A.
BPNTRE()       integer array of length KB such that
               BPNTRE(J)-BPNTRB(1) points to location in BINDX
               of the last block entry of the J-th block column
               of A.

LB             dimension of dense blocks composing A.

B()            rectangular array with first dimension LDB.

LDB            leading dimension of B

BETA           Scalar parameter

C()            rectangular array with first dimension LDC.

LDC            leading dimension of C

WORK()         scratch array of length LWORK. WORK is not
               referenced in the current version.

LWORK          length of WORK array. LWORK is not referenced
               in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

It is known that there exists another representation of the block sparse column format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block column in the arrays VAL and BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of block sparse column format the following calling sequence should be used

```
  CALL SBSCMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *           VAL, BINDX, IA, IA(2), LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

cbscsm - block sparse column format triangular solve

SYNOPSIS

```
 SUBROUTINE CBSCSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, BINDX, BPNTRB, BPNTRE, LB,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), LB,
*            LDB, LDC, LWORK
 INTEGER    BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(MB*LB*LB), VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CBSCSM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, BINDX, BPNTRB, BPNTRE, LB,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), LB,
*            LDB, LDC, LWORK
 INTEGER*8  BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(MB*LB*LB), VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where: BNNZ = BPNTRE(MB)- BPNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE BSCSM(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*      BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, N, UNITD, LB
 INTEGER, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL, DV
 COMPLEX, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE BSCSM_64(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, N, UNITD, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL, DV
 COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in block sparse column format
and  op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row block scaling)
                3 : Scale on right (column block scaling)

DV()          Array of the length MB*LB*LB consisting of the block
              entries of block diagonal matrix D where each
              block is stored in standard column-major form.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
```

```
                 1 : symmetric (A=A')
                 2 : Hermitian (A= CONJG(A'))
                 3 : Triangular
                 4 : Skew(Anti)-Symmetric (A=-A')
                 5 : Diagonal
                 6 : Skew-Hermitian (A= -CONJG(A'))

               Note: For the routine, DESCRA(1)=3 is only supported.

               DESCRA(2) upper/lower triangular indicator
                 1 : lower
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-identity blocks on the main diagonal
                 1 : identity diagonal blocks
                 2 : diagonal blocks are dense matrices
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices
VAL()          scalar array of length LB*LB*BNNZ consisting of the block
               entries stored column-major within each dense block.

BINDX()        integer array of length BNNZ consisting of the
               block row indices of the block entries of A.
               The block row indices MUST be sorted
               in increasing order for each block column.

BPNTRB()       integer array of length MB such that
               BPNTRB(J)-BPNTRB(1)+1 points to location in BINDX
               of the first block entry of the J-th block column of A.

BPNTRE()       integer array of length MB such that
               BPNTRE(J)-BPNTRB(1) points to location in BINDX
               of the last block entry of the J-th block column of A.

LB             dimension of dense blocks composing A.

B()            rectangular array with first dimension LDB.

LDB            leading dimension of B

BETA           Scalar parameter

C()            rectangular array with first dimension LDC.

LDC            leading dimension of C
```

WORK() scratch array of length LWORK.
On exit, if LWORK= -1, WORK(1) returns the optimum size of LWORK.

LWORK length of WORK array. LWORK should be at least MB*LB.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=MB*LB*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If *DESCRA*(3)=0,the lower or upper triangular part of each diagonal block is used by the routine depending on *DESCRA*(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might not be referenced in the BSC representation of a sparse matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense matrices and the LU factorization with partial pivoting is used by the routine. *WORK*(1)=0 on return if the factorization for all diagonal blocks has been completed

successfully, otherwise *WORK*(1) = -i where i is the block number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3 in this case.

6. It is known that there exists another representation of the block sparse column format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block column in the arrays VAL and BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of block sparse column format the following calling sequence should be used

```
  CALL SBSCSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, BINDX, IA, IA(2), LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

```
cbsrmm - block sparse row format matrix-matrix multiply
```

SYNOPSIS

```
 SUBROUTINE CBSRMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BPNTRB, BPNTRE, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, KB, DESCRA(5), LB,
*           LDB, LDC, LWORK
 INTEGER    BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CBSRMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BPNTRB, BPNTRE, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), LB,
*           LDB, LDC, LWORK
 INTEGER*8  BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where: BNNZ = BPNTRE(MB)-BPNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE BSRMM( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB,  KB, LB
 INTEGER, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL
 COMPLEX, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE BSRMM_64( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, KB, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL
 COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block sparse row format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix A is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
```

```
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

 VAL()          scalar array of length LB*LB*BNNZ consisting
                of the block  entries stored column-major within
                each dense block.

 BINDX()        integer array of length BNNZ consisting of the
                block column indices of the block entries of A.

 BPNTRB()       integer array of length MB such that
                BPNTRB(J)-BPNTRB(1)+1 points to location in BINDX
                of the first block entry of the J-th block row of A.
 BPNTRE()       integer array of length MB such that
                BPNTRE(J)-BPNTRB(1) points to location in BINDX
                of the last block entry of the J-th block row of A.

 LB             dimension of dense blocks composing A.

 B()            rectangular array with first dimension LDB.

 LDB            leading dimension of B

 BETA           Scalar parameter

 C()            rectangular array with first dimension LDC.

 LDC            leading dimension of C

 WORK()         scratch array of length LWORK. WORK is not
                referenced in the current version.

 LWORK          length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

It is known that there exists another representation of the block sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block row in the arrays VAL and BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of block sparse row format the following calling sequence should be used

```
  CALL SBSRMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *           VAL, BINDX, IA, IA(2), LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

```
cbsrsm - block sparse row format triangular solve
```

SYNOPSIS

```
 SUBROUTINE CBSRSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, BINDX, BPNTRB, BPNTRE, LB,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), LB,
*            LDB, LDC, LWORK
 INTEGER    BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(MB*LB*LB), VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CBSRSM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, BINDX, BPNTRB, BPNTRE, LB,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), LB,
*            LDB, LDC, LWORK
 INTEGER*8  BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(MB*LB*LB), VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where: BNNZ = BPNTRE(MB)-BPNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE BSRSM(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*      BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, N, UNITD, LB
 INTEGER, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL, DV
 COMPLEX, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE BSRSM_64(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, N, UNITD, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL, DV
 COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in block sparse row format
format and  op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row block scaling)
                3 : Scale on right (column block scaling)

DV()          Array of the length MB*LB*LB consisting of the block
              entries of block diagonal matrix D where each
              block is stored in standard column-major form.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
```

```
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))

                Note: For the routine, DESCRA(1)=3 is only supported.

                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-identity blocks on the main diagonal
                  1 : identity diagonal blocks
                  2 : diagonal blocks are dense matrices
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices
VAL()           scalar array of length LB*LB*BNNZ consisting of the block
                entries stored column-major within each dense block.

BINDX()         integer array of length BNNZ consisting of the
                block column indices of the block entries of A.
                The block column indices MUST be sorted
                in increasing order for each block row.

BPNTRB()        integer array of length MB such that
                BPNTRB(J)-BPNTRB(1)+1 points to location in BINDX
                of the first block entry of the J-th block row of A.

BPNTRE()        integer array of length MB such that
                BPNTRE(J)-BPNTRB(1) points to location in BINDX
                of the last block entry of the J-th block row of A.

LB              dimension of dense blocks composing A.

B()             rectangular array with first dimension LDB.

LDB             leading dimension of B

BETA            Scalar parameter

C()             rectangular array with first dimension LDC.

LDC             leading dimension of C
```

WORK()   scratch array of length LWORK.
On exit, if LWORK= -1, WORK(1) returns the optimum size of LWORK.

LWORK   length of WORK array. LWORK should be at least MB*LB.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=MB*LB*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If *DESCRA*(3)=0,the lower or upper triangular part of each diagonal block is used by the routine depending on *DESCRA*(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might not be referenced in the BSR representation of a sparse matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense matrices and the LU factorization with partial pivoting is used by the routine. *WORK*(1)=0 on return if the factorization for all diagonal blocks has been completed

successfully, otherwise *WORK*(1) = -i where i is the block number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3 in this case.

6. It is known that there exists another representation of the block sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block row in the arrays VAL and BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of block sparse row format the following calling sequence should be used

```
  CALL SBSRSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, BINDX, IA, IA(2), LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

```
ccnvcor - compute the convolution or correlation of  complex
vectors
```

SYNOPSIS

```
SUBROUTINE CCNVCOR(CNVCOR, FOUR, NX, X, IFX, INCX, NY, NPRE, M, Y,
      IFY, INC1Y, INC2Y, NZ, K, Z, IFZ, INC1Z, INC2Z, WORK, LWORK)

CHARACTER * 1 CNVCOR, FOUR
COMPLEX X(*), Y(*), Z(*), WORK(*)
INTEGER NX, IFX, INCX, NY, NPRE, M, IFY, INC1Y,  INC2Y,  NZ,
K, IFZ, INC1Z, INC2Z, LWORK

SUBROUTINE CCNVCOR_64(CNVCOR, FOUR, NX, X, IFX, INCX, NY, NPRE, M, Y,
      IFY, INC1Y, INC2Y, NZ, K, Z, IFZ, INC1Z, INC2Z, WORK, LWORK)

CHARACTER * 1 CNVCOR, FOUR
COMPLEX X(*), Y(*), Z(*), WORK(*)
INTEGER*8 NX, IFX, INCX, NY, NPRE, M, IFY, INC1Y, INC2Y, NZ,
K, IFZ, INC1Z, INC2Z, LWORK
```

F95 INTERFACE

```
SUBROUTINE CNVCOR(CNVCOR, FOUR, NX, X, IFX, [INCX], NY, NPRE, M, Y,
       IFY, INC1Y, INC2Y, NZ, K, Z, IFZ, INC1Z, INC2Z, WORK, [LWORK])

CHARACTER(LEN=1) :: CNVCOR, FOUR
COMPLEX, DIMENSION(:) :: X, Y, Z, WORK
INTEGER :: NX, IFX, INCX, NY, NPRE, M,  IFY,  INC1Y,  INC2Y,
NZ, K, IFZ, INC1Z, INC2Z, LWORK

SUBROUTINE CNVCOR_64(CNVCOR, FOUR, NX, X, IFX, [INCX], NY, NPRE, M,
       Y, IFY, INC1Y, INC2Y, NZ, K, Z, IFZ, INC1Z, INC2Z, WORK, [LWORK])
```

```
CHARACTER(LEN=1) :: CNVCOR, FOUR
COMPLEX, DIMENSION(:) :: X, Y, Z, WORK
INTEGER(8) :: NX, IFX, INCX, NY, NPRE, M, IFY, INC1Y, INC2Y,
NZ, K, IFZ, INC1Z, INC2Z, LWORK
```

C INTERFACE

```
#include <sunperf.h>

void ccnvcor(char cnvcor, char four, int nx, complex *x, int
          ifx,  int  incx,  int ny, int npre, int m, complex
          *y, int ify, int inc1y, int inc2y, int nz, int  k,
          complex *z, int ifz, int inc1z, int inc2z, complex
          *work, int lwork);
void ccnvcor_64(char cnvcor, char four, long nx, complex *x,
          long  ifx,  long incx, long ny, long npre, long m,
          complex *y, long ify, long inc1y, long inc2y, long
          nz, long k, complex *z, long ifz, long inc1z, long
          inc2z, complex *work, long lwork);
```

PURPOSE

ccnvcor computes the convolution or correlation of complex vectors.

ARGUMENTS

CNVCOR (input)
CHARACTER
'V' or 'v' if convolution is desired, 'R' or 'r' if correlation is desired.

FOUR (input)
CHARACTER
'T' or 't' if the Fourier transform method is to be used, 'D' or 'd' if the computation should be done directly from the definition. The Fourier transform method is generally faster, but it may introduce noticeable errors into certain results, notably when both the real and imaginary parts of the filter and data vectors consist entirely of integers or vectors where elements of either the filter vector or a given data vector differ significantly in magnitude from the 1-norm of the vector.

NX (input)

Length of the filter vector. NX >= 0. CCNVCOR will return immediately if NX = 0.

X (input) dimension(*)

Filter vector.

IFX (input)

Index of the first element of X. NX >= IFX >= 1.

INCX (input)

Stride between elements of the filter vector in X. INCX > 0.

NY (input)

Length of the input vectors. NY >= 0. CCNVCOR will return immediately if NY = 0.

NPRE (input)

The number of implicit zeros prepended to the Y vectors. NPRE >= 0.

M (input)

Number of input vectors. M >= 0. CCNVCOR will return immediately if M = 0.

Y (input) dimension(*)

Input vectors.

IFY (input)

Index of the first element of Y. NY >= IFY >= 1.

INC1Y (input)

Stride between elements of the input vectors in Y. INC1Y > 0.

INC2Y (input)

Stride between the input vectors in Y. INC2Y > 0.

NZ (input)

Length of the output vectors. NZ >= 0. CCNVCOR will return immediately if NZ = 0. See the Notes section below for information about how this argument interacts with NX and NY to control circular versus end-off shifting.

K (input)

Number of Z vectors. K >= 0. If K = 0 then CCNVCOR will return immediately. If K < M then only the first K input vectors will be processed. If K > M then M input vectors will be processed.

Z (output)
dimension(*)
Result vectors.

IFZ (input)
Index of the first element of Z. NZ >= IFZ >= 1.

INC1Z (input)
Stride between elements of the output vectors in Z. INC1Z > 0.

INC2Z (input)
Stride between the output vectors in Z. INC2Z > 0.

WORK (input/output)
(input/scratch) dimension(LWORK)
Scratch space. Before the first call to CCNVCOR with particular values of the integer arguments the first element of WORK must be set to zero. If WORK is written between calls to CCNVCOR or if CCNVCOR is called with different values of the integer arguments then the first element of WORK must again be set to zero before each call. If WORK has not been written and the same values of the integer arguments are used then the first element of WORK to zero. This can avoid certain initializations that store their results into WORK, and avoiding the initialization can make CCNVCOR run faster.

LWORK (input)
Length of WORK. LWORK >= 2*MAX(NX,NY+NPRE,NZ)+8.

Contents

NAME

```
ccnvcor2 - compute the convolution or correlation of complex
matrices
```

SYNOPSIS

```
SUBROUTINE CCNVCOR2(CNVCOR, METHOD, TRANSX, SCRATCHX, TRANSY,
      SCRATCHY, MX, NX, X, LDX, MY, NY, MPRE, NPRE, Y, LDY, MZ, NZ, Z,
      LDZ, WORK, LWORK)

CHARACTER * 1  CNVCOR,  METHOD,  TRANSX,  SCRATCHX,  TRANSY,
SCRATCHY
COMPLEX X(LDX,*), Y(LDY,*), Z(LDZ,*), WORK(*)
INTEGER MX, NX, LDX, MY, NY, MPRE, NPRE, LDY, MZ,  NZ,  LDZ,
LWORK

SUBROUTINE CCNVCOR2_64(CNVCOR, METHOD, TRANSX, SCRATCHX, TRANSY,
      SCRATCHY, MX, NX, X, LDX, MY, NY, MPRE, NPRE, Y, LDY, MZ, NZ, Z,
      LDZ, WORK, LWORK)

CHARACTER * 1  CNVCOR,  METHOD,  TRANSX,  SCRATCHX,  TRANSY,
SCRATCHY
COMPLEX X(LDX,*), Y(LDY,*), Z(LDZ,*), WORK(*)
INTEGER*8 MX, NX, LDX, MY, NY, MPRE, NPRE, LDY, MZ, NZ, LDZ,
LWORK
```

F95 INTERFACE

```
SUBROUTINE CNVCOR2(CNVCOR, METHOD, TRANSX, SCRATCHX, TRANSY,
       SCRATCHY, [MX], [NX], X, [LDX], [MY], [NY], MPRE, NPRE, Y, [LDY],
       [MZ], [NZ], Z, [LDZ], WORK, [LWORK])

CHARACTER(LEN=1)  ::  CNVCOR,  METHOD,   TRANSX,   SCRATCHX,
TRANSY, SCRATCHY
```

```
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: X, Y, Z
INTEGER :: MX, NX, LDX, MY, NY, MPRE,  NPRE,  LDY,  MZ,  NZ,
LDZ, LWORK

SUBROUTINE CNVCOR2_64(CNVCOR, METHOD, TRANSX, SCRATCHX, TRANSY,
       SCRATCHY, [MX], [NX], X, [LDX], [MY], [NY], MPRE, NPRE, Y, [LDY],
       [MZ], [NZ], Z, [LDZ], WORK, [LWORK])

CHARACTER(LEN=1)  ::  CNVCOR,  METHOD,   TRANSX,   SCRATCHX,
TRANSY, SCRATCHY
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: X, Y, Z
INTEGER(8) :: MX, NX, LDX, MY, NY, MPRE, NPRE, LDY, MZ,  NZ,
LDZ, LWORK
```

C INTERFACE

```
#include <sunperf.h>

void ccnvcor2(char cnvcor, char method,  char  transx,  char
          scratchx,  char transy, char scratchy, int mx, int
          nx, complex *x, int ldx, int my, int ny, int mpre,
          int  npre,  complex  *y,  int ldy, int mz, int nz,
          complex *z, int ldz, complex *work, int lwork);

void ccnvcor2_64(char cnvcor, char method, char transx, char
          scratchx,  char  transy,  char  scratchy, long mx,
          long nx, complex *x, long ldx, long my,  long  ny,
          long  mpre,  long npre, complex *y, long ldy, long
          mz, long nz, complex *z, long ldz, complex  *work,
          long lwork);
```

PURPOSE

ccnvcor2 computes the convolution or correlation of complex matrices.

ARGUMENTS

CNVCOR (input)
'V' or 'v' to compute convolution, 'R' or 'r' to compute correlation.

METHOD (input)
'T' or 't' if the Fourier transform method is to be used, 'D' or 'd' to compute directly from the definition.

```
TRANSX (input)
          'N' or 'n' if X is the filter matrix, 'T'  or  't'
          if transpose(X) is the filter matrix.

SCRATCHX (input)
          'N' or 'n' if X must be preserved, 'S' or 's' if X
          can  be  used as scratch space.  The contents of X
          are undefined after returning from a call in which
          X is allowed to be used for scratch.

TRANSY (input)
          'N' or 'n' if Y is the input matrix, 'T' or 't' if
          transpose(Y) is the input matrix.

SCRATCHY (input)
          'N' or 'n' if Y must be preserved, 'S' or 's' if Y
          can  be  used as scratch space.  The contents of Y
          are undefined after returning from a call in which
          Y is allowed to be used for scratch.

MX (input)
          Number of rows in the filter matrix.  MX >= 0.

NX (input)
          Number of columns in the filter matrix.  NX >= 0.

X (input)
          On entry, the filter matrix.  Unchanged on exit if
          SCRATCHX  is  'N'  or  'n',  undefined  on exit if
          SCRATCHX is 'S' or 's'.

LDX (input)
          Leading dimension of the array that  contains  the
          filter matrix.

MY (input)
          Number of rows in the input matrix.  MY >= 0.

NY (input)
          Number of columns in the input matrix.  NY >= 0.

MPRE (input)
          Number of implicit zeros to prepend to each row of
          the input matrix.  MPRE >= 0.

NPRE (input)
          Number of implicit zeros to prepend to each column
          of the input matrix.  NPRE >= 0.

Y (input)
```

Input matrix. Unchanged on exit if SCRATCHY is 'N' or 'n', undefined on exit if SCRATCHY is 'S' or 's'.

LDY (input)
Leading dimension of the array that contains the input matrix.

MZ (input)
Number of rows in the output matrix. MZ >= 0. CCNVCOR2 will return immediately if MZ = 0.

NZ (input)
Number of columns in the output matrix. NZ >= 0. CCNVCOR2 will return immediately if NZ = 0.

Z (output)
dimension(LDZ,*)
Result matrix.

LDZ (input)
Leading dimension of the array that contains the result matrix. LDZ >= MAX(1,MZ).

WORK (input/output)
(input/scratch) dimension(LWORK)
On entry for the first call to CCNVCOR2, WORK(1) must contain CMPLX(0.0,0.0). After the first call, WORK(1) must be set to CMPLX(0.0,0.0) iff WORK has been altered since the last call to this subroutine or if the sizes of the arrays have changed.

LWORK (input)
Length of the work vector. The upper bound of the workspace length requirement is 2 * (MYC + NYC) + 15, where MYC = MAX(MAX(MX,NX), MAX(MY,NY)+NPRE) and NYC = MAX(MAX(MX,NX), MAX(MY,NY)+MPRE). If LWORK indicates a workspace that is too small, the routine will allocate its own workspace. If the FFT is not used, the value of LWORK is unimportant.

Contents

NAME

ccoomm - coordinate matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE CCOOMM( TRANSA, M, N, K, ALPHA, DESCRA,
*            VAL, INDX, JNDX, NNZ,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, K, DESCRA(5), NNZ
*            LDB, LDC, LWORK
 INTEGER    INDX(NNZ), JNDX(NNZ)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CCOOMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*            VAL, INDX, JNDX, NNZ,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, K, DESCRA(5), NNZ
*            LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), JNDX(NNZ)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE COOMM( TRANSA, M, [N], K, ALPHA, DESCRA,
*            VAL, INDX, JNDX, NNZ, B, [LDB], BETA, C, [LDC],
*            [WORK], [LWORK] )
 INTEGER TRANSA, M, K,  NNZ
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, JNDX
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL
 COMPLEX, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE COOMM_64( TRANSA, M, [N], K, ALPHA, DESCRA,
*            VAL, INDX, JNDX, NNZ, B, [LDB], BETA, C, [LDC],
*            [WORK], [LWORK] )
 INTEGER*8 TRANSA, M, K,  NNZ
 INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, JNDX
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL
 COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in coordinate format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
```

```
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

 VAL()          scalar array of length NNZ consisting of the
                non-zero entries of A, in any order.

 INDX()         integer array of length NNZ consisting of the
                corresponding row indices of the entries of A.

 JNDX()         integer array of length NNZ consisting of the
                corresponding column indices of the entries of A.

 NNZ            number of non-zero elements in A.
 B()            rectangular array with first dimension LDB.

 LDB            leading dimension of B

 BETA           Scalar parameter

 C()            rectangular array with first dimension LDC.

 LDC            leading dimension of C

 WORK()         scratch array of length LWORK. WORK is not
                referenced in the current version.

 LWORK          length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS)
Standard", University of Tennessee, Knoxville, Tennessee,
1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

```
ccopy - Copy x to y
```

SYNOPSIS

```
SUBROUTINE CCOPY(N, X, INCX, Y, INCY)

COMPLEX X(*), Y(*)
INTEGER N, INCX, INCY

SUBROUTINE CCOPY_64(N, X, INCX, Y, INCY)

COMPLEX X(*), Y(*)
INTEGER*8 N, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE COPY([N], X, [INCX], Y, [INCY])

COMPLEX, DIMENSION(:) :: X, Y
INTEGER :: N, INCX, INCY

SUBROUTINE COPY_64([N], X, [INCX], Y, [INCY])

COMPLEX, DIMENSION(:) :: X, Y
INTEGER(8) :: N, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>

void ccopy(int n, complex *x,  int  incx,  complex  *y,  int
          incy);

void ccopy_64(long n, complex *x,  long  incx,  complex  *y,
```

```
          long incy);
```

PURPOSE

ccopy Copy x to y where x and y are n-vectors.

ARGUMENTS

```
N (input)
          On entry, N specifies the number  of  elements  in
          the  vector.   N must be at least one for the sub-
          routine to have any visible effect.  Unchanged  on
          exit.
X (input)
          of DIMENSION at least ( 1 + ( n - 1 )*abs( INCX  )
          ).   Before  entry,  the  incremented array X must
          contain the vector x.  Unchanged on exit.

INCX (input)
          On entry, INCX specifies  the  increment  for  the
          elements  of X.  INCX must not be zero.  Unchanged
          on exit.

Y (output)
          of DIMENSION at least ( 1 + ( m - 1 )*abs( INCY  )
          ).  On entry, the incremented array Y must contain
          the vector y. On exit, Y  is  overwritten  by  the
          vector x.

INCY (input)
          On entry, INCY specifies  the  increment  for  the
          elements  of Y.  INCY must not be zero.  Unchanged
          on exit.
```

Contents

NAME

```
ccscmm - compressed sparse column format matrix-matrix
multiply
```

SYNOPSIS

```
 SUBROUTINE CCSCMM( TRANSA, M, N, K, ALPHA, DESCRA,
*            VAL, INDX, PNTRB, PNTRE,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, K, DESCRA(5),
*            LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTRB(K), PNTRE(K)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CCSCMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*            VAL, INDX, PNTRB, PNTRE,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, K, DESCRA(5),
*            LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTRB(K), PNTRE(K)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTRE(K)-PNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE CSCMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER TRANSA, M, K
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL
```

```
 COMPLEX, DIMENSION(:, :) ::  B, C

 SUBROUTINE CSCMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER*8 TRANSA, M, K
 INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL
 COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in compressed sparse column format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                    ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
```

```
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices

 VAL()         scalar array of length NNZ consisting of nonzero
               entries of A.

 INDX()        integer array of length NNZ consisting of the row
               indices of nonzero entries of A.

 PNTRB()       integer array of length K such that PNTRB(J)-PNTRB(1)+1
               points to location in VAL of the first nonzero element
               in column J.
 PNTRE()       integer array of length K such that PNTRE(J)-PNTRB(1)
               points to location in VAL of the last nonzero element
               in column J.

 B()           rectangular array with first dimension LDB.

 LDB           leading dimension of B

 BETA          Scalar parameter

 C()           rectangular array with first dimension LDC.

 LDC           leading dimension of C

 WORK()        scratch array of length LWORK. WORK is not
               referenced in the current version.

 LWORK         length of WORK array. LWORK is not referenced
               in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS)
Standard", University of Tennessee, Knoxville, Tennessee,

1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

It is known that there exists another representation of the compressed sparse column format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each column in the arrays VAL and INDX is used instead of two arrays PNTRB and PNTRE. To use the routine with this kind of sparse column format the following calling sequence should be used

```
  SUBROUTINE SCSCMM( TRANSA, M, N, K, ALPHA, DESCRA,
 *           VAL, INDX, IA, IA(2), B, LDB, BETA,
 *           C, LDC, WORK, LWORK )
```

Contents

NAME

```
ccscsm - compressed sparse column format triangular solve
```

SYNOPSIS

```
 SUBROUTINE CCSCSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTRB(M), PNTRE(M)
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CCSCSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTRB(M), PNTRE(M)
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTRE(M)-PNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE CSCSM( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER TRANSA, M, UNITD
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL, DV
 COMPLEX, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE CSCSM_64(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8 TRANSA, M, UNITD
 INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL, DV
 COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a diagonal scaling matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in compressed sparse column
format and op( A )  is one  of
op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic column scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                  0 : general
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))

                Note: For the routine, DESCRA(1)=3 is only supported.

                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices
  VAL()         scalar array of length NNZ consisting of nonzero entries
                of A.

  INDX()        integer array of length NNZ consisting of the row indices
                of nonzero entries of A. (Row indices MUST be sorted in
                increasing order for each column).

  PNTRB()       integer array of length M such that PNTRB(J)-PNTRB(1)+1
                points to location in VAL of the first nonzero element
                in column J.

  PNTRE()       integer array of length M such that PNTRE(J)-PNTRB(1)
                points to location in VAL of the last nonzero element
                in column J.

  B()           rectangular array with first dimension LDB.

  LDB           leading dimension of B

  BETA          Scalar parameter

  C()           rectangular array with first dimension LDC.

  LDC           leading dimension of C

  WORK()        scratch array of length LWORK.

                On exit,  if LWORK = -1, WORK(1) returns the optimum  LWORK.
```

LWORK length of WORK array. LWORK should be at least M.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the columns of A such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the columns have been scaled. UNITD=3 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the column number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the CSC representation

of a sparse matrix. They are not used anyway in these cases.
But if UNITD=4, the unit diagonal elements MUST be
referenced in the CSC representation.

4. The routine can be applied for solving triangular systems
when the upper or lower triangle of the general sparse
matrix A is used. However *DESCRA*(1) must be equal to 3 in
this case.

5. It is known that there exists another representation of
the compressed sparse column format (see for example Y.Saad,
"Iterative Methods for Sparse Linear Systems", WPS, 1996).
Its data structure consists of three array instead of the
four used in the current implementation. The main
difference is that only one array, IA, containing the
pointers to the beginning of each column in the arrays VAL
and INDX is used instead of two arrays PNTRB and PNTRE. To
use the routine with this kind of sparse column format the
following calling sequence should be used

```
  SUBROUTINE SCSCSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, INDX, IA, IA(2), B, LDB, BETA,
 *           C, LDC, WORK, LWORK )
```

Contents

NAME

ccsrmm - compressed sparse row format matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE CCSRMM( TRANSA, M, N, K, ALPHA, DESCRA,
*            VAL, INDX, PNTRB, PNTRE,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, K, DESCRA(5),
*            LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTRB(M), PNTRE(M)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CCSRMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*            VAL, INDX, PNTRB, PNTRE,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, K, DESCRA(5),
*            LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTRB(M), PNTRE(M)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTRE(M)-PNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE CSRMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER TRANSA, M, K
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL
 COMPLEX, DIMENSION(:, :) ::  B, C
```

```
       SUBROUTINE CSRMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
      *   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
       INTEGER*8 TRANSA, M, K
       INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
       COMPLEX    ALPHA, BETA
       COMPLEX, DIMENSION(:) :: VAL
       COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
         C <- alpha op(A) B + beta C

 where ALPHA and BETA are scalar, C and B are dense matrices,
 A is a matrix represented in compressed sparse row format and
 op( A )  is one  of
 op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                    ( ' indicates matrix transpose)
```

ARGUMENTS

```
 TRANSA        Indicates how to operate with the sparse matrix
                 0 : operate with matrix
                 1 : operate with transpose matrix
                 2 : operate with the conjugate transpose of matrix.
                     2 is equivalent to 1 if matrix is real.

 M             Number of rows in matrix A

 N             Number of columns in matrix C

 K             Number of columns in matrix A

 ALPHA         Scalar parameter

 DESCRA()      Descriptor argument.  Five element integer array
               DESCRA(1) matrix structure
                 0 : general
                 1 : symmetric (A=A')
                 2 : Hermitian (A= CONJG(A'))
                 3 : Triangular
                 4 : Skew(Anti)-Symmetric (A=-A')
                 5 : Diagonal
                 6 : Skew-Hermitian (A= -CONJG(A'))
               DESCRA(2) upper/lower triangular indicator
                 1 : lower
                 2 : upper
```

```
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

 VAL()          scalar array of length NNZ consisting of nonzero
                entries of A.

 INDX()         integer array of length NNZ consisting of the
                column indices of nonzero entries of A.

 PNTRB()        integer array of length M such that PNTRB(J)-PNTRB(1)+1
                points to location in VAL of the first nonzero element
                in row J.
 PNTRE()        integer array of length M such that PNTRE(J)-PNTRB(1)
                points to location in VAL of the last nonzero element
                in row J.

 B()            rectangular array with first dimension LDB.

 LDB            leading dimension of B

 BETA           Scalar parameter

 C()            rectangular array with first dimension LDC.

 LDC            leading dimension of C

 WORK()         scratch array of length LWORK. WORK is not
                referenced in the current version.

 LWORK          length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS)
Standard", University of Tennessee, Knoxville, Tennessee,
1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

It is known that there exists another representation of the compressed sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each row in the arrays VAL and INDX is used instead of two arrays PNTRB and PNTRE. To use the routine with this kind of compressed sparse row format the following calling sequence should be used

```
  SUBROUTINE SCSRMM( TRANSA, M, N, K, ALPHA, DESCRA,
 *           VAL, INDX, IA, IA(2), B, LDB, BETA,
 *           C, LDC, WORK, LWORK )
```

Contents

NAME

```
ccsrsm - compressed sparse row format triangular solve
```

SYNOPSIS

```
 SUBROUTINE CCSRSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTRB(M), PNTRE(M)
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CCSRSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTRB(M), PNTRE(M)
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTRE(M)-PNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE CSRSM( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER TRANSA, M, UNITD
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL, DV
 COMPLEX, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE CSRSM_64(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8 TRANSA, M, UNITD
 INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL, DV
 COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a diagonal scaling matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in compressed sparse row
format and op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of
              the scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                  0 : general
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))

                Note: For the routine, only DESCRA(1)=3 is supported.

                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices
VAL()           scalar array of length NNZ consisting of nonzero entries
                of A.

INDX()          integer array of length NNZ consisting of the column
                indices of nonzero entries of A (column indices MUST be
                sorted in increasing order for each row)

PNTRB()         integer array of length M such that PNTRB(J)-PNTRB(1)+1
                points to location in VAL of the first nonzero element
                in row J.

PNTRE()         integer array of length M such that PNTRE(J)-PNTRB(1)
                points to location in VAL of the last nonzero element
                in row J.

B()             rectangular array with first dimension LDB.

LDB             leading dimension of B

BETA            Scalar parameter

C()             rectangular array with first dimension LDC.

LDC             leading dimension of C

WORK()          scratch array of length LWORK.

                On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.
```

LWORK length of WORK array. LWORK should be at least M.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the rows of A such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows have been scaled. UNITD=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the CSR representation of a sparse matrix. They are not used anyway in these cases.

But if UNITD=4, the unit diagonal elements MUST be referenced in the CSR representation.

4. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

5. It is known that there exists another representation of the compressed sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each row in the arrays VAL and INDX is used instead of two arrays PNTRB and PNTRE. To use the routine with this kind of compressed sparse row format the following calling sequence should be used

```
  SUBROUTINE SCSRSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, INDX, IA, IA(2), B, LDB, BETA, C,
 *            LDC, WORK, LWORK )
```

Contents

NAME

```
cdiamm - diagonal format matrix-matrix multiply
```

SYNOPSIS

```
 SUBROUTINE CDIAMM( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, LDA, IDIAG, NDIAG,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, K, DESCRA(5), LDA, NDIAG,
*           LDB, LDC, LWORK
 INTEGER    IDIAG(NDIAG)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(LDA,NDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CDIAMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, LDA, IDIAG, NDIAG,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, K, DESCRA(5), LDA, NDIAG,
*           LDB, LDC, LWORK
 INTEGER*8  IDIAG(NDIAG)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(LDA,NDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
  SUBROUTINE DIAMM(TRANSA, M, [N], K, ALPHA, DESCRA, VAL, [LDA],
*    IDIAG, NDIAG, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, K,  NDIAG
 INTEGER, DIMENSION(:) ::    DESCRA, IDIAG
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:, :) ::  VAL, B, C

  SUBROUTINE DIAMM_64(TRANSA, M, [N], K, ALPHA, DESCRA, VAL, [LDA],
*    IDIAG, NDIAG, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
```

```
 INTEGER*8    TRANSA, M, K,  NDIAG
 INTEGER*8, DIMENSION(:) ::    DESCRA, IDIAG
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:, :) ::  VAL, B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in diagonal format and op( A ) is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)

TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
                0 : non-unit
                1 : unit
              DESCRA(4) Array base  (NOT IMPLEMENTED)
                0 : C/C++ compatible
                1 : Fortran compatible
              DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                0 : unknown
                1 : no repeated indices
```

VAL()	two-dimensional LDA-by-NDIAG array such that VAL(:,I) consists of non-zero elements on diagonal IDIAG(I) of A. Diagonals in the lower triangular part of A are padded from the top, and those in the upper triangular part are padded from the bottom.
LDA	leading dimension of VAL, must be .GE. MIN(M,K)
IDIAG()	integer array of length NDIAG consisting of the corresponding diagonal offsets of the non-zero diagonals of A in VAL. Lower triangular diagonals have negative offsets, the main diagonal has offset 0, and upper triangular diagonals have positive offset.
NDIAG	number of non-zero diagonals in A.
B()	rectangular array with first dimension LDB.
LDB	leading dimension of B
BETA	Scalar parameter
C()	rectangular array with first dimension LDC.
LDC	leading dimension of C
WORK()	scratch array of length LWORK. WORK is not referenced in the current version.
LWORK	length of WORK array. LWORK is not referenced in the current version.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

```
cdiasm - diagonal format triangular solve
```

SYNOPSIS

```
 SUBROUTINE CDIASM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, LDA, IDIAG, NDIAG,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5), LDA, NDIAG,
*           LDB, LDC, LWORK
 INTEGER    IDIAG(NDIAG)
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(M), VAL(LDA,NDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CDIASM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, LDA, IDIAG, NDIAG,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5), LDA, NDIAG,
*           LDB, LDC, LWORK
 INTEGER*8  IDIAG(NDIAG)
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(M), VAL(LDA,NDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
  SUBROUTINE DIASM(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
*   [LDA], IDIAG, NDIAG, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, NDIAG
 INTEGER, DIMENSION(:) ::    DESCRA, IDIAG
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) ::  DV
 COMPLEX, DIMENSION(:, :) ::  VAL, B, C

  SUBROUTINE DIASM_64(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
```

```
*   [LDA], IDIAG, NDIAG, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, NDIAG
 INTEGER*8, DIMENSION(:) ::    DESCRA, IDIAG
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) ::  DV
 COMPLEX, DIMENSION(:, :) ::  VAL, B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C
```

where ALPHA and BETA are scalar, C and B are m by n dense matrices, D is a diagonal scaling matrix, A is a unit, or non-unit, upper or lower triangular matrix represented in diagonal format and op(A) is one of

```
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
```

```
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))
                Note: For the routine, only DESCRA(1)=3 is supported.

                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices
VAL()           two-dimensional LDA-by-NDIAG array such that VAL(:,I)
                consists of non-zero elements on diagonal IDIAG(I)
                of A.  Diagonals in the lower triangular part of A
                are padded from the top, and those in the upper
                triangular part are padded from the bottom.

LDA             leading dimension of VAL, must be .GE. MIN(M,K)

IDIAG()         integer array of length NDIAG consisting of the
                corresponding diagonal offsets of the non-zero
                diagonals of A in VAL.  Lower triangular diagonals
                have negative offsets, the main diagonal has offset
                0, and upper triangular diagonals have positive offset.
                Elements of IDIAG of MUST be sorted in  increasing order.

NDIAG           number of non-zero diagonals in A.

B()             rectangular array with first dimension LDB.

LDB             leading dimension of B

BETA            Scalar parameter

C()             rectangular array with first dimension LDC.

LDC             leading dimension of C

WORK()          scratch array of length LWORK.
                On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.
```

LWORK length of WORK array. LWORK should be at least M.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/
"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the rows of A such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows have been scaled. UNITD=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the DIA representation of a sparse matrix. They are not used anyway in these cases. But if UNITD=4, the unit diagonal elements MUST be referenced in the DIA representation.

4. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

```
cdotc - compute the dot product of two vectors conjg(x)  and
y.
```

SYNOPSIS

```
COMPLEX FUNCTION CDOTC(N, X, INCX, Y, INCY)

COMPLEX X(*), Y(*)
INTEGER N, INCX, INCY

COMPLEX FUNCTION CDOTC_64(N, X, INCX, Y, INCY)

COMPLEX X(*), Y(*)
INTEGER*8 N, INCX, INCY
```

F95 INTERFACE

```
COMPLEX FUNCTION DOTC([N], X, [INCX], Y, [INCY])

COMPLEX, DIMENSION(:) :: X, Y
INTEGER :: N, INCX, INCY

COMPLEX FUNCTION DOTC_64([N], X, [INCX], Y, [INCY])

COMPLEX, DIMENSION(:) :: X, Y
INTEGER(8) :: N, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>

complex cdotc(int n, complex *x, int incx, complex  *y,  int
          incy);
```

```
complex cdotc_64(long n, complex *x, long incx, complex  *y,
          long incy);
```

PURPOSE

cdotc compute the dot product of conjg(x) and y where x and y are n-vectors.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. If N is not positive then the function returns the value 0.0. Unchanged on exit.

X (input)
of DIMENSION at least (1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input)
of DIMENSION at least (1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. Unchanged on exit.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
cdotci - Compute the complex conjugated indexed dot product.
```

SYNOPSIS

```
 COMPLEX FUNCTION CDOTCI(NZ, X, INDX, Y)

 COMPLEX X(*), Y(*)
 INTEGER NZ
 INTEGER INDX(*)

 COMPLEX FUNCTION CDOTCI_64(NZ, X, INDX, Y)

 COMPLEX X(*), Y(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 COMPLEX FUNCTION DOTCI([NZ], X, INDX, Y)

 COMPLEX, DIMENSION(:) :: X, Y
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 COMPLEX FUNCTION DOTCI_64([NZ], X, INDX, Y)

 COMPLEX, DIMENSION(:) :: X, Y
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
CDOTCI Compute the complex conjugated indexed dot product of
a complex sparse vector x stored in compressed form with a
```

complex vector y in full storage form.

```
 dot = 0
 do i = 1, n
   dot = dot + conjg(x(i)) * y(indx(i))
 enddo
```

ARGUMENTS

NZ (input)
Number of elements in the compressed form.
Unchanged on exit.

X (input)
Vector in compressed form. Unchanged on exit.

INDX (input)
Vector containing the indices of the compressed form. It is assumed that the elements in INDX are distinct and greater than zero. Unchanged on exit.

Y (input)
Vector in full storage form. Only the elements corresponding to the indices in INDX will be accessed.

Contents

NAME

```
cdotu - compute the dot product of two vectors x and y.
```

SYNOPSIS

```
COMPLEX FUNCTION CDOTU(N, X, INCX, Y, INCY)

COMPLEX X(*), Y(*)
INTEGER N, INCX, INCY

COMPLEX FUNCTION CDOTU_64(N, X, INCX, Y, INCY)

COMPLEX X(*), Y(*)
INTEGER*8 N, INCX, INCY
```

F95 INTERFACE

```
COMPLEX FUNCTION DOT([N], X, [INCX], Y, [INCY])

COMPLEX, DIMENSION(:) :: X, Y
INTEGER :: N, INCX, INCY

COMPLEX FUNCTION DOT_64([N], X, [INCX], Y, [INCY])

COMPLEX, DIMENSION(:) :: X, Y
INTEGER(8) :: N, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>

complex cdotu(int n, complex *x, int incx, complex  *y,  int
          incy);

complex cdotu_64(long n, complex *x, long incx, complex  *y,
```

```
            long incy);
```

PURPOSE

cdotu compute the dot product of x and y where x and y are n-vectors.

ARGUMENTS

N (input)
: On entry, N specifies the number of elements in the vector. If N is not positive then the function returns the value 0.0. Unchanged on exit.

X (input)
: of DIMENSION at least (1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
: On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input)
: of DIMENSION at least (1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. Unchanged on exit.

INCY (input)
: On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
cdotui - Compute the complex unconjugated indexed dot
product.
```

SYNOPSIS

```
 COMPLEX FUNCTION CDOTCI(NZ, X, INDX, Y)

 COMPLEX X(*), Y(*)
 INTEGER NZ
 INTEGER INDX(*)

 COMPLEX FUNCTION CDOTCI_64(NZ, X, INDX, Y)

 COMPLEX X(*), Y(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 COMPLEX FUNCTION DOTCI([NZ], X, INDX, Y)

 COMPLEX, DIMENSION(:) :: X, Y
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 COMPLEX FUNCTION DOTCI_64([NZ], X, INDX, Y)

 COMPLEX, DIMENSION(:) :: X, Y
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
CDOTUI Compute the complex unconjugated indexed dot product
```

of a complex sparse vector x stored in compressed form with
a complex vector y in full storage form.

```
dot = 0
do i = 1, n
  dot = dot + x(i) * y(indx(i))
enddo
```

ARGUMENTS

NZ (input)
Number of elements in the compressed form.
Unchanged on exit.

X (input)
Vector in compressed form. Unchanged on exit.

INDX (input)
Vector containing the indices of the compressed form. It is assumed that the elements in INDX are distinct and greater than zero. Unchanged on exit.

Y (input)
Vector in full storage form. Only the elements corresponding to the indices in INDX will be accessed.

Contents

NAME

cellmm - Ellpack format matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE CELLMM( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, LDA, MAXNZ,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, K, DESCRA(5), LDA, MAXNZ,
*           LDB, LDC, LWORK
 INTEGER    INDX(LDA,MAXNZ)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(LDA,MAXNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CELLMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, LDA, MAXNZ,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, K, DESCRA(5), LDA, MAXNZ,
*           LDB, LDC, LWORK
 INTEGER*8  INDX(LDA,MAXNZ)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(LDA,MAXNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE ELLMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*        [LDA], MAXNZ, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, K, MAXNZ
 INTEGER, DIMENSION(:) ::  DESCRA
 INTEGER, DIMENSION(:, :) ::    INDX
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:, :) ::  VAL, B, C

 SUBROUTINE ELLMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
```

```
*        [LDA], MAXNZ, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, K, MAXNZ
 INTEGER*8, DIMENSION(:) ::  DESCRA
 INTEGER*8, DIMENSION(:, :) ::    INDX
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:, :) ::  VAL, B, C
```

DESCRIPTION

```
where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in  Ellpack format format and
op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
                0 : non-unit
                1 : unit
```

```
                 DESCRA(4) Array base  (NOT IMPLEMENTED)
                   0 : C/C++ compatible
                   1 : Fortran compatible
                 DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                   0 : unknown
                   1 : no repeated indices

 VAL()           two-dimensional LDA-by-MAXNZ array such that VAL(I,:)
                 consists of non-zero elements in row I of A, padded by
                 zero values if the row contains less than MAXNZ.

 INDX()          two-dimensional integer LDA-by-MAXNZ array such
                 INDX(I,:) consists of the column indices of the
                 nonzero elements in row I, padded by the integer
                 value I if the number of nonzeros is less than MAXNZ.

 LDA             leading dimension of VAL and INDX.

 MAXNZ           max number of nonzeros elements per row.

 B()             rectangular array with first dimension LDB.
 LDB             leading dimension of B

 BETA            Scalar parameter

 C()             rectangular array with first dimension LDC.

 LDC             leading dimension of C

 WORK()          scratch array of length LWORK. WORK is not
                 referenced in the current version.

 LWORK           length of WORK array. LWORK is not referenced
                 in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS)
Standard", University of Tennessee, Knoxville, Tennessee,
1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

```
cellsm - Ellpack format triangular solve
```

SYNOPSIS

```
  SUBROUTINE CELLSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, INDX, LDA, MAXNZ,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
  INTEGER    TRANSA, M, N, UNITD, DESCRA(5), LDA, MAXNZ,
 *           LDB, LDC, LWORK
  INTEGER    INDX(LDA,MAXNZ)
  COMPLEX    ALPHA, BETA
  COMPLEX    DV(M), VAL(LDA,MAXNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

  SUBROUTINE CELLSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, INDX, LDA, MAXNZ,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
  INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5), LDA, MAXNZ,
 *           LDB, LDC, LWORK
  INTEGER*8  INDX(LDA,MAXNZ)
  COMPLEX    ALPHA, BETA
  COMPLEX    DV(M), VAL(LDA,MAXNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
  SUBROUTINE ELLSM( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
 *    INDX, [LDA], MAXNZ, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
  INTEGER    TRANSA, M,  MAXNZ
  INTEGER, DIMENSION(:) ::  DESCRA
  INTEGER, DIMENSION(:, :) ::    INDX
  COMPLEX    ALPHA, BETA
  COMPLEX, DIMENSION(:) ::  DV
  COMPLEX, DIMENSION(:, :) ::  VAL, B, C
```

```
 SUBROUTINE ELLSM_64( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
*    INDX, [LDA], MAXNZ, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M,  MAXNZ
 INTEGER*8, DIMENSION(:) ::  DESCRA
 INTEGER*8, DIMENSION(:, :) ::    INDX
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) ::  DV
 COMPLEX, DIMENSION(:, :) ::  VAL, B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a diagonal scaling matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in Ellpack format and
op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                    0 : general
                    1 : symmetric (A=A')
                    2 : Hermitian (A= CONJG(A'))
                    3 : Triangular
                    4 : Skew(Anti)-Symmetric (A=-A')
                    5 : Diagonal
                    6 : Skew-Hermitian (A= -CONJG(A'))

                  Note: For the routine, only DESCRA(1)=3 is supported.

                  DESCRA(2) upper/lower triangular indicator
                    1 : lower
                    2 : upper
                  DESCRA(3) main diagonal type
                    0 : non-unit
                    1 : unit
                  DESCRA(4) Array base  (NOT IMPLEMENTED)
                    0 : C/C++ compatible
                    1 : Fortran compatible
                  DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                    0 : unknown
                    1 : no repeated indices
    VAL()         two-dimensional LDA-by-MAXNZ array such that VAL(I,:)
                  consists of non-zero elements in row I of A, padded by
                  zero values if the row contains less than MAXNZ.

    INDX()        two-dimensional integer LDA-by-MAXNZ array such
                  INDX(I,:) consists of the column indices of the
                  nonzero elements in row I, padded by the integer
                  value I if the number of nonzeros is less than MAXNZ.
                  The column indices MUST be sorted in increasing order
                  for each row.

    LDA           leading dimension of VAL and INDX.

    MAXNZ         max number of nonzeros elements per row.

    B()           rectangular array with first dimension LDB.

    LDB           leading dimension of B

    BETA          Scalar parameter

    C()           rectangular array with first dimension LDC.

    LDC           leading dimension of C

    WORK()        scratch array of length LWORK.

                  On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.
```

LWORK   length of WORK array. LWORK should be at least M.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the rows of A such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows have been scaled. UNITD=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the ELL representation of a sparse matrix. They are not used anyway in these cases.

But if UNITD=4, the unit diagonal elements MUST be referenced in the ELL representation.

4. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

```
cfft2b - compute a periodic sequence from its Fourier  coef-
ficients.   The  xFFT operations are unnormalized, so a call
of xFFT2F followed by a call of  xFFT2B  will  multiply  the
input sequence by M*N.
```

SYNOPSIS

```
SUBROUTINE CFFT2B(M, N, A, LDA, WORK, LWORK)

COMPLEX A(LDA,*)
INTEGER M, N, LDA, LWORK
REAL WORK(*)

SUBROUTINE CFFT2B_64(M, N, A, LDA, WORK, LWORK)

COMPLEX A(LDA,*)
INTEGER*8 M, N, LDA, LWORK
REAL WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT2B([M], [N], A, [LDA], WORK, LWORK)

COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, LWORK
REAL, DIMENSION(:) :: WORK

SUBROUTINE FFT2B_64([M], [N], A, [LDA], WORK, LWORK)

COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, LWORK
REAL, DIMENSION(:) :: WORK
```

C INTERFACE

```
#include <sunperf.h>

void cfft2b(int m, int n, complex *a, int lda, float  *work,
          int lwork);

void cfft2b_64(long m, long n, complex *a, long  lda,  float
          *work, long lwork);
```

ARGUMENTS

```
M (input) Number of rows to be transformed.   These  subrou-
          tines  are  most  efficient when M is a product of
          small primes.  M >= 0.
N (input) Number of columns to be transformed.   These  sub-
          routines are most efficient when N is a product of
          small primes.  N >= 0.

A (input/output)
          On entry, a two-dimensional array A(M,N) that con-
          tains the sequences to be transformed.

LDA (input)
          Leading dimension of the array containing the data
          to be transformed.  LDA >= M.

WORK (input)
          On input, workspace WORK must have  been  initial-
          ized by CFFT2I.

LWORK (input)
          The dimension of the array WORK.  LWORK >= (4 * (M
          + N) + 30)
```

Contents

NAME

```
cfft2f - compute the  Fourier  coefficients  of  a  periodic
sequence.   The  xFFT operations are unnormalized, so a call
of xFFT2F followed by a call of  xFFT2B  will  multiply  the
input sequence by M*N.
```

SYNOPSIS

```
SUBROUTINE CFFT2F(M, N, A, LDA, WORK, LWORK)

COMPLEX A(LDA,*)
INTEGER M, N, LDA, LWORK
REAL WORK(*)

SUBROUTINE CFFT2F_64(M, N, A, LDA, WORK, LWORK)

COMPLEX A(LDA,*)
INTEGER*8 M, N, LDA, LWORK
REAL WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT2F([M], [N], A, [LDA], WORK, LWORK)

COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, LWORK
REAL, DIMENSION(:) :: WORK

SUBROUTINE FFT2F_64([M], [N], A, [LDA], WORK, LWORK)

COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, LWORK
REAL, DIMENSION(:) :: WORK
```

C INTERFACE

```
#include <sunperf.h>

void cfft2f(int m, int n, complex *a, int lda, float  *work,
          int lwork);

void cfft2f_64(long m, long n, complex *a, long  lda,  float
          *work, long lwork);
```

ARGUMENTS

```
M (input) Number of rows to be transformed.   These  subrou-
          tines  are  most  efficient when M is a product of
          small primes.  M >= 0.
N (input) Number of columns to be transformed.   These  sub-
          routines are most efficient when N is a product of
          small primes.  N >= 0.

A (input/output)
          On entry, a two-dimensional array A(M,N) that con-
          tains the sequences to be transformed.

LDA (input)
          Leading dimension of the array containing the data
          to be transformed.  LDA >= M.

WORK (input)
          On input, workspace WORK must have  been  initial-
          ized by CFFT2I.

LWORK (input)
          The dimension of the array WORK.  LWORK >= (4 * (M
          + N) + 30)
```

Contents

NAME

```
cfft2i - initialize the array WSAVE, which is used  in  both
the forward and backward transforms.
```

SYNOPSIS

```
SUBROUTINE CFFT2I(M, N, WORK)

INTEGER M, N
REAL WORK(*)

SUBROUTINE CFFT2I_64(M, N, WORK)

INTEGER*8 M, N
REAL WORK(*)
```

F95 INTERFACE

```
SUBROUTINE CFFT2I(M, N, WORK)

INTEGER :: M, N
REAL, DIMENSION(:) :: WORK

SUBROUTINE CFFT2I_64(M, N, WORK)

INTEGER(8) :: M, N
REAL, DIMENSION(:) :: WORK
```

C INTERFACE

```
#include <sunperf.h>

void cfft2i(int m, int n, float *work);

void cfft2i_64(long m, long n, float *work);
```

ARGUMENTS

M (input) Number of rows to be transformed. M >= 0.

N (input) Number of columns to be transformed. N >= 0.

WORK (input/output)
On entry, an array of dimension (4 * (M + N) + 30) or greater. CFFT2I needs to be called only once to initialize array WORK before calling CFFT2F and/or CFFT2B if M, N and WORK remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
cfft3b - compute a periodic sequence from its Fourier  coef-
ficients.  The FFT operations are unnormalized, so a call of
CFFT3F followed by a call of CFFT3B will multiply the  input
sequence by M*N*K.
```

SYNOPSIS

```
SUBROUTINE CFFT3B(M, N, K, A, LDA, LD2A, WORK, LWORK)

COMPLEX A(LDA,LD2A,*)
INTEGER M, N, K, LDA, LD2A, LWORK
REAL WORK(*)

SUBROUTINE CFFT3B_64(M, N, K, A, LDA, LD2A, WORK, LWORK)

COMPLEX A(LDA,LD2A,*)
INTEGER*8 M, N, K, LDA, LD2A, LWORK
REAL WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT3B([M], [N], [K], A, [LDA], LD2A, WORK, LWORK)

COMPLEX, DIMENSION(:,:,:) :: A
INTEGER :: M, N, K, LDA, LD2A, LWORK
REAL, DIMENSION(:) :: WORK

SUBROUTINE FFT3B_64([M], [N], [K], A, [LDA], LD2A, WORK, LWORK)

COMPLEX, DIMENSION(:,:,:) :: A
INTEGER(8) :: M, N, K, LDA, LD2A, LWORK
REAL, DIMENSION(:) :: WORK
```

C INTERFACE

```
#include <sunperf.h>

void cfft3b(int m, int n, int k, complex *a,  int  lda,  int
          ld2a, float *work, int lwork);

void cfft3b_64(long m, long n, long k, complex *a, long lda,
          long ld2a, float *work, long lwork);
```

ARGUMENTS

```
M (input) Number of rows to be transformed.   These  subrou-
          tines  are  most  efficient when M is a product of
          small primes.  M >= 0.
N (input) Number of columns to be transformed.   These  sub-
          routines are most efficient when N is a product of
          small primes.  N >= 0.

K (input) Number of planes to be transformed.  These subrou-
          tines  are  most  efficient when K is a product of
          small primes.  K >= 0.

A (input/output)
          On entry, a three-dimensional array  A(LDA,LD2A,K)
          that contains the sequences to be transformed.

LDA (input)
          Leading dimension of the array containing the data
          to be transformed.  LDA >= M.

LD2A (input)
          Second dimension of the array containing the  data
          to be transformed.  LD2A >= N.

WORK (input)
          On input, workspace WORK must have  been  initial-
          ized by CFFT3I.

LWORK (input)
          The dimension of the array WORK.  LWORK >= (4*(M +
          N + K) + 45).
```

Contents

NAME

```
cfft3f - compute the  Fourier  coefficients  of  a  periodic
sequence.  The FFT operations are unnormalized, so a call of
CFFT3F followed by a call of CFFT3B will multiply the  input
sequence by M*N*K.
```

SYNOPSIS

```
SUBROUTINE CFFT3F(M, N, K, A, LDA, LD2A, WORK, LWORK)

COMPLEX A(LDA,LD2A,*)
INTEGER M, N, K, LDA, LD2A, LWORK
REAL WORK(*)

SUBROUTINE CFFT3F_64(M, N, K, A, LDA, LD2A, WORK, LWORK)

COMPLEX A(LDA,LD2A,*)
INTEGER*8 M, N, K, LDA, LD2A, LWORK
REAL WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT3F([M], [N], [K], A, [LDA], LD2A, WORK, LWORK)

COMPLEX, DIMENSION(:,:,:) :: A
INTEGER :: M, N, K, LDA, LD2A, LWORK
REAL, DIMENSION(:) :: WORK

SUBROUTINE FFT3F_64([M], [N], [K], A, [LDA], LD2A, WORK, LWORK)

COMPLEX, DIMENSION(:,:,:) :: A
INTEGER(8) :: M, N, K, LDA, LD2A, LWORK
REAL, DIMENSION(:) :: WORK
```

C INTERFACE

```
#include <sunperf.h>

void cfft3f(int m, int n, int k, complex *a,  int  lda,  int
          ld2a, float *work, int lwork);

void cfft3f_64(long m, long n, long k, complex *a, long lda,
          long ld2a, float *work, long lwork);
```

ARGUMENTS

M (input) Number of rows to be transformed. These subroutines are most efficient when M is a product of small primes. M >= 0.

N (input) Number of columns to be transformed. These subroutines are most efficient when N is a product of small primes. N >= 0.

K (input) Number of planes to be transformed. These subroutines are most efficient when K is a product of small primes. K >= 0.

A (input/output)
On entry, a three-dimensional array A(M,N,K) that contains the sequences to be transformed.

LDA (input)
Leading dimension of the array containing the data to be transformed. LDA >= M.

LD2A (input)
Second dimension of the array containing the data to be transformed. LD2A >= N.

WORK (input)
On input, workspace WORK must have been initialized by CFFT3I.

LWORK (input)
The dimension of the array WORK. LWORK >= (4*(M + N + K) + 45).

Contents

NAME

```
cfft3i - initialize the array WSAVE, which is used  in  both
CFFT3F and CFFT3B.
```

SYNOPSIS

```
SUBROUTINE CFFT3I(M, N, K, WORK)

INTEGER M, N, K
REAL WORK(*)

SUBROUTINE CFFT3I_64(M, N, K, WORK)

INTEGER*8 M, N, K
REAL WORK(*)
```

F95 INTERFACE

```
SUBROUTINE CFFT3I(M, N, K, WORK)

INTEGER :: M, N, K
REAL, DIMENSION(:) :: WORK

SUBROUTINE CFFT3I_64(M, N, K, WORK)

INTEGER(8) :: M, N, K
REAL, DIMENSION(:) :: WORK
```

C INTERFACE

```
#include <sunperf.h>

void cfft3i(int m, int n, int k, float *work);

void cfft3i_64(long m, long n, long k, float *work);
```

ARGUMENTS

M (input) Number of rows to be transformed. M >= 0.

N (input) Number of columns to be transformed. N >= 0.

K (input) Number of planes to be transformed. K >= 0.

WORK (input/output)
On entry, an array of dimension (4*(M + N + K) + 45) or greater. CFFT3I needs to be called only once to initialize array WORK before calling CFFT3F and/or CFFT3B if M, N, K and WORK remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
cfftb - compute a periodic sequence from its Fourier coeffi-
cients.   The  FFT operations are unnormalized, so a call of
CFFTF followed by a call of CFFTB will  multiply  the  input
sequence by N.
```

SYNOPSIS

```
SUBROUTINE CFFTB(N, X, WSAVE)

COMPLEX X(*)
INTEGER N
REAL WSAVE(*)

SUBROUTINE CFFTB_64(N, X, WSAVE)

COMPLEX X(*)
INTEGER*8 N
REAL WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE FFTB([N], X, WSAVE)

COMPLEX, DIMENSION(:) :: X
INTEGER :: N
REAL, DIMENSION(:) :: WSAVE

SUBROUTINE FFTB_64([N], X, WSAVE)

COMPLEX, DIMENSION(:) :: X
INTEGER(8) :: N
REAL, DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void cfftb(int n, complex *x, float *wsave);

void cfftb_64(long n, complex *x, float *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence to be  transformed.   These
          subroutines are most efficient when N is a product
          of small primes.  N >= 0.

X (input) On entry, an array  of  length  N  containing  the
          sequence to be transformed.

WSAVE (input/output)
          On entry, WSAVE must be an array of dimension (4 *
          N  + 15) or greater and must have been initialized
          by CFFTI.
```

Contents

NAME

```
cfftc -  initialize  the  trigonometric  weight  and  factor
tables  or  compute  the  Fast Fourier transform (forward or
inverse) of a complex sequence.
```

SYNOPSIS

```
SUBROUTINE CFFTC(IOPT, N, SCALE, X, Y, TRIGS, IFAC, WORK, LWORK, IERR)

INTEGER IOPT, N, IFAC(*), LWORK, IERR
COMPLEX X(*), Y(*)
REAL SCALE, TRIGS(*), WORK(*)

SUBROUTINE CFFTC_64(IOPT, N, SCALE, X, Y, TRIGS, IFAC, WORK, LWORK, IERR)

INTEGER*8 IOPT, N, IFAC(*), LWORK, IERR
REAL SCALE, TRIGS(*), WORK(*)
COMPLEX X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE FFT(IOPT, [N], [SCALE], X, Y, TRIGS, IFAC, WORK, [LWORK], IERR)

INTEGER*4, INTENT(IN) :: IOPT
INTEGER*4, INTENT(IN), OPTIONAL :: N, LWORK
REAL, INTENT(IN), OPTIONAL :: SCALE
COMPLEX, INTENT(IN), DIMENSION(:) :: X
COMPLEX, INTENT(OUT), DIMENSION(:) :: Y
REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
INTEGER*4, INTENT(INOUT), DIMENSION(:) :: IFAC
REAL, INTENT(OUT), DIMENSION(:) :: WORK
INTEGER*4, INTENT(OUT) :: IERR

SUBROUTINE FFT_64(IOPT, [N], [SCALE], X, Y, TRIGS, IFAC, WORK, [LWORK], IERR)
```

```
INTEGER(8), INTENT(IN) :: IOPT
INTEGER(8), INTENT(IN), OPTIONAL :: N, LWORK
REAL, INTENT(IN), OPTIONAL :: SCALE
COMPLEX, INTENT(IN), DIMENSION(:) :: X
COMPLEX, INTENT(OUT), DIMENSION(:) :: Y
REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
REAL, INTENT(OUT), DIMENSION(:) :: WORK
INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
#include <sunperf.h>

void cfftc_ (int *iopt, int *n, float  *scale,  complex  *x,
          complex  *y, float *trigs, int *ifac, float *work,
          int *lwork, int *ierr);

void cfftc_64_ (long *iopt, long *n, float  *scale,  complex
          *x,  complex  *y,  float *trigs, long *ifac, float
          *work, long *lwork, long *ierr);
```

PURPOSE

```
cfftc initializes the trigonometric weight and factor tables
or  computes the Fast Fourier transform (forward or inverse)
of a complex sequence as follows:

               N-1
Y(k) = scale * SUM  W*X(j)
               j=0

where
k ranges from 0 to N-1
i = sqrt(-1)
isign = 1 for inverse transform or -1 for forward transform
W = exp(isign*i*j*k*2*pi/N)
```

ARGUMENTS

```
IOPT (input)
          Integer specifying the operation to be performed:
          IOPT = 0 computes the trigonometric  weight  table
          and factor table
          IOPT = -1 computes forward FFT
          IOPT = +1 computes inverse FFT
```

N (input)
Integer specifying length of the input sequence X. N is most efficient when it is a product of small primes. N >= 0. Unchanged on exit.

SCALE (input)
Real scalar by which transform results are scaled. Unchanged on exit. SCALE is defaulted to 1.0 for F95 INTERFACE.

X (input) On entry, X is a complex array of dimension at least N that contains the sequence to be transformed.

Y (output)
Complex array of dimension at least N that contains the transform results. X and Y may be the same array starting at the same memory location. Otherwise, it is assumed that there is no overlap between X and Y in memory.

TRIGS (input/output)
Real array of length 2*N that contains the trigonometric weights. The weights are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1 or IOPT = -1. Unchanged on exit.

IFAC (input/output)
Integer array of dimension at least 128 that contains the factors of N. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls where IOPT = 1 or IOPT = -1. Unchanged on exit.

WORK (workspace)
Real array of dimension at least 2*N. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)
Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)
On exit, integer IERR has one of the following values:
0 = normal return
-1 = IOPT is not 0, 1 or -1
-2 = N < 0

```
-3 = (LWORK is not 0) and (LWORK is less than 2*N)
-4 = memory allocation for workspace failed
```

SEE ALSO

fft

Contents

NAME

```
     cfftc2 - initialize  the  trigonometric  weight  and  factor
     tables or compute the two-dimensional Fast Fourier Transform
     (forward or inverse) of a two-dimensional complex array.
```

SYNOPSIS

```
     SUBROUTINE CFFTC2(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     COMPLEX X(LDX, *), Y(LDY, *)
     REAL SCALE, TRIGS(*), WORK(*)

     SUBROUTINE CFFTC2_64(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER*8 IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     REAL SCALE, TRIGS(*), WORK(*)
     COMPLEX X(LDX, *), Y(LDY, *)
```

F95 INTERFACE

```
     SUBROUTINE FFT2(IOPT, [N1], [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
               IFAC, WORK, [LWORK], IERR)

     INTEGER*4, INTENT(IN) :: IOPT
     INTEGER*4, INTENT(IN), OPTIONAL :: N1, N2, LDX, LDY, LWORK
     REAL, INTENT(IN), OPTIONAL :: SCALE
     COMPLEX, INTENT(IN), DIMENSION(:,:) :: X
     COMPLEX, INTENT(OUT), DIMENSION(:,:) :: Y
     REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER*4, INTENT(INOUT), DIMENSION(:) :: IFAC
```

```
     REAL, INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER*4, INTENT(OUT) :: IERR

     SUBROUTINE FFT2_64(IOPT, [N1], [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
IFAC, WORK, [LWORK], IERR)

     INTEGER(8), INTENT(IN) :: IOPT
     INTEGER(8), INTENT(IN), OPTIONAL :: N1, N2, LDX, LDY, LWORK
     REAL, INTENT(IN), OPTIONAL :: SCALE
     COMPLEX, INTENT(IN), DIMENSION(:,:) :: X
     COMPLEX, INTENT(OUT), DIMENSION(:,:) :: Y
     REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL, INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
     #include <sunperf.h>
     void cfftc2_ (int *iopt, int *n1,  int  *n2,  float  *scale,
               complex  *x, int *ldx, complex *y, int *ldy, float
               *trigs, int *ifac, float *work,  int  *lwork,  int
               *ierr);

     void cfftc2_64_ (long  *iopt,  long  *n1,  long  *n2,  float
               *scale,  complex  *x,  long *ldx, complex *y, long
               *ldy, float *trigs, long *ifac, float *work,  long
               *lwork, long *ierr);
```

PURPOSE

cfftc2 initializes the trigonometric weight and factor tables or computes the two-dimensional Fast Fourier Transform (forward or inverse) of a two-dimensional complex array. In computing the two-dimensional FFT, one-dimensional FFTs are computed along the columns of the input array. One-dimensional FFTs are then computed along the rows of the intermediate results.

```
                   N2-1  N1-1
Y(k1,k2) = scale * SUM   SUM   W2*W1*X(j1,j2)
                   j2=0  j1=0
```

where
k1 ranges from 0 to N1-1 and k2 ranges from 0 to N2-1
i = sqrt(-1)
isign = 1 for inverse transform or -1 for forward transform
W1 = exp(isign*i*j1*k1*2*pi/N1)
W2 = exp(isign*i*j2*k2*2*pi/N2)

ARGUMENTS

IOPT (input)
Integer specifying the operation to be performed:
IOPT = 0 computes the trigonometric weight table and factor table
IOPT = -1 computes forward FFT
IOPT = +1 computes inverse FFT

N1 (input)
Integer specifying length of the transform in the first dimension. N1 is most efficient when it is a product of small primes. N1 >= 0. Unchanged on exit.

N2 (input)
Integer specifying length of the transform in the second dimension. N2 is most efficient when it is a product of small primes. N2 >= 0. Unchanged on exit.

SCALE (input)
Real scalar by which transform results are scaled. Unchanged on exit. SCALE is defaulted to 1.0 for F95 INTERFACE.

X (input) X is a complex array of dimensions (LDX, N2) that contains input data to be transformed.

LDX (input)
Leading dimension of X. LDX >= N1 Unchanged on exit.

Y (output)
Y is a complex array of dimensions (LDY, N2) that contains the transform results. X and Y can be the same array starting at the same memory location, in which case the input data are overwritten by their transform results. Otherwise, it is assumed that there is no overlap between X and Y in memory.

LDY (input)
Leading dimension of Y. If X and Y are the same array, LDY = LDX Else LDY >= N1 Unchanged on exit.

TRIGS (input/output)
Real array of length 2*(N1+N2) that contains the trigonometric weights. The weights are computed

when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1 or IOPT = -1. Unchanged on exit.

IFAC (input/output)
Integer array of dimension at least 2*128 that contains the factors of N1 and N2. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1 or IOPT = -1. Unchanged on exit.

WORK (workspace)
Real array of dimension at least 2*MAX(N1,N2)*NCPUS where NCPUS is the number of threads used to execute the routine. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)
Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)
On exit, integer IERR has one of the following values:
0 = normal return
-1 = IOPT is not 0, 1 or -1
-2 = N1 < 0
-3 = N2 < 0
-4 = (LDX < N1)
-5 = (LDY < N1) or (LDY not equal LDX when X and Y are same array)
-6 = (LWORK not equal 0) and (LWORK < 2*MAX(N1,N2)*NCPUS)
-7 = memory allocation failed

SEE ALSO

fft

CAUTIONS

On exit, entire output array Y(1:LDY, 1:N2) is overwritten.

Contents

NAME

```
     cfftc3 - initialize  the  trigonometric  weight  and  factor
     tables   or   compute  the  three-dimensional  Fast  Fourier
     Transform (forward or inverse) of a  three-dimensional  com-
     plex array.
```

SYNOPSIS

```
     SUBROUTINE CFFTC3(IOPT, N1, N2, N3, SCALE, X, LDX1, LDX2, Y, LDY1, LDY2,
TRIGS, IFAC, WORK, LWORK, IERR)

     INTEGER IOPT, N1, N2, N3, LDX1, LDX2, LDY1,  LDY2,  IFAC(*),
     LWORK, IERR
     COMPLEX X(LDX1, LDX2, *), Y(LDY1, LDY2, *)
     REAL SCALE, TRIGS(*), WORK(*)

     SUBROUTINE CFFTC3_64(IOPT, N1, N2, N3, SCALE, X, LDX1, LDX2, Y, LDY1, LDY2,
TRIGS, IFAC, WORK, LWORK, IERR)

     INTEGER*8 IOPT, N1, N2, N3, LDX1, LDX2, LDY1, LDY2, IFAC(*),
     LWORK, IERR
     COMPLEX X(LDX1, LDX2, *), Y(LDY1, LDY2, *)
     REAL SCALE, TRIGS(*), WORK(*)
```

F95 INTERFACE

```
     SUBROUTINE FFT3(IOPT, [N1], [N2], [N3], [SCALE], X, [LDX1], LDX2, Y, [LDY1],
LDY2, TRIGS,
               IFAC, WORK, [LWORK], IERR)

     INTEGER*4, INTENT(IN) :: IOPT, LDX2, LDY2
     INTEGER*4, INTENT(IN), OPTIONAL :: N1, N2, N3,  LDX1,  LDY1,
     LWORK
```

```
REAL, INTENT(IN), OPTIONAL :: SCALE
COMPLEX, INTENT(IN), DIMENSION(:,:) :: X
COMPLEX, INTENT(OUT), DIMENSION(:,:) :: Y
REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
INTEGER*4, INTENT(INOUT), DIMENSION(:) :: IFAC
REAL, INTENT(OUT), DIMENSION(:) :: WORK
INTEGER*4, INTENT(OUT) :: IERR

     SUBROUTINE FFT3_64(IOPT, [N1], [N2], [N3], [SCALE], X, [LDX1], LDX2, Y,
[LDY1], LDY2, TRIGS,
               IFAC, WORK, [LWORK], IERR)

INTEGER(8), INTENT(IN) :: IOPT, LDX2, LDY2
INTEGER(8), INTENT(IN), OPTIONAL :: N1, N2, N3, LDX1,  LDY1,
LWORK
REAL, INTENT(IN), OPTIONAL :: SCALE
COMPLEX, INTENT(IN), DIMENSION(:,:) :: X
COMPLEX, INTENT(OUT), DIMENSION(:,:) :: Y
REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
REAL, INTENT(OUT), DIMENSION(:) :: WORK
INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
#include <sunperf.h>

void cfftc3_ (int *iopt, int *n1, int *n2,  int  *n3,  float
          *scale,  complex *x, int *ldx1, int *ldx2, complex
          *y, int *ldy1, int *ldy2, float *trigs, int *ifac,
          float *work, int *lwork, int *ierr);

void cfftc3_64_ (long *iopt, long *n1, long *n2,  long  *n3,
          float  *scale, complex *x, long *ldx1, long *ldx2,
          complex *y, long *ldy1, long *ldy2, float  *trigs,
          long *ifac, float *work, long *lwork, long *ierr);
```

PURPOSE

cfftc3 initializes the trigonometric weight and factor tables or computes the three-dimensional Fast Fourier Transform (forward or inverse) of a three-dimensional complex array.

```
                      N3-1  N2-1  N1-1
Y(k1,k2,k3) = scale * SUM   SUM   SUM   W3*W2*W1*X(j1,j2,j3)
                      j3=0  j2=0  j1=0
```

where
k1 ranges from 0 to N1-1; k2 ranges from 0 to N2-1 and k3 ranges from 0 to N3-1

```
i = sqrt(-1)
isign = 1 for inverse transform or -1 for forward transform
W1 = exp(isign*i*j1*k1*2*pi/N1)
W2 = exp(isign*i*j2*k2*2*pi/N2)
W3 = exp(isign*i*j3*k3*2*pi/N3)
```

ARGUMENTS

```
IOPT (input)
          Integer specifying the operation to be performed:
          IOPT = 0 computes the trigonometric  weight  table
          and factor table
          IOPT = -1 computes forward FFT
          IOPT = +1 computes inverse FFT

N1 (input)
          Integer specifying length of the transform in  the
          first  dimension.  N1 is most efficient when it is
          a product of small primes.  N1 >= 0.  Unchanged on
          exit.

N2 (input)
          Integer specifying length of the transform in  the
          second dimension.  N2 is most efficient when it is
          a product of small primes.  N2 >= 0.  Unchanged on
          exit.

N3 (input)
          Integer specifying length of the transform in  the
          third  dimension.  N3 is most efficient when it is
          a product of small primes.  N3 >= 0.  Unchanged on
          exit.

SCALE (input)
          Real scalar by which transform results are scaled.
          Unchanged  on exit.  SCALE is defaulted to 1.0 for
          F95 INTERFACE.

X (input) X is a complex array of  dimensions  (LDX1,  LDX2,
          N3) that contains input data to be transformed.

LDX1 (input)
          first dimension of X.  LDX1  >=  N1  Unchanged  on
          exit.

LDX2 (input)
          second dimension of X.  LDX2 >=  N2  Unchanged  on
          exit.
```

Y (output)

Y is a complex array of dimensions (LDY1, LDY2, N3) that contains the transform results. X and Y can be the same array starting at the same memory location, in which case the input data are overwritten by their transform results. Otherwise, it is assumed that there is no overlap between X and Y in memory.

LDY1 (input)

first dimension of Y. If X and Y are the same array, LDY1 = LDX1 Else LDY1 >= N1 Unchanged on exit.

LDY2 (input)

second dimension of Y. If X and Y are the same array, LDY2 = LDX2 Else LDY2 >= N2 Unchanged on exit.

TRIGS (input/output)

Real array of length 2*(N1+N2+N3) that contains the trigonometric weights. The weights are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1 or IOPT = -1. Unchanged on exit.

IFAC (input/output)

Integer array of dimension at least 3*128 that contains the factors of N1, N2 and N3. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1 or IOPT = -1. Unchanged on exit.

WORK (workspace)

Real array of dimension at least (2*MAX(N,N2,N3) + 32*N3) * NCPUS where NCPUS is the number of threads used to execute the routine. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)

Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)

On exit, integer IERR has one of the following values:

0 = normal return

-1 = IOPT is not 0, 1 or -1

```
-2 = N1 < 0
-3 = N2 < 0
-4 = N3 < 0
-5 = (LDX1 < N1)
-6 = (LDX2 < N2)
-7 = (LDY1 < N1) or (LDY1 not equal  LDX1  when  X
and Y are same array)
-8 = (LDY2 < N2) or (LDY2 not equal  LDX2  when  X
and Y are same array)
-9  =  (LWORK  not   equal   0)   and   (LWORK   <
(2*MAX(N,N2,N3) + 16*N3) * NCPUS)
-10 = memory allocation failed
```

SEE ALSO

fft

CAUTIONS

This routine uses Y(N1+1:LDY1,:,:) as scratch space. Therefore, the original contents of this subarray will be lost upon returning from routine while subarray Y(1:N1,1:N2,1:N3) contains the transform results.

Contents

NAME

```
cfftcm - initialize  the  trigonometric  weight  and  factor
tables or compute the one-dimensional Fast Fourier Transform
(forward or inverse) of a set of data sequences stored in  a
two-dimensional complex array.
```

SYNOPSIS

```
     SUBROUTINE CFFTCM(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     COMPLEX X(LDX, *), Y(LDY, *)
     REAL SCALE, TRIGS(*), WORK(*)

     SUBROUTINE CFFTCM_64(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER*8 IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     REAL SCALE, TRIGS(*), WORK(*)
     COMPLEX X(LDX, *), Y(LDY, *)
```

F95 INTERFACE

```
     SUBROUTINE FFTM(IOPT, [N1], [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
               IFAC, WORK, [LWORK], IERR)

     INTEGER*4, INTENT(IN) :: IOPT
     INTEGER*4, INTENT(IN), OPTIONAL :: N1, N2, LDX, LDY, LWORK
     REAL, INTENT(IN), OPTIONAL :: SCALE
     COMPLEX, INTENT(IN), DIMENSION(:,:) :: X
     COMPLEX, INTENT(OUT), DIMENSION(:,:) :: Y
     REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER*4, INTENT(INOUT), DIMENSION(:) :: IFAC
```

```
     REAL, INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER*4, INTENT(OUT) :: IERR

     SUBROUTINE FFTM_64(IOPT, [N1], [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
IFAC, WORK, [LWORK], IERR)

     INTEGER(8), INTENT(IN) :: IOPT
     INTEGER(8), INTENT(IN), OPTIONAL :: N1, N2, LDX, LDY, LWORK
     REAL, INTENT(IN), OPTIONAL :: SCALE
     COMPLEX, INTENT(IN), DIMENSION(:,:) :: X
     COMPLEX, INTENT(OUT), DIMENSION(:,:) :: Y
     REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL, INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
     #include <sunperf.h>
     void cfftcm_ (int *iopt, int *n1,  int  *n2,  float  *scale,
               complex  *x, int *ldx, complex *y, int *ldy, float
               *trigs, int *ifac, float *work,  int  *lwork,  int
               *ierr);

     void cfftcm_64_ (long  *iopt,  long  *n1,  long  *n2,  float
               *scale,  complex  *x,  long *ldx, complex *y, long
               *ldy, float *trigs, long *ifac, float *work,  long
               *lwork, long *ierr);
```

PURPOSE

cfftcm initializes the trigonometric weight and factor tables or computes the one-dimensional Fast Fourier Transform (forward or inverse) of a set of data sequences stored in a two-dimensional complex array:

```
            N1-1
     Y(k,l) = SUM  W*X(j,l)
            j=0

     where
     k ranges from 0 to N1-1 and l ranges from 0 to N2-1
     i = sqrt(-1)
     isign = 1 for inverse transform or -1 for forward transform
     W = exp(isign*i*j*k*2*pi/N1)
```

ARGUMENTS

IOPT (input)
Integer specifying the operation to be performed:
IOPT = 0 computes the trigonometric weight table and factor table
IOPT = -1 computes forward FFT
IOPT = +1 computes inverse FFT

N1 (input)
Integer specifying length of the input sequences. N1 is most efficient when it is a product of small primes. N1 >= 0. Unchanged on exit.

N2 (input)
Integer specifying number of input sequences. N2 >= 0. Unchanged on exit.

SCALE (input)
Real scalar by which transform results are scaled. Unchanged on exit. SCALE is defaulted to 1.0 for F95 INTERFACE.

X (input) X is a complex array of dimensions (LDX, N2) that contains the sequences to be transformed stored in its columns.

LDX (input)
Leading dimension of X. LDX >= N1 Unchanged on exit.

Y (output)
Y is a complex array of dimensions (LDY, N2) that contains the transform results of the input sequences. X and Y can be the same array starting at the same memory location, in which case the input sequences are overwritten by their transform results. Otherwise, it is assumed that there is no overlap between X and Y in memory.

LDY (input)
Leading dimension of Y. If X and Y are the same array, LDY = LDX Else LDY >= N1 Unchanged on exit.

TRIGS (input/output)
Real array of length 2*N1 that contains the trigonometric weights. The weights are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1 or IOPT = -1. Unchanged on exit.

IFAC (input/output)

Integer array of dimension at least 128 that contains the factors of N1. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1 or IOPT = -1. Unchanged on exit.

WORK (workspace)

Real array of dimension at least 2*N1*NCPUS where NCPUS is the number of threads used to execute the routine. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)

Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)

On exit, integer IERR has one of the following values:
0 = normal return
-1 = IOPT is not 0, 1 or -1
-2 = N1 < 0
-3 = N2 < 0
-4 = (LDX < N1)
-5 = (LDY < N1) or (LDY not equal LDX when X and Y are same array)
-6 = (LWORK not equal 0) and (LWORK < 2*N1*NCPUS)
-7 = memory allocation failed

SEE ALSO

fft

Contents

NAME

```
cfftf - compute  the  Fourier  coefficients  of  a  periodic
sequence.  The FFT operations are unnormalized, so a call of
CFFTF followed by a call of CFFTB will  multiply  the  input
sequence by N.
```

SYNOPSIS

```
SUBROUTINE CFFTF(N, X, WSAVE)

COMPLEX X(*)
INTEGER N
REAL WSAVE(*)

SUBROUTINE CFFTF_64(N, X, WSAVE)

COMPLEX X(*)
INTEGER*8 N
REAL WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE FFTF([N], X, WSAVE)

COMPLEX, DIMENSION(:) :: X
INTEGER :: N
REAL, DIMENSION(:) :: WSAVE

SUBROUTINE FFTF_64([N], X, WSAVE)

COMPLEX, DIMENSION(:) :: X
INTEGER(8) :: N
REAL, DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void cfftf(int n, complex *x, float *wsave);

void cfftf_64(long n, complex *x, float *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence to be  transformed.   These
          subroutines are most efficient when N is a product
          of small primes.  N >= 0.

X (input) On entry, an array  of  length  N  containing  the
          sequence to be transformed.

WSAVE (input)
          On entry, WSAVE must be an array of dimension (4 *
          N  + 15) or greater and must have been initialized
          by CFFTI.
```

Contents

NAME

```
cffti - initialize the array WSAVE, which is  used  in  both
CFFTF and CFFTB.
```

SYNOPSIS

```
SUBROUTINE CFFTI(N, WSAVE)

INTEGER N
REAL WSAVE(*)

SUBROUTINE CFFTI_64(N, WSAVE)

INTEGER*8 N
REAL WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE CFFTI(N, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: WSAVE

SUBROUTINE CFFTI_64(N, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void cffti(int n, float *wsave);

void cffti_64(long n, float *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. N >= 0.

WSAVE (input/output)
On entry, an array of dimension (4 * N + 15) or greater. CFFTI needs to be called only once to initialize array WORK before calling CFFTF and/or CFFTB if N and WSAVE remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
cfftopt - compute the length of the closest fast FFT
```

SYNOPSIS

```
INTEGER FUNCTION CFFTOPT(LEN)

INTEGER LEN

INTEGER*8 FUNCTION CFFTOPT_64(LEN)

INTEGER*8 LEN
```

F95 INTERFACE

```
INTEGER FUNCTION CFFTOPT(LEN)

INTEGER :: LEN

INTEGER(8) FUNCTION CFFTOPT_64(LEN)

INTEGER(8) :: LEN
```

C INTERFACE

```
#include <sunperf.h>

int cfftopt(int len);

long cfftopt_64(long len);
```

PURPOSE

```
cfftopt computes the length of the closest fast  FFT.   Fast
```

Fourier transform algorithms, including those used in Performance Library, work best with vector lengths that are products of small primes. For example, an FFT of length 32=2**5 will run faster than an FFT of prime length 31 because 32 is a product of small primes and 31 is not. If your application is such that you can taper or zero pad your vector to a larger length then this function may help you select a better length and run your FFT faster.

CFFTOPT will return an integer no smaller than the input argument N that is the closest number that is the product of small primes. CFFTOPT will return 16 for an input of N=16 and return 18=2*3*3 for an input of N=17.

Note that the length computed here is not guaranteed to be optimal, only to be a product of small primes. Also, the value returned may change as the underlying FFTs become capable of handling larger primes. For example, passing in N=51 today will return 52=2*2*13 rather than 51=3*17 because the FFTs in Performance Library do not have fast radix 17 code. In the future, radix 17 code may be added and then N=51 will return 51.

Contents

NAME

```
cffts -  initialize  the  trigonometric  weight  and  factor
tables  or  compute  the inverse Fast Fourier Transform of a
complex sequence as follows.
```

SYNOPSIS

```
SUBROUTINE CFFTS(IOPT, N, SCALE, X, Y, TRIGS, IFAC, WORK, LWORK, IERR)

INTEGER IOPT, N, IFAC(*), LWORK, IERR
COMPLEX X(*)
REAL SCALE, Y(*), TRIGS(*), WORK(*)

SUBROUTINE CFFTS_64(IOPT, N, SCALE, X, Y, TRIGS, IFAC, WORK, LWORK, IERR)

INTEGER*8 IOPT, N, IFAC(*), LWORK, IERR
REAL SCALE, Y(*), TRIGS(*), WORK(*)
COMPLEX X(*)
```

F95 INTERFACE

```
SUBROUTINE FFT(IOPT, N, [SCALE], X, Y, TRIGS, IFAC, WORK, [LWORK], IERR)

INTEGER*4, INTENT(IN) :: IOPT, N
INTEGER*4, INTENT(IN), OPTIONAL :: LWORK
REAL, INTENT(IN), OPTIONAL :: SCALE
COMPLEX, INTENT(IN), DIMENSION(:) :: X
REAL, INTENT(OUT), DIMENSION(:) :: Y
REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
INTEGER*4, INTENT(INOUT), DIMENSION(:) :: IFAC
REAL, INTENT(OUT), DIMENSION(:) :: WORK
INTEGER*4, INTENT(OUT) :: IERR

SUBROUTINE FFT_64(IOPT, N, [SCALE], X, Y, TRIGS, IFAC, WORK, [LWORK], IERR)
```

```
INTEGER(8), INTENT(IN) :: IOPT, N
INTEGER(8), INTENT(IN), OPTIONAL :: LWORK
REAL, INTENT(IN), OPTIONAL :: SCALE
COMPLEX, INTENT(IN), DIMENSION(:) :: X
REAL, INTENT(OUT), DIMENSION(:) :: Y
REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
REAL, INTENT(OUT), DIMENSION(:) :: WORK
INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
#include <sunperf.h>

void cffts_ (int *iopt, int *n, float  *scale,  complex  *x,
          float  *y,  float  *trigs, int *ifac, float *work,
          int *lwork, int *ierr);

void cffts_64_ (long *iopt, long *n, float  *scale,  complex
          *x,  float  *y,  float  *trigs,  long *ifac, float
          *work, long *lwork, long *ierr);
```

PURPOSE

```
cffts initializes the trigonometric weight and factor tables
or  computes the inverse Fast Fourier Transform of a complex
sequence as follows:

               N-1
Y(k) = scale * SUM  W*X(j)
               j=0

where
k ranges from 0 to N-1
i = sqrt(-1)
isign = 1 for inverse transform or -1 for forward transform
W = exp(isign*i*j*k*2*pi/N)
In complex-to-real transform of length N, the  (N/2+1)  com-
plex  input  data  points  stored are the positive-frequency
half of the spectrum of the Discrete Fourier Transform.  The
other  half  can be obtained through complex conjugation and
therefore is not stored.  Furthermore, due to symmetries the
imaginary  of the component of X(0) and X(N/2) (if N is even
in the latter) is assumed to be zero and is not referenced.
```

ARGUMENTS

IOPT (input)
Integer specifying the operation to be performed:
IOPT = 0 computes the trigonometric weight table and factor table
IOPT = 1 computes inverse FFT

N (input)
Integer specifying length of the input sequence X. N is most efficient when it is a product of small primes. N >= 0. Unchanged on exit.

SCALE (input)
Real scalar by which transform results are scaled. Unchanged on exit. SCALE is defaulted to 1.0 for F95 INTERFACE.

X (input) On entry, X is a complex array whose first (N/2+1) elements are the input sequence to be transformed.

Y (output)
Real array of dimension at least N that contains the transform results. X and Y may be the same array starting at the same memory location. Otherwise, it is assumed that there is no overlap between X and Y in memory.

TRIGS (input/output)
Real array of length 2*N that contains the trigonometric weights. The weights are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1. Unchanged on exit.

IFAC (input/output)
Integer array of dimension at least 128 that contains the factors of N. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls where IOPT = 1. Unchanged on exit.

WORK (workspace)
Real array of dimension at least N. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)
Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)
On exit, integer IERR has one of the following

values:
0 = normal return
-1 = IOPT is not 0 or 1
-2 = N < 0
-3 = (LWORK is not 0) and (LWORK is less than N)
-4 = memory allocation for workspace failed

SEE ALSO

fft

Contents

NAME

```
cffts2 - initialize  the  trigonometric  weight  and  factor
tables  or  compute the two-dimensional inverse Fast Fourier
Transform of a two-dimensional complex array.
```

SYNOPSIS

```
     SUBROUTINE CFFTS2(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     COMPLEX X(LDX, *)
     REAL SCALE, Y(LDY, *), TRIGS(*), WORK(*)

     SUBROUTINE CFFTS2_64(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER*8 IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     COMPLEX X(LDX, *)
     REAL SCALE, Y(LDY, *), TRIGS(*), WORK(*)
```

F95 INTERFACE

```
     SUBROUTINE FFT2(IOPT, N1, [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
     &           IFAC, WORK, [LWORK], IERR)

     INTEGER*4, INTENT(IN) :: IOPT, N1
     INTEGER*4, INTENT(IN), OPTIONAL :: N2, LDX, LDY, LWORK
     REAL, INTENT(IN), OPTIONAL :: SCALE
     COMPLEX, INTENT(IN), DIMENSION(:,:) :: X
     REAL, INTENT(OUT), DIMENSION(:,:) :: Y
     REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER*4, INTENT(INOUT), DIMENSION(:) :: IFAC
```

```
     REAL, INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER*4, INTENT(OUT) :: IERR

     SUBROUTINE FFT2_64(IOPT, N1, [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS, IFAC,
WORK, [LWORK], IERR)

     INTEGER(8), INTENT(IN) :: IOPT, N1
     INTEGER(8), INTENT(IN), OPTIONAL :: N2, LDX, LDY, LWORK
     REAL, INTENT(IN), OPTIONAL :: SCALE
     COMPLEX, INTENT(IN), DIMENSION(:,:) :: X
     REAL, INTENT(OUT), DIMENSION(:,:) :: Y
     REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL, INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
     #include <sunperf.h>
     void cffts2_ (int *iopt, int *n1,  int  *n2,  float  *scale,
               complex  *x,  int  *ldx, float *y, int *ldy, float
               *trigs, int *ifac, float *work,  int  *lwork,  int
               *ierr);

     void cffts2_64_ (long  *iopt,  long  *n1,  long  *n2,  float
               *scale,  complex  *x,  long  *ldx,  float *y, long
               *ldy, float *trigs, long *ifac, float *work,  long
               *lwork, long *ierr);
```

PURPOSE

```
     cffts2  initializes  the  trigonometric  weight  and  factor
     tables  or computes the two-dimensional inverse Fast Fourier
     Transform of a two-dimensional complex array.  In  computing
     the  two-dimensional  FFT, one-dimensional FFTs are computed
     along the rows of the input array.  One-dimensional FFTs are
     then computed along the columns of the intermediate results.

                        N1-1  N2-1
     Y(k1,k2) = scale * SUM   SUM   W2*W1*X(j1,j2)
                        j1=0  j2=0

     where
     k1 ranges from 0 to N1-1 and k2 ranges from 0 to N2-1
     i = sqrt(-1)
     isign = 1 for inverse transform
     W1 = exp(isign*i*j1*k1*2*pi/N1)
     W2 = exp(isign*i*j2*k2*2*pi/N2)
     In complex-to-real transform of length N1, the (N1/2+1) com-
     plex  input  data  points  stored are the positive-frequency
```

half of the spectrum of the Discrete Fourier Transform. The other half can be obtained through complex conjugation and therefore is not stored.

ARGUMENTS

IOPT (input)
Integer specifying the operation to be performed:
IOPT = 0 computes the trigonometric weight table and factor table
IOPT = 1 computes inverse FFT

N1 (input)
Integer specifying length of the transform in the first dimension. N1 is most efficient when it is a product of small primes. N1 >= 0. Unchanged on exit.

N2 (input)
Integer specifying length of the transform in the second dimension. N2 is most efficient when it is a product of small primes. N2 >= 0. Unchanged on exit.

SCALE (input)
Real scalar by which transform results are scaled. Unchanged on exit. SCALE is defaulted to 1.0 for F95 INTERFACE.

X (input) X is a complex array of dimensions (LDX, N2) that contains input data to be transformed.

LDX (input)
Leading dimension of X. LDX >= (N1/2 + 1) Unchanged on exit.

Y (output)
Y is a real array of dimensions (LDY, N2) that contains the transform results. X and Y can be the same array starting at the same memory location, in which case the input data are overwritten by their transform results. Otherwise, it is assumed that there is no overlap between X and Y in memory.

LDY (input)
Leading dimension of Y. If X and Y are the same array, LDY = 2*LDX Else LDY >= 2*LDX and LDY must

be even. Unchanged on exit.

TRIGS (input/output)

Real array of length 2*(N1+N2) that contains the trigonometric weights. The weights are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1. Unchanged on exit.

IFAC (input/output)

Integer array of dimension at least 2*128 that contains the factors of N1 and N2. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1. Unchanged on exit.

WORK (workspace)

Real array of dimension at least MAX(N1,2*N2)*NCPUS, where NCPUS is the number of threads used to execute the routine. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)

Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)

On exit, integer IERR has one of the following values:

0 = normal return
-1 = IOPT is not 0, 1
-2 = N1 < 0
-3 = N2 < 0
-4 = (LDX < N1/2+1)
-5 = LDY not equal 2*LDX when X and Y are same array
-6 = (LDY < 2*LDX or LDY odd) when X and Y are same array
-7 = (LWORK not equal 0) and (LWORK < MAX(N1,2*N2)*NCPUS)
-8 = memory allocation failed

SEE ALSO

fft

CAUTIONS

Y(N1+1:LDY,:) is used as scratch space. Upon returning, the original contents of Y(N1+1:LDY,:) will be lost, whereas Y(1:N1,1:N2) contains the transform results.

Contents

NAME

```
     cffts3 - initialize  the  trigonometric  weight  and  factor
     tables or compute the three-dimensional inverse Fast Fourier
     Transform of a three-dimensional complex array.
```

SYNOPSIS

```
     SUBROUTINE CFFTS3(IOPT, N1, N2, N3, SCALE, X, LDX1, LDX2, Y, LDY1, LDY2,
TRIGS, IFAC, WORK, LWORK, IERR)

     INTEGER IOPT, N1, N2, N3, LDX1, LDX2, LDY1,  LDY2,  IFAC(*),
     LWORK, IERR
     COMPLEX X(LDX1, LDX2, *)
     REAL SCALE, TRIGS(*), WORK(*), Y(LDY1, LDY2, *)

     SUBROUTINE CFFTS3_64(IOPT, N1, N2, N3, SCALE, X, LDX1, LDX2, Y, LDY1, LDY2,
TRIGS, IFAC, WORK, LWORK, IERR)

     INTEGER*8 IOPT, N1, N2, N3, LDX1, LDX2, LDY1, LDY2, IFAC(*),
     LWORK, IERR
     COMPLEX X(LDX1, LDX2, *)
     REAL SCALE, TRIGS(*), WORK(*), Y(LDY1, LDY2, *)
```

F95 INTERFACE

```
     SUBROUTINE FFT3(IOPT, N1, [N2], [N3], [SCALE], X, [LDX1], LDX2, Y, [LDY1],
LDY2, TRIGS,
               IFAC, WORK, [LWORK], IERR)

     INTEGER*4, INTENT(IN) :: IOPT, N1, LDX2, LDY2
     INTEGER*4, INTENT(IN), OPTIONAL :: N2, N3, LDX1, LDY1, LWORK
     REAL, INTENT(IN), OPTIONAL :: SCALE
     COMPLEX, INTENT(IN), DIMENSION(:,:) :: X
```

```
     REAL, INTENT(OUT), DIMENSION(:,:) :: Y
     REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER*4, INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL, INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER*4, INTENT(OUT) :: IERR

     SUBROUTINE FFT3_64(IOPT, N1, [N2], [N3], [SCALE], X, [LDX1], LDX2, Y,
[LDY1], LDY2, TRIGS,
               IFAC, WORK, [LWORK], IERR)

     INTEGER(8), INTENT(IN) :: IOPT, N1, LDX2, LDY2
     INTEGER(8), INTENT(IN), OPTIONAL  ::  N2,  N3,  LDX1,  LDY1,
     LWORK
     REAL, INTENT(IN), OPTIONAL :: SCALE
     COMPLEX, INTENT(IN), DIMENSION(:,:) :: X
     REAL, INTENT(OUT), DIMENSION(:,:) :: Y
     REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL, INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
     #include <sunperf.h>

     void cffts3_ (int *iopt, int *n1, int *n2,  int  *n3,  float
               *scale,  complex  *x,  int *ldx1, int *ldx2, float
               *y, int *ldy1, int *ldy2, float *trigs, int *ifac,
               float *work, int *lwork, int *ierr);

     void cffts3_64_ (long *iopt, long *n1, long *n2,  long  *n3,
               float  *scale, complex *x, long *ldx1, long *ldx2,
               float *y, long *ldy1, long  *ldy2,  float  *trigs,
               long *ifac, float *work, long *lwork, long *ierr);
```

PURPOSE

```
     cffts3  initializes  the  trigonometric  weight  and  factor
     tables   or  computes  the  three-dimensional  inverse  Fast
     Fourier Transform of a three-dimensional complex array.

                           N3-1  N2-1  N1-1
     Y(k1,k2,k3) = scale * SUM   SUM   SUM   W3*W2*W1*X(j1,j2,j3)
                           j3=0  j2=0  j1=0

     where
     k1 ranges from 0 to N1-1; k2 ranges from 0 to  N2-1  and  k3
     ranges from 0 to N3-1
     i = sqrt(-1)
     isign = 1 for inverse transform
     W1 = exp(isign*i*j1*k1*2*pi/N1)
```

```
W2 = exp(isign*i*j2*k2*2*pi/N2)
W3 = exp(isign*i*j3*k3*2*pi/N3)
```

ARGUMENTS

```
IOPT (input)
          Integer specifying the operation to be performed:
          IOPT = 0 computes the trigonometric  weight  table
          and factor table
          IOPT = +1 computes inverse FFT

N1 (input)
          Integer specifying length of the transform in  the
          first  dimension.  N1 is most efficient when it is
          a product of small primes.  N1 >= 0.  Unchanged on
          exit.

N2 (input)
          Integer specifying length of the transform in  the
          second dimension.  N2 is most efficient when it is
          a product of small primes.  N2 >= 0.  Unchanged on
          exit.
N3 (input)
          Integer specifying length of the transform in  the
          third  dimension.  N3 is most efficient when it is
          a product of small primes.  N3 >= 0.  Unchanged on
          exit.

SCALE (input)
          Real scalar by which transform results are scaled.
          Unchanged  on exit.  SCALE is defaulted to 1.0 for
          F95 INTERFACE.

X (input) X is a complex array of  dimensions  (LDX1,  LDX2,
          N3) that contains input data to be transformed.

LDX1 (input)
          first dimension of X.  LDX1 >= N1/2+1 Unchanged on
          exit.

LDX2 (input)
          second dimension of X.  LDX2 >=  N2  Unchanged  on
          exit.

Y (output)
          Y is a complex array of  dimensions  (LDY1,  LDY2,
          N3)  that contains the transform results.  X and Y
          can be the same array starting at the same  memory
```

location, in which case the input data are overwritten by their transform results. Otherwise, it is assumed that there is no overlap between X and Y in memory.

LDY1 (input)
first dimension of Y. If X and Y are the same array, LDY1 = 2*LDX1 Else LDY1 >= 2*LDX1 and LDY1 is even Unchanged on exit.

LDY2 (input)
second dimension of Y. If X and Y are the same array, LDY2 = LDX2 Else LDY2 >= N2 Unchanged on exit.

TRIGS (input/output)
Real array of length 2*(N1+N2+N3) that contains the trigonometric weights. The weights are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1. Unchanged on exit.

IFAC (input/output)
Integer array of dimension at least 3*128 that contains the factors of N1, N2 and N3. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1. Unchanged on exit.

WORK (workspace)
Real array of dimension at least (MAX(N,2*N2,2*N3) + 16*N3) * NCPUS where NCPUS is the number of threads used to execute the routine. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)
Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)
On exit, integer IERR has one of the following values:
0 = normal return
-1 = IOPT is not 0 or 1
-2 = N1 < 0
-3 = N2 < 0
-4 = N3 < 0
-5 = (LDX1 < N1/2+1)
-6 = (LDX2 < N2)

-7 = LDY1 not equal 2*LDX1 when X and Y are same array
-8 = (LDY1 < 2*LDX1) or (LDY1 is odd) when X and Y are not same array
-9 = (LDY2 < N2) or (LDY2 not equal LDX2) when X and Y are same array
-10 = (LWORK not equal 0) and ((LWORK < MAX(N,2*N2,2*N3) + 16*N3)*NCPUS)
-11 = memory allocation failed

SEE ALSO

fft

CAUTIONS

This routine uses Y(N1+1:LDY1,:,:) as scratch space. Therefore, the original contents of this subarray will be lost upon returning from routine while subarray Y(1:N1,1:N2,1:N3) contains the transform results.

Contents

NAME

```
     cfftsm - initialize  the  trigonometric  weight  and  factor
     tables  or  compute the one-dimensional inverse Fast Fourier
     Transform of a set of complex data  sequences  stored  in  a
     two-dimensional array.
```

SYNOPSIS

```
     SUBROUTINE CFFTSM(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     COMPLEX X(LDX, *)
     REAL SCALE, Y(LDY, *), TRIGS(*), WORK(*)

     SUBROUTINE CFFTSM_64(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER*8 IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     REAL SCALE, Y(LDY,*), TRIGS(*), WORK(*)
     COMPLEX X(LDX, *)
```

F95 INTERFACE

```
     SUBROUTINE FFTM(IOPT, N1, [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
               IFAC, WORK, [LWORK], IERR)

     INTEGER*4, INTENT(IN) :: IOPT, N1
     INTEGER*4, INTENT(IN), OPTIONAL :: N2, LDX, LDY, LWORK
     REAL, INTENT(IN), OPTIONAL :: SCALE
     COMPLEX, INTENT(IN), DIMENSION(:,:) :: X
     REAL, INTENT(OUT), DIMENSION(:,:) :: Y
     REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER*4, INTENT(INOUT), DIMENSION(:) :: IFAC
```

```
     REAL, INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER*4, INTENT(OUT) :: IERR

     SUBROUTINE FFTM_64(IOPT, N1, [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS, IFAC,
WORK, [LWORK], IERR)

     INTEGER(8), INTENT(IN) :: IOPT, N1
     INTEGER(8), INTENT(IN), OPTIONAL :: N2, LDX, LDY, LWORK
     REAL, INTENT(IN), OPTIONAL :: SCALE
     COMPLEX, INTENT(IN), DIMENSION(:,:) :: X
     REAL, INTENT(OUT), DIMENSION(:,:) :: Y
     REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL, INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
     #include <sunperf.h>
     void cfftsm_ (int *iopt, int *n1,  int  *n2,  float  *scale,
               complex  *x,  int  *ldx, float *y, int *ldy, float
               *trigs, int *ifac, float *work,  int  *lwork,  int
               *ierr);

     void cfftsm_64_ (long  *iopt,  long  *n1,  long  *n2,  float
               *scale,  complex  *x,  long  *ldx,  float *y, long
               *ldy, float *trigs, long *ifac, float *work,  long
               *lwork, long *ierr);
```

PURPOSE

cfftsm initializes the trigonometric weight and factor tables or computes the one-dimensional inverse Fast Fourier Transform of a set of complex data sequences stored in a two-dimensional array:

```
                     N1-1
     Y(k,l) = scale * SUM  W*X(j,l)
                     j=0
```

where
k ranges from 0 to N1-1 and l ranges from 0 to N2-1
i = sqrt(-1)
isign = 1 for inverse transform
W = exp(isign*i*j*k*2*pi/N1)
In complex-to-real transform of length N1, the (N1/2+1) complex input data points stored are the positive-frequency half of the spectrum of the Discrete Fourier Transform. The other half can be obtained through complex conjugation and therefore is not stored. Furthermore, due to symmetries the

imaginary of the component of X(0,0:N2-1) and X(N1/2,0:N2-1) (if N1 is even in the latter) is assumed to be zero and is not referenced.

ARGUMENTS

IOPT (input)
Integer specifying the operation to be performed:
IOPT = 0 computes the trigonometric weight table and factor table
IOPT = 1 computes inverse FFT

N1 (input)
Integer specifying length of the input sequences. N1 is most efficient when it is a product of small primes. N1 >= 0. Unchanged on exit.

N2 (input)
Integer specifying number of input sequences. N2 >= 0. Unchanged on exit.

SCALE (input)
Real scalar by which transform results are scaled. Unchanged on exit. SCALE is defaulted to 1.0 for F95 INTERFACE.

X (input) X is a complex array of dimensions (LDX, N2) that contains the sequences to be transformed stored in its columns in X(0:N1/2, 0:N2-1).

LDX (input)
Leading dimension of X. LDX >= (N1/2+1) Unchanged on exit.

Y (output)
Y is a real array of dimensions (LDY, N2) that contains the transform results of the input sequences in Y(0:N1-1,0:N2-1). X and Y can be the same array starting at the same memory location, in which case the input sequences are overwritten by their transform results. Otherwise, it is assumed that there is no overlap between X and Y in memory.

LDY (input)
Leading dimension of Y. If X and Y are the same array, LDY = 2*LDX Else LDY >= N1 Unchanged on exit.

TRIGS (input/output)
Real array of length 2*N1 that contains the trigonometric weights. The weights are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1. Unchanged on exit.

IFAC (input/output)
Integer array of dimension at least 128 that contains the factors of N1. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1. Unchanged on exit.

WORK (workspace)
Real array of dimension at least N1. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)
Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)
On exit, integer IERR has one of the following values:
0 = normal return
-1 = IOPT is not 0 or 1
-2 = N1 < 0
-3 = N2 < 0
-4 = (LDX < N1/2+1)
-5 = (LDY < N1) or (LDY not equal 2*LDX when X and Y are same array)
-6 = (LWORK not equal 0) and (LWORK < N1)
-7 = memory allocation failed

SEE ALSO

fft

Contents

NAME

```
cgbbrd - reduce a complex general m-by-n band  matrix  A  to
real upper bidiagonal form B by a unitary transformation
```

SYNOPSIS

```
SUBROUTINE CGBBRD(VECT, M, N, NCC, KL, KU, AB, LDAB, D, E, Q, LDQ,
      PT, LDPT, C, LDC, WORK, RWORK, INFO)

CHARACTER * 1 VECT
COMPLEX AB(LDAB,*), Q(LDQ,*), PT(LDPT,*), C(LDC,*), WORK(*)
INTEGER M, N, NCC, KL, KU, LDAB, LDQ, LDPT, LDC, INFO
REAL D(*), E(*), RWORK(*)

SUBROUTINE CGBBRD_64(VECT, M, N, NCC, KL, KU, AB, LDAB, D, E, Q, LDQ,
      PT, LDPT, C, LDC, WORK, RWORK, INFO)

CHARACTER * 1 VECT
COMPLEX AB(LDAB,*), Q(LDQ,*), PT(LDPT,*), C(LDC,*), WORK(*)
INTEGER*8 M, N, NCC, KL, KU, LDAB, LDQ, LDPT, LDC, INFO
REAL D(*), E(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GBBRD(VECT, M, [N], [NCC], KL, KU, AB, [LDAB], D, E, Q,
       [LDQ], PT, [LDPT], C, [LDC], [WORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: VECT
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: AB, Q, PT, C
INTEGER :: M, N, NCC, KL, KU, LDAB, LDQ, LDPT, LDC, INFO
REAL, DIMENSION(:) :: D, E, RWORK

SUBROUTINE GBBRD_64(VECT, M, [N], [NCC], KL, KU, AB, [LDAB], D, E,
```

```
          Q, [LDQ], PT, [LDPT], C, [LDC], [WORK], [RWORK], [INFO])

   CHARACTER(LEN=1) :: VECT
   COMPLEX, DIMENSION(:) :: WORK
   COMPLEX, DIMENSION(:,:) :: AB, Q, PT, C
   INTEGER(8) :: M, N, NCC, KL, KU, LDAB, LDQ, LDPT, LDC, INFO
   REAL, DIMENSION(:) :: D, E, RWORK
```

C INTERFACE

```
   #include <sunperf.h>

   void cgbbrd(char vect, int m, int n, int ncc,  int  kl,  int
             ku,  complex  *ab,  int  ldab, float *d, float *e,
             complex *q, int ldq, complex *pt, int  ldpt,  com-
             plex *c, int ldc, int *info);
   void cgbbrd_64(char vect, long m, long n, long ncc, long kl,
             long  ku,  complex *ab, long ldab, float *d, float
             *e, complex *q, long ldq, complex *pt, long  ldpt,
             complex *c, long ldc, long *info);
```

PURPOSE

cgbbrd reduces a complex general m-by-n band matrix A to real upper bidiagonal form B by a unitary transformation: Q' * A * P = B.

The routine computes B, and optionally forms Q or P', or computes Q'*C for a given matrix C.

ARGUMENTS

VECT (input)
Specifies whether or not the matrices Q and P' are to be formed. = 'N': do not form Q or P';
= 'Q': form Q only;
= 'P': form P' only;
= 'B': form both.

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

NCC (input)
The number of columns of the matrix C. NCC >= 0.

KL (input)
The number of subdiagonals of the matrix A. KL >=

0.

KU (input)
The number of superdiagonals of the matrix A. KU >= 0.

AB (input/output)
On entry, the m-by-n band matrix A, stored in rows 1 to KL+KU+1. The j-th column of A is stored in the j-th column of the array AB as follows: AB(ku+1+i-j,j) = A(i,j) for max(1,j-ku)<=i<=min(m,j+kl). On exit, A is overwritten by values generated during the reduction.

LDAB (input)
The leading dimension of the array A. LDAB >= KL+KU+1.

D (output)
The diagonal elements of the bidiagonal matrix B.

E (output)
The superdiagonal elements of the bidiagonal matrix B.

Q (output)
If VECT = 'Q' or 'B', the m-by-m unitary matrix Q. If VECT = 'N' or 'P', the array Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= max(1,M) if VECT = 'Q' or 'B'; LDQ >= 1 otherwise.

PT (output)
If VECT = 'P' or 'B', the n-by-n unitary matrix P'. If VECT = 'N' or 'Q', the array PT is not referenced.

LDPT (input)
The leading dimension of the array PT. LDPT >= max(1,N) if VECT = 'P' or 'B'; LDPT >= 1 otherwise.

C (input/output)
On entry, an m-by-ncc matrix C. On exit, C is overwritten by Q'*C. C is not referenced if NCC = 0.

LDC (input)
The leading dimension of the array C. LDC >=

```
          max(1,M) if NCC > 0; LDC >= 1 if NCC = 0.

WORK (workspace)
          dimension(MAX(M,N))

RWORK (workspace)
          dimension(MAX(M,N))

INFO (output)
          = 0:  successful exit.
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value.
```

Contents

NAME

```
cgbcon - estimate the reciprocal of the condition number  of
a complex general band matrix A, in either the 1-norm or the
infinity-norm,
```

SYNOPSIS

```
SUBROUTINE CGBCON(NORM, N, NSUB, NSUPER, A, LDA, IPIVOT, ANORM,
      RCOND, WORK, WORK2, INFO)

CHARACTER * 1 NORM
COMPLEX A(LDA,*), WORK(*)
INTEGER N, NSUB, NSUPER, LDA, INFO
INTEGER IPIVOT(*)
REAL ANORM, RCOND
REAL WORK2(*)

SUBROUTINE CGBCON_64(NORM, N, NSUB, NSUPER, A, LDA, IPIVOT, ANORM,
      RCOND, WORK, WORK2, INFO)

CHARACTER * 1 NORM
COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, NSUB, NSUPER, LDA, INFO
INTEGER*8 IPIVOT(*)
REAL ANORM, RCOND
REAL WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GBCON(NORM, [N], NSUB, NSUPER, A, [LDA], IPIVOT, ANORM,
       RCOND, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM
COMPLEX, DIMENSION(:) :: WORK
```

```
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, NSUB, NSUPER, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: WORK2

SUBROUTINE GBCON_64(NORM, [N], NSUB, NSUPER, A, [LDA], IPIVOT, ANORM,
       RCOND, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, NSUB, NSUPER, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cgbcon(char norm, int n, int nsub, int nsuper,  complex
          *a,  int  lda,  int  *ipivot,  float  anorm, float
          *rcond, int *info);

void cgbcon_64(char norm, long n, long  nsub,  long  nsuper,
          complex  *a,  long lda, long *ipivot, float anorm,
          float *rcond, long *info);
```

PURPOSE

cgbcon estimates the reciprocal of the condition number of a complex general band matrix A, in either the 1-norm or the infinity-norm, using the LU factorization computed by CGBTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as

RCOND = 1 / (norm(A) * norm(inv(A))).

ARGUMENTS

NORM (input)
Specifies whether the 1-norm condition number or the infinity-norm condition number is required:
= '1' or 'O': 1-norm;
= 'I': Infinity-norm.

N (input) The order of the matrix A. N >= 0.

NSUB (input)
The number of subdiagonals within the band of A.
NSUB >= 0.

NSUPER (input)
The number of superdiagonals within the band of A.
NSUPER >= 0.

A (input) Details of the LU factorization of the band matrix A, as computed by CGBTRF. U is stored as an upper triangular band matrix with NSUB+NSUPER superdiagonals in rows 1 to NSUB+NSUPER+1, and the multipliers used during the factorization are stored in rows NSUB+NSUPER+2 to 2*NSUB+NSUPER+1.

LDA (input)
The leading dimension of the array A. LDA >= 2*NSUB+NSUPER+1.

IPIVOT (input)
The pivot indices; for 1 <= i <= N, row i of the matrix was interchanged with row IPIVOT(i).

ANORM (input)
If NORM = '1' or 'O', the 1-norm of the original matrix A. If NORM = 'I', the infinity-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(norm(A) * norm(inv(A))).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension (N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
cgbequ - compute row and column scalings intended to equili-
brate  an  M-by-N  band  matrix  A  and reduce its condition
number
```

SYNOPSIS

```
SUBROUTINE CGBEQU(M, N, KL, KU, A, LDA, R, C, ROWCND,
      COLCND, AMAX, INFO)

COMPLEX A(LDA,*)
INTEGER M, N, KL, KU, LDA, INFO
REAL ROWCND, COLCND, AMAX
REAL R(*), C(*)

SUBROUTINE CGBEQU_64(M, N, KL, KU, A, LDA, R, C,
      ROWCND, COLCND, AMAX, INFO)

COMPLEX A(LDA,*)
INTEGER*8 M, N, KL, KU, LDA, INFO
REAL ROWCND, COLCND, AMAX
REAL R(*), C(*)
```

F95 INTERFACE

```
SUBROUTINE GBEQU([M], [N], KL, KU, A, [LDA], R, C,
       ROWCND, COLCND, AMAX, [INFO])

COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, KL, KU, LDA, INFO
REAL :: ROWCND, COLCND, AMAX
REAL, DIMENSION(:) :: R, C

SUBROUTINE GBEQU_64([M], [N], KL, KU, A, [LDA], R, C,
```

```
          ROWCND, COLCND, AMAX, [INFO])

COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, KL, KU, LDA, INFO
REAL :: ROWCND, COLCND, AMAX
REAL, DIMENSION(:) :: R, C
```

C INTERFACE

```
#include <sunperf.h>

void cgbequ(int m, int n, int kl, int ku,  complex  *a,  int
          lda,  float  *r,  float  *c,  float *rowcnd, float
          *colcnd, float *amax, int *info);
void cgbequ_64(long m, long n, long kl, long ku, complex *a,
          long lda, float *r, float *c, float *rowcnd, float
          *colcnd, float *amax, long *info);
```

PURPOSE

cgbequ computes row and column scalings intended to equilibrate an M-by-N band matrix A and reduce its condition number. R returns the row scale factors and C the column scale factors, chosen to try to make the largest element in each row and column of the matrix B with elements B(i,j)=R(i)*A(i,j)*C(j) have absolute value 1.

R(i) and C(j) are restricted to be between SMLNUM = smallest safe number and BIGNUM = largest safe number. Use of these scaling factors is not guaranteed to reduce the condition number of A but works well in practice.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

KL (input)
The number of subdiagonals within the band of A. KL >= 0.

KU (input)
The number of superdiagonals within the band of A. KU >= 0.

A (input) The band matrix A, stored in rows 1 to KL+KU+1. The j-th column of A is stored in the j-th column of the array A as follows: A(ku+1+i-j,j) = A(i,j) for max(1,j-ku)<=i<=min(m,j+kl).

LDA (input)
The leading dimension of the array A. LDA >= KL+KU+1.

R (output)
If INFO = 0, or INFO > M, R contains the row scale factors for A.

C (output)
If INFO = 0, C contains the column scale factors for A.

ROWCND (output)
If INFO = 0 or INFO > M, ROWCND contains the ratio of the smallest R(i) to the largest R(i). If ROWCND >= 0.1 and AMAX is neither too large nor too small, it is not worth scaling by R.

COLCND (output)
If INFO = 0, COLCND contains the ratio of the smallest C(i) to the largest C(i). If COLCND >= 0.1, it is not worth scaling by C.

AMAX (output)
Absolute value of largest matrix element. If AMAX is very close to overflow or very close to underflow, the matrix should be scaled.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= M: the i-th row of A is exactly zero
> M: the (i-M)-th column of A is exactly zero

Contents

NAME

```
cgbmv - perform one of the matrix-vector operations    y  :=
alpha*A*x  +  beta*y, or y := alpha*A'*x + beta*y, or   y :=
alpha*conjg( A' )*x + beta*y
```

SYNOPSIS

```
SUBROUTINE CGBMV(TRANSA, M, N, NSUB, NSUPER, ALPHA, A, LDA, X, INCX,
      BETA, Y, INCY)

CHARACTER * 1 TRANSA
COMPLEX ALPHA, BETA
COMPLEX A(LDA,*), X(*), Y(*)
INTEGER M, N, NSUB, NSUPER, LDA, INCX, INCY

SUBROUTINE CGBMV_64(TRANSA, M, N, NSUB, NSUPER, ALPHA, A, LDA, X,
      INCX, BETA, Y, INCY)

CHARACTER * 1 TRANSA
COMPLEX ALPHA, BETA
COMPLEX A(LDA,*), X(*), Y(*)
INTEGER*8 M, N, NSUB, NSUPER, LDA, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE GBMV([TRANSA], [M], [N], NSUB, NSUPER, ALPHA, A, [LDA], X,
       [INCX], BETA, Y, [INCY])

CHARACTER(LEN=1) :: TRANSA
COMPLEX :: ALPHA, BETA
COMPLEX, DIMENSION(:) :: X, Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, NSUB, NSUPER, LDA, INCX, INCY
```

```
SUBROUTINE GBMV_64([TRANSA], [M], [N], NSUB, NSUPER, ALPHA, A, [LDA],
       X, [INCX], BETA, Y, [INCY])

CHARACTER(LEN=1) :: TRANSA
COMPLEX :: ALPHA, BETA
COMPLEX, DIMENSION(:) :: X, Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, NSUB, NSUPER, LDA, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>

void cgbmv(char transa, int m, int n, int nsub, int  nsuper,
          complex  *alpha,  complex *a, int lda, complex *x,
          int incx, complex *beta, complex *y, int incy);
void cgbmv_64(char transa, long m, long n, long  nsub,  long
          nsuper, complex *alpha, complex *a, long lda, com-
          plex *x, long incx,  complex  *beta,  complex  *y,
          long incy);
```

PURPOSE

cgbmv performs one of the matrix-vector operations y := alpha*A*x + beta*y, or y := alpha*A'*x + beta*y, or y := alpha*conjg(A')*x + beta*y where alpha and beta are scalars, x and y are vectors and A is an m by n band matrix, with nsub sub-diagonals and nsuper super-diagonals.

ARGUMENTS

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:
TRANSA = 'N' or 'n'   y := alpha*A*x + beta*y.
TRANSA = 'T' or 't'   y := alpha*A'*x + beta*y.
TRANSA = 'C' or 'c'   y := alpha*conjg(A')*x + beta*y.
Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

M (input)
On entry, M specifies the number of rows of the matrix A. M must be at least zero. Unchanged on exit.

N (input)

On entry, N specifies the number of columns of the matrix A. N must be at least zero. Unchanged on exit.

NSUB (input)
On entry, NSUB specifies the number of sub-diagonals of the matrix A. NSUB must satisfy 0 .le. NSUB. Unchanged on exit.

NSUPER (input)
On entry, NSUPER specifies the number of super-diagonals of the matrix A. NSUPER must satisfy 0 .le. NSUPER. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
Before entry, the leading (nsub + nsuper + 1) by n part of the array A must contain the matrix of coefficients, supplied column by column, with the leading diagonal of the matrix in row (nsuper + 1) of the array, the first super-diagonal starting at position 2 in row nsuper, the first sub-diagonal starting at position 1 in row (nsuper + 2), and so on. Elements in the array A that do not correspond to elements in the band matrix (such as the top left nsuper by nsuper triangle) are not referenced. The following program segment will transfer a band matrix from conventional full matrix storage to band storage:

```
    DO 20, J = 1, N
      K = NSUPER + 1 - J
      DO 10, I = MAX( 1, J - NSUPER ), MIN( M, J +
NSUB )
        A( K + I, J ) = matrix( I, J )
10    CONTINUE
20 CONTINUE
```

Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA must be at least (nsub + nsuper + 1). Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)) when TRANSA = 'N' or

'n' and at least (1 + (m - 1)*abs(INCX)) otherwise. Before entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. When BETA is supplied as zero then Y need not be set on input. Unchanged on exit.

Y (input/output)
(1 + (m - 1)*abs(INCY)) when TRANSA = 'N' or 'n' and at least (1 + (n - 1)*abs(INCY)) otherwise. Before entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
cgbrfs - improve the computed solution to a system of linear
equations  when  the  coefficient matrix is banded, and pro-
vides error bounds and  backward  error  estimates  for  the
solution
```

SYNOPSIS

```
SUBROUTINE CGBRFS(TRANSA, N, KL, KU, NRHS, A, LDA, AF, LDAF,
      IPIVOT, B, LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*)
REAL FERR(*), BERR(*), WORK2(*)

SUBROUTINE CGBRFS_64(TRANSA, N, KL, KU, NRHS, A, LDA, AF, LDAF,
      IPIVOT, B, LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
REAL FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GBRFS([TRANSA], [N], KL, KU, [NRHS], A, [LDA], AF,
       [LDAF], IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: TRANSA
COMPLEX, DIMENSION(:) :: WORK
```

```
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER :: N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE GBRFS_64([TRANSA], [N], KL, KU, [NRHS], A, [LDA],
       AF, [LDAF], IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: TRANSA
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cgbrfs(char transa, int n, int kl, int  ku,  int  nrhs,
          complex  *a,  int  lda, complex *af, int ldaf, int
          *ipivot, complex *b, int ldb, complex *x, int ldx,
          float *ferr, float *berr, int *info);

void cgbrfs_64(char transa, long n, long kl, long  ku,  long
          nrhs,  complex  *a,  long  lda,  complex *af, long
          ldaf, long *ipivot, complex *b, long ldb,  complex
          *x,  long  ldx,  float  *ferr,  float  *berr, long
          *info);
```

PURPOSE

cgbrfs improves the computed solution to a system of linear equations when the coefficient matrix is banded, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

```
TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A.  N >= 0.
```

KL (input)
The number of subdiagonals within the band of A. KL >= 0.

KU (input)
The number of superdiagonals within the band of A. KU >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The original band matrix A, stored in rows 1 to KL+KU+1. The j-th column of A is stored in the j-th column of the array A as follows: A(ku+1+i-j,j) = A(i,j) for max(1,j-ku)<=i<=min(n,j+kl).

LDA (input)
The leading dimension of the array A. LDA >= KL+KU+1.

AF (input)
Details of the LU factorization of the band matrix A, as computed by CGBTRF. U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1.

LDAF (input)
The leading dimension of the array AF. LDAF >= 2*KL*KU+1.

IPIVOT (input)
The pivot indices from CGBTRF; for 1<=i<=N, row i of the matrix was interchanged with row IPIVOT(i).

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by CGBTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >=

max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
cgbsv - compute the solution to a complex system  of  linear
equations  A  *  X  = B, where A is a band matrix of order N
with KL subdiagonals and KU superdiagonals, and X and B  are
N-by-NRHS matrices
```

SYNOPSIS

```
SUBROUTINE CGBSV(N, KL, KU, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, KL, KU, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE CGBSV_64(N, KL, KU, NRHS, A, LDA, IPIVOT, B, LDB,
      INFO)

COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, KL, KU, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE GBSV([N], KL, KU, [NRHS], A, [LDA], IPIVOT, B, [LDB],
       [INFO])

COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: N, KL, KU, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE GBSV_64([N], KL, KU, [NRHS], A, [LDA], IPIVOT, B,
       [LDB], [INFO])
```

```
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: N, KL, KU, NRHS, LDA, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void cgbsv(int n, int kl, int ku, int nrhs, complex *a,  int
          lda, int *ipivot, complex *b, int ldb, int *info);

void cgbsv_64(long n, long kl, long ku, long  nrhs,  complex
          *a,  long lda, long *ipivot, complex *b, long ldb,
          long *info);
```

PURPOSE

cgbsv computes the solution to a complex system of linear equations A * X = B, where A is a band matrix of order N with KL subdiagonals and KU superdiagonals, and X and B are N-by-NRHS matrices.

The LU decomposition with partial pivoting and row interchanges is used to factor A as A = L * U, where L is a product of permutation and unit lower triangular matrices with KL subdiagonals, and U is upper triangular with KL+KU superdiagonals. The factored form of A is then used to solve the system of equations A * X = B.

ARGUMENTS

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

KL (input)
The number of subdiagonals within the band of A. KL >= 0.

KU (input)
The number of superdiagonals within the band of A. KU >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input/output)
On entry, the matrix A in band storage, in rows

KL+1 to 2*KL+KU+1; rows 1 to KL of the array need not be set. The j-th column of A is stored in the j-th column of the array A as follows: A(KL+KU+1+i-j,j) = A(i,j) for max(1,j-KU)<=i<=min(N,j+KL) On exit, details of the factorization: U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1. See below for further details.

LDA (input)
The leading dimension of the array A. LDA >= 2*KL+KU+1.

IPIVOT (output)
The pivot indices that define the permutation matrix P; row i of the matrix was interchanged with row IPIVOT(i).

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly singular, and the solution has not been computed.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when M = N = 6, KL = 2, KU = 1:

On entry: On exit:

```
    *    *    *    +    +    +       *    *    *   u14   u25
u36
    *    *    +    +    +    +       *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
```

```
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
   a21  a32  a43  a54  a65   *      m21  m32  m43  m54   m65
*
   a31  a42  a53  a64   *    *      m31  m42  m53   m64    *
*
```

Array elements marked * are not used by the routine; elements marked + need not be set on entry, but are required by the routine to store elements of U because of fill-in resulting from the row interchanges.

Contents

NAME

```
cgbsvx - use the LU factorization to compute the solution to
a  complex  system of linear equations A * X = B, A**T * X =
B, or A**H * X = B,
```

SYNOPSIS

```
SUBROUTINE CGBSVX(FACT, TRANSA, N, KL, KU, NRHS, A, LDA, AF,
      LDAF, IPIVOT, EQUED, R, C, B, LDB, X, LDX, RCOND, FERR,
      BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, TRANSA, EQUED
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*)
REAL RCOND
REAL R(*), C(*), FERR(*), BERR(*), WORK2(*)

SUBROUTINE CGBSVX_64(FACT, TRANSA, N, KL, KU, NRHS, A, LDA, AF,
      LDAF, IPIVOT, EQUED, R, C, B, LDB, X, LDX, RCOND, FERR,
      BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, TRANSA, EQUED
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
REAL RCOND
REAL R(*), C(*), FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GBSVX(FACT, [TRANSA], [N], KL, KU, [NRHS], A, [LDA],
       AF, [LDAF], IPIVOT, EQUED, R, C, B, [LDB], X, [LDX],
       RCOND, FERR, BERR, [WORK], [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: FACT, TRANSA, EQUED
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER :: N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL :: RCOND
REAL, DIMENSION(:) :: R, C, FERR, BERR, WORK2

SUBROUTINE GBSVX_64(FACT, [TRANSA], [N], KL, KU, [NRHS], A,
       [LDA], AF, [LDAF], IPIVOT, EQUED, R, C, B, [LDB], X, [LDX],
       RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, TRANSA, EQUED
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL :: RCOND
REAL, DIMENSION(:) :: R, C, FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cgbsvx(char fact, char transa, int n, int kl,  int  ku,
          int  nrhs,  complex  *a, int lda, complex *af, int
          ldaf, int *ipivot, char equed, float *r, float *c,
          complex  *b,  int  ldb, complex *x, int ldx, float
          *rcond, float *ferr, float *berr, int *info);

void cgbsvx_64(char fact, char transa, long n, long kl, long
          ku,  long nrhs, complex *a, long lda, complex *af,
          long ldaf, long *ipivot,  char  equed,  float  *r,
          float  *c,  complex *b, long ldb, complex *x, long
          ldx, float *rcond, float *ferr, float *berr,  long
          *info);
```

PURPOSE

cgbsvx uses the LU factorization to compute the solution to a complex system of linear equations A * X = B, A**T * X = B, or A**H * X = B, where A is a band matrix of order N with KL subdiagonals and KU superdiagonals, and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed by this subroutine:

```
1. If FACT = 'E',  real  scaling  factors  are  computed  to
equilibrate
   the system:
      TRANS = 'N':  diag(R)*A*diag(C)     *inv(diag(C))*X  =
diag(R)*B
      TRANS = 'T': (diag(R)*A*diag(C))**T *inv(diag(R))*X  =
diag(C)*B
      TRANS = 'C': (diag(R)*A*diag(C))**H *inv(diag(R))*X  =
diag(C)*B
   Whether or not the system will be equilibrated depends on
the
   scaling of the matrix A, but if equilibration is used,  A
is
   overwritten by diag(R)*A*diag(C) and B by  diag(R)*B  (if
TRANS='N')
   or diag(C)*B (if TRANS = 'T' or 'C').
2. If FACT = 'N' or 'E', the LU  decomposition  is  used  to
factor the
   matrix A (after equilibration if FACT = 'E') as
      A = L * U,
   where L is a product of permutation and unit  lower  tri-
angular
   matrices with KL subdiagonals, and U is upper  triangular
with
   KL+KU superdiagonals.

3. If some U(i,i)=0, so that U is exactly singular, then the
routine
   returns with INFO = i. Otherwise, the factored form of  A
is used
   to estimate the condition number of the matrix A.  If the
   reciprocal of the condition number is less  than  machine
precision,
   INFO = N+1 is returned as  a  warning,  but  the  routine
still goes on
   to solve for X and  compute  error  bounds  as  described
below.

4. The system of equations is solved for X  using  the  fac-
tored form
   of A.

5. Iterative refinement is applied to improve  the  computed
solution
   matrix and calculate  error  bounds  and  backward  error
estimates
   for it.

6. If equilibration was used, the matrix X is  premultiplied
```

```
by
   diag(C) (if TRANS = 'N') or diag(R) (if TRANS  =  'T'  or
'C') so
   that it solves the original system before equilibration.
```

ARGUMENTS

```
FACT (input)
          Specifies whether or not the factored form of  the
          matrix A is supplied on entry, and if not, whether
          the matrix A should be equilibrated before  it  is
          factored.  = 'F':  On entry, AF and IPIVOT contain
          the factored form of A.  If EQUED is not 'N',  the
          matrix  A  has been equilibrated with scaling fac-
          tors given by R and C.  A, AF, and IPIVOT are  not
          modified.   = 'N':  The matrix A will be copied to
          AF and factored.
          = 'E':  The  matrix  A  will  be  equilibrated  if
          necessary, then copied to AF and factored.

TRANSA (input)
          Specifies the form of the system of  equations.  =
          'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

KL (input)
          The number of subdiagonals within the band  of  A.
          KL >= 0.

KU (input)
          The number of superdiagonals within the band of A.
          KU >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input/output)
          On entry, the matrix A in band storage, in rows  1
          to KL+KU+1.  The j-th column of A is stored in the
          j-th column of the array A as follows:   A(KU+1+i-
          j,j) = A(i,j) for max(1,j-KU)<=i<=min(N,j+kl)
```

If FACT = 'F' and EQUED is not 'N', then A must have been equilibrated by the scaling factors in R and/or C. A is not modified if FACT = 'F' or 'N', or if FACT = 'E' and EQUED = 'N' on exit.

On exit, if EQUED .ne. 'N', A is scaled as follows: EQUED = 'R': A := diag(R) * A
EQUED = 'C': A := A * diag(C)
EQUED = 'B': A := diag(R) * A * diag(C).

LDA (input)
The leading dimension of the array A. LDA >= KL+KU+1.

AF (input/output)
If FACT = 'F', then AF is an input argument and on entry contains details of the LU factorization of the band matrix A, as computed by CGBTRF. U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1. If EQUED .ne. 'N', then AF is the factored form of the equilibrated matrix A.

If FACT = 'N', then AF is an output argument and on exit returns details of the LU factorization of A.

If FACT = 'E', then AF is an output argument and on exit returns details of the LU factorization of the equilibrated matrix A (see the description of A for the form of the equilibrated matrix).

LDAF (input)
The leading dimension of the array AF. LDAF >= 2*KL+KU+1.

IPIVOT (input)
If FACT = 'F', then IPIVOT is an input argument and on entry contains the pivot indices from the factorization A = L*U as computed by CGBTRF; row i of the matrix was interchanged with row IPIVOT(i).

If FACT = 'N', then IPIVOT is an output argument and on exit contains the pivot indices from the factorization A = L*U of the original matrix A.

If FACT = 'E', then IPIVOT is an output argument and on exit contains the pivot indices from the factorization A = L*U of the equilibrated matrix

A.

EQUED (input)
Specifies the form of equilibration that was done.
= 'N': No equilibration (always true if FACT = 'N').
= 'R': Row equilibration, i.e., A has been premultiplied by diag(R). = 'C': Column equilibration, i.e., A has been postmultiplied by diag(C). = 'B': Both row and column equilibration, i.e., A has been replaced by diag(R) * A * diag(C). EQUED is an input argument if FACT = 'F'; otherwise, it is an output argument.

R (input/output)
The row scale factors for A. If EQUED = 'R' or 'B', A is multiplied on the left by diag(R); if EQUED = 'N' or 'C', R is not accessed. R is an input argument if FACT = 'F'; otherwise, R is an output argument. If FACT = 'F' and EQUED = 'R' or 'B', each element of R must be positive.

C (input/output)
The column scale factors for A. If EQUED = 'C' or 'B', A is multiplied on the right by diag(C); if EQUED = 'N' or 'R', C is not accessed. C is an input argument if FACT = 'F'; otherwise, C is an output argument. If FACT = 'F' and EQUED = 'C' or 'B', each element of C must be positive.

B (input/output)
On entry, the right hand side matrix B. On exit, if EQUED = 'N', B is not modified; if TRANSA = 'N' and EQUED = 'R' or 'B', B is overwritten by diag(R)*B; if TRANSA = 'T' or 'C' and EQUED = 'C' or 'B', B is overwritten by diag(C)*B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X to the original system of equations. Note that A and B are modified on exit if EQUED .ne. 'N', and the solution to the equilibrated system is inv(diag(C))*X if TRANSA = 'N' and EQUED = 'C' or 'B', or inv(diag(R))*X if TRANSA = 'T' or 'C' and EQUED = 'R' or 'B'.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A after equilibration (if done). If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N) On exit, WORK2(1) contains the reciprocal pivot growth factor norm(A)/norm(U). The "max absolute element" norm is used. If WORK2(1) is much less than 1, then the stability of the LU factorization of the (equilibrated) matrix A could be poor. This also means that the solution X, condition estimator RCOND, and forward error bound FERR could be unreliable. If factorization fails with 0<INFO<=N, then WORK2(1) contains the reciprocal pivot growth factor for the leading INFO columns of A.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is

<= N: U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
cgbtf2 - compute an LU factorization  of  a  complex  m-by-n
band matrix A using partial pivoting with row interchanges
```

SYNOPSIS

```
SUBROUTINE CGBTF2(M, N, KL, KU, AB, LDAB, IPIV, INFO)

COMPLEX AB(LDAB,*)
INTEGER M, N, KL, KU, LDAB, INFO
INTEGER IPIV(*)

SUBROUTINE CGBTF2_64(M, N, KL, KU, AB, LDAB, IPIV, INFO)

COMPLEX AB(LDAB,*)
INTEGER*8 M, N, KL, KU, LDAB, INFO
INTEGER*8 IPIV(*)
```

F95 INTERFACE

```
SUBROUTINE GBTF2([M], [N], KL, KU, AB, [LDAB], IPIV, [INFO])

COMPLEX, DIMENSION(:,:) :: AB
INTEGER :: M, N, KL, KU, LDAB, INFO
INTEGER, DIMENSION(:) :: IPIV

SUBROUTINE GBTF2_64([M], [N], KL, KU, AB, [LDAB], IPIV, [INFO])

COMPLEX, DIMENSION(:,:) :: AB
INTEGER(8) :: M, N, KL, KU, LDAB, INFO
INTEGER(8), DIMENSION(:) :: IPIV
```

C INTERFACE

```
#include <sunperf.h>

void cgbtf2(int m, int n, int kl, int ku, complex  *ab,  int
          ldab, int *ipiv, int *info);

void cgbtf2_64(long m, long n, long  kl,  long  ku,  complex
          *ab, long ldab, long *ipiv, long *info);
```

PURPOSE

cgbtf2 computes an LU factorization of a complex m-by-n band matrix A using partial pivoting with row interchanges.

This is the unblocked version of the algorithm, calling Level 2 BLAS.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

KL (input)
The number of subdiagonals within the band of A. KL >= 0.

KU (input)
The number of superdiagonals within the band of A. KU >= 0.

AB (input/output)
On entry, the matrix A in band storage, in rows KL+1 to 2*KL+KU+1; rows 1 to KL of the array need not be set. The j-th column of A is stored in the j-th column of the array AB as follows: AB(kl+ku+1+i-j,j) = A(i,j) for max(1,j-ku)<=i<=min(m,j+kl)

On exit, details of the factorization: U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1. See below for further details.

LDAB (input)
The leading dimension of the array AB. LDAB >=

2*KL+KU+1.

IPIV (output)
The pivot indices; for 1 <= i <= min(M,N), row i of the matrix was interchanged with row IPIV(i).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = +i, U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when M = N = 6, KL = 2, KU = 1:

```
On entry:                       On exit:

    *    *    *    +    +    +       *    *    *   u14   u25
u36
    *    *    +    +    +    +       *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
   a21  a32  a43  a54  a65   *      m21  m32  m43  m54   m65
*
   a31  a42  a53  a64   *    *      m31  m42  m53   m64    *
*
```

Array elements marked * are not used by the routine; elements marked + need not be set on entry, but are required by the routine to store elements of U, because of fill-in resulting from the row interchanges.

Contents

NAME

```
cgbtrf - compute an LU factorization  of  a  complex  m-by-n
band matrix A using partial pivoting with row interchanges
```

SYNOPSIS

```
SUBROUTINE CGBTRF(M, N, KL, KU, AB, LDAB, IPIVOT, INFO)

COMPLEX AB(LDAB,N)
INTEGER M, N, KL, KU, LDAB, INFO
INTEGER IPIVOT(MIN(M,N))

SUBROUTINE CGBTRF_64(M, N, KL, KU, AB, LDAB, IPIVOT, INFO)

COMPLEX AB(LDAB,N)
INTEGER*8 M, N, KL, KU, LDAB, INFO
INTEGER*8 IPIVOT(MIN(M,N))
```

F95 INTERFACE

```
SUBROUTINE GBTRF(M, [N], KL, KU, AB, [LDAB], IPIVOT, [INFO])

COMPLEX, DIMENSION(:,:) :: AB
INTEGER :: M, N, KL, KU, LDAB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE GBTRF_64(M, [N], KL, KU, AB, [LDAB], IPIVOT, [INFO])

COMPLEX, DIMENSION(:,:) :: AB
INTEGER(8) :: M, N, KL, KU, LDAB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void cgbtrf(int m, int n, int kl, int ku, complex  *ab,  int
          ldab, int *ipivot, int *info);

void cgbtrf_64(long m, long n, long  kl,  long  ku,  complex
          *ab, long ldab, long *ipivot, long *info);
```

PURPOSE

cgbtrf computes an LU factorization of a complex m-by-n band matrix A using partial pivoting with row interchanges.

This is the blocked version of the algorithm, calling Level 3 BLAS.

ARGUMENTS

M (input) Integer
The number of rows of the matrix A. M >= 0.

N (input) Integer
The number of columns of the matrix A. N >= 0.

KL (input) Integer
The number of subdiagonals within the band of A. KL >= 0.

KU (input) Integer
The number of superdiagonals within the band of A. KU >= 0.

AB (input/output) Complex array of dimension (LDAB,N).
On entry, the matrix A in band storage, in rows KL+1 to 2*KL+KU+1; rows 1 to KL of the array need not be set. The J-th column of A is stored in the J-th column of the array AB as follows: AB(KL+KU+1+I-J,J) = A(I,J) for MAX(1,J-KU)<=I<=MIN(M,J+KL)

On exit, details of the factorization: U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1. See below for further details.

LDAB (input) Integer
The leading dimension of the array AB. LDAB >= 2*KL+KU+1.

IPIVOT (output) Integer array of dimension MIN(M,N)
The pivot indices; for 1 <= I <= MIN(M,N), row I of the matrix was interchanged with row IPIVOT(I).

INFO (output) Integer
= 0: successful exit
< 0: if INFO = -I, the I-th argument had an illegal value
> 0: if INFO = +I, U(I,I) is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when M = N = 6, KL = 2, KU = 1:

```
On entry:                      On exit:

    *    *    *    +    +    +       *    *    *   u14   u25
u36
    *    *    +    +    +    +       *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
   a21  a32  a43  a54  a65   *      m21  m32  m43  m54   m65
*
   a31  a42  a53  a64   *    *      m31  m42  m53   m64    *
*
```

Array elements marked * are not used by the routine; elements marked + need not be set on entry, but are required by the routine to store elements of U because of fill-in resulting from the row interchanges.

Contents

NAME

```
cgbtrs - solve a system of linear equations  A * X = B, A**T
*  X = B, or A**H * X = B with a general band matrix A using
the LU factorization computed by CGBTRF
```

SYNOPSIS

```
SUBROUTINE CGBTRS(TRANSA, N, NSUB, NSUPER, NRHS, A, LDA, IPIVOT, B,
      LDB, INFO)

CHARACTER * 1 TRANSA
COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, NSUB, NSUPER, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE CGBTRS_64(TRANSA, N, NSUB, NSUPER, NRHS, A, LDA, IPIVOT,
      B, LDB, INFO)

CHARACTER * 1 TRANSA
COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, NSUB, NSUPER, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE GBTRS([TRANSA], [N], NSUB, NSUPER, [NRHS], A, [LDA],
       IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: TRANSA
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: N, NSUB, NSUPER, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE GBTRS_64([TRANSA], [N], NSUB, NSUPER, [NRHS], A, [LDA],
```

```
          IPIVOT, B, [LDB], [INFO])

   CHARACTER(LEN=1) :: TRANSA
   COMPLEX, DIMENSION(:,:) :: A, B
   INTEGER(8) :: N, NSUB, NSUPER, NRHS, LDA, LDB, INFO
   INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
   #include <sunperf.h>

   void cgbtrs(char transa, int n, int nsub,  int  nsuper,  int
             nrhs,  complex  *a,  int lda, int *ipivot, complex
             *b, int ldb, int *info);
   void cgbtrs_64(char transa, long n, long nsub, long  nsuper,
             long  nrhs,  complex  *a,  long lda, long *ipivot,
             complex *b, long ldb, long *info);
```

PURPOSE

cgbtrs solves a system of linear equations
A * X = B, A**T * X = B, or A**H * X = B with a general band matrix A using the LU factorization computed by CGBTRF.

ARGUMENTS

```
   TRANSA (input)
             Specifies the form of the system of equations.   =
             'N':  A * X = B     (No transpose)
             = 'T':  A**T * X = B  (Transpose)
             = 'C':  A**H * X = B  (Conjugate transpose)

             TRANSA is defaulted to 'N' for F95 INTERFACE.

   N (input) The order of the matrix A.  N >= 0.

   NSUB (input)
             The number of subdiagonals within the band  of  A.
             NSUB >= 0.

   NSUPER (input)
             The number of superdiagonals within the band of A.
             NSUPER >= 0.

   NRHS (input)
             The number of right hand sides, i.e.,  the  number
             of columns of the matrix B.  NRHS >= 0.
```

```
A (input) Details of the LU factorization of the band matrix
          A, as computed by CGBTRF.  U is stored as an upper
          triangular band matrix with NSUB+NSUPER superdiag-
          onals  in  rows 1 to NSUB+NSUPER+1, and the multi-
          pliers used during the factorization are stored in
          rows NSUB+NSUPER+2 to 2*NSUB+NSUPER+1.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          2*NSUB+NSUPER+1.
IPIVOT (input)
          The pivot indices; for 1 <= i <= N, row i  of  the
          matrix was interchanged with row IPIVOT(i).

B (input/output)
          On entry, the right hand side matrix B.  On  exit,
          the solution matrix X.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
cgebak - form the right or left eigenvectors  of  a  complex
general  matrix  by  backward transformation on the computed
eigenvectors of the balanced matrix output by CGEBAL
```

SYNOPSIS

```
SUBROUTINE CGEBAK(JOB, SIDE, N, ILO, IHI, SCALE, M, V, LDV, INFO)

CHARACTER * 1 JOB, SIDE
COMPLEX V(LDV,*)
INTEGER N, ILO, IHI, M, LDV, INFO
REAL SCALE(*)

SUBROUTINE CGEBAK_64(JOB, SIDE, N, ILO, IHI, SCALE, M, V, LDV, INFO)

CHARACTER * 1 JOB, SIDE
COMPLEX V(LDV,*)
INTEGER*8 N, ILO, IHI, M, LDV, INFO
REAL SCALE(*)
```

F95 INTERFACE

```
SUBROUTINE GEBAK(JOB, SIDE, [N], ILO, IHI, SCALE, [M], V, [LDV],
       [INFO])

CHARACTER(LEN=1) :: JOB, SIDE
COMPLEX, DIMENSION(:,:) :: V
INTEGER :: N, ILO, IHI, M, LDV, INFO
REAL, DIMENSION(:) :: SCALE

SUBROUTINE GEBAK_64(JOB, SIDE, [N], ILO, IHI, SCALE, [M], V, [LDV],
       [INFO])
```

```
CHARACTER(LEN=1) :: JOB, SIDE
COMPLEX, DIMENSION(:,:) :: V
INTEGER(8) :: N, ILO, IHI, M, LDV, INFO
REAL, DIMENSION(:) :: SCALE
```

C INTERFACE

```
#include <sunperf.h>

void cgebak(char job, char side, int n, int  ilo,  int  ihi,
          float  *scale,  int  m,  complex  *v, int ldv, int
          *info);

void cgebak_64(char job, char side, long n, long  ilo,  long
          ihi,  float  *scale, long m, complex *v, long ldv,
          long *info);
```

PURPOSE

cgebak forms the right or left eigenvectors of a complex general matrix by backward transformation on the computed eigenvectors of the balanced matrix output by CGEBAL.

ARGUMENTS

JOB (input)
Specifies the type of backward transformation required: = 'N', do nothing, return immediately; = 'P', do backward transformation for permutation only; = 'S', do backward transformation for scaling only; = 'B', do backward transformations for both permutation and scaling. JOB must be the same as the argument JOB supplied to CGEBAL.

SIDE (input)
= 'R': V contains right eigenvectors;
= 'L': V contains left eigenvectors.

N (input) The number of rows of the matrix V. N >= 0.

ILO (input)
The integer ILO determined by CGEBAL. 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

IHI (input)
The integer IHI determined by CGEBAL. 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

SCALE (input)
Details of the permutation and scaling factors, as returned by CGEBAL.

M (input) The number of columns of the matrix V. M >= 0.

V (input/output)
On entry, the matrix of right or left eigenvectors to be transformed, as returned by CHSEIN or CTREVC. On exit, V is overwritten by the transformed eigenvectors.

LDV (input)
The leading dimension of the array V. LDV >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

```
cgebal - balance a general complex matrix A
```

SYNOPSIS

```
SUBROUTINE CGEBAL(JOB, N, A, LDA, ILO, IHI, SCALE, INFO)

CHARACTER * 1 JOB
COMPLEX A(LDA,*)
INTEGER N, LDA, ILO, IHI, INFO
REAL SCALE(*)

SUBROUTINE CGEBAL_64(JOB, N, A, LDA, ILO, IHI, SCALE, INFO)

CHARACTER * 1 JOB
COMPLEX A(LDA,*)
INTEGER*8 N, LDA, ILO, IHI, INFO
REAL SCALE(*)
```

F95 INTERFACE

```
SUBROUTINE GEBAL(JOB, [N], A, [LDA], ILO, IHI, SCALE, [INFO])

CHARACTER(LEN=1) :: JOB
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, ILO, IHI, INFO
REAL, DIMENSION(:) :: SCALE

SUBROUTINE GEBAL_64(JOB, [N], A, [LDA], ILO, IHI, SCALE, [INFO])

CHARACTER(LEN=1) :: JOB
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, ILO, IHI, INFO
```

```
REAL, DIMENSION(:) :: SCALE
```

C INTERFACE

```
#include <sunperf.h>

void cgebal(char job, int n, complex *a, int lda, int  *ilo,
          int *ihi, float *scale, int *info);

void cgebal_64(char job, long n, complex *a, long lda,  long
          *ilo, long *ihi, float *scale, long *info);
```

PURPOSE

cgebal balances a general complex matrix A. This involves, first, permuting A by a similarity transformation to isolate eigenvalues in the first 1 to ILO-1 and last IHI+1 to N elements on the diagonal; and second, applying a diagonal similarity transformation to rows and columns ILO to IHI to make the rows and columns as close in norm as possible. Both steps are optional.

Balancing may reduce the 1-norm of the matrix, and improve the accuracy of the computed eigenvalues and/or eigenvectors.

ARGUMENTS

JOB (input)
Specifies the operations to be performed on A:
= 'N': none: simply set ILO = 1, IHI = N, SCALE(I) = 1.0 for i = 1,...,N; = 'P': permute only;
= 'S': scale only;
= 'B': both permute and scale.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the input matrix A. On exit, A is overwritten by the balanced matrix. If JOB = 'N', A is not referenced. See Further Details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

ILO (output)

ILO and IHI are set to integers such that on exit A(i,j) = 0 if i > j and j = 1,...,ILO-1 or I = IHI+1,...,N. If JOB = 'N' or 'S', ILO = 1 and IHI = N.

IHI (output)

ILO and IHI are set to integers such that on exit A(i,j) = 0 if i > j and j = 1,...,ILO-1 or I = IHI+1,...,N. If JOB = 'N' or 'S', ILO = 1 and IHI = N.

SCALE (output)

Details of the permutations and scaling factors applied to A. If P(j) is the index of the row and column interchanged with row and column j and D(j) is the scaling factor applied to row and column j, then SCALE(j) = P(j) for j = 1,...,ILO-1 = D(j) for j = ILO,...,IHI = P(j) for j = IHI+1,...,N. The order in which the interchanges are made is N to IHI+1, then 1 to ILO-1.

INFO (output)

= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

The permutations consist of row and column interchanges which put the matrix in the form

```
          ( T1   X   Y  )
  P A P = (  0   B   Z  )
          (  0   0   T2 )
```

where T1 and T2 are upper triangular matrices whose eigenvalues lie along the diagonal. The column indices ILO and IHI mark the starting and ending columns of the submatrix B. Balancing consists of applying a diagonal similarity transformation inv(D) * B * D to make the 1-norms of each row of B and its corresponding column nearly equal. The output matrix is

```
  ( T1     X*D          Y    )
  (  0  inv(D)*B*D  inv(D)*Z ).
  (  0      0           T2   )
```

Information about the permutations P and the diagonal matrix D is returned in the vector SCALE.

This subroutine is based on the EISPACK routine CBAL.

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sity of
  California at Berkeley, USA

Contents

NAME

```
cgebrd - reduce a general complex M-by-N matrix A  to  upper
or lower bidiagonal form B by a unitary transformation
```

SYNOPSIS

```
SUBROUTINE CGEBRD(M, N, A, LDA, D, E, TAUQ, TAUP, WORK, LWORK, INFO)

COMPLEX A(LDA,*), TAUQ(*), TAUP(*), WORK(*)
INTEGER M, N, LDA, LWORK, INFO
REAL D(*), E(*)

SUBROUTINE CGEBRD_64(M, N, A, LDA, D, E, TAUQ, TAUP, WORK, LWORK,
      INFO)

COMPLEX A(LDA,*), TAUQ(*), TAUP(*), WORK(*)
INTEGER*8 M, N, LDA, LWORK, INFO
REAL D(*), E(*)
```

F95 INTERFACE

```
SUBROUTINE GEBRD([M], [N], A, [LDA], D, E, TAUQ, TAUP, [WORK], [LWORK],
       [INFO])

COMPLEX, DIMENSION(:) :: TAUQ, TAUP, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, LWORK, INFO
REAL, DIMENSION(:) :: D, E

SUBROUTINE GEBRD_64([M], [N], A, [LDA], D, E, TAUQ, TAUP, [WORK],
       [LWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAUQ, TAUP, WORK
```

```
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, LWORK, INFO
REAL, DIMENSION(:) :: D, E
```

C INTERFACE

```
#include <sunperf.h>

void cgebrd(int m, int n, complex *a,  int  lda,  float  *d,
          float   *e,  complex  *tauq,  complex  *taup,  int
          *info);

void cgebrd_64(long m, long n, complex *a, long  lda,  float
          *d,  float  *e, complex *tauq, complex *taup, long
          *info);
```

PURPOSE

cgebrd reduces a general complex M-by-N matrix A to upper or lower bidiagonal form B by a unitary transformation: Q**H * A * P = B.

If m >= n, B is upper bidiagonal; if m < n, B is lower bidiagonal.

ARGUMENTS

M (input) The number of rows in the matrix A. M >= 0.

N (input) The number of columns in the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N general matrix to be reduced. On exit, if m >= n, the diagonal and the first superdiagonal are overwritten with the upper bidiagonal matrix B; the elements below the diagonal, with the array TAUQ, represent the unitary matrix Q as a product of elementary reflectors, and the elements above the first superdiagonal, with the array TAUP, represent the unitary matrix P as a product of elementary reflectors; if m < n, the diagonal and the first subdiagonal are overwritten with the lower bidiagonal matrix B; the elements below the first subdiagonal, with the array TAUQ, represent the unitary matrix Q as a product of elementary reflectors, and the elements above the diagonal, with the array TAUP, represent the unitary matrix P as a product of elementary reflec-

tors. See Further Details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

D (output)
The diagonal elements of the bidiagonal matrix B: D(i) = A(i,i).

E (output)
The off-diagonal elements of the bidiagonal matrix B: if m >= n, E(i) = A(i,i+1) for i = 1,2,...,n-1; if m < n, E(i) = A(i+1,i) for i = 1,2,...,m-1.

TAUQ (output)
The scalar factors of the elementary reflectors which represent the unitary matrix Q. See Further Details.

TAUP (output)
The scalar factors of the elementary reflectors which represent the unitary matrix P. See Further Details.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The length of the array WORK. LWORK >= max(1,M,N). For optimum performance LWORK >= (M+N)*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

The matrices Q and P are represented as products of elementary reflectors:

If m >= n,

```
   Q = H(1) H(2) . . . H(n)  and  P = G(1) G(2) . . . G(n-1)

Each H(i) and G(i) has the form:

   H(i) = I - tauq * v * v'  and G(i) = I - taup * u * u'

where tauq and taup are complex scalars, and  v  and  u  are
complex  vectors;  v(1:i-1)  =  0, v(i) = 1, and v(i+1:m) is
stored on exit in A(i+1:m,i); u(1:i) = 0, u(i+1)  =  1,  and
u(i+2:n)  is stored on exit in A(i,i+2:n); tauq is stored in
TAUQ(i) and taup in TAUP(i).
If m < n,

   Q = H(1) H(2) . . . H(m-1)  and  P = G(1) G(2) . . . G(m)

Each H(i) and G(i) has the form:

   H(i) = I - tauq * v * v'  and G(i) = I - taup * u * u'

where tauq and taup are complex scalars, and  v  and  u  are
complex  vectors;  v(1:i)  =  0, v(i+1) = 1, and v(i+2:m) is
stored on exit in A(i+2:m,i); u(1:i-1) = 0, u(i)  =  1,  and
u(i+1:n)  is stored on exit in A(i,i+1:n); tauq is stored in
TAUQ(i) and taup in TAUP(i).

The contents of A on exit are illustrated by  the  following
examples:

m = 6 and n = 5 (m > n):          m = 5 and n = 6 (m < n):

  (  d   e   u1  u1  u1 )           (  d   u1   u1   u1   u1
u1 )
  (  v1  d   e   u2  u2 )           (  e   d    u2   u2   u2
u2 )
  (  v1  v2  d   e   u3 )           (  v1  e    d    u3   u3
u3 )
  (  v1  v2  v3  d   e  )           (  v1  v2   e    d    u4
u4 )
  (  v1  v2  v3  v4  d  )           (  v1   v2   v3   e    d
u5 )
  (  v1  v2  v3  v4  v5 )

where d and e denote diagonal and off-diagonal  elements  of
B, vi denotes an element of the vector defining H(i), and ui
an element of the vector defining G(i).
```

Contents

NAME

```
cgecon - estimate the reciprocal of the condition number  of
a  general  complex  matrix  A,  in either the 1-norm or the
infinity-norm, using the LU factorization computed by CGETRF
```

SYNOPSIS

```
SUBROUTINE CGECON(NORM, N, A, LDA, ANORM, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 NORM
COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, INFO
REAL ANORM, RCOND
REAL WORK2(*)

SUBROUTINE CGECON_64(NORM, N, A, LDA, ANORM, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 NORM
COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, INFO
REAL ANORM, RCOND
REAL WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GECON(NORM, [N], A, [LDA], ANORM, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: NORM
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
REAL :: ANORM, RCOND
```

```
REAL, DIMENSION(:) :: WORK2

SUBROUTINE GECON_64(NORM, [N], A, [LDA], ANORM, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: NORM
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>
void cgecon(char norm, int n, complex  *a,  int  lda,  float
          anorm, float *rcond, int *info);

void cgecon_64(char norm, long  n,  complex  *a,  long  lda,
          float anorm, float *rcond, long *info);
```

PURPOSE

cgecon estimates the reciprocal of the condition number of a general complex matrix A, in either the 1-norm or the infinity-norm, using the LU factorization computed by CGETRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as

```
   RCOND = 1 / ( norm(A) * norm(inv(A)) ).
```

ARGUMENTS

```
NORM (input)
          Specifies whether the 1-norm condition  number  or
          the infinity-norm condition number is required:
          = '1' or 'O':  1-norm;
          = 'I':         Infinity-norm.

N (input) The order of the matrix A.  N >= 0.

A (input) The factors L and U from  the  factorization  A  =
          P*L*U as computed by CGETRF.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).
```

ANORM (input)
If NORM = '1' or 'O', the 1-norm of the original matrix A. If NORM = 'I', the infinity-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(norm(A) * norm(inv(A))).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
cgeequ - compute row and column scalings intended to equili-
brate an M-by-N matrix A and reduce its condition number
```

SYNOPSIS

```
SUBROUTINE CGEEQU(M, N, A, LDA, R, C, ROWCND, COLCND, AMAX,
      INFO)

COMPLEX A(LDA,*)
INTEGER M, N, LDA, INFO
REAL ROWCND, COLCND, AMAX
REAL R(*), C(*)

SUBROUTINE CGEEQU_64(M, N, A, LDA, R, C, ROWCND, COLCND,
      AMAX, INFO)

COMPLEX A(LDA,*)
INTEGER*8 M, N, LDA, INFO
REAL ROWCND, COLCND, AMAX
REAL R(*), C(*)
```

F95 INTERFACE

```
SUBROUTINE GEEQU([M], [N], A, [LDA], R, C, ROWCND, COLCND,
       AMAX, [INFO])

COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, INFO
REAL :: ROWCND, COLCND, AMAX
REAL, DIMENSION(:) :: R, C

SUBROUTINE GEEQU_64([M], [N], A, [LDA], R, C, ROWCND, COLCND,
       AMAX, [INFO])
```

```
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, INFO
REAL :: ROWCND, COLCND, AMAX
REAL, DIMENSION(:) :: R, C
```

C INTERFACE

```
#include <sunperf.h>

void cgeequ(int m, int n, complex *a,  int  lda,  float  *r,
          float  *c,  float  *rowcnd,  float  *colcnd, float
          *amax, int *info);

void cgeequ_64(long m, long n, complex *a, long  lda,  float
          *r,  float *c, float *rowcnd, float *colcnd, float
          *amax, long *info);
```

PURPOSE

cgeequ computes row and column scalings intended to equilibrate an M-by-N matrix A and reduce its condition number. R returns the row scale factors and C the column scale factors, chosen to try to make the largest element in each row and column of the matrix B with elements B(i,j)=R(i)*A(i,j)*C(j) have absolute value 1.

R(i) and C(j) are restricted to be between SMLNUM = smallest safe number and BIGNUM = largest safe number. Use of these scaling factors is not guaranteed to reduce the condition number of A but works well in practice.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input) The M-by-N matrix whose equilibration factors are to be computed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

R (output)
If INFO = 0 or INFO > M, R contains the row scale factors for A.

C (output)
If INFO = 0, C contains the column scale factors for A.

ROWCND (output)
If INFO = 0 or INFO > M, ROWCND contains the ratio of the smallest R(i) to the largest R(i). If ROWCND >= 0.1 and AMAX is neither too large nor too small, it is not worth scaling by R.

COLCND (output)
If INFO = 0, COLCND contains the ratio of the smallest C(i) to the largest C(i). If COLCND >= 0.1, it is not worth scaling by C.

AMAX (output)
Absolute value of largest matrix element. If AMAX is very close to overflow or very close to underflow, the matrix should be scaled.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= M: the i-th row of A is exactly zero
> M: the (i-M)-th column of A is exactly zero

Contents

NAME

```
cgees - compute for an N-by-N complex nonsymmetric matrix A,
the  eigenvalues,  the  Schur  form  T, and, optionally, the
matrix of Schur vectors Z
```

SYNOPSIS

```
SUBROUTINE CGEES(JOBZ, SORTEV, SELECT, N, A, LDA, NOUT, W, Z, LDZ,
      WORK, LDWORK, WORK2, WORK3, INFO)

CHARACTER * 1 JOBZ, SORTEV
COMPLEX A(LDA,*), W(*), Z(LDZ,*), WORK(*)
INTEGER N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL SELECT
LOGICAL WORK3(*)
REAL WORK2(*)

SUBROUTINE CGEES_64(JOBZ, SORTEV, SELECT, N, A, LDA, NOUT, W, Z, LDZ,
      WORK, LDWORK, WORK2, WORK3, INFO)

CHARACTER * 1 JOBZ, SORTEV
COMPLEX A(LDA,*), W(*), Z(LDZ,*), WORK(*)
INTEGER*8 N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL*8 SELECT
LOGICAL*8 WORK3(*)
REAL WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GEES(JOBZ, SORTEV, [SELECT], [N], A, [LDA], [NOUT], W, [Z], [LDZ],
       [WORK], [LDWORK], [WORK2], [WORK3], [INFO])

CHARACTER(LEN=1) :: JOBZ, SORTEV
COMPLEX, DIMENSION(:) :: W, WORK
```

```
COMPLEX, DIMENSION(:,:) :: A, Z
INTEGER :: N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL :: SELECT
LOGICAL, DIMENSION(:) :: WORK3
REAL, DIMENSION(:) :: WORK2

SUBROUTINE GEES_64(JOBZ, SORTEV, [SELECT], [N], A, [LDA], [NOUT], W, [Z],
       [LDZ], [WORK], [LDWORK], [WORK2], [WORK3], [INFO])

CHARACTER(LEN=1) :: JOBZ, SORTEV
COMPLEX, DIMENSION(:) :: W, WORK
COMPLEX, DIMENSION(:,:) :: A, Z
INTEGER(8) :: N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL(8) :: SELECT
LOGICAL(8), DIMENSION(:) :: WORK3
REAL, DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cgees(char jobz,  char  sortev,  int(*select)(complex),
          int n, complex *a, int lda, int *nout, complex *w,
          complex *z, int ldz, int *info);

void      cgees_64(char       jobz,       char       sortev,
          long(*select)(complex),  long  n, complex *a, long
          lda, long *nout, complex *w, complex *z, long ldz,
          long *info);
```

PURPOSE

cgees computes for an N-by-N complex nonsymmetric matrix A, the eigenvalues, the Schur form T, and, optionally, the matrix of Schur vectors Z. This gives the Schur factorization A = Z*T*(Z**H).

Optionally, it also orders the eigenvalues on the diagonal of the Schur form so that selected eigenvalues are at the top left. The leading columns of Z then form an orthonormal basis for the invariant subspace corresponding to the selected eigenvalues.

A complex matrix is in Schur form if it is upper triangular.

ARGUMENTS

JOBZ (input)
 = 'N': Schur vectors are not computed;

= 'V': Schur vectors are computed.

SORTEV (input)
Specifies whether or not to order the eigenvalues on the diagonal of the Schur form. = 'N': Eigenvalues are not ordered:
= 'S': Eigenvalues are ordered (see SELECT).

SELECT (input)
SELECT must be declared EXTERNAL in the calling subroutine. If SORTEV = 'S', SELECT is used to select eigenvalues to order to the top left of the Schur form. If SORTEV = 'N', SELECT is not referenced. The eigenvalue W(j) is selected if SELECT(W(j)) is true.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the N-by-N matrix A. On exit, A has been overwritten by its Schur form T.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

NOUT (output)
If SORTEV = 'N', NOUT = 0. If SORTEV = 'S', NOUT = number of eigenvalues for which SELECT is true.

W (output)
W contains the computed eigenvalues, in the same order that they appear on the diagonal of the output Schur form T.

Z (output)
If JOBZ = 'V', Z contains the unitary matrix Z of Schur vectors. If JOBZ = 'N', Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1; if JOBZ = 'V', LDZ >= N.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,2*N). For good performance, LDWORK must

generally be larger.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
dimension(N)

WORK3 (workspace)
dimension(N) Not referenced if SORTEV = 'N'.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, and i is
<= N: the QR algorithm failed to compute all the eigenvalues; elements 1:ILO-1 and i+1:N of W contain those eigenvalues which have converged; if JOBZ = 'V', Z contains the matrix which reduces A to its partially converged Schur form. = N+1: the eigenvalues could not be reordered because some eigenvalues were too close to separate (the problem is very ill-conditioned); = N+2: after reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the Schur form no longer satisfy SELECT = .TRUE.. This could also be caused by underflow due to scaling.

Contents

NAME

```
cgeesx - compute for an N-by-N complex  nonsymmetric  matrix
A,  the  eigenvalues, the Schur form T, and, optionally, the
matrix of Schur vectors Z
```

SYNOPSIS

```
SUBROUTINE CGEESX(JOBZ, SORTEV, SELECT, SENSE, N, A, LDA, NOUT, W, Z,
      LDZ, RCONE, RCONV, WORK, LDWORK, WORK2, BWORK3, INFO)

CHARACTER * 1 JOBZ, SORTEV, SENSE
COMPLEX A(LDA,*), W(*), Z(LDZ,*), WORK(*)
INTEGER N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL SELECT
LOGICAL BWORK3(*)
REAL RCONE, RCONV
REAL WORK2(*)

SUBROUTINE CGEESX_64(JOBZ, SORTEV, SELECT, SENSE, N, A, LDA, NOUT, W,
      Z, LDZ, RCONE, RCONV, WORK, LDWORK, WORK2, BWORK3, INFO)

CHARACTER * 1 JOBZ, SORTEV, SENSE
COMPLEX A(LDA,*), W(*), Z(LDZ,*), WORK(*)
INTEGER*8 N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL*8 SELECT
LOGICAL*8 BWORK3(*)
REAL RCONE, RCONV
REAL WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GEESX(JOBZ, SORTEV, [SELECT], SENSE, [N], A, [LDA], NOUT, W,
       [Z], [LDZ], RCONE, RCONV, [WORK], [LDWORK], [WORK2], [BWORK3],
       [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, SORTEV, SENSE
COMPLEX, DIMENSION(:) :: W, WORK
COMPLEX, DIMENSION(:,:) :: A, Z
INTEGER :: N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL :: SELECT
LOGICAL, DIMENSION(:) :: BWORK3
REAL :: RCONE, RCONV
REAL, DIMENSION(:) :: WORK2

SUBROUTINE GEESX_64(JOBZ, SORTEV, [SELECT], SENSE, [N], A, [LDA], NOUT,
       W, [Z], [LDZ], RCONE, RCONV, [WORK], [LDWORK], [WORK2], [BWORK3],
       [INFO])

CHARACTER(LEN=1) :: JOBZ, SORTEV, SENSE
COMPLEX, DIMENSION(:) :: W, WORK
COMPLEX, DIMENSION(:,:) :: A, Z
INTEGER(8) :: N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL(8) :: SELECT
LOGICAL(8), DIMENSION(:) :: BWORK3
REAL :: RCONE, RCONV
REAL, DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cgeesx(char jobz, char  sortev,  int(*select)(complex),
          char sense, int n, complex *a, int lda, int *nout,
          complex *w, complex *z,  int  ldz,  float  *rcone,
          float *rconv, int *info);

void      cgeesx_64(char      jobz,       char       sortev,
          long(*select)(complex),  char  sense, long n, com-
          plex *a, long lda, long *nout, complex *w, complex
          *z,  long  ldz,  float  *rcone, float *rconv, long
          *info);
```

PURPOSE

cgeesx computes for an N-by-N complex nonsymmetric matrix A, the eigenvalues, the Schur form T, and, optionally, the matrix of Schur vectors Z. This gives the Schur factorization A = Z*T*(Z**H).

Optionally, it also orders the eigenvalues on the diagonal of the Schur form so that selected eigenvalues are at the top left; computes a reciprocal condition number for the average of the selected eigenvalues (RCONDE); and computes a reciprocal condition number for the right invariant subspace corresponding to the selected eigenvalues (RCONDV). The leading columns of Z form an orthonormal basis for this invariant subspace.

For further explanation of the reciprocal condition numbers RCONDE and RCONDV, see Section 4.10 of the LAPACK Users' Guide (where these quantities are called s and sep respectively).

A complex matrix is in Schur form if it is upper triangular.

ARGUMENTS

JOBZ (input)
= 'N': Schur vectors are not computed;
= 'V': Schur vectors are computed.

SORTEV (input)
Specifies whether or not to order the eigenvalues on the diagonal of the Schur form. = 'N': Eigenvalues are not ordered;
= 'S': Eigenvalues are ordered (see SELECT).

SELECT (input)
SELECT must be declared EXTERNAL in the calling subroutine. If SORTEV = 'S', SELECT is used to select eigenvalues to order to the top left of the Schur form. If SORTEV = 'N', SELECT is not referenced. An eigenvalue W(j) is selected if SELECT(W(j)) is true.

SENSE (input)
Determines which reciprocal condition numbers are computed. = 'N': None are computed;
= 'E': Computed for average of selected eigenvalues only;
= 'V': Computed for selected right invariant subspace only;
= 'B': Computed for both. If SENSE = 'E', 'V' or 'B', SORTEV must equal 'S'.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the N-by-N matrix A. On exit, A is overwritten by its Schur form T.

LDA (input)
The leading dimension of the array A. LDA >=

max(1,N).

NOUT (output)
If SORTEV = 'N', NOUT = 0. If SORTEV = 'S', NOUT = number of eigenvalues for which SELECT is true.

W (output)
W contains the computed eigenvalues, in the same order that they appear on the diagonal of the output Schur form T.

Z (output)
If JOBZ = 'V', Z contains the unitary matrix Z of Schur vectors. If JOBZ = 'N', Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= N.

RCONE (output)
If SENSE = 'E' or 'B', RCONE contains the reciprocal condition number for the average of the selected eigenvalues. Not referenced if SENSE = 'N' or 'V'.

RCONV (output)
If SENSE = 'V' or 'B', RCONV contains the reciprocal condition number for the selected right invariant subspace. Not referenced if SENSE = 'N' or 'E'.

WORK (workspace)
dimension(LDWORK) On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,2*N). Also, if SENSE = 'E' or 'V' or 'B', LDWORK >= 2*NOUT*(N-NOUT), where NOUT is the number of selected eigenvalues computed by this routine. Note that 2*NOUT*(N-NOUT) <= N*N/2. For good performance, LDWORK must generally be larger.

WORK2 (workspace)
dimension(N)

BWORK3 (workspace)
dimension(N) Not referenced if SORTEV = 'N'.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value.

> 0: if INFO = i, and i is

<= N: the QR algorithm failed to compute all the eigenvalues; elements 1:ILO-1 and i+1:N of W contain those eigenvalues which have converged; if JOBZ = 'V', Z contains the transformation which reduces A to its partially converged Schur form.
= N+1: the eigenvalues could not be reordered because some eigenvalues were too close to separate (the problem is very ill-conditioned); = N+2: after reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the Schur form no longer satisfy SELECT=.TRUE. This could also be caused by underflow due to scaling.

Contents

NAME

```
cgeev - compute for an N-by-N complex nonsymmetric matrix A,
the  eigenvalues  and,  optionally,  the  left  and/or right
eigenvectors
```

SYNOPSIS

```
SUBROUTINE CGEEV(JOBVL, JOBVR, N, A, LDA, W, VL, LDVL, VR, LDVR,
      WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 JOBVL, JOBVR
COMPLEX A(LDA,*), W(*), VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER N, LDA, LDVL, LDVR, LDWORK, INFO
REAL WORK2(*)

SUBROUTINE CGEEV_64(JOBVL, JOBVR, N, A, LDA, W, VL, LDVL, VR, LDVR,
      WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 JOBVL, JOBVR
COMPLEX A(LDA,*), W(*), VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER*8 N, LDA, LDVL, LDVR, LDWORK, INFO
REAL WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GEEV(JOBVL, JOBVR, [N], A, [LDA], W, VL, [LDVL], VR, [LDVR],
       [WORK], [LDWORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBVL, JOBVR
COMPLEX, DIMENSION(:) :: W, WORK
COMPLEX, DIMENSION(:,:) :: A, VL, VR
INTEGER :: N, LDA, LDVL, LDVR, LDWORK, INFO
REAL, DIMENSION(:) :: WORK2
```

```
SUBROUTINE GEEV_64(JOBVL, JOBVR, [N], A, [LDA], W, VL, [LDVL], VR,
       [LDVR], [WORK], [LDWORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBVL, JOBVR
COMPLEX, DIMENSION(:) :: W, WORK
COMPLEX, DIMENSION(:,:) :: A, VL, VR
INTEGER(8) :: N, LDA, LDVL, LDVR, LDWORK, INFO
REAL, DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cgeev(char jobvl, char jobvr, int n,  complex  *a,  int
          lda,  complex  *w,  complex *vl, int ldvl, complex
          *vr, int ldvr, int *info);
void cgeev_64(char jobvl, char jobvr, long  n,  complex  *a,
          long lda, complex *w, complex *vl, long ldvl, com-
          plex *vr, long ldvr, long *info);
```

PURPOSE

cgeev computes for an N-by-N complex nonsymmetric matrix A, the eigenvalues and, optionally, the left and/or right eigenvectors.

The right eigenvector v(j) of A satisfies

A * v(j) = lambda(j) * v(j)

where lambda(j) is its eigenvalue.
The left eigenvector u(j) of A satisfies

u(j)**H * A = lambda(j) * u(j)**H

where u(j)**H denotes the conjugate transpose of u(j).

The computed eigenvectors are normalized to have Euclidean norm equal to 1 and largest component real.

ARGUMENTS

JOBVL (input)
= 'N': left eigenvectors of A are not computed;
= 'V': left eigenvectors of are computed.

JOBVR (input)
= 'N': right eigenvectors of A are not computed;
= 'V': right eigenvectors of A are computed.

N (input) The order of the matrix A. N >= 0.

```
A (input/output)
          On entry, the N-by-N matrix A.   On  exit,  A  has
          been overwritten.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

W (output)
          W contains the computed eigenvalues.

VL (input)
          If JOBVL = 'V', the  left  eigenvectors  u(j)  are
          stored  one after another in the columns of VL, in
          the same order as their eigenvalues.  If  JOBVL  =
          'N',  VL  is  not referenced.  u(j) = VL(:,j), the
          j-th column of VL.

LDVL (input)
          The leading dimension of the array VL.  LDVL >= 1;
          if JOBVL = 'V', LDVL >= N.

VR (input)
          If JOBVR = 'V', the right  eigenvectors  v(j)  are
          stored  one after another in the columns of VR, in
          the same order as their eigenvalues.  If  JOBVR  =
          'N',  VR  is  not referenced.  v(j) = VR(:,j), the
          j-th column of VR.

LDVR (input)
          The leading dimension of the array VR.  LDVR >= 1;
          if JOBVR = 'V', LDVR >= N.

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LDWORK.

LDWORK (input)
          The  dimension  of  the  array  WORK.   LDWORK  >=
          max(1,2*N).   For  good  performance,  LDWORK must
          generally be larger.

          If LDWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LDWORK is issued by XERBLA.

WORK2 (workspace)
          dimension(2*N)
```

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value.

> 0: if INFO = i, the QR algorithm failed to compute all the eigenvalues, and no eigenvectors have been computed; elements and i+1:N of W contain eigenvalues which have converged.

Contents

NAME

```
cgeevx - compute for an N-by-N complex  nonsymmetric  matrix
A,  the  eigenvalues  and, optionally, the left and/or right
eigenvectors
```

SYNOPSIS

```
SUBROUTINE CGEEVX(BALANC, JOBVL, JOBVR, SENSE, N, A, LDA, W, VL,
      LDVL, VR, LDVR, ILO, IHI, SCALE, ABNRM, RCONE, RCONV, WORK,
      LDWORK, WORK2, INFO)

CHARACTER * 1 BALANC, JOBVL, JOBVR, SENSE
COMPLEX A(LDA,*), W(*), VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER N, LDA, LDVL, LDVR, ILO, IHI, LDWORK, INFO
REAL ABNRM
REAL SCALE(*), RCONE(*), RCONV(*), WORK2(*)

SUBROUTINE CGEEVX_64(BALANC, JOBVL, JOBVR, SENSE, N, A, LDA, W, VL,
      LDVL, VR, LDVR, ILO, IHI, SCALE, ABNRM, RCONE, RCONV, WORK,
      LDWORK, WORK2, INFO)

CHARACTER * 1 BALANC, JOBVL, JOBVR, SENSE
COMPLEX A(LDA,*), W(*), VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER*8 N, LDA, LDVL, LDVR, ILO, IHI, LDWORK, INFO
REAL ABNRM
REAL SCALE(*), RCONE(*), RCONV(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GEEVX(BALANC, JOBVL, JOBVR, SENSE, [N], A, [LDA], W, VL,
       [LDVL], VR, [LDVR], ILO, IHI, SCALE, ABNRM, RCONE, RCONV, [WORK],
       LDWORK, [WORK2], [INFO])

CHARACTER(LEN=1) :: BALANC, JOBVL, JOBVR, SENSE
```

```
COMPLEX, DIMENSION(:) :: W, WORK
COMPLEX, DIMENSION(:,:) :: A, VL, VR
INTEGER :: N, LDA, LDVL, LDVR, ILO, IHI, LDWORK, INFO
REAL :: ABNRM
REAL, DIMENSION(:) :: SCALE, RCONE, RCONV, WORK2

SUBROUTINE GEEVX_64(BALANC, JOBVL, JOBVR, SENSE, [N], A, [LDA], W,
       VL, [LDVL], VR, [LDVR], ILO, IHI, SCALE, ABNRM, RCONE, RCONV,
       [WORK], LDWORK, [WORK2], [INFO])

CHARACTER(LEN=1) :: BALANC, JOBVL, JOBVR, SENSE
COMPLEX, DIMENSION(:) :: W, WORK
COMPLEX, DIMENSION(:,:) :: A, VL, VR
INTEGER(8) :: N, LDA, LDVL, LDVR, ILO, IHI, LDWORK, INFO
REAL :: ABNRM
REAL, DIMENSION(:) :: SCALE, RCONE, RCONV, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cgeevx (char, char, char,  char,  int,  complex*,  int,
          complex*,  complex*,  int,  complex*,  int,  int*,
          int*, float*, float*, float*, float*, int*);

void cgeevx_64 (char,  char,  char,  char,  long,  complex*,
          long,  complex*,  complex*,  long, complex*, long,
          long*,  long*,  float*,  float*,  float*,  float*,
          long*);
```

PURPOSE

cgeevx computes for an N-by-N complex nonsymmetric matrix A, the eigenvalues and, optionally, the left and/or right eigenvectors.

Optionally also, it computes a balancing transformation to improve the conditioning of the eigenvalues and eigenvectors (ILO, IHI, SCALE, and ABNRM), reciprocal condition numbers for the eigenvalues (RCONDE), and reciprocal condition numbers for the right
eigenvectors (RCONDV).

```
The right eigenvector v(j) of A satisfies
                 A * v(j) = lambda(j) * v(j)
where lambda(j) is its eigenvalue.
The left eigenvector u(j) of A satisfies
              u(j)**H * A = lambda(j) * u(j)**H
where u(j)**H denotes the conjugate transpose of u(j).
```

The computed eigenvectors are normalized to have Euclidean norm equal to 1 and largest component real.

Balancing a matrix means permuting the rows and columns to make it more nearly upper triangular, and applying a diagonal similarity transformation D * A * D**(-1), where D is a diagonal matrix, to make its rows and columns closer in norm and the condition numbers of its eigenvalues and eigenvectors smaller. The computed reciprocal condition numbers correspond to the balanced matrix. Permuting rows and columns will not change the condition numbers (in exact arithmetic) but diagonal scaling will. For further explanation of balancing, see section 4.10.2 of the LAPACK Users' Guide.

ARGUMENTS

BALANC (input)
Indicates how the input matrix should be diagonally scaled and/or permuted to improve the conditioning of its eigenvalues. = 'N': Do not diagonally scale or permute;
= 'P': Perform permutations to make the matrix more nearly upper triangular. Do not diagonally scale; = 'S': Diagonally scale the matrix, ie. replace A by D*A*D**(-1), where D is a diagonal matrix chosen to make the rows and columns of A more equal in norm. Do not permute; = 'B': Both diagonally scale and permute A.

Computed reciprocal condition numbers will be for the matrix after balancing and/or permuting. Permuting does not change condition numbers (in exact arithmetic), but balancing does.

JOBVL (input)
= 'N': left eigenvectors of A are not computed;
= 'V': left eigenvectors of A are computed. If SENSE = 'E' or 'B', JOBVL must = 'V'.

JOBVR (input)
= 'N': right eigenvectors of A are not computed;
= 'V': right eigenvectors of A are computed. If SENSE = 'E' or 'B', JOBVR must = 'V'.

SENSE (input)
Determines which reciprocal condition numbers are computed. = 'N': None are computed;
= 'E': Computed for eigenvalues only;

```
          = 'V': Computed for right eigenvectors only;
          = 'B': Computed for eigenvalues and  right  eigen-
          vectors.

          If SENSE = 'E' or 'B', both left and right  eigen-
          vectors  must  also  be  computed (JOBVL = 'V' and
          JOBVR = 'V').

N (input) The order of the matrix A. N >= 0.

A (input/output)
          On entry, the N-by-N matrix A.   On  exit,  A  has
          been  overwritten.  If JOBVL = 'V' or JOBVR = 'V',
          A contains the Schur form of the balanced  version
          of the matrix A.
LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

W (output)
          W contains the computed eigenvalues.

VL (input)
          If JOBVL = 'V', the  left  eigenvectors  u(j)  are
          stored  one after another in the columns of VL, in
          the same order as their eigenvalues.  If  JOBVL  =
          'N',  VL  is  not referenced.  u(j) = VL(:,j), the
          j-th column of VL.

LDVL (input)
          The leading dimension of the array VL.  LDVL >= 1;
          if JOBVL = 'V', LDVL >= N.

VR (input)
          If JOBVR = 'V', the right  eigenvectors  v(j)  are
          stored  one after another in the columns of VR, in
          the same order as their eigenvalues.  If  JOBVR  =
          'N',  VR  is  not referenced.  v(j) = VR(:,j), the
          j-th column of VR.

LDVR (input)
          The leading dimension of the array VR.  LDVR >= 1;
          if JOBVR = 'V', LDVR >= N.

ILO (output)
          ILO and IHI are integer values determined  when  A
          was  balanced.   The  balanced A(i,j) = 0 if I > J
          and J = 1,...,ILO-1 or I = IHI+1,...,N.

IHI (output)
```

ILO and IHI are integer values determined when A was balanced. The balanced A(i,j) = 0 if I > J and J = 1,...,ILO-1 or I = IHI+1,...,N.

SCALE (output)
Details of the permutations and scaling factors applied when balancing A. If P(j) is the index of the row and column interchanged with row and column j, and D(j) is the scaling factor applied to row and column j, then SCALE(J) = P(J), for J = 1,...,ILO-1 = D(J), for J = ILO,...,IHI = P(J) for J = IHI+1,...,N. The order in which the interchanges are made is N to IHI+1, then 1 to ILO-1.

ABNRM (output)
The one-norm of the balanced matrix (the maximum of the sum of absolute values of elements of any column).

RCONE (output)
RCONE(j) is the reciprocal condition number of the j-th eigenvalue.

RCONV (output)
RCONV(j) is the reciprocal condition number of the j-th right eigenvector.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (output)
The dimension of the array WORK. If SENSE = 'N' or 'E', LDWORK >= max(1,2*N), and if SENSE = 'V' or 'B', LDWORK >= N*N+2*N. For good performance, LDWORK must generally be larger.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-

gal value.
> 0: if INFO = i, the QR algorithm failed to compute all the eigenvalues, and no eigenvectors or condition numbers have been computed; elements 1:ILO-1 and i+1:N of W contain eigenvalues which have converged.

Contents

NAME

```
cgegs - routine is deprecated and has been replaced by  rou-
tine CGGES
```

SYNOPSIS

```
SUBROUTINE CGEGS(JOBVSL, JOBVSR, N, A, LDA, B, LDB, ALPHA, BETA, VSL,
      LDVSL, VSR, LDVSR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 JOBVSL, JOBVSR
COMPLEX A(LDA,*), B(LDB,*), ALPHA(*), BETA(*), VSL(LDVSL,*),
VSR(LDVSR,*), WORK(*)
INTEGER N, LDA, LDB, LDVSL, LDVSR, LDWORK, INFO
REAL WORK2(*)

SUBROUTINE CGEGS_64(JOBVSL, JOBVSR, N, A, LDA, B, LDB, ALPHA, BETA,
      VSL, LDVSL, VSR, LDVSR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 JOBVSL, JOBVSR
COMPLEX A(LDA,*), B(LDB,*), ALPHA(*), BETA(*), VSL(LDVSL,*),
VSR(LDVSR,*), WORK(*)
INTEGER*8 N, LDA, LDB, LDVSL, LDVSR, LDWORK, INFO
REAL WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GEGS(JOBVSL, JOBVSR, [N], A, [LDA], B, [LDB], ALPHA, BETA,
       VSL, [LDVSL], VSR, [LDVSR], [WORK], [LDWORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR
COMPLEX, DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX, DIMENSION(:,:) :: A, B, VSL, VSR
INTEGER :: N, LDA, LDB, LDVSL, LDVSR, LDWORK, INFO
REAL, DIMENSION(:) :: WORK2
```

```
SUBROUTINE GEGS_64(JOBVSL, JOBVSR, [N], A, [LDA], B, [LDB], ALPHA,
       BETA, VSL, [LDVSL], VSR, [LDVSR], [WORK], [LDWORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR
COMPLEX, DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX, DIMENSION(:,:) :: A, B, VSL, VSR
INTEGER(8) :: N, LDA, LDB, LDVSL, LDVSR, LDWORK, INFO
REAL, DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>
void cgegs(char jobvsl, char jobvsr, int n, complex *a,  int
          lda,  complex *b, int ldb, complex *alpha, complex
          *beta, complex *vsl, int ldvsl, complex *vsr,  int
          ldvsr, int *info);

void cgegs_64(char jobvsl, char jobvsr, long n, complex  *a,
          long  lda,  complex  *b, long ldb, complex *alpha,
          complex *beta, complex *vsl, long  ldvsl,  complex
          *vsr, long ldvsr, long *info);
```

PURPOSE

cgegs routine is deprecated and has been replaced by routine CGGES.

CGEGS computes for a pair of N-by-N complex nonsymmetric matrices A, B: the generalized eigenvalues (alpha, beta), the complex Schur form (A, B), and optionally left and/or right Schur vectors (VSL and VSR).

(If only the generalized eigenvalues are needed, use the driver CGEGV instead.)

A generalized eigenvalue for a pair of matrices (A,B) is, roughly speaking, a scalar w or a ratio alpha/beta = w, such that A - w*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0, and even for both being zero. A good beginning reference is the book, "Matrix Computations", by G. Golub & C. van Loan (Johns Hopkins U. Press)

The (generalized) Schur form of a pair of matrices is the result of multiplying both matrices on the left by one unitary matrix and both on the right by another unitary matrix, these two unitary matrices being chosen so as to bring the pair of matrices into upper triangular form with the diagonal elements of B being non-negative real numbers (this is also called complex Schur form.)

The left and right Schur vectors are the columns of VSL and VSR, respectively, where VSL and VSR are the unitary matrices
which reduce A and B to Schur form:

Schur form of (A,B) = ((VSL)**H A (VSR), (VSL)**H B (VSR))

ARGUMENTS

JOBVSL (input)
= 'N': do not compute the left Schur vectors;
= 'V': compute the left Schur vectors.

JOBVSR (input)
= 'N': do not compute the right Schur vectors;
= 'V': compute the right Schur vectors.

N (input) The order of the matrices A, B, VSL, and VSR. N >= 0.

A (input/output)
On entry, the first of the pair of matrices whose generalized eigenvalues and (optionally) Schur vectors are to be computed. On exit, the generalized Schur form of A.

LDA (input)
The leading dimension of A. LDA >= max(1,N).

B (input/output)
On entry, the second of the pair of matrices whose generalized eigenvalues and (optionally) Schur vectors are to be computed. On exit, the generalized Schur form of B.

LDB (input)
The leading dimension of B. LDB >= max(1,N).

ALPHA (output)
On exit, ALPHA(j)/BETA(j), j=1,...,N, will be the generalized eigenvalues. ALPHA(j), j=1,...,N and BETA(j), j=1,...,N are the diagonals of the complex Schur form (A,B) output by CGEGS. The BETA(j) will be non-negative real.

Note: the quotients ALPHA(j)/BETA(j) may easily over- or underflow, and BETA(j) may even be zero. Thus, the user should avoid naively computing the ratio alpha/beta. However, ALPHA will be always less than and usually comparable with norm(A) in magnitude, and BETA always less than and usually comparable with norm(B).

BETA (output)
See the description of ALPHA.

VSL (input)
If JOBVSL = 'V', VSL will contain the left Schur vectors. (See "Purpose", above.) Not referenced if JOBVSL = 'N'.

LDVSL (input)
The leading dimension of the matrix VSL. LDVSL >= 1, and if JOBVSL = 'V', LDVSL >= N.

VSR (input)
If JOBVSR = 'V', VSR will contain the right Schur vectors. (See "Purpose", above.) Not referenced if JOBVSR = 'N'.

LDVSR (input)
The leading dimension of the matrix VSR. LDVSR >= 1, and if JOBVSR = 'V', LDVSR >= N.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,2*N). For good performance, LDWORK must generally be larger. To compute the optimal value of LDWORK, call ILAENV to get blocksizes (for CGEQRF, CUNMQR, and CUNGQR.) Then compute: NB as the MAX of the blocksizes for CGEQRF, CUNMQR, and CUNGQR; the optimal LDWORK is N*(NB+1).

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
dimension(3*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
=1,...,N: The QZ iteration failed. (A,B) are not in Schur form, but ALPHA(j) and BETA(j) should be correct for j=INFO+1,...,N. > N: errors that usually indicate LAPACK problems:
=N+1: error return from CGGBAL
=N+2: error return from CGEQRF
=N+3: error return from CUNMQR
=N+4: error return from CUNGQR
=N+5: error return from CGGHRD
=N+6: error return from CHGEQZ (other than failed iteration) =N+7: error return from CGGBAK (computing VSL)
=N+8: error return from CGGBAK (computing VSR)
=N+9: error return from CLASCL (various places)

Contents

NAME

```
cgegv - routine is deprecated and has been replaced by  rou-
tine CGGEV
```

SYNOPSIS

```
SUBROUTINE CGEGV(JOBVL, JOBVR, N, A, LDA, B, LDB, ALPHA, BETA, VL,
      LDVL, VR, LDVR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 JOBVL, JOBVR
COMPLEX A(LDA,*), B(LDB,*), ALPHA(*),  BETA(*),  VL(LDVL,*),
VR(LDVR,*), WORK(*)
INTEGER N, LDA, LDB, LDVL, LDVR, LDWORK, INFO
REAL WORK2(*)

SUBROUTINE CGEGV_64(JOBVL, JOBVR, N, A, LDA, B, LDB, ALPHA, BETA, VL,
      LDVL, VR, LDVR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 JOBVL, JOBVR
COMPLEX A(LDA,*), B(LDB,*), ALPHA(*),  BETA(*),  VL(LDVL,*),
VR(LDVR,*), WORK(*)
INTEGER*8 N, LDA, LDB, LDVL, LDVR, LDWORK, INFO
REAL WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GEGV(JOBVL, JOBVR, [N], A, [LDA], B, [LDB], ALPHA, BETA,
       VL, [LDVL], VR, [LDVR], [WORK], [LDWORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBVL, JOBVR
COMPLEX, DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX, DIMENSION(:,:) :: A, B, VL, VR
INTEGER :: N, LDA, LDB, LDVL, LDVR, LDWORK, INFO
```

```
REAL, DIMENSION(:) :: WORK2

SUBROUTINE GEGV_64(JOBVL, JOBVR, [N], A, [LDA], B, [LDB], ALPHA,
       BETA, VL, [LDVL], VR, [LDVR], [WORK], [LDWORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBVL, JOBVR
COMPLEX, DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX, DIMENSION(:,:) :: A, B, VL, VR
INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, LDWORK, INFO
REAL, DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cgegv(char jobvl, char jobvr, int n,  complex  *a,  int
          lda,  complex *b, int ldb, complex *alpha, complex
          *beta, complex *vl, int  ldvl,  complex  *vr,  int
          ldvr, int *info);

void cgegv_64(char jobvl, char jobvr, long  n,  complex  *a,
          long  lda,  complex  *b, long ldb, complex *alpha,
          complex *beta, complex  *vl,  long  ldvl,  complex
          *vr, long ldvr, long *info);
```

PURPOSE

cgegv routine is deprecated and has been replaced by routine CGGEV.

CGEGV computes for a pair of N-by-N complex nonsymmetric matrices A and B, the generalized eigenvalues (alpha, beta), and optionally, the left and/or right generalized eigenvectors (VL and VR).

A generalized eigenvalue for a pair of matrices (A,B) is, roughly speaking, a scalar w or a ratio alpha/beta = w, such that A - w*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0, and even for both being zero. A good beginning reference is the book, "Matrix Computations", by G. Golub & C. van Loan (Johns Hopkins U. Press)

A right generalized eigenvector corresponding to a generalized eigenvalue w for a pair of matrices (A,B) is a vector r such that (A - w B) r = 0 . A left generalized eigenvector is a vector l such that l**H * (A - w B) = 0, where l**H is the
conjugate-transpose of l.

Note: this routine performs "full balancing" on A and B. See "Further Details", below.

ARGUMENTS

JOBVL (input)
= 'N': do not compute the left generalized eigenvectors;
= 'V': compute the left generalized eigenvectors.

JOBVR (input)
= 'N': do not compute the right generalized eigenvectors;
= 'V': compute the right generalized eigenvectors.

N (input) The order of the matrices A, B, VL, and VR. N >= 0.

A (input/output)
On entry, the first of the pair of matrices whose generalized eigenvalues and (optionally) generalized eigenvectors are to be computed. On exit, the contents will have been destroyed. (For a description of the contents of A on exit, see "Further Details", below.)

LDA (input)
The leading dimension of A. LDA >= max(1,N).

B (input/output)
On entry, the second of the pair of matrices whose generalized eigenvalues and (optionally) generalized eigenvectors are to be computed. On exit, the contents will have been destroyed. (For a description of the contents of B on exit, see "Further Details", below.)

LDB (input)
The leading dimension of B. LDB >= max(1,N).

ALPHA (output)
On exit, ALPHA(j)/VL(j), j=1,...,N, will be the generalized eigenvalues.

Note: the quotients ALPHA(j)/VL(j) may easily over- or underflow, and VL(j) may even be zero. Thus, the user should avoid naively computing the ratio alpha/beta. However, ALPHA will be always less than and usually comparable with norm(A) in magnitude, and VL always less than and usually comparable with norm(B).

VL (output)
If JOBVL = 'V', the left generalized eigenvectors. (See "Purpose", above.) Each eigenvector will be scaled so the largest component will have abs(real part) + abs(imag. part) = 1, *except* that for eigenvalues with alpha=beta=0, a zero vector will be returned as the corresponding eigenvector. Not referenced if JOBVL = 'N'.

BETA (output)
If JOBVL = 'V', the left generalized eigenvectors. (See "Purpose", above.) Each eigenvector will be scaled so the largest component will have abs(real part) + abs(imag. part) = 1, *except* that for eigenvalues with alpha=beta=0, a zero vector will be returned as the corresponding eigenvector. Not referenced if JOBVL = 'N'.

LDVL (input)
The leading dimension of the matrix VL. LDVL >= 1, and if JOBVL = 'V', LDVL >= N.

VR (output)
If JOBVR = 'V', the right generalized eigenvectors. (See "Purpose", above.) Each eigenvector will be scaled so the largest component will have abs(real part) + abs(imag. part) = 1, *except* that for eigenvalues with alpha=beta=0, a zero vector will be returned as the corresponding eigenvector. Not referenced if JOBVR = 'N'.

LDVR (input)
The leading dimension of the matrix VR. LDVR >= 1, and if JOBVR = 'V', LDVR >= N.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,2*N). For good performance, LDWORK must generally be larger. To compute the optimal value of LDWORK, call ILAENV to get blocksizes (for CGEQRF, CUNMQR, and CUNGQR.) Then compute: NB as the MAX of the blocksizes for CGEQRF, CUNMQR, and

CUNGQR; The optimal LDWORK is MAX(2*N, N*(NB+1)).

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
dimension(8*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
=1,...,N: The QZ iteration failed. No eigenvectors have been calculated, but ALPHA(j) and VL(j) should be correct for j=INFO+1,...,N. > N: errors that usually indicate LAPACK problems:
=N+1: error return from CGGBAL
=N+2: error return from CGEQRF
=N+3: error return from CUNMQR
=N+4: error return from CUNGQR
=N+5: error return from CGGHRD
=N+6: error return from CHGEQZ (other than failed iteration) =N+7: error return from CTGEVC
=N+8: error return from CGGBAK (computing VL)
=N+9: error return from CGGBAK (computing VR)
=N+10: error return from CLASCL (various calls)

FURTHER DETAILS

Balancing

This driver calls CGGBAL to both permute and scale rows and columns of A and B. The permutations PL and PR are chosen so that PL*A*PR and PL*B*R will be upper triangular except for the diagonal blocks A(i:j,i:j) and B(i:j,i:j), with i and j as close together as possible. The diagonal scaling matrices DL and DR are chosen so that the pair DL*PL*A*PR*DR, DL*PL*B*PR*DR have elements close to one (except for the elements that start out zero.)

After the eigenvalues and eigenvectors of the balanced matrices have been computed, CGGBAK transforms the eigenvectors back to what they would have been (in perfect arithmetic) if they had not been balanced.

Contents of A and B on Exit

-------- -- - --- - -- ----

If any eigenvectors are computed (either JOBVL='V' or JOBVR='V' or both), then on exit the arrays A and B will contain the complex Schur form[*] of the "balanced" versions of A and B. If no eigenvectors are computed, then only the diagonal blocks will be correct.

[*] In other words, upper triangular form.

Contents

NAME

```
cgehrd - reduce a complex general matrix A to upper  Hessen-
berg form H by a unitary similarity transformation
```

SYNOPSIS

```
SUBROUTINE CGEHRD(N, ILO, IHI, A, LDA, TAU, WORKIN, LWORKIN, INFO)

COMPLEX A(LDA,*), TAU(*), WORKIN(*)
INTEGER N, ILO, IHI, LDA, LWORKIN, INFO

SUBROUTINE CGEHRD_64(N, ILO, IHI, A, LDA, TAU, WORKIN, LWORKIN, INFO)

COMPLEX A(LDA,*), TAU(*), WORKIN(*)
INTEGER*8 N, ILO, IHI, LDA, LWORKIN, INFO
```

F95 INTERFACE

```
SUBROUTINE GEHRD([N], ILO, IHI, A, [LDA], TAU, [WORKIN], [LWORKIN],
       [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORKIN
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, ILO, IHI, LDA, LWORKIN, INFO

SUBROUTINE GEHRD_64([N], ILO, IHI, A, [LDA], TAU, [WORKIN], [LWORKIN],
       [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORKIN
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, ILO, IHI, LDA, LWORKIN, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cgehrd(int n, int ilo, int ihi, complex  *a,  int  lda,
          complex *tau, int *info);

void cgehrd_64(long n, long ilo, long ihi, complex *a,  long
          lda, complex *tau, long *info);
```

PURPOSE

cgehrd reduces a complex general matrix A to upper Hessenberg form H by a unitary similarity transformation: Q' * A * Q = H .

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

ILO (input)
It is assumed that A is already upper triangular in rows and columns 1:ILO-1 and IHI+1:N. ILO and IHI are normally set by a previous call to CGEBAL; otherwise they should be set to 1 and N respectively. See Further Details.

IHI (input)
See the description of ILO.

A (input/output)
On entry, the N-by-N general matrix to be reduced. On exit, the upper triangle and the first subdiagonal of A are overwritten with the upper Hessenberg matrix H, and the elements below the first subdiagonal, with the array TAU, represent the unitary matrix Q as a product of elementary reflectors. See Further Details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

TAU (output)
The scalar factors of the elementary reflectors (see Further Details). Elements 1:ILO-1 and IHI:N-1 of TAU are set to zero.

WORKIN (workspace)

On exit, if INFO = 0, WORKIN(1) returns the optimal LWORKIN.

LWORKIN (input)
The length of the array WORKIN. LWORKIN >= max(1,N). For optimum performance LWORKIN >= N*NB, where NB is the optimal blocksize.

If LWORKIN = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORKIN array, returns this value as the first entry of the WORKIN array, and no error message related to LWORKIN is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

The matrix Q is represented as a product of (ihi-ilo) elementary reflectors

```
   Q = H(ilo) H(ilo+1) . . . H(ihi-1).
```

Each H(i) has the form

```
   H(i) = I - tau * v * v'
```

where tau is a complex scalar, and v is a complex vector with v(1:i) = 0, v(i+1) = 1 and v(ihi+1:n) = 0; v(i+2:ihi) is stored on exit in A(i+2:ihi,i), and tau in TAU(i).

The contents of A are illustrated by the following example, with n = 7, ilo = 2 and ihi = 6:

```
on entry,                        on exit,

( a   a   a   a   a   a   a )    (  a   a   h    h    h    h
a  )  (      a   a   a   a   a   a )    (      a   h   h   h
h   a ) (     a   a   a   a   a   a )    (       h    h    h
h    h    h  ) (     a   a   a   a   a   a )    (      v2  h
h   h   h   h ) (     a   a   a   a   a   a )     (       v2
v3   h    h    h    h  )  (     a   a   a   a   a   a )    (
v2  v3  v4  h   h   h ) (                         a  )     (
a )
```

where a denotes an element of the original matrix A, h denotes a modified element of the upper Hessenberg matrix H, and vi denotes an element of the vector defining H(i).

Contents

NAME

```
cgelqf - compute an LQ factorization  of  a  complex  M-by-N
matrix A
```

SYNOPSIS

```
SUBROUTINE CGELQF(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, LDA, LDWORK, INFO

SUBROUTINE CGELQF_64(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, LDA, LDWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE GELQF([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, LDWORK, INFO

SUBROUTINE GELQF_64([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, LDWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void cgelqf(int m, int n, complex *a, int lda, complex *tau,
          int *info);

void cgelqf_64(long m, long n, complex *a, long lda, complex
          *tau, long *info);
```

PURPOSE

cgelqf computes an LQ factorization of a complex M-by-N matrix A: A = L * Q.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, the elements on and below the diagonal of the array contain the m-by-min(m,n) lower trapezoidal matrix L (L is lower triangular if m <= n); the elements above the diagonal, with the array TAU, represent the unitary matrix Q as a product of elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,M). For optimum performance LDWORK >= M*NB, where NB is the optimal blocksize.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

```
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

FURTHER DETAILS

```
The matrix Q is  represented  as  a  product  of  elementary
reflectors

   Q = H(k)' . . . H(2)' H(1)', where k = min(m,n).

Each H(i) has the form
   H(i) = I - tau * v * v'

where tau is a complex scalar, and v  is  a  complex  vector
with v(1:i-1) = 0 and v(i) = 1; conjg(v(i+1:n)) is stored on
exit in A(i,i+1:n), and tau in TAU(i).
```

Contents

NAME

cgels - solve overdetermined or underdetermined complex linear systems involving an M-by-N matrix A, or its conjugate-transpose, using a QR or LQ factorization of A

SYNOPSIS

```
SUBROUTINE CGELS(TRANSA, M, N, NRHS, A, LDA, B, LDB, WORK, LDWORK,
      INFO)

CHARACTER * 1 TRANSA
COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER M, N, NRHS, LDA, LDB, LDWORK, INFO

SUBROUTINE CGELS_64(TRANSA, M, N, NRHS, A, LDA, B, LDB, WORK, LDWORK,
      INFO)

CHARACTER * 1 TRANSA
COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER*8 M, N, NRHS, LDA, LDB, LDWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE GELS([TRANSA], [M], [N], [NRHS], A, [LDA], B, [LDB], [WORK],
       LDWORK, [INFO])

CHARACTER(LEN=1) :: TRANSA
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: M, N, NRHS, LDA, LDB, LDWORK, INFO

SUBROUTINE GELS_64([TRANSA], [M], [N], [NRHS], A, [LDA], B, [LDB],
       [WORK], LDWORK, [INFO])
```

```
CHARACTER(LEN=1) :: TRANSA
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: M, N, NRHS, LDA, LDB, LDWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cgels (char, int, int, int,  complex*,  int,  complex*,
          int, int*);

void cgels_64 (char, long, long, long, complex*, long,  com-
          plex*, long, long*);
```

PURPOSE

cgels solves overdetermined or underdetermined complex linear systems involving an M-by-N matrix A, or its conjugate-transpose, using a QR or LQ factorization of A. It is assumed that A has full rank.

The following options are provided:

1. If TRANS = 'N' and m >= n: find the least squares solution of
   an overdetermined system, i.e., solve the least squares problem
   minimize || B - A*X ||.

2. If TRANS = 'N' and m < n: find the minimum norm solution of
   an underdetermined system A * X = B.

3. If TRANS = 'C' and m >= n: find the minimum norm solution of
   an undetermined system A**H * X = B.

4. If TRANS = 'C' and m < n: find the least squares solution of
   an overdetermined system, i.e., solve the least squares problem
   minimize || B - A**H * X ||.

Several right hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the M-by-NRHS right hand side matrix B and the N-by-NRHS solution matrix X.

ARGUMENTS

TRANSA (input)
= 'N': the linear system involves A;
= 'C': the linear system involves A**H.

TRANSA is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input/output)
On entry, the M-by-N matrix A. if M >= N, A is overwritten by details of its QR factorization as returned by CGEQRF; if M < N, A is overwritten by details of its LQ factorization as returned by CGELQF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the matrix B of right hand side vectors, stored columnwise; B is M-by-NRHS if TRANSA = 'N', or N-by-NRHS if TRANSA = 'C'. On exit, B is overwritten by the solution vectors, stored columnwise: if TRANSA = 'N' and m >= n, rows 1 to n of B contain the least squares solution vectors; the residual sum of squares for the solution in each column is given by the sum of squares of elements N+1 to M in that column; if TRANSA = 'N' and m < n, rows 1 to N of B contain the minimum norm solution vectors; if TRANSA = 'C' and m >= n, rows 1 to M of B contain the minimum norm solution vectors; if TRANSA = 'C' and m < n, rows 1 to M of B contain the least squares solution vectors; the residual sum of squares for the solution in each column is given by the sum of squares of elements M+1 to N in that column.

LDB (input)
The leading dimension of the array B. LDB >= MAX(1,M,N).

```
WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LDWORK.

LDWORK (output)
          The dimension of the array WORK.  LDWORK  >=  max(
          1,  MN  +  max( MN, NRHS ) ).  For optimal perfor-
          mance, LDWORK >= max( 1, MN + max( MN,  NRHS  )*NB
          ).   where  MN  =  min(M,N)  and NB is the optimum
          block size.

          If LDWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LDWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
cgelsd - compute the minimum-norm solution to a real  linear
least squares problem
```

SYNOPSIS

```
SUBROUTINE CGELSD(M, N, NRHS, A, LDA, B, LDB, S, RCOND, RANK, WORK,
      LWORK, RWORK, IWORK, INFO)

COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER IWORK(*)
REAL RCOND
REAL S(*), RWORK(*)

SUBROUTINE CGELSD_64(M, N, NRHS, A, LDA, B, LDB, S, RCOND, RANK,
      WORK, LWORK, RWORK, IWORK, INFO)

COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER*8 M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER*8 IWORK(*)
REAL RCOND
REAL S(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GELSD([M], [N], [NRHS], A, [LDA], B, [LDB], S, RCOND,
       RANK, [WORK], [LWORK], [RWORK], [IWORK], [INFO])

COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
```

```
REAL :: RCOND
REAL, DIMENSION(:) :: S, RWORK

SUBROUTINE GELSD_64([M], [N], [NRHS], A, [LDA], B, [LDB], S, RCOND,
       RANK, [WORK], [LWORK], [RWORK], [IWORK], [INFO])

COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL :: RCOND
REAL, DIMENSION(:) :: S, RWORK
```

C INTERFACE

```
#include <sunperf.h>
void cgelsd(int m, int n, int nrhs,  complex  *a,  int  lda,
          complex  *b,  int  ldb, float *s, float rcond, int
          *rank, int *info);

void cgelsd_64(long m, long n, long nrhs, complex  *a,  long
          lda,  complex *b, long ldb, float *s, float rcond,
          long *rank, long *info);
```

PURPOSE

```
cgelsd computes the minimum-norm solution to a  real  linear
least squares problem:
    minimize 2-norm(| b - A*x |)
using the singular value decomposition (SVD) of A. A  is  an
M-by-N matrix which may be rank-deficient.

Several right hand side vectors b and solution vectors x can
be  handled in a single call; they are stored as the columns
of the M-by-NRHS right hand side matrix B and the  N-by-NRHS
solution matrix X.

The problem is solved in three steps:
(1) Reduce the coefficient matrix A to bidiagonal form with
    Householder tranformations, reducing the original  prob-
lem
    into a "bidiagonal least squares problem" (BLS)
(2) Solve the BLS using a divide and conquer approach.
(3) Apply back all the Householder tranformations to solve
    the original least squares problem.

The effective rank of A is determined by  treating  as  zero
those  singular  values  which are less than RCOND times the
largest singular value.
```

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, A has been destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the M-by-NRHS right hand side matrix B. On exit, B is overwritten by the N-by-NRHS solution matrix X. If m >= n and RANK = n, the residual sum-of-squares for the solution in the i-th column is given by the sum of squares of elements n+1:m in that column.

LDB (input)
The leading dimension of the array B. LDB >= max(1,M,N).

S (output)
The singular values of A in decreasing order. The condition number of A in the 2-norm = S(1)/S(min(m,n)).

RCOND (input)
RCOND is used to determine the effective rank of A. Singular values S(i) <= RCOND*S(1) are treated as zero. If RCOND < 0, machine precision is used instead.

RANK (output)
The effective rank of A, i.e., the number of singular values which are greater than RCOND*S(1).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 1. The exact minimum amount of workspace needed depends on M, N and NRHS. If M >= N, LWORK >= 2*N + N*NRHS. If M < N, LWORK >= 2*M + M*NRHS. For good performance, LWORK should generally be larger.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
If M >= N, LRWORK >= 8*N + 2*N*SMLSIZ + 8*N*NLVL + N*NRHS. If M < N, LRWORK >= 8*M + 2*M*SMLSIZ + 8*M*NLVL + M*NRHS. SMLSIZ is returned by ILAENV and is equal to the maximum size of the subproblems at the bottom of the computation tree (usually about 25), and NLVL = INT(LOG_2(MIN(M,N)/(SMLSIZ+1))) + 1

IWORK (workspace)
LIWORK >= 3 * MINMN * NLVL + 11 * MINMN, where MINMN = MIN(M,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: the algorithm for computing the SVD failed to converge; if INFO = i, i off-diagonal elements of an intermediate bidiagonal form did not converge to zero.

FURTHER DETAILS

Based on contributions by
Ming Gu and Ren-Cang Li, Computer Science Division, University of California at Berkeley, USA

Osni Marques, LBNL/NERSC, USA

Contents

NAME

```
cgelss - compute the minimum  norm  solution  to  a  complex
linear least squares problem
```

SYNOPSIS

```
SUBROUTINE CGELSS(M, N, NRHS, A, LDA, B, LDB, SING, RCOND, IRANK,
      WORK, LDWORK, WORK2, INFO)

COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER M, N, NRHS, LDA, LDB, IRANK, LDWORK, INFO
REAL RCOND
REAL SING(*), WORK2(*)

SUBROUTINE CGELSS_64(M, N, NRHS, A, LDA, B, LDB, SING, RCOND, IRANK,
      WORK, LDWORK, WORK2, INFO)

COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER*8 M, N, NRHS, LDA, LDB, IRANK, LDWORK, INFO
REAL RCOND
REAL SING(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GELSS([M], [N], [NRHS], A, [LDA], B, [LDB], SING, RCOND,
       IRANK, [WORK], [LDWORK], [WORK2], [INFO])

COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: M, N, NRHS, LDA, LDB, IRANK, LDWORK, INFO
REAL :: RCOND
REAL, DIMENSION(:) :: SING, WORK2

SUBROUTINE GELSS_64([M], [N], [NRHS], A, [LDA], B, [LDB], SING,
```

```
          RCOND, IRANK, [WORK], [LDWORK], [WORK2], [INFO])

    COMPLEX, DIMENSION(:) :: WORK
    COMPLEX, DIMENSION(:,:) :: A, B
    INTEGER(8) :: M, N, NRHS, LDA, LDB, IRANK, LDWORK, INFO
    REAL :: RCOND
    REAL, DIMENSION(:) :: SING, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cgelss(int m, int n, int nrhs,  complex  *a,  int  lda,
          complex *b, int ldb, float *sing, float rcond, int
          *irank, int *info);
void cgelss_64(long m, long n, long nrhs, complex  *a,  long
          lda,  complex  *b,  long  ldb,  float *sing, float
          rcond, long *irank, long *info);
```

PURPOSE

cgelss computes the minimum norm solution to a complex linear least squares problem:

Minimize 2-norm(| b - A*x |).

using the singular value decomposition (SVD) of A. A is an M-by-N matrix which may be rank-deficient.

Several right hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the M-by-NRHS right hand side matrix B and the N-by-NRHS solution matrix X.

The effective rank of A is determined by treating as zero those singular values which are less than RCOND times the largest singular value.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input/output)

On entry, the M-by-N matrix A. On exit, the first min(m,n) rows of A are overwritten with its right singular vectors, stored rowwise.

LDA (input)

The leading dimension of the array A. LDA >= max(1,M).

B (input/output)

On entry, the M-by-NRHS right hand side matrix B. On exit, B is overwritten by the N-by-NRHS solution matrix X. If m >= n and IRANK = n, the residual sum-of-squares for the solution in the i-th column is given by the sum of squares of elements n+1:m in that column.

LDB (input)

The leading dimension of the array B. LDB >= max(1,M,N).

SING (output)

The singular values of A in decreasing order. The condition number of A in the 2-norm = SING(1)/SING(min(m,n)).

RCOND (input)

RCOND is used to determine the effective rank of A. Singular values SING(i) <= RCOND*SING(1) are treated as zero. If RCOND < 0, machine precision is used instead.

IRANK (output)

The effective rank of A, i.e., the number of singular values which are greater than RCOND*SING(1).

WORK (workspace)

On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)

The dimension of the array WORK. LDWORK >= 1, and also: LDWORK >= 2*min(M,N) + max(M,N,NRHS) For good performance, LDWORK should generally be larger.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first

entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
dimension(5*min(M,N))

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: the algorithm for computing the SVD failed to converge; if INFO = i, i off-diagonal elements of an intermediate bidiagonal form did not converge to zero.

Contents

NAME

```
cgelsx - routine is deprecated and has been replaced by rou-
tine CGELSY
```

SYNOPSIS

```
SUBROUTINE CGELSX(M, N, NRHS, A, LDA, B, LDB, JPIVOT, RCOND, IRANK,
      WORK, WORK2, INFO)

COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER M, N, NRHS, LDA, LDB, IRANK, INFO
INTEGER JPIVOT(*)
REAL RCOND
REAL WORK2(*)

SUBROUTINE CGELSX_64(M, N, NRHS, A, LDA, B, LDB, JPIVOT, RCOND,
      IRANK, WORK, WORK2, INFO)

COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER*8 M, N, NRHS, LDA, LDB, IRANK, INFO
INTEGER*8 JPIVOT(*)
REAL RCOND
REAL WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GELSX([M], [N], [NRHS], A, [LDA], B, [LDB], JPIVOT, RCOND,
       IRANK, [WORK], [WORK2], [INFO])

COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: M, N, NRHS, LDA, LDB, IRANK, INFO
INTEGER, DIMENSION(:) :: JPIVOT
REAL :: RCOND
```

```
REAL, DIMENSION(:) :: WORK2

SUBROUTINE GELSX_64([M], [N], [NRHS], A, [LDA], B, [LDB], JPIVOT,
       RCOND, IRANK, [WORK], [WORK2], [INFO])

COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: M, N, NRHS, LDA, LDB, IRANK, INFO
INTEGER(8), DIMENSION(:) :: JPIVOT
REAL :: RCOND
REAL, DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>
void cgelsx(int m, int n, int nrhs,  complex  *a,  int  lda,
          complex *b, int ldb, int *jpivot, float rcond, int
          *irank, int *info);

void cgelsx_64(long m, long n, long nrhs, complex  *a,  long
          lda,  complex  *b,  long  ldb, long *jpivot, float
          rcond, long *irank, long *info);
```

PURPOSE

```
cgelsx routine is deprecated and has been replaced  by  rou-
tine CGELSY.

CGELSX computes  the  minimum-norm  solution  to  a  complex
linear least squares problem:
    minimize || A * X - B ||
using a complete orthogonal factorization of A.  A is an  M-
by-N matrix which may be rank-deficient.

Several right hand side vectors b and solution vectors x can
be  handled in a single call; they are stored as the columns
of the M-by-NRHS right hand side matrix B and the  N-by-NRHS
solution matrix X.

The routine first computes a QR  factorization  with  column
pivoting:
    A * P = Q * [ R11 R12 ]
                [  0  R22 ]
with R11 defined as  the  largest  leading  submatrix  whose
estimated  condition number is less than 1/RCOND.  The order
of R11, RANK, is the effective rank of A.

Then, R22 is considered to be negligible, and R12 is annihi-
lated by unitary transformations from the right, arriving at
the complete orthogonal factorization:
```

```
   A * P = Q * [ T11 0 ] * Z
               [  0  0 ]
The minimum-norm solution is then
   X = P * Z' [ inv(T11)*Q1'*B ]
              [        0       ]
where Q1 consists of the first RANK columns of Q.
```

ARGUMENTS

```
M (input) The number of rows of the matrix A.  M >= 0.

N (input) The number of columns of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of matrices B and X. NRHS >= 0.

A (input/output)
          On entry, the M-by-N matrix A.   On  exit,  A  has
          been   overwritten  by  details  of  its  complete
          orthogonal factorization.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,M).

B (input/output)
          On entry, the M-by-NRHS right hand side matrix  B.
          On exit, the N-by-NRHS solution matrix X.  If m >=
          n and IRANK = n, the residual  sum-of-squares  for
          the  solution  in  the i-th column is given by the
          sum of squares of elements N+1:M in that column.

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          max(1,M,N).

JPIVOT (input/output)
          On entry, if JPIVOT(i) .ne. 0, the i-th column  of
          A  is  an  initial  column, otherwise it is a free
          column.  Before the QR  factorization  of  A,  all
          initial  columns are permuted to the leading posi-
          tions; only the remaining free columns  are  moved
          as a result of column pivoting during the factori-
          zation.  On exit, if JPIVOT(i) = k, then the  i-th
          column of A*P was the k-th column of A.

RCOND (input)
```

RCOND is used to determine the effective rank of A, which is defined as the order of the largest leading triangular submatrix R11 in the QR factorization with pivoting of A, whose estimated condition number < 1/RCOND.

IRANK (output)
The effective rank of A, i.e., the order of the submatrix R11. This is the same as the order of the submatrix T11 in the complete orthogonal factorization of A.

WORK (workspace)
(min(M,N) + max(N, 2*min(M,N)+NRHS)),

WORK2 (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
cgelsy - compute the  minimum-norm  solution  to  a  complex
linear least squares problem
```

SYNOPSIS

```
SUBROUTINE CGELSY(M, N, NRHS, A, LDA, B, LDB, JPVT, RCOND, RANK,
      WORK, LWORK, RWORK, INFO)

COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER JPVT(*)
REAL RCOND
REAL RWORK(*)

SUBROUTINE CGELSY_64(M, N, NRHS, A, LDA, B, LDB, JPVT, RCOND, RANK,
      WORK, LWORK, RWORK, INFO)

COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER*8 M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER*8 JPVT(*)
REAL RCOND
REAL RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GELSY([M], [N], [NRHS], A, [LDA], B, [LDB], JPVT, RCOND,
       RANK, [WORK], [LWORK], [RWORK], [INFO])

COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER, DIMENSION(:) :: JPVT
```

```
REAL :: RCOND
REAL, DIMENSION(:) :: RWORK

SUBROUTINE GELSY_64([M], [N], [NRHS], A, [LDA], B, [LDB], JPVT,
       RCOND, RANK, [WORK], [LWORK], [RWORK], [INFO])

COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER(8), DIMENSION(:) :: JPVT
REAL :: RCOND
REAL, DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>
void cgelsy(int m, int n, int nrhs,  complex  *a,  int  lda,
          complex  *b,  int ldb, int *jpvt, float rcond, int
          *rank, int *info);

void cgelsy_64(long m, long n, long nrhs, complex  *a,  long
          lda,  complex  *b,  long  ldb,  long  *jpvt, float
          rcond, long *rank, long *info);
```

PURPOSE

```
cgelsy computes  the  minimum-norm  solution  to  a  complex
linear least squares problem:
    minimize || A * X - B ||
using a complete orthogonal factorization of A.  A is an  M-
by-N matrix which may be rank-deficient.

Several right hand side vectors b and solution vectors x can
be  handled in a single call; they are stored as the columns
of the M-by-NRHS right hand side matrix B and the  N-by-NRHS
solution matrix X.

The routine first computes a QR  factorization  with  column
pivoting:
    A * P = Q * [ R11 R12 ]
                [  0  R22 ]
with R11 defined as  the  largest  leading  submatrix  whose
estimated  condition number is less than 1/RCOND.  The order
of R11, RANK, is the effective rank of A.

Then, R22 is considered to be negligible, and R12 is annihi-
lated by unitary transformations from the right, arriving at
the complete orthogonal factorization:
   A * P = Q * [ T11 0 ] * Z
               [  0  0 ]
```

The minimum-norm solution is then
```
   X = P * Z' [ inv(T11)*Q1'*B ]
              [        0       ]
```
where Q1 consists of the first RANK columns of Q.

This routine is basically identical to the original xGELSX except three differences:
- o The permutation of matrix B (the right hand side) is faster and more simple.
- o The call to the subroutine xGEQPF has been substituted by the the call to the subroutine xGEQP3. This subroutine is a Blas-3 version of the QR factorization with column pivoting.
- o Matrix B (the right hand side) is updated with Blas-3.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of matrices B and X. NRHS >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, A has been overwritten by details of its complete orthogonal factorization.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the M-by-NRHS right hand side matrix B. On exit, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,M,N).

JPVT (input/output)
On entry, if JPVT(i) .ne. 0, the i-th column of A is permuted to the front of AP, otherwise column i is a free column. On exit, if JPVT(i) = k, then

the i-th column of A*P was the k-th column of A.

RCOND (input)
RCOND is used to determine the effective rank of A, which is defined as the order of the largest leading triangular submatrix R11 in the QR factorization with pivoting of A, whose estimated condition number < 1/RCOND.

RANK (output)
The effective rank of A, i.e., the order of the submatrix R11. This is the same as the order of the submatrix T11 in the complete orthogonal factorization of A.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. The unblocked strategy requires that: LWORK >= MN + MAX(2*MN, N+1, MN+NRHS) where MN = min(M,N). The block algorithm requires that: LWORK >= MN + MAX(2*MN, NB*(N+1), MN+MN*NB, MN+NB*NRHS) where NB is an upper bound on the blocksize returned by ILAENV for the routines CGEQP3, CTZRZF, CTZRQF, CUNMQR, and CUNMRZ.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

Based on contributions by
A. Petitet, Computer Science Dept., Univ. of Tenn., Knoxville, USA
E. Quintana-Orti, Depto. de Informatica, Universidad Jaime I, Spain

G. Quintana-Orti, Depto. de Informatica, Universidad Jaime
I, Spain

Contents

NAME

```
cgemm - perform one of the  matrix-matrix  operations  C  :=
alpha*op( A )*op( B ) + beta*C
```

SYNOPSIS

```
SUBROUTINE CGEMM(TRANSA, TRANSB, M, N, K, ALPHA, A, LDA, B, LDB,
      BETA, C, LDC)

CHARACTER * 1 TRANSA, TRANSB
COMPLEX ALPHA, BETA
COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER M, N, K, LDA, LDB, LDC

SUBROUTINE CGEMM_64(TRANSA, TRANSB, M, N, K, ALPHA, A, LDA, B, LDB,
      BETA, C, LDC)

CHARACTER * 1 TRANSA, TRANSB
COMPLEX ALPHA, BETA
COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER*8 M, N, K, LDA, LDB, LDC
```

F95 INTERFACE

```
SUBROUTINE GEMM([TRANSA], [TRANSB], [M], [N], [K], ALPHA, A, [LDA],
       B, [LDB], BETA, C, [LDC])

CHARACTER(LEN=1) :: TRANSA, TRANSB
COMPLEX :: ALPHA, BETA
COMPLEX, DIMENSION(:,:) :: A, B, C
INTEGER :: M, N, K, LDA, LDB, LDC

SUBROUTINE GEMM_64([TRANSA], [TRANSB], [M], [N], [K], ALPHA, A, [LDA],
       B, [LDB], BETA, C, [LDC])
```

```
CHARACTER(LEN=1) :: TRANSA, TRANSB
COMPLEX :: ALPHA, BETA
COMPLEX, DIMENSION(:,:) :: A, B, C
INTEGER(8) :: M, N, K, LDA, LDB, LDC
```

C INTERFACE

```
#include <sunperf.h>

void cgemm(char transa, char transb, int m, int  n,  int  k,
          complex  *alpha,  complex *a, int lda, complex *b,
          int ldb, complex *beta, complex *c, int ldc);

void cgemm_64(char transa, char transb, long m, long n, long
          k,  complex  *alpha, complex *a, long lda, complex
          *b, long ldb,  complex  *beta,  complex  *c,  long
          ldc);
```

PURPOSE

cgemm performs one of the matrix-matrix operations

C := alpha*op(A)*op(B) + beta*C

where op(X) is one of

op(X) = X   or   op(X) = X'   or   op(X) = conjg(X'), alpha and beta are scalars, and A, B and C are matrices, with op(A) an m by k matrix, op(B) a k by n matrix and C an m by n matrix.

ARGUMENTS

TRANSA (input)

On entry, TRANSA specifies the form of op(A) to be used in the matrix multiplication as follows:

TRANSA = 'N' or 'n', op(A) = A.

TRANSA = 'T' or 't', op(A) = A'.

TRANSA = 'C' or 'c', op(A) = conjg(A').

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

```
TRANSB (input)
          On entry, TRANSB specifies the form of op( B )  to
          be used in the matrix multiplication as follows:

          TRANSB = 'N' or 'n',  op( B ) = B.

          TRANSB = 'T' or 't',  op( B ) = B'.

          TRANSB = 'C' or 'c',  op( B ) = conjg( B' ).

          Unchanged on exit.

          TRANSB is defaulted to 'N' for F95 INTERFACE.

M (input)
          On entry,  M  specifies  the number  of  rows   of
          the   matrix op( A )  and of the  matrix  C.  M >=
          0.  Unchanged on exit.

N (input)
          On entry,  N  specifies the number  of columns  of
          the  matrix  op(  B ) and the number of columns of
          the matrix C. N >= 0.  Unchanged on exit.

K (input)
          On entry,  K  specifies  the number of columns  of
          the  matrix  op( A ) and the number of rows of the
          matrix op( B ). K >= 0.  Unchanged on exit.

ALPHA (input)
          On  entry,  ALPHA  specifies  the  scalar   alpha.
          Unchanged on exit.

A (input)
          COMPLEX array of DIMENSION ( LDA, ka ),  where  ka
          is K when TRANSA = 'N' or 'n', and is M otherwise.
          Before entry with TRANSA = 'N' or 'n', the leading
          M by K part of the array A must contain the matrix
          A, otherwise the leading K by M part of the  array
          A must contain the matrix A.  Unchanged on exit.

LDA (input)
          On entry, LDA specifies the first dimension  of  A
          as  declared  in  the  calling (sub) program. When
          TRANSA = 'N' or 'n' then LDA >= max(1, M),  other-
          wise LDA >= max(1, K).  Unchanged on exit.

B (input)
          COMPLEX array of DIMENSION ( LDB, kb ),  where  kb
          is  n  when  TRANSB = 'N' or 'n',  and is  k  oth-
```

erwise. Before entry with TRANSB = 'N' or 'n', the leading k by n part of the array B must contain the matrix B, otherwise the leading n by k part of the array B must contain the matrix B. Unchanged on exit.

LDB (input)
On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. When TRANSB = 'N' or 'n' then LDB >= max(1, k), otherwise LDB >= max(1, n). Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. When BETA is supplied as zero then C need not be set on input. Unchanged on exit.

C (input/output)
COMPLEX array of DIMENSION (LDC, n). Before entry, the leading m by n part of the array C must contain the matrix C, except when beta is zero, in which case C need not be set on entry. On exit, the array C is overwritten by the m by n matrix (alpha*op(A)*op(B) + beta*C).

LDC (input)
On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC >= max(1, m). Unchanged on exit.

Contents

NAME

```
cgemv - perform one of the matrix-vector operations    y  :=
alpha*A*x  +  beta*y, or y := alpha*A'*x + beta*y, or   y :=
alpha*conjg( A' )*x + beta*y
```

SYNOPSIS

```
SUBROUTINE CGEMV(TRANSA, M, N, ALPHA, A, LDA, X, INCX, BETA, Y, INCY)

CHARACTER * 1 TRANSA
COMPLEX ALPHA, BETA
COMPLEX A(LDA,*), X(*), Y(*)
INTEGER M, N, LDA, INCX, INCY

SUBROUTINE CGEMV_64(TRANSA, M, N, ALPHA, A, LDA, X, INCX, BETA, Y,
      INCY)

CHARACTER * 1 TRANSA
COMPLEX ALPHA, BETA
COMPLEX A(LDA,*), X(*), Y(*)
INTEGER*8 M, N, LDA, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE GEMV([TRANSA], [M], [N], ALPHA, A, [LDA], X, [INCX], BETA,
       Y, [INCY])

CHARACTER(LEN=1) :: TRANSA
COMPLEX :: ALPHA, BETA
COMPLEX, DIMENSION(:) :: X, Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, INCX, INCY

SUBROUTINE GEMV_64([TRANSA], [M], [N], ALPHA, A, [LDA], X, [INCX],
```

```
          BETA, Y, [INCY])

   CHARACTER(LEN=1) :: TRANSA
   COMPLEX :: ALPHA, BETA
   COMPLEX, DIMENSION(:) :: X, Y
   COMPLEX, DIMENSION(:,:) :: A
   INTEGER(8) :: M, N, LDA, INCX, INCY
```

C INTERFACE

```
   #include <sunperf.h>

   void cgemv(char transa, int m, int n, complex  *alpha,  com-
             plex  *a,  int  lda, complex *x, int incx, complex
             *beta, complex *y, int incy);
   void cgemv_64(char transa, long m, long n,  complex  *alpha,
             complex  *a, long lda, complex *x, long incx, com-
             plex *beta, complex *y, long incy);
```

PURPOSE

cgemv performs one of the matrix-vector operations y := alpha*A*x + beta*y, or y := alpha*A'*x + beta*y, or y := alpha*conjg(A')*x + beta*y where alpha and beta are scalars, x and y are vectors and A is an m by n matrix.

ARGUMENTS

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   y := alpha*A*x + beta*y.

TRANSA = 'T' or 't'   y := alpha*A'*x + beta*y.

TRANSA = 'C' or 'c'   y := alpha*conjg(A')*x + beta*y.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

M (input)
On entry, M specifies the number of rows of the matrix A. M >= 0. Unchanged on exit.

N (input)

On entry, N specifies the number of columns of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
Before entry, the leading m by n part of the array A must contain the matrix of coefficients. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, m). Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)) when TRANSA = 'N' or 'n' and at least (1 + (m - 1)*abs(INCX)) otherwise. Before entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. When BETA is supplied as zero then Y need not be set on input. Unchanged on exit.

Y (input/output)
(1 + (m - 1)*abs(INCY)) when TRANSA = 'N' or 'n' and at least (1 + (n - 1)*abs(INCY)) otherwise. Before entry with BETA non-zero, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
cgeqlf - compute a QL  factorization  of  a  complex  M-by-N
matrix A
```

SYNOPSIS

```
SUBROUTINE CGEQLF(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, LDA, LDWORK, INFO

SUBROUTINE CGEQLF_64(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, LDA, LDWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE GEQLF([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, LDWORK, INFO

SUBROUTINE GEQLF_64([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, LDWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void cgeqlf(int m, int n, complex *a, int lda, complex *tau,
          int *info);

void cgeqlf_64(long m, long n, complex *a, long lda, complex
          *tau, long *info);
```

PURPOSE

cgeqlf computes a QL factorization of a complex M-by-N matrix A: A = Q * L.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, if m >= n, the lower triangle of the subarray A(m-n+1:m,1:n) contains the N-by-N lower triangular matrix L; if m <= n, the elements on and below the (n-m)-th superdiagonal contain the M-by-N lower trapezoidal matrix L; the remaining elements, with the array TAU, represent the unitary matrix Q as a product of elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,N). For optimum performance LDWORK >= N*NB, where NB is the optimal blocksize.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first

entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The matrix Q is represented as a product of elementary reflectors

```
   Q = H(k) . . . H(2) H(1), where k = min(m,n).
```

Each H(i) has the form

```
   H(i) = I - tau * v * v'
```

where tau is a complex scalar, and v is a complex vector with v(m-k+i+1:m) = 0 and v(m-k+i) = 1; v(1:m-k+i-1) is stored on exit in A(1:m-k+i-1,n-k+i), and tau in TAU(i).

Contents

NAME

```
cgeqp3 - compute a QR factorization with column pivoting  of
a matrix A
```

SYNOPSIS

```
SUBROUTINE CGEQP3(M, N, A, LDA, JPVT, TAU, WORK, LWORK, RWORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, LDA, LWORK, INFO
INTEGER JPVT(*)
REAL RWORK(*)

SUBROUTINE CGEQP3_64(M, N, A, LDA, JPVT, TAU, WORK, LWORK, RWORK,
      INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, LDA, LWORK, INFO
INTEGER*8 JPVT(*)
REAL RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEQP3([M], [N], A, [LDA], JPVT, TAU, [WORK], [LWORK],
       [RWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, LWORK, INFO
INTEGER, DIMENSION(:) :: JPVT
REAL, DIMENSION(:) :: RWORK

SUBROUTINE GEQP3_64([M], [N], A, [LDA], JPVT, TAU, [WORK], [LWORK],
```

```
          [RWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, LWORK, INFO
INTEGER(8), DIMENSION(:) :: JPVT
REAL, DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>

void cgeqp3(int m, int n, complex *a, int  lda,  int  *jpvt,
          complex *tau, int *info);

void cgeqp3_64(long m, long n, complex *a,  long  lda,  long
          *jpvt, complex *tau, long *info);
```

PURPOSE

cgeqp3 computes a QR factorization with column pivoting of a matrix A: A*P = Q*R using Level 3 BLAS.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, the upper triangle of the array contains the min(M,N)-by-N upper trapezoidal matrix R; the elements below the diagonal, together with the array TAU, represent the unitary matrix Q as a product of min(M,N) elementary reflectors.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

JPVT (input/output)
On entry, if JPVT(J).ne.0, the J-th column of A is permuted to the front of A*P (a leading column); if JPVT(J)=0, the J-th column of A is a free column. On exit, if JPVT(J)=K, then the J-th column of A*P was the the K-th column of A.

```
TAU (output)
          The scalar factors of the elementary reflectors.

WORK (workspace)
          On exit, if INFO=0, WORK(1)  returns  the  optimal
          LWORK.

LWORK (input)
          The dimension of the array  WORK.  LWORK  >=  N+1.
          For optimal performance LWORK >= ( N+1 )*NB, where
          NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

RWORK (workspace)
          dimension(2*N)

INFO (output)
          = 0: successful exit.
          < 0: if INFO = -i, the i-th argument had an  ille-
          gal value.
```

FURTHER DETAILS

```
The matrix Q is  represented  as  a  product  of  elementary
reflectors

   Q = H(1) H(2) . . . H(k), where k = min(m,n).

Each H(i) has the form

   H(i) = I - tau * v * v'

where tau is a real/complex scalar, and v is a  real/complex
vector with v(1:i-1) = 0 and v(i) = 1; v(i+1:m) is stored on
exit in A(i+1:m,i), and tau in TAU(i).

Based on contributions by
  G. Quintana-Orti, Depto. de Informatica, Universidad Jaime
I, Spain
  X. Sun, Computer Science Dept., Duke University, USA
```

Contents

NAME

```
cgeqpf - routine is deprecated and has been replaced by rou-
tine CGEQP3
```

SYNOPSIS

```
SUBROUTINE CGEQPF(M, N, A, LDA, JPIVOT, TAU, WORK, WORK2, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, LDA, INFO
INTEGER JPIVOT(*)
REAL WORK2(*)

SUBROUTINE CGEQPF_64(M, N, A, LDA, JPIVOT, TAU, WORK, WORK2, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, LDA, INFO
INTEGER*8 JPIVOT(*)
REAL WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GEQPF([M], [N], A, [LDA], JPIVOT, TAU, [WORK], [WORK2],
       [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, INFO
INTEGER, DIMENSION(:) :: JPIVOT
REAL, DIMENSION(:) :: WORK2

SUBROUTINE GEQPF_64([M], [N], A, [LDA], JPIVOT, TAU, [WORK], [WORK2],
       [INFO])
```

```
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, INFO
INTEGER(8), DIMENSION(:) :: JPIVOT
REAL, DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cgeqpf(int m, int n, complex *a, int lda, int  *jpivot,
          complex *tau, int *info);

void cgeqpf_64(long m, long n, complex *a,  long  lda,  long
          *jpivot, complex *tau, long *info);
```

PURPOSE

cgeqpf routine is deprecated and has been replaced by routine CGEQP3.

CGEQPF computes a QR factorization with column pivoting of a complex M-by-N matrix A: A*P = Q*R.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0

A (input/output)
On entry, the M-by-N matrix A. On exit, the upper triangle of the array contains the min(M,N)-by-N upper triangular matrix R; the elements below the diagonal, together with the array TAU, represent the unitary matrix Q as a product of min(m,n) elementary reflectors.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

JPIVOT (input/output)
On entry, if JPIVOT(i) .ne. 0, the i-th column of A is permuted to the front of A*P (a leading column); if JPIVOT(i) = 0, the i-th column of A is a free column. On exit, if JPIVOT(i) = k, then

```
            the i-th column of A*P was the k-th column of A.

TAU (output)
            The scalar factors of the elementary reflectors.

WORK (workspace)
            dimension(N)

WORK2 (workspace)
            dimension(2*N)

INFO (output)
            = 0:  successful exit
            < 0:  if INFO =  -i,  the  i-th  argument  had  an
            illegal value
```

FURTHER DETAILS

```
The matrix Q is  represented  as  a  product  of  elementary
reflectors

   Q = H(1) H(2) . . . H(n)

Each H(i) has the form

   H = I - tau * v * v'

where tau is a complex scalar, and v  is  a  complex  vector
with  v(1:i-1)  = 0 and v(i) = 1; v(i+1:m) is stored on exit
in A(i+1:m,i).

The matrix P is represented in jpvt as follows: If
   jpvt(j) = i
then the jth column of P is the ith canonical unit vector.
```

Contents

NAME

```
cgeqrf - compute a QR  factorization  of  a  complex  M-by-N
matrix A
```

SYNOPSIS

```
SUBROUTINE CGEQRF(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, LDA, LDWORK, INFO

SUBROUTINE CGEQRF_64(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, LDA, LDWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE GEQRF([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, LDWORK, INFO

SUBROUTINE GEQRF_64([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, LDWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void cgeqrf(int m, int n, complex *a, int lda, complex *tau,
          int *info);

void cgeqrf_64(long m, long n, complex *a, long lda, complex
          *tau, long *info);
```

PURPOSE

cgeqrf computes a QR factorization of a complex M-by-N matrix A: A = Q * R.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, the elements on and above the diagonal of the array contain the min(M,N)-by-N upper trapezoidal matrix R (R is upper triangular if m >= n); the elements below the diagonal, with the array TAU, represent the unitary matrix Q as a product of min(m,n) elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,N). For optimum performance LDWORK >= N*NB, where NB is the optimal blocksize.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The matrix Q is represented as a product of elementary reflectors

```
Q = H(1) H(2) . . . H(k), where k = min(m,n).
```

Each H(i) has the form

```
H(i) = I - tau * v * v'
```

where tau is a complex scalar, and v is a complex vector with v(1:i-1) = 0 and v(i) = 1; v(i+1:m) is stored on exit in A(i+1:m,i), and tau in TAU(i).

Contents

NAME

```
cgerc - perform the rank 1 operation   A  :=  alpha*x*conjg(
y' ) + A
```

SYNOPSIS

```
SUBROUTINE CGERC(M, N, ALPHA, X, INCX, Y, INCY, A, LDA)

COMPLEX ALPHA
COMPLEX X(*), Y(*), A(LDA,*)
INTEGER M, N, INCX, INCY, LDA

SUBROUTINE CGERC_64(M, N, ALPHA, X, INCX, Y, INCY, A, LDA)

COMPLEX ALPHA
COMPLEX X(*), Y(*), A(LDA,*)
INTEGER*8 M, N, INCX, INCY, LDA
```

F95 INTERFACE

```
SUBROUTINE GERC([M], [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

COMPLEX :: ALPHA
COMPLEX, DIMENSION(:) :: X, Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, INCX, INCY, LDA

SUBROUTINE GERC_64([M], [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

COMPLEX :: ALPHA
COMPLEX, DIMENSION(:) :: X, Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, INCX, INCY, LDA
```

C INTERFACE

```
#include <sunperf.h>

void cgerc(int m, int n, complex  *alpha,  complex  *x,  int
          incx, complex *y, int incy, complex *a, int lda);

void cgerc_64(long m, long n, complex  *alpha,  complex  *x,
          long incx, complex *y, long incy, complex *a, long
          lda);
```

PURPOSE

cgerc performs the rank 1 operation A := alpha*x*conjg(y') + A where alpha is a scalar, x is an m element vector, y is an n element vector and A is an m by n matrix.

ARGUMENTS

M (input)
On entry, M specifies the number of rows of the matrix A. M >= 0. Unchanged on exit.

N (input)
On entry, N specifies the number of columns of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (m - 1)*abs(INCX)). Before entry, the incremented array X must contain the m element vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. Unchanged on exit.

INCY (input)
On entry, INCY specifies the increment for the

elements of Y. INCY must not be zero. Unchanged on exit.

A (input/output)

Before entry, the leading m by n part of the array A must contain the matrix of coefficients. On exit, A is overwritten by the updated matrix.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, m). Unchanged on exit.

Contents

NAME

```
cgerfs - improve the computed solution to a system of linear
equations and provides error bounds and backward error esti-
mates for the solution
```

SYNOPSIS

```
SUBROUTINE CGERFS(TRANSA, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*)
REAL FERR(*), BERR(*), WORK2(*)

SUBROUTINE CGERFS_64(TRANSA, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B,
      LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
REAL FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GERFS([TRANSA], [N], [NRHS], A, [LDA], AF, [LDAF], IPIVOT,
       B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: TRANSA
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
```

```
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE GERFS_64([TRANSA], [N], [NRHS], A, [LDA], AF, [LDAF],
       IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: TRANSA
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>
void cgerfs(char transa, int n, int nrhs,  complex  *a,  int
          lda,  complex  *af, int ldaf, int *ipivot, complex
          *b, int ldb, complex *x,  int  ldx,  float  *ferr,
          float *berr, int *info);

void cgerfs_64(char transa, long n, long nrhs,  complex  *a,
          long  lda,  complex  *af, long ldaf, long *ipivot,
          complex *b, long ldb, complex *x, long ldx,  float
          *ferr, float *berr, long *info);
```

PURPOSE

cgerfs improves the computed solution to a system of linear equations and provides error bounds and backward error estimates for the solution.

ARGUMENTS

```
TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.
```

A (input) The original N-by-N matrix A.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input)
The factors L and U from the factorization A = P*L*U as computed by CGETRF.

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

IPIVOT (input)
The pivot indices from CGETRF; for 1<=i<=N, row i of the matrix was interchanged with row IPIVOT(i).

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by CGETRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

```
WORK2 (workspace)
          dimension(N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO =  -i,  the  i-th  argument  had  an
          illegal value
```

Contents

NAME

```
cgerqf - compute an RQ factorization  of  a  complex  M-by-N
matrix A
```

SYNOPSIS

```
SUBROUTINE CGERQF(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, LDA, LDWORK, INFO

SUBROUTINE CGERQF_64(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, LDA, LDWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE GERQF([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, LDWORK, INFO

SUBROUTINE GERQF_64([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, LDWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void cgerqf(int m, int n, complex *a, int lda, complex *tau,
          int *info);

void cgerqf_64(long m, long n, complex *a, long lda, complex
          *tau, long *info);
```

PURPOSE

cgerqf computes an RQ factorization of a complex M-by-N matrix A: A = R * Q.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, if m <= n, the upper triangle of the subarray A(1:m,n-m+1:n) contains the M-by-M upper triangular matrix R; if m >= n, the elements on and above the (m-n)-th subdiagonal contain the M-by-N upper trapezoidal matrix R; the remaining elements, with the array TAU, represent the unitary matrix Q as a product of min(m,n) elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,M). For optimum performance LDWORK >= M*NB, where NB is the optimal blocksize.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first

entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The matrix Q is represented as a product of elementary reflectors

```
   Q = H(1)' H(2)' . . . H(k)', where k = min(m,n).
```

Each H(i) has the form

```
   H(i) = I - tau * v * v'
```

where tau is a complex scalar, and v is a complex vector with v(n-k+i+1:n) = 0 and v(n-k+i) = 1; conjg(v(1:n-k+i-1)) is stored on exit in A(m-k+i,1:n-k+i-1), and tau in TAU(i).

Contents

NAME

```
cgeru - perform the rank 1 operation   A := alpha*x*y' + A
```

SYNOPSIS

```
SUBROUTINE CGERU(M, N, ALPHA, X, INCX, Y, INCY, A, LDA)

COMPLEX ALPHA
COMPLEX X(*), Y(*), A(LDA,*)
INTEGER M, N, INCX, INCY, LDA

SUBROUTINE CGERU_64(M, N, ALPHA, X, INCX, Y, INCY, A, LDA)

COMPLEX ALPHA
COMPLEX X(*), Y(*), A(LDA,*)
INTEGER*8 M, N, INCX, INCY, LDA
```

F95 INTERFACE

```
SUBROUTINE GER([M], [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

COMPLEX :: ALPHA
COMPLEX, DIMENSION(:) :: X, Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, INCX, INCY, LDA

SUBROUTINE GER_64([M], [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

COMPLEX :: ALPHA
COMPLEX, DIMENSION(:) :: X, Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, INCX, INCY, LDA
```

C INTERFACE

```
#include <sunperf.h>

void cgeru(int m, int n, complex  *alpha,  complex  *x,  int
          incx, complex *y, int incy, complex *a, int lda);

void cgeru_64(long m, long n, complex  *alpha,  complex  *x,
          long incx, complex *y, long incy, complex *a, long
          lda);
```

PURPOSE

cgeru performs the rank 1 operation A := alpha*x*y' + A where alpha is a scalar, x is an m element vector, y is an n element vector and A is an m by n matrix.

ARGUMENTS

M (input)
On entry, M specifies the number of rows of the matrix A. M >= 0. Unchanged on exit.

N (input)
On entry, N specifies the number of columns of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (m - 1)*abs(INCX)). Before entry, the incremented array X must contain the m element vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. Unchanged on exit.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged

on exit.

A (input/output)

Before entry, the leading m by n part of the array A must contain the matrix of coefficients. On exit, A is overwritten by the updated matrix.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, m). Unchanged on exit.

Contents

NAME

```
cgesdd - compute the singular value decomposition (SVD) of a
complex  M-by-N  matrix  A,  optionally  computing  the left
and/or right singular vectors, by  using  divide-and-conquer
method
```

SYNOPSIS

```
SUBROUTINE CGESDD(JOBZ, M, N, A, LDA, S, U, LDU, VT, LDVT, WORK,
      LWORK, RWORK, IWORK, INFO)

CHARACTER * 1 JOBZ
COMPLEX A(LDA,*), U(LDU,*), VT(LDVT,*), WORK(*)
INTEGER M, N, LDA, LDU, LDVT, LWORK, INFO
INTEGER IWORK(*)
REAL S(*), RWORK(*)

SUBROUTINE CGESDD_64(JOBZ, M, N, A, LDA, S, U, LDU, VT, LDVT, WORK,
      LWORK, RWORK, IWORK, INFO)

CHARACTER * 1 JOBZ
COMPLEX A(LDA,*), U(LDU,*), VT(LDVT,*), WORK(*)
INTEGER*8 M, N, LDA, LDU, LDVT, LWORK, INFO
INTEGER*8 IWORK(*)
REAL S(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GESDD(JOBZ, [M], [N], A, [LDA], S, U, [LDU], VT, [LDVT],
       [WORK], [LWORK], [RWORK], [IWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ
COMPLEX, DIMENSION(:) :: WORK
```

```
COMPLEX, DIMENSION(:,:) :: A, U, VT
INTEGER :: M, N, LDA, LDU, LDVT, LWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: S, RWORK

SUBROUTINE GESDD_64(JOBZ, [M], [N], A, [LDA], S, U, [LDU], VT, [LDVT],
       [WORK], [LWORK], [RWORK], [IWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, U, VT
INTEGER(8) :: M, N, LDA, LDU, LDVT, LWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: S, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void cgesdd(char jobz, int m, int n, complex  *a,  int  lda,
          float  *s,  complex  *u, int ldu, complex *vt, int
          ldvt, int *info);

void cgesdd_64(char jobz, long m, long n, complex  *a,  long
          lda,  float *s, complex *u, long ldu, complex *vt,
          long ldvt, long *info);
```

PURPOSE

cgesdd computes the singular value decomposition (SVD) of a complex M-by-N matrix A, optionally computing the left and/or right singular vectors, by using divide-and-conquer method. The SVD is written

= U * SIGMA * conjugate-transpose(V)

where SIGMA is an M-by-N matrix which is zero except for its min(m,n) diagonal elements, U is an M-by-M unitary matrix, and V is an N-by-N unitary matrix. The diagonal elements of SIGMA are the singular values of A; they are real and non-negative, and are returned in descending order. The first min(m,n) columns of U and V are the left and right singular vectors of A.

Note that the routine returns VT = V**H, not V.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

JOBZ (input)
Specifies options for computing all or part of the matrix U:
= 'A': all M columns of U and all N rows of V**H are returned in the arrays U and VT; = 'S': the first min(M,N) columns of U and the first min(M,N) rows of V**H are returned in the arrays U and VT; = 'O': If M >= N, the first N columns of U are overwritten on the array A and all rows of V**H are returned in the array VT; otherwise, all columns of U are returned in the array U and the first M rows of V**H are overwritten in the array VT; = 'N': no columns of U or rows of V**H are computed.

M (input) The number of rows of the input matrix A. M >= 0.

N (input) The number of columns of the input matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, if JOBZ = 'O', A is overwritten with the first N columns of U (the left singular vectors, stored columnwise) if M >= N; A is overwritten with the first M rows of V**H (the right singular vectors, stored row-wise) otherwise. if JOBZ .ne. 'O', the contents of A are destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

S (output)
The singular values of A, sorted so that S(i) >= S(i+1).

U (output)
UCOL = M if JOBZ = 'A' or JOBZ = 'O' and M < N; UCOL = min(M,N) if JOBZ = 'S'. If JOBZ = 'A' or JOBZ = 'O' and M < N, U contains the M-by-M unitary matrix U; if JOBZ = 'S', U contains the first min(M,N) columns of U (the left singular vectors, stored columnwise); if JOBZ = 'O' and M >= N, or

JOBZ = 'N', U is not referenced.

LDU (input)

The leading dimension of the array U. LDU >= 1; if JOBZ = 'S' or 'A' or JOBZ = 'O' and M < N, LDU >= M.

VT (output)

If JOBZ = 'A' or JOBZ = 'O' and M >= N, VT contains the N-by-N unitary matrix V**H; if JOBZ = 'S', VT contains the first min(M,N) rows of V**H (the right singular vectors, stored rowwise); if JOBZ = 'O' and M < N, or JOBZ = 'N', VT is not referenced.

LDVT (input)

The leading dimension of the array VT. LDVT >= 1; if JOBZ = 'A' or JOBZ = 'O' and M >= N, LDVT >= N; if JOBZ = 'S', LDVT >= min(M,N).

WORK (workspace)

On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)

The dimension of the array WORK. LWORK >= 1. if JOBZ = 'N', LWORK >= 2*min(M,N)+max(M,N). if JOBZ = 'O', LWORK >= 2*min(M,N)*min(M,N)+2*min(M,N)+max(M,N). if JOBZ = 'S' or 'A', LWORK >= min(M,N)*min(M,N)+2*min(M,N)+max(M,N). For good performance, LWORK should generally be larger. If LWORK < 0 but other input arguments are legal, WORK(1) returns optimal LWORK.

RWORK (workspace)

If JOBZ = 'N', LRWORK >= 7*min(M,N). Otherwise, LRWORK >= 5*min(M,N)*min(M,N) + 5*min(M,N)

IWORK (workspace)

dimension(8*MIN(M,N))

INFO (output)

= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: The updating process of SBDSDC did not converge.

FURTHER DETAILS

Based on contributions by
Ming Gu and Huan Ren, Computer Science Division, University of
California at Berkeley, USA

Contents

NAME

```
cgesv - compute the solution to a complex system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE CGESV(N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE CGESV_64(N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE GESV([N], [NRHS], A, [LDA], IPIVOT, B, [LDB], [INFO])

COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: N, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE GESV_64([N], [NRHS], A, [LDA], IPIVOT, B, [LDB], [INFO])

COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>
```

```
void cgesv(int  n,  int  nrhs,  complex  *a,  int  lda,  int
          *ipivot, complex *b, int ldb, int *info);

void cgesv_64(long n, long nrhs, complex *a, long lda,  long
          *ipivot, complex *b, long ldb, long *info);
```

PURPOSE

cgesv computes the solution to a complex system of linear equations
   A * X = B, where A is an N-by-N matrix and X and B are N-by-NRHS matrices.

The LU decomposition with partial pivoting and row interchanges is used to factor A as
   A = P * L * U,
where P is a permutation matrix, L is unit lower triangular, and U is upper triangular. The factored form of A is then used to solve the system of equations A * X = B.

ARGUMENTS

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input/output)
On entry, the N-by-N coefficient matrix A. On exit, the factors L and U from the factorization A = P*L*U; the unit diagonal elements of L are not stored.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (output)
The pivot indices that define the permutation matrix P; row i of the matrix was interchanged with row IPIVOT(i).

B (input/output)
On entry, the N-by-NRHS matrix of right hand side

matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution could not be computed.

Contents

NAME

```
cgesvd - compute the singular value decomposition (SVD) of a
complex  M-by-N  matrix  A,  optionally  computing  the left
and/or right singular vectors
```

SYNOPSIS

```
SUBROUTINE CGESVD(JOBU, JOBVT, M, N, A, LDA, SING, U, LDU, VT, LDVT,
      WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 JOBU, JOBVT
COMPLEX A(LDA,*), U(LDU,*), VT(LDVT,*), WORK(*)
INTEGER M, N, LDA, LDU, LDVT, LDWORK, INFO
REAL SING(*), WORK2(*)

SUBROUTINE CGESVD_64(JOBU, JOBVT, M, N, A, LDA, SING, U, LDU, VT,
      LDVT, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 JOBU, JOBVT
COMPLEX A(LDA,*), U(LDU,*), VT(LDVT,*), WORK(*)
INTEGER*8 M, N, LDA, LDU, LDVT, LDWORK, INFO
REAL SING(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GESVD(JOBU, JOBVT, [M], [N], A, [LDA], SING, U, [LDU], VT,
       [LDVT], [WORK], [LDWORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBU, JOBVT
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, U, VT
INTEGER :: M, N, LDA, LDU, LDVT, LDWORK, INFO
REAL, DIMENSION(:) :: SING, WORK2
```

```
SUBROUTINE GESVD_64(JOBU, JOBVT, [M], [N], A, [LDA], SING, U, [LDU],
       VT, [LDVT], [WORK], [LDWORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBU, JOBVT
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, U, VT
INTEGER(8) :: M, N, LDA, LDU, LDVT, LDWORK, INFO
REAL, DIMENSION(:) :: SING, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cgesvd(char jobu, char jobvt, int m, int n, complex *a,
          int lda, float *sing, complex *u, int ldu, complex
          *vt, int ldvt, int *info);
void cgesvd_64(char jobu, char jobvt, long m, long  n,  com-
          plex  *a,  long lda, float *sing, complex *u, long
          ldu, complex *vt, long ldvt, long *info);
```

PURPOSE

cgesvd computes the singular value decomposition (SVD) of a complex M-by-N matrix A, optionally computing the left and/or right singular vectors. The SVD is written
 = U * SIGMA * conjugate-transpose(V)

where SIGMA is an M-by-N matrix which is zero except for its min(m,n) diagonal elements, U is an M-by-M unitary matrix, and V is an N-by-N unitary matrix. The diagonal elements of SIGMA are the singular values of A; they are real and non-negative, and are returned in descending order. The first min(m,n) columns of U and V are the left and right singular vectors of A.

Note that the routine returns V**H, not V.

ARGUMENTS

JOBU (input)
Specifies options for computing all or part of the matrix U:
= 'A': all M columns of U are returned in array U:
= 'S': the first min(m,n) columns of U (the left singular vectors) are returned in the array U; = 'O': the first min(m,n) columns of U (the left singular vectors) are overwritten on the array A;

= 'N': no columns of U (no left singular vectors) are computed.

JOBVT (input)
Specifies options for computing all or part of the matrix V**H:
= 'A': all N rows of V**H are returned in the array VT;
= 'S': the first min(m,n) rows of V**H (the right singular vectors) are returned in the array VT; = 'O': the first min(m,n) rows of V**H (the right singular vectors) are overwritten on the array A;
= 'N': no rows of V**H (no right singular vectors) are computed.

JOBVT and JOBU cannot both be 'O'.

M (input) The number of rows of the input matrix A. M >= 0.

N (input) The number of columns of the input matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, if JOBU = 'O',   A is overwritten with the first min(m,n) columns of U (the left singular vectors, stored columnwise); if JOBVT = 'O', A is overwritten with the first min(m,n) rows of V**H (the right singular vectors, stored rowwise); if JOBU .ne. 'O' and JOBVT .ne. 'O', the contents of A are destroyed.

LDA (input)
The leading dimension of the array A.   LDA >= max(1,M).

SING (output)
The singular values of A, sorted so that SING(i) >= SING(i+1).

U (input) (LDU,M) if JOBU = 'A' or (LDU,min(M,N)) if JOBU = 'S'. If JOBU = 'A', U contains the M-by-M unitary matrix U; if JOBU = 'S', U contains the first min(m,n) columns of U (the left singular vectors, stored columnwise); if JOBU = 'N' or 'O', U is not referenced.

LDU (input)
The leading dimension of the array U. LDU >= 1; if JOBU = 'S' or 'A', LDU >= M.

VT (input)

If JOBVT = 'A', VT contains the N-by-N unitary matrix V**H; if JOBVT = 'S', VT contains the first min(m,n) rows of V**H (the right singular vectors, stored rowwise); if JOBVT = 'N' or 'O', VT is not referenced.

LDVT (input)
The leading dimension of the array VT. LDVT >= 1; if JOBVT = 'A', LDVT >= N; if JOBVT = 'S', LDVT >= min(M,N).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= 1. LDWORK >= 2*MIN(M,N)+MAX(M,N) For good performance, LDWORK should generally be larger.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
DIMENSION(5*MIN(M,N)). On exit, if INFO > 0, WORK2(1:MIN(M,N)-1) contains the unconverged superdiagonal elements of an upper bidiagonal matrix B whose diagonal is in SING (not necessarily sorted). B satisfies A = U * B * VT, so it has the same singular values as A, and singular vectors related by U and VT.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if CBDSQR did not converge, INFO specifies how many superdiagonals of an intermediate bidiagonal form B did not converge to zero. See the description of WORK2 above for details.

Contents

NAME

```
cgesvx - use the LU factorization to compute the solution to
a complex system of linear equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE CGESVX(FACT, TRANSA, N, NRHS, A, LDA, AF, LDAF, IPIVOT,
      EQUED, R, C, B, LDB, X, LDX, RCOND, FERR, BERR, WORK,
      WORK2, INFO)

CHARACTER * 1 FACT, TRANSA, EQUED
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*)
REAL RCOND
REAL R(*), C(*), FERR(*), BERR(*), WORK2(*)

SUBROUTINE CGESVX_64(FACT, TRANSA, N, NRHS, A, LDA, AF, LDAF, IPIVOT,
      EQUED, R, C, B, LDB, X, LDX, RCOND, FERR, BERR, WORK,
      WORK2, INFO)

CHARACTER * 1 FACT, TRANSA, EQUED
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
REAL RCOND
REAL R(*), C(*), FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GESVX(FACT, [TRANSA], [N], [NRHS], A, [LDA], AF, [LDAF],
       IPIVOT, EQUED, R, C, B, [LDB], X, [LDX], RCOND, FERR,
       BERR, [WORK], [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: FACT, TRANSA, EQUED
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL :: RCOND
REAL, DIMENSION(:) :: R, C, FERR, BERR, WORK2

SUBROUTINE GESVX_64(FACT, [TRANSA], [N], [NRHS], A, [LDA], AF, [LDAF],
       IPIVOT, EQUED, R, C, B, [LDB], X, [LDX], RCOND, FERR,
       BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, TRANSA, EQUED
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL :: RCOND
REAL, DIMENSION(:) :: R, C, FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cgesvx(char fact, char transa, int n, int nrhs, complex
          *a,  int  lda, complex *af, int ldaf, int *ipivot,
          char equed, float *r, float *c,  complex  *b,  int
          ldb,  complex  *x,  int  ldx,  float *rcond, float
          *ferr, float *berr, int *info);

void cgesvx_64(char fact, char transa, long  n,  long  nrhs,
          complex *a, long lda, complex *af, long ldaf, long
          *ipivot, char equed, float *r, float  *c,  complex
          *b,  long ldb, complex *x, long ldx, float *rcond,
          float *ferr, float *berr, long *info);
```

PURPOSE

```
cgesvx uses the LU factorization to compute the solution  to
a complex system of linear equations
   A * X = B, where A is an N-by-N matrix and X  and  B  are
N-by-NRHS matrices.

Error bounds on the solution and a  condition  estimate  are
also provided.

The following steps are performed:

1. If FACT = 'E',  real  scaling  factors  are  computed  to
equilibrate
```

```
   the system:
      TRANS = 'N':  diag(R)*A*diag(C)     *inv(diag(C))*X  =
diag(R)*B
      TRANS = 'T': (diag(R)*A*diag(C))**T *inv(diag(R))*X  =
diag(C)*B
      TRANS = 'C': (diag(R)*A*diag(C))**H *inv(diag(R))*X  =
diag(C)*B
   Whether or not the system will be equilibrated depends on
the
   scaling of the matrix A, but if equilibration is used,  A
is
   overwritten by diag(R)*A*diag(C) and B by  diag(R)*B  (if
TRANS='N')
   or diag(C)*B (if TRANS = 'T' or 'C').

2. If FACT = 'N' or 'E', the LU  decomposition  is  used  to
factor the
   matrix A (after equilibration if FACT = 'E') as
      A = P * L * U,
   where P is a permutation matrix, L is a unit  lower  tri-
angular
   matrix, and U is upper triangular.

3. If some U(i,i)=0, so that U is exactly singular, then the
routine
   returns with INFO = i. Otherwise, the factored form of  A
is used
   to estimate the condition number of the matrix A.  If the
   reciprocal of the condition number is less  than  machine
precision,
   INFO = N+1 is returned as  a  warning,  but  the  routine
still goes on
   to solve for X and  compute  error  bounds  as  described
below.

4. The system of equations is solved for X  using  the  fac-
tored form
   of A.

5. Iterative refinement is applied to improve  the  computed
solution
   matrix and calculate  error  bounds  and  backward  error
estimates
   for it.

6. If equilibration was used, the matrix X is  premultiplied
by
   diag(C) (if TRANS = 'N') or diag(R) (if TRANS  =  'T'  or
'C') so
   that it solves the original system before equilibration.
```

ARGUMENTS

```
FACT (input)
          Specifies whether or not the factored form of  the
          matrix A is supplied on entry, and if not, whether
          the matrix A should be equilibrated before  it  is
          factored.  = 'F':  On entry, AF and IPIVOT contain
          the factored form of A.  If EQUED is not 'N',  the
          matrix  A  has been equilibrated with scaling fac-
          tors given by R and C.  A, AF, and IPIVOT are  not
          modified.   = 'N':  The matrix A will be copied to
          AF and factored.
          = 'E':  The  matrix  A  will  be  equilibrated  if
          necessary, then copied to AF and factored.

TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input/output)
          On entry, the N-by-N matrix A.  If FACT = 'F'  and
          EQUED  is  not  'N', then A must have been equili-
          brated by the scaling factors in R and/or C.  A is
          not  modified  if  FACT = 'F' or 'N', or if FACT =
          'E' and EQUED = 'N' on exit.

          On exit, if EQUED .ne. 'N', A is  scaled  as  fol-
          lows:  EQUED = 'R':  A := diag(R) * A
          EQUED = 'C':  A := A * diag(C)
          EQUED = 'B':  A := diag(R) * A * diag(C).

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

AF (input/output)
          If FACT = 'F', then AF is an input argument and on
```

entry contains the factors L and U from the factorization A = P*L*U as computed by CGETRF. If EQUED .ne. 'N', then AF is the factored form of the equilibrated matrix A.

If FACT = 'N', then AF is an output argument and on exit returns the factors L and U from the factorization A = P*L*U of the original matrix A.

If FACT = 'E', then AF is an output argument and on exit returns the factors L and U from the factorization A = P*L*U of the equilibrated matrix A (see the description of A for the form of the equilibrated matrix).

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

IPIVOT (input or output)
If FACT = 'F', then IPIVOT is an input argument and on entry contains the pivot indices from the factorization A = P*L*U as computed by CGETRF; row i of the matrix was interchanged with row IPIVOT(i).

If FACT = 'N', then IPIVOT is an output argument and on exit contains the pivot indices from the factorization A = P*L*U of the original matrix A.

If FACT = 'E', then IPIVOT is an output argument and on exit contains the pivot indices from the factorization A = P*L*U of the equilibrated matrix A.

EQUED (input/output)
Specifies the form of equilibration that was done. = 'N': No equilibration (always true if FACT = 'N').
= 'R': Row equilibration, i.e., A has been premultiplied by diag(R). = 'C': Column equilibration, i.e., A has been postmultiplied by diag(C). = 'B': Both row and column equilibration, i.e., A has been replaced by diag(R) * A * diag(C). EQUED is an input argument if FACT = 'F'; otherwise, it is an output argument.

R (input/output)
The row scale factors for A. If EQUED = 'R' or 'B', A is multiplied on the left by diag(R); if EQUED = 'N' or 'C', R is not accessed. R is an

input argument if FACT = 'F'; otherwise, R is an output argument. If FACT = 'F' and EQUED = 'R' or 'B', each element of R must be positive.

C (input/output)
The column scale factors for A. If EQUED = 'C' or 'B', A is multiplied on the right by diag(C); if EQUED = 'N' or 'R', C is not accessed. C is an input argument if FACT = 'F'; otherwise, C is an output argument. If FACT = 'F' and EQUED = 'C' or 'B', each element of C must be positive.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if EQUED = 'N', B is not modified; if TRANSA = 'N' and EQUED = 'R' or 'B', B is overwritten by diag(R)*B; if TRANSA = 'T' or 'C' and EQUED = 'C' or 'B', B is overwritten by diag(C)*B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X to the original system of equations. Note that A and B are modified on exit if EQUED .ne. 'N', and the solution to the equilibrated system is inv(diag(C))*X if TRANSA = 'N' and EQUED = 'C' or 'B', or inv(diag(R))*X if TRANSA = 'T' or 'C' and EQUED = 'R' or 'B'.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A after equilibration (if done). If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest ele-

ment in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)

The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)

dimension(2*N)

WORK2 (workspace)

dimension(2*N) On exit, WORK2(1) contains the reciprocal pivot growth factor norm(A)/norm(U). The "max absolute element" norm is used. If WORK2(1) is much less than 1, then the stability of the LU factorization of the (equilibrated) matrix A could be poor. This also means that the solution X, condition estimator RCOND, and forward error bound FERR could be unreliable. If factorization fails with 0<INFO<=N, then WORK2(1) contains the reciprocal pivot growth factor for the leading INFO columns of A.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, and i is

<= N: U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
cgetf2 - compute an LU factorization  of  a  general  m-by-n
matrix A using partial pivoting with row interchanges
```

SYNOPSIS

```
SUBROUTINE CGETF2(M, N, A, LDA, IPIV, INFO)

COMPLEX A(LDA,*)
INTEGER M, N, LDA, INFO
INTEGER IPIV(*)

SUBROUTINE CGETF2_64(M, N, A, LDA, IPIV, INFO)

COMPLEX A(LDA,*)
INTEGER*8 M, N, LDA, INFO
INTEGER*8 IPIV(*)
```

F95 INTERFACE

```
SUBROUTINE GETF2([M], [N], A, [LDA], IPIV, [INFO])

COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIV

SUBROUTINE GETF2_64([M], [N], A, [LDA], IPIV, [INFO])

COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIV
```

C INTERFACE

```
#include <sunperf.h>
```

```
void cgetf2(int m, int n, complex *a, int  lda,  int  *ipiv,
          int *info);

void cgetf2_64(long m, long n, complex *a,  long  lda,  long
          *ipiv, long *info);
```

PURPOSE

cgetf2 computes an LU factorization of a general m-by-n matrix A using partial pivoting with row interchanges.

The factorization has the form
   A = P * L * U
where P is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if m > n), and U is upper triangular (upper trapezoidal if m < n).

This is the right-looking Level 2 BLAS version of the algorithm.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the m by n matrix to be factored. On exit, the factors L and U from the factorization A = P*L*U; the unit diagonal elements of L are not stored.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

IPIV (output)
The pivot indices; for 1 <= i <= min(M,N), row i of the matrix was interchanged with row IPIV(i).

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, U(k,k) is exactly zero. The factorization has been completed, but the factor U is

exactly singular, and division by zero will occur if it is used to solve a system of equations.

Contents

NAME

```
cgetrf - compute an LU factorization  of  a  general  M-by-N
matrix A using partial pivoting with row interchanges
```

SYNOPSIS

```
SUBROUTINE CGETRF(M, N, A, LDA, IPIVOT, INFO)

COMPLEX A(LDA,*)
INTEGER M, N, LDA, INFO
INTEGER IPIVOT(*)

SUBROUTINE CGETRF_64(M, N, A, LDA, IPIVOT, INFO)

COMPLEX A(LDA,*)
INTEGER*8 M, N, LDA, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE GETRF([M], [N], A, [LDA], IPIVOT, [INFO])

COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE GETRF_64([M], [N], A, [LDA], IPIVOT, [INFO])

COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>
```

```
void cgetrf(int m, int n, complex *a, int lda, int  *ipivot,
          int *info);

void cgetrf_64(long m, long n, complex *a,  long  lda,  long
          *ipivot, long *info);
```

PURPOSE

cgetrf computes an LU factorization of a general M-by-N matrix A using partial pivoting with row interchanges.

The factorization has the form
   A = P * L * U
where P is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if m > n), and U is upper triangular (upper trapezoidal if m < n).

This is the right-looking Level 3 BLAS version of the algorithm.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix to be factored. On exit, the factors L and U from the factorization A = P*L*U; the unit diagonal elements of L are not stored.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

IPIVOT (output)
The pivot indices; for 1 <= i <= min(M,N), row i of the matrix was interchanged with row IPIVOT(i).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, U(i,i) is exactly zero. The factorization has been completed, but the factor U

is exactly singular, and division by zero will occur if it is used to solve a system of equations.

Contents

NAME

```
cgetri - compute the inverse of a matrix using the  LU  fac-
torization computed by CGETRF
```

SYNOPSIS

```
SUBROUTINE CGETRI(N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, LDWORK, INFO
INTEGER IPIVOT(*)

SUBROUTINE CGETRI_64(N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, LDWORK, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE GETRI([N], A, [LDA], IPIVOT, [WORK], [LDWORK], [INFO])

COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, LDWORK, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE GETRI_64([N], A, [LDA], IPIVOT, [WORK], [LDWORK], [INFO])

COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void cgetri(int n, complex *a, int  lda,  int  *ipivot,  int
          *info);

void cgetri_64(long n, complex *a, long lda,  long  *ipivot,
          long *info);
```

PURPOSE

cgetri computes the inverse of a matrix using the LU factorization computed by CGETRF.

This method inverts U and then computes inv(A) by solving the system inv(A)*L = inv(U) for inv(A).

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the factors L and U from the factorization A = P*L*U as computed by CGETRF. On exit, if INFO = 0, the inverse of the original matrix A.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (input)
The pivot indices from CGETRF; for 1<=i<=N, row i of the matrix was interchanged with row IPIVOT(i).

WORK (workspace)
On exit, if INFO=0, then WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,N). For optimal performance LDWORK >= N*NB, where NB is the optimal blocksize returned by ILAENV.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first

entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, U(i,i) is exactly zero; the matrix is singular and its inverse could not be computed.

Contents

NAME

```
cgetrs - solve a system of linear equations  A * X = B, A**T
*  X  =  B,  or  A**H * X = B with a general N-by-N matrix A
using the LU factorization computed by CGETRF
```

SYNOPSIS

```
SUBROUTINE CGETRS(TRANSA, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 TRANSA
COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE CGETRS_64(TRANSA, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 TRANSA
COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE GETRS([TRANSA], [N], [NRHS], A, [LDA], IPIVOT, B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: TRANSA
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: N, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE GETRS_64([TRANSA], [N], [NRHS], A, [LDA], IPIVOT, B, [LDB],
       [INFO])
```

```
CHARACTER(LEN=1) :: TRANSA
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void cgetrs(char transa, int n, int nrhs,  complex  *a,  int
          lda, int *ipivot, complex *b, int ldb, int *info);

void cgetrs_64(char transa, long n, long nrhs,  complex  *a,
          long lda, long *ipivot, complex *b, long ldb, long
          *info);
```

PURPOSE

cgetrs solves a system of linear equations
   A * X = B, A**T * X = B, or A**H * X = B with a general N-by-N matrix A using the LU factorization computed by CGETRF.

ARGUMENTS

TRANSA (input)
Specifies the form of the system of equations:
= 'N': A * X = B   (No transpose)
= 'T': A**T * X = B (Transpose)
= 'C': A**H * X = B (Conjugate transpose)

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The factors L and U from the factorization A = P*L*U as computed by CGETRF.

LDA (input)
The leading dimension of the array A.   LDA >= max(1,N).

IPIVOT (input)
The pivot indices from CGETRF; for 1<=i<=N, row i

of the matrix was interchanged with row IPIVOT(i).

B (input/output)
On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
cggbak - form the right or left eigenvectors  of  a  complex
generalized eigenvalue problem A*x = lambda*B*x, by backward
transformation on the computed eigenvectors of the  balanced
pair of matrices output by CGGBAL
```

SYNOPSIS

```
SUBROUTINE CGGBAK(JOB, SIDE, N, ILO, IHI, LSCALE, RSCALE, M, V, LDV,
      INFO)

CHARACTER * 1 JOB, SIDE
COMPLEX V(LDV,*)
INTEGER N, ILO, IHI, M, LDV, INFO
REAL LSCALE(*), RSCALE(*)

SUBROUTINE CGGBAK_64(JOB, SIDE, N, ILO, IHI, LSCALE, RSCALE, M, V,
      LDV, INFO)

CHARACTER * 1 JOB, SIDE
COMPLEX V(LDV,*)
INTEGER*8 N, ILO, IHI, M, LDV, INFO
REAL LSCALE(*), RSCALE(*)
```

F95 INTERFACE

```
SUBROUTINE GGBAK(JOB, SIDE, [N], ILO, IHI, LSCALE, RSCALE, [M], V,
       [LDV], [INFO])

CHARACTER(LEN=1) :: JOB, SIDE
COMPLEX, DIMENSION(:,:) :: V
INTEGER :: N, ILO, IHI, M, LDV, INFO
REAL, DIMENSION(:) :: LSCALE, RSCALE
```

```
SUBROUTINE GGBAK_64(JOB, SIDE, [N], ILO, IHI, LSCALE, RSCALE, [M], V,
       [LDV], [INFO])

CHARACTER(LEN=1) :: JOB, SIDE
COMPLEX, DIMENSION(:,:) :: V
INTEGER(8) :: N, ILO, IHI, M, LDV, INFO
REAL, DIMENSION(:) :: LSCALE, RSCALE
```

C INTERFACE

```
#include <sunperf.h>

void cggbak(char job, char side, int n, int  ilo,  int  ihi,
          float  *lscale,  float *rscale, int m, complex *v,
          int ldv, int *info);
void cggbak_64(char job, char side, long n, long  ilo,  long
          ihi, float *lscale, float *rscale, long m, complex
          *v, long ldv, long *info);
```

PURPOSE

cggbak forms the right or left eigenvectors of a complex generalized eigenvalue problem A*x = lambda*B*x, by backward transformation on the computed eigenvectors of the balanced pair of matrices output by CGGBAL.

ARGUMENTS

```
JOB (input)
          Specifies  the  type  of  backward  transformation
          required:
          = 'N':  do nothing, return immediately;
          = 'P':  do backward transformation for permutation
          only;
          = 'S':  do  backward  transformation  for  scaling
          only;
          = 'B':  do backward transformations for both  per-
          mutation and scaling.  JOB must be the same as the
          argument JOB supplied to CGGBAL.

SIDE (input)
          = 'R':  V contains right eigenvectors;
          = 'L':  V contains left eigenvectors.

N (input) The number of rows of the matrix V.  N >= 0.

ILO (input)
```

The integers ILO and IHI determined by CGGBAL. 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

IHI (input)
The integers ILO and IHI determined by CGGBAL. 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

LSCALE (input)
Details of the permutations and/or scaling factors applied to the left side of A and B, as returned by CGGBAL.

RSCALE (input)
Details of the permutations and/or scaling factors applied to the right side of A and B, as returned by CGGBAL.

M (input) The number of columns of the matrix V. M >= 0.

V (input/output)
On entry, the matrix of right or left eigenvectors to be transformed, as returned by CTGEVC. On exit, V is overwritten by the transformed eigenvectors.

LDV (input)
The leading dimension of the matrix V. LDV >= max(1,N).

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

See R.C. Ward, Balancing the generalized eigenvalue problem,
SIAM J. Sci. Stat. Comp. 2 (1981), 141-152.

Contents

NAME

```
cggbal - balance a pair of general complex matrices (A,B)
```

SYNOPSIS

```
SUBROUTINE CGGBAL(JOB, N, A, LDA, B, LDB, ILO, IHI, LSCALE, RSCALE,
      WORK, INFO)

CHARACTER * 1 JOB
COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, LDA, LDB, ILO, IHI, INFO
REAL LSCALE(*), RSCALE(*), WORK(*)

SUBROUTINE CGGBAL_64(JOB, N, A, LDA, B, LDB, ILO, IHI, LSCALE,
      RSCALE, WORK, INFO)

CHARACTER * 1 JOB
COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, LDA, LDB, ILO, IHI, INFO
REAL LSCALE(*), RSCALE(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGBAL(JOB, [N], A, [LDA], B, [LDB], ILO, IHI, LSCALE,
       RSCALE, [WORK], [INFO])

CHARACTER(LEN=1) :: JOB
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: N, LDA, LDB, ILO, IHI, INFO
REAL, DIMENSION(:) :: LSCALE, RSCALE, WORK

SUBROUTINE GGBAL_64(JOB, [N], A, [LDA], B, [LDB], ILO, IHI, LSCALE,
       RSCALE, [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOB
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: N, LDA, LDB, ILO, IHI, INFO
REAL, DIMENSION(:) :: LSCALE, RSCALE, WORK
```

C INTERFACE

```
#include <sunperf.h>

void cggbal(char job, int n, complex *a,  int  lda,  complex
          *b,  int  ldb,  int *ilo, int *ihi, float *lscale,
          float *rscale, int *info);

void cggbal_64(char job, long n, complex *a, long lda,  com-
          plex  *b,  long  ldb,  long *ilo, long *ihi, float
          *lscale, float *rscale, long *info);
```

PURPOSE

cggbal balances a pair of general complex matrices (A,B). This involves, first, permuting A and B by similarity transformations to isolate eigenvalues in the first 1 to ILO$-$1 and last IHI+1 to N elements on the diagonal; and second, applying a diagonal similarity transformation to rows and columns ILO to IHI to make the rows and columns as close in norm as possible. Both steps are optional.

Balancing may reduce the 1-norm of the matrices, and improve the accuracy of the computed eigenvalues and/or eigenvectors in the generalized eigenvalue problem A*x = lambda*B*x.

ARGUMENTS

```
JOB (input)
          Specifies the operations to be performed on A  and
          B:
          = 'N':  none:  simply  set  ILO  =  1,  IHI  =  N,
          LSCALE(I) = 1.0 and RSCALE(I) = 1.0 for i=1,...,N;
          = 'P':  permute only;
          = 'S':  scale only;
          = 'B':  both permute and scale.

N (input) The order of the matrices A and B.  N >= 0.

A (input/output)
          On entry, the input  matrix  A.   On  exit,  A  is
          overwritten by the balanced matrix.  If JOB = 'N',
```

A is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input) On entry, the input matrix B. On exit, B is overwritten by the balanced matrix. If JOB = 'N', B is not referenced.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

ILO (output)
ILO and IHI are set to integers such that on exit A(i,j) = 0 and B(i,j) = 0 if i > j and j = 1,...,ILO-1 or i = IHI+1,...,N. If JOB = 'N' or 'S', ILO = 1 and IHI = N.

IHI (output)
ILO and IHI are set to integers such that on exit A(i,j) = 0 and B(i,j) = 0 if i > j and j = 1,...,ILO-1 or i = IHI+1,...,N.

LSCALE (input)
Details of the permutations and scaling factors applied to the left side of A and B. If P(j) is the index of the row interchanged with row j, and D(j) is the scaling factor applied to row j, then LSCALE(j) = P(j) for J = 1,...,ILO-1 = D(j) for J = ILO,...,IHI = P(j) for J = IHI+1,...,N. The order in which the interchanges are made is N to IHI+1, then 1 to ILO-1.

RSCALE (input)
Details of the permutations and scaling factors applied to the right side of A and B. If P(j) is the index of the column interchanged with column j, and D(j) is the scaling factor applied to column j, then RSCALE(j) = P(j) for J = 1,...,ILO-1 = D(j) for J = ILO,...,IHI = P(j) for J = IHI+1,...,N. The order in which the interchanges are made is N to IHI+1, then 1 to ILO-1.

WORK (workspace)
dimension(6*N)

INFO (output)

```
= 0:  successful exit
< 0:  if INFO = -i, the i-th argument had an ille-
gal value.
```

FURTHER DETAILS

```
See R.C. WARD, Balancing the generalized eigenvalue problem,
               SIAM J. Sci. Stat. Comp. 2 (1981), 141-152.
```

Contents

NAME

```
cgges - compute for a pair of  N-by-N  complex  nonsymmetric
matrices (A,B), the generalized eigenvalues, the generalized
complex Schur form (S, T), and optionally left and/or  right
Schur vectors (VSL and VSR)
```

SYNOPSIS

```
SUBROUTINE CGGES(JOBVSL, JOBVSR, SORT, SELCTG, N, A, LDA, B, LDB,
      SDIM, ALPHA, BETA, VSL, LDVSL, VSR, LDVSR, WORK, LWORK, RWORK,
      BWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR, SORT
COMPLEX A(LDA,*), B(LDB,*), ALPHA(*), BETA(*), VSL(LDVSL,*),
VSR(LDVSR,*), WORK(*)
INTEGER N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, INFO
LOGICAL SELCTG
LOGICAL BWORK(*)
REAL RWORK(*)

SUBROUTINE CGGES_64(JOBVSL, JOBVSR, SORT, SELCTG, N, A, LDA, B, LDB,
      SDIM, ALPHA, BETA, VSL, LDVSL, VSR, LDVSR, WORK, LWORK, RWORK,
      BWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR, SORT
COMPLEX A(LDA,*), B(LDB,*), ALPHA(*), BETA(*), VSL(LDVSL,*),
VSR(LDVSR,*), WORK(*)
INTEGER*8 N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, INFO
LOGICAL*8 SELCTG
LOGICAL*8 BWORK(*)
REAL RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGES(JOBVSL, JOBVSR, SORT, [SELCTG], [N], A, [LDA], B, [LDB],
       SDIM, ALPHA, BETA, VSL, [LDVSL], VSR, [LDVSR], [WORK], [LWORK],
       [RWORK], [BWORK], [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR, SORT
COMPLEX, DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX, DIMENSION(:,:) :: A, B, VSL, VSR
INTEGER :: N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, INFO
LOGICAL :: SELCTG
LOGICAL, DIMENSION(:) :: BWORK
REAL, DIMENSION(:) :: RWORK

SUBROUTINE GGES_64(JOBVSL, JOBVSR, SORT, [SELCTG], [N], A, [LDA], B,
       [LDB], SDIM, ALPHA, BETA, VSL, [LDVSL], VSR, [LDVSR], [WORK],
       [LWORK], [RWORK], [BWORK], [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR, SORT
COMPLEX, DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX, DIMENSION(:,:) :: A, B, VSL, VSR
INTEGER(8) :: N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, INFO
LOGICAL(8) :: SELCTG
LOGICAL(8), DIMENSION(:) :: BWORK
REAL, DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>

void   cgges(char   jobvsl,   char   jobvsr,   char    sort,
          int(*selctg)(complex,complex),  int n, complex *a,
          int lda, complex *b, int ldb, int  *sdim,  complex
          *alpha,  complex  *beta,  complex *vsl, int ldvsl,
          complex *vsr, int ldvsr, int *info);

void  cgges_64(char  jobvsl,   char   jobvsr,   char   sort,
          long(*selctg)(complex,complex),  long  n,  complex
          *a, long lda, complex *b, long  ldb,  long  *sdim,
          complex  *alpha, complex *beta, complex *vsl, long
          ldvsl, complex *vsr, long ldvsr, long *info);
```

PURPOSE

cgges computes for a pair of N-by-N complex nonsymmetric matrices (A,B), the generalized eigenvalues, the generalized complex Schur form (S, T), and optionally left and/or right Schur vectors (VSL and VSR). This gives the generalized Schur factorization

```
(A,B) = ( (VSL)*S*(VSR)**H, (VSL)*T*(VSR)**H )
```

where (VSR)**H is the conjugate-transpose of VSR.

Optionally, it also orders the eigenvalues so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper triangular matrix S and the upper triangular matrix T. The leading columns of VSL and VSR then form an unitary basis for the corresponding left and right eigenspaces (deflating subspaces).

(If only the generalized eigenvalues are needed, use the driver CGGEV instead, which is faster.)

A generalized eigenvalue for a pair of matrices (A,B) is a scalar w or a ratio alpha/beta = w, such that A - w*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0, and even for both being zero.
A pair of matrices (S,T) is in generalized complex Schur form if S and T are upper triangular and, in addition, the diagonal elements of T are non-negative real numbers.

ARGUMENTS

JOBVSL (input)
= 'N': do not compute the left Schur vectors;
= 'V': compute the left Schur vectors.

JOBVSR (input)
= 'N': do not compute the right Schur vectors;
= 'V': compute the right Schur vectors.

SORT (input)
Specifies whether or not to order the eigenvalues on the diagonal of the generalized Schur form. = 'N': Eigenvalues are not ordered;
= 'S': Eigenvalues are ordered (see SELCTG).

SELCTG (input)
SELCTG must be declared EXTERNAL in the calling subroutine. If SORT = 'N', SELCTG is not referenced. If SORT = 'S', SELCTG is used to select eigenvalues to sort to the top left of the Schur form. An eigenvalue ALPHA(j)/BETA(j) is selected if SELCTG(ALPHA(j),BETA(j)) is true.

Note that a selected complex eigenvalue may no longer satisfy SELCTG(ALPHA(j),BETA(j)) = .TRUE. after ordering, since ordering may change the value of complex eigenvalues (especially if the eigenvalue is ill-conditioned), in this case INFO is set to N+2 (See INFO below).

N (input) The order of the matrices A, B, VSL, and VSR. N >= 0.

A (input/output)
On entry, the first of the pair of matrices. On exit, A has been overwritten by its generalized Schur form S.

LDA (input)
The leading dimension of A. LDA >= max(1,N).

B (input/output)
On entry, the second of the pair of matrices. On exit, B has been overwritten by its generalized Schur form T.

LDB (input)
The leading dimension of B. LDB >= max(1,N).

SDIM (output)
If SORT = 'N', SDIM = 0. If SORT = 'S', SDIM = number of eigenvalues (after sorting) for which SELCTG is true.

ALPHA (output)
On exit, ALPHA(j)/BETA(j), j=1,...,N, will be the generalized eigenvalues. ALPHA(j), j=1,...,N and BETA(j), j=1,...,N are the diagonals of the complex Schur form (A,B) output by CGGES. The BETA(j) will be non-negative real.

Note: the quotients ALPHA(j)/BETA(j) may easily over- or underflow, and BETA(j) may even be zero. Thus, the user should avoid naively computing the ratio alpha/beta. However, ALPHA will be always less than and usually comparable with norm(A) in magnitude, and BETA always less than and usually comparable with norm(B).

BETA (output)
See description of ALPHA.

VSL (input)
If JOBVSL = 'V', VSL will contain the left Schur vectors. Not referenced if JOBVSL = 'N'.

LDVSL (input)

The leading dimension of the matrix VSL. LDVSL >= 1, and if JOBVSL = 'V', LDVSL >= N.

VSR (input)
If JOBVSR = 'V', VSR will contain the right Schur vectors. Not referenced if JOBVSR = 'N'.

LDVSR (input)
The leading dimension of the matrix VSR. LDVSR >= 1, and if JOBVSR = 'V', LDVSR >= N.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,2*N). For good performance, LWORK must generally be larger.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
dimension(8*N)

BWORK (workspace)
dimension(N) Not referenced if SORT = 'N'.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
=1,...,N: The QZ iteration failed. (A,B) are not in Schur form, but ALPHA(j) and BETA(j) should be correct for j=INFO+1,...,N. > N: =N+1: other than QZ iteration failed in CHGEQZ
=N+2: after reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the Generalized Schur form no longer satisfy SELCTG=.TRUE. This could also be caused due to scaling. =N+3: reordering falied in CTGSEN.

Contents

NAME

```
cggesx - compute for a pair of N-by-N  complex  nonsymmetric
matrices  (A,B),  the  generalized  eigenvalues, the complex
Schur form (S,T),
```

SYNOPSIS

```
SUBROUTINE CGGESX(JOBVSL, JOBVSR, SORT, SELCTG, SENSE, N, A, LDA, B,
      LDB, SDIM, ALPHA, BETA, VSL, LDVSL, VSR, LDVSR, RCONDE, RCONDV,
      WORK, LWORK, RWORK, IWORK, LIWORK, BWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR, SORT, SENSE
COMPLEX A(LDA,*), B(LDB,*), ALPHA(*), BETA(*), VSL(LDVSL,*),
VSR(LDVSR,*), WORK(*)
INTEGER N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, LIWORK, INFO
INTEGER IWORK(*)
LOGICAL SELCTG
LOGICAL BWORK(*)
REAL RCONDE(*), RCONDV(*), RWORK(*)

SUBROUTINE CGGESX_64(JOBVSL, JOBVSR, SORT, SELCTG, SENSE, N, A, LDA,
      B, LDB, SDIM, ALPHA, BETA, VSL, LDVSL, VSR, LDVSR, RCONDE,
      RCONDV, WORK, LWORK, RWORK, IWORK, LIWORK, BWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR, SORT, SENSE
COMPLEX A(LDA,*), B(LDB,*), ALPHA(*), BETA(*), VSL(LDVSL,*),
VSR(LDVSR,*), WORK(*)
INTEGER*8 N, LDA, LDB, SDIM, LDVSL,  LDVSR,  LWORK,  LIWORK,
INFO
INTEGER*8 IWORK(*)
LOGICAL*8 SELCTG
LOGICAL*8 BWORK(*)
REAL RCONDE(*), RCONDV(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGESX(JOBVSL, JOBVSR, SORT, [SELCTG], SENSE, [N], A, [LDA],
       B, [LDB], SDIM, ALPHA, BETA, VSL, [LDVSL], VSR, [LDVSR], RCONDE,
       RCONDV, [WORK], [LWORK], [RWORK], [IWORK], [LIWORK], [BWORK],
       [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR, SORT, SENSE
COMPLEX, DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX, DIMENSION(:,:) :: A, B, VSL, VSR
INTEGER :: N, LDA, LDB, SDIM, LDVSL, LDVSR,  LWORK,  LIWORK,
INFO
INTEGER, DIMENSION(:) :: IWORK
LOGICAL :: SELCTG
LOGICAL, DIMENSION(:) :: BWORK
REAL, DIMENSION(:) :: RCONDE, RCONDV, RWORK
SUBROUTINE GGESX_64(JOBVSL, JOBVSR, SORT, [SELCTG], SENSE, [N], A, [LDA],
       B, [LDB], SDIM, ALPHA, BETA, VSL, [LDVSL], VSR, [LDVSR], RCONDE,
       RCONDV, [WORK], [LWORK], [RWORK], [IWORK], [LIWORK], [BWORK],
       [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR, SORT, SENSE
COMPLEX, DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX, DIMENSION(:,:) :: A, B, VSL, VSR
INTEGER(8) ::  N,  LDA,  LDB,  SDIM,  LDVSL,  LDVSR,  LWORK,
LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
LOGICAL(8) :: SELCTG
LOGICAL(8), DIMENSION(:) :: BWORK
REAL, DIMENSION(:) :: RCONDE, RCONDV, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void   cggesx(char   jobvsl,   char   jobvsr,   char   sort,
          int(*selctg)(complex,complex),  char sense, int n,
          complex *a, int lda,  complex  *b,  int  ldb,  int
          *sdim,  complex  *alpha,  complex  *beta,  complex
          *vsl, int ldvsl, complex *vsr,  int  ldvsr,  float
          *rconde, float *rcondv, int *info);

void  cggesx_64(char  jobvsl,  char   jobvsr,   char   sort,
          long(*selctg)(complex,complex),  char  sense, long
          n, complex *a, long lda,  complex  *b,  long  ldb,
          long *sdim, complex *alpha, complex *beta, complex
          *vsl, long ldvsl, complex *vsr, long ldvsr,  float
          *rconde, float *rcondv, long *info);
```

PURPOSE

cggesx computes for a pair of N-by-N complex nonsymmetric matrices (A,B), the generalized eigenvalues, the complex Schur form (S,T), and, optionally, the left and/or right matrices of Schur vectors (VSL and VSR). This gives the generalized Schur factorization A,B) = ((VSL) S (VSR)**H, (VSL) T (VSR)**H)

where (VSR)**H is the conjugate-transpose of VSR.

Optionally, it also orders the eigenvalues so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper triangular matrix S and the upper triangular matrix T; computes a reciprocal condition number for the average of the selected eigenvalues (RCONDE); and computes a reciprocal condition number for the right and left deflating subspaces corresponding to the selected eigenvalues (RCONDV). The leading columns of VSL and VSR then form an orthonormal basis for the corresponding left and right eigenspaces (deflating subspaces).

A generalized eigenvalue for a pair of matrices (A,B) is a scalar w or a ratio alpha/beta = w, such that A - w*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0 or for both being zero.

A pair of matrices (S,T) is in generalized complex Schur form if T is upper triangular with non-negative diagonal and S is upper triangular.

ARGUMENTS

JOBVSL (input)
        = 'N': do not compute the left Schur vectors;
        = 'V': compute the left Schur vectors.

JOBVSR (input)
        = 'N': do not compute the right Schur vectors;
        = 'V': compute the right Schur vectors.

SORT (input)
        Specifies whether or not to order the eigenvalues on the diagonal of the generalized Schur form. = 'N': Eigenvalues are not ordered;
        = 'S': Eigenvalues are ordered (see SELCTG).

SELCTG (input)

SELCTG must be declared EXTERNAL in the calling subroutine. If SORT = 'N', SELCTG is not referenced. If SORT = 'S', SELCTG is used to select eigenvalues to sort to the top left of the Schur form. Note that a selected complex eigenvalue may no longer satisfy SELCTG(ALPHA(j),BETA(j)) = .TRUE. after ordering, since ordering may change the value of complex eigenvalues (especially if the eigenvalue is ill-conditioned), in this case INFO is set to N+3 see INFO below).

SENSE (input)
Determines which reciprocal condition numbers are computed. = 'N' : None are computed;
= 'E' : Computed for average of selected eigenvalues only;
= 'V' : Computed for selected deflating subspaces only;
= 'B' : Computed for both. If SENSE = 'E', 'V', or 'B', SORT must equal 'S'.

N (input) The order of the matrices A, B, VSL, and VSR. N >= 0.

A (input/output)
On entry, the first of the pair of matrices. On exit, A has been overwritten by its generalized Schur form S.

LDA (input)
The leading dimension of A. LDA >= max(1,N).

B (input/output)
On entry, the second of the pair of matrices. On exit, B has been overwritten by its generalized Schur form T.

LDB (input)
The leading dimension of B. LDB >= max(1,N).

SDIM (output)
If SORT = 'N', SDIM = 0. If SORT = 'S', SDIM = number of eigenvalues (after sorting) for which SELCTG is true.

ALPHA (output)
On exit, ALPHA(j)/BETA(j), j=1,...,N, will be the generalized eigenvalues. ALPHA(j) and BETA(j),j=1,...,N are the diagonals of the complex Schur form (S,T). BETA(j) will be non-

negative real.

Note: the quotients ALPHA(j)/BETA(j) may easily over- or underflow, and BETA(j) may even be zero. Thus, the user should avoid naively computing the ratio alpha/beta. However, ALPHA will be always less than and usually comparable with norm(A) in magnitude, and BETA always less than and usually comparable with norm(B).

BETA (output)
See description of ALPHA.

VSL (input)
If JOBVSL = 'V', VSL will contain the left Schur vectors. Not referenced if JOBVSL = 'N'.

LDVSL (input)
The leading dimension of the matrix VSL. LDVSL >=1, and if JOBVSL = 'V', LDVSL >= N.

VSR (input)
If JOBVSR = 'V', VSR will contain the right Schur vectors. Not referenced if JOBVSR = 'N'.

LDVSR (input)
The leading dimension of the matrix VSR. LDVSR >= 1, and if JOBVSR = 'V', LDVSR >= N.

RCONDE (output)
If SENSE = 'E' or 'B', RCONDE(1) and RCONDE(2) contain the reciprocal condition numbers for the average of the selected eigenvalues. Not referenced if SENSE = 'N' or 'V'.

RCONDV (output)
If SENSE = 'V' or 'B', RCONDV(1) and RCONDV(2) contain the reciprocal condition number for the selected deflating subspaces. Not referenced if SENSE = 'N' or 'E'.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 2*N. If SENSE = 'E', 'V', or 'B', LWORK >= MAX(2*N, 2*SDIM*(N-SDIM)).

RWORK (workspace)

```
          dimension(8*N) Real workspace.
IWORK (workspace/output)
          Not referenced if SENSE = 'N'.  On exit, if INFO =
          0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
          The dimension of the array WORK. LIWORK >= N+2.

BWORK (workspace)
          dimension(N) Not referenced if SORT = 'N'.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value.
          = 1,...,N:  The QZ iteration  failed.   (A,B)  are
          not in Schur form, but ALPHA(j) and BETA(j) should
          be correct for j=INFO+1,...,N.  > N:  =N+1:  other
          than QZ iteration failed in CHGEQZ
          =N+2: after reordering, roundoff changed values of
          some  complex  eigenvalues  so that leading eigen-
          values in the Generalized  Schur  form  no  longer
          satisfy  SELCTG=.TRUE.   This could also be caused
          due  to  scaling.   =N+3:  reordering  failed   in
          CTGSEN.
```

Contents

NAME

```
cggev - compute for a pair of  N-by-N  complex  nonsymmetric
matrices (A,B), the generalized eigenvalues, and optionally,
the left and/or right generalized eigenvectors
```

SYNOPSIS

```
SUBROUTINE CGGEV(JOBVL, JOBVR, N, A, LDA, B, LDB, ALPHA, BETA, VL,
      LDVL, VR, LDVR, WORK, LWORK, RWORK, INFO)

CHARACTER * 1 JOBVL, JOBVR
COMPLEX A(LDA,*), B(LDB,*), ALPHA(*),  BETA(*),  VL(LDVL,*),
VR(LDVR,*), WORK(*)
INTEGER N, LDA, LDB, LDVL, LDVR, LWORK, INFO
REAL RWORK(*)

SUBROUTINE CGGEV_64(JOBVL, JOBVR, N, A, LDA, B, LDB, ALPHA, BETA, VL,
      LDVL, VR, LDVR, WORK, LWORK, RWORK, INFO)

CHARACTER * 1 JOBVL, JOBVR
COMPLEX A(LDA,*), B(LDB,*), ALPHA(*),  BETA(*),  VL(LDVL,*),
VR(LDVR,*), WORK(*)
INTEGER*8 N, LDA, LDB, LDVL, LDVR, LWORK, INFO
REAL RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGEV(JOBVL, JOBVR, [N], A, [LDA], B, [LDB], ALPHA, BETA,
       VL, [LDVL], VR, [LDVR], [WORK], [LWORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: JOBVL, JOBVR
COMPLEX, DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX, DIMENSION(:,:) :: A, B, VL, VR
INTEGER :: N, LDA, LDB, LDVL, LDVR, LWORK, INFO
```

```
REAL, DIMENSION(:) :: RWORK

SUBROUTINE GGEV_64(JOBVL, JOBVR, [N], A, [LDA], B, [LDB], ALPHA,
       BETA, VL, [LDVL], VR, [LDVR], [WORK], [LWORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: JOBVL, JOBVR
COMPLEX, DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX, DIMENSION(:,:) :: A, B, VL, VR
INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, LWORK, INFO
REAL, DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>
void cggev(char jobvl, char jobvr, int n,  complex  *a,  int
          lda,  complex *b, int ldb, complex *alpha, complex
          *beta, complex *vl, int  ldvl,  complex  *vr,  int
          ldvr, int *info);

void cggev_64(char jobvl, char jobvr, long  n,  complex  *a,
          long  lda,  complex  *b, long ldb, complex *alpha,
          complex *beta, complex  *vl,  long  ldvl,  complex
          *vr, long ldvr, long *info);
```

PURPOSE

cggev computes for a pair of N-by-N complex nonsymmetric matrices (A,B), the generalized eigenvalues, and optionally, the left and/or right generalized eigenvectors.

A generalized eigenvalue for a pair of matrices (A,B) is a scalar lambda or a ratio alpha/beta = lambda, such that A - lambda*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0, and even for both being zero.

The right generalized eigenvector v(j) corresponding to the generalized eigenvalue lambda(j) of (A,B) satisfies

```
A * v(j) = lambda(j) * B * v(j).
```

The left generalized eigenvector u(j) corresponding to the generalized eigenvalues lambda(j) of (A,B) satisfies

```
u(j)**H * A = lambda(j) * u(j)**H * B
```

where u(j)**H is the conjugate-transpose of u(j).

ARGUMENTS

JOBVL (input)
= 'N': do not compute the left generalized eigenvectors;
= 'V': compute the left generalized eigenvectors.

JOBVR (input)
= 'N': do not compute the right generalized eigenvectors;
= 'V': compute the right generalized eigenvectors.

N (input) The order of the matrices A, B, VL, and VR. N >= 0.

A (input/output)
On entry, the matrix A in the pair (A,B). On exit, A has been overwritten.

LDA (input)
The leading dimension of A. LDA >= max(1,N).

B (input/output)
On entry, the matrix B in the pair (A,B). On exit, B has been overwritten.

LDB (input)
The leading dimension of B. LDB >= max(1,N).

ALPHA (output)
On exit, ALPHA(j)/BETA(j), j=1,...,N, will be the generalized eigenvalues.

Note: the quotients ALPHA(j)/BETA(j) may easily over- or underflow, and BETA(j) may even be zero. Thus, the user should avoid naively computing the ratio alpha/beta. However, ALPHA will be always less than and usually comparable with norm(A) in magnitude, and BETA always less than and usually comparable with norm(B).

BETA (output)
See description of ALPHA.

VL (input)
If JOBVL = 'V', the left generalized eigenvectors u(j) are stored one after another in the columns of VL, in the same order as their eigenvalues. Each eigenvector will be scaled so the largest component will have abs(real part) + abs(imag.

part) = 1. Not referenced if JOBVL = 'N'.

LDVL (input)
The leading dimension of the matrix VL. LDVL >= 1, and if JOBVL = 'V', LDVL >= N.

VR (input)
If JOBVR = 'V', the right generalized eigenvectors v(j) are stored one after another in the columns of VR, in the same order as their eigenvalues. Each eigenvector will be scaled so the largest component will have abs(real part) + abs(imag. part) = 1. Not referenced if JOBVR = 'N'.

LDVR (input)
The leading dimension of the matrix VR. LDVR >= 1, and if JOBVR = 'V', LDVR >= N.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,2*N). For good performance, LWORK must generally be larger.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
dimension(8*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
=1,...,N: The QZ iteration failed. No eigenvectors have been calculated, but ALPHA(j) and BETA(j) should be correct for j=INFO+1,...,N. > N: =N+1: other then QZ iteration failed in SHGEQZ,
=N+2: error return from STGEVC.

Contents

NAME

```
cggevx - compute for a pair of N-by-N  complex  nonsymmetric
matrices  (A,B) the generalized eigenvalues, and optionally,
the left and/or right generalized eigenvectors
```

SYNOPSIS

```
SUBROUTINE CGGEVX(BALANC, JOBVL, JOBVR, SENSE, N, A, LDA, B, LDB,
      ALPHA, BETA, VL, LDVL, VR, LDVR, ILO, IHI, LSCALE, RSCALE, ABNRM,
      BBNRM, RCONDE, RCONDV, WORK, LWORK, RWORK, IWORK, BWORK, INFO)

CHARACTER * 1 BALANC, JOBVL, JOBVR, SENSE
COMPLEX A(LDA,*), B(LDB,*), ALPHA(*),  BETA(*),  VL(LDVL,*),
VR(LDVR,*), WORK(*)
INTEGER N, LDA, LDB, LDVL, LDVR, ILO, IHI, LWORK, INFO
INTEGER IWORK(*)
LOGICAL BWORK(*)
REAL ABNRM, BBNRM
REAL LSCALE(*), RSCALE(*), RCONDE(*), RCONDV(*), RWORK(*)

SUBROUTINE CGGEVX_64(BALANC, JOBVL, JOBVR, SENSE, N, A, LDA, B, LDB,
      ALPHA, BETA, VL, LDVL, VR, LDVR, ILO, IHI, LSCALE, RSCALE, ABNRM,
      BBNRM, RCONDE, RCONDV, WORK, LWORK, RWORK, IWORK, BWORK, INFO)

CHARACTER * 1 BALANC, JOBVL, JOBVR, SENSE
COMPLEX A(LDA,*), B(LDB,*), ALPHA(*),  BETA(*),  VL(LDVL,*),
VR(LDVR,*), WORK(*)
INTEGER*8 N, LDA, LDB, LDVL, LDVR, ILO, IHI, LWORK, INFO
INTEGER*8 IWORK(*)
LOGICAL*8 BWORK(*)
REAL ABNRM, BBNRM
REAL LSCALE(*), RSCALE(*), RCONDE(*), RCONDV(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGEVX(BALANC, JOBVL, JOBVR, SENSE, [N], A, [LDA], B, [LDB],
       ALPHA, BETA, VL, [LDVL], VR, [LDVR], ILO, IHI, LSCALE, RSCALE,
       ABNRM, BBNRM, RCONDE, RCONDV, [WORK], [LWORK], [RWORK], [IWORK],
       [BWORK], [INFO])

CHARACTER(LEN=1) :: BALANC, JOBVL, JOBVR, SENSE
COMPLEX, DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX, DIMENSION(:,:) :: A, B, VL, VR
INTEGER :: N, LDA, LDB, LDVL, LDVR, ILO, IHI, LWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
LOGICAL, DIMENSION(:) :: BWORK
REAL :: ABNRM, BBNRM
REAL, DIMENSION(:) :: LSCALE, RSCALE, RCONDE, RCONDV, RWORK
SUBROUTINE GGEVX_64(BALANC, JOBVL, JOBVR, SENSE, [N], A, [LDA], B,
       [LDB], ALPHA, BETA, VL, [LDVL], VR, [LDVR], ILO, IHI, LSCALE,
       RSCALE, ABNRM, BBNRM, RCONDE, RCONDV, [WORK], [LWORK], [RWORK],
       [IWORK], [BWORK], [INFO])

CHARACTER(LEN=1) :: BALANC, JOBVL, JOBVR, SENSE
COMPLEX, DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX, DIMENSION(:,:) :: A, B, VL, VR
INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, ILO, IHI, LWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
LOGICAL(8), DIMENSION(:) :: BWORK
REAL :: ABNRM, BBNRM
REAL, DIMENSION(:) :: LSCALE, RSCALE, RCONDE, RCONDV, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void cggevx(char balanc, char jobvl, char jobvr, char sense,
          int  n,  complex *a, int lda, complex *b, int ldb,
          complex *alpha, complex *beta,  complex  *vl,  int
          ldvl,  complex  *vr, int ldvr, int *ilo, int *ihi,
          float *lscale, float *rscale, float *abnrm,  float
          *bbnrm, float *rconde, float *rcondv, int *info);

void cggevx_64(char balanc, char  jobvl,  char  jobvr,  char
          sense,  long  n, complex *a, long lda, complex *b,
          long ldb, complex *alpha, complex  *beta,  complex
          *vl, long ldvl, complex *vr, long ldvr, long *ilo,
          long *ihi, float  *lscale,  float  *rscale,  float
          *abnrm,   float   *bbnrm,   float  *rconde,  float
          *rcondv, long *info);
```

PURPOSE

cggevx computes for a pair of N-by-N complex nonsymmetric matrices (A,B) the generalized eigenvalues, and optionally, the left and/or right generalized eigenvectors.

Optionally, it also computes a balancing transformation to improve the conditioning of the eigenvalues and eigenvectors (ILO, IHI, LSCALE, RSCALE, ABNRM, and BBNRM), reciprocal condition numbers for the eigenvalues (RCONDE), and reciprocal condition numbers for the right eigenvectors (RCONDV).

A generalized eigenvalue for a pair of matrices (A,B) is a scalar lambda or a ratio alpha/beta = lambda, such that A - lambda*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0, and even for both being zero.
The right eigenvector v(j) corresponding to the eigenvalue lambda(j) of (A,B) satisfies

```
                 A * v(j) = lambda(j) * B * v(j) .
```

The left eigenvector u(j) corresponding to the eigenvalue lambda(j) of (A,B) satisfies

```
                 u(j)**H * A  = lambda(j) * u(j)**H * B.
```

where u(j)**H is the conjugate-transpose of u(j).

ARGUMENTS

BALANC (input)
Specifies the balance option to be performed:
= 'N': do not diagonally scale or permute;
= 'P': permute only;
= 'S': scale only;
= 'B': both permute and scale. Computed reciprocal condition numbers will be for the matrices after permuting and/or balancing. Permuting does not change condition numbers (in exact arithmetic), but balancing does.

JOBVL (input)
= 'N': do not compute the left generalized eigenvectors;
= 'V': compute the left generalized eigenvectors.

JOBVR (input)
= 'N': do not compute the right generalized eigenvectors;
= 'V': compute the right generalized eigenvectors.

SENSE (input)

Determines which reciprocal condition numbers are computed. = 'N': none are computed;
= 'E': computed for eigenvalues only;
= 'V': computed for eigenvectors only;
= 'B': computed for eigenvalues and eigenvectors.

N (input) The order of the matrices A, B, VL, and VR. N >= 0.

A (input/output)
On entry, the matrix A in the pair (A,B). On exit, A has been overwritten. If JOBVL='V' or JOBVR='V' or both, then A contains the first part of the complex Schur form of the "balanced" versions of the input A and B.

LDA (input)
The leading dimension of A. LDA >= max(1,N).

B (input/output)
On entry, the matrix B in the pair (A,B). On exit, B has been overwritten. If JOBVL='V' or JOBVR='V' or both, then B contains the second part of the complex Schur form of the "balanced" versions of the input A and B.

LDB (input)
The leading dimension of B. LDB >= max(1,N).

ALPHA (output)
On exit, ALPHA(j)/BETA(j), j=1,...,N, will be the generalized eigenvalues.

Note: the quotient ALPHA(j)/BETA(j)) may easily over- or underflow, and BETA(j) may even be zero. Thus, the user should avoid naively computing the ratio ALPHA/BETA. However, ALPHA will be always less than and usually comparable with norm(A) in magnitude, and BETA always less than and usually comparable with norm(B).

BETA (output)
See description of ALPHA.

VL (output)
If JOBVL = 'V', the left generalized eigenvectors u(j) are stored one after another in the columns of VL, in the same order as their eigenvalues. Each eigenvector will be scaled so the largest component will have abs(real part) + abs(imag.

```
          part) = 1.  Not referenced if JOBVL = 'N'.

LDVL (input)
          The leading dimension of the matrix VL. LDVL >= 1,
          and if JOBVL = 'V', LDVL >= N.
VR (input)
          If JOBVR = 'V', the right generalized eigenvectors
          v(j)  are  stored one after another in the columns
          of VR, in the same  order  as  their  eigenvalues.
          Each  eigenvector  will  be  scaled so the largest
          component will have  abs(real  part)  +  abs(imag.
          part) = 1.  Not referenced if JOBVR = 'N'.

LDVR (input)
          The leading dimension of the matrix VR. LDVR >= 1,
          and if JOBVR = 'V', LDVR >= N.

ILO (output)
          ILO is an integer value such that on exit A(i,j) =
          0 and B(i,j) = 0 if i > j and j = 1,...,ILO-1 or i
          = IHI+1,...,N.  If BALANC = 'N' or 'S',  ILO  =  1
          and IHI = N.

IHI (output)
          IHI is an integer value such that on exit A(i,j) =
          0 and B(i,j) = 0 if i > j and j = 1,...,ILO-1 or i
          = IHI+1,...,N.  If BALANC = 'N' or 'S',  ILO  =  1
          and IHI = N.

LSCALE (output)
          Details of the permutations  and  scaling  factors
          applied  to the left side of A and B.  If PL(j) is
          the index of the row interchanged with row j,  and
          DL(j) is the scaling factor applied to row j, then
          LSCALE(j) = PL(j)  for j  =  1,...,ILO-1  =  DL(j)
          for  j = ILO,...,IHI = PL(j)  for j = IHI+1,...,N.
          The order in which the interchanges are made is  N
          to IHI+1, then 1 to ILO-1.

RSCALE (output)
          Details of the permutations  and  scaling  factors
          applied to the right side of A and B.  If PR(j) is
          the index of the column interchanged  with  column
          j,  and  DR(j)  is  the  scaling factor applied to
          column  j,  then  RSCALE(j)  =  PR(j)   for  j   =
          1,...,ILO-1  =  DR(j)  for j = ILO,...,IHI = PR(j)
          for j = IHI+1,...,N The order in which the  inter-
          changes are made is N to IHI+1, then 1 to ILO-1.

ABNRM (output)
```

The one-norm of the balanced matrix A.

BBNRM (output)
The one-norm of the balanced matrix B.

RCONDE (output)
If SENSE = 'E' or 'B', the reciprocal condition numbers of the selected eigenvalues, stored in consecutive elements of the array. If SENSE = 'V', RCONDE is not referenced.

RCONDV (output)
If JOB = 'V' or 'B', the estimated reciprocal condition numbers of the selected eigenvectors, stored in consecutive elements of the array. If the eigenvalues cannot be reordered to compute RCONDV(j), RCONDV(j) is set to 0; this can only occur when the true value would be very small anyway. If SENSE = 'E', RCONDV is not referenced. Not referenced if JOB = 'E'.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,2*N). If SENSE = 'N' or 'E', LWORK >= 2*N. If SENSE = 'V' or 'B', LWORK >= 2*N*N+2*N.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
dimension(6*N) Real workspace.

IWORK (workspace)
dimension(N+2) If SENSE = 'E', IWORK is not referenced.

BWORK (workspace)
dimension(N) If SENSE = 'N', BWORK is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
= 1,...,N: The QZ iteration failed. No eigenvec-

tors have been calculated, but ALPHA(j) and BETA(j) should be correct for j=INFO+1,...,N. > N: =N+1: other than QZ iteration failed in CHGEQZ.
=N+2: error return from CTGEVC.

FURTHER DETAILS

Balancing a matrix pair (A,B) includes, first, permuting rows and columns to isolate eigenvalues, second, applying diagonal similarity transformation to the rows and columns to make the rows and columns as close in norm as possible. The computed reciprocal condition numbers correspond to the balanced matrix. Permuting rows and columns will not change the condition numbers (in exact arithmetic) but diagonal scaling will. For further explanation of balancing, see section 4.11.1.2 of LAPACK Users' Guide.

An approximate error bound on the chordal distance between the i-th computed generalized eigenvalue w and the corresponding exact eigenvalue lambda is
hord(w, lambda) <= EPS * norm(ABNRM, BBNRM) / RCONDE(I)

An approximate error bound for the angle between the i-th computed eigenvector VL(i) or VR(i) is given by
PS * norm(ABNRM, BBNRM) / DIF(i).

For further explanation of the reciprocal condition numbers RCONDE and RCONDV, see section 4.11 of LAPACK User's Guide.

Contents

NAME

```
cggglm - solve a general  Gauss-Markov  linear  model  (GLM)
problem
```

SYNOPSIS

```
SUBROUTINE CGGGLM(N, M, P, A, LDA, B, LDB, D, X, Y, WORK, LDWORK,
      INFO)

COMPLEX A(LDA,*), B(LDB,*), D(*), X(*), Y(*), WORK(*)
INTEGER N, M, P, LDA, LDB, LDWORK, INFO

SUBROUTINE CGGGLM_64(N, M, P, A, LDA, B, LDB, D, X, Y, WORK, LDWORK,
      INFO)

COMPLEX A(LDA,*), B(LDB,*), D(*), X(*), Y(*), WORK(*)
INTEGER*8 N, M, P, LDA, LDB, LDWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE GGGLM([N], [M], [P], A, [LDA], B, [LDB], D, X, Y, [WORK],
       [LDWORK], [INFO])

COMPLEX, DIMENSION(:) :: D, X, Y, WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: N, M, P, LDA, LDB, LDWORK, INFO

SUBROUTINE GGGLM_64([N], [M], [P], A, [LDA], B, [LDB], D, X, Y, [WORK],
       [LDWORK], [INFO])

COMPLEX, DIMENSION(:) :: D, X, Y, WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: N, M, P, LDA, LDB, LDWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cggglm(int n, int m, int p, complex *a, int  lda,  com-
          plex  *b, int ldb, complex *d, complex *x, complex
          *y, int *info);

void cggglm_64(long n, long m, long p, complex *a, long lda,
          complex *b, long ldb, complex *d, complex *x, com-
          plex *y, long *info);
```

PURPOSE

```
cggglm solves a  general  Gauss-Markov  linear  model  (GLM)
problem:
        minimize || y ||_2   subject to   d = A*x + B*y
            x

where A is an N-by-M matrix, B is an N-by-P matrix, and d is
a given N-vector. It is assumed that M <= N <= M+P, and

           rank(A) = M    and    rank( A B ) = N.

Under these assumptions, the constrained equation is  always
consistent,  and  there is a unique solution x and a minimal
2-norm solution y, which is obtained using a generalized  QR
factorization of A and B.

In particular, if matrix B is square nonsingular,  then  the
problem  GLM  is equivalent to the following weighted linear
least squares problem

             minimize || inv(B)*(d-A*x) ||_2
                 x

where inv(B) denotes the inverse of B.
```

ARGUMENTS

```
N (input) The number of rows of the matrices A and B.  N  >=
          0.

M (input) The number of columns of the matrix A.  0 <= M  <=
          N.

P (input) The number of columns of the matrix B.  P >= N-M.
```

A (input/output)
On entry, the N-by-M matrix A. On exit, A is destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the N-by-P matrix B. On exit, B is destroyed.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

D (input/output)
On entry, D is the left hand side of the GLM equation. On exit, D is destroyed.

X (output)
On exit, X and Y are the solutions of the GLM problem.

Y (output)
On exit, X and Y are the solutions of the GLM problem.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,N+M+P). For optimum performance, LDWORK >= M+min(N,P)+max(N,P)*NB, where NB is an upper bound for the optimal blocksizes for CGEQRF, CGERQF, CUNMQR and CUNMRQ.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

cgghrd - reduce a pair of complex matrices (A,B) to generalized upper Hessenberg form using unitary transformations, where A is a general matrix and B is upper triangular

SYNOPSIS

```
SUBROUTINE CGGHRD(COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB, Q, LDQ,
      Z, LDZ, INFO)

CHARACTER * 1 COMPQ, COMPZ
COMPLEX A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*)
INTEGER N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO

SUBROUTINE CGGHRD_64(COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB, Q,
      LDQ, Z, LDZ, INFO)

CHARACTER * 1 COMPQ, COMPZ
COMPLEX A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*)
INTEGER*8 N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO
```

F95 INTERFACE

```
SUBROUTINE GGHRD(COMPQ, COMPZ, [N], ILO, IHI, A, [LDA], B, [LDB], Q,
       [LDQ], Z, [LDZ], [INFO])

CHARACTER(LEN=1) :: COMPQ, COMPZ
COMPLEX, DIMENSION(:,:) :: A, B, Q, Z
INTEGER :: N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO

SUBROUTINE GGHRD_64(COMPQ, COMPZ, [N], ILO, IHI, A, [LDA], B, [LDB],
       Q, [LDQ], Z, [LDZ], [INFO])
```

```
CHARACTER(LEN=1) :: COMPQ, COMPZ
COMPLEX, DIMENSION(:,:) :: A, B, Q, Z
INTEGER(8) :: N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cgghrd(char compq, char compz, int n, int ilo, int ihi,
          complex  *a, int lda, complex *b, int ldb, complex
          *q, int ldq, complex *z, int ldz, int *info);

void cgghrd_64(char compq, char compz,  long  n,  long  ilo,
          long  ihi,  complex *a, long lda, complex *b, long
          ldb, complex *q, long ldq, complex *z,  long  ldz,
          long *info);
```

PURPOSE

cgghrd reduces a pair of complex matrices (A,B) to generalized upper Hessenberg form using unitary transformations, where A is a general matrix and B is upper triangular: Q' * A * Z = H and Q' * B * Z = T, where H is upper Hessenberg, T is upper triangular, and Q and Z are unitary, and ' means conjugate transpose.

The unitary matrices Q and Z are determined as products of Givens rotations. They may either be formed explicitly, or they may be postmultiplied into input matrices Q1 and Z1, so that
1 * A * Z1' = (Q1*Q) * H * (Z1*Z)'

ARGUMENTS

COMPQ (input)
= 'N': do not compute Q;
= 'I': Q is initialized to the unit matrix, and the unitary matrix Q is returned; = 'V': Q must contain a unitary matrix Q1 on entry, and the product Q1*Q is returned.

COMPZ (input)
= 'N': do not compute Q;
= 'I': Q is initialized to the unit matrix, and the unitary matrix Q is returned; = 'V': Q must contain a unitary matrix Q1 on entry, and the product Q1*Q is returned.

N (input) The order of the matrices A and B. N >= 0.

ILO (input)
It is assumed that A is already upper triangular in rows and columns 1:ILO-1 and IHI+1:N. ILO and IHI are normally set by a previous call to CGGBAL; otherwise they should be set to 1 and N respectively. 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

IHI (input)
See description of ILO.

A (input/output)
On entry, the N-by-N general matrix to be reduced. On exit, the upper triangle and the first subdiagonal of A are overwritten with the upper Hessenberg matrix H, and the rest is set to zero.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the N-by-N upper triangular matrix B. On exit, the upper triangular matrix T = Q' B Z. The elements below the diagonal are set to zero.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

Q (input/output)
If COMPQ='N': Q is not referenced.
If COMPQ='I': on entry, Q need not be set, and on exit it contains the unitary matrix Q, where Q' is the product of the Givens transformations which are applied to A and B on the left. If COMPQ='V': on entry, Q must contain a unitary matrix Q1, and on exit this is overwritten by Q1*Q.

LDQ (input)
The leading dimension of the array Q. LDQ >= N if COMPQ='V' or 'I'; LDQ >= 1 otherwise.

Z (input/output)
If COMPZ='N': Z is not referenced.
If COMPZ='I': on entry, Z need not be set, and on exit it contains the unitary matrix Z, which is the product of the Givens transformations which

are applied to A and B on the right. If COMPZ='V': on entry, Z must contain a unitary matrix Z1, and on exit this is overwritten by Z1*Z.

LDZ (input)
The leading dimension of the array Z. LDZ >= N if COMPZ='V' or 'I'; LDZ >= 1 otherwise.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

This routine reduces A to Hessenberg and B to triangular form by an unblocked reduction, as described in _Matrix_Computations_, by Golub and van Loan (Johns Hopkins Press).

Contents

NAME

```
cgglse - solve the linear equality-constrained least squares
(LSE) problem
```

SYNOPSIS

```
SUBROUTINE CGGLSE(M, N, P, A, LDA, B, LDB, C, D, X, WORK, LDWORK,
      INFO)

COMPLEX A(LDA,*), B(LDB,*), C(*), D(*), X(*), WORK(*)
INTEGER M, N, P, LDA, LDB, LDWORK, INFO

SUBROUTINE CGGLSE_64(M, N, P, A, LDA, B, LDB, C, D, X, WORK, LDWORK,
      INFO)

COMPLEX A(LDA,*), B(LDB,*), C(*), D(*), X(*), WORK(*)
INTEGER*8 M, N, P, LDA, LDB, LDWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE GGLSE([M], [N], [P], A, [LDA], B, [LDB], C, D, X, [WORK],
       [LDWORK], [INFO])

COMPLEX, DIMENSION(:) :: C, D, X, WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: M, N, P, LDA, LDB, LDWORK, INFO

SUBROUTINE GGLSE_64([M], [N], [P], A, [LDA], B, [LDB], C, D, X, [WORK],
       [LDWORK], [INFO])

COMPLEX, DIMENSION(:) :: C, D, X, WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: M, N, P, LDA, LDB, LDWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cgglse(int m, int n, int p, complex *a, int  lda,  com-
          plex  *b, int ldb, complex *c, complex *d, complex
          *x, int *info);

void cgglse_64(long m, long n, long p, complex *a, long lda,
          complex *b, long ldb, complex *c, complex *d, com-
          plex *x, long *info);
```

PURPOSE

```
cgglse solves the linear equality-constrained least  squares
(LSE) problem:
        minimize || c - A*x ||_2   subject to   B*x = d

where A is an M-by-N matrix, B is a P-by-N matrix,  c  is  a
given  M-vector,  and  d  is a given P-vector. It is assumed
that
P <= N <= M+P, and

         rank(B) = P and  rank( ( A ) ) = N.
                              ( ( B ) )

These conditions ensure that the LSE problem  has  a  unique
solution, which is obtained using a GRQ factorization of the
matrices B and A.
```

ARGUMENTS

```
M (input) The number of rows of the matrix A.  M >= 0.

N (input) The number of columns of the matrices A and  B.  N
          >= 0.

P (input) The number of rows of the matrix B. 0 <= P <= N <=
          M+P.

A (input/output)
          On entry, the M-by-N matrix A.  On exit, A is des-
          troyed.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,M).
```

B (input/output)
On entry, the P-by-N matrix B. On exit, B is destroyed.

LDB (input)
The leading dimension of the array B. LDB >= max(1,P).

C (input/output)
On entry, C contains the right hand side vector for the least squares part of the LSE problem. On exit, the residual sum of squares for the solution is given by the sum of squares of elements N-P+1 to M of vector C.

D (input/output)
On entry, D contains the right hand side vector for the constrained equation. On exit, D is destroyed.

X (output)
On exit, X is the solution of the LSE problem.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,M+N+P). For optimum performance LDWORK >= P+min(M,N)+max(M,N)*NB, where NB is an upper bound for the optimal blocksizes for CGEQRF, CGERQF, CUNMQR and CUNMRQ.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

```
cggqrf - compute a generalized QR factorization of an N-by-M
matrix A and an N-by-P matrix B.
```

SYNOPSIS

```
SUBROUTINE CGGQRF(N, M, P, A, LDA, TAUA, B, LDB, TAUB, WORK, LWORK,
      INFO)

COMPLEX A(LDA,*), TAUA(*), B(LDB,*), TAUB(*), WORK(*)
INTEGER N, M, P, LDA, LDB, LWORK, INFO

SUBROUTINE CGGQRF_64(N, M, P, A, LDA, TAUA, B, LDB, TAUB, WORK,
      LWORK, INFO)

COMPLEX A(LDA,*), TAUA(*), B(LDB,*), TAUB(*), WORK(*)
INTEGER*8 N, M, P, LDA, LDB, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE GGQRF([N], [M], [P], A, [LDA], TAUA, B, [LDB], TAUB, [WORK],
       [LWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAUA, TAUB, WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: N, M, P, LDA, LDB, LWORK, INFO

SUBROUTINE GGQRF_64([N], [M], [P], A, [LDA], TAUA, B, [LDB], TAUB,
       [WORK], [LWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAUA, TAUB, WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: N, M, P, LDA, LDB, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cggqrf(int n, int m, int p, complex *a, int  lda,  com-
          plex  *taua,  complex  *b, int ldb, complex *taub,
          int *info);

void cggqrf_64(long n, long m, long p, complex *a, long lda,
          complex  *taua,  complex  *b,  long  ldb,  complex
          *taub, long *info);
```

PURPOSE

```
cggqrf computes a generalized QR factorization of an  N-by-M
matrix A and an N-by-P matrix B:
            A = Q*R,        B = Q*T*Z,

where Q is an N-by-N unitary matrix, Z is a  P-by-P  unitary
matrix, and R and T assume one of the forms:

if N >= M,  R = ( R11 ) M  ,   or if N < M,  R = ( R11   R12
) N,
                (  0  ) N-M                         N   M-N
                   M

where R11 is upper triangular, and

if N <= P,  T = ( 0  T12 ) N,   or if N > P,  T =  (  T11  )
N-P,
                 P-N  N                           ( T21 ) P
                                                     P

where T12 or T21 is upper triangular.

In particular, if B is square and nonsingular, the GQR  fac-
torization  of A and B implicitly gives the QR factorization
of inv(B)*A:

             inv(B)*A = Z'*(inv(T)*R)

where inv(B) denotes the inverse of the  matrix  B,  and  Z'
denotes the conjugate transpose of matrix Z.
```

ARGUMENTS

N (input) The number of rows of the matrices A and B. N >=

0.

M (input) The number of columns of the matrix A. M >= 0.

P (input) The number of columns of the matrix B. P >= 0.

A (input/output)
On entry, the N-by-M matrix A. On exit, the elements on and above the diagonal of the array contain the min(N,M)-by-M upper trapezoidal matrix R (R is upper triangular if N >= M); the elements below the diagonal, with the array TAUA, represent the unitary matrix Q as a product of min(N,M) elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

TAUA (output)
The scalar factors of the elementary reflectors which represent the unitary matrix Q (see Further Details).

B (input/output)
On entry, the N-by-P matrix B. On exit, if N <= P, the upper triangle of the subarray B(1:N,P-N+1:P) contains the N-by-N upper triangular matrix T; if N > P, the elements on and above the (N-P)-th subdiagonal contain the N-by-P upper trapezoidal matrix T; the remaining elements, with the array TAUB, represent the unitary matrix Z as a product of elementary reflectors (see Further Details).

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

TAUB (output)
The scalar factors of the elementary reflectors which represent the unitary matrix Z (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >=

max(1,N,M,P). For optimum performance LWORK >= max(N,M,P)*max(NB1,NB2,NB3), where NB1 is the optimal blocksize for the QR factorization of an N-by-M matrix, NB2 is the optimal blocksize for the RQ factorization of an N-by-P matrix, and NB3 is the optimal blocksize for a call of CUNMQR.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

The matrix Q is represented as a product of elementary reflectors

```
Q = H(1) H(2) . . . H(k), where k = min(n,m).
```

Each H(i) has the form

```
H(i) = I - taua * v * v'
```

where taua is a complex scalar, and v is a complex vector with v(1:i-1) = 0 and v(i) = 1; v(i+1:n) is stored on exit in A(i+1:n,i), and taua in TAUA(i).
To form Q explicitly, use LAPACK subroutine CUNGQR.
To use Q to update another matrix, use LAPACK subroutine CUNMQR.

The matrix Z is represented as a product of elementary reflectors

```
Z = H(1) H(2) . . . H(k), where k = min(n,p).
```

Each H(i) has the form

```
H(i) = I - taub * v * v'
```

where taub is a complex scalar, and v is a complex vector with v(p-k+i+1:p) = 0 and v(p-k+i) = 1; v(1:p-k+i-1) is stored on exit in B(n-k+i,1:p-k+i-1), and taub in TAUB(i).
To form Z explicitly, use LAPACK subroutine CUNGRQ.
To use Z to update another matrix, use LAPACK subroutine CUNMRQ.

Contents

NAME

```
cggrqf - compute a generalized RQ factorization of an M-by-N
matrix A and a P-by-N matrix B
```

SYNOPSIS

```
SUBROUTINE CGGRQF(M, P, N, A, LDA, TAUA, B, LDB, TAUB, WORK, LWORK,
      INFO)

COMPLEX A(LDA,*), TAUA(*), B(LDB,*), TAUB(*), WORK(*)
INTEGER M, P, N, LDA, LDB, LWORK, INFO

SUBROUTINE CGGRQF_64(M, P, N, A, LDA, TAUA, B, LDB, TAUB, WORK,
      LWORK, INFO)

COMPLEX A(LDA,*), TAUA(*), B(LDB,*), TAUB(*), WORK(*)
INTEGER*8 M, P, N, LDA, LDB, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE GGRQF([M], [P], [N], A, [LDA], TAUA, B, [LDB], TAUB, [WORK],
       [LWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAUA, TAUB, WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: M, P, N, LDA, LDB, LWORK, INFO

SUBROUTINE GGRQF_64([M], [P], [N], A, [LDA], TAUA, B, [LDB], TAUB,
       [WORK], [LWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAUA, TAUB, WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: M, P, N, LDA, LDB, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cggrqf(int m, int p, int n, complex *a, int  lda,  com-
          plex  *taua,  complex  *b, int ldb, complex *taub,
          int *info);

void cggrqf_64(long m, long p, long n, complex *a, long lda,
          complex  *taua,  complex  *b,  long  ldb,  complex
          *taub, long *info);
```

PURPOSE

```
cggrqf computes a generalized RQ factorization of an  M-by-N
matrix A and a P-by-N matrix B:
            A = R*Q,        B = Z*T*Q,

where Q is an N-by-N unitary matrix, Z is a  P-by-P  unitary
matrix, and R and T assume one of the forms:

if M <= N,  R = ( 0  R12 ) M,   or if M > N,  R =  (  R11  )
M-N,
                 N-M  M                           ( R21 ) N
                                                     N

where R12 or R21 is upper triangular, and

if P >= N,  T = ( T11 ) N  ,   or if P < N,  T = ( T11   T12
) P,
                (  0  ) P-N                         P   N-P
                   N

where T11 is upper triangular.

In particular, if B is square and nonsingular, the GRQ  fac-
torization  of A and B implicitly gives the RQ factorization
of A*inv(B):

             A*inv(B) = (R*inv(T))*Z'

where inv(B) denotes the inverse of the  matrix  B,  and  Z'
denotes the conjugate transpose of the matrix Z.
```

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

P (input) The number of rows of the matrix B. P >= 0.

N (input) The number of columns of the matrices A and B. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, if M <= N, the upper triangle of the subarray A(1:M,N-M+1:N) contains the M-by-M upper triangular matrix R; if M > N, the elements on and above the (M-N)-th subdiagonal contain the M-by-N upper trapezoidal matrix R; the remaining elements, with the array TAUA, represent the unitary matrix Q as a product of elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

TAUA (output)
The scalar factors of the elementary reflectors which represent the unitary matrix Q (see Further Details).

B (input/output)
On entry, the P-by-N matrix B. On exit, the elements on and above the diagonal of the array contain the min(P,N)-by-N upper trapezoidal matrix T (T is upper triangular if P >= N); the elements below the diagonal, with the array TAUB, represent the unitary matrix Z as a product of elementary reflectors (see Further Details).

LDB (input)
The leading dimension of the array B. LDB >= max(1,P).

TAUB (output)
The scalar factors of the elementary reflectors which represent the unitary matrix Z (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,N,M,P). For optimum performance LWORK >=

```
          max(N,M,P)*max(NB1,NB2,NB3),  where  NB1  is   the
          optimal  blocksize  for the RQ factorization of an
          M-by-N matrix, NB2 is the  optimal  blocksize  for
          the  QR  factorization of a P-by-N matrix, and NB3
          is the optimal blocksize for a call of CUNMRQ.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.
INFO (output)
          = 0:  successful exit
          < 0:  if INFO=-i, the i-th argument had an illegal
          value.
```

FURTHER DETAILS

```
The matrix Q is  represented  as  a  product  of  elementary
reflectors

   Q = H(1) H(2) . . . H(k), where k = min(m,n).

Each H(i) has the form

   H(i) = I - taua * v * v'

where taua is a complex scalar, and v is  a  complex  vector
with  v(n-k+i+1:n)  =  0  and  v(n-k+i) = 1; v(1:n-k+i-1) is
stored on exit in A(m-k+i,1:n-k+i-1), and taua in TAUA(i).
To form Q explicitly, use LAPACK subroutine CUNGRQ.
To use Q to update another  matrix,  use  LAPACK  subroutine
CUNMRQ.

The matrix Z is  represented  as  a  product  of  elementary
reflectors

   Z = H(1) H(2) . . . H(k), where k = min(p,n).

Each H(i) has the form

   H(i) = I - taub * v * v'

where taub is a complex scalar, and v is  a  complex  vector
with  v(1:i-1)  = 0 and v(i) = 1; v(i+1:p) is stored on exit
in B(i+1:p,i), and taub in TAUB(i).
To form Z explicitly, use LAPACK subroutine CUNGQR.
To use Z to update another  matrix,  use  LAPACK  subroutine
CUNMQR.
```

Contents

NAME

```
cggsvd - compute the generalized singular  value  decomposi-
tion (GSVD) of an M-by-N complex matrix A and P-by-N complex
matrix B
```

SYNOPSIS

```
SUBROUTINE CGGSVD(JOBU, JOBV, JOBQ, M, N, P, K, L, A, LDA, B, LDB,
      ALPHA, BETA, U, LDU, V, LDV, Q, LDQ, WORK, WORK2, IWORK3, INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
COMPLEX A(LDA,*), B(LDB,*),  U(LDU,*),  V(LDV,*),  Q(LDQ,*),
WORK(*)
INTEGER M, N, P, K, L, LDA, LDB, LDU, LDV, LDQ, INFO
INTEGER IWORK3(*)
REAL ALPHA(*), BETA(*), WORK2(*)

SUBROUTINE CGGSVD_64(JOBU, JOBV, JOBQ, M, N, P, K, L, A, LDA, B, LDB,
      ALPHA, BETA, U, LDU, V, LDV, Q, LDQ, WORK, WORK2, IWORK3, INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
COMPLEX A(LDA,*), B(LDB,*),  U(LDU,*),  V(LDV,*),  Q(LDQ,*),
WORK(*)
INTEGER*8 M, N, P, K, L, LDA, LDB, LDU, LDV, LDQ, INFO
INTEGER*8 IWORK3(*)
REAL ALPHA(*), BETA(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GGSVD(JOBU, JOBV, JOBQ, [M], [N], [P], K, L, A, [LDA], B,
       [LDB], ALPHA, BETA, U, [LDU], V, [LDV], Q, [LDQ], [WORK], [WORK2],
       IWORK3, [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
```

```
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B, U, V, Q
INTEGER :: M, N, P, K, L, LDA, LDB, LDU, LDV, LDQ, INFO
INTEGER, DIMENSION(:) :: IWORK3
REAL, DIMENSION(:) :: ALPHA, BETA, WORK2

SUBROUTINE GGSVD_64(JOBU, JOBV, JOBQ, [M], [N], [P], K, L, A, [LDA],
       B, [LDB], ALPHA, BETA, U, [LDU], V, [LDV], Q, [LDQ], [WORK],
       [WORK2], IWORK3, [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B, U, V, Q
INTEGER(8) :: M, N, P, K, L, LDA, LDB, LDU, LDV, LDQ, INFO
INTEGER(8), DIMENSION(:) :: IWORK3
REAL, DIMENSION(:) :: ALPHA, BETA, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cggsvd(char jobu, char jobv, char jobq, int m,  int  n,
          int  p,  int *k, int *l, complex *a, int lda, com-
          plex *b, int ldb, float *alpha, float *beta,  com-
          plex *u, int ldu, complex *v, int ldv, complex *q,
          int ldq, int *iwork3, int *info);

void cggsvd_64(char jobu, char jobv, char jobq, long m, long
          n, long p, long *k, long *l, complex *a, long lda,
          complex *b, long ldb, float *alpha,  float  *beta,
          complex  *u,  long ldu, complex *v, long ldv, com-
          plex *q, long ldq, long *iwork3, long *info);
```

PURPOSE

cggsvd computes the generalized singular value decomposition (GSVD) of an M-by-N complex matrix A and P-by-N complex matrix B:

```
      U'*A*Q = D1*( 0 R ),    V'*B*Q = D2*( 0 R )
```

where U, V and Q are unitary matrices, and Z' means the conjugate transpose of Z. Let K+L = the effective numerical rank of the matrix (A',B')', then R is a (K+L)-by-(K+L) nonsingular upper triangular matrix, D1 and D2 are M-by-(K+L) and P-by-(K+L) "diagonal" matrices and of the following structures, respectively:

If M-K-L >= 0,

```
                    K  L
```

```
       D1 =     K ( I  0 )
                L ( 0  C )
            M-K-L ( 0  0 )

                  K  L
       D2 =   L ( 0  S )
            P-L ( 0  0 )

                N-K-L  K    L
  ( 0 R ) = K (  0   R11  R12 )
            L (  0    0   R22 )
where

  C = diag( ALPHA(K+1), ... , ALPHA(K+L) ),
  S = diag( BETA(K+1),  ... , BETA(K+L) ),
  C**2 + S**2 = I.

  R is stored in A(1:K+L,N-K-L+1:N) on exit.
If M-K-L < 0,

                  K M-K K+L-M
       D1 =   K ( I  0    0   )
            M-K ( 0  C    0   )

                    K M-K K+L-M
       D2 =   M-K ( 0  S    0  )
            K+L-M ( 0  0    I  )
              P-L ( 0  0    0  )

                   N-K-L  K   M-K  K+L-M
  ( 0 R ) =     K ( 0    R11  R12  R13  )
              M-K ( 0     0   R22  R23  )
            K+L-M ( 0     0    0   R33  )

where

  C = diag( ALPHA(K+1), ... , ALPHA(M) ),
  S = diag( BETA(K+1),  ... , BETA(M) ),
  C**2 + S**2 = I.

  (R11 R12 R13 ) is stored in A(1:M, N-K-L+1:N), and R33  is
stored
  ( 0  R22 R23 )
  in B(M-K+1:L,N+M-K-L+1:N) on exit.

The routine computes C, S, R, and optionally the unitary
transformation matrices U, V and Q.

In particular, if B is an N-by-N  nonsingular  matrix,  then
the GSVD of A and B implicitly gives the SVD of A*inv(B):
```

```
                     A*inv(B) = U*(D1*inv(D2))*V'.
```

If (A',B')' has orthnormal columns, then the GSVD of A and B is also equal to the CS decomposition of A and B. Furthermore, the GSVD can be used to derive the solution of the eigenvalue problem:

```
                     A'*A x = lambda* B'*B x.
```

In some literature, the GSVD of A and B is presented in the form

```
                 U'*A*X = ( 0 D1 ),   V'*B*X = ( 0 D2 )
```

where U and V are orthogonal and X is nonsingular, and D1 and D2 are ``diagonal''. The former GSVD form can be converted to the latter form by taking the nonsingular matrix X as

```
                      X = Q*(  I   0    )
                            (  0 inv(R) )
```

ARGUMENTS

JOBU (input)
- = 'U': Unitary matrix U is computed;
- = 'N': U is not computed.

JOBV (input)
- = 'V': Unitary matrix V is computed;
- = 'N': V is not computed.

JOBQ (input)
- = 'Q': Unitary matrix Q is computed;
- = 'N': Q is not computed.

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrices A and B. N >= 0.

P (input) The number of rows of the matrix B. P >= 0.

K (output)
On exit, K and L specify the dimension of the subblocks described in Purpose. K + L = effective numerical rank of (A',B')'.

L (output)
On exit, K and L specify the dimension of the subblocks described in Purpose. K + L = effective numerical rank of (A',B')'.

A (input/output)

On entry, the M-by-N matrix A. On exit, A contains the triangular matrix R, or part of R. See Purpose for details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the P-by-N matrix B. On exit, B contains part of the triangular matrix R if M-K-L < 0. See Purpose for details.

LDB (input)
The leading dimension of the array B. LDB >= max(1,P).

ALPHA (output)
On exit, ALPHA and BETA contain the generalized singular value pairs of A and B; ALPHA(1:K) = 1,
ALPHA(1:K) = 1,
BETA(1:K) = 0, and if M-K-L >= 0, ALPHA(K+1:K+L) = C,
BETA(K+1:K+L) = S, or if M-K-L < 0, ALPHA(K+1:M)= C, ALPHA(M+1:K+L)= 0
BETA(K+1:M) = S, BETA(M+1:K+L) = 1 and
ALPHA(K+L+1:N) = 0
BETA(K+L+1:N) = 0

BETA (output)
See description of ALPHA.

U (output)
If JOBU = 'U', U contains the M-by-M unitary matrix U. If JOBU = 'N', U is not referenced.

LDU (input)
The leading dimension of the array U. LDU >= max(1,M) if JOBU = 'U'; LDU >= 1 otherwise.

V (output)
If JOBV = 'V', V contains the P-by-P unitary matrix V. If JOBV = 'N', V is not referenced.

LDV (input)
The leading dimension of the array V. LDV >= max(1,P) if JOBV = 'V'; LDV >= 1 otherwise.

Q (output)
If JOBQ = 'Q', Q contains the N-by-N unitary

matrix Q. If JOBQ = 'N', Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= max(1,N) if JOBQ = 'Q'; LDQ >= 1 otherwise.

WORK (workspace)
dimension(MAX(3*N,M,P)+N)

WORK2 (workspace)
dimension(2*N)

IWORK3 (output)
dimension(N) On exit, IWORK3 stores the sorting information. More precisely, the following loop will sort ALPHA for I = K+1, min(M,K+L) swap ALPHA(I) and ALPHA(IWORK3(I)) endfor such that ALPHA(1) >= ALPHA(2) >= ... >= ALPHA(N).

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = 1, the Jacobi-type procedure failed to converge. For further details, see subroutine CTGSJA.

Contents

NAME

```
cggsvp - compute unitary matrices U, V and Q such that    N-
K-L K L  U'*A*Q = K ( 0 A12 A13 ) if M-K-L >= 0
```

SYNOPSIS

```
SUBROUTINE CGGSVP(JOBU, JOBV, JOBQ, M, P, N, A, LDA, B, LDB, TOLA,
      TOLB, K, L, U, LDU, V, LDV, Q, LDQ, IWORK, RWORK, TAU, WORK,
      INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
COMPLEX A(LDA,*), B(LDB,*),  U(LDU,*),  V(LDV,*),  Q(LDQ,*),
TAU(*), WORK(*)
INTEGER M, P, N, LDA, LDB, K, L, LDU, LDV, LDQ, INFO
INTEGER IWORK(*)
REAL TOLA, TOLB
REAL RWORK(*)

SUBROUTINE CGGSVP_64(JOBU, JOBV, JOBQ, M, P, N, A, LDA, B, LDB, TOLA,
      TOLB, K, L, U, LDU, V, LDV, Q, LDQ, IWORK, RWORK, TAU, WORK,
      INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
COMPLEX A(LDA,*), B(LDB,*),  U(LDU,*),  V(LDV,*),  Q(LDQ,*),
TAU(*), WORK(*)
INTEGER*8 M, P, N, LDA, LDB, K, L, LDU, LDV, LDQ, INFO
INTEGER*8 IWORK(*)
REAL TOLA, TOLB
REAL RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGSVP(JOBU, JOBV, JOBQ, [M], [P], [N], A, [LDA], B, [LDB],
```

```
       TOLA, TOLB, K, L, U, [LDU], V, [LDV], Q, [LDQ], [IWORK], [RWORK],
       [TAU], [WORK], [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, B, U, V, Q
INTEGER :: M, P, N, LDA, LDB, K, L, LDU, LDV, LDQ, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL :: TOLA, TOLB
REAL, DIMENSION(:) :: RWORK

SUBROUTINE GGSVP_64(JOBU, JOBV, JOBQ, [M], [P], [N], A, [LDA], B,
       [LDB], TOLA, TOLB, K, L, U, [LDU], V, [LDV], Q, [LDQ], [IWORK],
       [RWORK], [TAU], [WORK], [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, B, U, V, Q
INTEGER(8) :: M, P, N, LDA, LDB, K, L, LDU, LDV, LDQ, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL :: TOLA, TOLB
REAL, DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>

void cggsvp(char jobu, char jobv, char jobq, int m,  int  p,
          int  n,  complex *a, int lda, complex *b, int ldb,
          float tola, float tolb, int *k,  int  *l,  complex
          *u,  int ldu, complex *v, int ldv, complex *q, int
          ldq, int *info);

void cggsvp_64(char jobu, char jobv, char jobq, long m, long
          p,  long n, complex *a, long lda, complex *b, long
          ldb, float tola, float tolb,  long  *k,  long  *l,
          complex  *u,  long ldu, complex *v, long ldv, com-
          plex *q, long ldq, long *info);
```

PURPOSE

```
cggsvp computes unitary matrices U, V and Q such that
              L ( 0     0   A23 )
          M-K-L ( 0     0    0  )

                 N-K-L  K    L
        =     K ( 0    A12  A13 )  if M-K-L < 0;
            M-K ( 0     0   A23 )

               N-K-L  K    L
```

```
 V'*B*Q =   L ( 0     0   B13 )
          P-L ( 0     0    0  )
```

where the K-by-K matrix A12 and L-by-L matrix B13 are non-singular upper triangular; A23 is L-by-L upper triangular if M-K-L >= 0, otherwise A23 is (M-K)-by-L upper trapezoidal. K+L = the effective numerical rank of the (M+P)-by-N matrix (A',B')'. Z' denotes the conjugate transpose of Z.

This decomposition is the preprocessing step for computing the Generalized Singular Value Decomposition (GSVD), see subroutine CGGSVD.

ARGUMENTS

```
JOBU (input)
          = 'U':  Unitary matrix U is computed;
          = 'N':  U is not computed.
JOBV (input)
          = 'V':  Unitary matrix V is computed;
          = 'N':  V is not computed.

JOBQ (input)
          = 'Q':  Unitary matrix Q is computed;
          = 'N':  Q is not computed.
```

M (input) The number of rows of the matrix A. M >= 0.

P (input) The number of rows of the matrix B. P >= 0.

N (input) The number of columns of the matrices A and B. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, A contains the triangular (or trapezoidal) matrix described in the Purpose section.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the P-by-N matrix B. On exit, B contains the triangular matrix described in the Purpose section.

LDB (input)

The leading dimension of the array B. LDB >= max(1,P).

TOLA (input)
TOLA and TOLB are the thresholds to determine the effective numerical rank of matrix B and a sub-block of A. Generally, they are set to TOLA = MAX(M,N)*norm(A)*MACHEPS, TOLB = MAX(P,N)*norm(B)*MACHEPS. The size of TOLA and TOLB may affect the size of backward errors of the decomposition.

TOLB (input)
See description of TOLA.

K (output)
On exit, K and L specify the dimension of the sub-blocks described in Purpose section. K + L = effective numerical rank of (A',B')'.

L (output)
See the description of K.

U (input) If JOBU = 'U', U contains the unitary matrix U. If JOBU = 'N', U is not referenced.

LDU (input)
The leading dimension of the array U. LDU >= max(1,M) if JOBU = 'U'; LDU >= 1 otherwise.

V (input) If JOBV = 'V', V contains the unitary matrix V. If JOBV = 'N', V is not referenced.

LDV (input)
The leading dimension of the array V. LDV >= max(1,P) if JOBV = 'V'; LDV >= 1 otherwise.

Q (input) If JOBQ = 'Q', Q contains the unitary matrix Q. If JOBQ = 'N', Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= max(1,N) if JOBQ = 'Q'; LDQ >= 1 otherwise.

IWORK (workspace)
dimension(N)

RWORK (workspace)
dimension(2*N)

TAU (workspace)

dimension(N)

WORK (workspace)
dimension(MAX(3*N,M,P))

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

The subroutine uses LAPACK subroutine CGEQPF for the QR factorization with column pivoting to detect the effective numerical rank of the a matrix. It may be replaced by a better rank determination strategy.

Contents

NAME

cgssco - General sparse solver condition number estimate.

SYNOPSIS

```
SUBROUTINE CGSSCO ( COND, HANDLE, IER )

INTEGER             IER
REAL                COND
DOUBLE PRECISION    HANDLE(150)
```

PURPOSE

CGSSCO - Condition number estimate.

PARAMETERS

```
COND      - REAL
        On exit, an estimate of the condition number of the
        factored matrix.  Must be called after the numerical
        factorization subroutine, CGSSFA().

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -700 : Invalid calling sequence - need to call CGSSFA first.
```

-710 : Condition number estimate not available (not implemented
for this HANDLE's matix type).

Contents

NAME

```
cgssda - Deallocate working storage for the general sparse
solver.
```

SYNOPSIS

```
 SUBROUTINE CGSSDA ( HANDLE, IER )

 INTEGER           IER
 DOUBLE PRECISION  HANDLE(150)
```

PURPOSE

CGSSDA - Deallocate dynamically allocated working storage.

PARAMETERS

```
HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         none
```

Contents

NAME

```
cgssfa - General sparse solver numeric factorization.
```

SYNOPSIS

```
 SUBROUTINE CGSSFA ( NEQNS, COLSTR, ROWIND, VALUES, HANDLE, IER )

 INTEGER            NEQNS, COLSTR(*), ROWIND(*), IER
 COMPLEX            VALUES(*)
 DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

CGSSFA - Numeric factorization of a sparse matrix.

PARAMETERS

```
NEQNS     - INTEGER
        On entry, NEQNS specifies the number of equations in
        coefficient matrix.  Unchanged on exit.

COLSTR(*) - INTEGER array
        On entry, COLSTR(*) is an array of size (NEQNS+1),
        containing the pointers of the matrix structure.
        Unchanged on exit.

ROWIND(*) - INTEGER array
        On entry, ROWIND(*) is an array of size
        COLSTR(NEQNS+1)-1, containing the indices of the
        matrix structure.  Unchanged on exit.

VALUES(*) - COMPLEX array
        On entry, VALUES(*) is an array of size
        COLSTR(NEQNS+1)-1, containing the numeric values of
```

```
        the sparse matrix to be factored.  Unchanged on
        exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER        - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -300 : Invalid calling sequence - need to call CGSSOR first.
         -301 : Failure to dynamically allocate memory.
         -666 : Internal error.
```

Contents

NAME

cgssfs - General sparse solver one call interface.

SYNOPSIS

```
 SUBROUTINE CGSSFS ( MTXTYP, PIVOT , NEQNS, COLSTR, ROWIND,
                     VALUES, NRHS  , RHS   , LDRHS , ORDMTHD,
                     OUTUNT, MSGLVL, HANDLE, IER )

 CHARACTER*2        MTXTYP
 CHARACTER*1        PIVOT
 INTEGER            NEQNS, COLSTR(*), ROWIND(*), NRHS, LDRHS,
                    OUTUNT, MSGLVL, IER
 CHARACTER*3        ORDMTHD
 COMPLEX            VALUES(*), RHS(*)
 DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

CGSSFS - General sparse solver one call interface.

PARAMETERS

```
MTXTYP    - CHARACTER*2
        On entry, MTXTYP specifies the coefficient matrix
        type.  Specifically, the valid options are:

         'sp' or 'SP' - symmetric structure, Hermitian positive definite
values
         'ss' or 'SS' - symmetric structure, symmetric values
         'su' or 'SU' - symmetric structure, unsymmetric values
         'uu' or 'UU' - unsymmetric structure, unsymmetric values

        Unchanged on exit.
```

```
PIVOT     - CHARACTER*1
        On entry, pivot specifies whether or not pivoting is
        used in the course of the numeric factorization.
        The valid options are:

         'n' or 'N' - no pivoting is used
         (Pivoting is not supported for this release).

        Unchanged on exit.

NEQNS     - INTEGER
        On entry, NEQNS specifies the number of equations in
        the coefficient matrix.  NEQNS must be at least one.
        Unchanged on exit.

COLSTR(*) - INTEGER array
        On entry, COLSTR(*) is an array of size (NEQNS+1),
        containing the pointers of the matrix structure.
        Unchanged on exit.
ROWIND(*) - INTEGER array
        On entry, ROWIND(*) is an array of size
        COLSTR(NEQNS+1)-1, containing the indices of the
        matrix structure.  Unchanged on exit.

VALUES(*) - COMPLEX array
        On entry, VALUES(*) is an array of size
        COLSTR(NEQNS+1)-1, containing the non-zero numeric
        values of the sparse matrix to be factored.
        Unchanged on exit.

NRHS      - INTEGER
        On entry, NRHS specifies the number of right hand
        sides to solve for.  Unchanged on exit.

RHS(*)    - COMPLEX array
        On entry, RHS(LDRHS,NRHS) contains the NRHS right
        hand sides.  On exit, it contains the solutions.

LDRHS     - INTEGER
        On entry, LDRHS specifies the leading dimension of
        the RHS array. Unchanged on exit.

ORDMTHD   - CHARACTER*3
        On entry, ORDMTHD specifies the fill-reducing
        ordering to be used by the sparse solver.
        Specifically, the valid options are:

         'nat' or 'NAT' - natural ordering (no ordering)
         'mmd' or 'MMD' - multiple minimum degree
```

```
         'gnd' or 'GND' - general nested dissection
         'uso' or 'USO' - user specified ordering (see CGSSUO)

        Unchanged on exit.

OUTUNT    - INTEGER
        Output unit.  Unchanged on exit.

MSGLVL    - INTEGER
        Message level.

         0 - no output from solver.
         (No messages supported for this release.)

        Unchanged on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array of containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.
IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -101 : Failure to dynamically allocate memory.
         -102 : Invalid matrix type.
         -103 : Invalid pivot option.
         -104 : Number of nonzeros is less than NEQNS.
         -105 : NEQNS < 1
         -201 : Failure to dynamically allocate memory.
         -301 : Failure to dynamically allocate memory.
         -401 : Failure to dynamically allocate memory.
         -402 : NRHS < 1
         -403 : NEQNS > LDRHS
         -666 : Internal error.
```

Contents

NAME

```
cgssin - Initialize the general sparse solver.
```

SYNOPSIS

```
 SUBROUTINE CGSSIN ( MTXTYP, PIVOT, NEQNS, COLSTR, ROWIND, OUTUNT,
                     MSGLVL, HANDLE, IER )

 CHARACTER*2       MTXTYP
 CHARACTER*1       PIVOT
 INTEGER           NEQNS, COLSTR(*), ROWIND(*), OUTUNT, MSGLVL, IER
 DOUBLE PRECISION  HANDLE(150)
```

PURPOSE

CGSSIN - Initialize the sparse solver and input the matrix
structure.

PARAMETERS

```
MTXTYP    - CHARACTER*2
        On entry, MTXTYP specifies the coefficient matrix
        type.  Specifically, the valid options are:

         'sp' or 'SP' - symmetric structure, Hermitian positive definite
values
         'ss' or 'SS' - symmetric structure, symmetric values
         'su' or 'SU' - symmetric structure, unsymmetric values
         'uu' or 'UU' - unsymmetric structure, unsymmetric values

        Unchanged on exit.

PIVOT     - CHARACTER*1
        On entry, PIVOT specifies whether or not pivoting is
```

```
        used in the course of the numeric factorization.
        The valid options are:

         'n' or 'N' - no pivoting is used
         (Pivoting is not supported for this release).

        Unchanged on exit.

NEQNS     - INTEGER
        On entry, NEQNS specifies the number of equations in
        the coefficient matrix.  NEQNS must be at least one.
        Unchanged on exit.

COLSTR(*) - INTEGER array
        On entry, COLSTR(*) is an array of size (NEQNS+1),
        containing the pointers of the matrix structure.
        Unchanged on exit.

ROWIND(*) - INTEGER array
        On entry, ROWIND(*) is an array of size
        COLSTR(NEQNS+1)-1, containing the indices of the
        matrix structure.  Unchanged on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

OUTUNT    - INTEGER
        Output unit.  Unchanged on exit.

MSGLVL    - INTEGER
        Message level.

         0 - no output from solver.
         (No messages supported for this release.)

        Unchanged on exit.

IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -101 : Failure to dynamically allocate memory.
         -102 : Invalid matrix type.
         -103 : Invalid pivot option.
         -104 : Number of nonzeros less than NEQNS.
         -105 : NEQNS < 1
```

Contents

NAME

```
cgssor - General sparse solver ordering and symbolic
factorization.
```

SYNOPSIS

```
 SUBROUTINE CGSSOR ( ORDMTHD, HANDLE, IER )

 CHARACTER*3       ORDMTHD
 INTEGER           IER
 DOUBLE PRECISION  HANDLE(150)
```

PURPOSE

CGSSOR - Orders and symbolically factors a sparse matrix.

PARAMETERS

```
ORDMTHD   - CHARACTER*3
        On entry, ORDMTHD specifies the fill-reducing
        ordering to be used by the sparse solver.
        Specifically, the valid options are:

         'nat' or 'NAT' - natural ordering (no ordering)
         'mmd' or 'MMD' - multiple minimum degree
         'gnd' or 'GND' - general nested dissection
         'uso' or 'USO' - user specified ordering (see CGSSUO)

        Unchanged on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
```

Modified on exit.

IER - INTEGER
Error number. If no error encountered, unchanged on exit. If error encountered, it is set to a non-zero integer. Error numbers set by this subroutine:

-200 : Invalid calling sequence - need to call CGSSIN first.
-201 : Failure to dynamically allocate memory.
-666 : Internal error.

Contents

NAME

```
cgssps - Print general sparse solver statics.
```

SYNOPSIS

```
 SUBROUTINE CGSSPS ( HANDLE, IER )

 INTEGER            IER
 DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

CGSSPS - Print solver statistics.

PARAMETERS

```
HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -800 : Invalid calling sequence - need to call CGSSSL first.
         -899 : Printed solver statistics not supported this release.
```

Contents

NAME

```
cgssrp - Return permutation used by the general sparse
solver.
```

SYNOPSIS

```
 SUBROUTINE CGSSRP ( PERM, HANDLE, IER )

 INTEGER            PERM(*), IER
 DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

CGSSRP - Returns the permutation used by the solver for the fill-reducing ordering.

PARAMETERS

```
PERM(NEQNS)  - INTEGER array
        Undefined on entry.  PERM(NEQNS) is the permutation
        array used by the sparse solver for the fill-
        reducing ordering.  Modified on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:
```

-600 : Invalid calling sequence - need to call CGSSOR first.

Contents

NAME

cgsssl - Solve routine for the general sparse solver.

SYNOPSIS

```
SUBROUTINE CGSSSL ( NRHS, RHS, LDRHS, HANDLE, IER )

INTEGER            NRHS, LDRHS, IER
COMPLEX            RHS(LDRHS,NRHS)
DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

CGSSSL - Triangular solve of a factored sparse matrix.

PARAMETERS

```
NRHS      - INTEGER
        On entry, NRHS specifies the number of right hand
        sides to solve for.  Unchanged on exit.

RHS(LDRHS,*) - COMPLEX array
        On entry, RHS(LDRHS,NRHS) contains the NRHS right
        hand sides.  On exit, it contains the solutions.

LDRHS     - INTEGER
        On entry, LDRHS specifies the leading dimension of
        the RHS array. Unchanged on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.
```

```
IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -400 : Invalid calling sequence - need to call CGSSFA first.
         -401 : Failure to dynamically allocate memory.
         -402 : NRHS < 1
         -403 : NEQNS > LDRHS
```

Contents

NAME

```
cgssuo - User supplied permutation for ordering used in the
general sparse solver.
```

SYNOPSIS

```
 SUBROUTINE CGSSUO ( PERM, HANDLE, IER )

 INTEGER            PERM(*), IER
 DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

CGSSUO - User supplied permutation for ordering. Must be
called after *CGSSIN*() (sparse solver initialization) and
before *CGSSOR*() (sparse solver ordering).

PARAMETERS

```
PERM(NEQNS)  - INTEGER array
        On entry, PERM(NEQNS) is a permutation array
        supplied by the user for the fill-reducing ordering.
        Unchanged on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:
```

-500 : Invalid calling sequence - need to call CGSSIN first.

Contents

NAME

```
cgtcon - estimate the reciprocal of the condition number  of
a complex tridiagonal matrix A using the LU factorization as
computed by CGTTRF
```

SYNOPSIS

```
SUBROUTINE CGTCON(NORM, N, LOW, DIAG, UP1, UP2, IPIVOT, ANORM, RCOND,
      WORK, INFO)

CHARACTER * 1 NORM
COMPLEX LOW(*), DIAG(*), UP1(*), UP2(*), WORK(*)
INTEGER N, INFO
INTEGER IPIVOT(*)
REAL ANORM, RCOND

SUBROUTINE CGTCON_64(NORM, N, LOW, DIAG, UP1, UP2, IPIVOT, ANORM,
      RCOND, WORK, INFO)

CHARACTER * 1 NORM
COMPLEX LOW(*), DIAG(*), UP1(*), UP2(*), WORK(*)
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
REAL ANORM, RCOND
```

F95 INTERFACE

```
SUBROUTINE GTCON(NORM, [N], LOW, DIAG, UP1, UP2, IPIVOT, ANORM,
       RCOND, [WORK], [INFO])

CHARACTER(LEN=1) :: NORM
COMPLEX, DIMENSION(:) :: LOW, DIAG, UP1, UP2, WORK
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT
```

```
REAL :: ANORM, RCOND

SUBROUTINE GTCON_64(NORM, [N], LOW, DIAG, UP1, UP2, IPIVOT, ANORM,
       RCOND, [WORK], [INFO])

CHARACTER(LEN=1) :: NORM
COMPLEX, DIMENSION(:) :: LOW, DIAG, UP1, UP2, WORK
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL :: ANORM, RCOND
```

C INTERFACE

```
#include <sunperf.h>
void cgtcon(char norm, int n, complex *low,  complex  *diag,
          complex  *up1,  complex  *up2,  int *ipivot, float
          anorm, float *rcond, int *info);

void cgtcon_64(char norm,  long  n,  complex  *low,  complex
          *diag,  complex  *up1, complex *up2, long *ipivot,
          float anorm, float *rcond, long *info);
```

PURPOSE

cgtcon estimates the reciprocal of the condition number of a complex tridiagonal matrix A using the LU factorization as computed by CGTTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

NORM (input)
Specifies whether the 1-norm condition number or the infinity-norm condition number is required:
= '1' or 'O': 1-norm;
= 'I':   Infinity-norm.

N (input) The order of the matrix A. N >= 0.

LOW (input)
The (n-1) multipliers that define the matrix L from the LU factorization of A as computed by CGTTRF.

DIAG (input)

The n diagonal elements of the upper triangular matrix U from the LU factorization of A.

UP1 (input)
The (n-1) elements of the first superdiagonal of U.

UP2 (input)
The (n-2) elements of the second superdiagonal of U.

IPIVOT (input)
The pivot indices; for 1 <= i <= n, row i of the matrix was interchanged with row IPIVOT(i). IPIVOT(i) will always be either i or i+1; IPIVOT(i) = i indicates a row interchange was not required.

ANORM (input)
If NORM = '1' or 'O', the 1-norm of the original matrix A. If NORM = 'I', the infinity-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-gal value

Contents

NAME

```
cgthr - Gathers specified elements from y into x.
```

SYNOPSIS

```
 SUBROUTINE CGTHR(NZ, Y, X, INDX)

 COMPLEX Y(*), X(*)
 INTEGER NZ
 INTEGER INDX(*)

 SUBROUTINE CGTHR_64(NZ, Y, X, INDX)

 COMPLEX Y(*), X(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 SUBROUTINE GTHR([NZ], Y, X, INDX)

 COMPLEX, DIMENSION(:) :: Y, X
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 SUBROUTINE GTHR_64([NZ], Y, X, INDX)

 COMPLEX, DIMENSION(:) :: Y, X
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
CGTHR - Gathers the specified elements from a vector y in
full storage form into a vector x in compressed form.  Only
```

the elements of y whose indices are listed in indx are referenced.

```
 do i = 1, n
   x(i) = y(indx(i))
 enddo
```

ARGUMENTS

NZ (input) - INTEGER
Number of elements in the compressed form. Unchanged on exit.

Y (input)
Vector in full storage form. Unchanged on exit.

X (output)
Vector in compressed form. Contains elements of y whose indices are listed in indx on exit.

INDX (input) - INTEGER
Vector containing the indices of the compressed form. It is assumed that the elements in INDX are distinct and greater than zero. Unchanged on exit.

Contents

NAME

```
cgthrz - Gather and zero.
```

SYNOPSIS

```
 SUBROUTINE CGTHRZ(NZ, Y, X, INDX)

 COMPLEX Y(*), X(*)
 INTEGER NZ
 INTEGER INDX(*)

 SUBROUTINE CGTHRZ_64(NZ, Y, X, INDX)

 COMPLEX Y(*), X(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 SUBROUTINE GTHRZ([NZ], Y, X, INDX)

 COMPLEX, DIMENSION(:) :: Y, X
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 SUBROUTINE GTHRZ_64([NZ], Y, X, INDX)

 COMPLEX, DIMENSION(:) :: Y, X
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
CGTHRZ - Gathers the specified elements from a vector y in
full storage form into a vector x in compressed form.  The
```

gathered elements of y are set to zero. Only the elements of y whose indices are listed in indx are referenced.

```
 do i = 1, n
   x(i) = y(indx(i))
   y(indx(i)) = 0
 enddo
```

ARGUMENTS

NZ (input) - INTEGER
Number of elements in the compressed form. Unchanged on exit.

Y (input/output)
Vector in full storage form. Gathered elements are set to zero.

X (output)
Vector in compressed form. Contains elements of y whose indices are listed in indx on exit.

INDX (input) - INTEGER
Vector containing the indices of the compressed form. It is assumed that the elements in INDX are distinct and greater than zero. Unchanged on exit.

Contents

NAME

cgtrfs - improve the computed solution to a system of linear equations when the coefficient matrix is tridiagonal, and provides error bounds and backward error estimates for the solution

SYNOPSIS

```
SUBROUTINE CGTRFS(TRANSA, N, NRHS, LOW, DIAG, UP, LOWF, DIAGF, UPF1,
      UPF2, IPIVOT, B, LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
COMPLEX LOW(*), DIAG(*), UP(*), LOWF(*), DIAGF(*),  UPF1(*),
UPF2(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*)
REAL FERR(*), BERR(*), WORK2(*)

SUBROUTINE CGTRFS_64(TRANSA, N, NRHS, LOW, DIAG, UP, LOWF, DIAGF,
      UPF1, UPF2, IPIVOT, B, LDB, X, LDX, FERR, BERR, WORK, WORK2,
      INFO)

CHARACTER * 1 TRANSA
COMPLEX LOW(*), DIAG(*), UP(*), LOWF(*), DIAGF(*),  UPF1(*),
UPF2(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
REAL FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GTRFS([TRANSA], [N], [NRHS], LOW, DIAG, UP, LOWF, DIAGF,
       UPF1, UPF2, IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK],
       [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: TRANSA
COMPLEX, DIMENSION(:) :: LOW, DIAG, UP, LOWF,  DIAGF,  UPF1,
UPF2, WORK
COMPLEX, DIMENSION(:,:) :: B, X
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE GTRFS_64([TRANSA], [N], [NRHS], LOW, DIAG, UP, LOWF,
       DIAGF, UPF1, UPF2, IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: TRANSA
COMPLEX, DIMENSION(:) :: LOW, DIAG, UP, LOWF,  DIAGF,  UPF1,
UPF2, WORK
COMPLEX, DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cgtrfs(char transa, int n, int nrhs, complex *low, com-
          plex  *diag,  complex  *up, complex *lowf, complex
          *diagf, complex *upf1, complex *upf2, int *ipivot,
          complex  *b,  int  ldb, complex *x, int ldx, float
          *ferr, float *berr, int *info);

void cgtrfs_64(char transa, long n, long nrhs, complex *low,
          complex *diag, complex *up, complex *lowf, complex
          *diagf,  complex  *upf1,   complex   *upf2,   long
          *ipivot,  complex  *b,  long ldb, complex *x, long
          ldx, float *ferr, float *berr, long *info);
```

PURPOSE

cgtrfs improves the computed solution to a system of linear equations when the coefficient matrix is tridiagonal, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

TRANSA (input)
Specifies the form of the system of equations:

```
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

LOW (input)
          The (n-1) subdiagonal elements of A.
DIAG (input)
          The diagonal elements of A.

UP (input)
          The (n-1) superdiagonal elements of A.

LOWF (input)
          The (n-1) multipliers that  define  the  matrix  L
          from  the  LU  factorization  of  A as computed by
          CGTTRF.

DIAGF (input)
          The n diagonal elements of  the  upper  triangular
          matrix U from the LU factorization of A.

UPF1 (input)
          The (n-1) elements of the first  superdiagonal  of
          U.

UPF2 (input)
          The (n-2) elements of the second superdiagonal  of
          U.

IPIVOT (input)
          The pivot indices; for 1 <= i <= n, row i  of  the
          matrix   was   interchanged  with  row  IPIVOT(i).
          IPIVOT(i)  will  always  be  either  i   or   i+1;
          IPIVOT(i)  = i indicates a row interchange was not
          required.

B (input) The right hand side matrix B.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).
```

X (input/output)
On entry, the solution matrix X, as computed by CGTTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
cgtsv - solve the equation   A*X = B,
```

SYNOPSIS

```
SUBROUTINE CGTSV(N, NRHS, LOW, DIAG, UP, B, LDB, INFO)

COMPLEX LOW(*), DIAG(*), UP(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO

SUBROUTINE CGTSV_64(N, NRHS, LOW, DIAG, UP, B, LDB, INFO)

COMPLEX LOW(*), DIAG(*), UP(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE GTSV([N], [NRHS], LOW, DIAG, UP, B, [LDB], [INFO])

COMPLEX, DIMENSION(:) :: LOW, DIAG, UP
COMPLEX, DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO

SUBROUTINE GTSV_64([N], [NRHS], LOW, DIAG, UP, B, [LDB], [INFO])

COMPLEX, DIMENSION(:) :: LOW, DIAG, UP
COMPLEX, DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cgtsv(int n, int nrhs,  complex  *low,  complex  *diag,
          complex *up, complex *b, int ldb, int *info);
```

```
void cgtsv_64(long  n,  long  nrhs,  complex  *low,  complex
          *diag,  complex  *up,  complex  *b, long ldb, long
          *info);
```

PURPOSE

cgtsv solves the equation

where A is an N-by-N tridiagonal matrix, by Gaussian elimination with partial pivoting.

Note that the equation A'*X = B may be solved by interchanging the order of the arguments DU and DL.

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

LOW (input/output)
On entry, LOW must contain the (n-1) subdiagonal elements of A. On exit, LOW is overwritten by the (n-2) elements of the second superdiagonal of the upper triangular matrix U from the LU factorization of A, in LOW(1), ..., LOW(n-2).

DIAG (input/output)
On entry, DIAG must contain the diagonal elements of A. On exit, DIAG is overwritten by the n diagonal elements of U.

UP (input/output)
On entry, UP must contain the (n-1) superdiagonal elements of A. On exit, UP is overwritten by the (n-1) elements of the first superdiagonal of U.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >=

max(1,N).

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, U(i,i) is exactly zero, and the solution has not been computed. The factorization has not been completed unless i = N.

Contents

NAME

```
cgtsvx - use the LU factorization to compute the solution to
a  complex  system of linear equations A * X = B, A**T * X =
B, or A**H * X = B,
```

SYNOPSIS

```
SUBROUTINE CGTSVX(FACT, TRANSA, N, NRHS, LOW, DIAG, UP, LOWF, DIAGF,
      UPF1, UPF2, IPIVOT, B, LDB, X, LDX, RCOND, FERR, BERR, WORK,
      WORK2, INFO)

CHARACTER * 1 FACT, TRANSA
COMPLEX LOW(*), DIAG(*), UP(*), LOWF(*), DIAGF(*),  UPF1(*),
UPF2(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*)
REAL RCOND
REAL FERR(*), BERR(*), WORK2(*)

SUBROUTINE CGTSVX_64(FACT, TRANSA, N, NRHS, LOW, DIAG, UP, LOWF,
      DIAGF, UPF1, UPF2, IPIVOT, B, LDB, X, LDX, RCOND, FERR, BERR,
      WORK, WORK2, INFO)

CHARACTER * 1 FACT, TRANSA
COMPLEX LOW(*), DIAG(*), UP(*), LOWF(*), DIAGF(*),  UPF1(*),
UPF2(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
REAL RCOND
REAL FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GTSVX(FACT, [TRANSA], [N], [NRHS], LOW, DIAG, UP, LOWF,
```

```
       DIAGF, UPF1, UPF2, IPIVOT, B, [LDB], X, [LDX], RCOND, FERR, BERR,
       [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, TRANSA
COMPLEX, DIMENSION(:) :: LOW, DIAG, UP, LOWF,  DIAGF,  UPF1,
UPF2, WORK
COMPLEX, DIMENSION(:,:) :: B, X
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL :: RCOND
REAL, DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE GTSVX_64(FACT, [TRANSA], [N], [NRHS], LOW, DIAG, UP, LOWF,
       DIAGF, UPF1, UPF2, IPIVOT, B, [LDB], X, [LDX], RCOND, FERR, BERR,
       [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, TRANSA
COMPLEX, DIMENSION(:) :: LOW, DIAG, UP, LOWF,  DIAGF,  UPF1,
UPF2, WORK
COMPLEX, DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL :: RCOND
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cgtsvx(char fact, char transa, int n, int nrhs, complex
          *low,  complex  *diag, complex *up, complex *lowf,
          complex *diagf, complex *upf1, complex *upf2,  int
          *ipivot, complex *b, int ldb, complex *x, int ldx,
          float  *rcond,  float  *ferr,  float  *berr,   int
          *info);

void cgtsvx_64(char fact, char transa, long  n,  long  nrhs,
          complex  *low, complex *diag, complex *up, complex
          *lowf,  complex  *diagf,  complex  *upf1,  complex
          *upf2, long *ipivot, complex *b, long ldb, complex
          *x, long ldx, float  *rcond,  float  *ferr,  float
          *berr, long *info);
```

PURPOSE

cgtsvx uses the LU factorization to compute the solution to a complex system of linear equations A * X = B, A**T * X = B, or A**H * X = B, where A is a tridiagonal matrix of order N and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'N', the LU decomposition is used to factor the matrix A
   as A = L * U, where L is a product of permutation and unit lower
   bidiagonal matrices and U is upper triangular with nonzeros in
   only the main diagonal and first two superdiagonals.

2. If some U(i,i)=0, so that U is exactly singular, then the routine
   returns with INFO = i. Otherwise, the factored form of A is used
   to estimate the condition number of the matrix A. If the
   reciprocal of the condition number is less than machine precision,
   INFO = N+1 is returned as a warning, but the routine still goes on
   to solve for X and compute error bounds as described below.

3. The system of equations is solved for X using the factored form
   of A.

4. Iterative refinement is applied to improve the computed solution
   matrix and calculate error bounds and backward error estimates
   for it.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of A has been supplied on entry. = 'F': LOWF, DIAGF, UPF1, UPF2, and IPIVOT contain the factored form of A; LOW, DIAG, UP, LOWF, DIAGF, UPF1, UPF2 and IPIVOT will not be modified. = 'N': The matrix will be copied to LOWF, DIAGF, and UPF1 and factored.

TRANSA (input)
Specifies the form of the system of equations:

```
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

LOW (input)
          The (n-1) subdiagonal elements of A.

DIAG (input)
          The n diagonal elements of A.

UP (input)
          The (n-1) superdiagonal elements of A.

LOWF (input/output)
          If FACT = 'F', then LOWF is an input argument  and
          on  entry  contains  the  (n-1)  multipliers  that
          define the matrix L from the LU factorization of A
          as computed by CGTTRF.

          If FACT = 'N', then LOWF is an output argument and
          on exit contains the (n-1) multipliers that define
          the matrix L from the LU factorization of A.

DIAGF (input/output)
          If FACT = 'F', then DIAGF is an input argument and
          on  entry  contains the n diagonal elements of the
          upper triangular matrix U from the  LU  factoriza-
          tion of A.

          If FACT = 'N', then DIAGF is  an  output  argument
          and  on  exit  contains the n diagonal elements of
          the upper triangular matrix U from the LU factori-
          zation of A.

UPF1 (input/output)
          If FACT = 'F', then UPF1 is an input argument  and
          on  entry contains the (n-1) elements of the first
          superdiagonal of U.

          If FACT = 'N', then UPF1 is an output argument and
          on  exit  contains the (n-1) elements of the first
          superdiagonal of U.
```

UPF2 (input/output)
If FACT = 'F', then UPF2 is an input argument and on entry contains the (n-2) elements of the second superdiagonal of U.

If FACT = 'N', then UPF2 is an output argument and on exit contains the (n-2) elements of the second superdiagonal of U.

IPIVOT (input/output)
If FACT = 'F', then IPIVOT is an input argument and on entry contains the pivot indices from the LU factorization of A as computed by CGTTRF.

If FACT = 'N', then IPIVOT is an output argument and on exit contains the pivot indices from the LU factorization of A; row i of the matrix was interchanged with row IPIVOT(i). IPIVOT(i) will always be either i or i+1; IPIVOT(i) = i indicates a row interchange was not required.

B (input) The N-by-NRHS right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A. If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest ele-

ment in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)

The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)

dimension(2*N)

WORK2 (workspace)

dimension(N)

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, and i is

<= N: U(i,i) is exactly zero. The factorization has not been completed unless i = N, but the factor U is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
cgttrf - compute an LU factorization of a complex  tridiago-
nal matrix A using elimination with partial pivoting and row
interchanges
```

SYNOPSIS

```
SUBROUTINE CGTTRF(N, LOW, DIAG, UP1, UP2, IPIVOT, INFO)

COMPLEX LOW(*), DIAG(*), UP1(*), UP2(*)
INTEGER N, INFO
INTEGER IPIVOT(*)

SUBROUTINE CGTTRF_64(N, LOW, DIAG, UP1, UP2, IPIVOT, INFO)

COMPLEX LOW(*), DIAG(*), UP1(*), UP2(*)
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE GTTRF([N], LOW, DIAG, UP1, UP2, IPIVOT, [INFO])

COMPLEX, DIMENSION(:) :: LOW, DIAG, UP1, UP2
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE GTTRF_64([N], LOW, DIAG, UP1, UP2, IPIVOT, [INFO])

COMPLEX, DIMENSION(:) :: LOW, DIAG, UP1, UP2
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void cgttrf(int n,  complex  *low,  complex  *diag,  complex
          *up1, complex *up2, int *ipivot, int *info);

void cgttrf_64(long n, complex *low, complex *diag,  complex
          *up1, complex *up2, long *ipivot, long *info);
```

PURPOSE

cgttrf computes an LU factorization of a complex tridiagonal matrix A using elimination with partial pivoting and row interchanges.

The factorization has the form
   A = L * U
where L is a product of permutation and unit lower bidiagonal matrices and U is upper triangular with nonzeros in only the main diagonal and first two superdiagonals.

ARGUMENTS

N (input) The order of the matrix A.

LOW (input/output)
On entry, LOW must contain the (n-1) sub-diagonal elements of A.

On exit, LOW is overwritten by the (n-1) multipliers that define the matrix L from the LU factorization of A.

DIAG (input/output)
On entry, DIAG must contain the diagonal elements of A.

On exit, DIAG is overwritten by the n diagonal elements of the upper triangular matrix U from the LU factorization of A.

UP1 (input/output)
On entry, UP1 must contain the (n-1) super-diagonal elements of A.

On exit, UP1 is overwritten by the (n-1) elements of the first super-diagonal of U.

UP2 (output)

On exit, UP2 is overwritten by the (n-2) elements of the second super-diagonal of U.

IPIVOT (output)

The pivot indices; for 1 <= i <= n, row i of the matrix was interchanged with row IPIVOT(i). IPIVOT(i) will always be either i or i+1; IPIVOT(i) = i indicates a row interchange was not required.

INFO (output)

= 0: successful exit

< 0: if INFO = -k, the k-th argument had an illegal value

> 0: if INFO = k, U(k,k) is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

Contents

NAME

```
cgttrs - solve one of the systems of equations  A * X  =  B,
A**T * X = B, or A**H * X = B,
```

SYNOPSIS

```
SUBROUTINE CGTTRS(TRANSA, N, NRHS, LOW, DIAG, UP1, UP2, IPIVOT, B,
      LDB, INFO)

CHARACTER * 1 TRANSA
COMPLEX LOW(*), DIAG(*), UP1(*), UP2(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE CGTTRS_64(TRANSA, N, NRHS, LOW, DIAG, UP1, UP2, IPIVOT, B,
      LDB, INFO)

CHARACTER * 1 TRANSA
COMPLEX LOW(*), DIAG(*), UP1(*), UP2(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE GTTRS([TRANSA], [N], [NRHS], LOW, DIAG, UP1, UP2, IPIVOT,
       B, [LDB], [INFO])

CHARACTER(LEN=1) :: TRANSA
COMPLEX, DIMENSION(:) :: LOW, DIAG, UP1, UP2
COMPLEX, DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE GTTRS_64([TRANSA], [N], [NRHS], LOW, DIAG, UP1, UP2,
```

```
          IPIVOT, B, [LDB], [INFO])

   CHARACTER(LEN=1) :: TRANSA
   COMPLEX, DIMENSION(:) :: LOW, DIAG, UP1, UP2
   COMPLEX, DIMENSION(:,:) :: B
   INTEGER(8) :: N, NRHS, LDB, INFO
   INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
   #include <sunperf.h>

   void cgttrs(char transa, int n, int nrhs, complex *low, com-
             plex   *diag,  complex  *up1,  complex  *up2,  int
             *ipivot, complex *b, int ldb, int *info);
   void cgttrs_64(char transa, long n, long nrhs, complex *low,
             complex  *diag,  complex  *up1, complex *up2, long
             *ipivot, complex *b, long ldb, long *info);
```

PURPOSE

cgttrs solves one of the systems of equations
   A * X = B, A**T * X = B, or A**H * X = B, with a tri-
diagonal matrix A using the LU factorization computed by
CGTTRF.

ARGUMENTS

TRANSA (input)
          Specifies the form of the system of equations.   =
          'N': A * X = B   (No transpose)
          = 'T': A**T * X = B (Transpose)
          = 'C': A**H * X = B (Conjugate transpose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A.

NRHS (input)
          The number of right hand sides, i.e., the number
          of columns of the matrix B. NRHS >= 0.

LOW (input)
          The (n-1) multipliers that define the matrix L
          from the LU factorization of A.

DIAG (input)
          The n diagonal elements of the upper triangular

matrix U from the LU factorization of A.

UP1 (input)
The (n-1) elements of the first super-diagonal of U.

UP2 (input)
The (n-2) elements of the second super-diagonal of U.

IPIVOT (input)
The pivot indices; for 1 <= i <= n, row i of the matrix was interchanged with row IPIVOT(i). IPIVOT(i) will always be either i or i+1; IPIVOT(i) = i indicates a row interchange was not required.

B (input/output)
On entry, the matrix of right hand side vectors B. On exit, B is overwritten by the solution vectors X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value

Contents

NAME

```
chbev - compute all the eigenvalues and, optionally,  eigen-
vectors of a complex Hermitian band matrix A
```

SYNOPSIS

```
SUBROUTINE CHBEV(JOBZ, UPLO, N, KD, A, LDA, W, Z, LDZ, WORK,
      WORK2, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX A(LDA,*), Z(LDZ,*), WORK(*)
INTEGER N, KD, LDA, LDZ, INFO
REAL W(*), WORK2(*)

SUBROUTINE CHBEV_64(JOBZ, UPLO, N, KD, A, LDA, W, Z, LDZ, WORK,
      WORK2, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX A(LDA,*), Z(LDZ,*), WORK(*)
INTEGER*8 N, KD, LDA, LDZ, INFO
REAL W(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HBEV(JOBZ, UPLO, [N], KD, A, [LDA], W, Z, [LDZ], [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, Z
INTEGER :: N, KD, LDA, LDZ, INFO
REAL, DIMENSION(:) :: W, WORK2

SUBROUTINE HBEV_64(JOBZ, UPLO, [N], KD, A, [LDA], W, Z, [LDZ],
```

```
        [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, Z
INTEGER(8) :: N, KD, LDA, LDZ, INFO
REAL, DIMENSION(:) :: W, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void chbev(char jobz, char uplo, int n, int kd, complex  *a,
          int  lda,  float  *w,  complex  *z,  int  ldz, int
          *info);
void chbev_64(char jobz, char uplo, long n, long kd, complex
          *a, long lda, float *w, complex *z, long ldz, long
          *info);
```

PURPOSE

chbev computes all the eigenvalues and, optionally, eigenvectors of a complex Hermitian band matrix A.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  KD >= 0.

A (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  band  matrix  A,  stored in the first KD+1
          rows of the array.  The j-th column of A is stored
          in  the j-th column of the array A as follows:  if
          UPLO = 'U', A(kd+1+i-j,j) =  A(i,j)  for  max(1,j-
          kd)<=i<=j;  if  UPLO = 'L', A(1+i-j,j)    = A(i,j)
```

for j<=i<=min(n,j+kd).

On exit, A is overwritten by values generated during the reduction to tridiagonal form. If UPLO = 'U', the first superdiagonal and the diagonal of the tridiagonal matrix T are returned in rows KD and KD+1 of A, and if UPLO = 'L', the diagonal and first subdiagonal of T are returned in the first two rows of A.

LDA (input)
The leading dimension of the array A. LDA >= KD + 1.

W (output)
If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, Z contains the orthonormal eigenvectors of the matrix A, with the i-th column of Z holding the eigenvector associated with W(i). If JOBZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
dimension(N)

WORK2 (workspace)
dimension(max(1,3*N-2))

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

Contents

NAME

```
chbevd - compute all the eigenvalues and, optionally, eigen-
vectors of a complex Hermitian band matrix A
```

SYNOPSIS

```
SUBROUTINE CHBEVD(JOBZ, UPLO, N, KD, AB, LDAB, W, Z, LDZ, WORK,
      LWORK, RWORK, LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX AB(LDAB,*), Z(LDZ,*), WORK(*)
INTEGER N, KD, LDAB, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER IWORK(*)
REAL W(*), RWORK(*)

SUBROUTINE CHBEVD_64(JOBZ, UPLO, N, KD, AB, LDAB, W, Z, LDZ, WORK,
      LWORK, RWORK, LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX AB(LDAB,*), Z(LDZ,*), WORK(*)
INTEGER*8 N, KD, LDAB, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
REAL W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HBEVD(JOBZ, UPLO, [N], KD, AB, [LDAB], W, Z, [LDZ], [WORK],
       [LWORK], [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: AB, Z
INTEGER :: N, KD, LDAB, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
```

```
REAL, DIMENSION(:) :: W, RWORK

SUBROUTINE HBEVD_64(JOBZ, UPLO, [N], KD, AB, [LDAB], W, Z, [LDZ],
       [WORK], [LWORK], [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: AB, Z
INTEGER(8) :: N, KD, LDAB, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>
void chbevd(char jobz, char uplo, int  n,  int  kd,  complex
          *ab,  int ldab, float *w, complex *z, int ldz, int
          *info);

void chbevd_64(char jobz, char uplo, long n, long  kd,  com-
          plex  *ab,  long  ldab, float *w, complex *z, long
          ldz, long *info);
```

PURPOSE

chbevd computes all the eigenvalues and, optionally, eigenvectors of a complex Hermitian band matrix A. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

AB (input/output)
On entry, the upper or lower triangle of the Hermitian band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).
On exit, AB is overwritten by values generated during the reduction to tridiagonal form. If UPLO = 'U', the first superdiagonal and the diagonal of the tridiagonal matrix T are returned in rows KD and KD+1 of AB, and if UPLO = 'L', the diagonal and first subdiagonal of T are returned in the first two rows of AB.

LDAB (input)
The leading dimension of the array AB. LDAB >= KD + 1.

W (output)
If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, Z contains the orthonormal eigenvectors of the matrix A, with the i-th column of Z holding the eigenvector associated with W(i). If JOBZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If N <= 1, LWORK must be at least 1. If JOBZ = 'N' and N > 1, LWORK must be at least N. If JOBZ = 'V' and N > 1, LWORK must be at least 2*N**2.

If LWORK = -1, then a workspace query is assumed;

the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
dimension (LRWORK) On exit, if INFO = 0, RWORK(1) returns the optimal LRWORK.

LRWORK (input)
The dimension of array RWORK. If N <= 1, LRWORK must be at least 1. If JOBZ = 'N' and N > 1, LRWORK must be at least N. If JOBZ = 'V' and N > 1, LRWORK must be at least 1 + 5*N + 2*N**2.

If LRWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the RWORK array, returns this value as the first entry of the RWORK array, and no error message related to LRWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of array IWORK. If JOBZ = 'N' or N <= 1, LIWORK must be at least 1. If JOBZ = 'V' and N > 1, LIWORK must be at least 3 + 5*N .

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

Contents

NAME

chbevx - compute selected eigenvalues and, optionally, eigenvectors of a complex Hermitian band matrix A

SYNOPSIS

```
SUBROUTINE CHBEVX(JOBZ, RANGE, UPLO, N, KD, A, LDA, Q, LDQ, VL,
      VU, IL, IU, ABTOL, NFOUND, W, Z, LDZ, WORK, WORK2, IWORK3, IFAIL,
      INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
COMPLEX A(LDA,*), Q(LDQ,*), Z(LDZ,*), WORK(*)
INTEGER N, KD, LDA, LDQ, IL, IU, NFOUND, LDZ, INFO
INTEGER IWORK3(*), IFAIL(*)
REAL VL, VU, ABTOL
REAL W(*), WORK2(*)

SUBROUTINE CHBEVX_64(JOBZ, RANGE, UPLO, N, KD, A, LDA, Q, LDQ, VL,
      VU, IL, IU, ABTOL, NFOUND, W, Z, LDZ, WORK, WORK2, IWORK3, IFAIL,
      INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
COMPLEX A(LDA,*), Q(LDQ,*), Z(LDZ,*), WORK(*)
INTEGER*8 N, KD, LDA, LDQ, IL, IU, NFOUND, LDZ, INFO
INTEGER*8 IWORK3(*), IFAIL(*)
REAL VL, VU, ABTOL
REAL W(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HBEVX(JOBZ, RANGE, UPLO, [N], KD, A, [LDA], Q, [LDQ],
       VL, VU, IL, IU, ABTOL, [NFOUND], W, Z, [LDZ], [WORK], [WORK2],
       [IWORK3], IFAIL, [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, Q, Z
INTEGER :: N, KD, LDA, LDQ, IL, IU, NFOUND, LDZ, INFO
INTEGER, DIMENSION(:) :: IWORK3, IFAIL
REAL :: VL, VU, ABTOL
REAL, DIMENSION(:) :: W, WORK2

SUBROUTINE HBEVX_64(JOBZ, RANGE, UPLO, [N], KD, A, [LDA], Q, [LDQ],
       VL, VU, IL, IU, ABTOL, [NFOUND], W, Z, [LDZ], [WORK], [WORK2],
       [IWORK3], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, Q, Z
INTEGER(8) :: N, KD, LDA, LDQ, IL, IU, NFOUND, LDZ, INFO
INTEGER(8), DIMENSION(:) :: IWORK3, IFAIL
REAL :: VL, VU, ABTOL
REAL, DIMENSION(:) :: W, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void chbevx(char jobz, char range, char uplo, int n, int kd,
          complex  *a,  int  lda, complex *q, int ldq, float
          vl, float vu, int il, int  iu,  float  abtol,  int
          *nfound,  float  *w,  complex  *z,  int  ldz,  int
          *ifail, int *info);

void chbevx_64(char jobz, char range,  char  uplo,  long  n,
          long  kd,  complex  *a, long lda, complex *q, long
          ldq, float vl, float vu, long il, long  iu,  float
          abtol,  long  *nfound,  float *w, complex *z, long
          ldz, long *ifail, long *info);
```

PURPOSE

chbevx computes selected eigenvalues and, optionally, eigenvectors of a complex Hermitian band matrix A. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;

= 'V': Compute eigenvalues and eigenvectors.

RANGE (input)

= 'A': all eigenvalues will be found;
= 'V': all eigenvalues in the half-open interval (VL,VU] will be found; = 'I': the IL-th through IU-th eigenvalues will be found.

UPLO (input)

= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

KD (input)

The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

A (input/output)

On entry, the upper or lower triangle of the Hermitian band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).

On exit, A is overwritten by values generated during the reduction to tridiagonal form.

LDA (input)

The leading dimension of the array A. LDA >= KD + 1.

Q (output)

If JOBZ = 'V', the N-by-N unitary matrix used in the reduction to tridiagonal form. If JOBZ = 'N', the array Q is not referenced.

LDQ (input)

The leading dimension of the array Q. If JOBZ = 'V', then LDQ >= max(1,N).

VL (input)

If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)

If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

ABTOL (input)
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing A to tri- diagonal form.

Eigenvalues will be computed most accurately when ABTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABTOL to 2*SLAMCH('S').

See "Computing Small Singular Values of Bidiagonal Matrices with Guaranteed High Relative Accuracy," by Demmel and Kahan, LAPACK Working Note #3.

NFOUND (output)
The total number of eigenvalues found. 0 <= NFOUND <= N. If RANGE = 'A', NFOUND = N, and if RANGE = 'I', NFOUND = IU-IL+1.

W (output)
The first NFOUND elements contain the selected eigenvalues in ascending order.

```
Z (input) If JOBZ = 'V', then if INFO = 0, the first  NFOUND
          columns  of Z contain the orthonormal eigenvectors
          of the matrix  A  corresponding  to  the  selected
          eigenvalues, with the i-th column of Z holding the
          eigenvector associated with W(i).  If an eigenvec-
          tor  fails to converge, then that column of Z con-
          tains the latest approximation to the eigenvector,
          and  the  index  of the eigenvector is returned in
          IFAIL.  If JOBZ = 'N', then Z is  not  referenced.
          Note:   the   user   must  ensure  that  at  least
          max(1,NFOUND) columns are supplied in the array Z;
          if  RANGE  = 'V', the exact value of NFOUND is not
          known in advance and an upper bound must be used.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
          dimension(N)

WORK2 (workspace)
          dimension(7*N)

IWORK3 (workspace)
          dimension(5*N)

IFAIL (output)
          If JOBZ = 'V', then if INFO = 0, the first  NFOUND
          elements  of  IFAIL  are  zero.  If INFO > 0, then
          IFAIL contains the  indices  of  the  eigenvectors
          that  failed  to  converge.   If  JOBZ = 'N', then
          IFAIL is not referenced.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, then i eigenvectors  failed  to
          converge.   Their  indices  are  stored  in  array
          IFAIL.
```

Contents

NAME

```
chbgst - reduce a complex Hermitian-definite banded general-
ized  eigenproblem  A*x  = lambda*B*x to standard form C*y =
lambda*y,
```

SYNOPSIS

```
SUBROUTINE CHBGST(VECT, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, X, LDX,
      WORK, RWORK, INFO)

CHARACTER * 1 VECT, UPLO
COMPLEX AB(LDAB,*), BB(LDBB,*), X(LDX,*), WORK(*)
INTEGER N, KA, KB, LDAB, LDBB, LDX, INFO
REAL RWORK(*)

SUBROUTINE CHBGST_64(VECT, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, X,
      LDX, WORK, RWORK, INFO)

CHARACTER * 1 VECT, UPLO
COMPLEX AB(LDAB,*), BB(LDBB,*), X(LDX,*), WORK(*)
INTEGER*8 N, KA, KB, LDAB, LDBB, LDX, INFO
REAL RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HBGST(VECT, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB], X,
       [LDX], [WORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: VECT, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: AB, BB, X
INTEGER :: N, KA, KB, LDAB, LDBB, LDX, INFO
REAL, DIMENSION(:) :: RWORK
```

```
SUBROUTINE HBGST_64(VECT, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB],
       X, [LDX], [WORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: VECT, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: AB, BB, X
INTEGER(8) :: N, KA, KB, LDAB, LDBB, LDX, INFO
REAL, DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>

void chbgst(char vect, char uplo, int n,  int  ka,  int  kb,
          complex *ab, int ldab, complex *bb, int ldbb, com-
          plex *x, int ldx, int *info);
void chbgst_64(char vect, char uplo, long n, long  ka,  long
          kb,  complex  *ab,  long  ldab,  complex *bb, long
          ldbb, complex *x, long ldx, long *info);
```

PURPOSE

chbgst reduces a complex Hermitian-definite banded generalized eigenproblem A*x = lambda*B*x to standard form C*y = lambda*y, such that C has the same bandwidth as A.

B must have been previously factorized as S**H*S by CPBSTF, using a split Cholesky factorization. A is overwritten by C = X**H*A*X, where X = S**(-1)*Q and Q is a unitary matrix chosen to preserve the bandwidth of A.

ARGUMENTS

VECT (input)
= 'N': do not form the transformation matrix X;
= 'V': form X.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrices A and B. N >= 0.

KA (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KA >= 0.

KB (input)
The number of superdiagonals of the matrix B if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KA >= KB >= 0.

AB (input/output)
On entry, the upper or lower triangle of the Hermitian band matrix A, stored in the first ka+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(ka+1+i-j,j) = A(i,j) for max(1,j-ka)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+ka).
On exit, the transformed matrix X**H*A*X, stored in the same format as A.

LDAB (input)
The leading dimension of the array AB. LDAB >= KA+1.

BB (input)
The banded factor S from the split Cholesky factorization of B, as returned by CPBSTF, stored in the first kb+1 rows of the array.

LDBB (input)
The leading dimension of the array BB. LDBB >= KB+1.

X (output)
If VECT = 'V', the n-by-n matrix X. If VECT = 'N', the array X is not referenced.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N) if VECT = 'V'; LDX >= 1 otherwise.

WORK (workspace)
dimension(N)

RWORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

chbgv - compute all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite banded eigenproblem, of the form A*x=(lambda)*B*x

SYNOPSIS

```
SUBROUTINE CHBGV(JOBZ, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, W, Z,
      LDZ, WORK, RWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX AB(LDAB,*), BB(LDBB,*), Z(LDZ,*), WORK(*)
INTEGER N, KA, KB, LDAB, LDBB, LDZ, INFO
REAL W(*), RWORK(*)

SUBROUTINE CHBGV_64(JOBZ, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, W, Z,
      LDZ, WORK, RWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX AB(LDAB,*), BB(LDBB,*), Z(LDZ,*), WORK(*)
INTEGER*8 N, KA, KB, LDAB, LDBB, LDZ, INFO
REAL W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HBGV(JOBZ, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB], W,
       Z, [LDZ], [WORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: AB, BB, Z
INTEGER :: N, KA, KB, LDAB, LDBB, LDZ, INFO
REAL, DIMENSION(:) :: W, RWORK
```

```
SUBROUTINE HBGV_64(JOBZ, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB],
       W, Z, [LDZ], [WORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: AB, BB, Z
INTEGER(8) :: N, KA, KB, LDAB, LDBB, LDZ, INFO
REAL, DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void chbgv(char jobz, char uplo, int n, int ka, int kb, com-
          plex  *ab,  int ldab, complex *bb, int ldbb, float
          *w, complex *z, int ldz, int *info);
void chbgv_64(char jobz, char uplo, long n,  long  ka,  long
          kb,  complex  *ab,  long  ldab,  complex *bb, long
          ldbb, float *w, complex *z, long ldz, long *info);
```

PURPOSE

chbgv computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite banded eigenproblem, of the form A*x=(lambda)*B*x. Here A and B are assumed to be Hermitian and banded, and B is also positive definite.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.

KA (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'. KA >= 0.

KB (input)
          The number of superdiagonals of the  matrix  B  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
```

= 'L'. KB >= 0.

AB (input/output)
On entry, the upper or lower triangle of the Hermitian band matrix A, stored in the first ka+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(ka+1+i-j,j) = A(i,j) for max(1,j-ka)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+ka).

On exit, the contents of AB are destroyed.

LDAB (input)
The leading dimension of the array AB. LDAB >= KA+1.

BB (input/output)
On entry, the upper or lower triangle of the Hermitian band matrix B, stored in the first kb+1 rows of the array. The j-th column of B is stored in the j-th column of the array BB as follows: if UPLO = 'U', BB(kb+1+i-j,j) = B(i,j) for max(1,j-kb)<=i<=j; if UPLO = 'L', BB(1+i-j,j) = B(i,j) for j<=i<=min(n,j+kb).

On exit, the factor S from the split Cholesky factorization B = S**H*S, as returned by CPBSTF.

LDBB (input)
The leading dimension of the array BB. LDBB >= KB+1.

W (output)
If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, Z contains the matrix Z of eigenvectors, with the i-th column of Z holding the eigenvector associated with W(i). The eigenvectors are normalized so that Z**H*B*Z = I. If JOBZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= N.

WORK (workspace)
dimension(N)

RWORK (workspace)
dimension(3*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is:
<= N: the algorithm failed to converge: i off-diagonal elements of an intermediate tridiagonal form did not converge to zero; > N: if INFO = N + i, for 1 <= i <= N, then CPBSTF
returned INFO = i: B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

Contents

NAME

```
chbgvd - compute all the eigenvalues,  and  optionally,  the
eigenvectors  of  a  complex  generalized Hermitian-definite
banded eigenproblem, of the form A*x=(lambda)*B*x
```

SYNOPSIS

```
SUBROUTINE CHBGVD(JOBZ, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, W, Z,
      LDZ, WORK, LWORK, RWORK, LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX AB(LDAB,*), BB(LDBB,*), Z(LDZ,*), WORK(*)
INTEGER N, KA, KB, LDAB, LDBB, LDZ, LWORK,  LRWORK,  LIWORK,
INFO
INTEGER IWORK(*)
REAL W(*), RWORK(*)

SUBROUTINE CHBGVD_64(JOBZ, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, W, Z,
      LDZ, WORK, LWORK, RWORK, LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX AB(LDAB,*), BB(LDBB,*), Z(LDZ,*), WORK(*)
INTEGER*8 N, KA, KB, LDAB, LDBB, LDZ, LWORK, LRWORK, LIWORK,
INFO
INTEGER*8 IWORK(*)
REAL W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HBGVD(JOBZ, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB], W,
       Z, [LDZ], [WORK], [LWORK], [RWORK], [LRWORK], [IWORK], [LIWORK],
       [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: AB, BB, Z
INTEGER :: N,  KA,  KB,  LDAB,  LDBB,  LDZ,  LWORK,  LRWORK,
LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: W, RWORK

SUBROUTINE HBGVD_64(JOBZ, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB],
       W, Z, [LDZ], [WORK], [LWORK], [RWORK], [LRWORK], [IWORK], [LIWORK],
       [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: AB, BB, Z
INTEGER(8) :: N, KA, KB, LDAB,  LDBB,  LDZ,  LWORK,  LRWORK,
LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void chbgvd(char jobz, char uplo, int n,  int  ka,  int  kb,
          complex  *ab,  int  ldab,  complex  *bb, int ldbb,
          float *w, complex *z, int ldz, int *info);

void chbgvd_64(char jobz, char uplo, long n, long  ka,  long
          kb,  complex  *ab,  long  ldab,  complex *bb, long
          ldbb, float *w, complex *z, long ldz, long *info);
```

PURPOSE

chbgvd computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite banded eigenproblem, of the form A*x=(lambda)*B*x. Here A and B are assumed to be Hermitian and banded, and B is also positive definite. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.

KA (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'. KA >= 0.
KB (input)
          The number of superdiagonals of the  matrix  B  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'. KB >= 0.

AB (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  band  matrix  A,  stored in the first ka+1
          rows of the array.  The j-th column of A is stored
          in the j-th column of the array AB as follows:  if
          UPLO = 'U', AB(ka+1+i-j,j) = A(i,j)  for  max(1,j-
          ka)<=i<=j;  if UPLO = 'L', AB(1+i-j,j)    = A(i,j)
          for j<=i<=min(n,j+ka).

          On exit, the contents of AB are destroyed.

LDAB (input)
          The leading dimension of the array  AB.   LDAB  >=
          KA+1.

BB (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  band  matrix  B,  stored in the first kb+1
          rows of the array.  The j-th column of B is stored
          in the j-th column of the array BB as follows:  if
          UPLO = 'U', BB(kb+1+i-j,j) = B(i,j)  for  max(1,j-
          kb)<=i<=j;  if UPLO = 'L', BB(1+i-j,j)    = B(i,j)
          for j<=i<=min(n,j+kb).

          On exit, the factor S from the split Cholesky fac-
          torization B = S**H*S, as returned by CPBSTF.

LDBB (input)
```

The leading dimension of the array BB. LDBB >= KB+1.

W (output)
If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, Z contains the matrix Z of eigenvectors, with the i-th column of Z holding the eigenvector associated with W(i). The eigenvectors are normalized so that Z**H*B*Z = I. If JOBZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= N.

WORK (workspace)
On exit, if INFO=0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If N <= 1, LWORK >= 1. If JOBZ = 'N' and N > 1, LWORK >= N. If JOBZ = 'V' and N > 1, LWORK >= 2*N**2.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
On exit, if INFO=0, RWORK(1) returns the optimal LRWORK.

LRWORK (input)
The dimension of array RWORK. If N <= 1, LRWORK >= 1. If JOBZ = 'N' and N > 1, LRWORK >= N. If JOBZ = 'V' and N > 1, LRWORK >= 1 + 5*N + 2*N**2.

If LRWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the RWORK array, returns this value as the first entry of the RWORK array, and no error message related to LRWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if INFO=0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of array IWORK. If JOBZ = 'N' or N <= 1, LIWORK >= 1. If JOBZ = 'V' and N > 1, LIWORK >= 3 + 5*N.

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is:
<= N: the algorithm failed to converge: i off-diagonal elements of an intermediate tridiagonal form did not converge to zero; > N: if INFO = N + i, for 1 <= i <= N, then CPBSTF
returned INFO = i: B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
Mark Fahey, Department of Mathematics, Univ. of Kentucky, USA

Contents

NAME

```
chbgvx - compute all the eigenvalues,  and  optionally,  the
eigenvectors  of  a  complex  generalized Hermitian-definite
banded eigenproblem, of the form A*x=(lambda)*B*x
```

SYNOPSIS

```
SUBROUTINE CHBGVX(JOBZ, RANGE, UPLO, N, KA, KB, AB, LDAB, BB, LDBB,
      Q, LDQ, VL, VU, IL, IU, ABSTOL, M, W, Z, LDZ, WORK, RWORK, IWORK,
      IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
COMPLEX AB(LDAB,*), BB(LDBB,*), Q(LDQ,*), Z(LDZ,*), WORK(*)
INTEGER N, KA, KB, LDAB, LDBB, LDQ, IL, IU, M, LDZ, INFO
INTEGER IWORK(*), IFAIL(*)
REAL VL, VU, ABSTOL
REAL W(*), RWORK(*)

SUBROUTINE CHBGVX_64(JOBZ, RANGE, UPLO, N, KA, KB, AB, LDAB, BB,
      LDBB, Q, LDQ, VL, VU, IL, IU, ABSTOL, M, W, Z, LDZ, WORK, RWORK,
      IWORK, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
COMPLEX AB(LDAB,*), BB(LDBB,*), Q(LDQ,*), Z(LDZ,*), WORK(*)
INTEGER*8 N, KA, KB, LDAB, LDBB, LDQ, IL, IU, M, LDZ, INFO
INTEGER*8 IWORK(*), IFAIL(*)
REAL VL, VU, ABSTOL
REAL W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HBGVX(JOBZ, RANGE, UPLO, [N], KA, KB, AB, [LDAB], BB,
       [LDBB], Q, [LDQ], VL, VU, IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK],
```

```
       [RWORK], [IWORK], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: AB, BB, Q, Z
INTEGER :: N, KA, KB, LDAB, LDBB, LDQ, IL, IU, M, LDZ, INFO
INTEGER, DIMENSION(:) :: IWORK, IFAIL
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: W, RWORK

SUBROUTINE HBGVX_64(JOBZ, RANGE, UPLO, [N], KA, KB, AB, [LDAB], BB,
       [LDBB], Q, [LDQ], VL, VU, IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK],
       [RWORK], [IWORK], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: AB, BB, Q, Z
INTEGER(8) :: N, KA, KB, LDAB, LDBB, LDQ, IL,  IU,  M,  LDZ,
INFO
INTEGER(8), DIMENSION(:) :: IWORK, IFAIL
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void chbgvx(char jobz, char range, char uplo, int n, int ka,
          int  kb,  complex  *ab, int ldab, complex *bb, int
          ldbb, complex *q, int ldq, float vl, float vu, int
          il,  int  iu, float abstol, int *m, float *w, com-
          plex *z, int ldz, int *ifail, int *info);

void chbgvx_64(char jobz, char range,  char  uplo,  long  n,
          long  ka, long kb, complex *ab, long ldab, complex
          *bb, long ldbb, complex *q, long  ldq,  float  vl,
          float vu, long il, long iu, float abstol, long *m,
          float *w, complex *z, long ldz, long *ifail,  long
          *info);
```

PURPOSE

chbgvx computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite banded eigenproblem, of the form A*x=(lambda)*B*x. Here A and B are assumed to be Hermitian and banded, and B is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either all eigenvalues, a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

RANGE (input)
          = 'A': all eigenvalues will be found;
          = 'V': all eigenvalues in the  half-open  interval
          (VL,VU]  will  be  found; = 'I': the IL-th through
          IU-th eigenvalues will be found.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.
N (input) The order of the matrices A and B.  N >= 0.

KA (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'. KA >= 0.

KB (input)
          The number of superdiagonals of the  matrix  B  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'. KB >= 0.

AB (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  band  matrix  A,  stored in the first ka+1
          rows of the array.  The j-th column of A is stored
          in the j-th column of the array AB as follows:  if
          UPLO = 'U', AB(ka+1+i-j,j) = A(i,j)  for  max(1,j-
          ka)<=i<=j;  if UPLO = 'L', AB(1+i-j,j)    = A(i,j)
          for j<=i<=min(n,j+ka).

          On exit, the contents of AB are destroyed.

LDAB (input)
          The leading dimension of the array  AB.   LDAB  >=
          KA+1.

BB (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  band  matrix  B,  stored in the first kb+1
          rows of the array.  The j-th column of B is stored
          in the j-th column of the array BB as follows:  if
          UPLO = 'U', BB(kb+1+i-j,j) = B(i,j)  for  max(1,j-
```

kb)<=i<=j; if UPLO = 'L', BB(1+i-j,j) = B(i,j) for j<=i<=min(n,j+kb).

On exit, the factor S from the split Cholesky factorization B = S**H*S, as returned by CPBSTF.

LDBB (input)
The leading dimension of the array BB. LDBB >= KB+1.

Q (output)
If JOBZ = 'V', the n-by-n matrix used in the reduction of A*x = (lambda)*B*x to standard form, i.e. C*x = (lambda)*x, and consequently C to tridiagonal form. If JOBZ = 'N', the array Q is not referenced.

LDQ (input)
The leading dimension of the array Q. If JOBZ = 'N', LDQ >= 1. If JOBZ = 'V', LDQ >= max(1,N).

VL (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

ABSTOL (input)
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

```
          ABSTOL + EPS *   max( |a|,|b| ) ,

          where EPS is the machine precision.  If ABSTOL  is
          less than or equal to zero, then  EPS*|T|  will be
          used in its place, where |T| is the 1-norm of  the
          tridiagonal matrix obtained by reducing AP to tri-
          diagonal form.
          Eigenvalues will be computed most accurately  when
          ABSTOL  is  set  to  twice the underflow threshold
          2*SLAMCH('S'), not zero.  If this routine  returns
          with INFO>0, indicating that some eigenvectors did
          not converge, try setting ABSTOL to 2*SLAMCH('S').

M (output)
          The total number of eigenvalues found.  0 <= M  <=
          N.  If RANGE = 'A', M = N, and if RANGE = 'I', M =
          IU-IL+1.

W (output)
          If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0,  Z  contains  the
          matrix  Z of eigenvectors, with the i-th column of
          Z holding the eigenvector  associated  with  W(i).
          The eigenvectors are normalized so that Z**H*B*Z =
          I.  If JOBZ = 'N', then Z is not referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= N.

WORK (workspace)
          dimension(N)

RWORK (workspace)
          dimension(7*N)

IWORK (workspace)
          dimension(5*N)

IFAIL (output)
          If JOBZ = 'V', then if INFO = 0, the first M  ele-
          ments  of IFAIL are zero.  If INFO > 0, then IFAIL
          contains the  indices  of  the  eigenvectors  that
          failed  to converge.  If JOBZ = 'N', then IFAIL is
          not referenced.

INFO (output)
          = 0:  successful exit
```

< 0: if INFO = -i, the i-th argument had an ille-
gal value
> 0: if INFO = i, and i is:
<= N: then i eigenvectors failed to converge.
Their indices are stored in array IFAIL. > N:
if INFO = N + i, for 1 <= i <= N, then CPBSTF
returned INFO = i: B is not positive definite.
The factorization of B could not be completed and
no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
Mark Fahey, Department of Mathematics, Univ. of Kentucky,
USA

Contents

NAME

```
chbmv - perform the matrix-vector operation   y := alpha*A*x
+ beta*y
```

SYNOPSIS

```
SUBROUTINE CHBMV(UPLO, N, K, ALPHA, A, LDA, X, INCX, BETA, Y,
      INCY)

CHARACTER * 1 UPLO
COMPLEX ALPHA, BETA
COMPLEX A(LDA,*), X(*), Y(*)
INTEGER N, K, LDA, INCX, INCY

SUBROUTINE CHBMV_64(UPLO, N, K, ALPHA, A, LDA, X, INCX, BETA, Y,
      INCY)

CHARACTER * 1 UPLO
COMPLEX ALPHA, BETA
COMPLEX A(LDA,*), X(*), Y(*)
INTEGER*8 N, K, LDA, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE HBMV(UPLO, [N], K, ALPHA, A, [LDA], X, [INCX], BETA,
       Y, [INCY])

CHARACTER(LEN=1) :: UPLO
COMPLEX :: ALPHA, BETA
COMPLEX, DIMENSION(:) :: X, Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, K, LDA, INCX, INCY

SUBROUTINE HBMV_64(UPLO, [N], K, ALPHA, A, [LDA], X, [INCX],
```

```
          BETA, Y, [INCY])

     CHARACTER(LEN=1) :: UPLO
     COMPLEX :: ALPHA, BETA
     COMPLEX, DIMENSION(:) :: X, Y
     COMPLEX, DIMENSION(:,:) :: A
     INTEGER(8) :: N, K, LDA, INCX, INCY
```

C INTERFACE

```
     #include <sunperf.h>

     void chbmv(char uplo, int n, int k, complex *alpha,  complex
               *a,  int lda, complex *x, int incx, complex *beta,
               complex *y, int incy);
     void chbmv_64(char uplo, long n,  long  k,  complex  *alpha,
               complex  *a, long lda, complex *x, long incx, com-
               plex *beta, complex *y, long incy);
```

PURPOSE

chbmv performs the matrix-vector operation y := alpha*A*x + beta*y where alpha and beta are scalars, x and y are n element vectors and A is an n by n hermitian band matrix, with k super-diagonals.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the band matrix A is being supplied as follows:

UPLO = 'U' or 'u'   The upper triangular part of A is being supplied.

UPLO = 'L' or 'l'   The lower triangular part of A is being supplied.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

K (input)
On entry, K specifies the number of super-diagonals of the matrix A. K must satisfy 0 .le.

K. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
Before entry with UPLO = 'U' or 'u', the leading (k + 1) by n part of the array A must contain the upper triangular band part of the hermitian matrix, supplied column by column, with the leading diagonal of the matrix in row (k + 1) of the array, the first super-diagonal starting at position 2 in row k, and so on. The top left k by k triangle of the array A is not referenced. The following program segment will transfer the upper triangular part of a hermitian band matrix from conventional full matrix storage to band storage:

```
    DO 20, J = 1, N
      M = K + 1 - J
      DO 10, I = MAX( 1, J - K ), J
        A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Before entry with UPLO = 'L' or 'l', the leading (k + 1) by n part of the array A must contain the lower triangular band part of the hermitian matrix, supplied column by column, with the leading diagonal of the matrix in row 1 of the array, the first sub-diagonal starting at position 1 in row 2, and so on. The bottom right k by k triangle of the array A is not referenced. The following program segment will transfer the lower triangular part of a hermitian band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = 1 - J
     DO 10, I = J, MIN( N, J + K )
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Note that the imaginary parts of the diagonal elements need not be set and are assumed to be zero. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= (k + 1). Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
chbtrd - reduce a complex Hermitian band matrix  A  to  real
symmetric   tridiagonal  form  T  by  a  unitary  similarity
transformation
```

SYNOPSIS

```
SUBROUTINE CHBTRD(VECT, UPLO, N, KD, AB, LDAB, D, E, Q, LDQ, WORK,
      INFO)

CHARACTER * 1 VECT, UPLO
COMPLEX AB(LDAB,*), Q(LDQ,*), WORK(*)
INTEGER N, KD, LDAB, LDQ, INFO
REAL D(*), E(*)

SUBROUTINE CHBTRD_64(VECT, UPLO, N, KD, AB, LDAB, D, E, Q, LDQ, WORK,
      INFO)

CHARACTER * 1 VECT, UPLO
COMPLEX AB(LDAB,*), Q(LDQ,*), WORK(*)
INTEGER*8 N, KD, LDAB, LDQ, INFO
REAL D(*), E(*)
```

F95 INTERFACE

```
SUBROUTINE HBTRD(VECT, UPLO, [N], KD, AB, [LDAB], D, E, Q, [LDQ],
       [WORK], [INFO])

CHARACTER(LEN=1) :: VECT, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: AB, Q
INTEGER :: N, KD, LDAB, LDQ, INFO
REAL, DIMENSION(:) :: D, E
```

```
SUBROUTINE HBTRD_64(VECT, UPLO, [N], KD, AB, [LDAB], D, E, Q, [LDQ],
       [WORK], [INFO])

CHARACTER(LEN=1) :: VECT, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: AB, Q
INTEGER(8) :: N, KD, LDAB, LDQ, INFO
REAL, DIMENSION(:) :: D, E
```

C INTERFACE

```
#include <sunperf.h>

void chbtrd(char vect, char uplo, int  n,  int  kd,  complex
          *ab, int ldab, float *d, float *e, complex *q, int
          ldq, int *info);
void chbtrd_64(char vect, char uplo, long n, long  kd,  com-
          plex  *ab,  long ldab, float *d, float *e, complex
          *q, long ldq, long *info);
```

PURPOSE

chbtrd reduces a complex Hermitian band matrix A to real symmetric tridiagonal form T by a unitary similarity transformation: Q**H * A * Q = T.

ARGUMENTS

```
VECT (input)
          = 'N':  do not form Q;
          = 'V':  form Q;
          = 'U':  update a matrix X, by forming X*Q.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  KD >= 0.

AB (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  band  matrix  A,  stored in the first KD+1
```

rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd). On exit, the diagonal elements of AB are overwritten by the diagonal elements of the tridiagonal matrix T; if KD > 0, the elements on the first superdiagonal (if UPLO = 'U') or the first subdiagonal (if UPLO = 'L') are overwritten by the off-diagonal elements of T; the rest of AB is overwritten by values generated during the reduction.

LDAB (input)
The leading dimension of the array AB. LDAB >= KD+1.

D (output)
The diagonal elements of the tridiagonal matrix T.

E (output)
The off-diagonal elements of the tridiagonal matrix T: E(i) = T(i,i+1) if UPLO = 'U'; E(i) = T(i+1,i) if UPLO = 'L'.

Q (input/output)
On entry, if VECT = 'U', then Q must contain an N-by-N matrix X; if VECT = 'N' or 'V', then Q need not be set.

On exit: if VECT = 'V', Q contains the N-by-N unitary matrix Q; if VECT = 'U', Q contains the product X*Q; if VECT = 'N', the array Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= 1, and LDQ >= N if VECT = 'V' or 'U'.

WORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

Modified by Linda Kaufman, Bell Labs.

Contents

NAME

```
checon - estimate the reciprocal of the condition number  of
a  complex  Hermitian  matrix  A using the factorization A =
U*D*U**H or A = L*D*L**H computed by CHETRF
```

SYNOPSIS

```
SUBROUTINE CHECON(UPLO, N, A, LDA, IPIVOT, ANORM, RCOND, WORK, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, INFO
INTEGER IPIVOT(*)
REAL ANORM, RCOND

SUBROUTINE CHECON_64(UPLO, N, A, LDA, IPIVOT, ANORM, RCOND, WORK,
      INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, INFO
INTEGER*8 IPIVOT(*)
REAL ANORM, RCOND
```

F95 INTERFACE

```
SUBROUTINE HECON(UPLO, [N], A, [LDA], IPIVOT, ANORM, RCOND, [WORK],
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT
```

```
REAL :: ANORM, RCOND

SUBROUTINE HECON_64(UPLO, [N], A, [LDA], IPIVOT, ANORM, RCOND, [WORK],
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL :: ANORM, RCOND
```

C INTERFACE

```
#include <sunperf.h>
void checon(char uplo, int  n,  complex  *a,  int  lda,  int
          *ipivot, float anorm, float *rcond, int *info);

void checon_64(char uplo, long n, complex *a, long lda, long
          *ipivot, float anorm, float *rcond, long *info);
```

PURPOSE

checon estimates the reciprocal of the condition number of a complex Hermitian matrix A using the factorization A = U*D*U**H or A = L*D*L**H computed by CHETRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**H;
= 'L': Lower triangular, form is A = L*D*L**H.

N (input) The order of the matrix A. N >= 0.

A (input) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CHETRF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CHETRF.

ANORM (input)
The 1-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
cheev - compute all eigenvalues and,  optionally,  eigenvec-
tors of a complex Hermitian matrix A
```

SYNOPSIS

```
SUBROUTINE CHEEV(JOBZ, UPLO, N, A, LDA, W, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, LDWORK, INFO
REAL W(*), WORK2(*)

SUBROUTINE CHEEV_64(JOBZ, UPLO, N, A, LDA, W, WORK, LDWORK, WORK2,
      INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, LDWORK, INFO
REAL W(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HEEV(JOBZ, UPLO, [N], A, [LDA], W, [WORK], [LDWORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: W, WORK2

SUBROUTINE HEEV_64(JOBZ, UPLO, [N], A, [LDA], W, [WORK], [LDWORK],
       [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: W, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cheev(char jobz, char uplo, int n, complex *a, int lda,
          float *w, int *info);

void cheev_64(char jobz, char uplo, long n, complex *a, long
          lda, float *w, long *info);
```

PURPOSE

cheev computes all eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the Hermitian matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of A contains the lower  triangular  part  of
          the  matrix  A.   On  exit, if JOBZ = 'V', then if
          INFO = 0, A contains the orthonormal  eigenvectors
          of  the matrix A.  If JOBZ = 'N', then on exit the
          lower triangle (if UPLO='L') or the upper triangle
          (if  UPLO='U')  of  A,  including the diagonal, is
          destroyed.

LDA (input)
```

The leading dimension of the array A. LDA >= max(1,N).

W (output)

If INFO = 0, the eigenvalues in ascending order.

WORK (workspace)

On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)

The length of the array WORK. LDWORK >= max(1,2*N-1). For optimal efficiency, LDWORK >= (NB+1)*N, where NB is the blocksize for CHETRD returned by ILAENV.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)

dimension(max(1,3*N-2))

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

Contents

NAME

```
cheevd - compute all eigenvalues and, optionally,  eigenvec-
tors of a complex Hermitian matrix A
```

SYNOPSIS

```
SUBROUTINE CHEEVD(JOBZ, UPLO, N, A, LDA, W, WORK, LWORK, RWORK,
      LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, LWORK, LRWORK, LIWORK, INFO
INTEGER IWORK(*)
REAL W(*), RWORK(*)

SUBROUTINE CHEEVD_64(JOBZ, UPLO, N, A, LDA, W, WORK, LWORK, RWORK,
      LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, LWORK, LRWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
REAL W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HEEVD(JOBZ, UPLO, [N], A, [LDA], W, [WORK], [LWORK],
       [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, LWORK, LRWORK, LIWORK, INFO
```

```
INTEGER, DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: W, RWORK

SUBROUTINE HEEVD_64(JOBZ, UPLO, [N], A, [LDA], W, [WORK], [LWORK],
       [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, LWORK, LRWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>
void cheevd(char jobz, char uplo, int  n,  complex  *a,  int
          lda, float *w, int *info);

void cheevd_64(char jobz, char uplo,  long  n,  complex  *a,
          long lda, float *w, long *info);
```

PURPOSE

cheevd computes all eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

JOBZ (input)
 = 'N': Compute eigenvalues only;
 = 'V': Compute eigenvalues and eigenvectors.

UPLO (input)
 = 'U': Upper triangle of A is stored;
 = 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A. On exit, if JOBZ = 'V', then if INFO = 0, A contains the orthonormal eigenvectors of the matrix A. If JOBZ = 'N', then on exit the lower triangle (if UPLO='L') or the upper triangle (if UPLO='U') of A, including the diagonal, is destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

W (output)
If INFO = 0, the eigenvalues in ascending order.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The length of the array WORK. If N <= 1, LWORK must be at least 1. If JOBZ = 'N' and N > 1, LWORK must be at least N + 1. If JOBZ = 'V' and N > 1, LWORK must be at least 2*N + N**2.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
dimension (LRWORK) On exit, if INFO = 0, RWORK(1) returns the optimal LRWORK.

LRWORK (input)
The dimension of the array RWORK. If N <= 1, LRWORK must be at least 1. If JOBZ = 'N' and N > 1, LRWORK must be at least N. If JOBZ = 'V' and N > 1, LRWORK must be at least 1 + 5*N + 2*N**2.

If LRWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the RWORK array, returns this value as the first entry of the RWORK array, and no error message

```
            related to LRWORK is issued by XERBLA.

    IWORK (workspace/output)
            On exit, if INFO = 0, IWORK(1) returns the optimal
            LIWORK.

    LIWORK (input)
            The dimension of the array  IWORK.   If  N  <=  1,
            LIWORK must be at least 1.  If JOBZ  = 'N' and N >
            1, LIWORK must be at least 1.  If JOBZ  = 'V'  and
            N > 1, LIWORK must be at least 3 + 5*N.

            If LIWORK = -1, then a workspace query is assumed;
            the  routine  only  calculates the optimal size of
            the IWORK array, returns this value as  the  first
            entry  of  the  IWORK  array, and no error message
            related to LIWORK is issued by XERBLA.

    INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value
            > 0:  if INFO = i, the algorithm  failed  to  con-
            verge;  i off-diagonal elements of an intermediate
            tridiagonal form did not converge to zero.
```

FURTHER DETAILS

```
    Based on contributions by
       Jeff Rutter, Computer  Science  Division,  University  of
    California
       at Berkeley, USA
```

Contents

NAME

cheevr - compute selected eigenvalues and, optionally, eigenvectors of a complex Hermitian tridiagonal matrix T

SYNOPSIS

```
SUBROUTINE CHEEVR(JOBZ, RANGE, UPLO, N, A, LDA, VL, VU, IL, IU,
      ABSTOL, M, W, Z, LDZ, ISUPPZ, WORK, LWORK, RWORK, LRWORK, IWORK,
      LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
COMPLEX A(LDA,*), Z(LDZ,*), WORK(*)
INTEGER N, LDA, IL, IU, M, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER ISUPPZ(*), IWORK(*)
REAL VL, VU, ABSTOL
REAL W(*), RWORK(*)

SUBROUTINE CHEEVR_64(JOBZ, RANGE, UPLO, N, A, LDA, VL, VU, IL, IU,
      ABSTOL, M, W, Z, LDZ, ISUPPZ, WORK, LWORK, RWORK, LRWORK, IWORK,
      LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
COMPLEX A(LDA,*), Z(LDZ,*), WORK(*)
INTEGER*8 N, LDA, IL, IU, M,  LDZ,  LWORK,  LRWORK,  LIWORK,
INFO
INTEGER*8 ISUPPZ(*), IWORK(*)
REAL VL, VU, ABSTOL
REAL W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HEEVR(JOBZ, RANGE, UPLO, [N], A, [LDA], VL, VU, IL, IU,
       ABSTOL, M, W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [RWORK], [LRWORK],
```

```
       [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, Z
INTEGER :: N, LDA, IL, IU, M, LDZ,  LWORK,  LRWORK,  LIWORK,
INFO
INTEGER, DIMENSION(:) :: ISUPPZ, IWORK
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: W, RWORK

SUBROUTINE HEEVR_64(JOBZ, RANGE, UPLO, [N], A, [LDA], VL, VU, IL, IU,
       ABSTOL, M, W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [RWORK], [LRWORK],
       [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, Z
INTEGER(8) :: N, LDA, IL, IU, M, LDZ, LWORK, LRWORK, LIWORK,
INFO
INTEGER(8), DIMENSION(:) :: ISUPPZ, IWORK
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void cheevr(char jobz, char range, char uplo, int n, complex
          *a,  int  lda, float vl, float vu, int il, int iu,
          float abstol, int *m, float *w,  complex  *z,  int
          ldz, int *isuppz, int *info);

void cheevr_64(char jobz, char range,  char  uplo,  long  n,
          complex *a, long lda, float vl, float vu, long il,
          long iu, float abstol, long *m, float *w,  complex
          *z, long ldz, long *isuppz, long *info);
```

PURPOSE

cheevr computes selected eigenvalues and, optionally, eigenvectors of a complex Hermitian tridiagonal matrix T. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Whenever possible, CHEEVR calls CSTEGR to compute the eigenspectrum using Relatively Robust Representations. CSTEGR computes eigenvalues by the dqds algorithm, while orthogonal eigenvectors are computed from various "good" L D

L^T representations (also known as Relatively Robust Representations). Gram-Schmidt orthogonalization is avoided as far as possible. More specifically, the various steps of the algorithm are as follows. For the i-th unreduced block of T,
   (a) Compute T - sigma_i = L_i D_i L_i^T, such that L_i D_i L_i^T
        is a relatively robust representation,
   (b) Compute the eigenvalues, lambda_j, of L_i D_i L_i^T to high
       relative accuracy by the dqds algorithm,
   (c) If there is a cluster of close eigenvalues, "choose" sigma_i
       close to the cluster, and go to step (a),
   (d) Given the approximate eigenvalue lambda_j of L_i D_i L_i^T,
       compute the corresponding eigenvector by forming a
       rank-revealing twisted factorization.
The desired accuracy of the output can be specified by the input parameter ABSTOL.

For more details, see "A new O(n^2) algorithm for the symmetric tridiagonal eigenvalue/eigenvector problem", by Inderjit Dhillon, Computer Science Division Technical Report No. UCB//CSD-97-971, UC Berkeley, May 1997.

Note 1 : CHEEVR calls CSTEGR when the full spectrum is requested on machines which conform to the ieee-754 floating point standard. CHEEVR calls SSTEBZ and CSTEIN on non-ieee machines and
when partial spectrum requests are made.

Normal execution of CSTEGR may create NaNs and infinities and hence may abort due to a floating point exception in environments which do not handle NaNs and infinities in the ieee standard default manner.

ARGUMENTS

JOBZ (input)
          = 'N': Compute eigenvalues only;
          = 'V': Compute eigenvalues and eigenvectors.

RANGE (input)
          = 'A': all eigenvalues will be found.
          = 'V': all eigenvalues in the half-open interval (VL,VU] will be found. = 'I': the IL-th through IU-th eigenvalues will be found.

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the Hermitian matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of A contains the lower  triangular  part  of
          the  matrix  A.   On  exit, the lower triangle (if
          UPLO='L') or the upper triangle (if  UPLO='U')  of
          A, including the diagonal, is destroyed.
LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

VL (input)
          If RANGE='V', the lower and upper  bounds  of  the
          interval  to be searched for eigenvalues. VL < VU.
          Not referenced if RANGE = 'A' or 'I'.

VU (input)
          If RANGE='V', the lower and upper  bounds  of  the
          interval  to be searched for eigenvalues. VL < VU.
          Not referenced if RANGE = 'A' or 'I'.

IL (input)
          If RANGE='I', the indices (in ascending order)  of
          the   smallest   and  largest  eigenvalues  to  be
          returned.  1 <= IL <= IU <= N, if N > 0;  IL  =  1
          and  IU  =  0 if N = 0.  Not referenced if RANGE =
          'A' or 'V'.

IU (input)
          If RANGE='I', the indices (in ascending order)  of
          the   smallest   and  largest  eigenvalues  to  be
          returned.  1 <= IL <= IU <= N, if N > 0;  IL  =  1
          and  IU  =  0 if N = 0.  Not referenced if RANGE =
          'A' or 'V'.

ABSTOL (input)
          The absolute error tolerance for the  eigenvalues.
          An approximate eigenvalue is accepted as converged
          when it is determined to lie in an interval  [a,b]
          of width less than or equal to
```

```
          ABSTOL + EPS *   max( |a|,|b| ) ,

          where EPS is the machine precision.  If ABSTOL  is
          less than or equal to zero, then  EPS*|T|  will be
          used in its place, where |T| is the 1-norm of  the
          tridiagonal  matrix obtained by reducing A to tri-
          diagonal form.

          See "Computing Small Singular Values of Bidiagonal
          Matrices  with Guaranteed High Relative Accuracy,"
          by Demmel and Kahan, LAPACK Working Note #3.

          If high relative accuracy is important, set ABSTOL
          to  SLAMCH(  'Safe  minimum'  ).   Doing  so  will
          guarantee that eigenvalues are  computed  to  high
          relative   accuracy   when   possible   in  future
          releases.  The current  code  does  not  make  any
          guarantees about high relative accuracy, but furu-
          tre releases will. See J. Barlow  and  J.  Demmel,
          "Computing  Accurate Eigensystems of Scaled Diago-
          nally Dominant Matrices", LAPACK Working Note  #7,
          for  a  discussion  of which matrices define their
          eigenvalues to high relative accuracy.

M (output)
          The total number of eigenvalues found.  0 <= M  <=
          N.  If RANGE = 'A', M = N, and if RANGE = 'I', M =
          IU-IL+1.

W (output)
          The first M elements contain the  selected  eigen-
          values in ascending order.

Z (output)
          If JOBZ = 'V', then if  INFO  =  0,  the  first  M
          columns  of Z contain the orthonormal eigenvectors
          of the matrix  A  corresponding  to  the  selected
          eigenvalues, with the i-th column of Z holding the
          eigenvector associated with W(i).  If JOBZ =  'N',
          then  Z  is  not  referenced.  Note: the user must
          ensure that at least max(1,M) columns are supplied
          in the array Z; if RANGE = 'V', the exact value of
          M is not known in advance and an upper bound  must
          be used.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).

ISUPPZ (output)
```

The support of the eigenvectors in Z, i.e., the indices indicating the nonzero elements in Z. The i-th eigenvector is nonzero only in elements ISUPPZ(2*i-1) through ISUPPZ(2*i).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The length of the array WORK. LWORK >= max(1,2*N). For optimal efficiency, LWORK >= (NB+1)*N, where NB is the max of the blocksize for CHETRD and for CUNMTR as returned by ILAENV.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
On exit, if INFO = 0, RWORK(1) returns the optimal (and minimal) LRWORK.

LRWORK (input)
The length of the array RWORK. LRWORK >= max(1,24*N).

If LRWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the RWORK array, returns this value as the first entry of the RWORK array, and no error message related to LRWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if INFO = 0, IWORK(1) returns the optimal (and minimal) LIWORK.

LIWORK (input)
The dimension of the array IWORK. LIWORK >= max(1,10*N).

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value
> 0: Internal error

FURTHER DETAILS

Based on contributions by
Inderjit Dhillon, IBM Almaden, USA
Osni Marques, LBNL/NERSC, USA
Ken Stanley, Computer Science Division, University of California at Berkeley, USA

Contents

NAME

```
cheevx  -  compute  selected  eigenvalues  and,  optionally,
eigenvectors of a complex Hermitian matrix A
```

SYNOPSIS

```
SUBROUTINE CHEEVX(JOBZ, RANGE, UPLO, N, A, LDA, VL, VU, IL, IU,
      ABTOL, NFOUND, W, Z, LDZ, WORK, LDWORK, WORK2, IWORK3, IFAIL,
      INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
COMPLEX A(LDA,*), Z(LDZ,*), WORK(*)
INTEGER N, LDA, IL, IU, NFOUND, LDZ, LDWORK, INFO
INTEGER IWORK3(*), IFAIL(*)
REAL VL, VU, ABTOL
REAL W(*), WORK2(*)

SUBROUTINE CHEEVX_64(JOBZ, RANGE, UPLO, N, A, LDA, VL, VU, IL, IU,
      ABTOL, NFOUND, W, Z, LDZ, WORK, LDWORK, WORK2, IWORK3, IFAIL,
      INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
COMPLEX A(LDA,*), Z(LDZ,*), WORK(*)
INTEGER*8 N, LDA, IL, IU, NFOUND, LDZ, LDWORK, INFO
INTEGER*8 IWORK3(*), IFAIL(*)
REAL VL, VU, ABTOL
REAL W(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HEEVX(JOBZ, RANGE, UPLO, [N], A, [LDA], VL, VU, IL, IU,
       ABTOL, [NFOUND], W, Z, [LDZ], [WORK], [LDWORK], [WORK2], [IWORK3],
       IFAIL, [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, Z
INTEGER :: N, LDA, IL, IU, NFOUND, LDZ, LDWORK, INFO
INTEGER, DIMENSION(:) :: IWORK3, IFAIL
REAL :: VL, VU, ABTOL
REAL, DIMENSION(:) :: W, WORK2

SUBROUTINE HEEVX_64(JOBZ, RANGE, UPLO, [N], A, [LDA], VL, VU, IL, IU,
       ABTOL, [NFOUND], W, Z, [LDZ], [WORK], [LDWORK], [WORK2], [IWORK3],
       IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, Z
INTEGER(8) :: N, LDA, IL, IU, NFOUND, LDZ, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK3, IFAIL
REAL :: VL, VU, ABTOL
REAL, DIMENSION(:) :: W, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cheevx(char jobz, char range, char uplo, int n, complex
          *a,  int  lda, float vl, float vu, int il, int iu,
          float abtol, int *nfound, float  *w,  complex  *z,
          int ldz, int *ifail, int *info);

void cheevx_64(char jobz, char range,  char  uplo,  long  n,
          complex *a, long lda, float vl, float vu, long il,
          long iu, float abtol, long *nfound, float *w, com-
          plex *z, long ldz, long *ifail, long *info);
```

PURPOSE

cheevx computes selected eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

RANGE (input)

= 'A': all eigenvalues will be found.
= 'V': all eigenvalues in the half-open interval (VL,VU] will be found. = 'I': the IL-th through IU-th eigenvalues will be found.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A. On exit, the lower triangle (if UPLO='L') or the upper triangle (if UPLO='U') of A, including the diagonal, is destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

VL (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

ABTOL (input)

The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing A to tridiagonal form.

Eigenvalues will be computed most accurately when ABTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABTOL to 2*SLAMCH('S').

See "Computing Small Singular Values of Bidiagonal Matrices with Guaranteed High Relative Accuracy," by Demmel and Kahan, LAPACK Working Note #3.

NFOUND (output)
The total number of eigenvalues found. 0 <= NFOUND <= N. If RANGE = 'A', NFOUND = N, and if RANGE = 'I', NFOUND = IU-IL+1.

W (output)
On normal exit, the first NFOUND elements contain the selected eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, the first NFOUND columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). If an eigenvector fails to converge, then that column of Z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in IFAIL. If JOBZ = 'N', then Z is not referenced. Note: the user must ensure that at least max(1,NFOUND) columns are supplied in the array Z; if RANGE = 'V', the exact value of NFOUND is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

```
WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LDWORK.

LDWORK (input)
          The  length  of  the  array   WORK.    LDWORK   >=
          max(1,2*N).   For  optimal  efficiency,  LDWORK >=
          (NB+1)*N, where NB is the max of the blocksize for
          CHETRD and for CUNMTR as returned by ILAENV.

          If LDWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LDWORK is issued by XERBLA.

WORK2 (workspace)
          dimension(7*N)

IWORK3 (workspace)
          dimension(5*N)

IFAIL (output)
          If JOBZ = 'V', then if INFO = 0, the first  NFOUND
          elements  of  IFAIL  are  zero.  If INFO > 0, then
          IFAIL contains the  indices  of  the  eigenvectors
          that  failed  to  converge.   If  JOBZ = 'N', then
          IFAIL is not referenced.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, then i eigenvectors  failed  to
          converge.   Their  indices  are  stored  in  array
          IFAIL.
```

Contents

NAME

```
chegs2 - reduce  a  complex  Hermitian-definite  generalized
eigenproblem to standard form
```

SYNOPSIS

```
SUBROUTINE CHEGS2(ITYPE, UPLO, N, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*)
INTEGER ITYPE, N, LDA, LDB, INFO

SUBROUTINE CHEGS2_64(ITYPE, UPLO, N, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 ITYPE, N, LDA, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE HEGS2(ITYPE, UPLO, N, A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: ITYPE, N, LDA, LDB, INFO

SUBROUTINE HEGS2_64(ITYPE, UPLO, N, A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: ITYPE, N, LDA, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void chegs2(int itype, char uplo, int  n,  complex  *a,  int
          lda, complex *b, int ldb, int *info);

void chegs2_64(long itype, char uplo, long  n,  complex  *a,
          long lda, complex *b, long ldb, long *info);
```

PURPOSE

chegs2 reduces a complex Hermitian-definite generalized eigenproblem to standard form.

If ITYPE = 1, the problem is A*x = lambda*B*x,
and A is overwritten by inv(U')*A*inv(U) or inv(L)*A*inv(L')
If ITYPE = 2 or 3, the problem is A*B*x = lambda*x or
B*A*x = lambda*x, and A is overwritten by U*A*U` or L'*A*L.

B must have been previously factorized as U'*U or L*L' by CPOTRF.

ARGUMENTS

ITYPE (input)
= 1: compute inv(U')*A*inv(U) or inv(L)*A*inv(L');
= 2 or 3: compute U*A*U' or L'*A*L.

UPLO (input)
Specifies whether the upper or lower triangular part of the Hermitian matrix A is stored, and how B has been factorized. = 'U': Upper triangular
= 'L': Lower triangular

N (input) The order of the matrices A and B. N >= 0.

A (input/output)
On entry, the Hermitian matrix A. If UPLO = 'U', the leading n by n upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading n by n lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, if INFO = 0, the transformed matrix, stored in the same format as A.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input) The triangular factor from the Cholesky factorization of B, as returned by CPOTRF.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

```
chegst - reduce  a  complex  Hermitian-definite  generalized
eigenproblem to standard form
```

SYNOPSIS

```
SUBROUTINE CHEGST(ITYPE, UPLO, N, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*)
INTEGER ITYPE, N, LDA, LDB, INFO

SUBROUTINE CHEGST_64(ITYPE, UPLO, N, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 ITYPE, N, LDA, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE HEGST(ITYPE, UPLO, N, A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: ITYPE, N, LDA, LDB, INFO

SUBROUTINE HEGST_64(ITYPE, UPLO, N, A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: ITYPE, N, LDA, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void chegst(int itype, char uplo, int  n,  complex  *a,  int
          lda, complex *b, int ldb, int *info);

void chegst_64(long itype, char uplo, long  n,  complex  *a,
          long lda, complex *b, long ldb, long *info);
```

PURPOSE

chegst reduces a complex Hermitian-definite generalized eigenproblem to standard form.

If ITYPE = 1, the problem is A*x = lambda*B*x,
and A is overwritten by inv(U**H)*A*inv(U) or inv(L)*A*inv(L**H)
If ITYPE = 2 or 3, the problem is A*B*x = lambda*x or
B*A*x = lambda*x, and A is overwritten by U*A*U**H or L**H*A*L.

B must have been previously factorized as U**H*U or L*L**H by CPOTRF.

ARGUMENTS

ITYPE (input)
= 1: compute inv(U**H)*A*inv(U) or inv(L)*A*inv(L**H);
= 2 or 3: compute U*A*U**H or L**H*A*L.

UPLO (input)
= 'U': Upper triangle of A is stored and B is factored as U**H*U; = 'L': Lower triangle of A is stored and B is factored as L*L**H.

N (input) The order of the matrices A and B. N >= 0.

A (input/output)
On entry, the Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, if INFO = 0, the transformed matrix,

```
          stored in the same format as A.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

B (input) The triangular factor from the Cholesky factoriza-
          tion of B, as returned by CPOTRF.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
chegv - compute all the  eigenvalues,  and  optionally,  the
eigenvectors  of  a  complex  generalized Hermitian-definite
eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x,
or B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE CHEGV(ITYPE, JOBZ, UPLO, N, A, LDA, B, LDB, W, WORK,
      LDWORK, WORK2, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER ITYPE, N, LDA, LDB, LDWORK, INFO
REAL W(*), WORK2(*)

SUBROUTINE CHEGV_64(ITYPE, JOBZ, UPLO, N, A, LDA, B, LDB, W, WORK,
      LDWORK, WORK2, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER*8 ITYPE, N, LDA, LDB, LDWORK, INFO
REAL W(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HEGV(ITYPE, JOBZ, UPLO, N, A, [LDA], B, [LDB], W, [WORK],
       [LDWORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: ITYPE, N, LDA, LDB, LDWORK, INFO
REAL, DIMENSION(:) :: W, WORK2
```

```
SUBROUTINE HEGV_64(ITYPE, JOBZ, UPLO, N, A, [LDA], B, [LDB], W, [WORK],
       [LDWORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: ITYPE, N, LDA, LDB, LDWORK, INFO
REAL, DIMENSION(:) :: W, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void chegv(int itype, char jobz, char uplo, int  n,  complex
          *a,  int  lda,  complex *b, int ldb, float *w, int
          *info);

void chegv_64(long itype, char jobz, char uplo, long n, com-
          plex *a, long lda, complex *b, long ldb, float *w,
          long *info);
```

PURPOSE

chegv computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be Hermitian and B is also positive definite.

ARGUMENTS

```
ITYPE (input)
          Specifies the problem type to be solved:
          = 1:  A*x = (lambda)*B*x
          = 2:  A*B*x = (lambda)*x
          = 3:  B*A*x = (lambda)*x

JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.
```

A (input/output)
On entry, the Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A.

On exit, if JOBZ = 'V', then if INFO = 0, A contains the matrix Z of eigenvectors. The eigenvectors are normalized as follows: if ITYPE = 1 or 2, Z**H*B*Z = I; if ITYPE = 3, Z**H*inv(B)*Z = I. If JOBZ = 'N', then on exit the upper triangle (if UPLO='U') or the lower triangle (if UPLO='L') of A, including the diagonal, is destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the Hermitian positive definite matrix B. If UPLO = 'U', the leading N-by-N upper triangular part of B contains the upper triangular part of the matrix B. If UPLO = 'L', the leading N-by-N lower triangular part of B contains the lower triangular part of the matrix B.

On exit, if INFO <= N, the part of B containing the matrix is overwritten by the triangular factor U or L from the Cholesky factorization B = U**H*U or B = L*L**H.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

W (output)
If INFO = 0, the eigenvalues in ascending order.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of the array WORK. LDWORK >= max(1,2*N-1). For optimal efficiency, LDWORK >= (NB+1)*N, where NB is the blocksize for CHETRD returned by ILAENV.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
dimension(max(1,3*N-2))

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: CPOTRF or CHEEV returned an error code:
<= N: if INFO = i, CHEEV failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero; > N: if INFO = N + i, for 1 <= i <= N, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

Contents

NAME

```
chegvd - compute all the eigenvalues,  and  optionally,  the
eigenvectors  of  a  complex  generalized Hermitian-definite
eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x,
or B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE CHEGVD(ITYPE, JOBZ, UPLO, N, A, LDA, B, LDB, W, WORK,
      LWORK, RWORK, LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER ITYPE, N, LDA, LDB, LWORK, LRWORK, LIWORK, INFO
INTEGER IWORK(*)
REAL W(*), RWORK(*)

SUBROUTINE CHEGVD_64(ITYPE, JOBZ, UPLO, N, A, LDA, B, LDB, W, WORK,
      LWORK, RWORK, LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER*8 ITYPE, N, LDA, LDB, LWORK, LRWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
REAL W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HEGVD(ITYPE, JOBZ, UPLO, [N], A, [LDA], B, [LDB], W, [WORK],
       [LWORK], [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: WORK
```

```
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: ITYPE, N, LDA, LDB, LWORK, LRWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: W, RWORK

SUBROUTINE HEGVD_64(ITYPE, JOBZ, UPLO, [N], A, [LDA], B, [LDB], W,
       [WORK], [LWORK], [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: ITYPE, N, LDA,  LDB,  LWORK,  LRWORK,  LIWORK,
INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void chegvd(int itype, char jobz, char uplo, int n,  complex
          *a,  int  lda,  complex *b, int ldb, float *w, int
          *info);

void chegvd_64(long itype, char jobz,  char  uplo,  long  n,
          complex  *a, long lda, complex *b, long ldb, float
          *w, long *info);
```

PURPOSE

chegvd computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be Hermitian and B is also positive definite. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

ITYPE (input)

```
          Specifies the problem type to be solved:
          = 1:  A*x = (lambda)*B*x
          = 2:  A*B*x = (lambda)*x
          = 3:  B*A*x = (lambda)*x

JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.
A (input/output)
          On entry, the Hermitian matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of A contains the lower  triangular  part  of
          the matrix A.

          On exit, if JOBZ = 'V', then if INFO = 0,  A  con-
          tains the matrix Z of eigenvectors.  The eigenvec-
          tors are normalized as follows:  if ITYPE =  1  or
          2,  Z**H*B*Z = I; if ITYPE = 3, Z**H*inv(B)*Z = I.
          If JOBZ = 'N', then on exit the upper triangle (if
          UPLO='U')  or  the lower triangle (if UPLO='L') of
          A, including the diagonal, is destroyed.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

B (input/output)
          On entry, the Hermitian matrix B.  If UPLO =  'U',
          the leading N-by-N upper triangular part of B con-
          tains the upper triangular part of the  matrix  B.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of B contains the lower  triangular  part  of
          the matrix B.

          On exit, if INFO <= N, the part  of  B  containing
          the matrix is overwritten by the triangular factor
          U or L from the Cholesky factorization B =  U**H*U
          or B = L*L**H.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).
```

W (output)
If INFO = 0, the eigenvalues in ascending order.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The length of the array WORK. If N <= 1, LWORK >= 1. If JOBZ = 'N' and N > 1, LWORK >= N + 1. If JOBZ = 'V' and N > 1, LWORK >= 2*N + N**2.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
On exit, if INFO = 0, RWORK(1) returns the optimal LRWORK.

LRWORK (input)
The dimension of the array RWORK. If N <= 1, LRWORK >= 1. If JOBZ = 'N' and N > 1, LRWORK >= N. If JOBZ = 'V' and N > 1, LRWORK >= 1 + 5*N + 2*N**2.

If LRWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the RWORK array, returns this value as the first entry of the RWORK array, and no error message related to LRWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of the array IWORK. If N <= 1, LIWORK >= 1. If JOBZ = 'N' and N > 1, LIWORK >= 1. If JOBZ = 'V' and N > 1, LIWORK >= 3 + 5*N.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: CPOTRF or CHEEVD returned an error code:

<= N: if INFO = i, CHEEVD failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero; > N: if INFO = N + i, for 1 <= i <= N, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
Mark Fahey, Department of Mathematics, Univ. of Kentucky, USA

Contents

NAME

```
chegvx  -  compute  selected  eigenvalues,  and  optionally,
eigenvectors  of  a  complex  generalized Hermitian-definite
eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x,
or B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE CHEGVX(ITYPE, JOBZ, RANGE, UPLO, N, A, LDA, B, LDB, VL,
      VU, IL, IU, ABSTOL, M, W, Z, LDZ, WORK, LWORK, RWORK, IWORK,
      IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
COMPLEX A(LDA,*), B(LDB,*), Z(LDZ,*), WORK(*)
INTEGER ITYPE, N, LDA, LDB, IL, IU, M, LDZ, LWORK, INFO
INTEGER IWORK(*), IFAIL(*)
REAL VL, VU, ABSTOL
REAL W(*), RWORK(*)

SUBROUTINE CHEGVX_64(ITYPE, JOBZ, RANGE, UPLO, N, A, LDA, B, LDB, VL,
      VU, IL, IU, ABSTOL, M, W, Z, LDZ, WORK, LWORK, RWORK, IWORK,
      IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
COMPLEX A(LDA,*), B(LDB,*), Z(LDZ,*), WORK(*)
INTEGER*8 ITYPE, N, LDA, LDB, IL, IU, M, LDZ, LWORK, INFO
INTEGER*8 IWORK(*), IFAIL(*)
REAL VL, VU, ABSTOL
REAL W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HEGVX(ITYPE, JOBZ, RANGE, UPLO, [N], A, [LDA], B, [LDB],
```

```
       VL, VU, IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK], [LWORK], [RWORK],
       [IWORK], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B, Z
INTEGER :: ITYPE, N, LDA, LDB, IL, IU, M, LDZ, LWORK, INFO
INTEGER, DIMENSION(:) :: IWORK, IFAIL
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: W, RWORK

SUBROUTINE HEGVX_64(ITYPE, JOBZ, RANGE, UPLO, [N], A, [LDA], B, [LDB],
       VL, VU, IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK], [LWORK], [RWORK],
       [IWORK], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B, Z
INTEGER(8) :: ITYPE, N, LDA, LDB, IL,  IU,  M,  LDZ,  LWORK,
INFO
INTEGER(8), DIMENSION(:) :: IWORK, IFAIL
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void chegvx(int itype, char jobz, char range, char uplo, int
          n, complex *a, int lda, complex *b, int ldb, float
          vl, float vu, int il, int iu,  float  abstol,  int
          *m, float *w, complex *z, int ldz, int *ifail, int
          *info);

void chegvx_64(long itype, char jobz, char range, char uplo,
          long  n,  complex  *a,  long lda, complex *b, long
          ldb, float vl, float vu, long il, long  iu,  float
          abstol,  long  *m, float *w, complex *z, long ldz,
          long *ifail, long *info);
```

PURPOSE

chegvx computes selected eigenvalues, and optionally, eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be Hermitian and B is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

```
ITYPE (input)
          Specifies the problem type to be solved:
          = 1:  A*x = (lambda)*B*x
          = 2:  A*B*x = (lambda)*x
          = 3:  B*A*x = (lambda)*x

JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

RANGE (input)
          = 'A': all eigenvalues will be found.
          = 'V': all eigenvalues in the  half-open  interval
          (VL,VU]  will  be found.  = 'I': the IL-th through
          IU-th eigenvalues will be found.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.

A (input/output)
          On entry, the Hermitian matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of A contains the lower  triangular  part  of
          the matrix A.

          On exit,  the lower triangle (if UPLO='L') or  the
          upper  triangle  (if UPLO='U') of A, including the
          diagonal, is destroyed.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

B (input/output)
          On entry, the Hermitian matrix B.  If UPLO =  'U',
          the leading N-by-N upper triangular part of B con-
          tains the upper triangular part of the  matrix  B.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of B contains the lower  triangular  part  of
          the matrix B.
```

```
          On exit, if INFO <= N, the part  of  B  containing
          the matrix is overwritten by the triangular factor
          U or L from the Cholesky factorization B =  U**H*U
          or B = L*L**H.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

VL (input)
          If RANGE='V', the lower and upper  bounds  of  the
          interval  to be searched for eigenvalues. VL < VU.
          Not referenced if RANGE = 'A' or 'I'.
VU (input)
          If RANGE='V', the lower and upper  bounds  of  the
          interval  to be searched for eigenvalues. VL < VU.
          Not referenced if RANGE = 'A' or 'I'.

IL (input)
          If RANGE='I', the indices (in ascending order)  of
          the   smallest   and  largest  eigenvalues  to  be
          returned.  1 <= IL <= IU <= N, if N > 0;  IL  =  1
          and  IU  =  0 if N = 0.  Not referenced if RANGE =
          'A' or 'V'.

IU (input)
          If RANGE='I', the indices (in ascending order)  of
          the   smallest   and  largest  eigenvalues  to  be
          returned.  1 <= IL <= IU <= N, if N > 0;  IL  =  1
          and  IU  =  0 if N = 0.  Not referenced if RANGE =
          'A' or 'V'.

ABSTOL (input)
          The absolute error tolerance for the  eigenvalues.
          An approximate eigenvalue is accepted as converged
          when it is determined to lie in an interval  [a,b]
          of width less than or equal to

          ABSTOL + EPS *   max( |a|,|b| ) ,

          where EPS is the machine precision.  If ABSTOL  is
          less than or equal to zero, then  EPS*|T|  will be
          used in its place, where |T| is the 1-norm of  the
          tridiagonal  matrix obtained by reducing A to tri-
          diagonal form.

          Eigenvalues will be computed most accurately  when
          ABSTOL  is  set  to  twice the underflow threshold
          2*SLAMCH('S'), not zero.  If this routine  returns
          with INFO>0, indicating that some eigenvectors did
```

not converge, try setting ABSTOL to 2*SLAMCH('S').

M (output)
The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)
The first M elements contain the selected eigenvalues in ascending order.

Z (output)
If JOBZ = 'N', then Z is not referenced. If JOBZ = 'V', then if INFO = 0, the first M columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). The eigenvectors are normalized as follows: if ITYPE = 1 or 2, Z**T*B*Z = I; if ITYPE = 3, Z**T*inv(B)*Z = I.

If an eigenvector fails to converge, then that column of Z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in IFAIL. Note: the user must ensure that at least max(1,M) columns are supplied in the array Z; if RANGE = 'V', the exact value of M is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace/output)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The length of the array WORK. LWORK >= max(1,2*N-1). For optimal efficiency, LWORK >= (NB+1)*N, where NB is the blocksize for CHETRD returned by ILAENV.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)

dimension(7*N)

IWORK (workspace)
dimension(5*N)

IFAIL (output)
If JOBZ = 'V', then if INFO = 0, the first M elements of IFAIL are zero. If INFO > 0, then IFAIL contains the indices of the eigenvectors that failed to converge. If JOBZ = 'N', then IFAIL is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: CPOTRF or CHEEVX returned an error code:
<= N: if INFO = i, CHEEVX failed to converge; i eigenvectors failed to converge. Their indices are stored in array IFAIL. > N: if INFO = N + i, for 1 <= i <= N, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
Mark Fahey, Department of Mathematics, Univ. of Kentucky, USA

Contents

NAME

```
chemm - perform one of the  matrix-matrix  operations  C  :=
alpha*A*B + beta*C or C := alpha*B*A + beta*C
```

SYNOPSIS

```
SUBROUTINE CHEMM(SIDE, UPLO, M, N, ALPHA, A, LDA, B, LDB, BETA, C,
      LDC)

CHARACTER * 1 SIDE, UPLO
COMPLEX ALPHA, BETA
COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER M, N, LDA, LDB, LDC

SUBROUTINE CHEMM_64(SIDE, UPLO, M, N, ALPHA, A, LDA, B, LDB, BETA, C,
      LDC)

CHARACTER * 1 SIDE, UPLO
COMPLEX ALPHA, BETA
COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER*8 M, N, LDA, LDB, LDC
```

F95 INTERFACE

```
SUBROUTINE HEMM(SIDE, UPLO, [M], [N], ALPHA, A, [LDA], B, [LDB],
       BETA, C, [LDC])

CHARACTER(LEN=1) :: SIDE, UPLO
COMPLEX :: ALPHA, BETA
COMPLEX, DIMENSION(:,:) :: A, B, C
INTEGER :: M, N, LDA, LDB, LDC

SUBROUTINE HEMM_64(SIDE, UPLO, [M], [N], ALPHA, A, [LDA], B, [LDB],
       BETA, C, [LDC])
```

```
CHARACTER(LEN=1) :: SIDE, UPLO
COMPLEX :: ALPHA, BETA
COMPLEX, DIMENSION(:,:) :: A, B, C
INTEGER(8) :: M, N, LDA, LDB, LDC
```

C INTERFACE

```
#include <sunperf.h>

void chemm(char side, char  uplo,  int  m,  int  n,  complex
          *alpha,  complex *a, int lda, complex *b, int ldb,
          complex *beta, complex *c, int ldc);

void chemm_64(char side, char uplo, long m, long n,  complex
          *alpha,  complex  *a,  long  lda, complex *b, long
          ldb, complex *beta, complex *c, long ldc);
```

PURPOSE

chemm performs one of the matrix-matrix operations C := alpha*A*B + beta*C or C := alpha*B*A + beta*C where alpha and beta are scalars, A is an hermitian matrix and B and C are m by n matrices.

ARGUMENTS

SIDE (input)
On entry, SIDE specifies whether the hermitian matrix A appears on the left or right in the operation as follows:

SIDE = 'L' or 'l'   C := alpha*A*B + beta*C,

SIDE = 'R' or 'r'   C := alpha*B*A + beta*C,

Unchanged on exit.

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the hermitian matrix A is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of the hermitian matrix is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of the hermitian matrix is to be referenced.

Unchanged on exit.

M (input)
On entry, M specifies the number of rows of the matrix C. M >= 0. Unchanged on exit.

N (input)
On entry, N specifies the number of columns of the matrix C. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.
A (input)
COMPLEX array of DIMENSION (LDA, ka), where ka is m when SIDE = 'L' or 'l' and is n otherwise.

Before entry with SIDE = 'L' or 'l', the m by m part of the array A must contain the hermitian matrix, such that when UPLO = 'U' or 'u', the leading m by m upper triangular part of the array A must contain the upper triangular part of the hermitian matrix and the strictly lower triangular part of A is not referenced, and when UPLO = 'L' or 'l', the leading m by m lower triangular part of the array A must contain the lower triangular part of the hermitian matrix and the strictly upper triangular part of A is not referenced.

Before entry with SIDE = 'R' or 'r', the n by n part of the array A must contain the hermitian matrix, such that when UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular part of the hermitian matrix and the strictly lower triangular part of A is not referenced, and when UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular part of the hermitian matrix and the strictly upper triangular part of A is not referenced.

Note that the imaginary parts of the diagonal elements need not be set, they are assumed to be zero. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When SIDE = 'L' or 'l' then LDA >= max(1, m), otherwise LDA >= max(1, n). Unchanged on exit.

B (input)

COMPLEX array of DIMENSION (LDB, n). Before entry, the leading m by n part of the array B must contain the matrix B. Unchanged on exit.

LDB (input)

On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. LDB must be at least max(1, m). Unchanged on exit.

BETA (input)

On entry, BETA specifies the scalar beta. When BETA is supplied as zero then C need not be set on input. Unchanged on exit.

C (input/output)

COMPLEX array of DIMENSION (LDC, n).

Before entry, the leading m by n part of the array C must contain the matrix C, except when beta is zero, in which case C need not be set on entry.

On exit, the array C is overwritten by the m by n updated matrix.

LDC (input)

On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC must be at least max(1, m). Unchanged on exit.

Contents

NAME

```
chemv - perform the matrix-vector operation   y := alpha*A*x
+ beta*y
```

SYNOPSIS

```
SUBROUTINE CHEMV(UPLO, N, ALPHA, A, LDA, X, INCX, BETA, Y, INCY)

CHARACTER * 1 UPLO
COMPLEX ALPHA, BETA
COMPLEX A(LDA,*), X(*), Y(*)
INTEGER N, LDA, INCX, INCY

SUBROUTINE CHEMV_64(UPLO, N, ALPHA, A, LDA, X, INCX, BETA, Y, INCY)

CHARACTER * 1 UPLO
COMPLEX ALPHA, BETA
COMPLEX A(LDA,*), X(*), Y(*)
INTEGER*8 N, LDA, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE HEMV(UPLO, [N], ALPHA, A, [LDA], X, [INCX], BETA, Y, [INCY])

CHARACTER(LEN=1) :: UPLO
COMPLEX :: ALPHA, BETA
COMPLEX, DIMENSION(:) :: X, Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, INCX, INCY

SUBROUTINE HEMV_64(UPLO, [N], ALPHA, A, [LDA], X, [INCX], BETA, Y,
       [INCY])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX :: ALPHA, BETA
COMPLEX, DIMENSION(:) :: X, Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>

void chemv(char uplo, int n, complex *alpha, complex *a, int
          lda,  complex *x, int incx, complex *beta, complex
          *y, int incy);

void chemv_64(char uplo, long n, complex *alpha, complex *a,
          long  lda,  complex  *x, long incx, complex *beta,
          complex *y, long incy);
```

PURPOSE

chemv performs the matrix-vector operation y := alpha*A*x + beta*y where alpha and beta are scalars, x and y are n element vectors and A is an n by n hermitian matrix.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array A is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of A is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of A is to be referenced.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
Before entry with UPLO = 'U' or 'u', the leading

n by n upper triangular part of the array A must contain the upper triangular part of the hermitian matrix and the strictly lower triangular part of A is not referenced. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular part of the hermitian matrix and the strictly upper triangular part of A is not referenced. Note that the imaginary parts of the diagonal elements need not be set and are assumed to be zero. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

X (input)

(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)

On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

BETA (input)

On entry, BETA specifies the scalar beta. When BETA is supplied as zero then Y need not be set on input. Unchanged on exit.

Y (input/output)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
cher - perform the hermitian rank 1 operation   A :=
alpha*x*conjg( x' ) + A
```

SYNOPSIS

```
SUBROUTINE CHER(UPLO, N, ALPHA, X, INCX, A, LDA)

CHARACTER * 1 UPLO
COMPLEX X(*), A(LDA,*)
INTEGER N, INCX, LDA
REAL ALPHA

SUBROUTINE CHER_64(UPLO, N, ALPHA, X, INCX, A, LDA)

CHARACTER * 1 UPLO
COMPLEX X(*), A(LDA,*)
INTEGER*8 N, INCX, LDA
REAL ALPHA
```

F95 INTERFACE

```
SUBROUTINE HER(UPLO, [N], ALPHA, X, [INCX], A, [LDA])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: X
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, INCX, LDA
REAL :: ALPHA

SUBROUTINE HER_64(UPLO, [N], ALPHA, X, [INCX], A, [LDA])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: X
```

```
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, INCX, LDA
REAL :: ALPHA
```

C INTERFACE

```
#include <sunperf.h>

void cher(char uplo, int n, float  alpha,  complex  *x,  int
          incx, complex *a, int lda);

void cher_64(char uplo, long n,  float  alpha,  complex  *x,
          long incx, complex *a, long lda);
```

PURPOSE

cher performs the hermitian rank 1 operation A := alpha*x*conjg(x') + A where alpha is a real scalar, x is an n element vector and A is an n by n hermitian matrix.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array A is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of A is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of A is to be referenced.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)

On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

A (input/output)

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular part of the hermitian matrix and the strictly lower triangular part of A is not referenced. On exit, the upper triangular part of the array A is overwritten by the upper triangular part of the updated matrix. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular part of the hermitian matrix and the strictly upper triangular part of A is not referenced. On exit, the lower triangular part of the array A is overwritten by the lower triangular part of the updated matrix. Note that the imaginary parts of the diagonal elements need not be set, they are assumed to be zero, and on exit they are set to zero.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

Contents

NAME

cher2 - perform the hermitian rank 2 operation A := alpha*x*conjg(y') + conjg(alpha)*y*conjg(x') + A

SYNOPSIS

```
SUBROUTINE CHER2(UPLO, N, ALPHA, X, INCX, Y, INCY, A, LDA)

CHARACTER * 1 UPLO
COMPLEX ALPHA
COMPLEX X(*), Y(*), A(LDA,*)
INTEGER N, INCX, INCY, LDA

SUBROUTINE CHER2_64(UPLO, N, ALPHA, X, INCX, Y, INCY, A, LDA)

CHARACTER * 1 UPLO
COMPLEX ALPHA
COMPLEX X(*), Y(*), A(LDA,*)
INTEGER*8 N, INCX, INCY, LDA
```

F95 INTERFACE

```
SUBROUTINE HER2(UPLO, [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

CHARACTER(LEN=1) :: UPLO
COMPLEX :: ALPHA
COMPLEX, DIMENSION(:) :: X, Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, INCX, INCY, LDA

SUBROUTINE HER2_64(UPLO, [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

CHARACTER(LEN=1) :: UPLO
COMPLEX :: ALPHA
```

```
COMPLEX, DIMENSION(:) :: X, Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, INCX, INCY, LDA
```

C INTERFACE

```
#include <sunperf.h>

void cher2(char uplo, int n, complex *alpha, complex *x, int
          incx, complex *y, int incy, complex *a, int lda);

void cher2_64(char uplo, long n, complex *alpha, complex *x,
          long incx, complex *y, long incy, complex *a, long
          lda);
```

PURPOSE

cher2 performs the hermitian rank 2 operation A := alpha*x*conjg(y') + conjg(alpha)*y*conjg(x') + A where alpha is a scalar, x and y are n element vectors and A is an n by n hermitian matrix.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array A is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of A is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of A is to be referenced.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element

vector x. Unchanged on exit.

INCX (input)

On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

Y (input)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. Unchanged on exit.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

A (input/output)

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular part of the hermitian matrix and the strictly lower triangular part of A is not referenced. On exit, the upper triangular part of the array A is overwritten by the upper triangular part of the updated matrix. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular part of the hermitian matrix and the strictly upper triangular part of A is not referenced. On exit, the lower triangular part of the array A is overwritten by the lower triangular part of the updated matrix. Note that the imaginary parts of the diagonal elements need not be set, they are assumed to be zero, and on exit they are set to zero.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

Contents

NAME

```
cher2k - perform one of the Hermitian rank 2k operations   C
:=  alpha*A*conjg(  B'  )  +  conjg( alpha )*B*conjg( A' ) +
beta*C or C := alpha*conjg( A' )*B + conjg(  alpha  )*conjg(
B' )*A + beta*C
```

SYNOPSIS

```
SUBROUTINE CHER2K(UPLO, TRANSA, N, K, ALPHA, A, LDA, B, LDB, BETA, C,
      LDC)

CHARACTER * 1 UPLO, TRANSA
COMPLEX ALPHA
COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER N, K, LDA, LDB, LDC
REAL BETA

SUBROUTINE CHER2K_64(UPLO, TRANSA, N, K, ALPHA, A, LDA, B, LDB, BETA,
      C, LDC)

CHARACTER * 1 UPLO, TRANSA
COMPLEX ALPHA
COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER*8 N, K, LDA, LDB, LDC
REAL BETA
```

F95 INTERFACE

```
SUBROUTINE HER2K(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], B, [LDB],
       BETA, C, [LDC])

CHARACTER(LEN=1) :: UPLO, TRANSA
COMPLEX :: ALPHA
COMPLEX, DIMENSION(:,:) :: A, B, C
```

```
INTEGER :: N, K, LDA, LDB, LDC
REAL :: BETA

SUBROUTINE HER2K_64(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], B,
       [LDB], BETA, C, [LDC])

CHARACTER(LEN=1) :: UPLO, TRANSA
COMPLEX :: ALPHA
COMPLEX, DIMENSION(:,:) :: A, B, C
INTEGER(8) :: N, K, LDA, LDB, LDC
REAL :: BETA
```

C INTERFACE

```
#include <sunperf.h>
void cher2k(char uplo, char transa, int n, int  k,  complex*
          alpha,  complex  *a, int lda, complex *b, int ldb,
          float beta, complex *c, int ldc);

void cher2k_64(char uplo, char transa, long n, long k,  com-
          plex  *alpha,  complex  *a,  long lda, complex *b,
          long ldb, float beta, complex *c, long ldc);
```

PURPOSE

cher2k performs one of the Hermitian rank 2k operations C := alpha*A*conjg(B') + conjg(alpha)*B*conjg(A') + beta*C or C := alpha*conjg(A')*B + conjg(alpha)*conjg(B')*A + beta*C where alpha and beta are scalars with beta real, C is an n by n Hermitian matrix and A and B are n by k matrices in the first case and k by n matrices in the second case.

ARGUMENTS

UPLO (input)

On entry, UPLO specifies whether the upper or lower triangular part of the array C is to be referenced as follows:

UPLO = 'U' or 'u' Only the upper triangular part of C is to be referenced.

UPLO = 'L' or 'l' Only the lower triangular part of C is to be referenced.

Unchanged on exit.

TRANSA (input)

On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   C := alpha*A*conjg(B') + conjg(alpha)*B*conjg(A') + beta*C.

TRANSA = 'C' or 'c'   C := alpha*conjg(A')*B + conjg(alpha)*conjg(B')*A + beta*C.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input)

On entry, N specifies the order of the matrix C. N must be at least zero. Unchanged on exit.

K (input)

On entry with TRANSA = 'N' or 'n', K specifies the number of columns of the matrices A and B, and on entry with TRANSA = 'C' or 'c', K specifies the number of rows of the matrices A and B. K must be at least zero. Unchanged on exit.

ALPHA (input)

On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)

COMPLEX array of DIMENSION (LDA, ka), where ka is k when TRANSA = 'N' or 'n', and is n otherwise. Before entry with TRANSA = 'N' or 'n', the leading n by k part of the array A must contain the matrix A, otherwise the leading k by n part of the array A must contain the matrix A. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDA must be at least max(1, n), otherwise LDA must be at least max(1, k). Unchanged on exit.

B (input)

COMPLEX array of DIMENSION (LDB, kb), where kb is k when TRANSA = 'N' or 'n', and is n otherwise. Before entry with TRANSA = 'N' or 'n', the leading n by k part of the array B must contain the matrix B, otherwise the leading k by n part of the array B must contain the matrix B. Unchanged on exit.

LDB (input)
On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDB must be at least max(1, n), otherwise LDB must be at least max(1, k). Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. Unchanged on exit.

C (input/output)
COMPLEX array of DIMENSION (LDC, n).

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array C must contain the upper triangular part of the Hermitian matrix and the strictly lower triangular part of C is not referenced. On exit, the upper triangular part of the array C is overwritten by the upper triangular part of the updated matrix.

Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array C must contain the lower triangular part of the Hermitian matrix and the strictly upper triangular part of C is not referenced. On exit, the lower triangular part of the array C is overwritten by the lower triangular part of the updated matrix.

Note that the imaginary parts of the diagonal elements need not be set, they are assumed to be zero, and on exit they are set to zero.

LDC (input)
On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC must be at least max(1, n). Unchanged on exit.

Contents

NAME

```
cherfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is Hermitian indefin-
ite, and provides error bounds and backward error  estimates
for the solution
```

SYNOPSIS

```
SUBROUTINE CHERFS(UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*)
REAL FERR(*), BERR(*), WORK2(*)

SUBROUTINE CHERFS_64(UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
REAL FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HERFS(UPLO, [N], [NRHS], A, [LDA], AF, [LDAF], IPIVOT, B,
       [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
```

```
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE HERFS_64(UPLO, [N], [NRHS], A, [LDA], AF, [LDAF], IPIVOT,
       B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cherfs(char uplo, int n, int nrhs, complex *a, int lda,
          complex  *af,  int  ldaf, int *ipivot, complex *b,
          int ldb, complex *x, int ldx, float  *ferr,  float
          *berr, int *info);

void cherfs_64(char uplo, long n,  long  nrhs,  complex  *a,
          long  lda,  complex  *af, long ldaf, long *ipivot,
          complex *b, long ldb, complex *x, long ldx,  float
          *ferr, float *berr, long *info);
```

PURPOSE

cherfs improves the computed solution to a system of linear equations when the coefficient matrix is Hermitian indefinite, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input)
The factored form of the matrix A. AF contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**H or A = L*D*L**H as computed by CHETRF.

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CHETRF.

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by CHETRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
cherk - perform one of the Hermitian rank k  operations    C
:=  alpha*A*conjg( A' ) + beta*C or C := alpha*conjg( A' )*A
+ beta*C
```

SYNOPSIS

```
SUBROUTINE CHERK(UPLO, TRANSA, N, K, ALPHA, A, LDA, BETA, C, LDC)

CHARACTER * 1 UPLO, TRANSA
COMPLEX A(LDA,*), C(LDC,*)
INTEGER N, K, LDA, LDC
REAL ALPHA, BETA

SUBROUTINE CHERK_64(UPLO, TRANSA, N, K, ALPHA, A, LDA, BETA, C, LDC)

CHARACTER * 1 UPLO, TRANSA
COMPLEX A(LDA,*), C(LDC,*)
INTEGER*8 N, K, LDA, LDC
REAL ALPHA, BETA
```

F95 INTERFACE

```
SUBROUTINE HERK(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], BETA, C,
       [LDC])

CHARACTER(LEN=1) :: UPLO, TRANSA
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER :: N, K, LDA, LDC
REAL :: ALPHA, BETA

SUBROUTINE HERK_64(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], BETA,
       C, [LDC])
```

```
CHARACTER(LEN=1) :: UPLO, TRANSA
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER(8) :: N, K, LDA, LDC
REAL :: ALPHA, BETA
```

C INTERFACE

```
#include <sunperf.h>

void cherk(char uplo, char  transa,  int  n,  int  k,  float
          alpha,  complex  *a,  int lda, float beta, complex
          *c, int ldc);

void cherk_64(char uplo, char transa, long n, long k,  float
          alpha,  complex  *a, long lda, float beta, complex
          *c, long ldc);
```

PURPOSE

cherk performs one of the Hermitian rank k operations C := alpha*A*conjg(A') + beta*C or C := alpha*conjg(A')*A + beta*C where alpha and beta are real scalars, C is an n by n Hermitian matrix and A is an n by k matrix in the first case and a k by n matrix in the second case.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array C is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of C is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of C is to be referenced.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   C := alpha*A*conjg(A') + beta*C.

TRANSA = 'C' or 'c'   C := alpha*conjg(A')*A +

beta*C.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input)
On entry, N specifies the order of the matrix C. N must be at least zero. Unchanged on exit.

K (input)
On entry with TRANSA = 'N' or 'n', K specifies the number of columns of the matrix A, and on entry with TRANSA = 'C' or 'c', K specifies the number of rows of the matrix A. K must be at least zero. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
COMPLEX array of DIMENSION (LDA, ka), where ka is k when TRANSA = 'N' or 'n', and is n otherwise. Before entry with TRANSA = 'N' or 'n', the leading n by k part of the array A must contain the matrix A, otherwise the leading k by n part of the array A must contain the matrix A. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDA must be at least max(1, n), otherwise LDA must be at least max(1, k). Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. Unchanged on exit.

C (input/output)
COMPLEX array of DIMENSION (LDC, n).

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array C must contain the upper triangular part of the Hermitian matrix and the strictly lower triangular part of C is not referenced. On exit, the upper triangular part of the array C is overwritten by the upper triangular part of the updated

matrix.

Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array C must contain the lower triangular part of the Hermitian matrix and the strictly upper triangular part of C is not referenced. On exit, the lower triangular part of the array C is overwritten by the lower triangular part of the updated matrix.

Note that the imaginary parts of the diagonal elements need not be set, they are assumed to be zero, and on exit they are set to zero.

LDC (input)

On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC must be at least max(1, n). Unchanged on exit.

Contents

NAME

```
chesv - compute the solution to a complex system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE CHESV(UPLO, N, NRHS, A, LDA, IPIVOT, B, LDB, WORK, LDWORK,
      INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER N, NRHS, LDA, LDB, LDWORK, INFO
INTEGER IPIVOT(*)

SUBROUTINE CHESV_64(UPLO, N, NRHS, A, LDA, IPIVOT, B, LDB, WORK,
      LDWORK, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER*8 N, NRHS, LDA, LDB, LDWORK, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE HESV(UPLO, [N], [NRHS], A, [LDA], IPIVOT, B, [LDB], [WORK],
       [LDWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: N, NRHS, LDA, LDB, LDWORK, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE HESV_64(UPLO, [N], [NRHS], A, [LDA], IPIVOT, B, [LDB],
```

```
          [WORK], [LDWORK], [INFO])

   CHARACTER(LEN=1) :: UPLO
   COMPLEX, DIMENSION(:) :: WORK
   COMPLEX, DIMENSION(:,:) :: A, B
   INTEGER(8) :: N, NRHS, LDA, LDB, LDWORK, INFO
   INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
   #include <sunperf.h>

   void chesv(char uplo, int n, int nrhs, complex *a, int  lda,
             int *ipivot, complex *b, int ldb, int *info);
   void chesv_64(char uplo, long n, long nrhs, complex *a, long
             lda,  long  *ipivot,  complex  *b,  long ldb, long
             *info);
```

PURPOSE

chesv computes the solution to a complex system of linear equations
A * X = B, where A is an N-by-N Hermitian matrix and X and B are N-by-NRHS matrices.

The diagonal pivoting method is used to factor A as
A = U * D * U**H, if UPLO = 'U', or
A = L * D * L**H, if UPLO = 'L',
where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is Hermitian and block diagonal with 1-by-1 and 2-by-2 diagonal blocks. The factored form of A is then used to solve the system of equations A * X = B.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input/output)

On entry, the Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, if INFO = 0, the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**H or A = L*D*L**H as computed by CHETRF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (output)
Details of the interchanges and the block structure of D, as determined by CHETRF. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged, and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of WORK. LDWORK >= 1, and for best performance LDWORK >= N*NB, where NB is the optimal blocksize for CHETRF.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, so the solution could not be computed.

Contents

NAME

```
chesvx - use the diagonal pivoting factorization to  compute
the solution to a complex system of linear equations A * X =
B,
```

SYNOPSIS

```
SUBROUTINE CHESVX(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B,
      LDB, X, LDX, RCOND, FERR, BERR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER IPIVOT(*)
REAL RCOND
REAL FERR(*), BERR(*), WORK2(*)

SUBROUTINE CHESVX_64(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT,
      B, LDB, X, LDX, RCOND, FERR, BERR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER*8 IPIVOT(*)
REAL RCOND
REAL FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HESVX(FACT, UPLO, [N], [NRHS], A, [LDA], AF, [LDAF],
       IPIVOT, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [LDWORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO
```

```
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL :: RCOND
REAL, DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE HESVX_64(FACT, UPLO, [N], [NRHS], A, [LDA], AF, [LDAF],
       IPIVOT, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [LDWORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL :: RCOND
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void chesvx(char fact, char uplo, int n, int  nrhs,  complex
          *a,  int  lda, complex *af, int ldaf, int *ipivot,
          complex *b, int ldb, complex *x,  int  ldx,  float
          *rcond, float *ferr, float *berr, int *info);

void chesvx_64(char fact, char uplo, long n, long nrhs, com-
          plex  *a,  long  lda, complex *af, long ldaf, long
          *ipivot, complex *b, long ldb,  complex  *x,  long
          ldx,  float *rcond, float *ferr, float *berr, long
          *info);
```

PURPOSE

chesvx uses the diagonal pivoting factorization to compute the solution to a complex system of linear equations A * X = B, where A is an N-by-N Hermitian matrix and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'N', the diagonal pivoting method is used to factor A.
   The form of the factorization is
      A = U * D * U**H, if UPLO = 'U', or

```
      A = L * D * L**H,  if UPLO = 'L',
   where U (or L) is a product of permutation and unit upper
(lower)
   triangular matrices, and D is Hermitian and block  diago-
nal with
   1-by-1 and 2-by-2 diagonal blocks.

2. If some D(i,i)=0, so that D is exactly singular, then the
routine
   returns with INFO = i. Otherwise, the factored form of  A
is used
   to estimate the condition number of the matrix A.  If the
   reciprocal of the condition number is less  than  machine
precision,
   INFO = N+1 is returned as  a  warning,  but  the  routine
still goes on
   to solve for X and  compute  error  bounds  as  described
below.
3. The system of equations is solved for X  using  the  fac-
tored form
   of A.

4. Iterative refinement is applied to improve  the  computed
solution
   matrix and calculate  error  bounds  and  backward  error
estimates
   for it.
```

ARGUMENTS

```
FACT (input)
          Specifies whether or not the factored  form  of  A
          has  been supplied on entry.  = 'F':  On entry, AF
          and IPIVOT contain the factored form of A.  A,  AF
          and  IPIVOT  will  not  be  modified.  = 'N':  The
          matrix A will be copied to AF and factored.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.
```

A (input) The Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input/output)
If FACT = 'F', then AF is an input argument and on entry contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**H or A = L*D*L**H as computed by CHETRF.

If FACT = 'N', then AF is an output argument and on exit returns the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**H or A = L*D*L**H.

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

IPIVOT (input or output)
If FACT = 'F', then IPIVOT is an input argument and on entry contains details of the interchanges and the block structure of D, as determined by CHETRF. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

If FACT = 'N', then IPIVOT is an output argument and on exit contains details of the interchanges and the block structure of D, as determined by CHETRF.

B (input) The N-by-NRHS right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A. If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of WORK. LDWORK >= 2*N, and for best performance LDWORK >= N*NB, where NB is the optimal blocksize for CHETRF.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first

entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, and i is
<= N: D(i,i) is exactly zero. The factorization has been completed but the factor D is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: D is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
chetf2 - compute the factorization of  a  complex  Hermitian
matrix A using the Bunch-Kaufman diagonal pivoting method
```

SYNOPSIS

```
SUBROUTINE CHETF2(UPLO, N, A, LDA, IPIV, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*)
INTEGER N, LDA, INFO
INTEGER IPIV(*)

SUBROUTINE CHETF2_64(UPLO, N, A, LDA, IPIV, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*)
INTEGER*8 N, LDA, INFO
INTEGER*8 IPIV(*)
```

F95 INTERFACE

```
SUBROUTINE HETF2(UPLO, [N], A, [LDA], IPIV, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIV

SUBROUTINE HETF2_64(UPLO, [N], A, [LDA], IPIV, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A
```

```
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIV
```

C INTERFACE

```
#include <sunperf.h>

void chetf2(char uplo, int  n,  complex  *a,  int  lda,  int
          *ipiv, int *info);

void chetf2_64(char uplo, long n, complex *a, long lda, long
          *ipiv, long *info);
```

PURPOSE

chetf2 computes the factorization of a complex Hermitian matrix A using the Bunch-Kaufman diagonal pivoting method:

```
   A = U*D*U'  or  A = L*D*L'
```

where U (or L) is a product of permutation and unit upper (lower) triangular matrices, U' is the conjugate transpose of U, and D is Hermitian and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

This is the unblocked version of the algorithm, calling Level 2 BLAS.

ARGUMENTS

UPLO (input)
Specifies whether the upper or lower triangular part of the Hermitian matrix A is stored:
= 'U': Upper triangular
= 'L': Lower triangular

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the Hermitian matrix A. If UPLO = 'U', the leading n-by-n upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, the block diagonal matrix D and the mul-

tipliers used to obtain the factor U or L (see below for further details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIV (output)
Details of the interchanges and the block structure of D. If IPIV(k) > 0, then rows and columns k and IPIV(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIV(k) = IPIV(k-1) < 0, then rows and columns k-1 and -IPIV(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIV(k) = IPIV(k+1) < 0, then rows and columns k+1 and -IPIV(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, D(k,k) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

1-96 - Based on modifications by
J. Lewis, Boeing Computer Services Company
A. Petitet, Computer Science Dept., Univ. of Tenn., Knoxville, USA

If UPLO = 'U', then A = U*D*U', where
U = P(n)*U(n)* ... *P(k)U(k)* ...,
i.e., U is a product of terms P(k)*U(k), where k decreases from n to 1 in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIV(k), and U(k) is a unit upper triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    v    0   )   k-s
   U(k) =  (   0    I    0   )   s
           (   0    0    I   )   n-k
              k-s   s   n-k
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(1:k-1,k). If s = 2, the upper triangle of D(k) overwrites A(k-1,k-1), A(k-1,k), and A(k,k), and v overwrites A(1:k-2,k-1:k).

If UPLO = 'L', then A = L*D*L', where

```
   L = P(1)*L(1)* ... *P(k)*L(k)* ...,
```

i.e., L is a product of terms P(k)*L(k), where k increases from 1 to n in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIV(k), and L(k) is a unit lower triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    0     0   )  k-1
   L(k) =  (   0    I     0   )  s
           (   0    v     I   )  n-k-s+1
              k-1   s  n-k-s+1
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(k+1:n,k). If s = 2, the lower triangle of D(k) overwrites A(k,k), A(k+1,k), and A(k+1,k+1), and v overwrites A(k+2:n,k:k+1).

Contents

NAME

```
chetrd - reduce a complex Hermitian matrix A  to  real  sym-
metric  tridiagonal form T by a unitary similarity transfor-
mation
```

SYNOPSIS

```
SUBROUTINE CHETRD(UPLO, N, A, LDA, D, E, TAU, WORK, LWORK, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER N, LDA, LWORK, INFO
REAL D(*), E(*)

SUBROUTINE CHETRD_64(UPLO, N, A, LDA, D, E, TAU, WORK, LWORK, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 N, LDA, LWORK, INFO
REAL D(*), E(*)
```

F95 INTERFACE

```
SUBROUTINE HETRD(UPLO, [N], A, [LDA], D, E, TAU, [WORK], [LWORK],
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, LWORK, INFO
REAL, DIMENSION(:) :: D, E

SUBROUTINE HETRD_64(UPLO, [N], A, [LDA], D, E, TAU, [WORK], [LWORK],
```

```
          [INFO])

     CHARACTER(LEN=1) :: UPLO
     COMPLEX, DIMENSION(:) :: TAU, WORK
     COMPLEX, DIMENSION(:,:) :: A
     INTEGER(8) :: N, LDA, LWORK, INFO
     REAL, DIMENSION(:) :: D, E
```

C INTERFACE

```
#include <sunperf.h>

void chetrd(char uplo, int n, complex *a, int lda, float *d,
          float *e, complex *tau, int *info);

void chetrd_64(char uplo, long  n,  complex  *a,  long  lda,
          float *d, float *e, complex *tau, long *info);
```

PURPOSE

chetrd reduces a complex Hermitian matrix A to real symmetric tridiagonal form T by a unitary similarity transformation: Q**H * A * Q = T.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input) On entry, the Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced. On exit, if UPLO = 'U', the diagonal and first superdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements above the first superdiagonal, with the array TAU, represent the unitary matrix Q as a product of elementary reflectors; if UPLO = 'L', the diagonal and first subdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements below the first subdiagonal, with the array TAU, represent the unitary matrix Q as a product of elementary reflectors. See Further Details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

D (output)
The diagonal elements of the tridiagonal matrix T: D(i) = A(i,i).

E (output)
The off-diagonal elements of the tridiagonal matrix T: E(i) = A(i,i+1) if UPLO = 'U', E(i) = A(i+1,i) if UPLO = 'L'.

TAU (output)
The scalar factors of the elementary reflectors (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 1. For optimum performance LWORK >= N*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

If UPLO = 'U', the matrix Q is represented as a product of elementary reflectors

Q = H(n-1) . . . H(2) H(1).

Each H(i) has the form

```
   H(i) = I - tau * v * v'

where tau is a complex scalar, and v  is  a  complex  vector
with  v(i+1:n)  = 0 and v(i) = 1; v(1:i-1) is stored on exit
in
A(1:i-1,i+1), and tau in TAU(i).

If UPLO = 'L', the matrix Q is represented as a  product  of
elementary reflectors

   Q = H(1) H(2) . . . H(n-1).

Each H(i) has the form

   H(i) = I - tau * v * v'

where tau is a complex scalar, and v  is  a  complex  vector
with  v(1:i)  = 0 and v(i+1) = 1; v(i+2:n) is stored on exit
in A(i+2:n,i), and tau in TAU(i).

The contents of A on exit are illustrated by  the  following
examples with n = 5:

if UPLO = 'U':                       if UPLO = 'L':

  (    d     e     v2    v3    v4   )                (     d
)
  (       d    e    v3    v4   )                (    e     d
)
  (           d    e    v4  )                (   v1   e     d
)
  (              d   e  )                (   v1   v2   e    d
)
  (                  d  )              (  v1  v2  v3  e    d
)

where d and e denote diagonal and off-diagonal  elements  of
T, and vi denotes an element of the vector defining H(i).
```

Contents

NAME

```
chetrf - compute the factorization of  a  complex  Hermitian
matrix A using the Bunch-Kaufman diagonal pivoting method
```

SYNOPSIS

```
SUBROUTINE CHETRF(UPLO, N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, LDWORK, INFO
INTEGER IPIVOT(*)

SUBROUTINE CHETRF_64(UPLO, N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, LDWORK, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE HETRF(UPLO, [N], A, [LDA], IPIVOT, [WORK], [LDWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, LDWORK, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE HETRF_64(UPLO, [N], A, [LDA], IPIVOT, [WORK], [LDWORK],
       [INFO])
```

```
CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void chetrf(char uplo, int  n,  complex  *a,  int  lda,  int
          *ipivot, int *info);

void chetrf_64(char uplo, long n, complex *a, long lda, long
          *ipivot, long *info);
```

PURPOSE

chetrf computes the factorization of a complex Hermitian matrix A using the Bunch-Kaufman diagonal pivoting method. The form of the factorization is

```
A = U*D*U**H  or  A = L*D*L**H
```

where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is Hermitian and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

This is the blocked version of the algorithm, calling Level 3 BLAS.

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the Hermitian matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A,
          and the strictly lower triangular part of A is not
          referenced.  If UPLO =  'L',  the  leading  N-by-N
          lower triangular part of A contains the lower tri-
          angular part of the matrix  A,  and  the  strictly
          upper triangular part of A is not referenced.
```

On exit, the block diagonal matrix D and the multipliers used to obtain the factor U or L (see below for further details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (output)
Details of the interchanges and the block structure of D. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of WORK. LDWORK >=1. For best performance LDWORK >= N*NB, where NB is the block size returned by ILAENV.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

```
If UPLO = 'U', then A = U*D*U', where
   U = P(n)*U(n)* ... *P(k)U(k)* ...,
i.e., U is a product of terms P(k)*U(k), where  k  decreases
from  n  to  1 in steps of 1 or 2, and D is a block diagonal
matrix with 1-by-1 and 2-by-2 diagonal blocks D(k).  P(k) is
a  permutation matrix as defined by IPIVOT(k), and U(k) is a
unit upper triangular matrix,  such  that  if  the  diagonal
```

block D(k) is of order s (s = 1 or 2), then

```
           (   I    v    0   )   k-s
   U(k) =  (   0    I    0   )   s
           (   0    0    I   )   n-k
              k-s   s   n-k
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(1:k-1,k). If s = 2, the upper triangle of D(k) overwrites A(k-1,k-1), A(k-1,k), and A(k,k), and v overwrites A(1:k-2,k-1:k).

If UPLO = 'L', then A = L*D*L', where
L = P(1)*L(1)* ... *P(k)*L(k)* ...,
i.e., L is a product of terms P(k)*L(k), where k increases from 1 to n in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIVOT(k), and L(k) is a unit lower triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    0     0   )  k-1
   L(k) =  (   0    I     0   )  s
           (   0    v     I   )  n-k-s+1
              k-1   s  n-k-s+1
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(k+1:n,k). If s = 2, the lower triangle of D(k) overwrites A(k,k), A(k+1,k), and A(k+1,k+1), and v overwrites A(k+2:n,k:k+1).

Contents

NAME

```
chetri - compute the inverse of a complex Hermitian indefin-
ite  matrix  A  using  the factorization A = U*D*U**H or A =
L*D*L**H computed by CHETRF
```

SYNOPSIS

```
SUBROUTINE CHETRI(UPLO, N, A, LDA, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, INFO
INTEGER IPIVOT(*)

SUBROUTINE CHETRI_64(UPLO, N, A, LDA, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE HETRI(UPLO, [N], A, [LDA], IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE HETRI_64(UPLO, [N], A, [LDA], IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void chetri(char uplo, int  n,  complex  *a,  int  lda,  int
          *ipivot, int *info);

void chetri_64(char uplo, long n, complex *a, long lda, long
          *ipivot, long *info);
```

PURPOSE

chetri computes the inverse of a complex Hermitian indefinite matrix A using the factorization A = U*D*U**H or A = L*D*L**H computed by CHETRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**H;
= 'L': Lower triangular, form is A = L*D*L**H.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CHETRF.

On exit, if INFO = 0, the (Hermitian) inverse of the original matrix. If UPLO = 'U', the upper triangular part of the inverse is formed and the part of A below the diagonal is not referenced; if UPLO = 'L' the lower triangular part of the inverse is formed and the part of A above the diagonal is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CHETRF.

WORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) = 0; the matrix is singular and its inverse could not be computed.

Contents

NAME

```
chetrs - solve a system of linear equations A*X = B  with  a
complex  Hermitian  matrix  A  using  the  factorization A =
U*D*U**H or A = L*D*L**H computed by CHETRF
```

SYNOPSIS

```
SUBROUTINE CHETRS(UPLO, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE CHETRS_64(UPLO, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE HETRS(UPLO, [N], [NRHS], A, [LDA], IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: N, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE HETRS_64(UPLO, [N], [NRHS], A, [LDA], IPIVOT, B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void chetrs(char uplo, int n, int nrhs, complex *a, int lda,
          int *ipivot, complex *b, int ldb, int *info);

void chetrs_64(char uplo, long n,  long  nrhs,  complex  *a,
          long lda, long *ipivot, complex *b, long ldb, long
          *info);
```

PURPOSE

chetrs solves a system of linear equations A*X = B with a complex Hermitian matrix A using the factorization A = U*D*U**H or A = L*D*L**H computed by CHETRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**H;
= 'L': Lower triangular, form is A = L*D*L**H.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CHETRF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CHETRF.

B (input/output)

On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
chgeqz - implement a single-shift version of the  QZ  method
for       finding       the       generalized       eigenvalues
w(i)=ALPHA(i)/BETA(i) of the equation   det( A-w(i) B ) =  0
If JOB='S', then the pair (A,B) is simultaneously reduced to
Schur form (i.e., A and B  are  both  upper  triangular)  by
applying one unitary tranformation (usually called Q) on the
left and another (usually called Z) on the right
```

SYNOPSIS

```
SUBROUTINE CHGEQZ(JOB, COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB,
      ALPHA, BETA, Q, LDQ, Z, LDZ, WORK, LWORK, RWORK, INFO)

CHARACTER * 1 JOB, COMPQ, COMPZ
COMPLEX A(LDA,*),  B(LDB,*),  ALPHA(*),  BETA(*),  Q(LDQ,*),
Z(LDZ,*), WORK(*)
INTEGER N, ILO, IHI, LDA, LDB, LDQ, LDZ, LWORK, INFO
REAL RWORK(*)

SUBROUTINE CHGEQZ_64(JOB, COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB,
      ALPHA, BETA, Q, LDQ, Z, LDZ, WORK, LWORK, RWORK, INFO)

CHARACTER * 1 JOB, COMPQ, COMPZ
COMPLEX A(LDA,*),  B(LDB,*),  ALPHA(*),  BETA(*),  Q(LDQ,*),
Z(LDZ,*), WORK(*)
INTEGER*8 N, ILO, IHI, LDA, LDB, LDQ, LDZ, LWORK, INFO
REAL RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HGEQZ(JOB, COMPQ, COMPZ, [N], ILO, IHI, A, [LDA], B, [LDB],
       ALPHA, BETA, Q, [LDQ], Z, [LDZ], [WORK], [LWORK], [RWORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOB, COMPQ, COMPZ
COMPLEX, DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX, DIMENSION(:,:) :: A, B, Q, Z
INTEGER :: N, ILO, IHI, LDA, LDB, LDQ, LDZ, LWORK, INFO
REAL, DIMENSION(:) :: RWORK

SUBROUTINE HGEQZ_64(JOB, COMPQ, COMPZ, [N], ILO, IHI, A, [LDA], B,
       [LDB], ALPHA, BETA, Q, [LDQ], Z, [LDZ], [WORK], [LWORK], [RWORK],
       [INFO])

CHARACTER(LEN=1) :: JOB, COMPQ, COMPZ
COMPLEX, DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX, DIMENSION(:,:) :: A, B, Q, Z
INTEGER(8) :: N, ILO, IHI, LDA, LDB, LDQ, LDZ, LWORK, INFO
REAL, DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>

void chgeqz(char job, char compq, char  compz,  int  n,  int
          ilo, int ihi, complex *a, int lda, complex *b, int
          ldb, complex *alpha, complex  *beta,  complex  *q,
          int ldq, complex *z, int ldz, int *info);

void chgeqz_64(char job, char compq,  char  compz,  long  n,
          long  ilo, long ihi, complex *a, long lda, complex
          *b, long ldb, complex *alpha, complex *beta,  com-
          plex  *q,  long  ldq,  complex  *z, long ldz, long
          *info);
```

PURPOSE

chgeqz implements a single-shift version of the QZ method for finding the generalized eigenvalues w(i)=ALPHA(i)/BETA(i) of the equation A are then ALPHA(1),...,ALPHA(N), and of B are BETA(1),...,BETA(N).

If JOB='S' and COMPQ and COMPZ are 'V' or 'I', then the unitary transformations used to reduce (A,B) are accumulated into the arrays Q and Z s.t.:
(in) A(in) Z(in)* = Q(out) A(out) Z(out)*

Ref: C.B. Moler & G.W. Stewart, "An Algorithm for Generalized Matrixigenvalue Problems", SIAM J. Numer. Anal., 10(1973),p. 241--256.

ARGUMENTS

```
JOB (input)
          = 'E': compute only ALPHA and BETA.  A and B  will
          not  necessarily  be  put  into  generalized Schur
          form.  = 'S': put A and B into  generalized  Schur
          form, as well as computing ALPHA and BETA.

COMPQ (input)
          = 'N': do not modify Q.
          = 'V': multiply the array Q on the  right  by  the
          conjugate  transpose  of the unitary tranformation
          that is applied to the left side of  A  and  B  to
          reduce them to Schur form.  = 'I': like COMPQ='V',
          except that Q will be initialized to the  identity
          first.

COMPZ (input)
          = 'N': do not modify Z.
          = 'V': multiply the array Z on the  right  by  the
          unitary tranformation that is applied to the right
          side of A and B to reduce them to Schur  form.   =
          'I':  like  COMPZ='V',  except that Z will be ini-
          tialized to the identity first.

N (input) The order of the matrices A, B, Q, and Z.  N >= 0.

ILO (input)
          It is assumed that A is already  upper  triangular
          in rows and columns 1:ILO-1 and IHI+1:N.  1 <= ILO
          <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

IHI (input)
          It is assumed that A is already  upper  triangular
          in rows and columns 1:ILO-1 and IHI+1:N.  1 <= ILO
          <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

A (input) On entry, the N-by-N upper  Hessenberg  matrix  A.
          Elements  below  the subdiagonal must be zero.  If
          JOB='S', then on exit  A  and  B  will  have  been
          simultaneously  reduced  to upper triangular form.
          If JOB='E', then on exit A  will  have  been  des-
          troyed.

LDA (input)
          The leading dimension of the array A.  LDA >= max(
          1, N ).

B (input) On entry, the N-by-N upper  triangular  matrix  B.
          Elements  below  the  diagonal  must  be zero.  If
          JOB='S', then on exit  A  and  B  will  have  been
```

simultaneously reduced to upper triangular form. If JOB='E', then on exit B will have been destroyed.

LDB (input)
The leading dimension of the array B. LDB >= max(1, N).

ALPHA (output)
The diagonal elements of A when the pair (A,B) has been reduced to Schur form. ALPHA(i)/BETA(i) i=1,...,N are the generalized eigenvalues.

BETA (output)
The diagonal elements of B when the pair (A,B) has been reduced to Schur form. ALPHA(i)/BETA(i) i=1,...,N are the generalized eigenvalues. A and B are normalized so that BETA(1),...,BETA(N) are non-negative real numbers.

Q (input/output)
If COMPQ='N', then Q will not be referenced. If COMPQ='V' or 'I', then the conjugate transpose of the unitary transformations which are applied to A and B on the left will be applied to the array Q on the right.

LDQ (input)
The leading dimension of the array Q. LDQ >= 1. If COMPQ='V' or 'I', then LDQ >= N.

Z (input/output)
If COMPZ='N', then Z will not be referenced. If COMPZ='V' or 'I', then the unitary transformations which are applied to A and B on the right will be applied to the array Z on the right.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1. If COMPZ='V' or 'I', then LDZ >= N.

WORK (workspace)
On exit, if INFO >= 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,N).

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of

the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
= 1,...,N: the QZ iteration did not converge. (A,B) is not in Schur form, but ALPHA(i) and BETA(i), i=INFO+1,...,N should be correct. = N+1,...,2*N: the shift calculation failed. (A,B) is not in Schur form, but ALPHA(i) and BETA(i), i=INFO-N+1,...,N should be correct. > 2*N: various "impossible" errors.

FURTHER DETAILS

We assume that complex ABS works as long as its value is less than overflow.

Contents

NAME

```
chpcon - estimate the reciprocal of the condition number  of
a  complex Hermitian packed matrix A using the factorization
A = U*D*U**H or A = L*D*L**H computed by CHPTRF
```

SYNOPSIS

```
SUBROUTINE CHPCON(UPLO, N, A, IPIVOT, ANORM, RCOND, WORK, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), WORK(*)
INTEGER N, INFO
INTEGER IPIVOT(*)
REAL ANORM, RCOND

SUBROUTINE CHPCON_64(UPLO, N, A, IPIVOT, ANORM, RCOND, WORK, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), WORK(*)
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
REAL ANORM, RCOND
```

F95 INTERFACE

```
SUBROUTINE HPCON(UPLO, N, A, IPIVOT, ANORM, RCOND, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A, WORK
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL :: ANORM, RCOND

SUBROUTINE HPCON_64(UPLO, N, A, IPIVOT, ANORM, RCOND, [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A, WORK
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL :: ANORM, RCOND
```

C INTERFACE

```
#include <sunperf.h>

void chpcon(char uplo, int n, complex *a, int *ipivot, float
          anorm, float *rcond, int *info);

void chpcon_64(char uplo, long n, complex *a, long  *ipivot,
          float anorm, float *rcond, long *info);
```

PURPOSE

chpcon estimates the reciprocal of the condition number of a complex Hermitian packed matrix A using the factorization A = U*D*U**H or A = L*D*L**H computed by CHPTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**H;
= 'L': Lower triangular, form is A = L*D*L**H.

N (input) The order of the matrix A. N >= 0.

A (input) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CHPTRF, stored as a packed triangular matrix.

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CHPTRF.

ANORM (input)
The 1-norm of the original matrix A.

```
RCOND (output)
          The reciprocal of  the  condition  number  of  the
          matrix  A, computed as RCOND = 1/(ANORM * AINVNM),
          where AINVNM is  an  estimate  of  the  1-norm  of
          inv(A) computed in this routine.

WORK (workspace)
          dimension(2*N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
chpev - compute all the eigenvalues and, optionally,  eigen-
vectors of a complex Hermitian matrix in packed storage
```

SYNOPSIS

```
SUBROUTINE CHPEV(JOBZ, UPLO, N, A, W, Z, LDZ, WORK, WORK2, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX A(*), Z(LDZ,*), WORK(*)
INTEGER N, LDZ, INFO
REAL W(*), WORK2(*)

SUBROUTINE CHPEV_64(JOBZ, UPLO, N, A, W, Z, LDZ, WORK, WORK2, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX A(*), Z(LDZ,*), WORK(*)
INTEGER*8 N, LDZ, INFO
REAL W(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HPEV(JOBZ, UPLO, N, A, W, Z, [LDZ], [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: A, WORK
COMPLEX, DIMENSION(:,:) :: Z
INTEGER :: N, LDZ, INFO
REAL, DIMENSION(:) :: W, WORK2

SUBROUTINE HPEV_64(JOBZ, UPLO, N, A, W, Z, [LDZ], [WORK], [WORK2],
       [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: A, WORK
COMPLEX, DIMENSION(:,:) :: Z
INTEGER(8) :: N, LDZ, INFO
REAL, DIMENSION(:) :: W, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void chpev(char jobz, char uplo, int n,  complex  *a,  float
          *w, complex *z, int ldz, int *info);

void chpev_64(char jobz, char  uplo,  long  n,  complex  *a,
          float *w, complex *z, long ldz, long *info);
```

PURPOSE

chpev computes all the eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix in packed storage.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the upper or lower triangle of the Hermitian matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

On exit, A is overwritten by values generated during the reduction to tridiagonal form. If UPLO = 'U', the diagonal and first superdiagonal of the tridiagonal matrix T overwrite the corresponding elements of A, and if UPLO = 'L', the diagonal and first subdiagonal of T overwrite the corresponding elements of A.

```
W (output)
          If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0,  Z  contains  the
          orthonormal eigenvectors of the matrix A, with the
          i-th column of Z holding the  eigenvector  associ-
          ated  with  W(i).   If  JOBZ  = 'N', then Z is not
          referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).
WORK (workspace)
          dimension(MAX(1,2*N-1))

WORK2 (workspace)
          dimension(max(1,3*N-2))

INFO (output)
          = 0:  successful exit.
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value.
          > 0:  if INFO = i, the algorithm  failed  to  con-
          verge;  i off-diagonal elements of an intermediate
          tridiagonal form did not converge to zero.
```

Contents

NAME

```
chpevd - compute all the eigenvalues and, optionally, eigen-
vectors of a complex Hermitian matrix A in packed storage
```

SYNOPSIS

```
SUBROUTINE CHPEVD(JOBZ, UPLO, N, AP, W, Z, LDZ, WORK, LWORK, RWORK,
      LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX AP(*), Z(LDZ,*), WORK(*)
INTEGER N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER IWORK(*)
REAL W(*), RWORK(*)

SUBROUTINE CHPEVD_64(JOBZ, UPLO, N, AP, W, Z, LDZ, WORK, LWORK,
      RWORK, LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX AP(*), Z(LDZ,*), WORK(*)
INTEGER*8 N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
REAL W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HPEVD(JOBZ, UPLO, N, AP, W, Z, [LDZ], [WORK], [LWORK],
       [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: AP, WORK
COMPLEX, DIMENSION(:,:) :: Z
INTEGER :: N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
```

```
REAL, DIMENSION(:) :: W, RWORK

SUBROUTINE HPEVD_64(JOBZ, UPLO, N, AP, W, Z, [LDZ], [WORK], [LWORK],
       [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: AP, WORK
COMPLEX, DIMENSION(:,:) :: Z
INTEGER(8) :: N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>
void chpevd(char jobz, char uplo, int n, complex *ap,  float
          *w, complex *z, int ldz, int *info);

void chpevd_64(char jobz, char uplo, long  n,  complex  *ap,
          float *w, complex *z, long ldz, long *info);
```

PURPOSE

chpevd computes all the eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A in packed storage. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.
```

AP (input/output)
On entry, the upper or lower triangle of the Hermitian matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

On exit, AP is overwritten by values generated during the reduction to tridiagonal form. If UPLO = 'U', the diagonal and first superdiagonal of the tridiagonal matrix T overwrite the corresponding elements of A, and if UPLO = 'L', the diagonal and first subdiagonal of T overwrite the corresponding elements of A.

W (output)
If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, Z contains the orthonormal eigenvectors of the matrix A, with the i-th column of Z holding the eigenvector associated with W(i). If JOBZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of array WORK. If N <= 1, LWORK must be at least 1. If JOBZ = 'N' and N > 1, LWORK must be at least N. If JOBZ = 'V' and N > 1, LWORK must be at least 2*N.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
dimension (LRWORK) On exit, if INFO = 0, RWORK(1) returns the optimal LRWORK.

LRWORK (input)
The dimension of array RWORK. If N <= 1,

LRWORK must be at least 1. If JOBZ = 'N' and N > 1, LRWORK must be at least N. If JOBZ = 'V' and N > 1, LRWORK must be at least 1 + 5*N + 2*N**2.

If LRWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the RWORK array, returns this value as the first entry of the RWORK array, and no error message related to LRWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of array IWORK. If JOBZ = 'N' or N <= 1, LIWORK must be at least 1. If JOBZ = 'V' and N > 1, LIWORK must be at least 3 + 5*N.

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

Contents

NAME

```
chpevx  -  compute  selected  eigenvalues  and,  optionally,
eigenvectors  of  a  complex  Hermitian  matrix  A in packed
storage
```

SYNOPSIS

```
SUBROUTINE CHPEVX(JOBZ, RANGE, UPLO, N, A, VL, VU, IL, IU, ABTOL,
      NFOUND, W, Z, LDZ, WORK, WORK2, IWORK3, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
COMPLEX A(*), Z(LDZ,*), WORK(*)
INTEGER N, IL, IU, NFOUND, LDZ, INFO
INTEGER IWORK3(*), IFAIL(*)
REAL VL, VU, ABTOL
REAL W(*), WORK2(*)

SUBROUTINE CHPEVX_64(JOBZ, RANGE, UPLO, N, A, VL, VU, IL, IU, ABTOL,
      NFOUND, W, Z, LDZ, WORK, WORK2, IWORK3, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
COMPLEX A(*), Z(LDZ,*), WORK(*)
INTEGER*8 N, IL, IU, NFOUND, LDZ, INFO
INTEGER*8 IWORK3(*), IFAIL(*)
REAL VL, VU, ABTOL
REAL W(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HPEVX(JOBZ, RANGE, UPLO, N, A, VL, VU, IL, IU, ABTOL,
       [NFOUND], W, Z, [LDZ], [WORK], [WORK2], [IWORK3], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX, DIMENSION(:) :: A, WORK
```

```
COMPLEX, DIMENSION(:,:) :: Z
INTEGER :: N, IL, IU, NFOUND, LDZ, INFO
INTEGER, DIMENSION(:) :: IWORK3, IFAIL
REAL :: VL, VU, ABTOL
REAL, DIMENSION(:) :: W, WORK2

SUBROUTINE HPEVX_64(JOBZ, RANGE, UPLO, N, A, VL, VU, IL, IU, ABTOL,
       [NFOUND], W, Z, [LDZ], [WORK], [WORK2], [IWORK3], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX, DIMENSION(:) :: A, WORK
COMPLEX, DIMENSION(:,:) :: Z
INTEGER(8) :: N, IL, IU, NFOUND, LDZ, INFO
INTEGER(8), DIMENSION(:) :: IWORK3, IFAIL
REAL :: VL, VU, ABTOL
REAL, DIMENSION(:) :: W, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void chpevx(char jobz, char range, char uplo, int n, complex
          *a,  float  vl,  float  vu,  int il, int iu, float
          abtol, int *nfound, float *w, complex *z, int ldz,
          int *ifail, int *info);

void chpevx_64(char jobz, char range,  char  uplo,  long  n,
          complex  *a, float vl, float vu, long il, long iu,
          float abtol, long *nfound, float *w,  complex  *z,
          long ldz, long *ifail, long *info);
```

PURPOSE

chpevx computes selected eigenvalues and, optionally, eigen- vectors of a complex Hermitian matrix A in packed storage. Eigenvalues/vectors can be selected by specifying either a range of values or a range of indices for the desired eigen- values.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

RANGE (input)
          = 'A': all eigenvalues will be found;
          = 'V': all eigenvalues in the  half-open  interval
          (VL,VU]  will  be  found; = 'I': the IL-th through
```

IU-th eigenvalues will be found.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the upper or lower triangle of the Hermitian matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.
On exit, A is overwritten by values generated during the reduction to tridiagonal form. If UPLO = 'U', the diagonal and first superdiagonal of the tridiagonal matrix T overwrite the corresponding elements of A, and if UPLO = 'L', the diagonal and first subdiagonal of T overwrite the corresponding elements of A.

VL (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

ABTOL (input)
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged

when it is determined to lie in an interval [a,b] of width less than or equal to

ABTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing A to tri-diagonal form.

Eigenvalues will be computed most accurately when ABTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABTOL to 2*SLAMCH('S').

See "Computing Small Singular Values of Bidiagonal Matrices with Guaranteed High Relative Accuracy," by Demmel and Kahan, LAPACK Working Note #3.

NFOUND (output)
The total number of eigenvalues found. 0 <= NFOUND <= N. If RANGE = 'A', NFOUND = N, and if RANGE = 'I', NFOUND = IU-IL+1.

W (output)
If INFO = 0, the selected eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, the first NFOUND columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). If an eigenvec-tor fails to converge, then that column of Z con-tains the latest approximation to the eigenvector, and the index of the eigenvector is returned in IFAIL. If JOBZ = 'N', then Z is not referenced. Note: the user must ensure that at least max(1,NFOUND) columns are supplied in the array Z; if RANGE = 'V', the exact value of NFOUND is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(7*N)

IWORK3 (workspace)
dimension(5*N)

IFAIL (output)
If JOBZ = 'V', then if INFO = 0, the first NFOUND elements of IFAIL are zero. If INFO > 0, then IFAIL contains the indices of the eigenvectors that failed to converge. If JOBZ = 'N', then IFAIL is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, then i eigenvectors failed to converge. Their indices are stored in array IFAIL.

Contents

NAME

```
chpgst - reduce  a  complex  Hermitian-definite  generalized
eigenproblem to standard form, using packed storage
```

SYNOPSIS

```
SUBROUTINE CHPGST(ITYPE, UPLO, N, AP, BP, INFO)

CHARACTER * 1 UPLO
COMPLEX AP(*), BP(*)
INTEGER ITYPE, N, INFO

SUBROUTINE CHPGST_64(ITYPE, UPLO, N, AP, BP, INFO)

CHARACTER * 1 UPLO
COMPLEX AP(*), BP(*)
INTEGER*8 ITYPE, N, INFO
```

F95 INTERFACE

```
SUBROUTINE HPGST(ITYPE, UPLO, N, AP, BP, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: AP, BP
INTEGER :: ITYPE, N, INFO

SUBROUTINE HPGST_64(ITYPE, UPLO, N, AP, BP, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: AP, BP
INTEGER(8) :: ITYPE, N, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void chpgst(int itype, char uplo, int n, complex  *ap,  com-
          plex *bp, int *info);

void chpgst_64(long itype, char uplo, long n,  complex  *ap,
          complex *bp, long *info);
```

PURPOSE

chpgst reduces a complex Hermitian-definite generalized eigenproblem to standard form, using packed storage.

If ITYPE = 1, the problem is A*x = lambda*B*x,
and A is overwritten by inv(U**H)*A*inv(U) or inv(L)*A*inv(L**H)
If ITYPE = 2 or 3, the problem is A*B*x = lambda*x or
B*A*x = lambda*x, and A is overwritten by U*A*U**H or L**H*A*L.

B must have been previously factorized as U**H*U or L*L**H by CPPTRF.

ARGUMENTS

ITYPE (input)
= 1: compute inv(U**H)*A*inv(U) or inv(L)*A*inv(L**H);
= 2 or 3: compute U*A*U**H or L**H*A*L.

UPLO (input)
= 'U': Upper triangle of A is stored and B is factored as U**H*U; = 'L': Lower triangle of A is stored and B is factored as L*L**H.

N (input) The order of the matrices A and B. N >= 0.

AP (input/output)
On entry, the upper or lower triangle of the Hermitian matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n.

On exit, if INFO = 0, the transformed matrix, stored in the same format as A.

BP (input)
The triangular factor from the Cholesky factorization of B, stored in the same format as A, as returned by CPPTRF.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
chpgv - compute all the  eigenvalues  and,  optionally,  the
eigenvectors  of  a  complex  generalized Hermitian-definite
eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x,
or B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE CHPGV(ITYPE, JOBZ, UPLO, N, A, B, W, Z, LDZ, WORK, WORK2,
      INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX A(*), B(*), Z(LDZ,*), WORK(*)
INTEGER ITYPE, N, LDZ, INFO
REAL W(*), WORK2(*)

SUBROUTINE CHPGV_64(ITYPE, JOBZ, UPLO, N, A, B, W, Z, LDZ, WORK,
      WORK2, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX A(*), B(*), Z(LDZ,*), WORK(*)
INTEGER*8 ITYPE, N, LDZ, INFO
REAL W(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HPGV(ITYPE, JOBZ, UPLO, N, A, B, W, Z, [LDZ], [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: A, B, WORK
COMPLEX, DIMENSION(:,:) :: Z
INTEGER :: ITYPE, N, LDZ, INFO
REAL, DIMENSION(:) :: W, WORK2
```

```
SUBROUTINE HPGV_64(ITYPE, JOBZ, UPLO, N, A, B, W, Z, [LDZ], [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: A, B, WORK
COMPLEX, DIMENSION(:,:) :: Z
INTEGER(8) :: ITYPE, N, LDZ, INFO
REAL, DIMENSION(:) :: W, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void chpgv(int itype, char jobz, char uplo, int  n,  complex
          *a, complex *b, float *w, complex *z, int ldz, int
          *info);

void chpgv_64(long itype, char jobz, char uplo, long n, com-
          plex  *a,  complex  *b, float *w, complex *z, long
          ldz, long *info);
```

PURPOSE

chpgv computes all the eigenvalues and, optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be Hermitian, stored in packed format, and B is also positive definite.

ARGUMENTS

```
ITYPE (input)
          Specifies the problem type to be solved:
          = 1:  A*x = (lambda)*B*x
          = 2:  A*B*x = (lambda)*x
          = 3:  B*A*x = (lambda)*x

JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.
```

```
A (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  matrix  A,  packed  columnwise in a linear
          array.  The j-th column of  A  is  stored  in  the
          array  A  as  follows:   if  UPLO = 'U', A(i + (j-
          1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i +
          (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

          On exit, the contents of A are destroyed.

B (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  matrix  B,  packed  columnwise in a linear
          array.  The j-th column of  B  is  stored  in  the
          array  B  as  follows:   if  UPLO = 'U', B(i + (j-
          1)*j/2) = B(i,j) for 1<=i<=j; if UPLO = 'L', B(i +
          (j-1)*(2*n-j)/2) = B(i,j) for j<=i<=n.

          On exit, the triangular factor U  or  L  from  the
          Cholesky  factorization  B = U**H*U or B = L*L**H,
          in the same storage format as B.

W (output)
          If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0,  Z  contains  the
          matrix  Z  of  eigenvectors.  The eigenvectors are
          normalized  as  follows:   if  ITYPE  =  1  or  2,
          Z**H*B*Z = I; if ITYPE = 3, Z**H*inv(B)*Z = I.  If
          JOBZ = 'N', then Z is not referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
          dimension(MAX(1,2*N-1))

WORK2 (workspace)
          dimension(MAX(1,3*N-2))

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  CPPTRF or CHPEV returned an error code:
          <= N:  if INFO = i, CHPEV failed  to  converge;  i
          off-diagonal elements of an intermediate tridiago-
          nal form did not convergeto zero; > N:   if INFO =
```

N + i, for 1 <= i <= n, then the leading minor of order i of B is not positive definite.   The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

Contents

NAME

```
chpgvd - compute all the eigenvalues  and,  optionally,  the
eigenvectors  of  a  complex  generalized Hermitian-definite
eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x,
or B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE CHPGVD(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, LDZ, WORK,
      LWORK, RWORK, LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX AP(*), BP(*), Z(LDZ,*), WORK(*)
INTEGER ITYPE, N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER IWORK(*)
REAL W(*), RWORK(*)

SUBROUTINE CHPGVD_64(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, LDZ, WORK,
      LWORK, RWORK, LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
COMPLEX AP(*), BP(*), Z(LDZ,*), WORK(*)
INTEGER*8 ITYPE, N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
REAL W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HPGVD(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, [LDZ], [WORK],
       [LWORK], [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: AP, BP, WORK
```

```
COMPLEX, DIMENSION(:,:) :: Z
INTEGER :: ITYPE, N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: W, RWORK

SUBROUTINE HPGVD_64(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, [LDZ],
       [WORK], [LWORK], [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX, DIMENSION(:) :: AP, BP, WORK
COMPLEX, DIMENSION(:,:) :: Z
INTEGER(8) :: ITYPE, N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void chpgvd(int itype, char jobz, char uplo, int n,  complex
          *ap,  complex  *bp, float *w, complex *z, int ldz,
          int *info);

void chpgvd_64(long itype, char jobz,  char  uplo,  long  n,
          complex  *ap,  complex  *bp, float *w, complex *z,
          long ldz, long *info);
```

PURPOSE

chpgvd computes all the eigenvalues and, optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be Hermitian, stored in packed format, and B is also positive definite.
If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

ITYPE (input)

```
          Specifies the problem type to be solved:
          = 1:  A*x = (lambda)*B*x
          = 2:  A*B*x = (lambda)*x
          = 3:  B*A*x = (lambda)*x

JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.
AP (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  matrix  A,  packed  columnwise in a linear
          array.  The j-th column of  A  is  stored  in  the
          array  AP  as  follows:  if UPLO = 'U', AP(i + (j-
          1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L',  AP(i
          + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

          On exit, the contents of AP are destroyed.

BP (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  matrix  B,  packed  columnwise in a linear
          array.  The j-th column of  B  is  stored  in  the
          array  BP  as  follows:  if UPLO = 'U', BP(i + (j-
          1)*j/2) = B(i,j) for 1<=i<=j; if UPLO = 'L',  BP(i
          + (j-1)*(2*n-j)/2) = B(i,j) for j<=i<=n.

          On exit, the triangular factor U  or  L  from  the
          Cholesky  factorization  B = U**H*U or B = L*L**H,
          in the same storage format as B.

W (output)
          If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0,  Z  contains  the
          matrix  Z  of  eigenvectors.  The eigenvectors are
          normalized  as  follows:   if  ITYPE  =  1  or  2,
          Z**H*B*Z = I; if ITYPE = 3, Z**H*inv(B)*Z = I.  If
          JOBZ = 'N', then Z is not referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
```

```
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The  dimension  of  array  WORK.   If  N   <=   1,
          LWORK  >= 1.  If JOBZ = 'N' and N > 1, LWORK >= N.
          If JOBZ = 'V' and N > 1, LWORK >= 2*N.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

RWORK (workspace)
          On exit, if INFO = 0, RWORK(1) returns the optimal
          LRWORK.

LRWORK (input)
          The  dimension  of  array  RWORK.   If  N  <=   1,
          LRWORK  >=  1.  If JOBZ = 'N' and N > 1, LRWORK >=
          N.  If JOBZ = 'V' and N > 1, LRWORK >= 1 +  5*N  +
          2*N**2.

          If LRWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the RWORK array, returns this value as  the  first
          entry  of  the  RWORK  array, and no error message
          related to LRWORK is issued by XERBLA.

IWORK (workspace/output)
          On exit, if INFO = 0, IWORK(1) returns the optimal
          LIWORK.

LIWORK (input)
          The dimension of array IWORK.  If JOBZ  = 'N' or N
          <=  1,  LIWORK  >=  1.   If JOBZ  = 'V' and N > 1,
          LIWORK >= 3 + 5*N.

          If LIWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the IWORK array, returns this value as  the  first
          entry  of  the  IWORK  array, and no error message
          related to LIWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  CPPTRF or CHPEVD returned an error code:
```

<= N: if INFO = i, CHPEVD failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not convergeto zero; > N: if INFO = N + i, for 1 <= i <= n, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
Mark Fahey, Department of Mathematics, Univ. of Kentucky, USA

Contents

NAME

```
chpgvx  -  compute  selected  eigenvalues  and,  optionally,
eigenvectors  of  a  complex  generalized Hermitian-definite
eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x,
or B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE CHPGVX(ITYPE, JOBZ, RANGE, UPLO, N, AP, BP, VL, VU, IL,
      IU, ABSTOL, M, W, Z, LDZ, WORK, RWORK, IWORK, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
COMPLEX AP(*), BP(*), Z(LDZ,*), WORK(*)
INTEGER ITYPE, N, IL, IU, M, LDZ, INFO
INTEGER IWORK(*), IFAIL(*)
REAL VL, VU, ABSTOL
REAL W(*), RWORK(*)

SUBROUTINE CHPGVX_64(ITYPE, JOBZ, RANGE, UPLO, N, AP, BP, VL, VU, IL,
      IU, ABSTOL, M, W, Z, LDZ, WORK, RWORK, IWORK, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
COMPLEX AP(*), BP(*), Z(LDZ,*), WORK(*)
INTEGER*8 ITYPE, N, IL, IU, M, LDZ, INFO
INTEGER*8 IWORK(*), IFAIL(*)
REAL VL, VU, ABSTOL
REAL W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HPGVX(ITYPE, JOBZ, RANGE, UPLO, N, AP, BP, VL, VU, IL,
       IU, ABSTOL, M, W, Z, [LDZ], [WORK], [RWORK], [IWORK], IFAIL,
       [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX, DIMENSION(:) :: AP, BP, WORK
COMPLEX, DIMENSION(:,:) :: Z
INTEGER :: ITYPE, N, IL, IU, M, LDZ, INFO
INTEGER, DIMENSION(:) :: IWORK, IFAIL
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: W, RWORK

SUBROUTINE HPGVX_64(ITYPE, JOBZ, RANGE, UPLO, N, AP, BP, VL, VU,
       IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK], [RWORK], [IWORK], IFAIL,
       [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX, DIMENSION(:) :: AP, BP, WORK
COMPLEX, DIMENSION(:,:) :: Z
INTEGER(8) :: ITYPE, N, IL, IU, M, LDZ, INFO
INTEGER(8), DIMENSION(:) :: IWORK, IFAIL
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void chpgvx(int itype, char jobz, char range, char uplo, int
          n,  complex  *ap, complex *bp, float vl, float vu,
          int il, int iu, float abstol, int  *m,  float  *w,
          complex *z, int ldz, int *ifail, int *info);

void chpgvx_64(long itype, char jobz, char range, char uplo,
          long  n, complex *ap, complex *bp, float vl, float
          vu, long il, long iu, float abstol, long *m, float
          *w,  complex  *z,  long  ldz,  long  *ifail,  long
          *info);
```

PURPOSE

chpgvx computes selected eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be Hermitian, stored in packed format, and B is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

```
ITYPE (input)
          Specifies the problem type to be solved:
          = 1:  A*x = (lambda)*B*x
          = 2:  A*B*x = (lambda)*x
          = 3:  B*A*x = (lambda)*x

JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

RANGE (input)
          = 'A': all eigenvalues will be found;
          = 'V': all eigenvalues in the  half-open  interval
          (VL,VU]  will  be  found; = 'I': the IL-th through
          IU-th eigenvalues will be found.
UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.

AP (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  matrix  A,  packed  columnwise in a linear
          array.  The j-th column of  A  is  stored  in  the
          array  AP  as  follows:  if UPLO = 'U', AP(i + (j-
          1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L',  AP(i
          + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

          On exit, the contents of AP are destroyed.

BP (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  matrix  B,  packed  columnwise in a linear
          array.  The j-th column of  B  is  stored  in  the
          array  BP  as  follows:  if UPLO = 'U', BP(i + (j-
          1)*j/2) = B(i,j) for 1<=i<=j; if UPLO = 'L',  BP(i
          + (j-1)*(2*n-j)/2) = B(i,j) for j<=i<=n.

          On exit, the triangular factor U  or  L  from  the
          Cholesky  factorization  B = U**H*U or B = L*L**H,
          in the same storage format as B.

VL (input)
          If RANGE='V', the lower and upper  bounds  of  the
          interval  to be searched for eigenvalues. VL < VU.
          Not referenced if RANGE = 'A' or 'I'.

VU (input)
```

If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

ABSTOL (input)
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABSTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABSTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing AP to tri-diagonal form.

Eigenvalues will be computed most accurately when ABSTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABSTOL to 2*SLAMCH('S').

M (output)
The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)
On normal exit, the first M elements contain the selected eigenvalues in ascending order.

Z (input) If JOBZ = 'N', then Z is not referenced. If JOBZ = 'V', then if INFO = 0, the first M columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). The eigenvectors are normalized as follows: if ITYPE = 1 or 2, Z**H*B*Z = I; if ITYPE = 3, Z**H*inv(B)*Z = I.

If an eigenvector fails to converge, then that column of Z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in IFAIL. Note: the user must ensure that at least max(1,M) columns are supplied in the array Z; if RANGE = 'V', the exact value of M is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
dimension(2*N)

RWORK (workspace)
dimension(7*N)

IWORK (workspace)
dimension(5*N)

IFAIL (output)
If JOBZ = 'V', then if INFO = 0, the first M elements of IFAIL are zero. If INFO > 0, then IFAIL contains the indices of the eigenvectors that failed to converge. If JOBZ = 'N', then IFAIL is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: CPPTRF or CHPEVX returned an error code:
<= N: if INFO = i, CHPEVX failed to converge; i eigenvectors failed to converge. Their indices are stored in array IFAIL. > N: if INFO = N + i, for 1 <= i <= n, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by

Mark Fahey, Department of Mathematics, Univ. of Kentucky,
USA

Contents

NAME

```
chpmv - perform the matrix-vector operation   y := alpha*A*x
+ beta*y
```

SYNOPSIS

```
SUBROUTINE CHPMV(UPLO, N, ALPHA, A, X, INCX, BETA, Y, INCY)

CHARACTER * 1 UPLO
COMPLEX ALPHA, BETA
COMPLEX A(*), X(*), Y(*)
INTEGER N, INCX, INCY

SUBROUTINE CHPMV_64(UPLO, N, ALPHA, A, X, INCX, BETA, Y, INCY)

CHARACTER * 1 UPLO
COMPLEX ALPHA, BETA
COMPLEX A(*), X(*), Y(*)
INTEGER*8 N, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE HPMV(UPLO, [N], ALPHA, A, X, [INCX], BETA, Y, [INCY])

CHARACTER(LEN=1) :: UPLO
COMPLEX :: ALPHA, BETA
COMPLEX, DIMENSION(:) :: A, X, Y
INTEGER :: N, INCX, INCY

SUBROUTINE HPMV_64(UPLO, [N], ALPHA, A, X, [INCX], BETA, Y, [INCY])

CHARACTER(LEN=1) :: UPLO
COMPLEX :: ALPHA, BETA
COMPLEX, DIMENSION(:) :: A, X, Y
```

```
INTEGER(8) :: N, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>

void chpmv(char uplo, int n,  complex  *alpha,  complex  *a,
          complex  *x,  int incx, complex *beta, complex *y,
          int incy);

void chpmv_64(char uplo, long n, complex *alpha, complex *a,
          complex  *x, long incx, complex *beta, complex *y,
          long incy);
```

PURPOSE

chpmv performs the matrix-vector operation y := alpha*A*x + beta*y where alpha and beta are scalars, x and y are n element vectors and A is an n by n hermitian matrix, supplied in packed form.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the matrix A is supplied in the packed array A as follows:

UPLO = 'U' or 'u'   The upper triangular part of A is supplied in A.

UPLO = 'L' or 'l'   The lower triangular part of A is supplied in A.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
((n*(n + 1))/2). Before entry with UPLO = 'U' or 'u', the array A must contain the upper triangular part of the hermitian matrix packed

sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(1, 2) and a(2, 2) respectively, and so on. Before entry with UPLO = 'L' or 'l', the array A must contain the lower triangular part of the hermitian matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(2, 1) and a(3, 1) respectively, and so on. Note that the imaginary parts of the diagonal elements need not be set and are assumed to be zero. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. When BETA is supplied as zero then Y need not be set on input. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

chpr - perform the hermitian rank 1 operation A := alpha*x*conjg(x') + A

SYNOPSIS

```
SUBROUTINE CHPR(UPLO, N, ALPHA, X, INCX, A)

CHARACTER * 1 UPLO
COMPLEX X(*), A(*)
INTEGER N, INCX
REAL ALPHA

SUBROUTINE CHPR_64(UPLO, N, ALPHA, X, INCX, A)

CHARACTER * 1 UPLO
COMPLEX X(*), A(*)
INTEGER*8 N, INCX
REAL ALPHA
```

F95 INTERFACE

```
SUBROUTINE HPR(UPLO, [N], ALPHA, X, [INCX], A)

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: X, A
INTEGER :: N, INCX
REAL :: ALPHA

SUBROUTINE HPR_64(UPLO, [N], ALPHA, X, [INCX], A)

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: X, A
INTEGER(8) :: N, INCX
```

```
REAL :: ALPHA
```

C INTERFACE

```
#include <sunperf.h>

void chpr(char uplo, int n, float  alpha,  complex  *x,  int
          incx, complex *a);

void chpr_64(char uplo, long n,  float  alpha,  complex  *x,
          long incx, complex *a);
```

PURPOSE

chpr performs the hermitian rank 1 operation A := alpha*x*conjg(x') + A where alpha is a real scalar, x is an n element vector and A is an n by n hermitian matrix, supplied in packed form.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the matrix A is supplied in the packed array A as follows:

UPLO = 'U' or 'u' The upper triangular part of A is supplied in A.

UPLO = 'L' or 'l' The lower triangular part of A is supplied in A.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)

On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

A (input/output)

((n*(n + 1))/2). Before entry with UPLO = 'U' or 'u', the array A must contain the upper triangular part of the hermitian matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(1, 2) and a(2, 2) respectively, and so on. On exit, the array A is overwritten by the upper triangular part of the updated matrix. Before entry with UPLO = 'L' or 'l', the array A must contain the lower triangular part of the hermitian matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(2, 1) and a(3, 1) respectively, and so on. On exit, the array A is overwritten by the lower triangular part of the updated matrix. Note that the imaginary parts of the diagonal elements need not be set, they are assumed to be zero, and on exit they are set to zero.

Contents

NAME

```
chpr2 - perform  the  Hermitian  rank  2  operation    A  :=
alpha*x*conjg( y' ) + conjg( alpha )*y*conjg( x' ) + A
```

SYNOPSIS

```
SUBROUTINE CHPR2(UPLO, N, ALPHA, X, INCX, Y, INCY, AP)

CHARACTER * 1 UPLO
COMPLEX ALPHA
COMPLEX X(*), Y(*), AP(*)
INTEGER N, INCX, INCY

SUBROUTINE CHPR2_64(UPLO, N, ALPHA, X, INCX, Y, INCY, AP)

CHARACTER * 1 UPLO
COMPLEX ALPHA
COMPLEX X(*), Y(*), AP(*)
INTEGER*8 N, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE HPR2(UPLO, [N], ALPHA, X, [INCX], Y, [INCY], AP)

CHARACTER(LEN=1) :: UPLO
COMPLEX :: ALPHA
COMPLEX, DIMENSION(:) :: X, Y, AP
INTEGER :: N, INCX, INCY

SUBROUTINE HPR2_64(UPLO, [N], ALPHA, X, [INCX], Y, [INCY], AP)

CHARACTER(LEN=1) :: UPLO
COMPLEX :: ALPHA
COMPLEX, DIMENSION(:) :: X, Y, AP
```

```
INTEGER(8) :: N, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>

void chpr2(char uplo, int n, complex *alpha, complex *x, int
          incx, complex *y, int incy, complex *ap);

void chpr2_64(char uplo, long n, complex *alpha, complex *x,
          long incx, complex *y, long incy, complex *ap);
```

PURPOSE

chpr2 performs the Hermitian rank 2 operation A := alpha*x*conjg(y') + conjg(alpha)*y*conjg(x') + A where alpha is a scalar, x and y are n element vectors and A is an n by n hermitian matrix, supplied in packed form.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the matrix A is supplied in the packed array AP as follows:

UPLO = 'U' or 'u' The upper triangular part of A is supplied in AP.

UPLO = 'L' or 'l' The lower triangular part of A is supplied in AP.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)

On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

Y (input)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. Unchanged on exit.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

AP (input/output)

((n*(n + 1))/2). Before entry with UPLO = 'U' or 'u', the array AP must contain the upper triangular part of the hermitian matrix packed sequentially, column by column, so that AP(1) contains a(1, 1), AP(2) and AP(3) contain a(1, 2) and a(2, 2) respectively, and so on. On exit, the array AP is overwritten by the upper triangular part of the updated matrix. Before entry with UPLO = 'L' or 'l', the array AP must contain the lower triangular part of the hermitian matrix packed sequentially, column by column, so that AP(1) contains a(1, 1), AP(2) and AP(3) contain a(2, 1) and a(3, 1) respectively, and so on. On exit, the array AP is overwritten by the lower triangular part of the updated matrix. Note that the imaginary parts of the diagonal elements need not be set, they are assumed to be zero, and on exit they are set to zero.

Contents

NAME

```
chprfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is Hermitian indefin-
ite and packed, and provides error bounds and backward error
estimates for the solution
```

SYNOPSIS

```
SUBROUTINE CHPRFS(UPLO, N, NRHS, A, AF, IPIVOT, B, LDB, X, LDX, FERR,
      BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*)
REAL FERR(*), BERR(*), WORK2(*)

SUBROUTINE CHPRFS_64(UPLO, N, NRHS, A, AF, IPIVOT, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
REAL FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HPRFS(UPLO, N, [NRHS], A, AF, IPIVOT, B, [LDB], X, [LDX],
       FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A, AF, WORK
COMPLEX, DIMENSION(:,:) :: B, X
```

```
    INTEGER :: N, NRHS, LDB, LDX, INFO
    INTEGER, DIMENSION(:) :: IPIVOT
    REAL, DIMENSION(:) :: FERR, BERR, WORK2

    SUBROUTINE HPRFS_64(UPLO, N, [NRHS], A, AF, IPIVOT, B, [LDB], X,
           [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

    CHARACTER(LEN=1) :: UPLO
    COMPLEX, DIMENSION(:) :: A, AF, WORK
    COMPLEX, DIMENSION(:,:) :: B, X
    INTEGER(8) :: N, NRHS, LDB, LDX, INFO
    INTEGER(8), DIMENSION(:) :: IPIVOT
    REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
    #include <sunperf.h>

    void chprfs(char uplo, int n, int nrhs, complex *a,  complex
              *af, int *ipivot, complex *b, int ldb, complex *x,
              int ldx, float *ferr, float *berr, int *info);

    void chprfs_64(char uplo, long n,  long  nrhs,  complex  *a,
              complex  *af,  long *ipivot, complex *b, long ldb,
              complex *x, long ldx, float  *ferr,  float  *berr,
              long *info);
```

PURPOSE

chprfs improves the computed solution to a system of linear equations when the coefficient matrix is Hermitian indefinite and packed, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

```
    UPLO (input)
              = 'U':  Upper triangle of A is stored;
              = 'L':  Lower triangle of A is stored.

    N (input) The order of the matrix A.  N >= 0.

    NRHS (input)
              The number of right hand sides, i.e.,  the  number
              of columns of the matrices B and X.  NRHS >= 0.

    A (input) The upper  or  lower  triangle  of  the  Hermitian
              matrix  A,  packed  columnwise  in a linear array.
              The j-th column of A is stored in the array  A  as
```

follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

AF (input)
The factored form of the matrix A. AF contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**H or A = L*D*L**H as computed by CHPTRF, stored as a packed triangular matrix.

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CHPTRF.

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by CHPTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)

```
            dimension(N)

INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value
```

Contents

NAME

```
chpsv - compute the solution to a complex system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE CHPSV(UPLO, N, NRHS, A, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE CHPSV_64(UPLO, N, NRHS, A, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE HPSV(UPLO, N, [NRHS], A, IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A
COMPLEX, DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE HPSV_64(UPLO, N, [NRHS], A, IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX, DIMENSION(:) :: A
COMPLEX, DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void chpsv(char uplo, int  n,  int  nrhs,  complex  *a,  int
          *ipivot, complex *b, int ldb, int *info);

void chpsv_64(char uplo, long n, long nrhs, complex *a, long
          *ipivot, complex *b, long ldb, long *info);
```

PURPOSE

chpsv computes the solution to a complex system of linear equations
A * X = B, where A is an N-by-N Hermitian matrix stored in packed format and X and B are N-by-NRHS matrices.

The diagonal pivoting method is used to factor A as
A = U * D * U**H, if UPLO = 'U', or
A = L * D * L**H, if UPLO = 'L',
where U (or L) is a product of permutation and unit upper (lower) triangular matrices, D is Hermitian and block diagonal with 1-by-1 and 2-by-2 diagonal blocks. The factored form of A is then used to solve the system of equations A * X = B.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input/output)
On entry, the upper or lower triangle of the Hermitian matrix A, packed columnwise in a linear array. The j-th column of A is stored in the

array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n. See below for further details.

On exit, the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**H or A = L*D*L**H as computed by CHPTRF, stored as a packed triangular matrix in the same storage format as A.

IPIVOT (output)
Details of the interchanges and the block structure of D, as determined by CHPTRF. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged, and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, so the solution could not be computed.

FURTHER DETAILS

The packed storage scheme is illustrated by the following example when N = 4, UPLO = 'U':

Two-dimensional storage of the Hermitian matrix A:

```
   a11 a12 a13 a14
```

```
       a22 a23 a24
           a33 a34     (aij = conjg(aji))
               a44
```

Packed storage of the upper triangle of A:

```
A = [ a11, a12, a22, a13, a23, a33, a14, a24, a34, a44 ]
```

Contents

NAME

```
chpsvx  -  use  the  diagonal  pivoting  factorization  A  =
U*D*U**H  or  A = L*D*L**H to compute the solution to a com-
plex system of linear equations A * X = B, where A is an  N-
by-N  Hermitian  matrix  stored in packed format and X and B
are N-by-NRHS matrices
```

SYNOPSIS

```
SUBROUTINE CHPSVX(FACT, UPLO, N, NRHS, A, AF, IPIVOT, B, LDB, X, LDX,
      RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*)
REAL RCOND
REAL FERR(*), BERR(*), WORK2(*)

SUBROUTINE CHPSVX_64(FACT, UPLO, N, NRHS, A, AF, IPIVOT, B, LDB, X,
      LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
REAL RCOND
REAL FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HPSVX(FACT, UPLO, N, [NRHS], A, AF, IPIVOT, B, [LDB], X,
       [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: FACT, UPLO
COMPLEX, DIMENSION(:) :: A, AF, WORK
COMPLEX, DIMENSION(:,:) :: B, X
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL :: RCOND
REAL, DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE HPSVX_64(FACT, UPLO, N, [NRHS], A, AF, IPIVOT, B, [LDB],
       X, [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO
COMPLEX, DIMENSION(:) :: A, AF, WORK
COMPLEX, DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL :: RCOND
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void chpsvx(char fact, char uplo, int n, int  nrhs,  complex
          *a, complex *af, int *ipivot, complex *b, int ldb,
          complex *x, int ldx, float  *rcond,  float  *ferr,
          float *berr, int *info);

void chpsvx_64(char fact, char uplo, long n, long nrhs, com-
          plex  *a,  complex  *af, long *ipivot, complex *b,
          long ldb, complex  *x,  long  ldx,  float  *rcond,
          float *ferr, float *berr, long *info);
```

PURPOSE

chpsvx uses the diagonal pivoting factorization A = U*D*U**H or A = L*D*L**H to compute the solution to a complex system of linear equations A * X = B, where A is an N-by-N Hermitian matrix stored in packed format and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'N', the diagonal pivoting method is used to factor A as

```
      A = U * D * U**H,  if UPLO = 'U', or
```

```
      A = L * D * L**H,  if UPLO = 'L',
   where U (or L) is a product of permutation and unit upper
(lower)
   triangular matrices and D is Hermitian and block diagonal
with
   1-by-1 and 2-by-2 diagonal blocks.

2. If some D(i,i)=0, so that D is exactly singular, then the
routine
   returns with INFO = i. Otherwise, the factored form of  A
is used
   to estimate the condition number of the matrix A.  If the
   reciprocal of the condition number is less  than  machine
precision,
   INFO = N+1 is returned as  a  warning,  but  the  routine
still goes on
   to solve for X and  compute  error  bounds  as  described
below.
3. The system of equations is solved for X  using  the  fac-
tored form
   of A.

4. Iterative refinement is applied to improve  the  computed
solution
   matrix and calculate  error  bounds  and  backward  error
estimates
   for it.
```

ARGUMENTS

```
FACT (input)
          Specifies whether or not the factored  form  of  A
          has  been supplied on entry.  = 'F':  On entry, AF
          and IPIVOT contain the factored form of A.  AF and
          IPIVOT will not be modified.  = 'N':  The matrix A
          will be copied to AF and factored.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.
```

A (input) The upper or lower triangle of the Hermitian matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n. See below for further details.

AF (input/output)
If FACT = 'F', then AF is an input argument and on entry contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**H or A = L*D*L**H as computed by CHPTRF, stored as a packed triangular matrix in the same storage format as A.
If FACT = 'N', then AF is an output argument and on exit contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**H or A = L*D*L**H as computed by CHPTRF, stored as a packed triangular matrix in the same storage format as A.

IPIVOT (input or output)
If FACT = 'F', then IPIVOT is an input argument and on entry contains details of the interchanges and the block structure of D, as determined by CHPTRF. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

If FACT = 'N', then IPIVOT is an output argument and on exit contains details of the interchanges and the block structure of D, as determined by CHPTRF.

B (input) The N-by-NRHS right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X.

```
LDX (input)
          The leading dimension of  the  array  X.   LDX  >=
          max(1,N).

RCOND (output)
          The estimate of the reciprocal condition number of
          the  matrix  A.  If RCOND is less than the machine
          precision (in  particular,  if  RCOND  =  0),  the
          matrix  is  singular  to  working precision.  This
          condition is indicated by a return code of INFO  >
          0.

FERR (output)
          The estimated forward error bound for  each  solu-
          tion  vector X(j) (the j-th column of the solution
          matrix  X).   If  XTRUE  is  the   true   solution
          corresponding  to  X(j),  FERR(j)  is an estimated
          upper bound for the magnitude of the largest  ele-
          ment in (X(j) - XTRUE) divided by the magnitude of
          the largest element in X(j).  The estimate  is  as
          reliable  as the estimate for RCOND, and is almost
          always a slight overestimate of the true error.

BERR (output)
          The componentwise relative backward error of  each
          solution  vector X(j) (i.e., the smallest relative
          change in any element of A or B that makes X(j) an
          exact solution).

WORK (workspace)
          dimension(2*N)

WORK2 (workspace)
          dimension(N)

INFO (output)
          = 0: successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
          gal value
          > 0:  if INFO = i, and i is
          <= N:  D(i,i) is exactly zero.  The  factorization
          has  been  completed  but  the factor D is exactly
          singular, so the solution and error  bounds  could
          not  be computed. RCOND = 0 is returned.  = N+1: D
          is nonsingular, but RCOND  is  less  than  machine
          precision,  meaning that the matrix is singular to
          working precision.  Nevertheless, the solution and
          error  bounds  are  computed  because  there are a
          number of situations where the  computed  solution
```

can be more accurate than the value of RCOND would suggest.

FURTHER DETAILS

The packed storage scheme is illustrated by the following example when N = 4, UPLO = 'U':

Two-dimensional storage of the Hermitian matrix A:

```
   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = conjg(aji))
               a44
```

Packed storage of the upper triangle of A:

A = [a11, a12, a22, a13, a23, a33, a14, a24, a34, a44]

Contents

NAME

```
chptrd - reduce a  complex  Hermitian  matrix  A  stored  in
packed  form  to real symmetric tridiagonal form T by a uni-
tary similarity transformation
```

SYNOPSIS

```
SUBROUTINE CHPTRD(UPLO, N, AP, D, E, TAU, INFO)

CHARACTER * 1 UPLO
COMPLEX AP(*), TAU(*)
INTEGER N, INFO
REAL D(*), E(*)

SUBROUTINE CHPTRD_64(UPLO, N, AP, D, E, TAU, INFO)

CHARACTER * 1 UPLO
COMPLEX AP(*), TAU(*)
INTEGER*8 N, INFO
REAL D(*), E(*)
```

F95 INTERFACE

```
SUBROUTINE HPTRD(UPLO, N, AP, D, E, TAU, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: AP, TAU
INTEGER :: N, INFO
REAL, DIMENSION(:) :: D, E

SUBROUTINE HPTRD_64(UPLO, N, AP, D, E, TAU, [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX, DIMENSION(:) :: AP, TAU
INTEGER(8) :: N, INFO
REAL, DIMENSION(:) :: D, E
```

C INTERFACE

```
#include <sunperf.h>

void chptrd(char uplo, int n, complex *ap, float  *d,  float
          *e, complex *tau, int *info);

void chptrd_64(char uplo, long n,  complex  *ap,  float  *d,
          float *e, complex *tau, long *info);
```

PURPOSE

chptrd reduces a complex Hermitian matrix A stored in packed form to real symmetric tridiagonal form T by a unitary similarity transformation: Q**H * A * Q = T.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

AP (input)
On entry, the upper or lower triangle of the Hermitian matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n. On exit, if UPLO = 'U', the diagonal and first superdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements above the first superdiagonal, with the array TAU, represent the unitary matrix Q as a product of elementary reflectors; if UPLO = 'L', the diagonal and first subdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements below the first subdiagonal, with the array TAU, represent the unitary matrix Q as a product of elementary reflectors. See Further Details.

```
D (output)
          The diagonal elements of the tridiagonal matrix T:
          D(i) = A(i,i).

E (output)
          The  off-diagonal  elements  of  the   tridiagonal
          matrix  T:   E(i) = A(i,i+1) if UPLO = 'U', E(i) =
          A(i+1,i) if UPLO = 'L'.

TAU (output)
          The scalar factors of  the  elementary  reflectors
          (see Further Details).

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

FURTHER DETAILS

```
If UPLO = 'U', the matrix Q is represented as a  product  of
elementary reflectors

   Q = H(n-1) . . . H(2) H(1).

Each H(i) has the form

   H(i) = I - tau * v * v'

where tau is a complex scalar, and v  is  a  complex  vector
with  v(i+1:n)  = 0 and v(i) = 1; v(1:i-1) is stored on exit
in AP,  overwriting  A(1:i-1,i+1),  and  tau  is  stored  in
TAU(i).

If UPLO = 'L', the matrix Q is represented as a  product  of
elementary reflectors

   Q = H(1) H(2) . . . H(n-1).

Each H(i) has the form

   H(i) = I - tau * v * v'

where tau is a complex scalar, and v  is  a  complex  vector
with  v(1:i)  = 0 and v(i+1) = 1; v(i+2:n) is stored on exit
in AP, overwriting A(i+2:n,i), and tau is stored in TAU(i).
```

Contents

NAME

```
chptrf - compute the factorization of  a  complex  Hermitian
packed  matrix  A  using the Bunch-Kaufman diagonal pivoting
method
```

SYNOPSIS

```
SUBROUTINE CHPTRF(UPLO, N, A, IPIVOT, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*)
INTEGER N, INFO
INTEGER IPIVOT(*)

SUBROUTINE CHPTRF_64(UPLO, N, A, IPIVOT, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*)
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE HPTRF(UPLO, N, A, IPIVOT, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE HPTRF_64(UPLO, N, A, IPIVOT, [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX, DIMENSION(:) :: A
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void chptrf(char uplo, int n, complex *a, int  *ipivot,  int
          *info);

void chptrf_64(char uplo, long n, complex *a, long  *ipivot,
          long *info);
```

PURPOSE

chptrf computes the factorization of a complex Hermitian packed matrix A using the Bunch-Kaufman diagonal pivoting method:

```
A = U*D*U**H  or  A = L*D*L**H
```

where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is Hermitian and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  matrix  A,  packed  columnwise in a linear
          array.  The j-th column of  A  is  stored  in  the
          array  A  as  follows:   if  UPLO = 'U', A(i + (j-
          1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i +
          (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n.

          On exit, the block diagonal matrix D and the  mul-
          tipliers  used to obtain the factor U or L, stored
          as a packed triangular matrix overwriting  A  (see
          below for further details).

IPIVOT (output)
```

```
          Details of the interchanges and the  block  struc-
          ture  of  D.   If  IPIVOT(k)  >  0,  then rows and
          columns k  and  IPIVOT(k)  were  interchanged  and
          D(k,k)  is a 1-by-1 diagonal block.  If UPLO = 'U'
          and IPIVOT(k) = IPIVOT(k-1) <  0,  then  rows  and
          columns  k-1  and -IPIVOT(k) were interchanged and
          D(k-1:k,k-1:k) is a  2-by-2  diagonal  block.   If
          UPLO  =  'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then
          rows and columns k+1 and  -IPIVOT(k)  were  inter-
          changed  and  D(k:k+1,k:k+1)  is a 2-by-2 diagonal
          block.

INFO (output)
          = 0: successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
          gal value
          > 0: if INFO = i, D(i,i)  is  exactly  zero.   The
          factorization  has  been  completed, but the block
          diagonal matrix D is exactly singular,  and  divi-
          sion  by  zero will occur if it is used to solve a
          system of equations.
```

FURTHER DETAILS

```
5-96 - Based on modifications by J. Lewis,  Boeing  Computer
Services
       Company

If UPLO = 'U', then A = U*D*U', where
   U = P(n)*U(n)* ... *P(k)U(k)* ...,
i.e., U is a product of terms P(k)*U(k), where  k  decreases
from  n  to  1 in steps of 1 or 2, and D is a block diagonal
matrix with 1-by-1 and 2-by-2 diagonal blocks D(k).  P(k) is
a  permutation matrix as defined by IPIVOT(k), and U(k) is a
unit upper triangular matrix,  such  that  if  the  diagonal
block D(k) is of order s (s = 1 or 2), then

           (   I    v    0   )   k-s
   U(k) =  (   0    I    0   )   s
           (   0    0    I   )   n-k
              k-s   s   n-k

If s = 1, D(k) overwrites A(k,k), and  v  overwrites  A(1:k-
1,k).   If s = 2, the upper triangle of D(k) overwrites A(k-
1,k-1), A(k-1,k), and A(k,k), and  v  overwrites  A(1:k-2,k-
1:k).

If UPLO = 'L', then A = L*D*L', where
   L = P(1)*L(1)* ... *P(k)*L(k)* ...,
i.e., L is a product of terms P(k)*L(k), where  k  increases
```

from 1 to n in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIVOT(k), and L(k) is a unit lower triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    0     0   )  k-1
   L(k) =  (   0    I     0   )  s
           (   0    v     I   )  n-k-s+1
              k-1   s  n-k-s+1
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(k+1:n,k). If s = 2, the lower triangle of D(k) overwrites A(k,k), A(k+1,k), and A(k+1,k+1), and v overwrites A(k+2:n,k:k+1).

Contents

NAME

```
chptri - compute the inverse of a complex Hermitian indefin-
ite  matrix  A in packed storage using the factorization A =
U*D*U**H or A = L*D*L**H computed by CHPTRF
```

SYNOPSIS

```
SUBROUTINE CHPTRI(UPLO, N, A, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), WORK(*)
INTEGER N, INFO
INTEGER IPIVOT(*)

SUBROUTINE CHPTRI_64(UPLO, N, A, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), WORK(*)
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE HPTRI(UPLO, N, A, IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A, WORK
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE HPTRI_64(UPLO, N, A, IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A, WORK
```

```
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void chptri(char uplo, int n, complex *a, int  *ipivot,  int
          *info);

void chptri_64(char uplo, long n, complex *a, long  *ipivot,
          long *info);
```

PURPOSE

chptri computes the inverse of a complex Hermitian indefinite matrix A in packed storage using the factorization A = U*D*U**H or A = L*D*L**H computed by CHPTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**H;
= 'L': Lower triangular, form is A = L*D*L**H.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CHPTRF, stored as a packed triangular matrix.

On exit, if INFO = 0, the (Hermitian) inverse of the original matrix, stored as a packed triangular matrix. The j-th column of inv(A) is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = inv(A)(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = inv(A)(i,j) for j<=i<=n.

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CHPTRF.

WORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) = 0; the matrix is singular and its inverse could not be computed.

Contents

NAME

```
chptrs - solve a system of linear equations A*X = B  with  a
complex Hermitian matrix A stored in packed format using the
factorization A = U*D*U**H  or  A  =  L*D*L**H  computed  by
CHPTRF
```

SYNOPSIS

```
SUBROUTINE CHPTRS(UPLO, N, NRHS, A, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE CHPTRS_64(UPLO, N, NRHS, A, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE HPTRS(UPLO, N, [NRHS], A, IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A
COMPLEX, DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE HPTRS_64(UPLO, N, [NRHS], A, IPIVOT, B, [LDB], [INFO])
```

```
CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A
COMPLEX, DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void chptrs(char uplo, int n,  int  nrhs,  complex  *a,  int
          *ipivot, complex *b, int ldb, int *info);

void chptrs_64(char uplo, long n,  long  nrhs,  complex  *a,
          long *ipivot, complex *b, long ldb, long *info);
```

PURPOSE

chptrs solves a system of linear equations A*X = B with a complex Hermitian matrix A stored in packed format using the factorization A = U*D*U**H or A = L*D*L**H computed by CHPTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**H;
= 'L': Lower triangular, form is A = L*D*L**H.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CHPTRF, stored as a packed triangular matrix.

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CHPTRF.

B (input/output)
On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
chsein - use  inverse  iteration  to  find  specified  right
and/or  left  eigenvectors  of  a  complex  upper Hessenberg
matrix H
```

SYNOPSIS

```
SUBROUTINE CHSEIN(SIDE, EIGSRC, INITV, SELECT, N, H, LDH, W, VL,
      LDVL, VR, LDVR, MM, M, WORK, RWORK, IFAILL, IFAILR, INFO)

CHARACTER * 1 SIDE, EIGSRC, INITV
COMPLEX H(LDH,*), W(*), VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER N, LDH, LDVL, LDVR, MM, M, INFO
INTEGER IFAILL(*), IFAILR(*)
LOGICAL SELECT(*)
REAL RWORK(*)

SUBROUTINE CHSEIN_64(SIDE, EIGSRC, INITV, SELECT, N, H, LDH, W, VL,
      LDVL, VR, LDVR, MM, M, WORK, RWORK, IFAILL, IFAILR, INFO)

CHARACTER * 1 SIDE, EIGSRC, INITV
COMPLEX H(LDH,*), W(*), VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER*8 N, LDH, LDVL, LDVR, MM, M, INFO
INTEGER*8 IFAILL(*), IFAILR(*)
LOGICAL*8 SELECT(*)
REAL RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HSEIN(SIDE, EIGSRC, INITV, SELECT, [N], H, [LDH], W, VL,
       [LDVL], VR, [LDVR], MM, M, [WORK], [RWORK], IFAILL, IFAILR, [INFO])

CHARACTER(LEN=1) :: SIDE, EIGSRC, INITV
```

```
COMPLEX, DIMENSION(:) :: W, WORK
COMPLEX, DIMENSION(:,:) :: H, VL, VR
INTEGER :: N, LDH, LDVL, LDVR, MM, M, INFO
INTEGER, DIMENSION(:) :: IFAILL, IFAILR
LOGICAL, DIMENSION(:) :: SELECT
REAL, DIMENSION(:) :: RWORK

SUBROUTINE HSEIN_64(SIDE, EIGSRC, INITV, SELECT, [N], H, [LDH], W,
       VL, [LDVL], VR, [LDVR], MM, M, [WORK], [RWORK], IFAILL, IFAILR,
       [INFO])

CHARACTER(LEN=1) :: SIDE, EIGSRC, INITV
COMPLEX, DIMENSION(:) :: W, WORK
COMPLEX, DIMENSION(:,:) :: H, VL, VR
INTEGER(8) :: N, LDH, LDVL, LDVR, MM, M, INFO
INTEGER(8), DIMENSION(:) :: IFAILL, IFAILR
LOGICAL(8), DIMENSION(:) :: SELECT
REAL, DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>

void chsein(char side, char eigsrc, char initv, int *select,
          int  n,  complex  *h, int ldh, complex *w, complex
          *vl, int ldvl, complex *vr, int ldvr, int mm,  int
          *m, int *ifaill, int *ifailr, int *info);

void chsein_64(char side,  char  eigsrc,  char  initv,  long
          *select, long n, complex *h, long ldh, complex *w,
          complex *vl, long ldvl, complex  *vr,  long  ldvr,
          long mm, long *m, long *ifaill, long *ifailr, long
          *info);
```

PURPOSE

chsein uses inverse iteration to find specified right and/or left eigenvectors of a complex upper Hessenberg matrix H.

The right eigenvector x and the left eigenvector y of the matrix H corresponding to an eigenvalue w are defined by:

```
H * x = w * x,     y**h * H = w * y**h
```

where y**h denotes the conjugate transpose of the vector y.

ARGUMENTS

SIDE (input)

```
          = 'R': compute right eigenvectors only;
          = 'L': compute left eigenvectors only;
          = 'B': compute both right and left eigenvectors.

EIGSRC (input)
          Specifies the source of eigenvalues supplied in W:
          = 'Q': the eigenvalues were  found  using  CHSEQR;
          thus,  if  H has zero subdiagonal elements, and so
          is block-triangular, then the j-th eigenvalue  can
          be  assumed  to be an eigenvalue of the block con-
          taining the j-th row/column.  This property allows
          CHSEIN  to  perform  inverse iteration on just one
          diagonal block.  = 'N': no assumptions are made on
          the  correspondence between eigenvalues and diago-
          nal blocks.  In this case, CHSEIN must always per-
          form inverse iteration using the whole matrix H.

INITV (input)
          = 'N': no initial vectors are supplied;
          = 'U': user-supplied initial vectors are stored in
          the arrays VL and/or VR.

SELECT (input)
          Specifies the  eigenvectors  to  be  computed.  To
          select the eigenvector corresponding to the eigen-
          value W(j), SELECT(j) must be set to .TRUE..

N (input) The order of the matrix H.  N >= 0.

H (input) The upper Hessenberg matrix H.

LDH (input)
          The leading dimension of  the  array  H.   LDH  >=
          max(1,N).

W (input/output)
          On entry, the eigenvalues of H.  On exit, the real
          parts  of  W  may  have  been  altered since close
          eigenvalues are perturbed  slightly  in  searching
          for independent eigenvectors.

VL (input/output)
          On entry, if INITV = 'U' and SIDE = 'L' or 'B', VL
          must  contain  starting  vectors  for  the inverse
          iteration for the left eigenvectors; the  starting
          vector  for  each  eigenvector must be in the same
          column in which the eigenvector  will  be  stored.
          On  exit, if SIDE = 'L' or 'B', the left eigenvec-
          tors specified by SELECT will be  stored  consecu-
          tively  in the columns of VL, in the same order as
```

their eigenvalues. If SIDE = 'R', VL is not referenced.

LDVL (input)
The leading dimension of the array VL. LDVL >= max(1,N) if SIDE = 'L' or 'B'; LDVL >= 1 otherwise.

VR (input/output)
On entry, if INITV = 'U' and SIDE = 'R' or 'B', VR must contain starting vectors for the inverse iteration for the right eigenvectors; the starting vector for each eigenvector must be in the same column in which the eigenvector will be stored. On exit, if SIDE = 'R' or 'B', the right eigenvectors specified by SELECT will be stored consecutively in the columns of VR, in the same order as their eigenvalues. If SIDE = 'L', VR is not referenced.

LDVR (input)
The leading dimension of the array VR. LDVR >= max(1,N) if SIDE = 'R' or 'B'; LDVR >= 1 otherwise.

MM (input)
The number of columns in the arrays VL and/or VR. MM >= M.

M (output)
The number of columns in the arrays VL and/or VR required to store the eigenvectors (= the number of .TRUE. elements in SELECT).

WORK (workspace)
dimension(N*N)

RWORK (workspace)
dimension(N)

IFAILL (output)
If SIDE = 'L' or 'B', IFAILL(i) = j > 0 if the left eigenvector in the i-th column of VL (corresponding to the eigenvalue w(j)) failed to converge; IFAILL(i) = 0 if the eigenvector converged satisfactorily. If SIDE = 'R', IFAILL is not referenced.

IFAILR (output)
If SIDE = 'R' or 'B', IFAILR(i) = j > 0 if the

right eigenvector in the i-th column of VR (corresponding to the eigenvalue w(j)) failed to converge; IFAILR(i) = 0 if the eigenvector converged satisfactorily. If SIDE = 'L', IFAILR is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, i is the number of eigenvectors which failed to converge; see IFAILL and IFAILR for further details.

FURTHER DETAILS

Each eigenvector is normalized so that the element of largest magnitude has magnitude 1; here the magnitude of a complex number (x,y) is taken to be |x|+|y|.

Contents

NAME

```
chseqr - compute the eigenvalues of a complex upper  Hessen-
berg  matrix  H,  and, optionally, the matrices T and Z from
the Schur decomposition H = Z T Z**H, where T  is  an  upper
triangular  matrix  (the  Schur  form), and Z is the unitary
matrix of Schur vectors
```

SYNOPSIS

```
SUBROUTINE CHSEQR(JOB, COMPZ, N, ILO, IHI, H, LDH, W, Z, LDZ, WORK,
      LWORK, INFO)

CHARACTER * 1 JOB, COMPZ
COMPLEX H(LDH,*), W(*), Z(LDZ,*), WORK(*)
INTEGER N, ILO, IHI, LDH, LDZ, LWORK, INFO

SUBROUTINE CHSEQR_64(JOB, COMPZ, N, ILO, IHI, H, LDH, W, Z, LDZ,
      WORK, LWORK, INFO)

CHARACTER * 1 JOB, COMPZ
COMPLEX H(LDH,*), W(*), Z(LDZ,*), WORK(*)
INTEGER*8 N, ILO, IHI, LDH, LDZ, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE HSEQR(JOB, COMPZ, N, ILO, IHI, H, [LDH], W, Z, [LDZ],
       [WORK], LWORK, [INFO])

CHARACTER(LEN=1) :: JOB, COMPZ
COMPLEX, DIMENSION(:) :: W, WORK
COMPLEX, DIMENSION(:,:) :: H, Z
INTEGER :: N, ILO, IHI, LDH, LDZ, LWORK, INFO

SUBROUTINE HSEQR_64(JOB, COMPZ, N, ILO, IHI, H, [LDH], W, Z, [LDZ],
```

```
          [WORK], LWORK, [INFO])

   CHARACTER(LEN=1) :: JOB, COMPZ
   COMPLEX, DIMENSION(:) :: W, WORK
   COMPLEX, DIMENSION(:,:) :: H, Z
   INTEGER(8) :: N, ILO, IHI, LDH, LDZ, LWORK, INFO
```

C INTERFACE

```
   #include <sunperf.h>

   void chseqr (char, char, int, int, int, complex*, int,  com-
             plex*, complex*, int, int*);

   void chseqr_64 (char,  char,  long,  long,  long,  complex*,
             long, complex*, complex*, long, long*);
```

PURPOSE

chseqr computes the eigenvalues of a complex upper Hessenberg matrix H, and, optionally, the matrices T and Z from the Schur decomposition H = Z T Z**H, where T is an upper triangular matrix (the Schur form), and Z is the unitary matrix of Schur vectors.

Optionally Z may be postmultiplied into an input unitary matrix Q, so that this routine can give the Schur factorization of a matrix A which has been reduced to the Hessenberg form H by the unitary matrix Q: A = Q*H*Q**H = (QZ)*T*(QZ)**H.

ARGUMENTS

JOB (input)
= 'E': compute eigenvalues only;
= 'S': compute eigenvalues and the Schur form T.

COMPZ (input)
= 'N': no Schur vectors are computed;
= 'I': Z is initialized to the unit matrix and the matrix Z of Schur vectors of H is returned; = 'V': Z must contain an unitary matrix Q on entry, and the product Q*Z is returned.

N (input) The order of the matrix H. N >= 0.

ILO (input)
It is assumed that H is already upper triangular

```
          in  rows  and columns 1:ILO-1 and IHI+1:N. ILO and
          IHI are normally set by a previous call to CGEBAL,
          and  then  passed to CGEHRD when the matrix output
          by CGEBAL is reduced to Hessenberg form. Otherwise
          ILO and IHI should be set to 1 and N respectively.
          1 <= ILO <= IHI <= N, if N > 0; ILO=1  and  IHI=0,
          if N=0.

IHI (input)
          See the description of ILO.

H (input/output)
          On entry, the upper Hessenberg matrix H.  On exit,
          if  JOB  =  'S',  H  contains the upper triangular
          matrix T from the Schur decomposition  (the  Schur
          form).  If  JOB  =  'E',  the  contents  of  H are
          unspecified on exit.

LDH (input)
          The leading dimension  of  the  array  H.  LDH  >=
          max(1,N).

W (output)
          The computed eigenvalues. If JOB = 'S', the eigen-
          values  are  stored  in  the  same order as on the
          diagonal of the Schur form  returned  in  H,  with
          W(i) = H(i,i).

Z (input) If COMPZ = 'N': Z is not referenced.
          If COMPZ = 'I': on entry, Z need not be  set,  and
          on  exit,  Z  contains the unitary matrix Z of the
          Schur vectors of H.  If COMPZ = 'V':  on  entry  Z
          must  contain an N-by-N matrix Q, which is assumed
          to be equal to the unit matrix except for the sub-
          matrix Z(ILO:IHI,ILO:IHI); on exit Z contains Q*Z.
          Normally Q is  the  unitary  matrix  generated  by
          CUNGHR  after  the call to CGEHRD which formed the
          Hessenberg matrix H.

LDZ (input)
          The leading dimension of  the  array  Z.   LDZ  >=
          max(1,N)  if  COMPZ  = 'I' or 'V'; LDZ >= 1 other-
          wise.

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (output)
          The  dimension  of  the  array  WORK.   LWORK   >=
```

max(1,N).

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, CHSEQR failed to compute all the eigenvalues in a total of 30*(IHI-ILO+1) iterations; elements 1:ilo-1 and i+1:n of W contain those eigenvalues which have been successfully computed.

Contents

NAME

cjadmm - Jagged diagonal matrix-matrix multiply (modified Ellpack)

SYNOPSIS

```
 SUBROUTINE CJADMM( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, PNTR, MAXNZ, IPERM,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, K, DESCRA(5), MAXNZ,
*           LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTR(MAXNZ+1), IPERM(M)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CJADMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, PNTR, MAXNZ, IPERM,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, K, DESCRA(5), MAXNZ,
*           LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTR(MAXNZ+1), IPERM(M)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

where NNZ=PNTR(MAXNZ+1)-PNTR(1)+1 is the number of non-zero elements
```

F95 INTERFACE

```
 SUBROUTINE JADMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*    PNTR, MAXNZ, IPERM, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, K, MAXNZ
 INTEGER, DIMENSION(:) ::  DESCRA, INDX, PNTR, IPERM
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) ::  VAL
```

```
 COMPLEX, DIMENSION(:, :) ::  B, C

 SUBROUTINE JADMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*    PNTR, MAXNZ, IPERM, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8     TRANSA, M, K, MAXNZ
 INTEGER*8, DIMENSION(:) ::  DESCRA, INDX, PNTR, IPERM
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) ::  VAL
 COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in jagged-diagonal format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
```

```
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices

VAL()          array of length NNZ consisting of entries of A.
               VAL can be viewed as a column major ordering of a
               row permutation of the Ellpack representation of A,
               where the Ellpack representation is permuted so that
               the rows are non-increasing in the number of nonzero
               entries.  Values added for padding in Ellpack are
               not included in the Jagged-Diagonal format.

INDX()         array of length NNZ consisting of the column indices
               of the corresponding entries in VAL.
PNTR()         array of length MAXNZ+1, where PNTR(I)-PNTR(1)+1
               points to the location in VAL of the first element
               in the row-permuted Ellpack represenation of A.

MAXNZ          max number of nonzeros elements per row.

IPERM()        integer array of length M such that I = IPERM(I'),
               where row I in the original Ellpack representation
               corresponds to row I' in the permuted representation.
               If IPERM(1) = 0, it is assumed by convention that
               IPERM(I) = I. IPERM is used to determine the order
               in which rows of C are updated.

B()            rectangular array with first dimension LDB.

LDB            leading dimension of B

BETA           Scalar parameter

C()            rectangular array with first dimension LDC.

LDC            leading dimension of C

WORK()         scratch array of length LWORK. WORK is not
               referenced in the current version.

LWORK          length of WORK array. LWORK is not referenced
               in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

cjadrp - right permutation of a jagged diagonal matrix

SYNOPSIS

```
 SUBROUTINE CJADRP( TRANSP, M, K, VAL, INDX, PNTR, MAXNZ,
*                   IPERM, WORK, LWORK )
 INTEGER    TRANSP, M, K, MAXNZ, LWORK
 INTEGER    INDX(*), PNTR(MAXNZ+1), IPERM(K), WORK(LWORK)
 COMPLEX    VAL(*)

 SUBROUTINE CJADRP_64( TRANSP, M, K, VAL, INDX, PNTR, MAXNZ,
*                   IPERM, WORK, LWORK )
 INTEGER*8  TRANSP, M, K, MAXNZ, LWORK
 INTEGER*8  INDX(*), PNTR(MAXNZ+1), IPERM(K), WORK(LWORK)
 COMPLEX    VAL(*)
```

F95 INTERFACE

```
 SUBROUTINE JADRP( TRANSP, M, K, VAL, INDX, PNTR, MAXNZ,
*                   IPERM, [WORK], [LWORK] )
 INTEGER TRANSP, M, K,  MAXNZ
 INTEGER, DIMENSION(:) :: INDX, PNTR, IPERM
 COMPLEX, DIMENSION(:) :: VAL

 SUBROUTINE JADRP_64( TRANSP, M, K, VAL, INDX, PNTR, MAXNZ,
*                   IPERM, [WORK], [LWORK] )
 INTEGER*8 TRANSP, M, K,  MAXNZ
 INTEGER*8, DIMENSION(:) :: INDX, PNTR, IPERM
 COMPLEX, DIMENSION(:) :: VAL
```

DESCRIPTION

A <- A P
A <- A P'
(' indicates matrix transpose)

where permutation P is represented by an integer vector IPERM, such that IPERM(I) is equal to the position of the only nonzero element in row I of permutation matrix P.

NOTE: In order to get a symetrically permuted jagged diagonal matrix P A P', one can explicitly permute the columns P A by calling

```
SJADRP(0, M, M, VAL, INDX, PNTR, MAXNZ, IPERM, WORK, LWORK)
```

where parameters VAL, INDX, PNTR, MAXNZ, IPERM are the representation of A in the jagged diagonal format. The operation makes sense if the original matrix A is square.

ARGUMENTS

TRANSP — Indicates how to operate with the permutation matrix
  0 : operate with matrix
  1 : operate with transpose matrix

M — Number of rows in matrix A

K — Number of columns in matrix A

VAL() — array of length PNTR(MAXNZ+1)-PNTR(1) consisting of entries of A. VAL can be viewed as a column major ordering of a row permutation of the Ellpack representation of A, where the Ellpack representation is permuted so that the rows are non-increasing in the number of nonzero entries. Values added for padding in Ellpack are not included in the Jagged-Diagonal format.

INDX() — array of length PNTR(MAXNZ+1)-PNTR(1) consisting of the column indices of the corresponding entries in VAL.

PNTR() — array of length MAXNZ+1, where PNTR(I)-PNTR(1)+1 points to the location in VAL of the first element in the row-permuted Ellpack represenation of A.

MAXNZ — max number of nonzeros elements per row.

IPERM() — integer array of length K such that I = IPERM(I').

```
                Array IPERM represents a permutation P, such that
                IPERM(I) is equal to the position of the only nonzero
                element in row I of permutation matrix P.
                For example, if
                               | 0 0 1 |
                           P  =| 1 0 0 |
                               | 0 1 0 |
                then IPERM = (3, 1, 2).

 WORK()         scratch array of length LWORK.  LWORK should be at
                least K.

 LWORK          length of WORK array

                If LWORK = -1, then a workspace query is assumed;
                the routine only calculates the optimal size of the
                WORK array, returns this value as the first entry of
                the WORK array, and no error message related to LWORK
                is issued by XERBLA.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

cjadsm - Jagged-diagonal format triangular solve

SYNOPSIS

```
 SUBROUTINE CJADSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, INDX, PNTR, MAXNZ, IPERM,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5), MAXNZ,
*            LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTR(MAXNZ+1), IPERM(M)
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CJADSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, INDX, PNTR, MAXNZ, IPERM,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5), MAXNZ,
*            LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTR(MAXNZ+1), IPERM(M)
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

where NNZ=PNTR(MAXNZ+1)-PNTR(1)+1 is the number of non-zero elements
```

F95 INTERFACE

```
 SUBROUTINE JADSM(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*    PNTR, MAXNZ, IPERM, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, MAXNZ
 INTEGER, DIMENSION(:) ::  DESCRA, INDX, PNTR, IPERM
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) ::  VAL, DV
 COMPLEX, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE JADSM_64(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*    PNTR, MAXNZ, IPERM, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, MAXNZ
 INTEGER*8, DIMENSION(:) ::  DESCRA, INDX, PNTR, IPERM
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) ::  VAL, DV
 COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a diagonal scaling matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in jagged-diagonal format and
op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                  0 : general
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))
                Note: For the routine, DESCRA(1)=3 is only supported.

                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices
VAL()           array of length NNZ consisting of entries of A.
                VAL can be viewed as a column major ordering of a
                row permutation of the Ellpack representation of A,
                where the Ellpack representation is permuted so that
                the rows are non-increasing in the number of nonzero
                entries.  Values added for padding in Ellpack are
                not included in the Jagged-Diagonal format.

INDX()          array of length NNZ consisting of the column indices
                of the corresponding entries in VAL.

PNTR()          array of length MAXNZ+1, where PNTR(I)-PNTR(1)+1
                points to the location in VAL of the first element
                in the row-permuted Ellpack represenation of A.

MAXNZ           max number of nonzeros elements per row.

IPERM()         integer array of length M such that I = IPERM(I'),
                where row I in the original Ellpack representation
                corresponds to row I' in the permuted representation.
                If IPERM(1)=0,   it's assumed by convention that
                IPERM(I)=I. IPERM is used to determine the order
                in which rows of C are updated.

B()             rectangular array with first dimension LDB.

LDB             leading dimension of B

BETA            Scalar parameter
```

```
C()            rectangular array with first dimension LDC.

LDC            leading dimension of C

WORK()         scratch array of length LWORK.
               On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.

LWORK          length of WORK array.  LWORK should be at least 2*M.

               For good performance, LWORK should generally be larger.
               For optimum performance on multiple processors, LWORK
               >=2*M*N_CPUS where N_CPUS is the maximum number of
               processors available to the program.

               If LWORK=0, the routine is to allocate workspace needed.

               If LWORK = -1, then a workspace query is assumed; the
               routine only calculates the optimum size of the WORK
               array, returns this value as the first entry of the WORK
               array, and no error message related to LWORK is issued
               by XERBLA.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the rows of A such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows have been scaled. UNITD=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the JAD representation of a sparse matrix. They are not used anyway in these cases. But if UNITD=4, the unit diagonal elements MUST be referenced in the JAD representation.

4. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

```
clarz - applie a complex elementary reflector H to a complex
M-by-N matrix C, from either the left or the right
```

SYNOPSIS

```
SUBROUTINE CLARZ(SIDE, M, N, L, V, INCV, TAU, C, LDC, WORK)

CHARACTER * 1 SIDE
COMPLEX TAU
COMPLEX V(*), C(LDC,*), WORK(*)
INTEGER M, N, L, INCV, LDC

SUBROUTINE CLARZ_64(SIDE, M, N, L, V, INCV, TAU, C, LDC, WORK)

CHARACTER * 1 SIDE
COMPLEX TAU
COMPLEX V(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, L, INCV, LDC
```

F95 INTERFACE

```
SUBROUTINE LARZ(SIDE, [M], [N], L, V, [INCV], TAU, C, [LDC], [WORK])

CHARACTER(LEN=1) :: SIDE
COMPLEX :: TAU
COMPLEX, DIMENSION(:) :: V, WORK
COMPLEX, DIMENSION(:,:) :: C
INTEGER :: M, N, L, INCV, LDC

SUBROUTINE LARZ_64(SIDE, [M], [N], L, V, [INCV], TAU, C, [LDC], [WORK])

CHARACTER(LEN=1) :: SIDE
```

```
COMPLEX :: TAU
COMPLEX, DIMENSION(:) :: V, WORK
COMPLEX, DIMENSION(:,:) :: C
INTEGER(8) :: M, N, L, INCV, LDC
```

C INTERFACE

```
#include <sunperf.h>

void clarz(char side, int m, int n, int l, complex  *v,  int
          incv, complex *tau, complex *c, int ldc);

void clarz_64(char side, long m, long n, long l, complex *v,
          long incv, complex *tau, complex *c, long ldc);
```

PURPOSE

clarz applies a complex elementary reflector H to a complex M-by-N matrix C, from either the left or the right. H is represented in the form

H = I - tau * v * v'

where tau is a complex scalar and v is a complex vector.

If tau = 0, then H is taken to be the unit matrix.

To apply H' (the conjugate transpose of H), supply conjg(tau) instead tau.

H is a product of k elementary reflectors as returned by CTZRZF.

ARGUMENTS

SIDE (input)
= 'L': form H * C
= 'R': form C * H

M (input) The number of rows of the matrix C.

N (input) The number of columns of the matrix C.

L (input) The number of entries of the vector V containing the meaningful part of the Householder vectors. If SIDE = 'L', M >= L >= 0, if SIDE = 'R', N >= L >= 0.

V (input) The vector v in the representation of H as returned by CTZRZF. V is not used if TAU = 0.

INCV (input)
The increment between elements of v. INCV <> 0.

TAU (input)
The value tau in the representation of H.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by the matrix H * C if SIDE = 'L', or C * H if SIDE = 'R'.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
(N) if SIDE = 'L' or (M) if SIDE = 'R'

FURTHER DETAILS

Based on contributions by
A. Petitet, Computer Science Dept., Univ. of Tenn., Knoxville, USA

Contents

NAME

```
clarzb - applie a complex block reflector H or its transpose
H**H  to a complex distributed M-by-N C from the left or the
right
```

SYNOPSIS

```
SUBROUTINE CLARZB(SIDE, TRANS, DIRECT, STOREV, M, N, K, L, V, LDV, T,
      LDT, C, LDC, WORK, LDWORK)

CHARACTER * 1 SIDE, TRANS, DIRECT, STOREV
COMPLEX V(LDV,*), T(LDT,*), C(LDC,*), WORK(LDWORK,*)
INTEGER M, N, K, L, LDV, LDT, LDC, LDWORK

SUBROUTINE CLARZB_64(SIDE, TRANS, DIRECT, STOREV, M, N, K, L, V, LDV,
      T, LDT, C, LDC, WORK, LDWORK)

CHARACTER * 1 SIDE, TRANS, DIRECT, STOREV
COMPLEX V(LDV,*), T(LDT,*), C(LDC,*), WORK(LDWORK,*)
INTEGER*8 M, N, K, L, LDV, LDT, LDC, LDWORK
```

F95 INTERFACE

```
SUBROUTINE LARZB(SIDE, TRANS, DIRECT, STOREV, [M], [N], K, L, V, [LDV],
       T, [LDT], C, [LDC], [WORK], [LDWORK])

CHARACTER(LEN=1) :: SIDE, TRANS, DIRECT, STOREV
COMPLEX, DIMENSION(:,:) :: V, T, C, WORK
INTEGER :: M, N, K, L, LDV, LDT, LDC, LDWORK

SUBROUTINE LARZB_64(SIDE, TRANS, DIRECT, STOREV, [M], [N], K, L, V,
       [LDV], T, [LDT], C, [LDC], [WORK], [LDWORK])
```

```
CHARACTER(LEN=1) :: SIDE, TRANS, DIRECT, STOREV
COMPLEX, DIMENSION(:,:) :: V, T, C, WORK
INTEGER(8) :: M, N, K, L, LDV, LDT, LDC, LDWORK
```

C INTERFACE

```
#include <sunperf.h>

void clarzb(char side, char trans, char direct, char storev,
          int  m,  int n, int k, int l, complex *v, int ldv,
          complex *t, int ldt,  complex  *c,  int  ldc,  int
          ldwork);

void clarzb_64(char side,  char  trans,  char  direct,  char
          storev,  long  m,  long n, long k, long l, complex
          *v, long ldv, complex *t, long  ldt,  complex  *c,
          long ldc, long ldwork);
```

PURPOSE

clarzb applies a complex block reflector H or its transpose H**H to a complex distributed M-by-N C from the left or the right.

Currently, only STOREV = 'R' and DIRECT = 'B' are supported.

ARGUMENTS

SIDE (input)
= 'L': apply H or H' from the Left
= 'R': apply H or H' from the Right

TRANS (input)
= 'N': apply H (No transpose)
= 'C': apply H' (Conjugate transpose)

DIRECT (input)
Indicates how H is formed from a product of elementary reflectors = 'F': H = H(1) H(2) . . . H(k) (Forward, not supported yet)
= 'B': H = H(k) . . . H(2) H(1) (Backward)

STOREV (input)
Indicates how the vectors which define the elementary reflectors are stored:
= 'C': Columnwise (not supported yet)
= 'R': Rowwise

M (input) The number of rows of the matrix C.

N (input) The number of columns of the matrix C.

K (input) The order of the matrix T (= the number of elementary reflectors whose product defines the block reflector).

L (input) The number of columns of the matrix V containing the meaningful part of the Householder reflectors. If SIDE = 'L', M >= L >= 0, if SIDE = 'R', N >= L >= 0.

V (input) If STOREV = 'C', NV = K; if STOREV = 'R', NV = L.

LDV (input)
The leading dimension of the array V. If STOREV = 'C', LDV >= L; if STOREV = 'R', LDV >= K.

T (input) The triangular K-by-K matrix T in the representation of the block reflector.

LDT (input)
The leading dimension of the array T. LDT >= K.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by H*C or H'*C or C*H or C*H'.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
dimension(MAX(M,N),K)

LDWORK (input)
The leading dimension of the array WORK. If SIDE = 'L', LDWORK >= max(1,N); if SIDE = 'R', LDWORK >= max(1,M).

FURTHER DETAILS

Based on contributions by
A. Petitet, Computer Science Dept., Univ. of Tenn., Knoxville, USA

Contents

NAME

```
clarzt - form the triangular factor T  of  a  complex  block
reflector H of order > n, which is defined as a product of k
elementary reflectors
```

SYNOPSIS

```
SUBROUTINE CLARZT(DIRECT, STOREV, N, K, V, LDV, TAU, T, LDT)

CHARACTER * 1 DIRECT, STOREV
COMPLEX V(LDV,*), TAU(*), T(LDT,*)
INTEGER N, K, LDV, LDT

SUBROUTINE CLARZT_64(DIRECT, STOREV, N, K, V, LDV, TAU, T, LDT)

CHARACTER * 1 DIRECT, STOREV
COMPLEX V(LDV,*), TAU(*), T(LDT,*)
INTEGER*8 N, K, LDV, LDT
```

F95 INTERFACE

```
SUBROUTINE LARZT(DIRECT, STOREV, N, K, V, [LDV], TAU, T, [LDT])

CHARACTER(LEN=1) :: DIRECT, STOREV
COMPLEX, DIMENSION(:) :: TAU
COMPLEX, DIMENSION(:,:) :: V, T
INTEGER :: N, K, LDV, LDT

SUBROUTINE LARZT_64(DIRECT, STOREV, N, K, V, [LDV], TAU, T, [LDT])

CHARACTER(LEN=1) :: DIRECT, STOREV
COMPLEX, DIMENSION(:) :: TAU
COMPLEX, DIMENSION(:,:) :: V, T
```

```
INTEGER(8) :: N, K, LDV, LDT
```

C INTERFACE

```
#include <sunperf.h>

void clarzt(char direct, char storev, int n, int k,  complex
          *v, int ldv, complex *tau, complex *t, int ldt);

void clarzt_64(char direct, char storev,  long  n,  long  k,
          complex  *v,  long  ldv, complex *tau, complex *t,
          long ldt);
```

PURPOSE

clarzt forms the triangular factor T of a complex block reflector H of order > n, which is defined as a product of k elementary reflectors.

If DIRECT = 'F', H = H(1) H(2) . . . H(k) and T is upper triangular;

If DIRECT = 'B', H = H(k) . . . H(2) H(1) and T is lower triangular.

If STOREV = 'C', the vector which defines the elementary reflector H(i) is stored in the i-th column of the array V, and

H = I - V * T * V'

If STOREV = 'R', the vector which defines the elementary reflector H(i) is stored in the i-th row of the array V, and

H = I - V' * T * V

Currently, only STOREV = 'R' and DIRECT = 'B' are supported.

ARGUMENTS

DIRECT (input)
Specifies the order in which the elementary reflectors are multiplied to form the block reflector:
= 'F': H = H(1) H(2) . . . H(k) (Forward, not supported yet)
= 'B': H = H(k) . . . H(2) H(1) (Backward)

```
STOREV (input)
          Specifies how the vectors which define the elemen-
          tary  reflectors  are  stored  (see  also  Further
          Details):
          = 'R': rowwise

N (input) The order of the block reflector H. N >= 0.

K (input) The order of the triangular factor T (= the number
          of elementary reflectors). K >= 1.

V (input) (LDV,K) if STOREV = 'C' (LDV,N) if  STOREV  =  'R'
          The matrix V. See further details.

LDV (input)
          The leading dimension of the array V.  If STOREV =
          'C', LDV >= max(1,N); if STOREV = 'R', LDV >= K.

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i).

T (input) The k by  k  triangular  factor  T  of  the  block
          reflector.   If  DIRECT = 'F', T is upper triangu-
          lar; if DIRECT = 'B', T is lower  triangular.  The
          rest of the array is not used.

LDT (input)
          The leading dimension of the array T. LDT >= K.
```

FURTHER DETAILS

```
Based on contributions by
  A. Petitet, Computer Science Dept., Univ. of Tenn.,  Knox-
ville, USA

The shape of the matrix V and the  storage  of  the  vectors
which  define  the H(i) is best illustrated by the following
example with n = 5 and k = 3. The elements equal  to  1  are
not  stored;  the  corresponding array elements are modified
but restored on exit. The rest of the array is not used.

DIRECT = 'F' and STOREV =  'C':          DIRECT  =  'F'  and
STOREV = 'R':

                                              ______V_____
       (    v1    v2    v3    )                            /
( v1 v2 v3 )                        ( v1 v1 v1 v1 v1 . . . . 1
)
   V = ( v1 v2 v3 )                        ( v2 v2 v2 v2 v2  .
```

```
. . 1   )
       ( v1 v2 v3 )                       ( v3 v3 v3 v3 v3  .
. 1     )
       ( v1 v2 v3 )
          .  .  .
          1  .  .
             1  .
                1

DIRECT = 'B' and STOREV =  'C':           DIRECT  =  'B'  and
STOREV = 'R':

                                                    ______V_____
          1                                              /
.  1                            ( 1 . . . . v1 v1 v1 v1 v1 )
          .  .  1                         ( . 1 . . .  v2  v2
v2 v2 v2 )
          .  .  .                         ( . . 1 . .  v3  v3
v3 v3 v3 )
          .  .  .
       ( v1 v2 v3 )
   V = ( v1 v2 v3 )
       ( v1 v2 v3 )
```

Contents

NAME

```
clatzm - routine is deprecated and has been replaced by rou-
tine CUNMRZ
```

SYNOPSIS

```
SUBROUTINE CLATZM(SIDE, M, N, V, INCV, TAU, C1, C2, LDC, WORK)

CHARACTER * 1 SIDE
COMPLEX TAU
COMPLEX V(*), C1(LDC,*), C2(LDC,*), WORK(*)
INTEGER M, N, INCV, LDC

SUBROUTINE CLATZM_64(SIDE, M, N, V, INCV, TAU, C1, C2, LDC, WORK)

CHARACTER * 1 SIDE
COMPLEX TAU
COMPLEX V(*), C1(LDC,*), C2(LDC,*), WORK(*)
INTEGER*8 M, N, INCV, LDC
```

F95 INTERFACE

```
SUBROUTINE LATZM(SIDE, [M], [N], V, [INCV], TAU, C1, C2, [LDC], [WORK])

CHARACTER(LEN=1) :: SIDE
COMPLEX :: TAU
COMPLEX, DIMENSION(:) :: V, WORK
COMPLEX, DIMENSION(:,:) :: C1, C2
INTEGER :: M, N, INCV, LDC

SUBROUTINE LATZM_64(SIDE, [M], [N], V, [INCV], TAU, C1, C2, [LDC],
       [WORK])

CHARACTER(LEN=1) :: SIDE
```

```
COMPLEX :: TAU
COMPLEX, DIMENSION(:) :: V, WORK
COMPLEX, DIMENSION(:,:) :: C1, C2
INTEGER(8) :: M, N, INCV, LDC
```

C INTERFACE

```
#include <sunperf.h>

void clatzm(char side, int m, int n, complex *v,  int  incv,
          complex *tau, complex *c1, complex *c2, int ldc);

void clatzm_64(char side, long m, long n, complex  *v,  long
          incv, complex *tau, complex *c1, complex *c2, long
          ldc);
```

PURPOSE

```
clatzm routine is deprecated and has been replaced  by  rou-
tine CUNMRZ.

CLATZM applies a Householder matrix generated by CTZRQF to a
matrix.

Let P = I - tau*u*u',   u = ( 1 ),
                            ( v )
where v is an (m-1) vector if SIDE = 'L', or a (n-1)  vector
if SIDE = 'R'.

If SIDE equals 'L', let
       C = [ C1 ] 1
           [ C2 ] m-1
             n
Then C is overwritten by P*C.

If SIDE equals 'R', let
       C = [ C1, C2 ] m
              1  n-1
Then C is overwritten by C*P.
```

ARGUMENTS

```
SIDE (input)
          = 'L': form P * C
          = 'R': form C * P

M (input) The number of rows of the matrix C.
```

N (input) The number of columns of the matrix C.

V (input) (1 + (M-1)*abs(INCV)) if SIDE = 'L' (1 + (N-1)*abs(INCV)) if SIDE = 'R' The vector v in the representation of P. V is not used if TAU = 0.

INCV (input)
The increment between elements of v. INCV <> 0

TAU (input)
The value tau in the representation of P.

C1 (input/output)
(LDC,N) if SIDE = 'L' (M,1) if SIDE = 'R' On entry, the n-vector C1 if SIDE = 'L', or the m-vector C1 if SIDE = 'R'.

On exit, the first row of P*C if SIDE = 'L', or the first column of C*P if SIDE = 'R'.

C2 (input/output)
(LDC, N) if SIDE = 'L' (LDC, N-1) if SIDE = 'R' On entry, the (m - 1) x n matrix C2 if SIDE = 'L', or the m x (n - 1) matrix C2 if SIDE = 'R'.

On exit, rows 2:m of P*C if SIDE = 'L', or columns 2:m of C*P if SIDE = 'R'.

LDC (input)
The leading dimension of the arrays C1 and C2. LDC >= max(1,M).

WORK (workspace)
(N) if SIDE = 'L' (M) if SIDE = 'R'

Contents

NAME

```
cosqb - synthesize a Fourier sequence from  its  representa-
tion in terms of a cosine series with odd wave numbers.  The
COSQ operations are unnormalized inverses of themselves,  so
a  call  to  COSQF followed by a call to COSQB will multiply
the input sequence by 4 * N.
```

SYNOPSIS

```
SUBROUTINE COSQB(N, X, WSAVE)

INTEGER N
REAL X(*), WSAVE(*)

SUBROUTINE COSQB_64(N, X, WSAVE)

INTEGER*8 N
REAL X(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE COSQB(N, X, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: X, WSAVE

SUBROUTINE COSQB_64(N, X, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: X, WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void cosqb(int n, float *x, float *wsave);
```

```
void cosqb_64(long n, float *x, float *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence to be  transformed.   These
          subroutines are most efficient when N is a product
          of small primes.  N >= 0.

X (input/output)
          On entry, an array  of  length  N  containing  the
          sequence   to   be   transformed.   On  exit,  the
          quarter-wave cosine synthesis of the input.
WSAVE (input)
          On entry, an array with dimension of at least (3 *
          N + 15) that has been initialized by COSQI.
```

Contents

NAME

cosqf - compute the Fourier coefficients in a cosine series representation with only odd wave numbers. The COSQ operations are unnormalized inverses of themselves, so a call to COSQF followed by a call to COSQB will multiply the input sequence by 4 * N.

SYNOPSIS

```
SUBROUTINE COSQF(N, X, WSAVE)

INTEGER N
REAL X(*), WSAVE(*)

SUBROUTINE COSQF_64(N, X, WSAVE)

INTEGER*8 N
REAL X(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE COSQF(N, X, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: X, WSAVE

SUBROUTINE COSQF_64(N, X, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: X, WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void cosqf(int n, float *x, float *wsave);
```

```
void cosqf_64(long n, float *x, float *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence to be  transformed.   These
          subroutines are most efficient when N is a product
          of small primes.  N >= 0.

X (input/output)
          On entry, an array  of  length  N  containing  the
          sequence   to   be   transformed.   On  exit,  the
          quarter-wave cosine transform of the input.
WSAVE (input)
          On entry, an array with dimension of at least   (3
          * N + 15) that has been initialized by COSQI.
```

Contents

NAME

```
cosqi - initialize the array WSAVE, which is  used  in  both
COSQF and COSQB.
```

SYNOPSIS

```
SUBROUTINE COSQI(N, WSAVE)

INTEGER N
REAL WSAVE(*)

SUBROUTINE COSQI_64(N, WSAVE)

INTEGER*8 N
REAL WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE COSQI(N, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: WSAVE

SUBROUTINE COSQI_64(N, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void cosqi(int n, float *wsave);

void cosqi_64(long n, float *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. The method is most efficient when N is a product of small primes.

WSAVE (input)
On entry, an array of dimension (3 * N + 15) or greater. COSQI needs to be called only once to initialize WSAVE before calling COSQF and/or COSQB if N and WSAVE remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

cost - compute the discrete Fourier cosine transform of an even sequence. The COST transforms are unnormalized inverses of themselves, so a call of COST followed by another call of COST will multiply the input sequence by 2 * (N-1).

SYNOPSIS

```
SUBROUTINE COST(N, X, WSAVE)

INTEGER N
REAL X(*), WSAVE(*)

SUBROUTINE COST_64(N, X, WSAVE)

INTEGER*8 N
REAL X(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE COST(N, X, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: X, WSAVE

SUBROUTINE COST_64(N, X, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: X, WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void cost(int n, float *x, float *wsave);
```

```
void cost_64(long n, float *x, float *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence to be  transformed.   These
          subroutines  are  most  efficient  when N - 1 is a
          product of small primes.  N >= 2.

X (input/output)
          On entry, an array  of  length  N  containing  the
          sequence  to  be transformed.  On exit, the cosine
          transform of the input.
WSAVE (input)
          On entry, an array with dimension of at least   (3
          * N + 15), initialized by COSTI.
```

Contents

NAME

```
costi - initialize the array WSAVE, which is used in COST.
```

SYNOPSIS

```
SUBROUTINE COSTI(N, WSAVE)

INTEGER N
REAL WSAVE(*)

SUBROUTINE COSTI_64(N, WSAVE)

INTEGER*8 N
REAL WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE COSTI(N, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: WSAVE

SUBROUTINE COSTI_64(N, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void costi(int n, float *wsave);

void costi_64(long n, float *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. The method is most efficient when N - 1 is a product of small primes. N >= 2.

WSAVE (input)
On entry, an array of dimension (3 * N + 15) or greater. COSTI is called once to initialize WSAVE before calling COST and need not be called again between calls to COST if N and WSAVE remain unchanged. Thus, subsequent transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
cpbcon - estimate the reciprocal of the condition number (in
the  1-norm)  of  a complex Hermitian positive definite band
matrix using the Cholesky factorization A = U**H*U  or  A  =
L*L**H computed by CPBTRF
```

SYNOPSIS

```
SUBROUTINE CPBCON(UPLO, N, KD, A, LDA, ANORM, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER N, KD, LDA, INFO
REAL ANORM, RCOND
REAL WORK2(*)

SUBROUTINE CPBCON_64(UPLO, N, KD, A, LDA, ANORM, RCOND, WORK,
      WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, KD, LDA, INFO
REAL ANORM, RCOND
REAL WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE PBCON(UPLO, [N], KD, A, [LDA], ANORM, RCOND, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
```

```
INTEGER :: N, KD, LDA, INFO
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: WORK2

SUBROUTINE PBCON_64(UPLO, [N], KD, A, [LDA], ANORM, RCOND, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, KD, LDA, INFO
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cpbcon(char uplo, int n, int kd, complex *a,  int  lda,
          float anorm, float *rcond, int *info);

void cpbcon_64(char uplo, long n, long kd, complex *a,  long
          lda, float anorm, float *rcond, long *info);
```

PURPOSE

cpbcon estimates the reciprocal of the condition number (in the 1-norm) of a complex Hermitian positive definite band matrix using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPBTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
= 'U': Upper triangular factor stored in A;
= 'L': Lower triangular factor stored in A.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of sub-diagonals if UPLO = 'L'. KD >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H of the band matrix A, stored in the first KD+1 rows of the array. The j-th column of U or L is stored in the j-th column of the array A as follows: if UPLO ='U', A(kd+1+i-j,j) = U(i,j) for max(1,j-kd)<=i<=j; if UPLO ='L', A(1+i-j,j) = L(i,j) for j<=i<=min(n,j+kd).

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

ANORM (input)
The 1-norm (or infinity-norm) of the Hermitian band matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
cpbequ - compute row and column scalings intended to equili-
brate a Hermitian positive definite band matrix A and reduce
its condition number (with respect to the two-norm)
```

SYNOPSIS

```
SUBROUTINE CPBEQU(UPLO, N, KD, A, LDA, SCALE, SCOND, AMAX, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*)
INTEGER N, KD, LDA, INFO
REAL SCOND, AMAX
REAL SCALE(*)

SUBROUTINE CPBEQU_64(UPLO, N, KD, A, LDA, SCALE, SCOND, AMAX,
      INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*)
INTEGER*8 N, KD, LDA, INFO
REAL SCOND, AMAX
REAL SCALE(*)
```

F95 INTERFACE

```
SUBROUTINE PBEQU(UPLO, [N], KD, A, [LDA], SCALE, SCOND, AMAX,
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, KD, LDA, INFO
REAL :: SCOND, AMAX
REAL, DIMENSION(:) :: SCALE
```

```
SUBROUTINE PBEQU_64(UPLO, [N], KD, A, [LDA], SCALE, SCOND, AMAX,
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, KD, LDA, INFO
REAL :: SCOND, AMAX
REAL, DIMENSION(:) :: SCALE
```

C INTERFACE

```
#include <sunperf.h>

void cpbequ(char uplo, int n, int kd, complex *a,  int  lda,
          float  *scale,  float  *scond,  float  *amax,  int
          *info);

void cpbequ_64(char uplo, long n, long kd, complex *a,  long
          lda, float *scale, float *scond, float *amax, long
          *info);
```

PURPOSE

cpbequ computes row and column scalings intended to equilibrate a Hermitian positive definite band matrix A and reduce its condition number (with respect to the two-norm). S contains the scale factors, S(i) = 1/sqrt(A(i,i)), chosen so that the scaled matrix B with elements B(i,j) = S(i)*A(i,j)*S(j) has ones on the diagonal. This choice of S puts the condition number of B within a factor N of the smallest possible condition number over all possible diagonal scalings.

ARGUMENTS

UPLO (input)
- = 'U': Upper triangular of A is stored;
- = 'L': Lower triangular of A is stored.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

A (input) The upper or lower triangle of the Hermitian band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

SCALE (output)
If INFO = 0, SCALE contains the scale factors for A.

SCOND (output)
If INFO = 0, SCALE contains the ratio of the smallest SCALE(i) to the largest SCALE(i). If SCOND >= 0.1 and AMAX is neither too large nor too small, it is not worth scaling by SCALE.

AMAX (output)
Absolute value of largest matrix element. If AMAX is very close to overflow or very close to underflow, the matrix should be scaled.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, the i-th diagonal element is nonpositive.

Contents

NAME

```
cpbrfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is Hermitian positive
definite and banded, and provides error bounds and  backward
error estimates for the solution
```

SYNOPSIS

```
SUBROUTINE CPBRFS(UPLO, N, KD, NRHS, A, LDA, AF, LDAF, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, KD, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL FERR(*), BERR(*), WORK2(*)

SUBROUTINE CPBRFS_64(UPLO, N, KD, NRHS, A, LDA, AF, LDAF, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, KD, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE PBRFS(UPLO, [N], KD, [NRHS], A, [LDA], AF, [LDAF], B,
       [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER :: N, KD, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

```
SUBROUTINE PBRFS_64(UPLO, [N], KD, [NRHS], A, [LDA], AF, [LDAF],
       B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, KD, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cpbrfs(char uplo, int n, int kd, int nrhs, complex  *a,
          int  lda,  complex  *af, int ldaf, complex *b, int
          ldb, complex  *x,  int  ldx,  float  *ferr,  float
          *berr, int *info);

void cpbrfs_64(char uplo, long n, long kd, long  nrhs,  com-
          plex *a, long lda, complex *af, long ldaf, complex
          *b, long ldb, complex *x, long ldx,  float  *ferr,
          float *berr, long *info);
```

PURPOSE

cpbrfs improves the computed solution to a system of linear equations when the coefficient matrix is Hermitian positive definite and banded, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The upper or lower triangle of the Hermitian band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

AF (input)
The triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H of the band matrix A as computed by CPBTRF, in the same storage format as A (see A).

LDAF (input)
The leading dimension of the array AF. LDAF >= KD+1.

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by CPBTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative

```
          change in any element of A or B that makes X(j) an
          exact solution).

WORK (workspace)
          dimension(2*N)

WORK2 (workspace)
          dimension(N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
cpbstf - compute a split Cholesky factorization of a complex
Hermitian positive definite band matrix A
```

SYNOPSIS

```
SUBROUTINE CPBSTF(UPLO, N, KD, AB, LDAB, INFO)

CHARACTER * 1 UPLO
COMPLEX AB(LDAB,*)
INTEGER N, KD, LDAB, INFO

SUBROUTINE CPBSTF_64(UPLO, N, KD, AB, LDAB, INFO)

CHARACTER * 1 UPLO
COMPLEX AB(LDAB,*)
INTEGER*8 N, KD, LDAB, INFO
```

F95 INTERFACE

```
SUBROUTINE PBSTF(UPLO, [N], KD, AB, [LDAB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: AB
INTEGER :: N, KD, LDAB, INFO

SUBROUTINE PBSTF_64(UPLO, [N], KD, AB, [LDAB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: AB
INTEGER(8) :: N, KD, LDAB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cpbstf(char uplo, int n, int kd, complex *ab, int ldab,
          int *info);

void cpbstf_64(char uplo, long n, long kd, complex *ab, long
          ldab, long *info);
```

PURPOSE

```
cpbstf computes a split Cholesky factorization of a  complex
Hermitian positive definite band matrix A.

This routine is designed to  be  used  in  conjunction  with
CHBGST.
The factorization has the form  A = S**H*S   where  S  is  a
band  matrix  of  the  same bandwidth as A and the following
structure:

  S = ( U    )
      ( M  L )

where U is upper triangular of order m = (n+kd)/2, and L  is
lower triangular of order n-m.
```

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  KD >= 0.

AB (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  band  matrix  A,  stored in the first kd+1
          rows of the array.  The j-th column of A is stored
          in the j-th column of the array AB as follows:  if
          UPLO = 'U', AB(kd+1+i-j,j) = A(i,j)  for  max(1,j-
          kd)<=i<=j;  if UPLO = 'L', AB(1+i-j,j)    = A(i,j)
          for j<=i<=min(n,j+kd).
```

On exit, if INFO = 0, the factor S from the split Cholesky factorization A = S**H*S. See Further Details.

LDAB (input)
The leading dimension of the array AB. LDAB >= KD+1.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the factorization could not be completed, because the updated element a(i,i) was negative; the matrix A is not positive definite.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when N = 7, KD = 2:

```
S = ( s11  s12  s13                     )
    (      s22  s23  s24                )
    (           s33  s34                )
    (                s44                )
    (           s53  s54  s55           )
    (                s64  s65  s66      )
    (                     s75  s76  s77 )
```

If UPLO = 'U', the array AB holds:

```
on entry:                          on exit:

 *    *   a13  a24  a35  a46  a57   *    *   s13  s24   s53'
s64' s75'
 *   a12  a23  a34  a45  a56  a67   *   s12  s23  s34   s54'
s65'  s76'  a11  a22  a33  a44  a55  a66  a77  s11  s22  s33
s44  s55  s66  s77
```

If UPLO = 'L', the array AB holds:

```
on entry:                          on exit:

a11  a22  a33  a44  a55  a66  a77  s11  s22  s33   s44   s55
s66   s77  a21  a32  a43  a54  a65  a76   *   s12' s23' s34'
s54  s65  s76   * a31  a42  a53  a64  a64    *     *    s13'
s24' s53  s64  s75   *    *
```

Array elements marked * are not used by the routine; s12' denotes conjg(s12); the diagonal elements of S are real.

Contents

NAME

```
cpbsv - compute the solution to a complex system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE CPBSV(UPLO, N, NDIAG, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, NDIAG, NRHS, LDA, LDB, INFO

SUBROUTINE CPBSV_64(UPLO, N, NDIAG, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, NDIAG, NRHS, LDA, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE PBSV(UPLO, [N], NDIAG, [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: N, NDIAG, NRHS, LDA, LDB, INFO

SUBROUTINE PBSV_64(UPLO, [N], NDIAG, [NRHS], A, [LDA], B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: N, NDIAG, NRHS, LDA, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cpbsv(char uplo, int n, int ndiag,  int  nrhs,  complex
          *a, int lda, complex *b, int ldb, int *info);

void cpbsv_64(char uplo, long n, long ndiag, long nrhs, com-
          plex  *a,  long  lda,  complex  *b, long ldb, long
          *info);
```

PURPOSE

cpbsv computes the solution to a complex system of linear equations
A * X = B, where A is an N-by-N Hermitian positive definite band matrix and X and B are N-by-NRHS matrices.
The Cholesky decomposition is used to factor A as
A = U**H * U, if UPLO = 'U', or
A = L * L**H, if UPLO = 'L',
where U is an upper triangular band matrix, and L is a lower triangular band matrix, with the same number of superdiagonals or subdiagonals as A. The factored form of A is then used to solve the system of equations A * X = B.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NDIAG (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. NDIAG >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input/output)
On entry, the upper or lower triangle of the Hermitian band matrix A, stored in the first NDIAG+1 rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if

UPLO = 'U', A(NDIAG+1+i-j,j) = A(i,j) for max(1,j-NDIAG)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(N,j+NDIAG). See below for further details.

On exit, if INFO = 0, the triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H of the band matrix A, in the same storage format as A.

LDA (input)
The leading dimension of the array A. LDA >= NDIAG+1.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when N = 6, NDIAG = 2, and UPLO = 'U':

```
On entry:                       On exit:

    *    *   a13  a24  a35  a46      *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
```

Similarly, if UPLO = 'L' the format of A is as follows:

```
On entry:                       On exit:

   a11  a22  a33  a44  a55  a66     l11  l22  l33  l44   l55
```

```
l66
   a21  a32  a43  a54  a65   *      l21  l32  l43  l54   l65
*
   a31  a42  a53  a64   *    *      l31  l42  l53   l64    *
*
```

Array elements marked * are not used by the routine.

Contents

NAME

```
cpbsvx - use the Cholesky factorization A = U**H*U  or  A  =
L*L**H to compute the solution to a complex system of linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE CPBSVX(FACT, UPLO, N, NDIAG, NRHS, A, LDA, AF, LDAF,
      EQUED, S, B, LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2,
      INFO)

CHARACTER * 1 FACT, UPLO, EQUED
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NDIAG, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL RCOND
REAL S(*), FERR(*), BERR(*), WORK2(*)

SUBROUTINE CPBSVX_64(FACT, UPLO, N, NDIAG, NRHS, A, LDA, AF, LDAF,
      EQUED, S, B, LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2,
      INFO)

CHARACTER * 1 FACT, UPLO, EQUED
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NDIAG, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL RCOND
REAL S(*), FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE PBSVX(FACT, UPLO, [N], NDIAG, [NRHS], A, [LDA], AF, [LDAF],
       EQUED, S, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK],
       [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: FACT, UPLO, EQUED
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER :: N, NDIAG, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL :: RCOND
REAL, DIMENSION(:) :: S, FERR, BERR, WORK2

SUBROUTINE PBSVX_64(FACT, UPLO, [N], NDIAG, [NRHS], A, [LDA], AF,
       [LDAF], EQUED, S, B, [LDB], X, [LDX], RCOND, FERR, BERR,
       [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, NDIAG, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL :: RCOND
REAL, DIMENSION(:) :: S, FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cpbsvx(char fact, char uplo,  int  n,  int  ndiag,  int
          nrhs,  complex *a, int lda, complex *af, int ldaf,
          char equed, float *s, complex *b, int ldb, complex
          *x,  int  ldx,  float  *rcond,  float *ferr, float
          *berr, int *info);

void cpbsvx_64(char fact, char uplo,  long  n,  long  ndiag,
          long nrhs, complex *a, long lda, complex *af, long
          ldaf, char equed, float *s, complex *b, long  ldb,
          complex  *x,  long ldx, float *rcond, float *ferr,
          float *berr, long *info);
```

PURPOSE

cpbsvx uses the Cholesky factorization A = U**H*U or A = L*L**H to compute the solution to a complex system of linear equations
   A * X = B, where A is an N-by-N Hermitian positive definite band matrix and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'E', real scaling factors are computed to equilibrate
   the system:
      diag(S) * A * diag(S) * inv(diag(S)) * X = diag(S) * B

Whether or not the system will be equilibrated depends on the
scaling of the matrix A, but if equilibration is used, A is
overwritten by diag(S)*A*diag(S) and B by diag(S)*B.

2. If FACT = 'N' or 'E', the Cholesky decomposition is used to
factor the matrix A (after equilibration if FACT = 'E') as
A = U**H * U, if UPLO = 'U', or
A = L * L**H, if UPLO = 'L',
where U is an upper triangular band matrix, and L is a lower
triangular band matrix.

3. If the leading i-by-i principal minor is not positive definite,
then the routine returns with INFO = i. Otherwise, the factored
form of A is used to estimate the condition number of the matrix
A. If the reciprocal of the condition number is less than machine
precision, INFO = N+1 is returned as a warning, but the routine
still goes on to solve for X and compute error bounds as
described below.

4. The system of equations is solved for X using the factored form
of A.

5. Iterative refinement is applied to improve the computed solution
matrix and calculate error bounds and backward error estimates
for it.

6. If equilibration was used, the matrix X is premultiplied by
diag(S) so that it solves the original system before
equilibration.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of the

matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored. = 'F': On entry, AF contains the factored form of A. If EQUED = 'Y', the matrix A has been equilibrated with scaling factors given by S. A and AF will not be modified. = 'N': The matrix A will be copied to AF and factored.
= 'E': The matrix A will be equilibrated if necessary, then copied to AF and factored.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NDIAG (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. NDIAG >= 0.

NRHS (input)
The number of right-hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input/output)
On entry, the upper or lower triangle of the Hermitian band matrix A, stored in the first NDIAG+1 rows of the array, except if FACT = 'F' and EQUED = 'Y', then A must contain the equilibrated matrix diag(S)*A*diag(S). The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(NDIAG+1+i-j,j) = A(i,j) for max(1,j-NDIAG)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(N,j+NDIAG). See below for further details.

On exit, if FACT = 'E' and EQUED = 'Y', A is overwritten by diag(S)*A*diag(S).

LDA (input)
The leading dimension of the array A. LDA >= NDIAG+1.

AF (input/output)
If FACT = 'F', then AF is an input argument and on entry contains the triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H of the band matrix A, in the same storage

format as A (see A). If EQUED = 'Y', then AF is the factored form of the equilibrated matrix A.

If FACT = 'N', then AF is an output argument and on exit returns the triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H.

If FACT = 'E', then AF is an output argument and on exit returns the triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H of the equilibrated matrix A (see the description of A for the form of the equilibrated matrix).

LDAF (input)
The leading dimension of the array AF. LDAF >= NDIAG+1.

EQUED (input)
Specifies the form of equilibration that was done.
= 'N': No equilibration (always true if FACT = 'N').
= 'Y': Equilibration was done, i.e., A has been replaced by diag(S) * A * diag(S). EQUED is an input argument if FACT = 'F'; otherwise, it is an output argument.

S (input/output)
The scale factors for A; not accessed if EQUED = 'N'. S is an input argument if FACT = 'F'; otherwise, S is an output argument. If FACT = 'F' and EQUED = 'Y', each element of S must be positive.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if EQUED = 'N', B is not modified; if EQUED = 'Y', B is overwritten by diag(S) * B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X to the original system of equations. Note that if EQUED = 'Y', A and B are modified on exit, and the solution to the equilibrated system is inv(diag(S))*X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A after equilibration (if done). If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

FURTHER DETAILS

```
The band storage scheme  is  illustrated  by  the  following
example, when N = 6, NDIAG = 2, and UPLO = 'U':

Two-dimensional storage of the Hermitian matrix A:

   a11  a12  a13
        a22  a23  a24
             a33  a34  a35
                  a44  a45  a46
                       a55  a56
   (aij=conjg(aji))         a66

Band storage of the upper triangle of A:

    *    *   a13  a24  a35  a46
    *   a12  a23  a34  a45  a56
   a11  a22  a33  a44  a55  a66

Similarly, if UPLO = 'L' the format of A is as follows:

   a11  a22  a33  a44  a55  a66
   a21  a32  a43  a54  a65   *
   a31  a42  a53  a64   *    *

Array elements marked * are not used by the routine.
```

Contents

NAME

```
cpbtf2 - compute the Cholesky  factorization  of  a  complex
Hermitian positive definite band matrix A
```

SYNOPSIS

```
SUBROUTINE CPBTF2(UPLO, N, KD, AB, LDAB, INFO)

CHARACTER * 1 UPLO
COMPLEX AB(LDAB,*)
INTEGER N, KD, LDAB, INFO

SUBROUTINE CPBTF2_64(UPLO, N, KD, AB, LDAB, INFO)

CHARACTER * 1 UPLO
COMPLEX AB(LDAB,*)
INTEGER*8 N, KD, LDAB, INFO
```

F95 INTERFACE

```
SUBROUTINE PBTF2(UPLO, [N], KD, AB, [LDAB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: AB
INTEGER :: N, KD, LDAB, INFO

SUBROUTINE PBTF2_64(UPLO, [N], KD, AB, [LDAB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: AB
INTEGER(8) :: N, KD, LDAB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cpbtf2(char uplo, int n, int kd, complex *ab, int ldab,
          int *info);

void cpbtf2_64(char uplo, long n, long kd, complex *ab, long
          ldab, long *info);
```

PURPOSE

cpbtf2 computes the Cholesky factorization of a complex Hermitian positive definite band matrix A.

The factorization has the form
```
   A = U' * U ,  if UPLO = 'U', or
   A = L  * L',  if UPLO = 'L',
```
where U is an upper triangular matrix, U' is the conjugate transpose of U, and L is lower triangular.

This is the unblocked version of the algorithm, calling Level 2 BLAS.

ARGUMENTS

UPLO (input)
Specifies whether the upper or lower triangular part of the Hermitian matrix A is stored:
= 'U': Upper triangular
= 'L': Lower triangular

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of super-diagonals of the matrix A if UPLO = 'U', or the number of sub-diagonals if UPLO = 'L'. KD >= 0.

AB (input/output)
On entry, the upper or lower triangle of the Hermitian band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', AB(1+i-j,j)   = A(i,j) for j<=i<=min(n,j+kd).

On exit, if INFO = 0, the triangular factor U or L

from the Cholesky factorization A = U'*U or A = L*L' of the band matrix A, in the same storage format as A.

LDAB (input)
The leading dimension of the array AB. LDAB >= KD+1.

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, the leading minor of order k is not positive definite, and the factorization could not be completed.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when N = 6, KD = 2, and UPLO = 'U':

```
On entry:                       On exit:

    *    *   a13  a24  a35  a46      *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
```

Similarly, if UPLO = 'L' the format of A is as follows:

```
On entry:                       On exit:

   a11  a22  a33  a44  a55  a66     l11  l22  l33  l44   l55
l66
   a21  a32  a43  a54  a65   *      l21  l32  l43  l54   l65
*
   a31  a42  a53  a64   *    *      l31  l42  l53   l64    *
*
```

Array elements marked * are not used by the routine.

Contents

NAME

cpbtrf - compute the Cholesky factorization of a complex Hermitian positive definite band matrix A

SYNOPSIS

```
SUBROUTINE CPBTRF(UPLO, N, KD, A, LDA, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*)
INTEGER N, KD, LDA, INFO

SUBROUTINE CPBTRF_64(UPLO, N, KD, A, LDA, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*)
INTEGER*8 N, KD, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE PBTRF(UPLO, [N], KD, A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, KD, LDA, INFO

SUBROUTINE PBTRF_64(UPLO, [N], KD, A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, KD, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cpbtrf(char uplo, int n, int kd, complex *a,  int  lda,
          int *info);

void cpbtrf_64(char uplo, long n, long kd, complex *a,  long
          lda, long *info);
```

PURPOSE

cpbtrf computes the Cholesky factorization of a complex Hermitian positive definite band matrix A.

The factorization has the form

```
   A = U**H * U,  if UPLO = 'U', or
   A = L  * L**H,  if UPLO = 'L',
```

where U is an upper triangular matrix and L is lower triangular.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

A (input/output)
On entry, the upper or lower triangle of the Hermitian band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).

On exit, if INFO = 0, the triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H of the band matrix A, in the same storage format as A.

LDA (input)

The leading dimension of the array A. LDA >= KD+1.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i is not positive definite, and the factorization could not be completed.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when N = 6, KD = 2, and UPLO = 'U':

```
On entry:                       On exit:

    *    *   a13  a24  a35  a46      *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
```

Similarly, if UPLO = 'L' the format of A is as follows:

```
On entry:                       On exit:

   a11  a22  a33  a44  a55  a66     l11  l22  l33  l44   l55
l66
   a21  a32  a43  a54  a65   *      l21  l32  l43  l54   l65
*
   a31  a42  a53  a64   *    *      l31  l42  l53   l64    *
*
```

Array elements marked * are not used by the routine.

Contributed by
Peter Mayes and Giuseppe Radicati, IBM ECSEC, Rome, March 23, 1989

Contents

NAME

```
cpbtrs - solve a system of linear equations A*X = B  with  a
Hermitian positive definite band matrix A using the Cholesky
factorization A = U**H*U or A = L*L**H computed by CPBTRF
```

SYNOPSIS

```
SUBROUTINE CPBTRS(UPLO, N, KD, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, KD, NRHS, LDA, LDB, INFO

SUBROUTINE CPBTRS_64(UPLO, N, KD, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, KD, NRHS, LDA, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE PBTRS(UPLO, [N], KD, [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: N, KD, NRHS, LDA, LDB, INFO

SUBROUTINE PBTRS_64(UPLO, [N], KD, [NRHS], A, [LDA], B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: N, KD, NRHS, LDA, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cpbtrs(char uplo, int n, int kd, int nrhs, complex  *a,
          int lda, complex *b, int ldb, int *info);

void cpbtrs_64(char uplo, long n, long kd, long  nrhs,  com-
          plex  *a,  long  lda,  complex  *b, long ldb, long
          *info);
```

PURPOSE

cpbtrs solves a system of linear equations A*X = B with a Hermitian positive definite band matrix A using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPBTRF.

ARGUMENTS

UPLO (input)
  = 'U': Upper triangular factor stored in A;
  = 'L': Lower triangular factor stored in A.

N (input) The order of the matrix A. N >= 0.

KD (input)
  The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

NRHS (input)
  The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H of the band matrix A, stored in the first KD+1 rows of the array. The j-th column of U or L is stored in the j-th column of the array A as follows: if UPLO ='U',   A(kd+1+i-j,j)   =   U(i,j)   for max(1,j-kd)<=i<=j; if UPLO ='L', A(1+i-j,j)   = L(i,j) for j<=i<=min(n,j+kd).

LDA (input)
  The leading dimension of the array A.   LDA >= KD+1.

B (input/output)

On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)

The leading dimension of the array B. LDB >= max(1,N).

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
cpocon - estimate the reciprocal of the condition number (in
the  1-norm) of a complex Hermitian positive definite matrix
using the Cholesky factorization A = U**H*U or  A  =  L*L**H
computed by CPOTRF
```

SYNOPSIS

```
SUBROUTINE CPOCON(UPLO, N, A, LDA, ANORM, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, INFO
REAL ANORM, RCOND
REAL WORK2(*)

SUBROUTINE CPOCON_64(UPLO, N, A, LDA, ANORM, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, INFO
REAL ANORM, RCOND
REAL WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE POCON(UPLO, [N], A, [LDA], ANORM, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
```

```
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: WORK2

SUBROUTINE POCON_64(UPLO, [N], A, [LDA], ANORM, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>
void cpocon(char uplo, int n, complex  *a,  int  lda,  float
          anorm, float *rcond, int *info);

void cpocon_64(char uplo, long  n,  complex  *a,  long  lda,
          float anorm, float *rcond, long *info);
```

PURPOSE

cpocon estimates the reciprocal of the condition number (in the 1-norm) of a complex Hermitian positive definite matrix using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPOTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H, as computed by CPOTRF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

ANORM (input)
The 1-norm (or infinity-norm) of the Hermitian matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
cpoequ - compute row and column scalings intended to equili-
brate  a Hermitian positive definite matrix A and reduce its
condition number (with respect to the two-norm)
```

SYNOPSIS

```
SUBROUTINE CPOEQU(N, A, LDA, SCALE, SCOND, AMAX, INFO)

COMPLEX A(LDA,*)
INTEGER N, LDA, INFO
REAL SCOND, AMAX
REAL SCALE(*)

SUBROUTINE CPOEQU_64(N, A, LDA, SCALE, SCOND, AMAX, INFO)

COMPLEX A(LDA,*)
INTEGER*8 N, LDA, INFO
REAL SCOND, AMAX
REAL SCALE(*)
```

F95 INTERFACE

```
SUBROUTINE POEQU([N], A, [LDA], SCALE, SCOND, AMAX, [INFO])

COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
REAL :: SCOND, AMAX
REAL, DIMENSION(:) :: SCALE

SUBROUTINE POEQU_64([N], A, [LDA], SCALE, SCOND, AMAX, [INFO])

COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
```

```
REAL :: SCOND, AMAX
REAL, DIMENSION(:) :: SCALE
```

C INTERFACE

```
#include <sunperf.h>

void cpoequ(int n, complex *a, int lda, float *scale,  float
          *scond, float *amax, int *info);

void cpoequ_64(long n, complex *a, long lda,  float  *scale,
          float *scond, float *amax, long *info);
```

PURPOSE

cpoequ computes row and column scalings intended to equilibrate a Hermitian positive definite matrix A and reduce its condition number (with respect to the two-norm). S contains the scale factors, S(i) = 1/sqrt(A(i,i)), chosen so that the scaled matrix B with elements B(i,j) = S(i)*A(i,j)*S(j) has ones on the diagonal. This choice of S puts the condition number of B within a factor N of the smallest possible condition number over all possible diagonal scalings.

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

A (input) The N-by-N Hermitian positive definite matrix whose scaling factors are to be computed. Only the diagonal elements of A are referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

SCALE (output)
If INFO = 0, SCALE contains the scale factors for A.

SCOND (output)
If INFO = 0, SCALE contains the ratio of the smallest SCALE(i) to the largest SCALE(i). If SCOND >= 0.1 and AMAX is neither too large nor too small, it is not worth scaling by SCALE.

AMAX (output)

Absolute value of largest matrix element. If AMAX is very close to overflow or very close to underflow, the matrix should be scaled.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the i-th diagonal element is nonpositive.

Contents

NAME

```
cporfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is Hermitian positive
definite,
```

SYNOPSIS

```
SUBROUTINE CPORFS(UPLO, N, NRHS, A, LDA, AF, LDAF, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL FERR(*), BERR(*), WORK2(*)

SUBROUTINE CPORFS_64(UPLO, N, NRHS, A, LDA, AF, LDAF, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE PORFS(UPLO, [N], [NRHS], A, [LDA], AF, [LDAF], B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

```
SUBROUTINE PORFS_64(UPLO, [N], [NRHS], A, [LDA], AF, [LDAF], B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cporfs(char uplo, int n, int nrhs, complex *a, int lda,
          complex  *af,  int ldaf, complex *b, int ldb, com-
          plex *x, int ldx, float *ferr,  float  *berr,  int
          *info);

void cporfs_64(char uplo, long n,  long  nrhs,  complex  *a,
          long lda, complex *af, long ldaf, complex *b, long
          ldb, complex *x,  long  ldx,  float  *ferr,  float
          *berr, long *info);
```

PURPOSE

cporfs improves the computed solution to a system of linear equations when the coefficient matrix is Hermitian positive definite, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular

part of the matrix A, and the strictly upper tri-
angular part of A is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input)
The triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H, as computed by CPOTRF.

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by CPOTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-
gal value

Contents

NAME

```
cposv - compute the solution to a complex system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE CPOSV(UPLO, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, NRHS, LDA, LDB, INFO

SUBROUTINE CPOSV_64(UPLO, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, NRHS, LDA, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE POSV(UPLO, [N], [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: N, NRHS, LDA, LDB, INFO

SUBROUTINE POSV_64(UPLO, [N], [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void cposv(char uplo, int n, int nrhs, complex *a, int  lda,
          complex *b, int ldb, int *info);

void cposv_64(char uplo, long n, long nrhs, complex *a, long
          lda, complex *b, long ldb, long *info);
```

PURPOSE

cposv computes the solution to a complex system of linear equations
   A * X = B, where A is an N-by-N Hermitian positive definite matrix and X and B are N-by-NRHS matrices.

The Cholesky decomposition is used to factor A as
   A = U**H* U, if UPLO = 'U', or
   A = L * L**H, if UPLO = 'L',
where U is an upper triangular matrix and L is a lower triangular matrix.   The factored form of A is then used to solve the system of equations A * X = B.

ARGUMENTS

UPLO (input)
          = 'U': Upper triangle of A is stored;
          = 'L': Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NRHS (input)
          The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input/output)
          On entry, the Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

          On exit, if INFO = 0, the factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed.

Contents

NAME

```
cposvx - use the Cholesky factorization A = U**H*U  or  A  =
L*L**H to compute the solution to a complex system of linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE CPOSVX(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, EQUED,
      S, B, LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO, EQUED
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL RCOND
REAL S(*), FERR(*), BERR(*), WORK2(*)

SUBROUTINE CPOSVX_64(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, EQUED,
      S, B, LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO, EQUED
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL RCOND
REAL S(*), FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE POSVX(FACT, UPLO, [N], [NRHS], A, [LDA], AF, [LDAF],
       EQUED, S, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
```

```
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL :: RCOND
REAL, DIMENSION(:) :: S, FERR, BERR, WORK2

SUBROUTINE POSVX_64(FACT, UPLO, [N], [NRHS], A, [LDA], AF, [LDAF],
       EQUED, S, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL :: RCOND
REAL, DIMENSION(:) :: S, FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cposvx(char fact, char uplo, int n, int  nrhs,  complex
          *a,  int  lda,  complex *af, int ldaf, char equed,
          float *s, complex *b, int  ldb,  complex  *x,  int
          ldx,  float  *rcond, float *ferr, float *berr, int
          *info);

void cposvx_64(char fact, char uplo, long n, long nrhs, com-
          plex  *a,  long  lda, complex *af, long ldaf, char
          equed, float *s, complex *b, long ldb, complex *x,
          long  ldx, float *rcond, float *ferr, float *berr,
          long *info);
```

PURPOSE

cposvx uses the Cholesky factorization A = U**H*U or A = L*L**H to compute the solution to a complex system of linear equations
   A * X = B, where A is an N-by-N Hermitian positive definite matrix and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'E', real scaling factors are computed to equilibrate
   the system:
      diag(S) * A * diag(S) * inv(diag(S)) * X = diag(S) * B
   Whether or not the system will be equilibrated depends on the
   scaling of the matrix A, but if equilibration is used, A

is
overwritten by diag(S)*A*diag(S) and B by diag(S)*B.

2. If FACT = 'N' or 'E', the Cholesky decomposition is used
to
factor the matrix A (after equilibration if FACT = 'E')
as
A = U**H* U, if UPLO = 'U', or
A = L * L**H, if UPLO = 'L',
where U is an upper triangular matrix and L is a lower
triangular
matrix.

3. If the leading i-by-i principal minor is not positive
definite,
then the routine returns with INFO = i. Otherwise, the
factored
form of A is used to estimate the condition number of the
matrix
A. If the reciprocal of the condition number is less
than machine
precision, INFO = N+1 is returned as a warning, but the
routine
still goes on to solve for X and compute error bounds as
described below.

4. The system of equations is solved for X using the fac-
tored form
of A.

5. Iterative refinement is applied to improve the computed
solution
matrix and calculate error bounds and backward error
estimates
for it.

6. If equilibration was used, the matrix X is premultiplied
by
diag(S) so that it solves the original system before
equilibration.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of the
matrix A is supplied on entry, and if not, whether
the matrix A should be equilibrated before it is
factored. = 'F': On entry, AF contains the fac-

```
          tored form of A.  If EQUED = 'Y', the matrix A has
          been equilibrated with scaling factors given by S.
          A and AF will not be modified.  = 'N':  The matrix
          A will be copied to AF and factored.
          = 'E':  The  matrix  A  will  be  equilibrated  if
          necessary, then copied to AF and factored.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.
A (input/output)
          On entry, the Hermitian matrix A, except if FACT =
          'F'  and  EQUED  =  'Y',  then  A must contain the
          equilibrated matrix diag(S)*A*diag(S).  If UPLO  =
          'U', the leading N-by-N upper triangular part of A
          contains the upper triangular part of  the  matrix
          A,  and the strictly lower triangular part of A is
          not referenced.  If UPLO = 'L', the leading N-by-N
          lower triangular part of A contains the lower tri-
          angular part of the matrix  A,  and  the  strictly
          upper  triangular  part of A is not referenced.  A
          is not modified if FACT = 'F' or 'N', or if FACT =
          'E' and EQUED = 'N' on exit.

          On exit, if FACT = 'E'  and  EQUED  =  'Y',  A  is
          overwritten by diag(S)*A*diag(S).

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

AF (input/output)
          If FACT = 'F', then AF is an input argument and on
          entry  contains  the triangular factor U or L from
          the Cholesky factorization  A  =  U**H*U  or  A  =
          L*L**H, in the same storage format as A.  If EQUED
          .ne. 'N', then AF is  the  factored  form  of  the
          equilibrated matrix diag(S)*A*diag(S).

          If FACT = 'N', then AF is an output  argument  and
          on  exit returns the triangular factor U or L from
          the Cholesky factorization  A  =  U**H*U  or  A  =
          L*L**H of the original matrix A.
```

If FACT = 'E', then AF is an output argument and on exit returns the triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H of the equilibrated matrix A (see the description of A for the form of the equilibrated matrix).

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

EQUED (input)
Specifies the form of equilibration that was done.
= 'N': No equilibration (always true if FACT = 'N').
= 'Y': Equilibration was done, i.e., A has been replaced by diag(S) * A * diag(S). EQUED is an input argument if FACT = 'F'; otherwise, it is an output argument.

S (input/output)
The scale factors for A; not accessed if EQUED = 'N'. S is an input argument if FACT = 'F'; otherwise, S is an output argument. If FACT = 'F' and EQUED = 'Y', each element of S must be positive.

B (input/output)
On entry, the N-by-NRHS righthand side matrix B. On exit, if EQUED = 'N', B is not modified; if EQUED = 'Y', B is overwritten by diag(S) * B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X to the original system of equations. Note that if EQUED = 'Y', A and B are modified on exit, and the solution to the equilibrated system is inv(diag(S))*X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A after equilibration (if done). If

RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
cpotf2 - compute the Cholesky  factorization  of  a  complex
Hermitian positive definite matrix A
```

SYNOPSIS

```
SUBROUTINE CPOTF2(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*)
INTEGER N, LDA, INFO

SUBROUTINE CPOTF2_64(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*)
INTEGER*8 N, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE POTF2(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO

SUBROUTINE POTF2_64(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void cpotf2(char uplo, int  n,  complex  *a,  int  lda,  int
          *info);

void cpotf2_64(char uplo, long n, complex *a, long lda, long
          *info);
```

PURPOSE

cpotf2 computes the Cholesky factorization of a complex Hermitian positive definite matrix A.

The factorization has the form

```
   A = U' * U ,  if UPLO = 'U', or
   A = L  * L',  if UPLO = 'L',
```

where U is an upper triangular matrix and L is lower triangular.

This is the unblocked version of the algorithm, calling Level 2 BLAS.

ARGUMENTS

UPLO (input)
Specifies whether the upper or lower triangular part of the Hermitian matrix A is stored. = 'U': Upper triangular
= 'L': Lower triangular

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the Hermitian matrix A. If UPLO = 'U', the leading n by n upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading n by n lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, if INFO = 0, the factor U or L from the Cholesky factorization A = U'*U or A = L*L'.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, the leading minor of order k is not positive definite, and the factorization could not be completed.

Contents

NAME

```
cpotrf - compute the Cholesky  factorization  of  a  complex
Hermitian positive definite matrix A
```

SYNOPSIS

```
SUBROUTINE CPOTRF(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*)
INTEGER N, LDA, INFO

SUBROUTINE CPOTRF_64(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*)
INTEGER*8 N, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE POTRF(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO

SUBROUTINE POTRF_64(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void cpotrf(char uplo, int  n,  complex  *a,  int  lda,  int
          *info);

void cpotrf_64(char uplo, long n, complex *a, long lda, long
          *info);
```

PURPOSE

cpotrf computes the Cholesky factorization of a complex Hermitian positive definite matrix A.

```
The factorization has the form
   A = U**H * U,  if UPLO = 'U', or
   A = L  * L**H,  if UPLO = 'L',
where U is an upper triangular matrix and L  is  lower  tri-
angular.
```

This is the block version of the algorithm, calling Level 3 BLAS.

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.
```

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, if INFO = 0, the factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H.

LDA (input)
The leading dimension of the array A.   LDA >= max(1,N).

INFO (output)

= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i is not positive definite, and the factorization could not be completed.

Contents

NAME

```
cpotri - compute the inverse of a complex Hermitian positive
definite  matrix  A  using  the  Cholesky  factorization A =
U**H*U or A = L*L**H computed by CPOTRF
```

SYNOPSIS

```
SUBROUTINE CPOTRI(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*)
INTEGER N, LDA, INFO

SUBROUTINE CPOTRI_64(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*)
INTEGER*8 N, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE POTRI(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO

SUBROUTINE POTRI_64(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cpotri(char uplo, int  n,  complex  *a,  int  lda,  int
          *info);

void cpotri_64(char uplo, long n, complex *a, long lda, long
          *info);
```

PURPOSE

cpotri computes the inverse of a complex Hermitian positive definite matrix A using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPOTRF.

ARGUMENTS

UPLO (input)
- = 'U': Upper triangle of A is stored;
- = 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H, as computed by CPOTRF. On exit, the upper or lower triangle of the (Hermitian) inverse of A, overwriting the input factor U or L.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

INFO (output)
- = 0: successful exit
- < 0: if INFO = -i, the i-th argument had an illegal value
- > 0: if INFO = i, the (i,i) element of the factor U or L is zero, and the inverse could not be computed.

Contents

NAME

cpotrs - solve a system of linear equations A*X = B with a Hermitian positive definite matrix A using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPOTRF

SYNOPSIS

```
SUBROUTINE CPOTRS(UPLO, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, NRHS, LDA, LDB, INFO

SUBROUTINE CPOTRS_64(UPLO, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, NRHS, LDA, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE POTRS(UPLO, [N], [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: N, NRHS, LDA, LDB, INFO

SUBROUTINE POTRS_64(UPLO, [N], [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cpotrs(char uplo, int n, int nrhs, complex *a, int lda,
          complex *b, int ldb, int *info);

void cpotrs_64(char uplo, long n,  long  nrhs,  complex  *a,
          long lda, complex *b, long ldb, long *info);
```

PURPOSE

cpotrs solves a system of linear equations A*X = B with a Hermitian positive definite matrix A using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPOTRF.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H, as computed by CPOTRF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
cppcon - estimate the reciprocal of the condition number (in
the  1-norm) of a complex Hermitian positive definite packed
matrix using the Cholesky factorization A = U**H*U  or  A  =
L*L**H computed by CPPTRF
```

SYNOPSIS

```
SUBROUTINE CPPCON(UPLO, N, A, ANORM, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), WORK(*)
INTEGER N, INFO
REAL ANORM, RCOND
REAL WORK2(*)

SUBROUTINE CPPCON_64(UPLO, N, A, ANORM, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), WORK(*)
INTEGER*8 N, INFO
REAL ANORM, RCOND
REAL WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE PPCON(UPLO, N, A, ANORM, RCOND, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A, WORK
INTEGER :: N, INFO
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: WORK2
```

```
SUBROUTINE PPCON_64(UPLO, N, A, ANORM, RCOND, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A, WORK
INTEGER(8) :: N, INFO
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cppcon(char uplo, int n, complex *a, float anorm, float
          *rcond, int *info);
void cppcon_64(char uplo, long n, complex *a,  float  anorm,
          float *rcond, long *info);
```

PURPOSE

cppcon estimates the reciprocal of the condition number (in the 1-norm) of a complex Hermitian positive definite packed matrix using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPPTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H, packed columnwise in a linear array. The j-th column of U or L is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = U(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = L(i,j) for j<=i<=n.

ANORM (input)
The 1-norm (or infinity-norm) of the Hermitian matrix A.

```
RCOND (output)
          The reciprocal of  the  condition  number  of  the
          matrix  A, computed as RCOND = 1/(ANORM * AINVNM),
          where AINVNM is  an  estimate  of  the  1-norm  of
          inv(A) computed in this routine.

WORK (workspace)
          dimension(2*N)

WORK2 (workspace)
          dimension(N)
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
cppequ - compute row and column scalings intended to equili-
brate  a  Hermitian  positive  definite  matrix  A in packed
storage and reduce its condition number (with respect to the
two-norm)
```

SYNOPSIS

```
SUBROUTINE CPPEQU(UPLO, N, A, SCALE, SCOND, AMAX, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*)
INTEGER N, INFO
REAL SCOND, AMAX
REAL SCALE(*)

SUBROUTINE CPPEQU_64(UPLO, N, A, SCALE, SCOND, AMAX, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*)
INTEGER*8 N, INFO
REAL SCOND, AMAX
REAL SCALE(*)
```

F95 INTERFACE

```
SUBROUTINE PPEQU(UPLO, [N], A, SCALE, SCOND, AMAX, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A
INTEGER :: N, INFO
REAL :: SCOND, AMAX
REAL, DIMENSION(:) :: SCALE
```

```
SUBROUTINE PPEQU_64(UPLO, [N], A, SCALE, SCOND, AMAX, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A
INTEGER(8) :: N, INFO
REAL :: SCOND, AMAX
REAL, DIMENSION(:) :: SCALE
```

C INTERFACE

```
#include <sunperf.h>

void cppequ(char uplo, int  n,  complex  *a,  float  *scale,
          float *scond, float *amax, int *info);
void cppequ_64(char uplo, long n, complex *a, float  *scale,
          float *scond, float *amax, long *info);
```

PURPOSE

cppequ computes row and column scalings intended to equilibrate a Hermitian positive definite matrix A in packed storage and reduce its condition number (with respect to the two-norm). S contains the scale factors, S(i)=1/sqrt(A(i,i)), chosen so that the scaled matrix B with elements B(i,j)=S(i)*A(i,j)*S(j) has ones on the diagonal. This choice of S puts the condition number of B within a factor N of the smallest possible condition number over all possible diagonal scalings.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input) The upper or lower triangle of the Hermitian matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n.

SCALE (output)
If INFO = 0, SCALE contains the scale factors for A.

SCOND (output)
If INFO = 0, SCALE contains the ratio of the smallest SCALE(i) to the largest SCALE(i). If SCOND >= 0.1 and AMAX is neither too large nor too small, it is not worth scaling by SCALE.

AMAX (output)
Absolute value of largest matrix element. If AMAX is very close to overflow or very close to underflow, the matrix should be scaled.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the i-th diagonal element is nonpositive.

Contents

NAME

```
cpprfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is Hermitian positive
definite and packed, and provides error bounds and  backward
error estimates for the solution
```

SYNOPSIS

```
SUBROUTINE CPPRFS(UPLO, N, NRHS, A, AF, B, LDB, X, LDX, FERR, BERR,
      WORK, WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
REAL FERR(*), BERR(*), WORK2(*)

SUBROUTINE CPPRFS_64(UPLO, N, NRHS, A, AF, B, LDB, X, LDX, FERR,
      BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
REAL FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE PPRFS(UPLO, N, [NRHS], A, AF, B, [LDB], X, [LDX], FERR,
       BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A, AF, WORK
COMPLEX, DIMENSION(:,:) :: B, X
INTEGER :: N, NRHS, LDB, LDX, INFO
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

```
SUBROUTINE PPRFS_64(UPLO, N, [NRHS], A, AF, B, [LDB], X, [LDX], FERR,
       BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A, AF, WORK
COMPLEX, DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cpprfs(char uplo, int n, int nrhs, complex *a,  complex
          *af,  complex  *b,  int  ldb, complex *x, int ldx,
          float *ferr, float *berr, int *info);

void cpprfs_64(char uplo, long n,  long  nrhs,  complex  *a,
          complex  *af,  complex  *b,  long ldb, complex *x,
          long ldx, float *ferr, float *berr, long *info);
```

PURPOSE

cpprfs improves the computed solution to a system of linear equations when the coefficient matrix is Hermitian positive definite and packed, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The upper or lower triangle of the Hermitian matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n.

```
AF (input)
          The triangular factor U or  L  from  the  Cholesky
          factorization  A  =  U**H*U or A = L*L**H, as com-
          puted by SPPTRF/CPPTRF,  packed  columnwise  in  a
          linear array in the same format as A (see A).

B (input) The right hand side matrix B.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).
X (input/output)
          On entry, the solution matrix X,  as  computed  by
          CPPTRS.  On exit, the improved solution matrix X.

LDX (input)
          The leading dimension of  the  array  X.   LDX  >=
          max(1,N).

FERR (output)
          The estimated forward error bound for  each  solu-
          tion  vector X(j) (the j-th column of the solution
          matrix  X).   If  XTRUE  is  the   true   solution
          corresponding  to  X(j),  FERR(j)  is an estimated
          upper bound for the magnitude of the largest  ele-
          ment in (X(j) - XTRUE) divided by the magnitude of
          the largest element in X(j).  The estimate  is  as
          reliable  as the estimate for RCOND, and is almost
          always a slight overestimate of the true error.

BERR (output)
          The componentwise relative backward error of  each
          solution  vector X(j) (i.e., the smallest relative
          change in any element of A or B that makes X(j) an
          exact solution).

WORK (workspace)
          dimension(2*N)

WORK2 (workspace)
          dimension(N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
cppsv - compute the solution to a complex system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE CPPSV(UPLO, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO

SUBROUTINE CPPSV_64(UPLO, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE PPSV(UPLO, N, [NRHS], A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A
COMPLEX, DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO

SUBROUTINE PPSV_64(UPLO, N, [NRHS], A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A
COMPLEX, DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cppsv(char uplo, int n, int nrhs, complex  *a,  complex
          *b, int ldb, int *info);

void cppsv_64(char uplo, long n, long nrhs, complex *a, com-
          plex *b, long ldb, long *info);
```

PURPOSE

```
cppsv computes the solution to a complex  system  of  linear
equations
   A * X = B, where A is an N-by-N Hermitian positive defin-
ite matrix stored in packed format and X and B are N-by-NRHS
matrices.

The Cholesky decomposition is used to factor A as
   A = U**H* U,  if UPLO = 'U', or
   A = L * L**H,  if UPLO = 'L',
where U is an upper triangular matrix and L is a lower  tri-
angular  matrix.   The  factored  form  of A is then used to
solve the system of equations A * X = B.
```

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

A (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  matrix  A,  packed  columnwise in a linear
          array.  The j-th column of  A  is  stored  in  the
          array  A  as  follows:   if  UPLO = 'U', A(i + (j-
          1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i +
          (j-1)*(2n-j)/2)  =  A(i,j) for j<=i<=n.  See below
          for further details.
```

```
            On exit, if INFO = 0, the factor U or L  from  the
            Cholesky  factorization  A = U**H*U or A = L*L**H,
            in the same storage format as A.

B (input/output)
            On entry, the N-by-NRHS right hand side matrix  B.
            On  exit,  if  INFO  =  0,  the N-by-NRHS solution
            matrix X.

LDB (input)
            The leading dimension of  the  array  B.   LDB  >=
            max(1,N).

INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value
            > 0:  if INFO = i, the leading minor of order i of
            A  is  not positive definite, so the factorization
            could not be completed, and the solution  has  not
            been computed.
```

FURTHER DETAILS

```
The packed storage scheme is illustrated  by  the  following
example when N = 4, UPLO = 'U':

Two-dimensional storage of the Hermitian matrix A:

   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = conjg(aji))
               a44

Packed storage of the upper triangle of A:

A = [ a11, a12, a22, a13, a23, a33, a14, a24, a34, a44 ]
```

Contents

NAME

```
cppsvx - use the Cholesky factorization A = U**H*U  or  A  =
L*L**H to compute the solution to a complex system of linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE CPPSVX(FACT, UPLO, N, NRHS, A, AF, EQUED, S, B, LDB,
      X, LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO, EQUED
COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
REAL RCOND
REAL S(*), FERR(*), BERR(*), WORK2(*)

SUBROUTINE CPPSVX_64(FACT, UPLO, N, NRHS, A, AF, EQUED, S, B,
      LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO, EQUED
COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
REAL RCOND
REAL S(*), FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE PPSVX(FACT, UPLO, [N], [NRHS], A, AF, EQUED, S, B,
       [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
COMPLEX, DIMENSION(:) :: A, AF, WORK
COMPLEX, DIMENSION(:,:) :: B, X
```

```
INTEGER :: N, NRHS, LDB, LDX, INFO
REAL :: RCOND
REAL, DIMENSION(:) :: S, FERR, BERR, WORK2

SUBROUTINE PPSVX_64(FACT, UPLO, [N], [NRHS], A, AF, EQUED, S, B,
       [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
COMPLEX, DIMENSION(:) :: A, AF, WORK
COMPLEX, DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
REAL :: RCOND
REAL, DIMENSION(:) :: S, FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>
void cppsvx(char fact, char uplo, int n, int  nrhs,  complex
          *a, complex *af, char equed, float *s, complex *b,
          int ldb, complex *x, int ldx, float *rcond,  float
          *ferr, float *berr, int *info);

void cppsvx_64(char fact, char uplo, long n, long nrhs, com-
          plex  *a,  complex *af, char equed, float *s, com-
          plex *b, long ldb, complex  *x,  long  ldx,  float
          *rcond, float *ferr, float *berr, long *info);
```

PURPOSE

cppsvx uses the Cholesky factorization A = U**H*U or A = L*L**H to compute the solution to a complex system of linear equations
   A * X = B, where A is an N-by-N Hermitian positive definite matrix stored in packed format and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'E', real scaling factors are computed to equilibrate
   the system:
      diag(S) * A * diag(S) * inv(diag(S)) * X = diag(S) * B
   Whether or not the system will be equilibrated depends on the
   scaling of the matrix A, but if equilibration is used, A is
   overwritten by diag(S)*A*diag(S) and B by diag(S)*B.

2. If FACT = 'N' or 'E', the Cholesky decomposition is used to
factor the matrix A (after equilibration if FACT = 'E') as
A = U'* U , if UPLO = 'U', or
A = L * L', if UPLO = 'L',
where U is an upper triangular matrix, L is a lower triangular
matrix, and ' indicates conjugate transpose.

3. If the leading i-by-i principal minor is not positive definite,
then the routine returns with INFO = i. Otherwise, the factored
form of A is used to estimate the condition number of the matrix
A. If the reciprocal of the condition number is less than machine
precision, INFO = N+1 is returned as a warning, but the routine
still goes on to solve for X and compute error bounds as
described below.

4. The system of equations is solved for X using the factored form
of A.

5. Iterative refinement is applied to improve the computed solution
matrix and calculate error bounds and backward error estimates
for it.

6. If equilibration was used, the matrix X is premultiplied by
diag(S) so that it solves the original system before
equilibration.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored. = 'F': On entry, AF contains the factored form of A. If EQUED = 'Y', the matrix A has been equilibrated with scaling factors given by S.

```
          A and AF will not be modified.  = 'N':  The matrix
          A will be copied to AF and factored.
          = 'E':  The  matrix  A  will  be  equilibrated  if
          necessary, then copied to AF and factored.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input/output)
          On entry, the  upper  or  lower  triangle  of  the
          Hermitian  matrix A, packed columnwise in a linear
          array, except if FACT = 'F' and EQUED = 'Y',  then
          A    must    contain   the   equilibrated   matrix
          diag(S)*A*diag(S).  The j-th column of A is stored
          in  the  array A as follows:  if UPLO = 'U', A(i +
          (j-1)*j/2) = A(i,j) for 1<=i<=j; if  UPLO  =  'L',
          A(i  +  (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n.  See
          below for further details.  A is not  modified  if
          FACT  =  'F'  or 'N', or if FACT = 'E' and EQUED =
          'N' on exit.

          On exit, if FACT = 'E'  and  EQUED  =  'Y',  A  is
          overwritten by diag(S)*A*diag(S).

AF (input/output)
          If FACT = 'F', then AF is an input argument and on
          entry  contains  the triangular factor U or L from
          the Cholesky factorization  A  =  U**H*U  or  A  =
          L*L**H, in the same storage format as A.  If EQUED
          .ne. 'N', then AF is  the  factored  form  of  the
          equilibrated matrix A.

          If FACT = 'N', then AF is an output  argument  and
          on  exit returns the triangular factor U or L from
          the Cholesky factorization  A  =  U**H*U  or  A  =
          L*L**H of the original matrix A.

          If FACT = 'E', then AF is an output  argument  and
          on  exit returns the triangular factor U or L from
          the Cholesky factorization  A  =  U**H*U  or  A  =
          L*L**H  of  the  equilibrated  matrix  A  (see the
          description of A for the form of the  equilibrated
```

matrix).

EQUED (input)
Specifies the form of equilibration that was done.
= 'N': No equilibration (always true if FACT = 'N').
= 'Y': Equilibration was done, i.e., A has been replaced by diag(S) * A * diag(S). EQUED is an input argument if FACT = 'F'; otherwise, it is an output argument.

S (input/output)
The scale factors for A; not accessed if EQUED = 'N'. S is an input argument if FACT = 'F'; otherwise, S is an output argument. If FACT = 'F' and EQUED = 'Y', each element of S must be positive.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if EQUED = 'N', B is not modified; if EQUED = 'Y', B is overwritten by diag(S) * B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X to the original system of equations. Note that if EQUED = 'Y', A and B are modified on exit, and the solution to the equilibrated system is inv(diag(S))*X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A after equilibration (if done). If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest ele-

ment in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)

The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)

dimension(2*N)

WORK2 (workspace)

dimension(N)

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, and i is

<= N: the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

FURTHER DETAILS

The packed storage scheme is illustrated by the following example when N = 4, UPLO = 'U':

Two-dimensional storage of the Hermitian matrix A:

```
   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = conjg(aji))
               a44
```

Packed storage of the upper triangle of A:

A = [a11, a12, a22, a13, a23, a33, a14, a24, a34, a44]

Contents

NAME

```
cpptrf - compute the Cholesky  factorization  of  a  complex
Hermitian positive definite matrix A stored in packed format
```

SYNOPSIS

```
SUBROUTINE CPPTRF(UPLO, N, A, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*)
INTEGER N, INFO

SUBROUTINE CPPTRF_64(UPLO, N, A, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*)
INTEGER*8 N, INFO
```

F95 INTERFACE

```
SUBROUTINE PPTRF(UPLO, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A
INTEGER :: N, INFO

SUBROUTINE PPTRF_64(UPLO, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A
INTEGER(8) :: N, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cpptrf(char uplo, int n, complex *a, int *info);

void cpptrf_64(char uplo, long n, complex *a, long *info);
```

PURPOSE

```
cpptrf computes the Cholesky factorization of a complex Her-
mitian positive definite matrix A stored in packed format.

The factorization has the form
   A = U**H * U,  if UPLO = 'U', or
   A = L  * L**H,  if UPLO = 'L',
where U is an upper triangular matrix and L  is  lower  tri-
angular.
```

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  matrix  A,  packed  columnwise in a linear
          array.  The j-th column of  A  is  stored  in  the
          array  A  as  follows:   if  UPLO = 'U', A(i + (j-
          1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i +
          (j-1)*(2n-j)/2)  =  A(i,j) for j<=i<=n.  See below
          for further details.

          On exit, if INFO = 0, the triangular factor U or L
          from  the Cholesky factorization A = U**H*U or A =
          L*L**H, in the same storage format as A.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, the leading minor of order i is
          not positive definite, and the factorization could
          not be completed.
```

FURTHER DETAILS

```
The packed storage scheme is illustrated  by  the  following
example when N = 4, UPLO = 'U':

Two-dimensional storage of the Hermitian matrix A:

   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = conjg(aji))
               a44

Packed storage of the upper triangle of A:

A = [ a11, a12, a22, a13, a23, a33, a14, a24, a34, a44 ]
```

Contents

NAME

```
cpptri - compute the inverse of a complex Hermitian positive
definite  matrix  A  using  the  Cholesky  factorization A =
U**H*U or A = L*L**H computed by CPPTRF
```

SYNOPSIS

```
SUBROUTINE CPPTRI(UPLO, N, A, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*)
INTEGER N, INFO

SUBROUTINE CPPTRI_64(UPLO, N, A, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*)
INTEGER*8 N, INFO
```

F95 INTERFACE

```
SUBROUTINE PPTRI(UPLO, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A
INTEGER :: N, INFO

SUBROUTINE PPTRI_64(UPLO, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A
INTEGER(8) :: N, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cpptri(char uplo, int n, complex *a, int *info);

void cpptri_64(char uplo, long n, complex *a, long *info);
```

PURPOSE

cpptri computes the inverse of a complex Hermitian positive definite matrix A using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPPTRF.

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangular factor is stored in A;
          = 'L':  Lower triangular factor is stored in A.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the triangular factor U or  L  from  the
          Cholesky  factorization  A = U**H*U or A = L*L**H,
          packed columnwise as a  linear  array.   The  j-th
          column  of U or L is stored in the array A as fol-
          lows:  if UPLO = 'U', A(i +  (j-1)*j/2)  =  U(i,j)
          for  1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2)
          = L(i,j) for j<=i<=n.

          On exit, the upper or lower triangle of the  (Her-
          mitian) inverse of A, overwriting the input factor
          U or L.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, the (i,i) element of the factor
          U  or L is zero, and the inverse could not be com-
          puted.
```

Contents

NAME

```
cpptrs - solve a system of linear equations A*X = B  with  a
Hermitian positive definite matrix A in packed storage using
the Cholesky factorization A = U**H*U or A = L*L**H computed
by CPPTRF
```

SYNOPSIS

```
SUBROUTINE CPPTRS(UPLO, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO

SUBROUTINE CPPTRS_64(UPLO, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE PPTRS(UPLO, N, [NRHS], A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A
COMPLEX, DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO

SUBROUTINE PPTRS_64(UPLO, N, [NRHS], A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A
COMPLEX, DIMENSION(:,:) :: B
```

```
INTEGER(8) :: N, NRHS, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cpptrs(char uplo, int n, int nrhs, complex *a,  complex
          *b, int ldb, int *info);

void cpptrs_64(char uplo, long n,  long  nrhs,  complex  *a,
          complex *b, long ldb, long *info);
```

PURPOSE

cpptrs solves a system of linear equations A*X = B with a Hermitian positive definite matrix A in packed storage using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPPTRF.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H, packed columnwise in a linear array. The j-th column of U or L is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = U(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = L(i,j) for j<=i<=n.

B (input/output)
On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)

= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-
gal value

Contents

NAME

```
cptcon - compute the reciprocal of the condition number  (in
the  1-norm) of a complex Hermitian positive definite tridi-
agonal matrix using the factorization A = L*D*L**H  or  A  =
U**H*D*U computed by CPTTRF
```

SYNOPSIS

```
SUBROUTINE CPTCON(N, DIAG, OFFD, ANORM, RCOND, WORK, INFO)

COMPLEX OFFD(*)
INTEGER N, INFO
REAL ANORM, RCOND
REAL DIAG(*), WORK(*)

SUBROUTINE CPTCON_64(N, DIAG, OFFD, ANORM, RCOND, WORK, INFO)

COMPLEX OFFD(*)
INTEGER*8 N, INFO
REAL ANORM, RCOND
REAL DIAG(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PTCON([N], DIAG, OFFD, ANORM, RCOND, [WORK], [INFO])

COMPLEX, DIMENSION(:) :: OFFD
INTEGER :: N, INFO
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: DIAG, WORK

SUBROUTINE PTCON_64([N], DIAG, OFFD, ANORM, RCOND, [WORK], [INFO])
```

```
COMPLEX, DIMENSION(:) :: OFFD
INTEGER(8) :: N, INFO
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: DIAG, WORK
```

C INTERFACE

```
#include <sunperf.h>

void cptcon(int n, float *diag, complex *offd, float  anorm,
          float *rcond, int *info);

void cptcon_64(long n, float  *diag,  complex  *offd,  float
          anorm, float *rcond, long *info);
```

PURPOSE

cptcon computes the reciprocal of the condition number (in the 1-norm) of a complex Hermitian positive definite tridiagonal matrix using the factorization A = L*D*L**H or A = U**H*D*U computed by CPTTRF.

Norm(inv(A)) is computed by a direct method, and the reciprocal of the condition number is computed as

RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

DIAG (input)
The n diagonal elements of the diagonal matrix DIAG from the factorization of A, as computed by CPTTRF.

OFFD (input)
The (n-1) off-diagonal elements of the unit bidiagonal factor U or L from the factorization of A, as computed by CPTTRF.

ANORM (input)
The 1-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is the 1-norm of inv(A) computed in this routine.

WORK (workspace)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The method used is described in Nicholas J. Higham, "Efficient Algorithms for Computing the Condition Number of a Tridiagonal Matrix", SIAM J. Sci. Stat. Comput., Vol. 7, No. 1, January 1986.

Contents

NAME

```
cpteqr - compute all eigenvalues and, optionally,  eigenvec-
tors  of a symmetric positive definite tridiagonal matrix by
first factoring the matrix using  SPTTRF  and  then  calling
CBDSQR to compute the singular values of the bidiagonal fac-
tor
```

SYNOPSIS

```
SUBROUTINE CPTEQR(COMPZ, N, D, E, Z, LDZ, WORK, INFO)

CHARACTER * 1 COMPZ
COMPLEX Z(LDZ,*)
INTEGER N, LDZ, INFO
REAL D(*), E(*), WORK(*)

SUBROUTINE CPTEQR_64(COMPZ, N, D, E, Z, LDZ, WORK, INFO)

CHARACTER * 1 COMPZ
COMPLEX Z(LDZ,*)
INTEGER*8 N, LDZ, INFO
REAL D(*), E(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PTEQR(COMPZ, [N], D, E, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
COMPLEX, DIMENSION(:,:) :: Z
INTEGER :: N, LDZ, INFO
REAL, DIMENSION(:) :: D, E, WORK

SUBROUTINE PTEQR_64(COMPZ, [N], D, E, Z, [LDZ], [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: COMPZ
COMPLEX, DIMENSION(:,:) :: Z
INTEGER(8) :: N, LDZ, INFO
REAL, DIMENSION(:) :: D, E, WORK
```

C INTERFACE

```
#include <sunperf.h>

void cpteqr(char compz, int n, float *d, float  *e,  complex
          *z, int ldz, int *info);

void cpteqr_64(char compz, long n, float *d, float *e,  com-
          plex *z, long ldz, long *info);
```

PURPOSE

cpteqr computes all eigenvalues and, optionally, eigenvectors of a symmetric positive definite tridiagonal matrix by first factoring the matrix using SPTTRF and then calling CBDSQR to compute the singular values of the bidiagonal factor.

This routine computes the eigenvalues of the positive definite tridiagonal matrix to high relative accuracy. This means that if the eigenvalues range over many orders of magnitude in size, then the small eigenvalues and corresponding eigenvectors will be computed more accurately than, for example, with the standard QR method.

The eigenvectors of a full or band positive definite Hermitian matrix can also be found if CHETRD, CHPTRD, or CHBTRD has been used to reduce this matrix to tridiagonal form. (The reduction to tridiagonal form, however, may preclude the possibility of obtaining high relative accuracy in the small eigenvalues of the original matrix, if these eigenvalues range over many orders of magnitude.)

ARGUMENTS

COMPZ (input)
= 'N': Compute eigenvalues only.
= 'V': Compute eigenvectors of original Hermitian matrix also. Array Z contains the unitary matrix used to reduce the original matrix to tridiagonal form. = 'I': Compute eigenvectors of tridiagonal matrix also.

```
N (input) The order of the matrix.  N >= 0.

D (input/output)
          On entry, the n diagonal elements of the tridiago-
          nal matrix.  On normal exit, D contains the eigen-
          values, in descending order.

E (input/output)
          On entry, the (n-1) subdiagonal  elements  of  the
          tridiagonal  matrix.   On  exit,  E  has been des-
          troyed.

Z (input) On entry, if COMPZ = 'V', the unitary matrix  used
          in the reduction to tridiagonal form.  On exit, if
          COMPZ = 'V', the orthonormal eigenvectors  of  the
          original  Hermitian  matrix;  if  COMPZ = 'I', the
          orthonormal  eigenvectors   of   the   tridiagonal
          matrix.   If  INFO  >  0  on  exit, Z contains the
          eigenvectors  associated  with  only  the   stored
          eigenvalues.   If   COMPZ  =  'N',  then  Z is not
          referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if COMPZ = 'V' or 'I', LDZ >= max(1,N).

WORK (workspace)
          dimension(4*N)

INFO (output)
          = 0:  successful exit.
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value.
          > 0:  if INFO = i, and i is:  <= N   the  Cholesky
          factorization of the matrix could not be performed
          because the i-th principal minor was not  positive
          definite.   > N   the SVD algorithm failed to con-
          verge; if INFO = N+i, i off-diagonal  elements  of
          the bidiagonal factor did not converge to zero.
```

Contents

NAME

```
cptrfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is Hermitian positive
definite and tridiagonal,  and  provides  error  bounds  and
backward error estimates for the solution
```

SYNOPSIS

```
SUBROUTINE CPTRFS(UPLO, N, NRHS, DIAG, OFFD, DIAGF, OFFDF, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX OFFD(*), OFFDF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
REAL DIAG(*), DIAGF(*), FERR(*), BERR(*), WORK2(*)

SUBROUTINE CPTRFS_64(UPLO, N, NRHS, DIAG, OFFD, DIAGF, OFFDF, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX OFFD(*), OFFDF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
REAL DIAG(*), DIAGF(*), FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE PTRFS(UPLO, [N], [NRHS], DIAG, OFFD, DIAGF, OFFDF, B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: OFFD, OFFDF, WORK
COMPLEX, DIMENSION(:,:) :: B, X
INTEGER :: N, NRHS, LDB, LDX, INFO
REAL, DIMENSION(:) :: DIAG, DIAGF, FERR, BERR, WORK2
```

```
SUBROUTINE PTRFS_64(UPLO, [N], [NRHS], DIAG, OFFD, DIAGF, OFFDF, B,
       [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: OFFD, OFFDF, WORK
COMPLEX, DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
REAL, DIMENSION(:) :: DIAG, DIAGF, FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cptrfs(char uplo, int n, int nrhs, float *diag, complex
          *offd,  float  *diagf, complex *offdf, complex *b,
          int ldb, complex *x, int ldx, float  *ferr,  float
          *berr, int *info);

void cptrfs_64(char uplo, long n, long  nrhs,  float  *diag,
          complex  *offd, float *diagf, complex *offdf, com-
          plex *b, long ldb, complex  *x,  long  ldx,  float
          *ferr, float *berr, long *info);
```

PURPOSE

cptrfs improves the computed solution to a system of linear equations when the coefficient matrix is Hermitian positive definite and tridiagonal, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

UPLO (input)
Specifies whether the superdiagonal or the subdiagonal of the tridiagonal matrix A is stored and the form of the factorization:
= 'U': OFFD is the superdiagonal of A, and A = U**H*DIAG*U;
= 'L': OFFD is the subdiagonal of A, and A = L*DIAG*L**H. (The two forms are equivalent if A is real.)

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

DIAG (input)
The n real diagonal elements of the tridiagonal matrix A.

OFFD (input)
The (n-1) off-diagonal elements of the tridiagonal matrix A (see UPLO).

DIAGF (input)
The n diagonal elements of the diagonal matrix DIAG from the factorization computed by CPTTRF.

OFFDF (input)
The (n-1) off-diagonal elements of the unit bidiagonal factor U or L from the factorization computed by CPTTRF (see UPLO).

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by CPTTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j).

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(N)

WORK2 (workspace)

```
          dimension(N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
cptsv - compute the solution to a complex system  of  linear
equations  A*X  = B, where A is an N-by-N Hermitian positive
definite tridiagonal matrix,  and  X  and  B  are  N-by-NRHS
matrices.
```

SYNOPSIS

```
SUBROUTINE CPTSV(N, NRHS, DIAG, SUB, B, LDB, INFO)

COMPLEX SUB(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO
REAL DIAG(*)

SUBROUTINE CPTSV_64(N, NRHS, DIAG, SUB, B, LDB, INFO)

COMPLEX SUB(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
REAL DIAG(*)
```

F95 INTERFACE

```
SUBROUTINE PTSV([N], [NRHS], DIAG, SUB, B, [LDB], [INFO])

COMPLEX, DIMENSION(:) :: SUB
COMPLEX, DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO
REAL, DIMENSION(:) :: DIAG

SUBROUTINE PTSV_64([N], [NRHS], DIAG, SUB, B, [LDB], [INFO])

COMPLEX, DIMENSION(:) :: SUB
COMPLEX, DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
```

```
REAL, DIMENSION(:) :: DIAG
```

C INTERFACE

```
#include <sunperf.h>

void cptsv(int n, int nrhs, float *diag, complex *sub,  com-
          plex *b, int ldb, int *info);

void cptsv_64(long n, long nrhs, float *diag, complex  *sub,
          complex *b, long ldb, long *info);
```

PURPOSE

cptsv computes the solution to a complex system of linear equations A*X = B, where A is an N-by-N Hermitian positive definite tridiagonal matrix, and X and B are N-by-NRHS matrices.

A is factored as A = L*D*L**H, and the factored form of A is then used to solve the system of equations.

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

DIAG (input/output)
On entry, the n diagonal elements of the tridiagonal matrix A. On exit, the n diagonal elements of the diagonal matrix DIAG from the factorization A = L*DIAG*L**H.

SUB (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix A. On exit, the (n-1) subdiagonal elements of the unit bidiagonal factor L from the L*DIAG*L**H factorization of A. SUB can also be regarded as the superdiagonal of the unit bidiagonal factor U from the U**H*DIAG*U factorization of A.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution

matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i is not positive definite, and the solution has not been computed. The factorization has not been completed unless i = N.

Contents

NAME

```
cptsvx - use the factorization A = L*D*L**H to  compute  the
solution  to  a  complex system of linear equations A*X = B,
where A is an N-by-N Hermitian positive definite tridiagonal
matrix and X and B are N-by-NRHS matrices
```

SYNOPSIS

```
SUBROUTINE CPTSVX(FACT, N, NRHS, DIAG, SUB, DIAGF, SUBF, B, LDB, X,
      LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT
COMPLEX SUB(*), SUBF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
REAL RCOND
REAL DIAG(*), DIAGF(*), FERR(*), BERR(*), WORK2(*)

SUBROUTINE CPTSVX_64(FACT, N, NRHS, DIAG, SUB, DIAGF, SUBF, B, LDB,
      X, LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT
COMPLEX SUB(*), SUBF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
REAL RCOND
REAL DIAG(*), DIAGF(*), FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE PTSVX(FACT, [N], [NRHS], DIAG, SUB, DIAGF, SUBF, B, [LDB],
       X, [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT
COMPLEX, DIMENSION(:) :: SUB, SUBF, WORK
COMPLEX, DIMENSION(:,:) :: B, X
```

```
INTEGER :: N, NRHS, LDB, LDX, INFO
REAL :: RCOND
REAL, DIMENSION(:) :: DIAG, DIAGF, FERR, BERR, WORK2

SUBROUTINE PTSVX_64(FACT, [N], [NRHS], DIAG, SUB, DIAGF, SUBF, B,
       [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT
COMPLEX, DIMENSION(:) :: SUB, SUBF, WORK
COMPLEX, DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
REAL :: RCOND
REAL, DIMENSION(:) :: DIAG, DIAGF, FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cptsvx(char fact, int n, int nrhs, float *diag, complex
          *sub, float *diagf, complex *subf, complex *b, int
          ldb, complex *x,  int  ldx,  float  *rcond,  float
          *ferr, float *berr, int *info);

void cptsvx_64(char fact, long n, long  nrhs,  float  *diag,
          complex *sub, float *diagf, complex *subf, complex
          *b, long ldb, complex *x, long ldx, float  *rcond,
          float *ferr, float *berr, long *info);
```

PURPOSE

```
cptsvx uses the factorization A = L*D*L**H  to  compute  the
solution  to  a  complex system of linear equations A*X = B,
where A is an N-by-N Hermitian positive definite tridiagonal
matrix and X and B are N-by-NRHS matrices.

Error bounds on the solution and a  condition  estimate  are
also provided.

The following steps are performed:

1. If FACT = 'N', the matrix A is factored as A =  L*D*L**H,
where L
   is a unit lower bidiagonal matrix and D is diagonal.  The
   factorization can also be regarded as having the form
   A = U**H*D*U.

2. If the leading i-by-i principal  minor  is  not  positive
definite,
   then the routine returns with INFO =  i.  Otherwise,  the
factored
   form of A is used to estimate the condition number of the
```

matrix
A. If the reciprocal of the condition number is less than machine
precision, INFO = N+1 is returned as a warning, but the routine
still goes on to solve for X and compute error bounds as described below.

3. The system of equations is solved for X using the factored form
of A.

4. Iterative refinement is applied to improve the computed solution
matrix and calculate error bounds and backward error estimates
for it.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of the matrix A is supplied on entry. = 'F': On entry, DIAGF and SUBF contain the factored form of A. DIAG, SUB, DIAGF, and SUBF will not be modified. = 'N': The matrix A will be copied to DIAGF and SUBF and factored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

DIAG (input)
The n diagonal elements of the tridiagonal matrix A.

SUB (input)
The (n-1) subdiagonal elements of the tridiagonal matrix A.

DIAGF (input/output)
If FACT = 'F', then DIAGF is an input argument and on entry contains the n diagonal elements of the diagonal matrix DIAG from the L*DIAG*L**H factorization of A. If FACT = 'N', then DIAGF is an output argument and on exit contains the n diagonal

elements of the diagonal matrix DIAG from the L*DIAG*L**H factorization of A.

SUBF (input/output)
If FACT = 'F', then SUBF is an input argument and on entry contains the (n-1) subdiagonal elements of the unit bidiagonal factor L from the L*DIAG*L**H factorization of A. If FACT = 'N', then SUBF is an output argument and on exit contains the (n-1) subdiagonal elements of the unit bidiagonal factor L from the L*DIAG*L**H factorization of A.

B (input) The N-by-NRHS right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The reciprocal condition number of the matrix A. If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j).

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
cpttrf - compute the L*D*L' factorization of a complex  Her-
mitian positive definite tridiagonal matrix A
```

SYNOPSIS

```
SUBROUTINE CPTTRF(N, DIAG, OFFD, INFO)

COMPLEX OFFD(*)
INTEGER N, INFO
REAL DIAG(*)

SUBROUTINE CPTTRF_64(N, DIAG, OFFD, INFO)

COMPLEX OFFD(*)
INTEGER*8 N, INFO
REAL DIAG(*)
```

F95 INTERFACE

```
SUBROUTINE PTTRF([N], DIAG, OFFD, [INFO])

COMPLEX, DIMENSION(:) :: OFFD
INTEGER :: N, INFO
REAL, DIMENSION(:) :: DIAG

SUBROUTINE PTTRF_64([N], DIAG, OFFD, [INFO])

COMPLEX, DIMENSION(:) :: OFFD
INTEGER(8) :: N, INFO
REAL, DIMENSION(:) :: DIAG
```

C INTERFACE

```
#include <sunperf.h>
```

```
void cpttrf(int n, float *diag, complex *offd, int *info);

void cpttrf_64(long n,  float  *diag,  complex  *offd,  long
          *info);
```

PURPOSE

cpttrf computes the L*D*L' factorization of a complex Hermitian positive definite tridiagonal matrix A. The factorization may also be regarded as having the form A = U'*D*U.

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

DIAG (input/output)
On entry, the n diagonal elements of the tridiagonal matrix A. On exit, the n diagonal elements of the diagonal matrix DIAG from the L*DIAG*L' factorization of A.

OFFD (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix A. On exit, the (n-1) subdiagonal elements of the unit bidiagonal factor L from the L*DIAG*L' factorization of A. OFFD can also be regarded as the superdiagonal of the unit bidiagonal factor U from the U'*DIAG*U factorization of A.

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, the leading minor of order k is not positive definite; if k < N, the factorization could not be completed, while if k = N, the factorization was completed, but DIAG(N) = 0.

Contents

NAME

```
cpttrs - solve a tridiagonal system of the form  A * X  =  B
using the factorization A = U'*D*U or A = L*D*L' computed by
CPTTRF
```

SYNOPSIS

```
SUBROUTINE CPTTRS(UPLO, N, NRHS, DIAG, OFFD, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX OFFD(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO
REAL DIAG(*)

SUBROUTINE CPTTRS_64(UPLO, N, NRHS, DIAG, OFFD, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX OFFD(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
REAL DIAG(*)
```

F95 INTERFACE

```
SUBROUTINE PTTRS(UPLO, [N], [NRHS], DIAG, OFFD, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: OFFD
COMPLEX, DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO
REAL, DIMENSION(:) :: DIAG

SUBROUTINE PTTRS_64(UPLO, [N], [NRHS], DIAG, OFFD, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX, DIMENSION(:) :: OFFD
COMPLEX, DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
REAL, DIMENSION(:) :: DIAG
```

C INTERFACE

```
#include <sunperf.h>

void cpttrs(char uplo, int n, int nrhs, float *diag, complex
          *offd, complex *b, int ldb, int *info);

void cpttrs_64(char uplo, long n, long  nrhs,  float  *diag,
          complex *offd, complex *b, long ldb, long *info);
```

PURPOSE

cpttrs solves a tridiagonal system of the form
A * X = B using the factorization A = U'*D*U or A = L*D*L' computed by CPTTRF. D is a diagonal matrix specified in the vector D, U (or L) is a unit bidiagonal matrix whose superdiagonal (subdiagonal) is specified in the vector E, and X and B are N by NRHS matrices.

ARGUMENTS

UPLO (input)
Specifies the form of the factorization and whether the vector OFFD is the superdiagonal of the upper bidiagonal factor U or the subdiagonal of the lower bidiagonal factor L. = 'U': A = U'*DIAG*U, OFFD is the superdiagonal of U
= 'L': A = L*DIAG*L', OFFD is the subdiagonal of L

N (input) The order of the tridiagonal matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

DIAG (input)
The n diagonal elements of the diagonal matrix DIAG from the factorization A = U'*DIAG*U or A = L*DIAG*L'.

OFFD (input/output)
If UPLO = 'U', the (n-1) superdiagonal elements of

the unit bidiagonal factor U from the factorization A = U'*DIAG*U. If UPLO = 'L', the (n-1) subdiagonal elements of the unit bidiagonal factor L from the factorization A = L*DIAG*L'.

B (input/output)
On entry, the right hand side vectors B for the system of linear equations. On exit, the solution vectors, X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value

Contents

NAME

```
cptts2 - solve a tridiagonal system of the form  A * X  =  B
using the factorization A = U'*D*U or A = L*D*L' computed by
CPTTRF
```

SYNOPSIS

```
SUBROUTINE CPTTS2(IUPLO, N, NRHS, D, E, B, LDB)

COMPLEX E(*), B(LDB,*)
INTEGER IUPLO, N, NRHS, LDB
REAL D(*)

SUBROUTINE CPTTS2_64(IUPLO, N, NRHS, D, E, B, LDB)

COMPLEX E(*), B(LDB,*)
INTEGER*8 IUPLO, N, NRHS, LDB
REAL D(*)
```

F95 INTERFACE

```
SUBROUTINE CPTTS2(IUPLO, N, NRHS, D, E, B, LDB)

COMPLEX, DIMENSION(:) :: E
COMPLEX, DIMENSION(:,:) :: B
INTEGER :: IUPLO, N, NRHS, LDB
REAL, DIMENSION(:) :: D

SUBROUTINE CPTTS2_64(IUPLO, N, NRHS, D, E, B, LDB)

COMPLEX, DIMENSION(:) :: E
COMPLEX, DIMENSION(:,:) :: B
INTEGER(8) :: IUPLO, N, NRHS, LDB
REAL, DIMENSION(:) :: D
```

C INTERFACE

```
#include <sunperf.h>

void cptts2(int iuplo, int n, int nrhs,  float  *d,  complex
          *e, complex *b, int ldb);

void cptts2_64(long iuplo, long n, long nrhs, float *d, com-
          plex *e, complex *b, long ldb);
```

PURPOSE

cptts2 solves a tridiagonal system of the form
   A * X = B using the factorization A = U'*D*U or A =
L*D*L' computed by CPTTRF. D is a diagonal matrix specified
in the vector D, U (or L) is a unit bidiagonal matrix whose
superdiagonal (subdiagonal) is specified in the vector E,
and X and B are N by NRHS matrices.

ARGUMENTS

IUPLO (input)
Specifies the form of the factorization and whether the vector E is the superdiagonal of the upper bidiagonal factor U or the subdiagonal of the lower bidiagonal factor L. = 1: A = U'*D*U, E is the superdiagonal of U
= 0: A = L*D*L', E is the subdiagonal of L

N (input) The order of the tridiagonal matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

D (input) The n diagonal elements of the diagonal matrix D from the factorization A = U'*D*U or A = L*D*L'.

E (input) If IUPLO = 1, the (n-1) superdiagonal elements of the unit bidiagonal factor U from the factorization A = U'*D*U. If IUPLO = 0, the (n-1) subdiagonal elements of the unit bidiagonal factor L from the factorization A = L*D*L'.

B (input/output)
On entry, the right hand side vectors B for the system of linear equations. On exit, the solution

vectors, X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

Contents

NAME

```
crot - apply a plane rotation, where the cos (C) is real and
the sin (S) is complex, and the vectors X and Y are complex
```

SYNOPSIS

```
SUBROUTINE CROT(N, X, INCX, Y, INCY, C, S)

COMPLEX S
COMPLEX X(*), Y(*)
INTEGER N, INCX, INCY
REAL C

SUBROUTINE CROT_64(N, X, INCX, Y, INCY, C, S)

COMPLEX S
COMPLEX X(*), Y(*)
INTEGER*8 N, INCX, INCY
REAL C
```

F95 INTERFACE

```
SUBROUTINE ROT([N], X, [INCX], Y, [INCY], C, S)

COMPLEX :: S
COMPLEX, DIMENSION(:) :: X, Y
INTEGER :: N, INCX, INCY
REAL :: C

SUBROUTINE ROT_64([N], X, [INCX], Y, [INCY], C, S)

COMPLEX :: S
COMPLEX, DIMENSION(:) :: X, Y
INTEGER(8) :: N, INCX, INCY
```

```
REAL :: C
```

C INTERFACE

```
#include <sunperf.h>

void crot(int n, complex *x, int incx, complex *y, int incy,
          float c, complex *s);

void crot_64(long n, complex *x, long incx, complex *y, long
          incy, float c, complex *s);
```

PURPOSE

crot applies a plane rotation, where the cos (C) is real and the sin (S) is complex, and the vectors X and Y are complex.

ARGUMENTS

```
N (input)
          The number of elements in the vectors X and Y.

X (input/output)
          On input, the vector X.  On output, X is overwrit-
          ten with C*X + S*Y.

INCX (input)
          The increment  between  successive  values  of  Y.
          INCX <> 0.

Y (input/output)
          On input, the vector Y.  On output, Y is overwrit-
          ten with -CONJG(S)*X + C*Y.

INCY (input)
          The increment  between  successive  values  of  Y.
          INCY <> 0.

C (input)

S (input)
               C and S define a rotation
               [  C          S  ]
               [ -conjg(S)   C  ]

               where C*C + S*CONJG(S) = 1.0.
```

Contents

NAME

```
crotg - Construct a Given's plane rotation
```

SYNOPSIS

```
SUBROUTINE CROTG(A, B, C, S)

COMPLEX A, B, S
REAL C

SUBROUTINE CROTG_64(A, B, C, S)

COMPLEX A, B, S
REAL C
```

F95 INTERFACE

```
SUBROUTINE ROTG(A, B, C, S)

COMPLEX :: A, B, S
REAL :: C

SUBROUTINE ROTG_64(A, B, C, S)

COMPLEX :: A, B, S
REAL :: C
```

C INTERFACE

```
#include <sunperf.h>

void crotg(complex *a, complex *b, float *c, complex *s);

void crotg_64(complex *a, complex *b, float *c, complex *s);
```

PURPOSE

crotg Construct a Given's plane rotation that will annihilate an element of a vector.

ARGUMENTS

A (input/output)
: On entry, A contains the entry in the first vector that corresponds to the element to be annihilated in the second vector. On exit, contains the nonzero element of the rotated vector.

B (input)
: On entry, B contains the entry to be annihilated in the second vector. Unchanged on exit.

C (output)
: On exit, C and S are the elements of the rotation matrix that will be applied to annihilate B.

S (output)
: On exit, C and S are the elements of the rotation matrix that will be applied to annihilate B.

Contents

NAME

```
cscal - Compute y := alpha * y
```

SYNOPSIS

```
SUBROUTINE CSCAL(N, ALPHA, Y, INCY)

COMPLEX ALPHA
COMPLEX Y(*)
INTEGER N, INCY

SUBROUTINE CSCAL_64(N, ALPHA, Y, INCY)

COMPLEX ALPHA
COMPLEX Y(*)
INTEGER*8 N, INCY
```

F95 INTERFACE

```
SUBROUTINE SCAL([N], ALPHA, Y, [INCY])

COMPLEX :: ALPHA
COMPLEX, DIMENSION(:) :: Y
INTEGER :: N, INCY

SUBROUTINE SCAL_64([N], ALPHA, Y, [INCY])

COMPLEX :: ALPHA
COMPLEX, DIMENSION(:) :: Y
INTEGER(8) :: N, INCY
```

C INTERFACE

```
#include <sunperf.h>
```

```
void cscal(int n, complex *alpha, complex *y, int incy);

void cscal_64(long  n,  complex  *alpha,  complex  *y,  long
          incy);
```

PURPOSE

cscal Compute y := alpha * y where alpha is a scalar and y is an n-vector.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the sub- routine to have any visible effect. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
csctr - Scatters elements from x into y.
```

SYNOPSIS

```
 SUBROUTINE CSCTR(NZ, X, INDX, Y)

 COMPLEX X(*), Y(*)
 INTEGER NZ
 INTEGER INDX(*)

 SUBROUTINE CSCTR_64(NZ, X, INDX, Y)

 COMPLEX X(*), Y(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 SUBROUTINE SCTR([NZ], X, INDX, Y)

 COMPLEX, DIMENSION(:) :: X, Y
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 SUBROUTINE SCTR_64([NZ], X, INDX, Y)

 COMPLEX, DIMENSION(:) :: X, Y
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
CSCTR - Scatters the components of a sparse vector x stored
in compressed form into specified components of a vector y
```

in full storage form.

```
 do i = 1, n
   y(indx(i)) = x(i)
 enddo
```

ARGUMENTS

```
NZ (input) - INTEGER
        Number of elements in the compressed form.
        Unchanged on exit.

X (input)
        Vector containing the values to be scattered from
        compressed form into full storage form.  Unchanged
        on exit.

INDX (input) - INTEGER
        Vector containing the indices of the compressed
        form.  It is assumed that the elements in INDX are
        distinct and greater than zero.  Unchanged on exit.

Y (output)
        Vector whose elements specified by indx have been
        set to the corresponding entries of x.  Only the
        elements corresponding to the indices in indx have
        been modified.
```

Contents

NAME

```
cskymm - Skyline format matrix-matrix multiply
```

SYNOPSIS

```
 SUBROUTINE CSKYMM( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, PNTR, B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, K, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER    PNTR(*),
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CSKYMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, PNTR, B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, K, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER*8  PNTR(*),
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTR(K+1)-PNTR(1) (upper triangular)
       NNZ = PNTR(M+1)-PNTR(1) (lower triangular)
       PNTR() size = (K+1) (upper triangular)
       PNTR() size = (M+1) (lower triangular)
```

F95 INTERFACE

```
 SUBROUTINE SKYMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL,
*    PNTR,  B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, K
 INTEGER, DIMENSION(:) ::  DESCRA,  PNTR
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) ::  VAL
```

```
 COMPLEX, DIMENSION(:, :) ::  B, C

 SUBROUTINE SKYMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL,
*    PNTR,  B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, K
 INTEGER*8, DIMENSION(:) ::  DESCRA,  PNTR
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) ::  VAL
 COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in skyline format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general (NOT SUPPORTED)
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
```

```
                   2 : upper
                 DESCRA(3) main diagonal type
                   0 : non-unit
                   1 : unit
                 DESCRA(4) Array base  (NOT IMPLEMENTED)
                   0 : C/C++ compatible
                   1 : Fortran compatible
                 DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                   0 : unknown
                   1 : no repeated indices

 VAL()           array contain the nonzeros of A in skyline profile form.
                 Row-oriented if DESCRA(2) = 1 (lower triangular),
                 column oriented if DESCRA(2) = 2 (upper triangular).

 PNTR()          integer array of length M+1 (lower triangular) or
                 K+1 (upper triangular) such that PNTR(I)-PNTR(1)+1
                 points to the location in VAL of the first element of
                 the skyline profile in row (column) I.

 B()             rectangular array with first dimension LDB.
 LDB             leading dimension of B

 BETA            Scalar parameter

 C()             rectangular array with first dimension LDC.

 LDC             leading dimension of C

 WORK()          scratch array of length LWORK. WORK is not
                 referenced in the current version.

 LWORK           length of WORK array. LWORK is not referenced
                 in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS)
Standard", University of Tennessee, Knoxville, Tennessee,
1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

The SKY data structure is not supported for a general matrix structure (*DESCRA*(1)=0).

Also not supported:

1. lower triangular matrix A of size m by n where m > n
2. upper triangular matrix A of size m by n where m < n

Contents

NAME

```
cskysm - Skyline format triangular solve
```

SYNOPSIS

```
 SUBROUTINE CSKYSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, PNTR,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5),
*            LDB, LDC, LWORK
 INTEGER    PNTR(*),
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CSKYSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, PNTR,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5),
*            LDB, LDC, LWORK
 INTEGER*8  PNTR(*),
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTR(M+1)-PNTR(1) (upper triangular)
       NNZ = PNTR(K+1)-PNTR(1) (lower triangular)
       PNTR() size = (M+1) (upper triangular)
       PNTR() size = (K+1) (lower triangular)
```

F95 INTERFACE

```
 SUBROUTINE SKYSM( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
*    PNTR,  B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, UNITD
 INTEGER, DIMENSION(:) ::  DESCRA,  PNTR
```

```
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) ::  VAL, DV
 COMPLEX, DIMENSION(:, :) ::  B, C

 SUBROUTINE SKYSM_64( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA,
*    VAL, PNTR,  B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, UNITD
 INTEGER*8, DIMENSION(:) ::  DESCRA,  PNTR
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) ::  VAL, DV
 COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C
where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a diagonal scaling matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in skyline format and
op( A )  is one  of

 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' )).
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row or column scaling (see section
                    NOTES for further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter
```

```
DESCRA()       Descriptor argument.  Five element integer array
               DESCRA(1) matrix structure
                 0 : general
                 1 : symmetric (A=A')
                 2 : Hermitian (A= CONJG(A'))
                 3 : Triangular
                 4 : Skew(Anti)-Symmetric (A=-A')
                 5 : Diagonal
                 6 : Skew-Hermitian (A= -CONJG(A'))

               Note: For the routine, DESCRA(1)=3 is only supported.
               DESCRA(2) upper/lower triangular indicator
                 1 : lower
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices

VAL()          array contain the nonzeros of A in skyline profile form.
               Row-oriented if DESCRA(2) = 1 (lower triangular),
               column oriented if DESCRA(2) = 2 (upper triangular).

PNTR()         integer array of length M+1 (lower triangular) or
               K+1 (upper triangular) such that PNTR(I)-PNTR(1)+1
               points to the location in VAL of the first element of
               the skyline profile in row (column) I.

B()            rectangular array with first dimension LDB.

LDB            leading dimension of B

BETA           Scalar parameter

C()            rectangular array with first dimension LDC.

LDC            leading dimension of C

WORK()         scratch array of length LWORK.
               On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.

LWORK          length of WORK array.  LWORK should be at least M.

               For good performance, LWORK should generally be larger.
```

For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. Also not supported:
   a. lower triangular matrix A of size m by n where m > n
   b. upper triangular matrix A of size m by n where m < n

2. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

3. If UNITD =4, the routine scales the rows of A if *DESCRA*(2)=1 and the columns of A if *DESCRA*(2)=2 such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in this particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows (columns) have been scaled. UNITD=2 if *DESCRA*(2)=1 and UNITD=3 if *DESCRA*(2)=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row (column) number which 2-norm is exactly zero.

4. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements

might or might not be referenced in the SKY representation of a sparse matrix. They are not used anyway in these cases. But if UNITD=4, the unit diagonal elements MUST be referenced in the SKY representation.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

```
cspcon - estimate the reciprocal of the condition number (in
the 1-norm) of a complex symmetric packed matrix A using the
factorization A = U*D*U**T  or  A  =  L*D*L**T  computed  by
CSPTRF
```

SYNOPSIS

```
SUBROUTINE CSPCON(UPLO, N, AP, IPIVOT, ANORM, RCOND, WORK, INFO)

CHARACTER * 1 UPLO
COMPLEX AP(*), WORK(*)
INTEGER N, INFO
INTEGER IPIVOT(*)
REAL ANORM, RCOND

SUBROUTINE CSPCON_64(UPLO, N, AP, IPIVOT, ANORM, RCOND, WORK, INFO)

CHARACTER * 1 UPLO
COMPLEX AP(*), WORK(*)
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
REAL ANORM, RCOND
```

F95 INTERFACE

```
SUBROUTINE SPCON(UPLO, N, AP, IPIVOT, ANORM, RCOND, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: AP, WORK
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL :: ANORM, RCOND
```

```
SUBROUTINE SPCON_64(UPLO, N, AP, IPIVOT, ANORM, RCOND, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: AP, WORK
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL :: ANORM, RCOND
```

C INTERFACE

```
#include <sunperf.h>

void cspcon(char uplo, int  n,  complex  *ap,  int  *ipivot,
          float anorm, float *rcond, int *info);
void cspcon_64(char uplo, long n, complex *ap, long *ipivot,
          float anorm, float *rcond, long *info);
```

PURPOSE

cspcon estimates the reciprocal of the condition number (in the 1-norm) of a complex symmetric packed matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by CSPTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

AP (input)
Complex array, dimension (N*(N+1)/2) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CSPTRF, stored as a packed triangular matrix.

IPIVOT (input)
Integer array, dimension (N) Details of the interchanges and the block structure of D as determined by CSPTRF.

ANORM (input)
The 1-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
Complex array, dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
csprfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is symmetric indefin-
ite and packed, and provides error bounds and backward error
estimates for the solution
```

SYNOPSIS

```
SUBROUTINE CSPRFS(UPLO, N, NRHS, A, AF, IPIVOT, B, LDB, X, LDX, FERR,
      BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*)
REAL FERR(*), BERR(*), WORK2(*)

SUBROUTINE CSPRFS_64(UPLO, N, NRHS, A, AF, IPIVOT, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
REAL FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE SPRFS(UPLO, N, [NRHS], A, AF, IPIVOT, B, [LDB], X, [LDX],
       FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A, AF, WORK
COMPLEX, DIMENSION(:,:) :: B, X
```

```
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE SPRFS_64(UPLO, N, [NRHS], A, AF, IPIVOT, B, [LDB], X,
       [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: A, AF, WORK
COMPLEX, DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void csprfs(char uplo, int n, int nrhs, complex *a,  complex
          *af, int *ipivot, complex *b, int ldb, complex *x,
          int ldx, float *ferr, float *berr, int *info);

void csprfs_64(char uplo, long n,  long  nrhs,  complex  *a,
          complex  *af,  long *ipivot, complex *b, long ldb,
          complex *x, long ldx, float  *ferr,  float  *berr,
          long *info);
```

PURPOSE

csprfs improves the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite and packed, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

UPLO (input)
          = 'U': Upper triangle of A is stored;
          = 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
          The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) Complex array, dimension (N*(N+1)/2) The upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of

A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

AF (input)
Complex array, dimension (N*(N+1)/2) The factored form of the matrix A. AF contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by CSPTRF, stored as a packed triangular matrix.

IPIVOT (input)
Integer array, dimension (N) Details of the interchanges and the block structure of D as determined by CSPTRF.

B (input) Complex array, dimension (LDB,NRHS) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
Complex array, dimension (LDX,NRHS) On entry, the solution matrix X, as computed by CSPTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
Real array, dimension (NRHS) The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
Real array, dimension (NRHS) The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solu-

tion).

WORK (workspace)
Complex array, dimension(2*N)

WORK2 (workspace)
Integer array, dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
cspsv - compute the solution to a complex system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE CSPSV(UPLO, N, NRHS, AP, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX AP(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE CSPSV_64(UPLO, N, NRHS, AP, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX AP(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE SPSV(UPLO, N, [NRHS], AP, IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: AP
COMPLEX, DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE SPSV_64(UPLO, N, [NRHS], AP, IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX, DIMENSION(:) :: AP
COMPLEX, DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void cspsv(char uplo, int n,  int  nrhs,  complex  *ap,  int
          *ipivot, complex *b, int ldb, int *info);

void cspsv_64(char uplo, long n,  long  nrhs,  complex  *ap,
          long *ipivot, complex *b, long ldb, long *info);
```

PURPOSE

cspsv computes the solution to a complex system of linear equations
   A * X = B, where A is an N-by-N symmetric matrix stored in packed format and X and B are N-by-NRHS matrices.

The diagonal pivoting method is used to factor A as
   A = U * D * U**T, if UPLO = 'U', or
   A = L * D * L**T, if UPLO = 'L',
where U (or L) is a product of permutation and unit upper (lower) triangular matrices, D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks. The factored form of A is then used to solve the system of equations A * X = B.

ARGUMENTS

UPLO (input)
 = 'U': Upper triangle of A is stored;
 = 'L': Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NRHS (input)
 The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

AP (input/output)
 Complex array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th

column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n. See below for further details.

On exit, the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by CSPTRF, stored as a packed triangular matrix in the same storage format as A.

IPIVOT (output)

Integer array, dimension (N) Details of the interchanges and the block structure of D, as determined by CSPTRF. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged, and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

B (input/output)

Complex array, dimension (LDB,NRHS) On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)

The leading dimension of the array B. LDB >= max(1,N).

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, so the solution could not be computed.

FURTHER DETAILS

The packed storage scheme is illustrated by the following example when N = 4, UPLO = 'U':

Two-dimensional storage of the symmetric matrix A:

```
   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = aji)
               a44

Packed storage of the upper triangle of A:

AP = [ a11, a12, a22, a13, a23, a33, a14, a24, a34, a44 ]
```

Contents

NAME

```
cspsvx  -  use  the  diagonal  pivoting  factorization  A  =
U*D*U**T  or  A = L*D*L**T to compute the solution to a com-
plex system of linear equations A * X = B, where A is an  N-
by-N  symmetric  matrix  stored in packed format and X and B
are N-by-NRHS matrices
```

SYNOPSIS

```
SUBROUTINE CSPSVX(FACT, UPLO, N, NRHS, A, AF, IPIVOT, B, LDB, X, LDX,
      RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*)
REAL RCOND
REAL FERR(*), BERR(*), WORK2(*)

SUBROUTINE CSPSVX_64(FACT, UPLO, N, NRHS, A, AF, IPIVOT, B, LDB, X,
      LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
REAL RCOND
REAL FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE SPSVX(FACT, UPLO, N, [NRHS], A, AF, IPIVOT, B, [LDB], X,
       [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: FACT, UPLO
COMPLEX, DIMENSION(:) :: A, AF, WORK
COMPLEX, DIMENSION(:,:) :: B, X
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL :: RCOND
REAL, DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE SPSVX_64(FACT, UPLO, N, [NRHS], A, AF, IPIVOT, B, [LDB],
       X, [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO
COMPLEX, DIMENSION(:) :: A, AF, WORK
COMPLEX, DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL :: RCOND
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void cspsvx(char fact, char uplo, int n, int  nrhs,  complex
          *a, complex *af, int *ipivot, complex *b, int ldb,
          complex *x, int ldx, float  *rcond,  float  *ferr,
          float *berr, int *info);

void cspsvx_64(char fact, char uplo, long n, long nrhs, com-
          plex  *a,  complex  *af, long *ipivot, complex *b,
          long ldb, complex  *x,  long  ldx,  float  *rcond,
          float *ferr, float *berr, long *info);
```

PURPOSE

cspsvx uses the diagonal pivoting factorization A = U*D*U**T or A = L*D*L**T to compute the solution to a complex system of linear equations A * X = B, where A is an N-by-N symmetric matrix stored in packed format and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'N', the diagonal pivoting method is used to factor A as

```
      A = U * D * U**T,  if UPLO = 'U', or
```

A = L * D * L**T, if UPLO = 'L',
where U (or L) is a product of permutation and unit upper (lower)
triangular matrices and D is symmetric and block diagonal with
1-by-1 and 2-by-2 diagonal blocks.

2. If some D(i,i)=0, so that D is exactly singular, then the routine
returns with INFO = i. Otherwise, the factored form of A is used
to estimate the condition number of the matrix A. If the
reciprocal of the condition number is less than machine precision,
INFO = N+1 is returned as a warning, but the routine still goes on
to solve for X and compute error bounds as described below.

3. The system of equations is solved for X using the factored form
of A.

4. Iterative refinement is applied to improve the computed solution
matrix and calculate error bounds and backward error estimates
for it.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of A has been supplied on entry. = 'F': On entry, AF and IPIVOT contain the factored form of A. A, AF and IPIVOT will not be modified. = 'N': The matrix A will be copied to AF and factored.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) Complex array, dimension (N*(N+1)/2) The upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n. See below for further details.

AF (input/output)

Complex array, dimension (N*(N+1)/2) If FACT = 'F', then AF is an input argument and on entry contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by CSPTRF, stored as a packed triangular matrix in the same storage format as A. If FACT = 'N', then AF is an output argument and on exit contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by CSPTRF, stored as a packed triangular matrix in the same storage format as A.

IPIVOT (input or output)

Integer array, dimension (N) If FACT = 'F', then IPIVOT is an input argument and on entry contains details of the interchanges and the block structure of D, as determined by CSPTRF. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

If FACT = 'N', then IPIVOT is an output argument and on exit contains details of the interchanges and the block structure of D, as determined by CSPTRF.

B (input) Complex array, dimension (LDB,NRHS) The N-by-NRHS right hand side matrix B.

LDB (input)

The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X.

LDX (input)
Complex array, dimension (LDX,NRHS) The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A. If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
Complex array, dimension (NRHS) The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
Complex array, dimension (NRHS) The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
Complex array, dimension(2*N)

WORK2 (workspace)
Integer array, dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: D(i,i) is exactly zero. The factorization has been completed but the factor D is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: D

is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

FURTHER DETAILS

The packed storage scheme is illustrated by the following example when N = 4, UPLO = 'U':

Two-dimensional storage of the symmetric matrix A:

```
   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = aji)
               a44
```

Packed storage of the upper triangle of A:

A = [a11, a12, a22, a13, a23, a33, a14, a24, a34, a44]

Contents

NAME

```
csptrf - compute the factorization of  a  complex  symmetric
matrix  A  stored  in  packed format using the Bunch-Kaufman
diagonal pivoting method
```

SYNOPSIS

```
SUBROUTINE CSPTRF(UPLO, N, AP, IPIVOT, INFO)

CHARACTER * 1 UPLO
COMPLEX AP(*)
INTEGER N, INFO
INTEGER IPIVOT(*)

SUBROUTINE CSPTRF_64(UPLO, N, AP, IPIVOT, INFO)

CHARACTER * 1 UPLO
COMPLEX AP(*)
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE SPTRF(UPLO, N, AP, IPIVOT, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: AP
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE SPTRF_64(UPLO, N, AP, IPIVOT, [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX, DIMENSION(:) :: AP
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void csptrf(char uplo, int n, complex *ap, int *ipivot,  int
          *info);

void csptrf_64(char uplo, long n, complex *ap, long *ipivot,
          long *info);
```

PURPOSE

csptrf computes the factorization of a complex symmetric matrix A stored in packed format using the Bunch-Kaufman diagonal pivoting method:

```
A = U*D*U**T  or  A = L*D*L**T
```

where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

AP (input/output)
Complex array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n.

On exit, the block diagonal matrix D and the multipliers used to obtain the factor U or L, stored as a packed triangular matrix overwriting A (see below for further details).

IPIVOT (output)
Integer array, dimension (N) Details of the interchanges and the block structure of D. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

5-96 - Based on modifications by J. Lewis, Boeing Computer Services
Company

If UPLO = 'U', then A = U*D*U', where
   U = P(n)*U(n)* ... *P(k)U(k)* ...,
i.e., U is a product of terms P(k)*U(k), where k decreases from n to 1 in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIVOT(k), and U(k) is a unit upper triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    v    0   )   k-s
   U(k) =  (   0    I    0   )   s
           (   0    0    I   )   n-k
              k-s   s   n-k
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(1:k-1,k). If s = 2, the upper triangle of D(k) overwrites A(k-1,k-1), A(k-1,k), and A(k,k), and v overwrites A(1:k-2,k-1:k).

If UPLO = 'L', then A = L*D*L', where
   L = P(1)*L(1)* ... *P(k)*L(k)* ...,

i.e., L is a product of terms P(k)*L(k), where k increases from 1 to n in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIVOT(k), and L(k) is a unit lower triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    0     0   )  k-1
   L(k) =  (   0    I     0   )  s
           (   0    v     I   )  n-k-s+1
              k-1   s  n-k-s+1
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(k+1:n,k). If s = 2, the lower triangle of D(k) overwrites A(k,k), A(k+1,k), and A(k+1,k+1), and v overwrites A(k+2:n,k:k+1).

Contents

NAME

```
csptri - compute the inverse of a complex symmetric indefin-
ite  matrix  A in packed storage using the factorization A =
U*D*U**T or A = L*D*L**T computed by CSPTRF
```

SYNOPSIS

```
SUBROUTINE CSPTRI(UPLO, N, AP, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
COMPLEX AP(*), WORK(*)
INTEGER N, INFO
INTEGER IPIVOT(*)

SUBROUTINE CSPTRI_64(UPLO, N, AP, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
COMPLEX AP(*), WORK(*)
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE SPTRI(UPLO, N, AP, IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: AP, WORK
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE SPTRI_64(UPLO, N, AP, IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: AP, WORK
```

```
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void csptri(char uplo, int n, complex *ap, int *ipivot,  int
          *info);

void csptri_64(char uplo, long n, complex *ap, long *ipivot,
          long *info);
```

PURPOSE

csptri computes the inverse of a complex symmetric indefinite matrix A in packed storage using the factorization A = U*D*U**T or A = L*D*L**T computed by CSPTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

AP (input/output)
Complex array, dimension (N*(N+1)/2) On entry, the block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CSPTRF, stored as a packed triangular matrix.

On exit, if INFO = 0, the (symmetric) inverse of the original matrix, stored as a packed triangular matrix. The j-th column of inv(A) is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = inv(A)(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2n-j)/2) = inv(A)(i,j) for j<=i<=n.

IPIVOT (input)
Integer array, dimension (N) Details of the interchanges and the block structure of D as determined by CSPTRF.

WORK (workspace)

Complex array, dimension (N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) = 0; the matrix is singular and its inverse could not be computed.

Contents

NAME

```
csptrs - solve a system of linear equations A*X = B  with  a
complex symmetric matrix A stored in packed format using the
factorization A = U*D*U**T  or  A  =  L*D*L**T  computed  by
CSPTRF
```

SYNOPSIS

```
SUBROUTINE CSPTRS(UPLO, N, NRHS, AP, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX AP(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE CSPTRS_64(UPLO, N, NRHS, AP, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX AP(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE SPTRS(UPLO, N, [NRHS], AP, IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: AP
COMPLEX, DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE SPTRS_64(UPLO, N, [NRHS], AP, IPIVOT, B, [LDB], [INFO])
```

```
CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: AP
COMPLEX, DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void csptrs(char uplo, int n, int  nrhs,  complex  *ap,  int
          *ipivot, complex *b, int ldb, int *info);

void csptrs_64(char uplo, long n, long  nrhs,  complex  *ap,
          long *ipivot, complex *b, long ldb, long *info);
```

PURPOSE

csptrs solves a system of linear equations A*X = B with a complex symmetric matrix A stored in packed format using the factorization A = U*D*U**T or A = L*D*L**T computed by CSPTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

AP (input)
Complex array, dimension (N*(N+1)/2) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CSPTRF, stored as a packed triangular matrix.

IPIVOT (input)
Integer array, dimension (N) Details of the interchanges and the block structure of D as determined by CSPTRF.

B (input/output)
Complex array, dimension (LDB,NRHS) On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
csrot - Apply a plane rotation.
```

SYNOPSIS

```
SUBROUTINE CSROT(N, X, INCX, Y, INCY, C, S)

REAL C, S
COMPLEX X(*), Y(*)
INTEGER N, INCX, INCY

SUBROUTINE CSROT_64(N, X, INCX, Y, INCY, C, S)

REAL C, S
COMPLEX X(*), Y(*)
INTEGER*8 N, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE ROT([N], X, [INCX], Y, [INCY], C, S)

REAL :: C, S
COMPLEX, DIMENSION(:) :: X, Y
INTEGER :: N, INCX, INCY

SUBROUTINE ROT_64([N], X, [INCX], Y, [INCY], C, S)

REAL :: C, S
COMPLEX, DIMENSION(:) :: X, Y
INTEGER(8) :: N, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>
```

```
void csrot(int n, complex *x,  int  incx,  complex  *y,  int
          incy, float c, float s);

void csrot_64(long n, complex *x,  long  incx,  complex  *y,
          long incy, float c, float s);
```

PURPOSE

csrot Apply a plane rotation, where the cos and sin (c and s) are real and the vectors x and y are complex.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input/output)
Before entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input/output)
On entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

C (input)
On entry, the cosine. Unchanged on exit.

S (input)
On entry, the sin. Unchanged on exit.

Contents

NAME

```
csscal - Compute y := alpha * y
```

SYNOPSIS

```
SUBROUTINE CSSCAL(N, ALPHA, Y, INCY)

COMPLEX Y(*)
INTEGER N, INCY
REAL ALPHA

SUBROUTINE CSSCAL_64(N, ALPHA, Y, INCY)

COMPLEX Y(*)
INTEGER*8 N, INCY
REAL ALPHA
```

F95 INTERFACE

```
SUBROUTINE SCAL([N], ALPHA, Y, [INCY])

COMPLEX, DIMENSION(:) :: Y
INTEGER :: N, INCY
REAL :: ALPHA

SUBROUTINE SCAL_64([N], ALPHA, Y, [INCY])

COMPLEX, DIMENSION(:) :: Y
INTEGER(8) :: N, INCY
REAL :: ALPHA
```

C INTERFACE

```
#include <sunperf.h>
```

```
void csscal(int n, float alpha, complex *y, int incy);

void csscal_64(long n, float alpha, complex *y, long incy);
```

PURPOSE

csscal Compute y := alpha * y where alpha is a scalar and y is an n-vector.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
cstedc - compute all eigenvalues and, optionally,  eigenvec-
tors  of a symmetric tridiagonal matrix using the divide and
conquer method
```

SYNOPSIS

```
SUBROUTINE CSTEDC(COMPZ, N, D, E, Z, LDZ, WORK, LWORK, RWORK, LRWORK,
      IWORK, LIWORK, INFO)

CHARACTER * 1 COMPZ
COMPLEX Z(LDZ,*), WORK(*)
INTEGER N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER IWORK(*)
REAL D(*), E(*), RWORK(*)

SUBROUTINE CSTEDC_64(COMPZ, N, D, E, Z, LDZ, WORK, LWORK, RWORK,
      LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 COMPZ
COMPLEX Z(LDZ,*), WORK(*)
INTEGER*8 N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
REAL D(*), E(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEDC(COMPZ, [N], D, E, Z, [LDZ], [WORK], [LWORK], [RWORK],
       [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: Z
```

```
INTEGER :: N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: D, E, RWORK

SUBROUTINE STEDC_64(COMPZ, [N], D, E, Z, [LDZ], [WORK], [LWORK],
       [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: Z
INTEGER(8) :: N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: D, E, RWORK
```

C INTERFACE

```
#include <sunperf.h>
void cstedc(char compz, int n, float *d, float  *e,  complex
          *z, int ldz, int *info);

void cstedc_64(char compz, long n, float *d, float *e,  com-
          plex *z, long ldz, long *info);
```

PURPOSE

cstedc computes all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the divide and conquer method. The eigenvectors of a full or band complex Hermitian matrix can also be found if CHETRD or CHPTRD or CHBTRD has been used to reduce this matrix to tridiagonal form.

This code makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none. See SLAED3 for details.

ARGUMENTS

```
COMPZ (input)
          = 'N':  Compute eigenvalues only.
          = 'I':  Compute eigenvectors of tridiagonal matrix
          also.
          = 'V':  Compute eigenvectors of original Hermitian
          matrix  also.   On  entry,  Z contains the unitary
```

matrix used to reduce the original matrix to tri-
diagonal form.

N (input) The dimension of the symmetric tridiagonal matrix. N >= 0.

D (input/output)
On entry, the diagonal elements of the tridiagonal matrix. On exit, if INFO = 0, the eigenvalues in ascending order.

E (input/output)
On entry, the subdiagonal elements of the tridiagonal matrix. On exit, E has been destroyed.

Z (input) On entry, if COMPZ = 'V', then Z contains the unitary matrix used in the reduction to tridiagonal form. On exit, if INFO = 0, then if COMPZ = 'V', Z contains the orthonormal eigenvectors of the original Hermitian matrix, and if COMPZ = 'I', Z contains the orthonormal eigenvectors of the symmetric tridiagonal matrix. If COMPZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1. If eigenvectors are desired, then LDZ >= max(1,N).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If COMPZ = 'N' or 'I', or N <= 1, LWORK must be at least 1. If COMPZ = 'V' and N > 1, LWORK must be at least N*N.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
dimension (LRWORK) On exit, if INFO = 0, RWORK(1) returns the optimal LRWORK.

LRWORK (input)
The dimension of the array RWORK. If COMPZ = 'N' or N <= 1, LRWORK must be at least 1. If COMPZ =

'V' and N > 1, LRWORK must be at least 1 + 3*N + 2*N*lg N + 3*N**2 , where lg(N) = smallest integer k such that 2**k >= N. If COMPZ = 'I' and N > 1, LRWORK must be at least 1 + 4*N + 2*N**2 .

If LRWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the RWORK array, returns this value as the first entry of the RWORK array, and no error message related to LRWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of the array IWORK. If COMPZ = 'N' or N <= 1, LIWORK must be at least 1. If COMPZ = 'V' or N > 1, LIWORK must be at least 6 + 6*N + 5*N*lg N. If COMPZ = 'I' or N > 1, LIWORK must be at least 3 + 5*N .

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: The algorithm failed to compute an eigenvalue while working on the submatrix lying in rows and columns INFO/(N+1) through mod(INFO,N+1).

FURTHER DETAILS

Based on contributions by
Jeff Rutter, Computer Science Division, University of California
at Berkeley, USA

Contents

NAME

```
cstegr - Compute T-sigma_i = L_i D_i L_i^T,  such  that  L_i
D_i L_i^T  is a relatively robust representation
```

SYNOPSIS

```
SUBROUTINE CSTEGR(JOBZ, RANGE, N, D, E, VL, VU, IL, IU, ABSTOL, M, W,
      Z, LDZ, ISUPPZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE
COMPLEX Z(LDZ,*)
INTEGER N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER ISUPPZ(*), IWORK(*)
REAL VL, VU, ABSTOL
REAL D(*), E(*), W(*), WORK(*)

SUBROUTINE CSTEGR_64(JOBZ, RANGE, N, D, E, VL, VU, IL, IU, ABSTOL, M,
      W, Z, LDZ, ISUPPZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE
COMPLEX Z(LDZ,*)
INTEGER*8 N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER*8 ISUPPZ(*), IWORK(*)
REAL VL, VU, ABSTOL
REAL D(*), E(*), W(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEGR(JOBZ, RANGE, [N], D, E, VL, VU, IL, IU, ABSTOL, M,
       W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE
COMPLEX, DIMENSION(:,:) :: Z
```

```
INTEGER :: N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: ISUPPZ, IWORK
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: D, E, W, WORK

SUBROUTINE STEGR_64(JOBZ, RANGE, [N], D, E, VL, VU, IL, IU, ABSTOL,
       M, W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE
COMPLEX, DIMENSION(:,:) :: Z
INTEGER(8) :: N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: ISUPPZ, IWORK
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: D, E, W, WORK
```

C INTERFACE

```
#include <sunperf.h>

void cstegr(char jobz, char range, int n,  float  *d,  float
          *e,  float  vl,  float  vu,  int il, int iu, float
          abstol, int *m, float *w, complex *z, int ldz, int
          *isuppz, int *info);

void cstegr_64(char jobz, char  range,  long  n,  float  *d,
          float  *e,  float  vl, float vu, long il, long iu,
          float abstol, long *m, float *w, complex *z,  long
          ldz, long *isuppz, long *info);
```

PURPOSE

```
cstegr b) Compute the  eigenvalues,  lambda_j,  of  L_i  D_i
L_i^T to high
       relative accuracy by the dqds algorithm,
   (c) If there is a cluster of close eigenvalues,  "choose"
sigma_i
       close to the cluster, and go to step (a),
   (d) Given the approximate eigenvalue lambda_j of L_i  D_i
L_i^T,
       compute the corresponding eigenvector by forming a
       rank-revealing twisted factorization.
The desired accuracy of the output can be specified  by  the
input parameter ABSTOL.

For more details, see "A new O(n^2) algorithm for  the  sym-
metric   tridiagonal   eigenvalue/eigenvector  problem",  by
Inderjit Dhillon, Computer Science Division Technical Report
No. UCB/CSD-97-971, UC Berkeley, May 1997.

Note 1 : Currently CSTEGR is only set up to find ALL  the  n
eigenvalues and eigenvectors of T in O(n^2) time
```

Note 2 : Currently the routine CSTEIN is called when an appropriate sigma_i cannot be chosen in step (c) above. CSTEIN invokes modified Gram-Schmidt when eigenvalues are close.

Note 3 : CSTEGR works only on machines which follow ieee-754 floating-point standard in their handling of infinities and NaNs. Normal execution of CSTEGR may create NaNs and infinities and hence may abort due to a floating point exception in environments which do not conform to the ieee standard.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.
RANGE (input)
          = 'A': all eigenvalues will be found.
          = 'V': all eigenvalues in the  half-open  interval
          (VL,VU]  will  be found.  = 'I': the IL-th through
          IU-th eigenvalues will be found.

N (input) The order of the matrix.  N >= 0.

D (input/output)
          On entry, the n diagonal elements of the tridiago-
          nal matrix T. On exit, D is overwritten.

E (input/output)
          On entry, the (n-1) subdiagonal  elements  of  the
          tridiagonal  matrix  T  in elements 1 to N-1 of E;
          E(N) need not be set.  On exit, E is overwritten.

VL (input)
          If RANGE='V', the lower and upper  bounds  of  the
          interval  to be searched for eigenvalues. VL < VU.
          Not referenced if RANGE = 'A' or 'I'.

VU (input)
          If RANGE='V', the lower and upper  bounds  of  the
          interval  to be searched for eigenvalues. VL < VU.
          Not referenced if RANGE = 'A' or 'I'.

IL (input)
          If RANGE='I', the indices (in ascending order)  of
          the   smallest   and   largest   eigenvalues  to  be
          returned.  1 <= IL <= IU <= N, if N > 0;  IL  =  1
          and  IU  =  0 if N = 0.  Not referenced if RANGE =
          'A' or 'V'.
```

IU (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

ABSTOL (input)
The absolute error tolerance for the eigenvalues/eigenvectors. IF JOBZ = 'V', the eigenvalues and eigenvectors output have residual norms bounded by ABSTOL, and the dot products between different eigenvectors are bounded by ABSTOL. If ABSTOL is less than N*EPS*|T|, then N*EPS*|T| will be used in its place, where EPS is the machine precision and |T| is the 1-norm of the tridiagonal matrix. The eigenvalues are computed to an accuracy of EPS*|T| irrespective of ABSTOL. If high relative accuracy is important, set ABSTOL to DLAMCH('Safe minimum'). See Barlow and Demmel "Computing Accurate Eigensystems of Scaled Diagonally Dominant Matrices", LAPACK Working Note #7 for a discussion of which matrices define their eigenvalues to high relative accuracy.

M (output)
The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)
The first M elements contain the selected eigenvalues in ascending order.

Z (input/output)
If JOBZ = 'V', then if INFO = 0, the first M columns of Z contain the orthonormal eigenvectors of the matrix T corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). If JOBZ = 'N', then Z is not referenced. Note: the user must ensure that at least max(1,M) columns are supplied in the array Z; if RANGE = 'V', the exact value of M is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

```
ISUPPZ (output)
          The support of the eigenvectors in  Z,  i.e.,  the
          indices  indicating the nonzero elements in Z. The
          i-th  eigenvector  is  nonzero  only  in  elements
          ISUPPZ( 2*i-1 ) through ISUPPZ( 2*i ).
WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          (and minimal) LWORK.

LWORK (input)
          The  dimension  of  the  array  WORK.   LWORK   >=
          max(1,18*N)

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

IWORK (workspace/output)
          On exit, if INFO = 0, IWORK(1) returns the optimal
          LIWORK.

LIWORK (input)
          The dimension  of  the  array  IWORK.   LIWORK  >=
          max(1,10*N)

          If LIWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the IWORK array, returns this value as  the  first
          entry  of  the  IWORK  array, and no error message
          related to LIWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = 1, internal error  in  SLARRE,  if
          INFO = 2, internal error in CLARRV.
```

FURTHER DETAILS

```
Based on contributions by
   Inderjit Dhillon, IBM Almaden, USA
   Osni Marques, LBNL/NERSC, USA
   Ken Stanley, Computer Science Division, University of
     California at Berkeley, USA
```

Contents

NAME

```
cstein - compute the eigenvectors of a real symmetric tridi-
agonal  matrix  T  corresponding  to  specified eigenvalues,
using inverse iteration
```

SYNOPSIS

```
SUBROUTINE CSTEIN(N, D, E, M, W, IBLOCK, ISPLIT, Z, LDZ, WORK, IWORK,
      IFAIL, INFO)

COMPLEX Z(LDZ,*)
INTEGER N, M, LDZ, INFO
INTEGER IBLOCK(*), ISPLIT(*), IWORK(*), IFAIL(*)
REAL D(*), E(*), W(*), WORK(*)

SUBROUTINE CSTEIN_64(N, D, E, M, W, IBLOCK, ISPLIT, Z, LDZ, WORK,
      IWORK, IFAIL, INFO)

COMPLEX Z(LDZ,*)
INTEGER*8 N, M, LDZ, INFO
INTEGER*8 IBLOCK(*), ISPLIT(*), IWORK(*), IFAIL(*)
REAL D(*), E(*), W(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEIN([N], D, E, [M], W, IBLOCK, ISPLIT, Z, [LDZ], [WORK],
       [IWORK], IFAIL, [INFO])

COMPLEX, DIMENSION(:,:) :: Z
INTEGER :: N, M, LDZ, INFO
INTEGER, DIMENSION(:) :: IBLOCK, ISPLIT, IWORK, IFAIL
REAL, DIMENSION(:) :: D, E, W, WORK

SUBROUTINE STEIN_64([N], D, E, [M], W, IBLOCK, ISPLIT, Z, [LDZ],
```

```
        [WORK], [IWORK], IFAIL, [INFO])

 COMPLEX, DIMENSION(:,:) :: Z
 INTEGER(8) :: N, M, LDZ, INFO
 INTEGER(8), DIMENSION(:) :: IBLOCK, ISPLIT, IWORK, IFAIL
 REAL, DIMENSION(:) :: D, E, W, WORK
```

C INTERFACE

```
#include <sunperf.h>

void cstein(int n, float *d, float *e, int m, float *w,  int
          *iblock,  int  *isplit,  complex  *z, int ldz, int
          *ifail, int *info);
void cstein_64(long n, float *d, float *e, long m, float *w,
          long  *iblock, long *isplit, complex *z, long ldz,
          long *ifail, long *info);
```

PURPOSE

cstein computes the eigenvectors of a real symmetric tridiagonal matrix T corresponding to specified eigenvalues, using inverse iteration.

The maximum number of iterations allowed for each eigenvector is specified by an internal parameter MAXITS (currently set to 5).

Although the eigenvectors are real, they are stored in a complex array, which may be passed to CUNMTR or CUPMTR for back
transformation to the eigenvectors of a complex Hermitian matrix which was reduced to tridiagonal form.

ARGUMENTS

N (input) The order of the matrix. N >= 0.

D (input) The n diagonal elements of the tridiagonal matrix T.

E (input) The (n-1) subdiagonal elements of the tridiagonal matrix T, stored in elements 1 to N-1; E(N) need not be set.

M (input) The number of eigenvectors to be found. 0 <= M <= N.

W (input) The first M elements of W contain the eigenvalues for which eigenvectors are to be computed. The eigenvalues should be grouped by split-off block and ordered from smallest to largest within the block. (The output array W from SSTEBZ with ORDER = 'B' is expected here.)

IBLOCK (input)
The submatrix indices associated with the corresponding eigenvalues in W; IBLOCK(i)=1 if eigenvalue W(i) belongs to the first submatrix from the top, =2 if W(i) belongs to the second submatrix, etc. (The output array IBLOCK from SSTEBZ is expected here.)

ISPLIT (input)
The splitting points, at which T breaks up into submatrices. The first submatrix consists of rows/columns 1 to ISPLIT(1), the second of rows/columns ISPLIT(1)+1 through ISPLIT(2), etc. (The output array ISPLIT from SSTEBZ is expected here.)

Z (output)
The computed eigenvectors. The eigenvector associated with the eigenvalue W(i) is stored in the i-th column of Z. Any vector which fails to converge is set to its current iterate after MAXITS iterations. The imaginary parts of the eigenvectors are set to zero.

LDZ (input)
The leading dimension of the array Z. LDZ >= max(1,N).

WORK (workspace)
dimension(5*N)

IWORK (workspace)
dimension(N)

IFAIL (output)
On normal exit, all elements of IFAIL are zero. If one or more eigenvectors fail to converge after MAXITS iterations, then their indices are stored in array IFAIL.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-

gal value
> 0: if INFO = i, then i eigenvectors failed to converge in MAXITS iterations. Their indices are stored in array IFAIL.

Contents

NAME

```
csteqr - compute all eigenvalues and, optionally,  eigenvec-
tors of a symmetric tridiagonal matrix using the implicit QL
or QR method
```

SYNOPSIS

```
SUBROUTINE CSTEQR(COMPZ, N, D, E, Z, LDZ, WORK, INFO)

CHARACTER * 1 COMPZ
COMPLEX Z(LDZ,*)
INTEGER N, LDZ, INFO
REAL D(*), E(*), WORK(*)

SUBROUTINE CSTEQR_64(COMPZ, N, D, E, Z, LDZ, WORK, INFO)

CHARACTER * 1 COMPZ
COMPLEX Z(LDZ,*)
INTEGER*8 N, LDZ, INFO
REAL D(*), E(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEQR(COMPZ, [N], D, E, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
COMPLEX, DIMENSION(:,:) :: Z
INTEGER :: N, LDZ, INFO
REAL, DIMENSION(:) :: D, E, WORK

SUBROUTINE STEQR_64(COMPZ, [N], D, E, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
COMPLEX, DIMENSION(:,:) :: Z
```

```
INTEGER(8) :: N, LDZ, INFO
REAL, DIMENSION(:) :: D, E, WORK
```

C INTERFACE

```
#include <sunperf.h>

void csteqr(char compz, int n, float *d, float  *e,  complex
          *z, int ldz, int *info);

void csteqr_64(char compz, long n, float *d, float *e,  com-
          plex *z, long ldz, long *info);
```

PURPOSE

csteqr computes all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the implicit QL or QR method. The eigenvectors of a full or band complex Hermitian matrix can also be found if CHETRD or CHPTRD or CHBTRD has been used to reduce this matrix to tridiagonal form.

ARGUMENTS

COMPZ (input)
= 'N': Compute eigenvalues only.
= 'V': Compute eigenvalues and eigenvectors of the original Hermitian matrix. On entry, Z must contain the unitary matrix used to reduce the original matrix to tridiagonal form. = 'I': Compute eigenvalues and eigenvectors of the tridiagonal matrix. Z is initialized to the identity matrix.

N (input) The order of the matrix. N >= 0.

D (input/output)
On entry, the diagonal elements of the tridiagonal matrix. On exit, if INFO = 0, the eigenvalues in ascending order.

E (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix. On exit, E has been destroyed.

Z (input) On entry, if COMPZ = 'V', then Z contains the unitary matrix used in the reduction to tridiagonal form. On exit, if INFO = 0, then if COMPZ =

```
          'V',  Z  contains  the orthonormal eigenvectors of
          the original Hermitian matrix, and if COMPZ = 'I',
          Z  contains  the  orthonormal  eigenvectors of the
          symmetric tridiagonal matrix.   If  COMPZ  =  'N',
          then Z is not referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and  if  eigenvectors  are  desired,  then  LDZ >=
          max(1,N).

WORK (workspace)
          dimension(max(1,2*N-2)) If COMPZ = 'N', then  WORK
          is not referenced.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  the algorithm has failed  to  find  all  the
          eigenvalues in a total of 30*N iterations; if INFO
          = i, then i elements of E have  not  converged  to
          zero;  on  exit, D and E contain the elements of a
          symmetric tridiagonal matrix  which  is  unitarily
          similar to the original matrix.
```

Contents

NAME

```
cstsv - compute the solution to a complex system  of  linear
equations  A  *  X  =  B  where A is a Hermitian tridiagonal
matrix
```

SYNOPSIS

```
SUBROUTINE CSTSV(N, NRHS, L, D, SUBL, B, LDB, IPIV, INFO)

COMPLEX L(*), D(*), SUBL(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO
INTEGER IPIV(*)

SUBROUTINE CSTSV_64(N, NRHS, L, D, SUBL, B, LDB, IPIV, INFO)

COMPLEX L(*), D(*), SUBL(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIV(*)
```

F95 INTERFACE

```
SUBROUTINE STSV([N], [NRHS], L, D, SUBL, B, [LDB], IPIV, [INFO])

COMPLEX, DIMENSION(:) :: L, D, SUBL
COMPLEX, DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIV

SUBROUTINE STSV_64([N], [NRHS], L, D, SUBL, B, [LDB], IPIV, [INFO])

COMPLEX, DIMENSION(:) :: L, D, SUBL
COMPLEX, DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIV
```

C INTERFACE

```
#include <sunperf.h>

void cstsv(int n, int nrhs, complex *l, complex *d,  complex
          *subl, complex *b, int ldb, int *ipiv, int *info);

void cstsv_64(long n, long nrhs,  complex  *l,  complex  *d,
          complex  *subl,  complex *b, long ldb, long *ipiv,
          long *info);
```

PURPOSE

cstsv computes the solution to a complex system of linear equations A * X = B where A is a Hermitian tridiagonal matrix.

ARGUMENTS

N (input)
The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides in B.

L (input/output)
COMPLEX array, dimension (N)
On entry, the n-1 subdiagonal elements of the tridiagonal matrix A. On exit, part of the factorization of A.

D (input/output)
REAL array, dimension (N)
On entry, the n diagonal elements of the tridiagonal matrix A. On exit, the n diagonal elements of the diagonal matrix D from the factorization of A.

SUBL (output)
COMPLEX array, dimension (N)
On exit, part of the factorization of A.

B (input/output)
The columns of B contain the right hand sides.

LDB (input)
The leading dimension of B as specified in a type or DIMENSION statement.

IPIV (output)
INTEGER array, dimension (N)
On exit, the pivot indices of the factorization.

INFO (output)
INTEGER
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(k,k) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular and division by zero will occur if it is used to solve a system of equations.

Contents

NAME

csttrf - compute the factorization of a complex Hermitian tridiagonal matrix A

SYNOPSIS

```
SUBROUTINE CSTTRF(N, L, D, SUBL, IPIV, INFO)

COMPLEX L(*), D(*), SUBL(*)
INTEGER N, INFO
INTEGER IPIV(*)

SUBROUTINE CSTTRF_64(N, L, D, SUBL, IPIV, INFO)

COMPLEX L(*), D(*), SUBL(*)
INTEGER*8 N, INFO
INTEGER*8 IPIV(*)
```

F95 INTERFACE

```
SUBROUTINE STTRF([N], L, D, SUBL, IPIV, [INFO])

COMPLEX, DIMENSION(:) :: L, D, SUBL
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIV

SUBROUTINE STTRF_64([N], L, D, SUBL, IPIV, [INFO])

COMPLEX, DIMENSION(:) :: L, D, SUBL
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIV
```

C INTERFACE

```
#include <sunperf.h>
```

```
void csttrf(int n, complex *l, complex  *d,  complex  *subl,
          int *ipiv, int *info);

void csttrf_64(long  n,  complex  *l,  complex  *d,  complex
          *subl, long *ipiv, long *info);
```

PURPOSE

csttrf computes the L*D*L**H factorization of a complex Hermitian tridiagonal matrix A.

ARGUMENTS

```
N (input)  INTEGER
          The order of the matrix A.  N >= 0.

L (input/output)
           COMPLEX array, dimension (N)
          On entry, the n-1 subdiagonal elements of the tri-
          diagonal  matrix A.  On exit, part of the factori-
          zation of A.

D (input/output)
           REAL array, dimension (N)
          On entry, the n diagonal elements of the tridiago-
          nal matrix A.  On exit, the n diagonal elements of
          the diagonal matrix D from the factorization of A.

SUBL (output)
           COMPLEX array, dimension (N)
          On exit, part of the factorization of A.

IPIV (output)
           INTEGER array, dimension (N)
          On exit, the pivot indices of the factorization.

INFO (output)
           INTEGER
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, D(k,k) is  exactly  zero.   The
          factorization  has  been  completed, but the block
          diagonal matrix D is exactly singular and division
          by zero will occur if it is used to solve a system
          of equations.
```

Contents

NAME

```
csttrs - computes the solution to a complex system of linear
equations A * X = B
```

SYNOPSIS

```
SUBROUTINE CSTTRS(N, NRHS, L, D, SUBL, B, LDB, IPIV, INFO)

COMPLEX L(*), D(*), SUBL(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO
INTEGER IPIV(*)

SUBROUTINE CSTTRS_64(N, NRHS, L, D, SUBL, B, LDB, IPIV, INFO)

COMPLEX L(*), D(*), SUBL(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIV(*)
```

F95 INTERFACE

```
SUBROUTINE STTRS([N], [NRHS], L, D, SUBL, B, [LDB], IPIV, [INFO])

COMPLEX, DIMENSION(:) :: L, D, SUBL
COMPLEX, DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIV

SUBROUTINE STTRS_64([N], [NRHS], L, D, SUBL, B, [LDB], IPIV, [INFO])

COMPLEX, DIMENSION(:) :: L, D, SUBL
COMPLEX, DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIV
```

C INTERFACE

```
#include <sunperf.h>

void csttrs(int n, int nrhs, complex *l, complex *d, complex
          *subl, complex *b, int ldb, int *ipiv, int *info);

void csttrs_64(long n, long nrhs, complex  *l,  complex  *d,
          complex  *subl,  complex *b, long ldb, long *ipiv,
          long *info);
```

PURPOSE

csttrs computes the solution to a complex system of linear equations A * X = B, where A is an N-by-N symmetric tridiagonal matrix and X and B are N-by-NRHS matrices.

ARGUMENTS

N (input) INTEGER
The order of the matrix A. N >= 0.

NRHS (input)
INTEGER
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

L (input) COMPLEX array, dimension (N-1)
On entry, the subdiagonal elements of LL and DD.

D (input) COMPLEX array, dimension (N)
On entry, the diagonal elements of DD.

SUBL (input)
COMPLEX array, dimension (N-2)
On entry, the second subdiagonal elements of LL.

B (input/output)
COMPLEX array, dimension (LDB, NRHS)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
INTEGER
The leading dimension of the array B. LDB >= max(1, N)

IPIV (output)
INTEGER array, dimension (N)
Details of the interchanges and block pivot. If IPIV(K) > 0, 1 by 1 pivot, and if IPIV(K) = K + 1 an interchange done; If IPIV(K) < 0, 2 by 2 pivot, no interchange required.

INFO (output)
INTEGER
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value

Contents

NAME

```
cswap - Exchange vectors x and y.
```

SYNOPSIS

```
SUBROUTINE CSWAP(N, X, INCX, Y, INCY)

COMPLEX X(*), Y(*)
INTEGER N, INCX, INCY

SUBROUTINE CSWAP_64(N, X, INCX, Y, INCY)

COMPLEX X(*), Y(*)
INTEGER*8 N, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE SWAP([N], X, [INCX], Y, [INCY])

COMPLEX, DIMENSION(:) :: X, Y
INTEGER :: N, INCX, INCY

SUBROUTINE SWAP_64([N], X, [INCX], Y, [INCY])

COMPLEX, DIMENSION(:) :: X, Y
INTEGER(8) :: N, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>

void cswap(int n, complex *x,  int  incx,  complex  *y,  int
          incy);

void cswap_64(long n, complex *x,  long  incx,  complex  *y,
```

```
            long incy);
```

PURPOSE

cswap Exchange x and y where x and y are n-vectors.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input/output)
(1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. On exit, the y vector.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. On exit, the x vector.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
csycon - estimate the reciprocal of the condition number (in
the  1-norm)  of a complex symmetric matrix A using the fac-
torization A = U*D*U**T or A = L*D*L**T computed by CSYTRF
```

SYNOPSIS

```
SUBROUTINE CSYCON(UPLO, N, A, LDA, IPIVOT, ANORM, RCOND, WORK, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, INFO
INTEGER IPIVOT(*)
REAL ANORM, RCOND

SUBROUTINE CSYCON_64(UPLO, N, A, LDA, IPIVOT, ANORM, RCOND, WORK,
      INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, INFO
INTEGER*8 IPIVOT(*)
REAL ANORM, RCOND
```

F95 INTERFACE

```
SUBROUTINE SYCON(UPLO, [N], A, [LDA], IPIVOT, ANORM, RCOND, [WORK],
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT
```

```
REAL :: ANORM, RCOND

SUBROUTINE SYCON_64(UPLO, [N], A, [LDA], IPIVOT, ANORM, RCOND, [WORK],
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL :: ANORM, RCOND
```

C INTERFACE

```
#include <sunperf.h>
void csycon(char uplo, int  n,  complex  *a,  int  lda,  int
          *ipivot, float anorm, float *rcond, int *info);

void csycon_64(char uplo, long n, complex *a, long lda, long
          *ipivot, float anorm, float *rcond, long *info);
```

PURPOSE

csycon estimates the reciprocal of the condition number (in the 1-norm) of a complex symmetric matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by CSYTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

A (input) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CSYTRF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CSYTRF.

ANORM (input)
The 1-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
csymm - perform one of the matrix-matrix operations    C  :=
alpha*A*B + beta*C or C := alpha*B*A + beta*C
```

SYNOPSIS

```
SUBROUTINE CSYMM(SIDE, UPLO, M, N, ALPHA, A, LDA, B, LDB, BETA, C,
      LDC)

CHARACTER * 1 SIDE, UPLO
COMPLEX ALPHA, BETA
COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER M, N, LDA, LDB, LDC

SUBROUTINE CSYMM_64(SIDE, UPLO, M, N, ALPHA, A, LDA, B, LDB, BETA, C,
      LDC)

CHARACTER * 1 SIDE, UPLO
COMPLEX ALPHA, BETA
COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER*8 M, N, LDA, LDB, LDC
```

F95 INTERFACE

```
SUBROUTINE SYMM(SIDE, UPLO, [M], [N], ALPHA, A, [LDA], B, [LDB],
       BETA, C, [LDC])

CHARACTER(LEN=1) :: SIDE, UPLO
COMPLEX :: ALPHA, BETA
COMPLEX, DIMENSION(:,:) :: A, B, C
INTEGER :: M, N, LDA, LDB, LDC

SUBROUTINE SYMM_64(SIDE, UPLO, [M], [N], ALPHA, A, [LDA], B, [LDB],
       BETA, C, [LDC])
```

```
CHARACTER(LEN=1) :: SIDE, UPLO
COMPLEX :: ALPHA, BETA
COMPLEX, DIMENSION(:,:) :: A, B, C
INTEGER(8) :: M, N, LDA, LDB, LDC
```

C INTERFACE

```
#include <sunperf.h>

void csymm(char side, char  uplo,  int  m,  int  n,  complex
          *alpha,  complex *a, int lda, complex *b, int ldb,
          complex *beta, complex *c, int ldc);

void csymm_64(char side, char uplo, long m, long n,  complex
          *alpha,  complex  *a,  long  lda, complex *b, long
          ldb, complex *beta, complex *c, long ldc);
```

PURPOSE

csymm performs one of the matrix-matrix operations C := alpha*A*B + beta*C or C := alpha*B*A + beta*C where alpha and beta are scalars, A is a symmetric matrix and B and C are m by n matrices.

ARGUMENTS

SIDE (input)
On entry, SIDE specifies whether the symmetric matrix A appears on the left or right in the operation as follows:

SIDE = 'L' or 'l'   C := alpha*A*B + beta*C,

SIDE = 'R' or 'r'   C := alpha*B*A + beta*C,

Unchanged on exit.

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the symmetric matrix A is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of the symmetric matrix is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of the symmetric matrix is to be referenced.

Unchanged on exit.

M (input)
On entry, M specifies the number of rows of the matrix C. M >= 0. Unchanged on exit.

N (input)
On entry, N specifies the number of columns of the matrix C. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
COMPLEX array of DIMENSION (LDA, ka), where ka is m when SIDE = 'L' or 'l' and is n otherwise.

Before entry with SIDE = 'L' or 'l', the m by m part of the array A must contain the symmetric matrix, such that when UPLO = 'U' or 'u', the leading m by m upper triangular part of the array A must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of A is not referenced, and when UPLO = 'L' or 'l', the leading m by m lower triangular part of the array A must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of A is not referenced.

Before entry with SIDE = 'R' or 'r', the n by n part of the array A must contain the symmetric matrix, such that when UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of A is not referenced, and when UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of A is not referenced.

Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When

SIDE = 'L' or 'l' then LDA >= max(1, m), otherwise LDA >= max(1, n). Unchanged on exit.

B (input)
COMPLEX array of DIMENSION (LDB, n). Before entry, the leading m by n part of the array B must contain the matrix B. Unchanged on exit.

LDB (input)
On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. LDB >= max(1, m). Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. When BETA is supplied as zero then C need not be set on input. Unchanged on exit.

C (input/output)
COMPLEX array of DIMENSION (LDC, n). Before entry, the leading m by n part of the array C must contain the matrix C, except when beta is zero, in which case C need not be set on entry. On exit, the array C is overwritten by the m by n updated matrix.

LDC (input)
On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC >= max(1, m). Unchanged on exit.

Contents

NAME

```
csyr2k - perform one of the symmetric rank 2k operations   C
:=  alpha*A*B'  +  alpha*B*A'  + beta*C or C := alpha*A'*B +
alpha*B'*A + beta*C
```

SYNOPSIS

```
SUBROUTINE CSYR2K(UPLO, TRANSA, N, K, ALPHA, A, LDA, B, LDB, BETA, C,
      LDC)

CHARACTER * 1 UPLO, TRANSA
COMPLEX ALPHA, BETA
COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER N, K, LDA, LDB, LDC

SUBROUTINE CSYR2K_64(UPLO, TRANSA, N, K, ALPHA, A, LDA, B, LDB, BETA,
      C, LDC)

CHARACTER * 1 UPLO, TRANSA
COMPLEX ALPHA, BETA
COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER*8 N, K, LDA, LDB, LDC
```

F95 INTERFACE

```
SUBROUTINE SYR2K(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], B, [LDB],
       BETA, C, [LDC])

CHARACTER(LEN=1) :: UPLO, TRANSA
COMPLEX :: ALPHA, BETA
COMPLEX, DIMENSION(:,:) :: A, B, C
INTEGER :: N, K, LDA, LDB, LDC

SUBROUTINE SYR2K_64(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], B,
```

```
          [LDB], BETA, C, [LDC])

   CHARACTER(LEN=1) :: UPLO, TRANSA
   COMPLEX :: ALPHA, BETA
   COMPLEX, DIMENSION(:,:) :: A, B, C
   INTEGER(8) :: N, K, LDA, LDB, LDC
```

C INTERFACE

```
   #include <sunperf.h>

   void csyr2k(char uplo, char transa, int n,  int  k,  complex
             *alpha,  complex *a, int lda, complex *b, int ldb,
             complex *beta, complex *c, int ldc);
   void csyr2k_64(char uplo, char transa, long n, long k,  com-
             plex  *alpha,  complex  *a,  long lda, complex *b,
             long ldb, complex *beta, complex *c, long ldc);
```

PURPOSE

csyr2k performs one of the symmetric rank 2k operations C := alpha*A*B' + alpha*B*A' + beta*C or C := alpha*A'*B + alpha*B'*A + beta*C where alpha and beta are scalars, C is an n by n symmetric matrix and A and B are n by k matrices in the first case and k by n matrices in the second case.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array C is to be referenced as follows:

UPLO = 'U' or 'u' Only the upper triangular part of C is to be referenced.

UPLO = 'L' or 'l' Only the lower triangular part of C is to be referenced.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n' C := alpha*A*B' + alpha*B*A' + beta*C.

TRANSA = 'T' or 't'   C := alpha*A'*B + alpha*B'*A + beta*C.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input)

On entry, N specifies the order of the matrix C. N must be at least zero. Unchanged on exit.

K (input)

On entry with TRANSA = 'N' or 'n', K specifies the number of columns of the matrices A and B, and on entry with TRANSA = 'T' or 't', K specifies the number of rows of the matrices A and B. K must be at least zero. Unchanged on exit.

ALPHA (input)

On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)

COMPLEX array of DIMENSION (LDA, ka), where ka is k when TRANSA = 'N' or 'n', and is n otherwise. Before entry with TRANSA = 'N' or 'n', the leading n by k part of the array A must contain the matrix A, otherwise the leading k by n part of the array A must contain the matrix A. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDA must be at least max(1, n), otherwise LDA must be at least max(1, k). Unchanged on exit.

B (input)

COMPLEX array of DIMENSION (LDB, kb), where kb is k when TRANSA = 'N' or 'n', and is n otherwise. Before entry with TRANSA = 'N' or 'n', the leading n by k part of the array B must contain the matrix B, otherwise the leading k by n part of the array B must contain the matrix B. Unchanged on exit.

LDB (input)

On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDB must be at least max(1, n), otherwise LDB must be at least max(1, k). Unchanged on exit.

BETA (input)

On entry, BETA specifies the scalar beta. Unchanged on exit.

C (input/output)

COMPLEX array of DIMENSION (LDC, n).

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array C must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of C is not referenced. On exit, the upper triangular part of the array C is overwritten by the upper triangular part of the updated matrix.

Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array C must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of C is not referenced. On exit, the lower triangular part of the array C is overwritten by the lower triangular part of the updated matrix.

LDC (input)

On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC must be at least max(1, n). Unchanged on exit.

Contents

NAME

```
csyrfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is symmetric indefin-
ite, and provides error bounds and backward error  estimates
for the solution
```

SYNOPSIS

```
SUBROUTINE CSYRFS(UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*)
REAL FERR(*), BERR(*), WORK2(*)

SUBROUTINE CSYRFS_64(UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
REAL FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE SYRFS(UPLO, [N], [NRHS], A, [LDA], AF, [LDAF], IPIVOT, B,
       [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
```

```
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE SYRFS_64(UPLO, [N], [NRHS], A, [LDA], AF, [LDAF], IPIVOT,
       B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void csyrfs(char uplo, int n, int nrhs, complex *a, int lda,
          complex  *af,  int  ldaf, int *ipivot, complex *b,
          int ldb, complex *x, int ldx, float  *ferr,  float
          *berr, int *info);

void csyrfs_64(char uplo, long n,  long  nrhs,  complex  *a,
          long  lda,  complex  *af, long ldaf, long *ipivot,
          complex *b, long ldb, complex *x, long ldx,  float
          *ferr, float *berr, long *info);
```

PURPOSE

csyrfs improves the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input) The symmetric matrix A.  If UPLO = 'U', the  lead-
          ing N-by-N upper triangular part of A contains the
```

upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input)
The factored form of the matrix A. AF contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by CSYTRF.

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CSYTRF.

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by CSYTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

```
BERR (output)
          The componentwise relative backward error of  each
          solution  vector X(j) (i.e., the smallest relative
          change in any element of A or B that makes X(j) an
          exact solution).
WORK (workspace)
          dimension(2*N)

WORK2 (workspace)
          dimension(N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
csyrk - perform one of the symmetric rank k  operations    C
:= alpha*A*A' + beta*C or C := alpha*A'*A + beta*C
```

SYNOPSIS

```
SUBROUTINE CSYRK(UPLO, TRANSA, N, K, ALPHA, A, LDA, BETA, C, LDC)

CHARACTER * 1 UPLO, TRANSA
COMPLEX ALPHA, BETA
COMPLEX A(LDA,*), C(LDC,*)
INTEGER N, K, LDA, LDC

SUBROUTINE CSYRK_64(UPLO, TRANSA, N, K, ALPHA, A, LDA, BETA, C, LDC)

CHARACTER * 1 UPLO, TRANSA
COMPLEX ALPHA, BETA
COMPLEX A(LDA,*), C(LDC,*)
INTEGER*8 N, K, LDA, LDC
```

F95 INTERFACE

```
SUBROUTINE SYRK(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], BETA, C,
       [LDC])

CHARACTER(LEN=1) :: UPLO, TRANSA
COMPLEX :: ALPHA, BETA
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER :: N, K, LDA, LDC

SUBROUTINE SYRK_64(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], BETA,
       C, [LDC])

CHARACTER(LEN=1) :: UPLO, TRANSA
```

```
COMPLEX :: ALPHA, BETA
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER(8) :: N, K, LDA, LDC
```

C INTERFACE

```
#include <sunperf.h>

void csyrk(char uplo, char transa, int  n,  int  k,  complex
          *alpha,  complex  *a, int lda, complex *beta, com-
          plex *c, int ldc);

void csyrk_64(char uplo, char transa, long n, long  k,  com-
          plex  *alpha, complex *a, long lda, complex *beta,
          complex *c, long ldc);
```

PURPOSE

csyrk performs one of the symmetric rank k operations C := alpha*A*A' + beta*C or C := alpha*A'*A + beta*C where alpha and beta are scalars, C is an n by n symmetric matrix and A is an n by k matrix in the first case and a k by n matrix in the second case.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array C is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of C is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of C is to be referenced.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   C := alpha*A*A' + beta*C.

TRANSA = 'T' or 't'   C := alpha*A'*A + beta*C.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input)
On entry, N specifies the order of the matrix C. N must be at least zero. Unchanged on exit.

K (input)
On entry with TRANSA = 'N' or 'n', K specifies the number of columns of the matrix A, and on entry with TRANSA = 'T' or 't', K specifies the number of rows of the matrix A. K must be at least zero. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
COMPLEX array of DIMENSION (LDA, ka), where ka is k when TRANSA = 'N' or 'n', and is n otherwise. Before entry with TRANSA = 'N' or 'n', the leading n by k part of the array A must contain the matrix A, otherwise the leading k by n part of the array A must contain the matrix A. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDA must be at least max(1, n), otherwise LDA must be at least max(1, k). Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. Unchanged on exit.

C (input/output)
COMPLEX array of DIMENSION (LDC, n).

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array C must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of C is not referenced. On exit, the upper triangular part of the array C is overwritten by the upper triangular part of the updated matrix.

Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array C must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of C is not referenced. On exit, the lower triangular part of the array C is overwritten by the lower triangular part of the updated matrix.

LDC (input)

On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC must be at least max(1, n). Unchanged on exit.

Contents

NAME

```
csysv - compute the solution to a complex system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE CSYSV(UPLO, N, NRHS, A, LDA, IPIV, B, LDB, WORK, LWORK,
      INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER N, NRHS, LDA, LDB, LWORK, INFO
INTEGER IPIV(*)

SUBROUTINE CSYSV_64(UPLO, N, NRHS, A, LDA, IPIV, B, LDB, WORK, LWORK,
      INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER*8 N, NRHS, LDA, LDB, LWORK, INFO
INTEGER*8 IPIV(*)
```

F95 INTERFACE

```
SUBROUTINE SYSV(UPLO, [N], [NRHS], A, [LDA], IPIV, B, [LDB], [WORK],
       [LWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: N, NRHS, LDA, LDB, LWORK, INFO
INTEGER, DIMENSION(:) :: IPIV

SUBROUTINE SYSV_64(UPLO, [N], [NRHS], A, [LDA], IPIV, B, [LDB], [WORK],
```

```
          [LWORK], [INFO])

    CHARACTER(LEN=1) :: UPLO
    COMPLEX, DIMENSION(:) :: WORK
    COMPLEX, DIMENSION(:,:) :: A, B
    INTEGER(8) :: N, NRHS, LDA, LDB, LWORK, INFO
    INTEGER(8), DIMENSION(:) :: IPIV
```

C INTERFACE

```
    #include <sunperf.h>

    void csysv(char uplo, int n, int nrhs, complex *a, int  lda,
              int *ipiv, complex *b, int ldb, int *info);
    void csysv_64(char uplo, long n, long nrhs, complex *a, long
              lda,  long  *ipiv,  complex  *b,  long  ldb,  long
              *info);
```

PURPOSE

```
    csysv computes the solution to a complex  system  of  linear
    equations
       A * X = B, where A is an N-by-N symmetric  matrix  and  X
    and B are N-by-NRHS matrices.

    The diagonal pivoting method is used to factor A as
       A = U * D * U**T,  if UPLO = 'U', or
       A = L * D * L**T,  if UPLO = 'L',
    where U (or L) is a product of permutation  and  unit  upper
    (lower)  triangular  matrices,  and D is symmetric and block
    diagonal with 1-by-1 and 2-by-2 diagonal blocks.   The  fac-
    tored  form  of  A is then used to solve the system of equa-
    tions A * X = B.
```

ARGUMENTS

```
    UPLO (input)
              = 'U':  Upper triangle of A is stored;
              = 'L':  Lower triangle of A is stored.

    N (input) The number of linear equations, i.e., the order of
              the matrix A.  N >= 0.

    NRHS (input)
              The number of right hand sides, i.e.,  the  number
              of columns of the matrix B.  NRHS >= 0.

    A (input/output)
```

On entry, the symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, if INFO = 0, the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by CSYTRF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIV (output)
Details of the interchanges and the block structure of D, as determined by CSYTRF. If IPIV(k) > 0, then rows and columns k and IPIV(k) were interchanged, and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIV(k) = IPIV(k-1) < 0, then rows and columns k-1 and -IPIV(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIV(k) = IPIV(k+1) < 0, then rows and columns k+1 and -IPIV(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The length of WORK. LWORK >= 1, and for best performance LWORK >= N*NB, where NB is the optimal blocksize for CSYTRF.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of

the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, so the solution could not be computed.

Contents

NAME

```
csysvx - use the diagonal pivoting factorization to  compute
the solution to a complex system of linear equations A * X =
B,
```

SYNOPSIS

```
SUBROUTINE CSYSVX(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B,
      LDB, X, LDX, RCOND, FERR, BERR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER IPIVOT(*)
REAL RCOND
REAL FERR(*), BERR(*), WORK2(*)

SUBROUTINE CSYSVX_64(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT,
      B, LDB, X, LDX, RCOND, FERR, BERR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
COMPLEX A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER*8 IPIVOT(*)
REAL RCOND
REAL FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE SYSVX(FACT, UPLO, [N], [NRHS], A, [LDA], AF, [LDAF],
       IPIVOT, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [LDWORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO
```

```
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL :: RCOND
REAL, DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE SYSVX_64(FACT, UPLO, [N], [NRHS], A, [LDA], AF, [LDAF],
       IPIVOT, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [LDWORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL :: RCOND
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void csysvx(char fact, char uplo, int n, int  nrhs,  complex
          *a,  int  lda, complex *af, int ldaf, int *ipivot,
          complex *b, int ldb, complex *x,  int  ldx,  float
          *rcond, float *ferr, float *berr, int *info);

void csysvx_64(char fact, char uplo, long n, long nrhs, com-
          plex  *a,  long  lda, complex *af, long ldaf, long
          *ipivot, complex *b, long ldb,  complex  *x,  long
          ldx,  float *rcond, float *ferr, float *berr, long
          *info);
```

PURPOSE

csysvx uses the diagonal pivoting factorization to compute the solution to a complex system of linear equations A * X = B, where A is an N-by-N symmetric matrix and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'N', the diagonal pivoting method is used to factor A.
   The form of the factorization is
      A = U * D * U**T, if UPLO = 'U', or

```
      A = L * D * L**T,  if UPLO = 'L',
   where U (or L) is a product of permutation and unit upper
(lower)
   triangular matrices, and D is symmetric and block  diago-
nal with
   1-by-1 and 2-by-2 diagonal blocks.

2. If some D(i,i)=0, so that D is exactly singular, then the
routine
   returns with INFO = i. Otherwise, the factored form of  A
is used
   to estimate the condition number of the matrix A.  If the
   reciprocal of the condition number is less  than  machine
precision,
   INFO = N+1 is returned as  a  warning,  but  the  routine
still goes on
   to solve for X and  compute  error  bounds  as  described
below.
3. The system of equations is solved for X  using  the  fac-
tored form
   of A.

4. Iterative refinement is applied to improve  the  computed
solution
   matrix and calculate  error  bounds  and  backward  error
estimates
   for it.
```

ARGUMENTS

```
FACT (input)
          Specifies whether or not the factored  form  of  A
          has  been supplied on entry.  = 'F':  On entry, AF
          and IPIVOT contain the factored form of A.  A,  AF
          and  IPIVOT  will  not  be  modified.  = 'N':  The
          matrix A will be copied to AF and factored.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.
```

```
A (input) The symmetric matrix A.  If UPLO = 'U', the  lead-
          ing N-by-N upper triangular part of A contains the
          upper triangular part of the  matrix  A,  and  the
          strictly  lower triangular part of A is not refer-
          enced.  If UPLO = 'L', the  leading  N-by-N  lower
          triangular part of A contains the lower triangular
          part of the matrix A, and the strictly upper  tri-
          angular part of A is not referenced.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

AF (input/output)
          If FACT = 'F', then AF is an input argument and on
          entry contains the block diagonal matrix D and the
          multipliers used to obtain the factor U or L  from
          the  factorization A = U*D*U**T or A = L*D*L**T as
          computed by CSYTRF.

          If FACT = 'N', then AF is an output  argument  and
          on  exit  returns  the block diagonal matrix D and
          the multipliers used to obtain the factor U  or  L
          from  the  factorization  A  =  U*D*U**T  or  A  =
          L*D*L**T.

LDAF (input)
          The leading dimension of the array  AF.   LDAF  >=
          max(1,N).

IPIVOT (input or output)
          If FACT = 'F', then IPIVOT is  an  input  argument
          and  on entry contains details of the interchanges
          and the block structure of  D,  as  determined  by
          CSYTRF.  If IPIVOT(k) > 0, then rows and columns k
          and IPIVOT(k) were interchanged and  D(k,k)  is  a
          1-by-1   diagonal   block.   If  UPLO  =  'U'  and
          IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns
          k-1  and  -IPIVOT(k)  were  interchanged  and D(k-
          1:k,k-1:k) is a 2-by-2 diagonal block.  If UPLO  =
          'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and
          columns k+1 and -IPIVOT(k) were  interchanged  and
          D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

          If FACT = 'N', then IPIVOT is an  output  argument
          and  on  exit contains details of the interchanges
          and the block structure of  D,  as  determined  by
          CSYTRF.

B (input) The N-by-NRHS right hand side matrix B.
```

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A. If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of WORK. LDWORK >= 2*N, and for best performance LDWORK >= N*NB, where NB is the optimal blocksize for CSYTRF.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first

entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: D(i,i) is exactly zero. The factorization has been completed but the factor D is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: D is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
csytf2 - compute the factorization of  a  complex  symmetric
matrix A using the Bunch-Kaufman diagonal pivoting method
```

SYNOPSIS

```
SUBROUTINE CSYTF2(UPLO, N, A, LDA, IPIV, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*)
INTEGER N, LDA, INFO
INTEGER IPIV(*)

SUBROUTINE CSYTF2_64(UPLO, N, A, LDA, IPIV, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*)
INTEGER*8 N, LDA, INFO
INTEGER*8 IPIV(*)
```

F95 INTERFACE

```
SUBROUTINE SYTF2(UPLO, [N], A, [LDA], IPIV, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIV

SUBROUTINE SYTF2_64(UPLO, [N], A, [LDA], IPIV, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A
```

```
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIV
```

C INTERFACE

```
#include <sunperf.h>

void csytf2(char uplo, int  n,  complex  *a,  int  lda,  int
          *ipiv, int *info);

void csytf2_64(char uplo, long n, complex *a, long lda, long
          *ipiv, long *info);
```

PURPOSE

csytf2 computes the factorization of a complex symmetric matrix A using the Bunch-Kaufman diagonal pivoting method:

A = U*D*U' or A = L*D*L'

where U (or L) is a product of permutation and unit upper (lower) triangular matrices, U' is the transpose of U, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

This is the unblocked version of the algorithm, calling Level 2 BLAS.

ARGUMENTS

UPLO (input)
Specifies whether the upper or lower triangular part of the symmetric matrix A is stored:
= 'U': Upper triangular
= 'L': Lower triangular

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the symmetric matrix A. If UPLO = 'U', the leading n-by-n upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, the block diagonal matrix D and the mul-

tipliers used to obtain the factor U or L (see below for further details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIV (output)
Details of the interchanges and the block structure of D. If IPIV(k) > 0, then rows and columns k and IPIV(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIV(k) = IPIV(k-1) < 0, then rows and columns k-1 and -IPIV(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIV(k) = IPIV(k+1) < 0, then rows and columns k+1 and -IPIV(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, D(k,k) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

1-96 - Based on modifications by J. Lewis, Boeing Computer Services
Company

If UPLO = 'U', then A = U*D*U', where
U = P(n)*U(n)* ... *P(k)U(k)* ...,
i.e., U is a product of terms P(k)*U(k), where k decreases from n to 1 in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIV(k), and U(k) is a unit upper triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    v    0   )   k-s
   U(k) =  (   0    I    0   )   s
           (   0    0    I   )   n-k
              k-s   s   n-k
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(1:k-

```
1,k).   If s = 2, the upper triangle of D(k) overwrites A(k-
1,k-1), A(k-1,k), and A(k,k), and  v  overwrites  A(1:k-2,k-
1:k).

If UPLO = 'L', then A = L*D*L', where
   L = P(1)*L(1)* ... *P(k)*L(k)* ...,
i.e., L is a product of terms P(k)*L(k), where  k  increases
from  1  to  n in steps of 1 or 2, and D is a block diagonal
matrix with 1-by-1 and 2-by-2 diagonal blocks D(k).  P(k) is
a  permutation  matrix  as defined by IPIV(k), and L(k) is a
unit lower triangular matrix,  such  that  if  the  diagonal
block D(k) is of order s (s = 1 or 2), then

           (   I    0     0   )  k-1
   L(k) =  (   0    I     0   )  s
           (   0    v     I   )  n-k-s+1
              k-1   s  n-k-s+1
If  s  =  1,  D(k)  overwrites  A(k,k),  and  v   overwrites
A(k+1:n,k).  If s = 2, the lower triangle of D(k) overwrites
A(k,k),  A(k+1,k),  and   A(k+1,k+1),   and   v   overwrites
A(k+2:n,k:k+1).
```

Contents

NAME

```
csytrf - compute the factorization of  a  complex  symmetric
matrix A using the Bunch-Kaufman diagonal pivoting method
```

SYNOPSIS

```
SUBROUTINE CSYTRF(UPLO, N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, LDWORK, INFO
INTEGER IPIVOT(*)

SUBROUTINE CSYTRF_64(UPLO, N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, LDWORK, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE SYTRF(UPLO, [N], A, [LDA], IPIVOT, [WORK], [LDWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, LDWORK, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE SYTRF_64(UPLO, [N], A, [LDA], IPIVOT, [WORK], [LDWORK],
       [INFO])
```

```
CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void csytrf(char uplo, int  n,  complex  *a,  int  lda,  int
          *ipivot, int *info);

void csytrf_64(char uplo, long n, complex *a, long lda, long
          *ipivot, long *info);
```

PURPOSE

csytrf computes the factorization of a complex symmetric matrix A using the Bunch-Kaufman diagonal pivoting method. The form of the factorization is

A = U*D*U**T or A = L*D*L**T

where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with with 1-by-1 and 2-by-2 diagonal blocks.

This is the blocked version of the algorithm, calling Level 3 BLAS.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, the block diagonal matrix D and the multipliers used to obtain the factor U or L (see below for further details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (output)
Details of the interchanges and the block structure of D. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of WORK. LDWORK >=1. For best performance LDWORK >= N*NB, where NB is the block size returned by ILAENV.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

If UPLO = 'U', then A = U*D*U', where

U = P(n)*U(n)* ... *P(k)U(k)* ...,
i.e., U is a product of terms P(k)*U(k), where k decreases from n to 1 in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIVOT(k), and U(k) is a unit upper triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    v    0   )   k-s
   U(k) =  (   0    I    0   )   s
           (   0    0    I   )   n-k
              k-s   s   n-k
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(1:k-1,k). If s = 2, the upper triangle of D(k) overwrites A(k-1,k-1), A(k-1,k), and A(k,k), and v overwrites A(1:k-2,k-1:k).

If UPLO = 'L', then A = L*D*L', where
L = P(1)*L(1)* ... *P(k)*L(k)* ...,
i.e., L is a product of terms P(k)*L(k), where k increases from 1 to n in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIVOT(k), and L(k) is a unit lower triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    0     0   )  k-1
   L(k) =  (   0    I     0   )  s
           (   0    v     I   )  n-k-s+1
              k-1   s  n-k-s+1
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(k+1:n,k). If s = 2, the lower triangle of D(k) overwrites A(k,k), A(k+1,k), and A(k+1,k+1), and v overwrites A(k+2:n,k:k+1).

Contents

NAME

```
csytri - compute the inverse of a complex symmetric indefin-
ite  matrix  A  using  the factorization A = U*D*U**T or A =
L*D*L**T computed by CSYTRF
```

SYNOPSIS

```
SUBROUTINE CSYTRI(UPLO, N, A, LDA, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, INFO
INTEGER IPIVOT(*)

SUBROUTINE CSYTRI_64(UPLO, N, A, LDA, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE SYTRI(UPLO, [N], A, [LDA], IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE SYTRI_64(UPLO, [N], A, [LDA], IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void csytri(char uplo, int  n,  complex  *a,  int  lda,  int
          *ipivot, int *info);

void csytri_64(char uplo, long n, complex *a, long lda, long
          *ipivot, long *info);
```

PURPOSE

csytri computes the inverse of a complex symmetric indefinite matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by CSYTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CSYTRF.

On exit, if INFO = 0, the (symmetric) inverse of the original matrix. If UPLO = 'U', the upper triangular part of the inverse is formed and the part of A below the diagonal is not referenced; if UPLO = 'L' the lower triangular part of the inverse is formed and the part of A above the diagonal is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

```
IPIVOT (input)
          Details of the interchanges and the  block  struc-
          ture of D as determined by CSYTRF.

WORK (workspace)
          dimension(2*N)

INFO (output)
          = 0: successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
          gal value
          > 0: if INFO = i, D(i,i) = 0; the matrix is singu-
          lar and its inverse could not be computed.
```

Contents

NAME

```
csytrs - solve a system of linear equations A*X = B  with  a
complex  symmetric  matrix  A  using  the  factorization A =
U*D*U**T or A = L*D*L**T computed by CSYTRF
```

SYNOPSIS

```
SUBROUTINE CSYTRS(UPLO, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE CSYTRS_64(UPLO, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE SYTRS(UPLO, [N], [NRHS], A, [LDA], IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: N, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE SYTRS_64(UPLO, [N], [NRHS], A, [LDA], IPIVOT, B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void csytrs(char uplo, int n, int nrhs, complex *a, int lda,
          int *ipivot, complex *b, int ldb, int *info);

void csytrs_64(char uplo, long n,  long  nrhs,  complex  *a,
          long lda, long *ipivot, complex *b, long ldb, long
          *info);
```

PURPOSE

csytrs solves a system of linear equations A*X = B with a complex symmetric matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by CSYTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CSYTRF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CSYTRF.

B (input/output)

On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

ctbcon - estimate the reciprocal of the condition number of a triangular band matrix A, in either the 1-norm or the infinity-norm

SYNOPSIS

```
SUBROUTINE CTBCON(NORM, UPLO, DIAG, N, KD, A, LDA, RCOND, WORK,
      WORK2, INFO)

CHARACTER * 1 NORM, UPLO, DIAG
COMPLEX A(LDA,*), WORK(*)
INTEGER N, KD, LDA, INFO
REAL RCOND
REAL WORK2(*)

SUBROUTINE CTBCON_64(NORM, UPLO, DIAG, N, KD, A, LDA, RCOND, WORK,
      WORK2, INFO)

CHARACTER * 1 NORM, UPLO, DIAG
COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, KD, LDA, INFO
REAL RCOND
REAL WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE TBCON(NORM, UPLO, DIAG, [N], KD, A, [LDA], RCOND, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, KD, LDA, INFO
```

```
REAL :: RCOND
REAL, DIMENSION(:) :: WORK2

SUBROUTINE TBCON_64(NORM, UPLO, DIAG, [N], KD, A, [LDA], RCOND,
       [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, KD, LDA, INFO
REAL :: RCOND
REAL, DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>
void ctbcon(char norm, char uplo, char diag, int n, int  kd,
          complex *a, int lda, float *rcond, int *info);

void ctbcon_64(char norm, char uplo, char diag, long n, long
          kd,  complex  *a,  long  lda,  float  *rcond, long
          *info);
```

PURPOSE

ctbcon estimates the reciprocal of the condition number of a triangular band matrix A, in either the 1-norm or the infinity-norm.

The norm of A is computed and an estimate is obtained for norm(inv(A)), then the reciprocal of the condition number is computed as
   RCOND = 1 / (norm(A) * norm(inv(A))).

ARGUMENTS

```
NORM (input)
          Specifies whether the 1-norm condition  number  or
          the infinity-norm condition number is required:
          = '1' or 'O':  1-norm;
          = 'I':         Infinity-norm.

UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

DIAG (input)
          = 'N':  A is non-unit triangular;
```

```
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals  or  subdiagonals  of
          the triangular band matrix A.  KD >= 0.

A (input) The upper  or  lower  triangular  band  matrix  A,
          stored  in  the  first kd+1 rows of the array. The
          j-th column of A is stored in the j-th  column  of
          the  array A as follows:  if UPLO = 'U', A(kd+1+i-
          j,j) = A(i,j) for  max(1,j-kd)<=i<=j;  if  UPLO  =
          'L', A(1+i-j,j)    = A(i,j) for j<=i<=min(n,j+kd).
          If DIAG = 'U', the diagonal elements of A are  not
          referenced and are assumed to be 1.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          KD+1.

RCOND (output)
          The reciprocal of  the  condition  number  of  the
          matrix   A,  computed  as  RCOND  =  1/(norm(A)  *
          norm(inv(A))).

WORK (workspace)
          dimension(2*N)

WORK2 (workspace)
          dimension(N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

ctbmv - perform one of the matrix-vector operations x := A*x, or x := A'*x, or x := conjg(A')*x

SYNOPSIS

```
SUBROUTINE CTBMV(UPLO, TRANSA, DIAG, N, K, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(LDA,*), Y(*)
INTEGER N, K, LDA, INCY

SUBROUTINE CTBMV_64(UPLO, TRANSA, DIAG, N, K, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(LDA,*), Y(*)
INTEGER*8 N, K, LDA, INCY
```

F95 INTERFACE

```
SUBROUTINE TBMV(UPLO, [TRANSA], DIAG, [N], K, A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, K, LDA, INCY

SUBROUTINE TBMV_64(UPLO, [TRANSA], DIAG, [N], K, A, [LDA], Y,
       [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, K, LDA, INCY
```

C INTERFACE

```
#include <sunperf.h>

void ctbmv(char uplo, char transa, char diag, int n, int  k,
          complex *a, int lda, complex *y, int incy);

void ctbmv_64(char uplo, char transa,  char  diag,  long  n,
          long  k,  complex  *a,  long lda, complex *y, long
          incy);
```

PURPOSE

ctbmv performs one of the matrix-vector operations x := A*x, or x := A'*x, or x := conjg(A')*x where x is an n element vector and A is an n by n unit, or non-unit, upper or lower triangular band matrix, with (k + 1) diagonals.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   x := A*x.

TRANSA = 'T' or 't'   x := A'*x.

TRANSA = 'C' or 'c'   x := conjg(A')*x.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)

On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit triangular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

N (input)

On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

K (input)

On entry with UPLO = 'U' or 'u', K specifies the number of super-diagonals of the matrix A. On entry with UPLO = 'L' or 'l', K specifies the number of sub-diagonals of the matrix A. K >= 0. Unchanged on exit.

A (input)

Before entry with UPLO = 'U' or 'u', the leading (k + 1) by n part of the array A must contain the upper triangular band part of the matrix of coefficients, supplied column by column, with the leading diagonal of the matrix in row (k + 1) of the array, the first super-diagonal starting at position 2 in row k, and so on. The top left k by k triangle of the array A is not referenced. The following program segment will transfer an upper triangular band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = K + 1 - J
     DO 10, I = MAX( 1, J - K ), J
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Before entry with UPLO = 'L' or 'l', the leading (k + 1) by n part of the array A must contain the lower triangular band part of the matrix of coefficients, supplied column by column, with the leading diagonal of the matrix in row 1 of the array, the first sub-diagonal starting at position 1 in row 2, and so on. The bottom right k by k triangle of the array A is not referenced. The following program segment will transfer a lower

triangular band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = 1 - J
     DO 10, I = J, MIN( N, J + K )
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Note that when DIAG = 'U' or 'u' the elements of the array A corresponding to the diagonal elements of the matrix are not referenced, but are assumed to be unity. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= (k + 1). Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector x. On exit, Y is overwritten with the tran- formed vector x.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
ctbrfs - provide error bounds and backward  error  estimates
for the solution to a system of linear equations with a tri-
angular band coefficient matrix
```

SYNOPSIS

```
SUBROUTINE CTBRFS(UPLO, TRANSA, DIAG, N, KD, NRHS, A, LDA, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(LDA,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, KD, NRHS, LDA, LDB, LDX, INFO
REAL FERR(*), BERR(*), WORK2(*)

SUBROUTINE CTBRFS_64(UPLO, TRANSA, DIAG, N, KD, NRHS, A, LDA, B,
      LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(LDA,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, KD, NRHS, LDA, LDB, LDX, INFO
REAL FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE TBRFS(UPLO, [TRANSA], DIAG, [N], KD, [NRHS], A, [LDA],
       B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B, X
INTEGER :: N, KD, NRHS, LDA, LDB, LDX, INFO
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

```
SUBROUTINE TBRFS_64(UPLO, [TRANSA], DIAG, [N], KD, [NRHS], A, [LDA],
       B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B, X
INTEGER(8) :: N, KD, NRHS, LDA, LDB, LDX, INFO
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void ctbrfs(char uplo, char transa, char diag,  int  n,  int
          kd, int nrhs, complex *a, int lda, complex *b, int
          ldb, complex  *x,  int  ldx,  float  *ferr,  float
          *berr, int *info);

void ctbrfs_64(char uplo, char transa, char  diag,  long  n,
          long  kd, long nrhs, complex *a, long lda, complex
          *b, long ldb, complex *x, long ldx,  float  *ferr,
          float *berr, long *info);
```

PURPOSE

ctbrfs provides error bounds and backward error estimates for the solution to a system of linear equations with a triangular band coefficient matrix.

The solution matrix X must be computed by CTBTRS or some other means before entering this routine. CTBRFS does not do iterative refinement because doing so cannot improve the backward error.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.
```

DIAG (input)
= 'N': A is non-unit triangular;
= 'U': A is unit triangular.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals or subdiagonals of the triangular band matrix A. KD >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The upper or lower triangular band matrix A, stored in the first kd+1 rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd). If DIAG = 'U', the diagonal elements of A are not referenced and are assumed to be 1.

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input) The solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)

```
            The componentwise relative backward error of  each
            solution  vector X(j) (i.e., the smallest relative
            change in any element of A or B that makes X(j) an
            exact solution).

WORK (workspace)
            dimension(2*N)
WORK2 (workspace)
            dimension(N)

INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value
```

Contents

NAME

```
ctbsv - solve one of the systems of equations   A*x = b,  or
A'*x = b, or conjg( A' )*x = b
```

SYNOPSIS

```
SUBROUTINE CTBSV(UPLO, TRANSA, DIAG, N, K, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(LDA,*), Y(*)
INTEGER N, K, LDA, INCY

SUBROUTINE CTBSV_64(UPLO, TRANSA, DIAG, N, K, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(LDA,*), Y(*)
INTEGER*8 N, K, LDA, INCY
```

F95 INTERFACE

```
SUBROUTINE TBSV(UPLO, [TRANSA], DIAG, [N], K, A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, K, LDA, INCY

SUBROUTINE TBSV_64(UPLO, [TRANSA], DIAG, [N], K, A, [LDA], Y,
       [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, K, LDA, INCY
```

C INTERFACE

```
#include <sunperf.h>

void ctbsv(char uplo, char transa, char diag, int n, int  k,
          complex *a, int lda, complex *y, int incy);

void ctbsv_64(char uplo, char transa,  char  diag,  long  n,
          long  k,  complex  *a,  long lda, complex *y, long
          incy);
```

PURPOSE

ctbsv solves one of the systems of equations A*x = b, or A'*x = b, or conjg(A')*x = b where b and x are n element vectors and A is an n by n unit, or non-unit, upper or lower triangular band matrix, with (k + 1) diagonals.

No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u' A is an upper triangular matrix.

UPLO = 'L' or 'l' A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the equations to be solved as follows:

TRANSA = 'N' or 'n' A*x = b.

TRANSA = 'T' or 't' A'*x = b.

TRANSA = 'C' or 'c' conjg(A')*x = b.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit triangular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

K (input)
On entry with UPLO = 'U' or 'u', K specifies the number of super-diagonals of the matrix A. On entry with UPLO = 'L' or 'l', K specifies the number of sub-diagonals of the matrix A. K >= 0. Unchanged on exit.

A (input)
Before entry with UPLO = 'U' or 'u', the leading (k + 1) by n part of the array A must contain the upper triangular band part of the matrix of coefficients, supplied column by column, with the leading diagonal of the matrix in row (k + 1) of the array, the first super-diagonal starting at position 2 in row k, and so on. The top left k by k triangle of the array A is not referenced. The following program segment will transfer an upper triangular band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = K + 1 - J
     DO 10, I = MAX( 1, J - K ), J
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Before entry with UPLO = 'L' or 'l', the leading (k + 1) by n part of the array A must contain the lower triangular band part of the matrix of coefficients, supplied column by column, with the

leading diagonal of the matrix in row 1 of the array, the first sub-diagonal starting at position 1 in row 2, and so on. The bottom right k by k triangle of the array A is not referenced. The following program segment will transfer a lower triangular band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = 1 - J
     DO 10, I = J, MIN( N, J + K )
       A( M + I, J ) = matrix( I, J )
10  CONTINUE
20 CONTINUE
```

Note that when DIAG = 'U' or 'u' the elements of the array A corresponding to the diagonal elements of the matrix are not referenced, but are assumed to be unity. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= (k + 1). Unchanged on exit.

Y (input/output)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element right-hand side vector b. On exit, Y is overwritten with the solution vector x.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
ctbtrs - solve a triangular system of the form   A * X =  B,
A**T * X = B, or A**H * X = B,
```

SYNOPSIS

```
SUBROUTINE CTBTRS(UPLO, TRANSA, DIAG, N, KD, NRHS, A, LDA, B, LDB,
      INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, KD, NRHS, LDA, LDB, INFO

SUBROUTINE CTBTRS_64(UPLO, TRANSA, DIAG, N, KD, NRHS, A, LDA, B,
      LDB, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, KD, NRHS, LDA, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE TBTRS(UPLO, TRANSA, DIAG, [N], KD, [NRHS], A, [LDA], B,
       [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: N, KD, NRHS, LDA, LDB, INFO

SUBROUTINE TBTRS_64(UPLO, TRANSA, DIAG, [N], KD, [NRHS], A, [LDA],
       B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:,:) :: A, B
```

```
INTEGER(8) :: N, KD, NRHS, LDA, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void ctbtrs(char uplo, char transa, char diag,  int  n,  int
          kd, int nrhs, complex *a, int lda, complex *b, int
          ldb, int *info);

void ctbtrs_64(char uplo, char transa, char  diag,  long  n,
          long  kd, long nrhs, complex *a, long lda, complex
          *b, long ldb, long *info);
```

PURPOSE

ctbtrs solves a triangular system of the form

where A is a triangular band matrix of order N, and B is an N-by-NRHS matrix. A check is made to verify that A is non-singular.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals  or  subdiagonals  of
          the triangular band matrix A.  KD >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.
```

A (input) The upper or lower triangular band matrix A, stored in the first kd+1 rows of A. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd). If DIAG = 'U', the diagonal elements of A are not referenced and are assumed to be 1.

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

B (input/output)
On entry, the right hand side matrix B. On exit, if INFO = 0, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the i-th diagonal element of A is zero, indicating that the matrix is singular and the solutions X have not been computed.

Contents

NAME

```
ctgevc - compute some or all of the right and/or  left  gen-
eralized  eigenvectors of a pair of complex upper triangular
matrices (A,B)
```

SYNOPSIS

```
SUBROUTINE CTGEVC(SIDE, HOWMNY, SELECT, N, A, LDA, B, LDB, VL, LDVL,
      VR, LDVR, MM, M, WORK, RWORK, INFO)

CHARACTER * 1 SIDE, HOWMNY
COMPLEX A(LDA,*), B(LDB,*), VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER N, LDA, LDB, LDVL, LDVR, MM, M, INFO
LOGICAL SELECT(*)
REAL RWORK(*)

SUBROUTINE CTGEVC_64(SIDE, HOWMNY, SELECT, N, A, LDA, B, LDB, VL,
      LDVL, VR, LDVR, MM, M, WORK, RWORK, INFO)

CHARACTER * 1 SIDE, HOWMNY
COMPLEX A(LDA,*), B(LDB,*), VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER*8 N, LDA, LDB, LDVL, LDVR, MM, M, INFO
LOGICAL*8 SELECT(*)
REAL RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE TGEVC(SIDE, HOWMNY, SELECT, [N], A, [LDA], B, [LDB], VL,
       [LDVL], VR, [LDVR], MM, M, [WORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, HOWMNY
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B, VL, VR
INTEGER :: N, LDA, LDB, LDVL, LDVR, MM, M, INFO
```

```
LOGICAL, DIMENSION(:) :: SELECT
REAL, DIMENSION(:) :: RWORK

SUBROUTINE TGEVC_64(SIDE, HOWMNY, SELECT, [N], A, [LDA], B, [LDB],
       VL, [LDVL], VR, [LDVR], MM, M, [WORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, HOWMNY
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B, VL, VR
INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, MM, M, INFO
LOGICAL(8), DIMENSION(:) :: SELECT
REAL, DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>
void ctgevc(char side, char howmny, int *select, int n, com-
          plex  *a,  int  lda,  complex *b, int ldb, complex
          *vl, int ldvl, complex *vr, int ldvr, int mm,  int
          *m, int *info);

void ctgevc_64(char side, char howmny, long *select, long n,
          complex  *a,  long lda, complex *b, long ldb, com-
          plex *vl, long ldvl, complex *vr, long ldvr,  long
          mm, long *m, long *info);
```

PURPOSE

ctgevc computes some or all of the right and/or left generalized eigenvectors of a pair of complex upper triangular matrices (A,B).

The right generalized eigenvector x and the left generalized eigenvector y of (A,B) corresponding to a generalized eigenvalue w are defined by:

        (A - wB) * x = 0 and y**H * (A - wB) = 0

where y**H denotes the conjugate tranpose of y.

If an eigenvalue w is determined by zero diagonal elements of both A and B, a unit vector is returned as the corresponding eigenvector.

If all eigenvectors are requested, the routine may either return the matrices X and/or Y of right or left eigenvectors of (A,B), or the products Z*X and/or Q*Y, where Z and Q are input unitary matrices. If (A,B) was obtained from the generalized Schur factorization of an original pair of matrices
   (A0,B0) = (Q*A*Z**H,Q*B*Z**H),

then Z*X and Q*Y are the matrices of right or left eigenvectors of A.

ARGUMENTS

SIDE (input)
= 'R': compute right eigenvectors only;
= 'L': compute left eigenvectors only;
= 'B': compute both right and left eigenvectors.

HOWMNY (input)
= 'A': compute all right and/or left eigenvectors;
= 'B': compute all right and/or left eigenvectors, and backtransform them using the input matrices supplied in VR and/or VL; = 'S': compute selected right and/or left eigenvectors, specified by the logical array SELECT.

SELECT (input)
If HOWMNY='S', SELECT specifies the eigenvectors to be computed. If HOWMNY='A' or 'B', SELECT is not referenced. To select the eigenvector corresponding to the j-th eigenvalue, SELECT(j) must be set to .TRUE..

N (input) The order of the matrices A and B. N >= 0.

A (input) The upper triangular matrix A.

LDA (input)
The leading dimension of array A. LDA >= max(1,N).

B (input) The upper triangular matrix B. B must have real diagonal elements.

LDB (input)
The leading dimension of array B. LDB >= max(1,N).

VL (input/output)
On entry, if SIDE = 'L' or 'B' and HOWMNY = 'B', VL must contain an N-by-N matrix Q (usually the unitary matrix Q of left Schur vectors returned by CHGEQZ). On exit, if SIDE = 'L' or 'B', VL contains: if HOWMNY = 'A', the matrix Y of left eigenvectors of (A,B); if HOWMNY = 'B', the matrix Q*Y; if HOWMNY = 'S', the left eigenvectors of

(A,B) specified by SELECT, stored consecutively in the columns of VL, in the same order as their eigenvalues. If SIDE = 'R', VL is not referenced.

LDVL (input)
The leading dimension of array VL. LDVL >= max(1,N) if SIDE = 'L' or 'B'; LDVL >= 1 otherwise.

VR (input/output)
On entry, if SIDE = 'R' or 'B' and HOWMNY = 'B', VR must contain an N-by-N matrix Q (usually the unitary matrix Z of right Schur vectors returned by CHGEQZ). On exit, if SIDE = 'R' or 'B', VR contains: if HOWMNY = 'A', the matrix X of right eigenvectors of (A,B); if HOWMNY = 'B', the matrix Z*X; if HOWMNY = 'S', the right eigenvectors of (A,B) specified by SELECT, stored consecutively in the columns of VR, in the same order as their eigenvalues. If SIDE = 'L', VR is not referenced.

LDVR (input)
The leading dimension of the array VR. LDVR >= max(1,N) if SIDE = 'R' or 'B'; LDVR >= 1 otherwise.

MM (input)
The number of columns in the arrays VL and/or VR. MM >= M.

M (output)
The number of columns in the arrays VL and/or VR actually used to store the eigenvectors. If HOWMNY = 'A' or 'B', M is set to N. Each selected eigenvector occupies one column.

WORK (workspace)
dimension(2*N)

RWORK (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

```
ctgexc - reorder the generalized Schur  decomposition  of  a
complex  matrix  pair  (A,B),  using  an unitary equivalence
transformation (A, B) := Q * (A, B) * Z', so that the diago-
nal block of (A, B) with row index IFST is moved to row ILST
```

SYNOPSIS

```
SUBROUTINE CTGEXC(WANTQ, WANTZ, N, A, LDA, B, LDB, Q, LDQ, Z, LDZ,
      IFST, ILST, INFO)

COMPLEX A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*)
INTEGER N, LDA, LDB, LDQ, LDZ, IFST, ILST, INFO
LOGICAL WANTQ, WANTZ

SUBROUTINE CTGEXC_64(WANTQ, WANTZ, N, A, LDA, B, LDB, Q, LDQ, Z, LDZ,
      IFST, ILST, INFO)

COMPLEX A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*)
INTEGER*8 N, LDA, LDB, LDQ, LDZ, IFST, ILST, INFO
LOGICAL*8 WANTQ, WANTZ
```

F95 INTERFACE

```
SUBROUTINE TGEXC(WANTQ, WANTZ, [N], A, [LDA], B, [LDB], Q, [LDQ], Z,
       [LDZ], IFST, ILST, [INFO])

COMPLEX, DIMENSION(:,:) :: A, B, Q, Z
INTEGER :: N, LDA, LDB, LDQ, LDZ, IFST, ILST, INFO
LOGICAL :: WANTQ, WANTZ

SUBROUTINE TGEXC_64(WANTQ, WANTZ, [N], A, [LDA], B, [LDB], Q, [LDQ],
       Z, [LDZ], IFST, ILST, [INFO])
```

```
COMPLEX, DIMENSION(:,:) :: A, B, Q, Z
INTEGER(8) :: N, LDA, LDB, LDQ, LDZ, IFST, ILST, INFO
LOGICAL(8) :: WANTQ, WANTZ
```

C INTERFACE

```
#include <sunperf.h>

void ctgexc(int wantq, int wantz, int  n,  complex  *a,  int
          lda,  complex  *b,  int  ldb, complex *q, int ldq,
          complex *z, int ldz, int  *ifst,  int  *ilst,  int
          *info);

void ctgexc_64(long wantq, long wantz, long n,  complex  *a,
          long  lda,  complex *b, long ldb, complex *q, long
          ldq, complex *z, long ldz, long *ifst, long *ilst,
          long *info);
```

PURPOSE

ctgexc reorders the generalized Schur decomposition of a complex matrix pair (A,B), using an unitary equivalence transformation (A, B) := Q * (A, B) * Z', so that the diagonal block of (A, B) with row index IFST is moved to row ILST.

(A, B) must be in generalized Schur canonical form, that is, A and B are both upper triangular.

Optionally, the matrices Q and Z of generalized Schur vectors are updated.

```
Q(in) * A(in) * Z(in)' = Q(out) * A(out) * Z(out)'
Q(in) * B(in) * Z(in)' = Q(out) * B(out) * Z(out)'
```

ARGUMENTS

WANTQ (input)

WANTZ (input)

N (input) The order of the matrices A and B. N >= 0.

A (input/output)
On entry, the upper triangular matrix A in the pair (A, B). On exit, the updated matrix A.

```
LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,N).

B (input/output)
          On entry, the upper triangular  matrix  B  in  the
          pair (A, B).  On exit, the updated matrix B.

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          max(1,N).

Q (input/output)
          On entry, if WANTQ = .TRUE., the unitary matrix Q.
          On  exit,  the  updated  matrix  Q.   If  WANTQ  =
          .FALSE., Q is not referenced.

LDQ (input)
          The leading dimension of the array Q. LDQ >= 1; If
          WANTQ = .TRUE., LDQ >= N.

Z (input/output)
          On entry, if WANTZ = .TRUE., the unitary matrix Z.
          On  exit,  the  updated  matrix  Z.   If  WANTZ  =
          .FALSE., Z is not referenced.

LDZ (input)
          The leading dimension of the array Z. LDZ >= 1; If
          WANTZ = .TRUE., LDZ >= N.

IFST (input/output)
          Specify the reordering of the diagonal  blocks  of
          (A, B).  The block with row index IFST is moved to
          row ILST, by a sequence of swapping between  adja-
          cent blocks.

ILST (input/output)
          See the description of IFST.

INFO (output)
          =0:  Successful exit.
          <0:  if INFO = -i, the i-th argument had an  ille-
          gal value.
          =1:  The transformed matrix pair (A, B)  would  be
          too  far  from generalized Schur form; the problem
          is ill- conditioned. (A, B)  may  have  been  par-
          tially reordered, and ILST points to the first row
          of the current position of the block being moved.
```

FURTHER DETAILS

Based on contributions by
Bo Kagstrom and Peter Poromaa, Department of Computing Science,
Umea University, S-901 87 Umea, Sweden.

[1] B. Kagstrom; A Direct Method for Reordering Eigenvalues in the
Generalized Real Schur Form of a Regular Matrix Pair (A, B), in
M.S. Moonen et al (eds), Linear Algebra for Large Scale and
Real-Time Applications, Kluwer Academic Publ. 1993, pp 195-218.

[2] B. Kagstrom and P. Poromaa; Computing Eigenspaces with Specified
Eigenvalues of a Regular Matrix Pair (A, B) and Condi- tion
Estimation: Theory, Algorithms and Software, Report
UMINF - 94.04, Department of Computing Science, Umea University,
S-901 87 Umea, Sweden, 1994. Also as LAPACK Working Note 87.
To appear in Numerical Algorithms, 1996.

[3] B. Kagstrom and P. Poromaa, LAPACK-Style Algorithms and Software
for Solving the Generalized Sylvester Equation and Estimating the
Separation between Regular Matrix Pairs, Report UMINF - 93.23,
Department of Computing Science, Umea University, S-901 87 Umea,
Sweden, December 1993, Revised April 1994, Also as LAPACK working
Note 75. To appear in ACM Trans. on Math. Software, Vol 22, No 1,
1996.

Contents

NAME

```
ctgsen - reorder the generalized Schur  decomposition  of  a
complex   matrix  pair  (A,  B)  (in  terms  of  an  unitary
equivalence trans- formation Q' * (A, B) *  Z),  so  that  a
selected cluster of eigenvalues appears in the leading diag-
onal blocks of the pair (A,B)
```

SYNOPSIS

```
SUBROUTINE CTGSEN(IJOB, WANTQ, WANTZ, SELECT, N, A, LDA, B, LDB,
      ALPHA, BETA, Q, LDQ, Z, LDZ, M, PL, PR, DIF, WORK, LWORK, IWORK,
      LIWORK, INFO)

COMPLEX A(LDA,*),  B(LDB,*),  ALPHA(*),  BETA(*),  Q(LDQ,*),
Z(LDZ,*), WORK(*)
INTEGER IJOB, N, LDA, LDB, LDQ, LDZ, M, LWORK, LIWORK, INFO
INTEGER IWORK(*)
LOGICAL WANTQ, WANTZ
LOGICAL SELECT(*)
REAL PL, PR
REAL DIF(*)

SUBROUTINE CTGSEN_64(IJOB, WANTQ, WANTZ, SELECT, N, A, LDA, B, LDB,
      ALPHA, BETA, Q, LDQ, Z, LDZ, M, PL, PR, DIF, WORK, LWORK, IWORK,
      LIWORK, INFO)

COMPLEX A(LDA,*),  B(LDB,*),  ALPHA(*),  BETA(*),  Q(LDQ,*),
Z(LDZ,*), WORK(*)
INTEGER*8 IJOB, N, LDA, LDB, LDQ,  LDZ,  M,  LWORK,  LIWORK,
INFO
INTEGER*8 IWORK(*)
LOGICAL*8 WANTQ, WANTZ
```

```
LOGICAL*8 SELECT(*)
REAL PL, PR
REAL DIF(*)
```

F95 INTERFACE

```
SUBROUTINE TGSEN(IJOB, WANTQ, WANTZ, SELECT, [N], A, [LDA], B, [LDB],
       ALPHA, BETA, Q, [LDQ], Z, [LDZ], M, PL, PR, DIF, [WORK], [LWORK],
       [IWORK], [LIWORK], [INFO])

COMPLEX, DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX, DIMENSION(:,:) :: A, B, Q, Z
INTEGER :: IJOB, N, LDA, LDB, LDQ, LDZ,  M,  LWORK,  LIWORK,
INFO
INTEGER, DIMENSION(:) :: IWORK
LOGICAL :: WANTQ, WANTZ
LOGICAL, DIMENSION(:) :: SELECT
REAL :: PL, PR
REAL, DIMENSION(:) :: DIF
SUBROUTINE TGSEN_64(IJOB, WANTQ, WANTZ, SELECT, [N], A, [LDA], B,
       [LDB], ALPHA, BETA, Q, [LDQ], Z, [LDZ], M, PL, PR, DIF, [WORK],
       [LWORK], [IWORK], [LIWORK], [INFO])

COMPLEX, DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX, DIMENSION(:,:) :: A, B, Q, Z
INTEGER(8) :: IJOB, N, LDA, LDB, LDQ, LDZ, M, LWORK, LIWORK,
INFO
INTEGER(8), DIMENSION(:) :: IWORK
LOGICAL(8) :: WANTQ, WANTZ
LOGICAL(8), DIMENSION(:) :: SELECT
REAL :: PL, PR
REAL, DIMENSION(:) :: DIF
```

C INTERFACE

```
#include <sunperf.h>

void ctgsen(int ijob, int wantq, int wantz, int *select, int
          n,  complex *a, int lda, complex *b, int ldb, com-
          plex *alpha, complex *beta, complex *q,  int  ldq,
          complex *z, int ldz, int *m, float *pl, float *pr,
          float *dif, int *info);

void ctgsen_64(long  ijob,  long  wantq,  long  wantz,  long
          *select, long n, complex *a, long lda, complex *b,
          long ldb, complex *alpha, complex  *beta,  complex
          *q, long ldq, complex *z, long ldz, long *m, float
          *pl, float *pr, float *dif, long *info);
```

PURPOSE

ctgsen reorders the generalized Schur decomposition of a complex matrix pair (A, B) (in terms of an unitary equivalence trans- formation Q' * (A, B) * Z), so that a selected cluster of eigenvalues appears in the leading diag- onal blocks of the pair (A,B). The leading columns of Q and Z form unitary bases of the corresponding left and right eigenspaces (deflating subspaces). (A, B) must be in gen- eralized Schur canonical form, that is, A and B are both upper triangular.

CTGSEN also computes the generalized eigenvalues

w(j)= ALPHA(j) / BETA(j)

of the reordered matrix pair (A, B).

Optionally, the routine computes estimates of reciprocal condition numbers for eigenvalues and eigenspaces. These are Difu[(A11,B11), (A22,B22)] and Difl[(A11,B11), (A22,B22)], i.e. the separation(s) between the matrix pairs (A11, B11) and (A22,B22) that correspond to the selected cluster and the eigenvalues outside the cluster, resp., and norms of "projections" onto left and right eigenspaces w.r.t. the selected cluster in the (1,1)-block.

ARGUMENTS

IJOB (input)
Specifies whether condition numbers are required for the cluster of eigenvalues (PL and PR) or the deflating subspaces (Difu and Difl):
=0: Only reorder w.r.t. SELECT. No extras.
=1: Reciprocal of norms of "projections" onto left and right eigenspaces w.r.t. the selected cluster (PL and PR). =2: Upper bounds on Difu and Difl. F-norm-based estimate
(DIF(1:2)).
=3: Estimate of Difu and Difl. 1-norm-based esti- mate
(DIF(1:2)). About 5 times as expensive as IJOB = 2. =4: Compute PL, PR and DIF (i.e. 0, 1 and 2 above): Economic version to get it all. =5: Com- pute PL, PR and DIF (i.e. 0, 1 and 3 above)

WANTQ (input)

WANTZ (input)

```
SELECT (input)
          SELECT specifies the eigenvalues in  the  selected
          cluster.  To  select an eigenvalue w(j), SELECT(j)
          must be set to

N (input) The order of the matrices A and B. N >= 0.

A (input/output)
          On entry, the upper triangular matrix A,  in  gen-
          eralized  Schur  canonical  form.   On  exit, A is
          overwritten by the reordered matrix A.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,N).
B (input/output)
          On entry, the upper triangular matrix B,  in  gen-
          eralized  Schur  canonical  form.   On  exit, B is
          overwritten by the reordered matrix B.

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          max(1,N).

ALPHA (output)
          The diagonal elements of A  and  B,  respectively,
          when  the  pair (A,B) has been reduced to general-
          ized Schur form.  ALPHA(i)/BETA(i)  i=1,...,N  are
          the generalized eigenvalues.

BETA (output)
          See the description of ALPHA.

Q (input/output)
          On entry, if  WANTQ  =  .TRUE.,  Q  is  an  N-by-N
          matrix.  On exit, Q has been postmultiplied by the
          left unitary transformation matrix  which  reorder
          (A,  B); The leading M columns of Q form orthonor-
          mal bases for the specified pair  of  left  eigen-
          spaces (deflating subspaces).  If WANTQ = .FALSE.,
          Q is not referenced.

LDQ (input)
          The leading dimension of the array Q.  LDQ  >=  1.
          If WANTQ = .TRUE., LDQ >= N.

Z (input/output)
          On entry, if  WANTZ  =  .TRUE.,  Z  is  an  N-by-N
          matrix.  On exit, Z has been postmultiplied by the
          left unitary transformation matrix  which  reorder
```

(A, B); The leading M columns of Z form orthonormal bases for the specified pair of left eigenspaces (deflating subspaces). If WANTZ = .FALSE., Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1. If WANTZ = .TRUE., LDZ >= N.

M (output)
The dimension of the specified pair of left and right eigenspaces, (deflating subspaces) 0 <= M <= N.

PL (output)
IF IJOB = 1, 4, or 5, PL, PR are lower bounds on the reciprocal of the norm of "projections" onto left and right eigenspace with respect to the selected cluster.
0 < PL, PR <= 1. If M = 0 or M = N, PL = PR = 1.
If IJOB = 0, 2, or 3 PL, PR are not referenced.

PR (output)
See the description of PL.

DIF (output)
If IJOB >= 2, DIF(1:2) store the estimates of Difu and Difl.
If IJOB = 2 or 4, DIF(1:2) are F-norm-based upper bounds on
Difu and Difl. If IJOB = 3 or 5, DIF(1:2) are 1-norm-based estimates of Difu and Difl, computed using reversed communication with CLACON. If M = 0 or N, DIF(1:2) = F-norm([A, B]). If IJOB = 0 or 1, DIF is not referenced.

WORK (workspace)
If IJOB = 0, WORK is not referenced. Otherwise, on exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 1 If IJOB = 1, 2 or 4, LWORK >= 2*M*(N-M) If IJOB = 3 or 5, LWORK >= 4*M*(N-M)

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace/output)
If IJOB = 0, IWORK is not referenced. Otherwise, on exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of the array IWORK. LIWORK >= 1. If IJOB = 1, 2 or 4, LIWORK >= N+2; If IJOB = 3 or 5, LIWORK >= MAX(N+2, 2*M*(N-M));

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
=0: Successful exit.
<0: If INFO = -i, the i-th argument had an illegal value.
=1: Reordering of (A, B) failed because the transformed matrix pair (A, B) would be too far from generalized Schur form; the problem is very ill-conditioned. (A, B) may have been partially reordered. If requested, 0 is returned in DIF(*), PL and PR.

FURTHER DETAILS

CTGSEN first collects the selected eigenvalues by computing unitary U and W that move them to the top left corner of (A, B). In other words, the selected eigenvalues are the eigenvalues of (A11, B11) in

```
U'*(A, B)*W = (A11 A12) (B11 B12) n1
              ( 0  A22),( 0  B22) n2
                n1  n2    n1  n2
```

where N = n1+n2 and U' means the conjugate transpose of U. The first n1 columns of U and W span the specified pair of left and right eigenspaces (deflating subspaces) of (A, B).

If (A, B) has been obtained from the generalized real Schur decomposition of a matrix pair (C, D) = Q*(A, B)*Z', then the reordered generalized Schur form of (C, D) is given by

```
(C, D) = (Q*U)*(U'*(A, B)*W)*(Z*W)',
```

and the first n1 columns of Q*U and Z*W span the corresponding deflating subspaces of (C, D) (Q and Z store Q*U and Z*W, resp.).

Note that if the selected eigenvalue is sufficiently ill-conditioned, then its value may differ significantly from its value before reordering.

The reciprocal condition numbers of the left and right eigenspaces spanned by the first n1 columns of U and W (or Q*U and Z*W) may be returned in DIF(1:2), corresponding to Difu and Difl, resp.

The Difu and Difl are defined as:
ifu[(A11, B11), (A22, B22)] = sigma-min(Zu)
and

where sigma-min(Zu) is the smallest singular value of the (2*n1*n2)-by-(2*n1*n2) matrix

```
u = [ kron(In2, A11)  -kron(A22', In1) ]
          [ kron(In2, B11)  -kron(B22', In1) ].
```

Here, Inx is the identity matrix of size nx and A22' is the transpose of A22. kron(X, Y) is the Kronecker product between the matrices X and Y.

When DIF(2) is small, small changes in (A, B) can cause large changes in the deflating subspace. An approximate (asymptotic) bound on the maximum angular error in the computed deflating subspaces is PS * norm((A, B)) / DIF(2),

where EPS is the machine precision.

The reciprocal norm of the projectors on the left and right eigenspaces associated with (A11, B11) may be returned in PL and PR. They are computed as follows. First we compute L and R so that P*(A, B)*Q is block diagonal, where

```
 = ( I -L ) n1           Q = ( I R ) n1
         ( 0  I ) n2    and        ( 0 I ) n2
           n1 n2                    n1 n2
```

and (L, R) is the solution to the generalized Sylvester equation 11*R - L*A22 = -A12

Then PL = (F-norm(L)**2+1)**(-1/2) and PR = (F-norm(R)**2+1)**(-1/2). An approximate (asymptotic) bound on the average absolute error of the selected eigenvalues is
EPS * norm((A, B)) / PL.

There are also global error bounds which valid for perturbations up to a certain restriction: A lower bound (x) on the smallest F-norm(E,F) for which an eigenvalue of (A11, B11) may move and coalesce with an eigenvalue of (A22, B22) under perturbation (E,F), (i.e. (A + E, B + F), is

x = min(Difu,Difl)/((1/(PL*PL)+1/(PR*PR))**(1/2)+2*max(1/PL,1/PR)).
An approximate bound on x can be computed from DIF(1:2), PL and PL.

If y = (F-norm(E,F) / x) <= 1, the angles between the perturbed (L', R') and unperturbed (L, R) left and right deflating subspaces associated with the selected cluster in the (1,1)-blocks can be bounded as

max-angle(L, L') <= arctan(y * PL / (1 - y * (1 - PL * PL)**(1/2))
max-angle(R, R') <= arctan(y * PR / (1 - y * (1 - PR * PR)**(1/2))

See LAPACK User's Guide section 4.11 or the following references for more information.

Note that if the default method for computing the Frobenius-norm- based estimate DIF is not wanted (see CLATDF), then the parameter IDIFJB (see below) should be changed from 3 to 4 (routine CLATDF (IJOB = 2 will be used)). See CTGSYL for more details.

Based on contributions by
Bo Kagstrom and Peter Poromaa, Department of Computing Science,
Umea University, S-901 87 Umea, Sweden.

References
==========

[1] B. Kagstrom; A Direct Method for Reordering Eigenvalues in the
Generalized Real Schur Form of a Regular Matrix Pair (A, B), in
M.S. Moonen et al (eds), Linear Algebra for Large Scale and
Real-Time Applications, Kluwer Academic Publ. 1993, pp 195-218.

[2] B. Kagstrom and P. Poromaa; Computing Eigenspaces with Specified
Eigenvalues of a Regular Matrix Pair (A, B) and Condition
Estimation: Theory, Algorithms and Software, Report

UMINF - 94.04, Department of Computing Science, Umea University,
S-901 87 Umea, Sweden, 1994. Also as LAPACK Working Note 87.
To appear in Numerical Algorithms, 1996.

[3] B. Kagstrom and P. Poromaa, LAPACK-Style Algorithms and Software
for Solving the Generalized Sylvester Equation and Estimating the
Separation between Regular Matrix Pairs, Report UMINF - 93.23,
Department of Computing Science, Umea University, S-901 87 Umea,
Sweden, December 1993, Revised April 1994, Also as LAPACK working
Note 75. To appear in ACM Trans. on Math. Software, Vol 22, No 1,
1996.

Contents

NAME

ctgsja - compute the generalized singular value decomposition (GSVD) of two complex upper triangular (or trapezoidal) matrices A and B

SYNOPSIS

```
SUBROUTINE CTGSJA(JOBU, JOBV, JOBQ, M, P, N, K, L, A, LDA, B, LDB,
      TOLA, TOLB, ALPHA, BETA, U, LDU, V, LDV, Q, LDQ, WORK, NCYCLE,
      INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
COMPLEX A(LDA,*), B(LDB,*),  U(LDU,*),  V(LDV,*),  Q(LDQ,*),
WORK(*)
INTEGER M, P, N, K, L, LDA, LDB, LDU, LDV, LDQ, NCYCLE, INFO
REAL TOLA, TOLB
REAL ALPHA(*), BETA(*)

SUBROUTINE CTGSJA_64(JOBU, JOBV, JOBQ, M, P, N, K, L, A, LDA, B, LDB,
      TOLA, TOLB, ALPHA, BETA, U, LDU, V, LDV, Q, LDQ, WORK, NCYCLE,
      INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
COMPLEX A(LDA,*), B(LDB,*),  U(LDU,*),  V(LDV,*),  Q(LDQ,*),
WORK(*)
INTEGER*8 M, P, N, K, L, LDA, LDB, LDU,  LDV,  LDQ,  NCYCLE,
INFO
REAL TOLA, TOLB
REAL ALPHA(*), BETA(*)
```

F95 INTERFACE

```
SUBROUTINE TGSJA(JOBU, JOBV, JOBQ, [M], [P], [N], K, L, A, [LDA], B,
       [LDB], TOLA, TOLB, ALPHA, BETA, U, [LDU], V, [LDV], Q, [LDQ],
```

```
       [WORK], NCYCLE, [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B, U, V, Q
INTEGER :: M, P, N, K, L, LDA, LDB, LDU, LDV,  LDQ,  NCYCLE,
INFO
REAL :: TOLA, TOLB
REAL, DIMENSION(:) :: ALPHA, BETA

SUBROUTINE TGSJA_64(JOBU, JOBV, JOBQ, [M], [P], [N], K, L, A, [LDA],
       B, [LDB], TOLA, TOLB, ALPHA, BETA, U, [LDU], V, [LDV], Q, [LDQ],
       [WORK], NCYCLE, [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B, U, V, Q
INTEGER(8) :: M, P, N, K, L, LDA, LDB, LDU, LDV,  LDQ,  NCY-
CLE, INFO
REAL :: TOLA, TOLB
REAL, DIMENSION(:) :: ALPHA, BETA
```

C INTERFACE

```
#include <sunperf.h>

void ctgsja(char jobu, char jobv, char jobq, int m,  int  p,
          int  n, int k, int l, complex *a, int lda, complex
          *b, int ldb, float tola, float tolb, float *alpha,
          float  *beta, complex *u, int ldu, complex *v, int
          ldv, complex *q, int ldq, int *ncycle, int *info);

void ctgsja_64(char jobu, char jobv, char jobq, long m, long
          p,  long  n, long k, long l, complex *a, long lda,
          complex *b, long  ldb,  float  tola,  float  tolb,
          float  *alpha,  float *beta, complex *u, long ldu,
          complex *v, long ldv, complex *q, long  ldq,  long
          *ncycle, long *info);
```

PURPOSE

ctgsja computes the generalized singular value decomposition (GSVD) of two complex upper triangular (or trapezoidal) matrices A and B.

On entry, it is assumed that matrices A and B have the following forms, which may be obtained by the preprocessing subroutine CGGSVP from a general M-by-N matrix A and P-by-N matrix B:

```
                 N-K-L  K    L
   A =    K ( 0    A12  A13 ) if M-K-L >= 0;
          L ( 0     0   A23 )
      M-K-L ( 0     0    0  )

               N-K-L  K    L
   A =  K ( 0    A12  A13 ) if M-K-L < 0;
      M-K ( 0     0   A23 )

               N-K-L  K    L
   B =  L ( 0     0   B13 )
      P-L ( 0     0    0  )

where the K-by-K matrix A12 and L-by-L matrix B13  are  non-
singular upper triangular; A23 is L-by-L upper triangular if
M-K-L >= 0, otherwise A23 is (M-K)-by-L upper trapezoidal.

On exit,
       U'*A*Q = D1*( 0 R ),    V'*B*Q = D2*( 0 R ),

where U, V and Q are unitary matrices, Z' denotes the conju-
gate  transpose  of  Z,  R is a nonsingular upper triangular
matrix, and D1 and D2 are ``diagonal'' matrices,  which  are
of the following structures:

If M-K-L >= 0,

                    K  L
       D1 =     K ( I  0 )
                L ( 0  C )
            M-K-L ( 0  0 )

                   K  L
       D2 = L   ( 0  S )
            P-L ( 0  0 )

                N-K-L  K    L
  ( 0 R ) = K (  0   R11  R12 ) K
            L (  0    0   R22 ) L

where

  C = diag( ALPHA(K+1), ... , ALPHA(K+L) ),
  S = diag( BETA(K+1),  ... , BETA(K+L) ),
  C**2 + S**2 = I.

  R is stored in A(1:K+L,N-K-L+1:N) on exit.

If M-K-L < 0,
```

```
               K M-K K+L-M
    D1 =   K ( I  0    0   )
         M-K ( 0  C    0   )

                 K M-K K+L-M
    D2 =   M-K ( 0  S    0   )
         K+L-M ( 0  0    I   )
           P-L ( 0  0    0   )

               N-K-L  K   M-K  K+L-M

          M-K ( 0     0   R22  R23  )
        K+L-M ( 0     0    0   R33  )
```

```
where
C = diag( ALPHA(K+1), ... , ALPHA(M) ),
S = diag( BETA(K+1),  ... , BETA(M) ),
C**2 + S**2 = I.

R = ( R11 R12 R13 ) is stored in A(1:M, N-K-L+1:N)  and  R33
is stored
    (  0  R22 R23 )
in B(M-K+1:L,N+M-K-L+1:N) on exit.

The computation of the unitary transformation matrices U,  V
or  Q  is  optional.   These  matrices  may either be formed
explicitly,  or  they  may  be  postmultiplied  into   input
matrices U1, V1, or Q1.

CTGSJA essentially uses a variant of Kogbetliantz  algorithm
to reduce min(L,M-K)-by-L triangular (or trapezoidal) matrix
A23 and L-by-L matrix B13 to the form:
   U1'*A13*Q1 = C1*R1; V1'*B13*Q1 = S1*R1,
where U1, V1 and Q1 are unitary matrix, and Z' is the conju-
gate transpose of Z.  C1 and S1 are diagonal matrices satis-
fying
   C1**2 + S1**2 = I,
and R1 is an L-by-L nonsingular upper triangular matrix.
```

ARGUMENTS

```
JOBU (input)
          = 'U':  U must contain  a  unitary  matrix  U1  on
          entry, and the product U1*U is returned; = 'I':  U
          is initialized to the unit matrix, and the unitary
          matrix U is returned; = 'N':  U is not computed.

JOBV (input)
          = 'V':  V must contain  a  unitary  matrix  V1  on
```

entry, and the product V1*V is returned; = 'I': V is initialized to the unit matrix, and the unitary matrix V is returned; = 'N': V is not computed.

JOBQ (input)
= 'Q': Q must contain a unitary matrix Q1 on entry, and the product Q1*Q is returned; = 'I': Q is initialized to the unit matrix, and the unitary matrix Q is returned; = 'N': Q is not computed.

M (input) The number of rows of the matrix A. M >= 0.

P (input) The number of rows of the matrix B. P >= 0.

N (input) The number of columns of the matrices A and B. N >= 0.

K (input) K and L specify the subblocks in the input matrices A and B:
A23 = A(K+1:MIN(K+L,M),N-L+1:N) and B13 = B(1:L,,N-L+1:N) of A and B, whose GSVD is going to be computed by CTGSJA. See the Further Details section below.

L (input) See the description of K.

A (input/output)
On entry, the M-by-N matrix A. On exit, A(N-K+1:N,1:MIN(K+L,M)) contains the triangular matrix R or part of R. See Purpose for details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the P-by-N matrix B. On exit, if necessary, B(M-K+1:L,N+M-K-L+1:N) contains a part of R. See Purpose for details.

LDB (input)
The leading dimension of the array B. LDB >= max(1,P).

TOLA (input)
TOLA and TOLB are the convergence criteria for the Jacobi- Kogbetliantz iteration procedure. Generally, they are the same as used in the preprocessing step, say TOLA = MAX(M,N)*norm(A)*MACHEPS, TOLB = MAX(P,N)*norm(B)*MACHEPS.

TOLB (input)
See the description of TOLA.

ALPHA (output)
On exit, ALPHA and BETA contain the generalized singular value pairs of A and B; ALPHA(1:K) = 1, BETA(1:K) = 0, and if M-K-L >= 0, ALPHA(K+1:K+L) = diag(C),
BETA(K+1:K+L) = diag(S), or if M-K-L < 0, ALPHA(K+1:M)= C, ALPHA(M+1:K+L)= 0
BETA(K+1:M) = S, BETA(M+1:K+L) = 1. Furthermore, if K+L < N, ALPHA(K+L+1:N) = 0
BETA(K+L+1:N) = 0.

BETA (output)
See the description of ALPHA.

U (input) On entry, if JOBU = 'U', U must contain a matrix U1 (usually the unitary matrix returned by CGGSVP). On exit, if JOBU = 'I', U contains the unitary matrix U; if JOBU = 'U', U contains the product U1*U. If JOBU = 'N', U is not referenced.

LDU (input)
The leading dimension of the array U. LDU >= max(1,M) if JOBU = 'U'; LDU >= 1 otherwise.

V (input) On entry, if JOBV = 'V', V must contain a matrix V1 (usually the unitary matrix returned by CGGSVP). On exit, if JOBV = 'I', V contains the unitary matrix V; if JOBV = 'V', V contains the product V1*V. If JOBV = 'N', V is not referenced.

LDV (input)
The leading dimension of the array V. LDV >= max(1,P) if JOBV = 'V'; LDV >= 1 otherwise.

Q (input) On entry, if JOBQ = 'Q', Q must contain a matrix Q1 (usually the unitary matrix returned by CGGSVP). On exit, if JOBQ = 'I', Q contains the unitary matrix Q; if JOBQ = 'Q', Q contains the product Q1*Q. If JOBQ = 'N', Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= max(1,N) if JOBQ = 'Q'; LDQ >= 1 otherwise.

WORK (workspace)
dimension(2*N)

NCYCLE (output)
The number of cycles required for convergence.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
= 1: the procedure does not converge after MAXIT cycles.

Contents

NAME

```
ctgsna - estimate reciprocal condition numbers for specified
eigenvalues and/or eigenvectors of a matrix pair (A, B)
```

SYNOPSIS

```
SUBROUTINE CTGSNA(JOB, HOWMNT, SELECT, N, A, LDA, B, LDB, VL, LDVL,
      VR, LDVR, S, DIF, MM, M, WORK, LWORK, IWORK, INFO)

CHARACTER * 1 JOB, HOWMNT
COMPLEX A(LDA,*), B(LDB,*), VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER N, LDA, LDB, LDVL, LDVR, MM, M, LWORK, INFO
INTEGER IWORK(*)
LOGICAL SELECT(*)
REAL S(*), DIF(*)

SUBROUTINE CTGSNA_64(JOB, HOWMNT, SELECT, N, A, LDA, B, LDB, VL,
      LDVL, VR, LDVR, S, DIF, MM, M, WORK, LWORK, IWORK, INFO)

CHARACTER * 1 JOB, HOWMNT
COMPLEX A(LDA,*), B(LDB,*), VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER*8 N, LDA, LDB, LDVL, LDVR, MM, M, LWORK, INFO
INTEGER*8 IWORK(*)
LOGICAL*8 SELECT(*)
REAL S(*), DIF(*)
```

F95 INTERFACE

```
SUBROUTINE TGSNA(JOB, HOWMNT, SELECT, [N], A, [LDA], B, [LDB], VL,
       [LDVL], VR, [LDVR], S, DIF, MM, M, [WORK], [LWORK], [IWORK],
       [INFO])

CHARACTER(LEN=1) :: JOB, HOWMNT
```

```
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B, VL, VR
INTEGER :: N, LDA, LDB, LDVL, LDVR, MM, M, LWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
LOGICAL, DIMENSION(:) :: SELECT
REAL, DIMENSION(:) :: S, DIF

SUBROUTINE TGSNA_64(JOB, HOWMNT, SELECT, [N], A, [LDA], B, [LDB], VL,
       [LDVL], VR, [LDVR], S, DIF, MM, M, [WORK], [LWORK], [IWORK],
       [INFO])

CHARACTER(LEN=1) :: JOB, HOWMNT
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B, VL, VR
INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, MM, M, LWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
LOGICAL(8), DIMENSION(:) :: SELECT
REAL, DIMENSION(:) :: S, DIF
```

C INTERFACE

```
#include <sunperf.h>

void ctgsna(char job, char howmnt, int *select, int n,  com-
          plex  *a,  int  lda,  complex *b, int ldb, complex
          *vl, int ldvl, complex *vr, int  ldvr,  float  *s,
          float *dif, int mm, int *m, int *info);

void ctgsna_64(char job, char howmnt, long *select, long  n,
          complex  *a,  long lda, complex *b, long ldb, com-
          plex *vl, long ldvl, complex *vr, long ldvr, float
          *s, float *dif, long mm, long *m, long *info);
```

PURPOSE

ctgsna estimates reciprocal condition numbers for specified eigenvalues and/or eigenvectors of a matrix pair (A, B).

(A, B) must be in generalized Schur canonical form, that is, A and B are both upper triangular.

ARGUMENTS

```
JOB (input)
          Specifies whether condition numbers  are  required
          for eigenvalues (S) or eigenvectors (DIF):
          = 'E': for eigenvalues only (S);
          = 'V': for eigenvectors only (DIF);
          = 'B': for both eigenvalues  and  eigenvectors  (S
```

and DIF).

HOWMNT (input)
= 'A': compute condition numbers for all eigen-pairs;
= 'S': compute condition numbers for selected eigenpairs specified by the array SELECT.

SELECT (input)
If HOWMNT = 'S', SELECT specifies the eigenpairs for which condition numbers are required. To select condition numbers for the corresponding j-th eigenvalue and/or eigenvector, SELECT(j) must be set to .TRUE.. If HOWMNT = 'A', SELECT is not referenced.

N (input) The order of the square matrix pair (A, B). N >= 0.

A (input) The upper triangular matrix A in the pair (A,B).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input) The upper triangular matrix B in the pair (A, B).

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

VL (input)
If JOB = 'E' or 'B', VL must contain left eigen-vectors of (A, B), corresponding to the eigenpairs specified by HOWMNT and SELECT. The eigenvectors must be stored in consecutive columns of VL, as returned by CTGEVC. If JOB = 'V', VL is not referenced.

LDVL (input)
The leading dimension of the array VL. LDVL >= 1; and If JOB = 'E' or 'B', LDVL >= N.

VR (input)
If JOB = 'E' or 'B', VR must contain right eigen-vectors of (A, B), corresponding to the eigenpairs specified by HOWMNT and SELECT. The eigenvectors must be stored in consecutive columns of VR, as returned by CTGEVC. If JOB = 'V', VR is not referenced.

LDVR (input)
The leading dimension of the array VR. LDVR >= 1; If JOB = 'E' or 'B', LDVR >= N.

S (output)
If JOB = 'E' or 'B', the reciprocal condition numbers of the selected eigenvalues, stored in consecutive elements of the array. If JOB = 'V', S is not referenced.

DIF (output)
If JOB = 'V' or 'B', the estimated reciprocal condition numbers of the selected eigenvectors, stored in consecutive elements of the array. If the eigenvalues cannot be reordered to compute DIF(j), DIF(j) is set to 0; this can only occur when the true value would be very small anyway. For each eigenvalue/vector specified by SELECT, DIF stores a Frobenius norm-based estimate of Difl. If JOB = 'E', DIF is not referenced.

MM (input)
The number of elements in the arrays S and DIF. MM >= M.

M (output)
The number of elements of the arrays S and DIF used to store the specified condition numbers; for each selected eigenvalue one element is used. If HOWMNT = 'A', M is set to N.

WORK (workspace)
If JOB = 'E', WORK is not referenced. Otherwise, on exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 1. If JOB = 'V' or 'B', LWORK >= 2*N*N.

IWORK (workspace)
dimension(N+2) If JOB = 'E', IWORK is not referenced.

INFO (output)
= 0: Successful exit
< 0: If INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The reciprocal of the condition number of the i-th generalized eigenvalue w = (a, b) is defined as

```
        S(I)   =   (|v'Au|**2    +    |v'Bu|**2)**(1/2)    /
(norm(u)*norm(v))
```

where u and v are the right and left eigenvectors of (A, B) corresponding to w; |z| denotes the absolute value of the complex number, and norm(u) denotes the 2-norm of the vector u. The pair (a, b) corresponds to an eigenvalue w = a/b (= v'Au/v'Bu) of the matrix pair (A, B). If both a and b equal zero, then (A,B) is singular and S(I) = -1 is returned.

An approximate error bound on the chordal distance between the i-th computed generalized eigenvalue w and the corresponding exact eigenvalue lambda is

```
        chord(w, lambda) <=   EPS * norm(A, B) / S(I),
```

where EPS is the machine precision.

The reciprocal of the condition number of the right eigenvector u and left eigenvector v corresponding to the generalized eigenvalue w is defined as follows. Suppose

```
                 (A, B) = ( a   *  ) ( b  *  )  1
                          ( 0  A22 ),( 0 B22 )  n-1
                            1  n-1     1 n-1
```

Then the reciprocal condition number DIF(I) is

```
        Difl[(a, b), (A22, B22)]  = sigma-min( Zl )
```

where sigma-min(Zl) denotes the smallest singular value of

```
       Zl = [ kron(a, In-1) -kron(1, A22) ]
            [ kron(b, In-1) -kron(1, B22) ].
```

Here In-1 is the identity matrix of size n-1 and X' is the conjugate transpose of X. kron(X, Y) is the Kronecker product between the matrices X and Y.

We approximate the smallest singular value of Zl with an upper bound. This is done by CLATDF.

An approximate error bound for a computed eigenvector VL(i) or VR(i) is given by

```
                    EPS * norm(A, B) / DIF(i).
```

See ref. [2-3] for more details and further references.

Based on contributions by
Bo Kagstrom and Peter Poromaa, Department of Computing Science,
Umea University, S-901 87 Umea, Sweden.

References
==========

[1] B. Kagstrom; A Direct Method for Reordering Eigenvalues in the Generalized Real Schur Form of a Regular Matrix Pair (A, B), in M.S. Moonen et al (eds), Linear Algebra for Large Scale and Real-Time Applications, Kluwer Academic Publ. 1993, pp 195-218.

[2] B. Kagstrom and P. Poromaa; Computing Eigenspaces with Specified Eigenvalues of a Regular Matrix Pair (A, B) and Condition Estimation: Theory, Algorithms and Software, Report UMINF - 94.04, Department of Computing Science, Umea University, S-901 87 Umea, Sweden, 1994. Also as LAPACK Working Note 87. To appear in Numerical Algorithms, 1996.

[3] B. Kagstrom and P. Poromaa, LAPACK-Style Algorithms and Software for Solving the Generalized Sylvester Equation and Estimating the Separation between Regular Matrix Pairs, Report UMINF - 93.23, Department of Computing Science, Umea University, S-901 87 Umea, Sweden, December 1993, Revised April 1994, Also as LAPACK Working Note 75. To appear in ACM Trans. on Math. Software, Vol 22, No 1, 1996.

Contents

NAME

```
ctgsyl - solve the generalized Sylvester equation
```

SYNOPSIS

```
SUBROUTINE CTGSYL(TRANS, IJOB, M, N, A, LDA, B, LDB, C, LDC, D, LDD,
      E, LDE, F, LDF, SCALE, DIF, WORK, LWORK, IWORK, INFO)

CHARACTER * 1 TRANS
COMPLEX A(LDA,*), B(LDB,*),  C(LDC,*),  D(LDD,*),  E(LDE,*),
F(LDF,*), WORK(*)
INTEGER IJOB, M, N, LDA, LDB, LDC,  LDD,  LDE,  LDF,  LWORK,
INFO
INTEGER IWORK(*)
REAL SCALE, DIF

SUBROUTINE CTGSYL_64(TRANS, IJOB, M, N, A, LDA, B, LDB, C, LDC, D,
      LDD, E, LDE, F, LDF, SCALE, DIF, WORK, LWORK, IWORK, INFO)

CHARACTER * 1 TRANS
COMPLEX A(LDA,*), B(LDB,*),  C(LDC,*),  D(LDD,*),  E(LDE,*),
F(LDF,*), WORK(*)
INTEGER*8 IJOB, M, N, LDA, LDB, LDC, LDD, LDE,  LDF,  LWORK,
INFO
INTEGER*8 IWORK(*)
REAL SCALE, DIF
```

F95 INTERFACE

```
SUBROUTINE TGSYL(TRANS, IJOB, [M], [N], A, [LDA], B, [LDB], C, [LDC],
       D, [LDD], E, [LDE], F, [LDF], SCALE, DIF, [WORK], [LWORK], [IWORK],
       [INFO])
```

```
CHARACTER(LEN=1) :: TRANS
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B, C, D, E, F
INTEGER :: IJOB, M, N, LDA, LDB, LDC, LDD, LDE, LDF,  LWORK,
INFO
INTEGER, DIMENSION(:) :: IWORK
REAL :: SCALE, DIF

SUBROUTINE TGSYL_64(TRANS, IJOB, [M], [N], A, [LDA], B, [LDB], C,
       [LDC], D, [LDD], E, [LDE], F, [LDF], SCALE, DIF, [WORK], [LWORK],
       [IWORK], [INFO])

CHARACTER(LEN=1) :: TRANS
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B, C, D, E, F
INTEGER(8) :: IJOB, M, N, LDA,  LDB,  LDC,  LDD,  LDE,  LDF,
LWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL :: SCALE, DIF
```

C INTERFACE

```
#include <sunperf.h>

void ctgsyl(char trans, int ijob, int m, int n, complex  *a,
          int lda, complex *b, int ldb, complex *c, int ldc,
          complex *d, int ldd, complex *e, int lde,  complex
          *f, int ldf, float *scale, float *dif, int *info);

void ctgsyl_64(char trans, long ijob, long m, long  n,  com-
          plex  *a,  long lda, complex *b, long ldb, complex
          *c, long ldc, complex *d, long  ldd,  complex  *e,
          long  lde,  complex  *f,  long  ldf, float *scale,
          float *dif, long *info);
```

PURPOSE

ctgsyl solves the generalized Sylvester equation:

```
            A * R - L * B = scale * C            (1)
            D * R - L * E = scale * F
```

where R and L are unknown m-by-n matrices, (A, D), (B, E) and (C, F) are given matrix pairs of size m-by-m, n-by-n and m-by-n, respectively, with complex entries. A, B, D and E are upper triangular (i.e., (A,D) and (B,E) in generalized Schur form).

The solution (R, L) overwrites (C, F). 0 <= SCALE <= 1 is an output scaling factor chosen to avoid overflow.

In matrix notation (1) is equivalent to solve Zx = scale*b, where Z is defined as

```
       Z = [ kron(In, A)  -kron(B', Im) ]        (2)
           [ kron(In, D)  -kron(E', Im) ],
```

Here Ix is the identity matrix of size x and X' is the conjugate transpose of X. Kron(X, Y) is the Kronecker product between the matrices X and Y.

If TRANS = 'C', y in the conjugate transposed system Z'*y = scale*b is solved for, which is equivalent to solve for R and L in

```
            A' * R + D' * L = scale * C           (3)
            R * B' + L * E' = scale * -F
```

This case (TRANS = 'C') is used to compute an one-norm-based estimate of Dif[(A,D), (B,E)], the separation between the matrix pairs (A,D) and (B,E), using CLACON.

If IJOB >= 1, CTGSYL computes a Frobenius norm-based estimate of Dif[(A,D),(B,E)]. That is, the reciprocal of a lower bound on the reciprocal of the smallest singular value of Z.

This is a level-3 BLAS algorithm.

ARGUMENTS

TRANS (input)
= 'N': solve the generalized sylvester equation (1).
= 'C': solve the "conjugate transposed" system (3).

IJOB (input)
Specifies what kind of functionality to be performed. =0: solve (1) only.
=1: The functionality of 0 and 3.
=2: The functionality of 0 and 4.
=3: Only an estimate of Dif[(A,D), (B,E)] is computed. (look ahead strategy is used). =4: Only an estimate of Dif[(A,D), (B,E)] is computed. (CGECON on sub-systems is used). Not referenced if TRANS = 'C'.

M (input) The order of the matrices A and D, and the row

```
          dimension of the matrices C, F, R and L.

N (input) The order of the matrices B and E, and the  column
          dimension of the matrices C, F, R and L.

A (input) The upper triangular matrix A.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1, M).

B (input) The upper triangular matrix B.

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          max(1, N).

C (input/output)
          On entry, C contains the  right-hand-side  of  the
          first  matrix equation in (1) or (3).  On exit, if
          IJOB = 0, 1 or 2, C has been  overwritten  by  the
          solution  R.  If  IJOB = 3 or 4 and TRANS = 'N', C
          holds R, the solution achieved during the computa-
          tion of the Dif-estimate.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1, M).

D (input) The upper triangular matrix D.

LDD (input)
          The leading dimension  of  the  array  D.  LDD  >=
          max(1, M).

E (input) The upper triangular matrix E.

LDE (input)
          The leading dimension  of  the  array  E.  LDE  >=
          max(1, N).

F (input/output)
          On entry, F contains the  right-hand-side  of  the
          second matrix equation in (1) or (3).  On exit, if
          IJOB = 0, 1 or 2, F has been  overwritten  by  the
          solution  L.  If  IJOB = 3 or 4 and TRANS = 'N', F
          holds L, the solution achieved during the computa-
          tion of the Dif-estimate.

LDF (input)
```

```
            The leading dimension  of  the  array  F.  LDF  >=
            max(1, M).

DIF (output)
            On exit SCALE is the reciprocal of a  lower  bound
            of  the reciprocal of the Dif-function, i.e. SCALE
            is an upper bound of Dif[(A,D),  (B,E)]  =  sigma-
            min(Z), where Z as in (2).  If IJOB = 0 or TRANS =
            'C', SCALE is not referenced.

SCALE (output)
            On exit SCALE is the reciprocal of a  lower  bound
            of  the reciprocal of the Dif-function, i.e. SCALE
            is an upper bound of Dif[(A,D),  (B,E)]  =  sigma-
            min(Z), where Z as in (2).  If IJOB = 0 or TRANS =
            'C', SCALE is not referenced.

WORK (workspace)
            If IJOB = 0, WORK is not  referenced.   Otherwise,
            on  exit,  if  INFO=0  then  WORK(1)  returns  the
            optimal LWORK.

LWORK (input)
            The dimension of the array WORK. LWORK > = 1.   If
            IJOB = 1 or 2 and TRANS = 'N', LWORK >= 2*M*N.

            If LWORK = -1, then a workspace query is  assumed;
            the  routine  only  calculates the optimal size of
            the WORK array, returns this value  as  the  first
            entry  of  the  WORK  array,  and no error message
            related to LWORK is issued by XERBLA.

IWORK (workspace)
            If IJOB = 0, IWORK is not referenced.

INFO (output)
            =0: successful exit
            <0: If INFO = -i, the i-th argument had an illegal
            value.
            >0: (A, D) and (B, E) have common  or  very  close
            eigenvalues.
```

FURTHER DETAILS

```
Based on contributions by
   Bo Kagstrom and Peter Poromaa,  Department  of  Computing
Science,
   Umea University, S-901 87 Umea, Sweden.

[1] B. Kagstrom and P. Poromaa, LAPACK-Style Algorithms  and
```

Software for Solving the Generalized Sylvester Equation and Estimating the Separation between Regular Matrix Pairs, Report UMINF - 93.23, Department of Computing Science, Umea University, S-901 87 Umea, Sweden, December 1993, Revised April 1994, Also as LAPACK Working Note 75. To appear in ACM Trans. on Math. Software, Vol 22, No 1, 1996.

[2] B. Kagstrom, A Perturbation Analysis of the Generalized Sylvester Equation (AR - LB, DR - LE) = (C, F), SIAM J. Matrix Anal. Appl., 15(4):1045-1060, 1994.

[3] B. Kagstrom and L. Westin, Generalized Schur Methods with Condition Estimators for Solving the Generalized Sylvester Equation, IEEE Transactions on Automatic Control, Vol. 34, No. 7, July 1989, pp 745-751.

Contents

NAME

ctpcon - estimate the reciprocal of the condition number of a packed triangular matrix A, in either the 1-norm or the infinity-norm

SYNOPSIS

```
SUBROUTINE CTPCON(NORM, UPLO, DIAG, N, A, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 NORM, UPLO, DIAG
COMPLEX A(*), WORK(*)
INTEGER N, INFO
REAL RCOND
REAL WORK2(*)

SUBROUTINE CTPCON_64(NORM, UPLO, DIAG, N, A, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 NORM, UPLO, DIAG
COMPLEX A(*), WORK(*)
INTEGER*8 N, INFO
REAL RCOND
REAL WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE TPCON(NORM, UPLO, DIAG, N, A, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
COMPLEX, DIMENSION(:) :: A, WORK
INTEGER :: N, INFO
REAL :: RCOND
REAL, DIMENSION(:) :: WORK2
```

```
SUBROUTINE TPCON_64(NORM, UPLO, DIAG, N, A, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
COMPLEX, DIMENSION(:) :: A, WORK
INTEGER(8) :: N, INFO
REAL :: RCOND
REAL, DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>

void ctpcon(char norm, char uplo, char diag, int n,  complex
          *a, float *rcond, int *info);
void ctpcon_64(char norm, char uplo, char diag, long n, com-
          plex *a, float *rcond, long *info);
```

PURPOSE

ctpcon estimates the reciprocal of the condition number of a packed triangular matrix A, in either the 1-norm or the infinity-norm.

The norm of A is computed and an estimate is obtained for norm(inv(A)), then the reciprocal of the condition number is computed as

RCOND = 1 / (norm(A) * norm(inv(A))).

ARGUMENTS

NORM (input)
Specifies whether the 1-norm condition number or the infinity-norm condition number is required:
= '1' or 'O': 1-norm;
= 'I': Infinity-norm.

UPLO (input)
= 'U': A is upper triangular;
= 'L': A is lower triangular.

DIAG (input)
= 'N': A is non-unit triangular;
= 'U': A is unit triangular.

N (input) The order of the matrix A. N >= 0.

A (input) The upper or lower triangular matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n. If DIAG = 'U', the diagonal elements of A are not referenced and are assumed to be 1.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(norm(A) * norm(inv(A))).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

ctpmv - perform one of the matrix-vector operations x := A*x, or x := A'*x, or x := conjg(A')*x

SYNOPSIS

```
SUBROUTINE CTPMV(UPLO, TRANSA, DIAG, N, A, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(*), Y(*)
INTEGER N, INCY

SUBROUTINE CTPMV_64(UPLO, TRANSA, DIAG, N, A, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(*), Y(*)
INTEGER*8 N, INCY
```

F95 INTERFACE

```
SUBROUTINE TPMV(UPLO, [TRANSA], DIAG, [N], A, Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: A, Y
INTEGER :: N, INCY

SUBROUTINE TPMV_64(UPLO, [TRANSA], DIAG, [N], A, Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: A, Y
INTEGER(8) :: N, INCY
```

C INTERFACE

```
#include <sunperf.h>
```

```
void ctpmv(char uplo, char transa, char diag, int n, complex
          *a, complex *y, int incy);

void ctpmv_64(char uplo, char transa,  char  diag,  long  n,
          complex *a, complex *y, long incy);
```

PURPOSE

ctpmv performs one of the matrix-vector operations x := A*x, or x := A'*x, or x := conjg(A')*x where x is an n element vector and A is an n by n unit, or non-unit, upper or lower triangular matrix, supplied in packed form.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   x := A*x.

TRANSA = 'T' or 't'   x := A'*x.

TRANSA = 'C' or 'c'   x := conjg(A')*x.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit tri-

angular.

DIAG = 'N' or 'n'   A is not assumed to be unit
triangular.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A.
N >= 0. Unchanged on exit.

A (input)
((n*(n + 1))/2). Before entry with UPLO =
'U' or 'u', the array A must contain the upper
triangular matrix packed sequentially, column by
column, so that A(1) contains a(1, 1), A(2)
and A(3) contain a(1, 2) and a(2, 2) respec-
tively, and so on. Before entry with UPLO = 'L'
or 'l', the array A must contain the lower tri-
angular matrix packed sequentially, column by
column, so that A(1) contains a(1, 1), A(2)
and A(3) contain a(2, 1) and a(3, 1) respec-
tively, and so on. Note that when DIAG = 'U' or
'u', the diagonal elements of A are not refer-
enced, but are assumed to be unity. Unchanged on
exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). Before entry, the
incremented array Y must contain the n element
vector x. On exit, Y is overwritten with the tran-
formed vector x.

INCY (input)
On entry, INCY specifies the increment for the
elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
ctprfs - provide error bounds and backward  error  estimates
for the solution to a system of linear equations with a tri-
angular packed coefficient matrix
```

SYNOPSIS

```
SUBROUTINE CTPRFS(UPLO, TRANSA, DIAG, N, NRHS, A, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
REAL FERR(*), BERR(*), WORK2(*)

SUBROUTINE CTPRFS_64(UPLO, TRANSA, DIAG, N, NRHS, A, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
REAL FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE TPRFS(UPLO, [TRANSA], DIAG, N, [NRHS], A, B, [LDB], X,
       [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: A, WORK
COMPLEX, DIMENSION(:,:) :: B, X
INTEGER :: N, NRHS, LDB, LDX, INFO
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

```
SUBROUTINE TPRFS_64(UPLO, [TRANSA], DIAG, N, [NRHS], A, B, [LDB], X,
       [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: A, WORK
COMPLEX, DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void ctprfs(char uplo, char transa, char diag,  int  n,  int
          nrhs, complex *a, complex *b, int ldb, complex *x,
          int ldx, float *ferr, float *berr, int *info);
void ctprfs_64(char uplo, char transa, char  diag,  long  n,
          long  nrhs, complex *a, complex *b, long ldb, com-
          plex *x, long ldx, float *ferr, float *berr,  long
          *info);
```

PURPOSE

ctprfs provides error bounds and backward error estimates for the solution to a system of linear equations with a triangular packed coefficient matrix.

The solution matrix X must be computed by CTPTRS or some other means before entering this routine. CTPRFS does not do iterative refinement because doing so cannot improve the backward error.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
          = 'N':  A is non-unit triangular;
```

```
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input) The upper or lower  triangular  matrix  A,  packed
          columnwise  in a linear array.  The j-th column of
          A is stored in the array A as follows:  if UPLO  =
          'U',  A(i  +  (j-1)*j/2)  = A(i,j) for 1<=i<=j; if
          UPLO = 'L', A(i +  (j-1)*(2n-j)/2)  =  A(i,j)  for
          j<=i<=n.   If DIAG = 'U', the diagonal elements of
          A are not referenced and are assumed to be 1.

B (input) The right hand side matrix B.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

X (input) The solution matrix X.

LDX (input)
          The leading dimension of  the  array  X.   LDX  >=
          max(1,N).

FERR (output)
          The estimated forward error bound for  each  solu-
          tion  vector X(j) (the j-th column of the solution
          matrix  X).   If  XTRUE  is  the   true   solution
          corresponding  to  X(j),  FERR(j)  is an estimated
          upper bound for the magnitude of the largest  ele-
          ment in (X(j) - XTRUE) divided by the magnitude of
          the largest element in X(j).  The estimate  is  as
          reliable  as the estimate for RCOND, and is almost
          always a slight overestimate of the true error.

BERR (output)
          The componentwise relative backward error of  each
          solution  vector X(j) (i.e., the smallest relative
          change in any element of A or B that makes X(j) an
          exact solution).

WORK (workspace)
          dimension(2*N)

WORK2 (workspace)
          dimension(N)
```

```
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
ctpsv - solve one of the systems of equations   A*x = b,  or
A'*x = b, or conjg( A' )*x = b
```

SYNOPSIS

```
SUBROUTINE CTPSV(UPLO, TRANSA, DIAG, N, A, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(*), Y(*)
INTEGER N, INCY

SUBROUTINE CTPSV_64(UPLO, TRANSA, DIAG, N, A, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(*), Y(*)
INTEGER*8 N, INCY
```

F95 INTERFACE

```
SUBROUTINE TPSV(UPLO, [TRANSA], DIAG, [N], A, Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: A, Y
INTEGER :: N, INCY

SUBROUTINE TPSV_64(UPLO, [TRANSA], DIAG, [N], A, Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: A, Y
INTEGER(8) :: N, INCY
```

C INTERFACE

```
#include <sunperf.h>
```

```
void ctpsv(char uplo, char transa, char diag, int n, complex
          *a, complex *y, int incy);

void ctpsv_64(char uplo, char transa,  char  diag,  long  n,
          complex *a, complex *y, long incy);
```

PURPOSE

ctpsv solves one of the systems of equations A*x = b, or A'*x = b, or conjg(A')*x = b where b and x are n element vectors and A is an n by n unit, or non-unit, upper or lower triangular matrix, supplied in packed form.

No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the equations to be solved as follows:

TRANSA = 'N' or 'n'   A*x = b.

TRANSA = 'T' or 't'   A'*x = b.

TRANSA = 'C' or 'c'   conjg(A')*x = b.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)

On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u' A is assumed to be unit triangular.

DIAG = 'N' or 'n' A is not assumed to be unit triangular.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

A (input)
((n*(n + 1))/2). Before entry with UPLO = 'U' or 'u', the array A must contain the upper triangular matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(1, 2) and a(2, 2) respectively, and so on. Before entry with UPLO = 'L' or 'l', the array A must contain the lower triangular matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(2, 1) and a(3, 1) respectively, and so on. Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced, but are assumed to be unity. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element right-hand side vector b. On exit, Y is overwritten with the solution vector x.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

ctptri - compute the inverse of a complex upper or lower triangular matrix A stored in packed format

SYNOPSIS

```
SUBROUTINE CTPTRI(UPLO, DIAG, N, A, INFO)

CHARACTER * 1 UPLO, DIAG
COMPLEX A(*)
INTEGER N, INFO

SUBROUTINE CTPTRI_64(UPLO, DIAG, N, A, INFO)

CHARACTER * 1 UPLO, DIAG
COMPLEX A(*)
INTEGER*8 N, INFO
```

F95 INTERFACE

```
SUBROUTINE TPTRI(UPLO, DIAG, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
COMPLEX, DIMENSION(:) :: A
INTEGER :: N, INFO

SUBROUTINE TPTRI_64(UPLO, DIAG, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
COMPLEX, DIMENSION(:) :: A
INTEGER(8) :: N, INFO
```

C INTERFACE

```
#include <sunperf.h>

void ctptri(char uplo, char diag, int  n,  complex  *a,  int
          *info);

void ctptri_64(char uplo, char diag,  long  n,  complex  *a,
          long *info);
```

PURPOSE

ctptri computes the inverse of a complex upper or lower triangular matrix A stored in packed format.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the upper or lower triangular matrix  A,
          stored  columnwise  in  a  linear array.  The j-th
          column of A is stored in the array A  as  follows:
          if  UPLO  =  'U',  A(i  +  (j-1)*j/2) = A(i,j) for
          1<=i<=j; if UPLO = 'L', A(i + (j-1)*((2*n-j)/2)  =
          A(i,j)   for   j<=i<=n.   See  below  for  further
          details.  On exit, the (triangular) inverse of the
          original  matrix,  in the same packed storage for-
          mat.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, A(i,i) is  exactly  zero.   The
          triangular  matrix is singular and its inverse can
          not be computed.
```

FURTHER DETAILS

A triangular matrix A can be transferred to packed storage using one of the following program segments:

UPLO = 'U':

```
      JC = 1
      DO 2 J = 1, N
         DO 1 I = 1, J
            A(JC+I-1) = A(I,J)
    1    CONTINUE
         JC = JC + J
    2 CONTINUE
```

UPLO = 'L':

```
      JC = 1
      DO 2 J = 1, N
         DO 1 I = J, N
            A(JC+I-J) = A(I,J)
    1    CONTINUE
         JC = JC + N - J + 1
    2 CONTINUE
```

Contents

NAME

```
ctptrs - solve a triangular system of the form   A * X =  B,
A**T * X = B, or A**H * X = B,
```

SYNOPSIS

```
SUBROUTINE CTPTRS(UPLO, TRANSA, DIAG, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO

SUBROUTINE CTPTRS_64(UPLO, TRANSA, DIAG, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE TPTRS(UPLO, TRANSA, DIAG, N, [NRHS], A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: A
COMPLEX, DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO

SUBROUTINE TPTRS_64(UPLO, TRANSA, DIAG, N, [NRHS], A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: A
COMPLEX, DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void ctptrs(char uplo, char transa, char diag,  int  n,  int
          nrhs, complex *a, complex *b, int ldb, int *info);

void ctptrs_64(char uplo, char transa, char  diag,  long  n,
          long  nrhs, complex *a, complex *b, long ldb, long
          *info);
```

PURPOSE

ctptrs solves a triangular system of the form

where A is a triangular matrix of order N stored in packed format, and B is an N-by-NRHS matrix. A check is made to verify that A is nonsingular.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

A (input) The upper or lower  triangular  matrix  A,  packed
          columnwise  in a linear array.  The j-th column of
          A is stored in the array A as follows:  if UPLO  =
          'U',  A(i  +  (j-1)*j/2)  = A(i,j) for 1<=i<=j; if
          UPLO = 'L', A(i + (j-1)*(2*n-j)/2)  =  A(i,j)  for
          j<=i<=n.
```

B (input/output)
On entry, the right hand side matrix B. On exit, if INFO = 0, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the i-th diagonal element of A is zero, indicating that the matrix is singular and the solutions X have not been computed.

Contents

NAME

ctrans - transpose and scale source matrix

SYNOPSIS

```
SUBROUTINE CTRANS(PLACE, SCALE, SOURCE, M, N, DEST)

CHARACTER * 1 PLACE
COMPLEX SCALE
COMPLEX SOURCE(*), DEST(*)
INTEGER M, N

SUBROUTINE CTRANS_64(PLACE, SCALE, SOURCE, M, N, DEST)

CHARACTER * 1 PLACE
COMPLEX SCALE
COMPLEX SOURCE(*), DEST(*)
INTEGER*8 M, N
```

F95 INTERFACE

```
SUBROUTINE TRANS([PLACE], SCALE, SOURCE, M, N, [DEST])

CHARACTER(LEN=1) :: PLACE
COMPLEX :: SCALE
COMPLEX, DIMENSION(:) :: SOURCE, DEST
INTEGER :: M, N

SUBROUTINE TRANS_64([PLACE], SCALE, SOURCE, M, N, [DEST])

CHARACTER(LEN=1) :: PLACE
COMPLEX :: SCALE
COMPLEX, DIMENSION(:) :: SOURCE, DEST
INTEGER(8) :: M, N
```

C INTERFACE

```
#include <sunperf.h>

void ctrans(char place, complex *scale, complex *source, int
          m, int n, complex *dest);

void ctrans_64(char place, complex *scale, complex  *source,
          long m, long n, complex *dest);
```

PURPOSE

ctrans scales and transposes the source matrix. The N2 x N1 result is written into SOURCE when PLACE = 'I' or 'i', and DEST when PLACE = 'O' or 'o'.

PLACE = 'I' or 'i': SOURCE = SCALE * SOURCE'

PLACE = 'O' or 'o': DEST = SCALE * SOURCE'

ARGUMENTS

PLACE (input)
Type of transpose. 'I' or 'i' for in-place, 'O' or 'o' for out-of-place. 'I' is default.

SCALE (input)
Scale factor on the SOURCE matrix.

SOURCE (input/output)
on input. Array of (N, M) on output if in-place transpose.

M (input)
Number of rows in the SOURCE matrix on input.

N (input)
Number of columns in the SOURCE matrix on input.

DEST (output)
Scaled and transposed SOURCE matrix if out-of-place transpose. Not referenced if in-place transpose.

Contents

NAME

ctrcon - estimate the reciprocal of the condition number of a triangular matrix A, in either the 1-norm or the infinity-norm

SYNOPSIS

```
SUBROUTINE CTRCON(NORM, UPLO, DIAG, N, A, LDA, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 NORM, UPLO, DIAG
COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, INFO
REAL RCOND
REAL WORK2(*)

SUBROUTINE CTRCON_64(NORM, UPLO, DIAG, N, A, LDA, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 NORM, UPLO, DIAG
COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, INFO
REAL RCOND
REAL WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE TRCON(NORM, UPLO, DIAG, [N], A, [LDA], RCOND, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
```

```
REAL :: RCOND
REAL, DIMENSION(:) :: WORK2

SUBROUTINE TRCON_64(NORM, UPLO, DIAG, [N], A, [LDA], RCOND, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
REAL :: RCOND
REAL, DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>
void ctrcon(char norm, char uplo, char diag, int n,  complex
          *a, int lda, float *rcond, int *info);

void ctrcon_64(char norm, char uplo, char diag, long n, com-
          plex *a, long lda, float *rcond, long *info);
```

PURPOSE

ctrcon estimates the reciprocal of the condition number of a triangular matrix A, in either the 1-norm or the infinity-norm.

The norm of A is computed and an estimate is obtained for norm(inv(A)), then the reciprocal of the condition number is computed as

```
   RCOND = 1 / ( norm(A) * norm(inv(A)) ).
```

ARGUMENTS

```
NORM (input)
          Specifies whether the 1-norm condition  number  or
          the infinity-norm condition number is required:
          = '1' or 'O':  1-norm;
          = 'I':         Infinity-norm.

UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.
```

N (input) The order of the matrix A. N >= 0.

A (input) The triangular matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of the array A contains the upper triangular matrix, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of the array A contains the lower triangular matrix, and the strictly upper triangular part of A is not referenced. If DIAG = 'U', the diagonal elements of A are also not referenced and are assumed to be 1.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(norm(A) * norm(inv(A))).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
ctrevc - compute some or all of the right and/or left eigen-
vectors of a complex upper triangular matrix T
```

SYNOPSIS

```
SUBROUTINE CTREVC(SIDE, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR,
      LDVR, MM, M, WORK, RWORK, INFO)

CHARACTER * 1 SIDE, HOWMNY
COMPLEX T(LDT,*), VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER N, LDT, LDVL, LDVR, MM, M, INFO
LOGICAL SELECT(*)
REAL RWORK(*)

SUBROUTINE CTREVC_64(SIDE, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR,
      LDVR, MM, M, WORK, RWORK, INFO)

CHARACTER * 1 SIDE, HOWMNY
COMPLEX T(LDT,*), VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER*8 N, LDT, LDVL, LDVR, MM, M, INFO
LOGICAL*8 SELECT(*)
REAL RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE TREVC(SIDE, HOWMNY, SELECT, [N], T, [LDT], VL, [LDVL], VR,
       [LDVR], MM, M, [WORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, HOWMNY
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: T, VL, VR
INTEGER :: N, LDT, LDVL, LDVR, MM, M, INFO
```

```
LOGICAL, DIMENSION(:) :: SELECT
REAL, DIMENSION(:) :: RWORK

SUBROUTINE TREVC_64(SIDE, HOWMNY, SELECT, [N], T, [LDT], VL, [LDVL],
       VR, [LDVR], MM, M, [WORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, HOWMNY
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: T, VL, VR
INTEGER(8) :: N, LDT, LDVL, LDVR, MM, M, INFO
LOGICAL(8), DIMENSION(:) :: SELECT
REAL, DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>
void ctrevc(char side, char howmny, int *select, int n, com-
          plex  *t,  int ldt, complex *vl, int ldvl, complex
          *vr, int ldvr, int mm, int *m, int *info);

void ctrevc_64(char side, char howmny, long *select, long n,
          complex *t, long ldt, complex *vl, long ldvl, com-
          plex *vr,  long  ldvr,  long  mm,  long  *m,  long
          *info);
```

PURPOSE

ctrevc computes some or all of the right and/or left eigen-vectors of a complex upper triangular matrix T.

The right eigenvector x and the left eigenvector y of T corresponding to an eigenvalue w are defined by:

```
             T*x = w*x,     y'*T = w*y'
```

where y' denotes the conjugate transpose of the vector y.

If all eigenvectors are requested, the routine may either return the matrices X and/or Y of right or left eigenvectors of T, or the products Q*X and/or Q*Y, where Q is an input unitary
matrix. If T was obtained from the Schur factorization of an original matrix A = Q*T*Q', then Q*X and Q*Y are the matrices of right or left eigenvectors of A.

ARGUMENTS

SIDE (input)

= 'R': compute right eigenvectors only;
= 'L': compute left eigenvectors only;
= 'B': compute both right and left eigenvectors.

HOWMNY (input)
= 'A': compute all right and/or left eigenvectors;
= 'B': compute all right and/or left eigenvectors, and backtransform them using the input matrices supplied in VR and/or VL; = 'S': compute selected right and/or left eigenvectors, specified by the logical array SELECT.

SELECT (input/output)
If HOWMNY = 'S', SELECT specifies the eigenvectors to be computed. If HOWMNY = 'A' or 'B', SELECT is not referenced. To select the eigenvector corresponding to the j-th eigenvalue, SELECT(j) must be set to .TRUE..

N (input) The order of the matrix T. N >= 0.

T (input/output)
The upper triangular matrix T. T is modified, but restored on exit.

LDT (input)
The leading dimension of the array T. LDT >= max(1,N).

VL (input/output)
On entry, if SIDE = 'L' or 'B' and HOWMNY = 'B', VL must contain an N-by-N matrix Q (usually the unitary matrix Q of Schur vectors returned by CHSEQR). On exit, if SIDE = 'L' or 'B', VL contains: if HOWMNY = 'A', the matrix Y of left eigenvectors of T; VL is lower triangular. The i-th column VL(i) of VL is the eigenvector corresponding to T(i,i). if HOWMNY = 'B', the matrix Q*Y; if HOWMNY = 'S', the left eigenvectors of T specified by SELECT, stored consecutively in the columns of VL, in the same order as their eigenvalues. If SIDE = 'R', VL is not referenced.

LDVL (input)
The leading dimension of the array VL. LDVL >= max(1,N) if SIDE = 'L' or 'B'; LDVL >= 1 otherwise.

VR (input/output)

On entry, if SIDE = 'R' or 'B' and HOWMNY = 'B', VR must contain an N-by-N matrix Q (usually the unitary matrix Q of Schur vectors returned by CHSEQR). On exit, if SIDE = 'R' or 'B', VR contains: if HOWMNY = 'A', the matrix X of right eigenvectors of T; VR is upper triangular. The i-th column VR(i) of VR is the eigenvector corresponding to T(i,i). if HOWMNY = 'B', the matrix Q*X; if HOWMNY = 'S', the right eigenvectors of T specified by SELECT, stored consecutively in the columns of VR, in the same order as their eigenvalues. If SIDE = 'L', VR is not referenced.

LDVR (input)
The leading dimension of the array VR. LDVR >= max(1,N) if SIDE = 'R' or 'B'; LDVR >= 1 otherwise.

MM (input)
The number of columns in the arrays VL and/or VR. MM >= M.

M (output)
The number of columns in the arrays VL and/or VR actually used to store the eigenvectors. If HOWMNY = 'A' or 'B', M is set to N. Each selected eigenvector occupies one column.

WORK (workspace)
dimension(2*N)

RWORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The algorithm used in this program is basically backward (forward) substitution, with scaling to make the the code robust against possible overflow.

Each eigenvector is normalized so that the element of largest magnitude has magnitude 1; here the magnitude of a complex number (x,y) is taken to be |x| + |y|.

Contents

NAME

```
ctrexc - reorder the Schur factorization of a complex matrix
A  =  Q*T*Q**H,  so  that the diagonal element of T with row
index IFST is moved to row ILST
```

SYNOPSIS

```
SUBROUTINE CTREXC(COMPQ, N, T, LDT, Q, LDQ, IFST, ILST, INFO)

CHARACTER * 1 COMPQ
COMPLEX T(LDT,*), Q(LDQ,*)
INTEGER N, LDT, LDQ, IFST, ILST, INFO

SUBROUTINE CTREXC_64(COMPQ, N, T, LDT, Q, LDQ, IFST, ILST, INFO)

CHARACTER * 1 COMPQ
COMPLEX T(LDT,*), Q(LDQ,*)
INTEGER*8 N, LDT, LDQ, IFST, ILST, INFO
```

F95 INTERFACE

```
SUBROUTINE TREXC(COMPQ, [N], T, [LDT], Q, [LDQ], IFST, ILST, [INFO])

CHARACTER(LEN=1) :: COMPQ
COMPLEX, DIMENSION(:,:) :: T, Q
INTEGER :: N, LDT, LDQ, IFST, ILST, INFO

SUBROUTINE TREXC_64(COMPQ, [N], T, [LDT], Q, [LDQ], IFST, ILST, [INFO])

CHARACTER(LEN=1) :: COMPQ
COMPLEX, DIMENSION(:,:) :: T, Q
INTEGER(8) :: N, LDT, LDQ, IFST, ILST, INFO
```

C INTERFACE

```
#include <sunperf.h>

void ctrexc(char compq, int n, complex *t, int ldt,  complex
          *q, int ldq, int ifst, int ilst, int *info);

void ctrexc_64(char compq, long n,  complex  *t,  long  ldt,
          complex  *q,  long ldq, long ifst, long ilst, long
          *info);
```

PURPOSE

ctrexc reorders the Schur factorization of a complex matrix A = Q*T*Q**H, so that the diagonal element of T with row index IFST is moved to row ILST.
The Schur form T is reordered by a unitary similarity transformation Z**H*T*Z, and optionally the matrix Q of Schur vectors is updated by postmultplying it with Z.

ARGUMENTS

COMPQ (input)
  = 'V': update the matrix Q of Schur vectors;
  = 'N': do not update Q.

N (input) The order of the matrix T. N >= 0.

T (input/output)
  On entry, the upper triangular matrix T. On exit, the reordered upper triangular matrix.

LDT (input)
  The leading dimension of the array T. LDT >= max(1,N).

Q (input) On entry, if COMPQ = 'V', the matrix Q of Schur vectors. On exit, if COMPQ = 'V', Q has been postmultiplied by the unitary transformation matrix Z which reorders T. If COMPQ = 'N', Q is not referenced.

LDQ (input)
  The leading dimension of the array Q. LDQ >= max(1,N).

IFST (input)
  Specify the reordering of the diagonal elements of T: The element with row index IFST is moved to

row ILST by a sequence of transpositions between adjacent elements. 1 <= IFST <= N; 1 <= ILST <= N.

ILST (input)
See the description of IFST.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
ctrmm - perform one of the matrix-matrix operations    B  :=
alpha*op(  A  )*B, or B := alpha*B*op( A )  where alpha is a
scalar, B is an m by n matrix, A is  a  unit,  or  non-unit,
upper or lower triangular matrix and op( A ) is one of   op(
A ) = A or op( A ) = A' or op( A ) = conjg( A' )
```

SYNOPSIS

```
SUBROUTINE CTRMM(SIDE, UPLO, TRANSA, DIAG, M, N, ALPHA, A, LDA, B,
      LDB)

CHARACTER * 1 SIDE, UPLO, TRANSA, DIAG
COMPLEX ALPHA
COMPLEX A(LDA,*), B(LDB,*)
INTEGER M, N, LDA, LDB

SUBROUTINE CTRMM_64(SIDE, UPLO, TRANSA, DIAG, M, N, ALPHA, A, LDA, B,
      LDB)

CHARACTER * 1 SIDE, UPLO, TRANSA, DIAG
COMPLEX ALPHA
COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 M, N, LDA, LDB
```

F95 INTERFACE

```
SUBROUTINE TRMM(SIDE, UPLO, [TRANSA], DIAG, [M], [N], ALPHA, A, [LDA],
       B, [LDB])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANSA, DIAG
COMPLEX :: ALPHA
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: M, N, LDA, LDB
```

```
SUBROUTINE TRMM_64(SIDE, UPLO, [TRANSA], DIAG, [M], [N], ALPHA, A,
       [LDA], B, [LDB])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANSA, DIAG
COMPLEX :: ALPHA
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: M, N, LDA, LDB
```

C INTERFACE

```
#include <sunperf.h>

void ctrmm(char side, char uplo, char transa, char diag, int
          m,  int  n,  complex  *alpha, complex *a, int lda,
          complex *b, int ldb);
void ctrmm_64(char side, char uplo, char transa, char  diag,
          long  m,  long n, complex *alpha, complex *a, long
          lda, complex *b, long ldb);
```

PURPOSE

ctrmm performs one of the matrix-matrix operations B := alpha*op(A)*B, or B := alpha*B*op(A) where alpha is a scalar, B is an m by n matrix, A is a unit, or non-unit, upper or lower triangular matrix and op(A) is one of op(A) = A or op(A) = A' or op(A) = conjg(A')

ARGUMENTS

SIDE (input)
On entry, SIDE specifies whether op(A) multiplies B from the left or right as follows:

SIDE = 'L' or 'l'   B := alpha*op(A)*B.

SIDE = 'R' or 'r'   B := alpha*B*op(A).

Unchanged on exit.

UPLO (input)
On entry, UPLO specifies whether the matrix A is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular

```
          matrix.

          Unchanged on exit.

TRANSA (input)
          On entry, TRANSA specifies the form of op( A )  to
          be used in the matrix multiplication as follows:

          TRANSA = 'N' or 'n'   op( A ) = A.

          TRANSA = 'T' or 't'   op( A ) = A'.

          TRANSA = 'C' or 'c'   op( A ) = conjg( A' ).

          Unchanged on exit.

          TRANSA is defaulted to 'N' for F95 INTERFACE.
DIAG (input)
          On entry, DIAG specifies whether or not A is  unit
          triangular as follows:

          DIAG = 'U' or 'u'   A is assumed to be  unit  tri-
          angular.

          DIAG = 'N' or 'n'   A is not assumed  to  be  unit
          triangular.

          Unchanged on exit.

M (input)
          On entry, M specifies the number of rows of  B.  M
          >= 0.  Unchanged on exit.

N (input)
          On entry, N specifies the number of columns of  B.
          N >= 0.  Unchanged on exit.

ALPHA (input)
          On entry,  ALPHA specifies the scalar  alpha. When
          alpha  is  zero  then   A is not referenced and  B
          need not be set before entry.  Unchanged on exit.

A (input)
          COMPLEX array of DIMENSION ( LDA, k ), where k  is
          m when  SIDE = 'L' or 'l'  and is  n  when  SIDE =
          'R' or 'r'.

          Before entry  with  UPLO = 'U' or 'u',  the  lead-
          ing   k by k upper triangular part of the array  A
          must contain the upper triangular matrix  and  the
```

strictly lower triangular part of A is not referenced.

Before entry with UPLO = 'L' or 'l', the leading k by k lower triangular part of the array A must contain the lower triangular matrix and the strictly upper triangular part of A is not referenced.

Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced either, but are assumed to be unity.

Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When SIDE = 'L' or 'l' then LDA >= max(1,M), when SIDE = 'R' or 'r' then LDA >= max(1,N). Unchanged on exit.

B (input/output)

COMPLEX array of DIMENSION (LDB, n). Before entry, the leading M by N part of the array B must contain the matrix B, and on exit is overwritten by the transformed matrix.

LDB (input)

On entry, LDB specifies the first dimension of B as declared in the calling subprogram. LDB must be at least max(1,M). Unchanged on exit.

Contents

NAME

ctrmv - perform one of the matrix-vector operations x := A*x, or x := A'*x, or x := conjg(A')*x

SYNOPSIS

```
SUBROUTINE CTRMV(UPLO, TRANSA, DIAG, N, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(LDA,*), Y(*)
INTEGER N, LDA, INCY

SUBROUTINE CTRMV_64(UPLO, TRANSA, DIAG, N, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(LDA,*), Y(*)
INTEGER*8 N, LDA, INCY
```

F95 INTERFACE

```
SUBROUTINE TRMV(UPLO, [TRANSA], DIAG, [N], A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, INCY

SUBROUTINE TRMV_64(UPLO, [TRANSA], DIAG, [N], A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INCY
```

C INTERFACE

```
#include <sunperf.h>

void ctrmv(char uplo, char transa, char diag, int n, complex
          *a, int lda, complex *y, int incy);

void ctrmv_64(char uplo, char transa,  char  diag,  long  n,
          complex *a, long lda, complex *y, long incy);
```

PURPOSE

ctrmv performs one of the matrix-vector operations x := A*x, or x := A'*x, or x := conjg(A')*x where x is an n element vector and A is an n by n unit, or non-unit, upper or lower triangular matrix.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u' A is an upper triangular matrix.

UPLO = 'L' or 'l' A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n' x := A*x.

TRANSA = 'T' or 't' x := A'*x.

TRANSA = 'C' or 'c' x := conjg(A')*x.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit triangular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

N (input)

On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

A (input)

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular matrix and the strictly lower triangular part of A is not referenced. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular matrix and the strictly upper triangular part of A is not referenced. Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced either, but are assumed to be unity. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

Y (input/output)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector x. On exit, Y is overwritten with the tranformed vector x.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
ctrrfs - provide error bounds and backward  error  estimates
for the solution to a system of linear equations with a tri-
angular coefficient matrix
```

SYNOPSIS

```
SUBROUTINE CTRRFS(UPLO, TRANSA, DIAG, N, NRHS, A, LDA, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(LDA,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDA, LDB, LDX, INFO
REAL FERR(*), BERR(*), WORK2(*)

SUBROUTINE CTRRFS_64(UPLO, TRANSA, DIAG, N, NRHS, A, LDA, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(LDA,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDA, LDB, LDX, INFO
REAL FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE TRRFS(UPLO, [TRANSA], DIAG, [N], [NRHS], A, [LDA], B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B, X
INTEGER :: N, NRHS, LDA, LDB, LDX, INFO
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

```
SUBROUTINE TRRFS_64(UPLO, [TRANSA], DIAG, [N], [NRHS], A, [LDA], B,
       [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: WORK
COMPLEX, DIMENSION(:,:) :: A, B, X
INTEGER(8) :: N, NRHS, LDA, LDB, LDX, INFO
REAL, DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void ctrrfs(char uplo, char transa, char diag,  int  n,  int
          nrhs,  complex  *a,  int lda, complex *b, int ldb,
          complex *x, int ldx, float *ferr, float *berr, int
          *info);

void ctrrfs_64(char uplo, char transa, char  diag,  long  n,
          long  nrhs, complex *a, long lda, complex *b, long
          ldb, complex *x,  long  ldx,  float  *ferr,  float
          *berr, long *info);
```

PURPOSE

ctrrfs provides error bounds and backward error estimates for the solution to a system of linear equations with a triangular coefficient matrix.

The solution matrix X must be computed by CTRTRS or some other means before entering this routine. CTRRFS does not do iterative refinement because doing so cannot improve the backward error.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.
```

```
DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input) The triangular matrix A.  If UPLO = 'U', the lead-
          ing  N-by-N  upper  triangular part of the array A
          contains the  upper  triangular  matrix,  and  the
          strictly   lower  triangular  part  of  A  is  not
          referenced.  If UPLO =  'L',  the  leading  N-by-N
          lower  triangular part of the array A contains the
          lower triangular matrix, and  the  strictly  upper
          triangular part of A is not referenced.  If DIAG =
          'U', the diagonal  elements  of  A  are  also  not
          referenced and are assumed to be 1.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

B (input) The right hand side matrix B.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

X (input) The solution matrix X.

LDX (input)
          The leading dimension of  the  array  X.   LDX  >=
          max(1,N).

FERR (output)
          The estimated forward error bound for  each  solu-
          tion  vector X(j) (the j-th column of the solution
          matrix  X).   If  XTRUE  is  the   true   solution
          corresponding  to  X(j),  FERR(j)  is an estimated
          upper bound for the magnitude of the largest  ele-
          ment in (X(j) - XTRUE) divided by the magnitude of
          the largest element in X(j).  The estimate  is  as
          reliable  as the estimate for RCOND, and is almost
          always a slight overestimate of the true error.

BERR (output)
          The componentwise relative backward error of  each
```

solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
ctrsen - reorder the Schur factorization of a complex matrix
A  =  Q*T*Q**H,  so  that  a selected cluster of eigenvalues
appears in the leading positions  on  the  diagonal  of  the
upper triangular matrix T, and the leading columns of Q form
an orthonormal basis of the  corresponding  right  invariant
subspace
```

SYNOPSIS

```
SUBROUTINE CTRSEN(JOB, COMPQ, SELECT, N, T, LDT, Q, LDQ, W, M, S,
      SEP, WORK, LWORK, INFO)

CHARACTER * 1 JOB, COMPQ
COMPLEX T(LDT,*), Q(LDQ,*), W(*), WORK(*)
INTEGER N, LDT, LDQ, M, LWORK, INFO
LOGICAL SELECT(*)
REAL S, SEP

SUBROUTINE CTRSEN_64(JOB, COMPQ, SELECT, N, T, LDT, Q, LDQ, W, M, S,
      SEP, WORK, LWORK, INFO)

CHARACTER * 1 JOB, COMPQ
COMPLEX T(LDT,*), Q(LDQ,*), W(*), WORK(*)
INTEGER*8 N, LDT, LDQ, M, LWORK, INFO
LOGICAL*8 SELECT(*)
REAL S, SEP
```

F95 INTERFACE

```
SUBROUTINE TRSEN(JOB, COMPQ, SELECT, [N], T, [LDT], Q, [LDQ], W, M,
       S, SEP, [WORK], [LWORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOB, COMPQ
COMPLEX, DIMENSION(:) :: W, WORK
COMPLEX, DIMENSION(:,:) :: T, Q
INTEGER :: N, LDT, LDQ, M, LWORK, INFO
LOGICAL, DIMENSION(:) :: SELECT
REAL :: S, SEP

SUBROUTINE TRSEN_64(JOB, COMPQ, SELECT, [N], T, [LDT], Q, [LDQ], W,
       M, S, SEP, [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: JOB, COMPQ
COMPLEX, DIMENSION(:) :: W, WORK
COMPLEX, DIMENSION(:,:) :: T, Q
INTEGER(8) :: N, LDT, LDQ, M, LWORK, INFO
LOGICAL(8), DIMENSION(:) :: SELECT
REAL :: S, SEP
```

C INTERFACE

```
#include <sunperf.h>

void ctrsen(char job, char compq, int *select, int  n,  com-
          plex *t, int ldt, complex *q, int ldq, complex *w,
          int *m, float *s, float *sep, int *info);

void ctrsen_64(char job, char compq, long *select,  long  n,
          complex  *t,  long ldt, complex *q, long ldq, com-
          plex *w, long  *m,  float  *s,  float  *sep,  long
          *info);
```

PURPOSE

ctrsen reorders the Schur factorization of a complex matrix A = Q*T*Q**H, so that a selected cluster of eigenvalues appears in the leading positions on the diagonal of the upper triangular matrix T, and the leading columns of Q form an orthonormal basis of the corresponding right invariant subspace.

Optionally the routine computes the reciprocal condition numbers of the cluster of eigenvalues and/or the invariant subspace.

ARGUMENTS

JOB (input)
Specifies whether condition numbers are required for the cluster of eigenvalues (S) or the invariant subspace (SEP):

```
          = 'N': none;
          = 'E': for eigenvalues only (S);
          = 'V': for invariant subspace only (SEP);
          = 'B': for both eigenvalues and invariant subspace
          (S and SEP).

COMPQ (input)
          = 'V': update the matrix Q of Schur vectors;
          = 'N': do not update Q.

SELECT (input)
          SELECT specifies the eigenvalues in  the  selected
          cluster.  To select the j-th eigenvalue, SELECT(j)
          must be set to .TRUE..

N (input) The order of the matrix T. N >= 0.
T (input/output)
          On entry, the upper triangular matrix T.  On exit,
          T  is  overwritten by the reordered matrix T, with
          the selected eigenvalues as the  leading  diagonal
          elements.

LDT (input)
          The leading dimension  of  the  array  T.  LDT  >=
          max(1,N).

Q (input) On entry, if COMPQ = 'V', the matrix  Q  of  Schur
          vectors.   On  exit,  if  COMPQ  = 'V', Q has been
          postmultiplied  by  the   unitary   transformation
          matrix  which reorders T; the leading M columns of
          Q form an  orthonormal  basis  for  the  specified
          invariant  subspace.   If  COMPQ  =  'N', Q is not
          referenced.

LDQ (input)
          The leading dimension of the array Q.  LDQ  >=  1;
          and if COMPQ = 'V', LDQ >= N.

W (output)
          The reordered eigenvalues of T, in the same  order
          as they appear on the diagonal of T.

M (output)
          The dimension of the specified invariant subspace.
          0 <= M <= N.

S (output)
          If JOB = 'E' or 'B', S is a  lower  bound  on  the
          reciprocal condition number for the selected clus-
          ter of eigenvalues.  S  cannot  underestimate  the
```

true reciprocal condition number by more than a factor of sqrt(N). If M = 0 or N, S = 1. If JOB = 'N' or 'V', S is not referenced.

SEP (output)
If JOB = 'V' or 'B', SEP is the estimated reciprocal condition number of the specified invariant subspace. If M = 0 or N, SEP = norm(T). If JOB = 'N' or 'E', SEP is not referenced.

WORK (workspace)
If JOB = 'N', WORK is not referenced. Otherwise, on exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If JOB = 'N', LWORK >= 1; if JOB = 'E', LWORK = M*(N-M); if JOB = 'V' or 'B', LWORK >= 2*M*(N-M).

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

CTRSEN first collects the selected eigenvalues by computing a unitary transformation Z to move them to the top left corner of T. In other words, the selected eigenvalues are the eigenvalues of T11 in:

```
        Z'*T*Z = ( T11 T12 ) n1
                 (  0  T22 ) n2
                    n1  n2
```

where N = n1+n2 and Z' means the conjugate transpose of Z. The first n1 columns of Z span the specified invariant subspace of T.

If T has been obtained from the Schur factorization of a matrix A = Q*T*Q', then the reordered Schur factorization of A is given by A = (Q*Z)*(Z'*T*Z)*(Q*Z)', and the first n1 columns of Q*Z span the corresponding invariant subspace of A.

The reciprocal condition number of the average of the eigenvalues of T11 may be returned in S. S lies between 0 (very badly conditioned) and 1 (very well conditioned). It is computed as follows. First we compute R so that

```
P = ( I  R ) n1
    ( 0  0 ) n2
      n1 n2
```

is the projector on the invariant subspace associated with T11. R is the solution of the Sylvester equation:

```
T11*R - R*T22 = T12.
```

Let F-norm(M) denote the Frobenius-norm of M and 2-norm(M) denote the two-norm of M. Then S is computed as the lower bound

```
(1 + F-norm(R)**2)**(-1/2)
```

on the reciprocal of 2-norm(P), the true reciprocal condition number. S cannot underestimate 1 / 2-norm(P) by more than a factor of sqrt(N).

An approximate error bound for the computed average of the eigenvalues of T11 is

```
EPS * norm(T) / S
```

where EPS is the machine precision.

The reciprocal condition number of the right invariant subspace spanned by the first n1 columns of Z (or of Q*Z) is returned in SEP. SEP is defined as the separation of T11 and T22:

```
sep( T11, T22 ) = sigma-min( C )
```

where sigma-min(C) is the smallest singular value of the n1*n2-by-n1*n2 matrix

```
C  = kprod( I(n2), T11 ) - kprod( transpose(T22), I(n1) )
```

I(m) is an m by m identity matrix, and kprod denotes the Kronecker product. We estimate sigma-min(C) by the reciprocal of an estimate of the 1-norm of inverse(C). The true reciprocal 1-norm of inverse(C) cannot differ from sigma-min(C) by more than a factor of sqrt(n1*n2).

When SEP is small, small changes in T can cause large changes in the invariant subspace. An approximate bound on the maximum angular error in the computed right invariant subspace is

EPS * norm(T) / SEP

Contents

NAME

```
ctrsm - solve one of  the  matrix  equations  op(  A  )*X  =
alpha*B, or X*op( A ) = alpha*B
```

SYNOPSIS

```
SUBROUTINE CTRSM(SIDE, UPLO, TRANSA, DIAG, M, N, ALPHA, A, LDA, B,
      LDB)

CHARACTER * 1 SIDE, UPLO, TRANSA, DIAG
COMPLEX ALPHA
COMPLEX A(LDA,*), B(LDB,*)
INTEGER M, N, LDA, LDB

SUBROUTINE CTRSM_64(SIDE, UPLO, TRANSA, DIAG, M, N, ALPHA, A, LDA, B,
      LDB)

CHARACTER * 1 SIDE, UPLO, TRANSA, DIAG
COMPLEX ALPHA
COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 M, N, LDA, LDB
```

F95 INTERFACE

```
SUBROUTINE TRSM(SIDE, UPLO, [TRANSA], DIAG, [M], [N], ALPHA, A, [LDA],
       B, [LDB])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANSA, DIAG
COMPLEX :: ALPHA
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: M, N, LDA, LDB

SUBROUTINE TRSM_64(SIDE, UPLO, [TRANSA], DIAG, [M], [N], ALPHA, A,
       [LDA], B, [LDB])
```

```
CHARACTER(LEN=1) :: SIDE, UPLO, TRANSA, DIAG
COMPLEX :: ALPHA
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: M, N, LDA, LDB
```

C INTERFACE

```
#include <sunperf.h>

void ctrsm(char side, char uplo, char transa, char diag, int
          m,  int  n,  complex  *alpha, complex *a, int lda,
          complex *b, int ldb);

void ctrsm_64(char side, char uplo, char transa, char  diag,
          long  m,  long n, complex *alpha, complex *a, long
          lda, complex *b, long ldb);
```

PURPOSE

ctrsm solves one of the matrix equations op(A)*X = alpha*B, or X*op(A) = alpha*B where alpha is a scalar, X and B are m by n matrices, A is a unit, or non-unit, upper or lower triangular matrix and op(A) is one of

op(A) = A or op(A) = A' or op(A) = conjg(A').

The matrix X is overwritten on B.

ARGUMENTS

SIDE (input)
On entry, SIDE specifies whether op(A) appears on the left or right of X as follows:

SIDE = 'L' or 'l'   op(A)*X = alpha*B.

SIDE = 'R' or 'r'   X*op(A) = alpha*B.

Unchanged on exit.

UPLO (input)
On entry, UPLO specifies whether the matrix A is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l' A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the form of op(A) to be used in the matrix multiplication as follows:

TRANSA = 'N' or 'n' op(A) = A.

TRANSA = 'T' or 't' op(A) = A'.

TRANSA = 'C' or 'c' op(A) = conjg(A').

Unchanged on exit.
TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u' A is assumed to be unit triangular.

DIAG = 'N' or 'n' A is not assumed to be unit triangular.

Unchanged on exit.

M (input)
On entry, M specifies the number of rows of B. M >= 0. Unchanged on exit.

N (input)
On entry, N specifies the number of columns of B. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. When alpha is zero then A is not referenced and B need not be set before entry. Unchanged on exit.

A (input)
COMPLEX array of DIMENSION (LDA, k), where k is m when SIDE = 'L' or 'l' and is n when SIDE = 'R' or 'r'.

Before entry with UPLO = 'U' or 'u', the leading k by k upper triangular part of the array A must contain the upper triangular matrix and the strictly lower triangular part of A is not referenced.

Before entry with UPLO = 'L' or 'l', the leading k by k lower triangular part of the array A must contain the lower triangular matrix and the strictly upper triangular part of A is not referenced.

Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced either, but are assumed to be unity.
Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When SIDE = 'L' or 'l' then LDA >= max(1,M), when SIDE = 'R' or 'r' then LDA >= max(1,N). Unchanged on exit.

B (input/output)
COMPLEX array of DIMENSION (LDB, n).
Before entry, the leading M by N part of the array B must contain the right-hand side matrix B, and on exit is overwritten by the solution matrix X.

LDB (input)
On entry, LDB specifies the first dimension of B as declared in the calling subprogram. LDB >= max(1,M). Unchanged on exit.

Contents

NAME

```
ctrsna - estimate reciprocal condition numbers for specified
eigenvalues  and/or  right  eigenvectors  of a complex upper
triangular matrix T (or of any matrix Q*T*Q**H with  Q  uni-
tary)
```

SYNOPSIS

```
SUBROUTINE CTRSNA(JOB, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR, LDVR,
      S, SEP, MM, M, WORK, LDWORK, WORK1, INFO)

CHARACTER * 1 JOB, HOWMNY
COMPLEX T(LDT,*), VL(LDVL,*), VR(LDVR,*), WORK(LDWORK,*)
INTEGER N, LDT, LDVL, LDVR, MM, M, LDWORK, INFO
LOGICAL SELECT(*)
REAL S(*), SEP(*), WORK1(*)

SUBROUTINE CTRSNA_64(JOB, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR,
      LDVR, S, SEP, MM, M, WORK, LDWORK, WORK1, INFO)

CHARACTER * 1 JOB, HOWMNY
COMPLEX T(LDT,*), VL(LDVL,*), VR(LDVR,*), WORK(LDWORK,*)
INTEGER*8 N, LDT, LDVL, LDVR, MM, M, LDWORK, INFO
LOGICAL*8 SELECT(*)
REAL S(*), SEP(*), WORK1(*)
```

F95 INTERFACE

```
SUBROUTINE TRSNA(JOB, HOWMNY, SELECT, [N], T, [LDT], VL, [LDVL], VR,
       [LDVR], S, SEP, MM, M, [WORK], [LDWORK], [WORK1], [INFO])

CHARACTER(LEN=1) :: JOB, HOWMNY
COMPLEX, DIMENSION(:,:) :: T, VL, VR, WORK
```

```
INTEGER :: N, LDT, LDVL, LDVR, MM, M, LDWORK, INFO
LOGICAL, DIMENSION(:) :: SELECT
REAL, DIMENSION(:) :: S, SEP, WORK1

SUBROUTINE TRSNA_64(JOB, HOWMNY, SELECT, [N], T, [LDT], VL, [LDVL],
       VR, [LDVR], S, SEP, MM, M, [WORK], [LDWORK], [WORK1], [INFO])

CHARACTER(LEN=1) :: JOB, HOWMNY
COMPLEX, DIMENSION(:,:) :: T, VL, VR, WORK
INTEGER(8) :: N, LDT, LDVL, LDVR, MM, M, LDWORK, INFO
LOGICAL(8), DIMENSION(:) :: SELECT
REAL, DIMENSION(:) :: S, SEP, WORK1
```

C INTERFACE

```
#include <sunperf.h>
void ctrsna(char job, char howmny, int *select, int n,  com-
          plex  *t,  int ldt, complex *vl, int ldvl, complex
          *vr, int ldvr, float *s, float *sep, int  mm,  int
          *m, int ldwork, int *info);

void ctrsna_64(char job, char howmny, long *select, long  n,
          complex *t, long ldt, complex *vl, long ldvl, com-
          plex *vr, long ldvr, float *s,  float  *sep,  long
          mm, long *m, long ldwork, long *info);
```

PURPOSE

ctrsna estimates reciprocal condition numbers for specified eigenvalues and/or right eigenvectors of a complex upper triangular matrix T (or of any matrix Q*T*Q**H with Q unitary).

ARGUMENTS

JOB (input)
Specifies whether condition numbers are required for eigenvalues (S) or eigenvectors (SEP):
= 'E': for eigenvalues only (S);
= 'V': for eigenvectors only (SEP);
= 'B': for both eigenvalues and eigenvectors (S and SEP).

HOWMNY (input)
= 'A': compute condition numbers for all eigenpairs;
= 'S': compute condition numbers for selected eigenpairs specified by the array SELECT.

```
SELECT (input)
          If HOWMNY = 'S', SELECT specifies  the  eigenpairs
          for  which  condition  numbers  are  required.  To
          select condition numbers for the  j-th  eigenpair,
          SELECT(j) must be set to .TRUE..  If HOWMNY = 'A',
          SELECT is not referenced.

N (input) The order of the matrix T. N >= 0.

T (input) The upper triangular matrix T.

LDT (input)
          The leading dimension  of  the  array  T.  LDT  >=
          max(1,N).

VL (input)
          If JOB = 'E' or 'B', VL must contain  left  eigen-
          vectors  of T (or of any Q*T*Q**H with Q unitary),
          corresponding  to  the  eigenpairs  specified   by
          HOWMNY and SELECT. The eigenvectors must be stored
          in consecutive  columns  of  VL,  as  returned  by
          CHSEIN  or CTREVC.  If JOB = 'V', VL is not refer-
          enced.

LDVL (input)
          The leading dimension of the array VL.  LDVL >= 1;
          and if JOB = 'E' or 'B', LDVL >= N.

VR (input)
          If JOB = 'E' or 'B', VR must contain right  eigen-
          vectors  of T (or of any Q*T*Q**H with Q unitary),
          corresponding  to  the  eigenpairs  specified   by
          HOWMNY and SELECT. The eigenvectors must be stored
          in consecutive  columns  of  VR,  as  returned  by
          CHSEIN  or CTREVC.  If JOB = 'V', VR is not refer-
          enced.

LDVR (input)
          The leading dimension of the array VR.  LDVR >= 1;
          and if JOB = 'E' or 'B', LDVR >= N.

S (output)
          If JOB = 'E'  or  'B',  the  reciprocal  condition
          numbers  of  the  selected  eigenvalues, stored in
          consecutive elements  of  the  array.  Thus  S(j),
          SEP(j),  and  the  j-th  columns  of VL and VR all
          correspond to the same eigenpair (but not in  gen-
          eral the j-th eigenpair, unless all eigenpairs are
          selected).  If JOB = 'V', S is not referenced.
```

```
SEP (output)
          If JOB = 'V' or 'B', the estimated reciprocal con-
          dition   numbers  of  the  selected  eigenvectors,
          stored in consecutive elements of the  array.   If
          JOB = 'E', SEP is not referenced.

MM (input)
          The number of elements in the arrays S (if  JOB  =
          'E'  or  'B') and/or SEP (if JOB = 'V' or 'B'). MM
          >= M.

M (output)
          The number of elements of the arrays S and/or  SEP
          actually  used  to  store  the estimated condition
          numbers.  If HOWMNY = 'A', M is set to N.

WORK (workspace)
          dimension(LDWORK,N+1) If JOB = 'E',  WORK  is  not
          referenced.

LDWORK (input)
          The leading dimension of the array  WORK.   LDWORK
          >= 1; and if JOB = 'V' or 'B', LDWORK >= N.

WORK1 (workspace)
          dimension(N) If JOB = 'E',  WORK1  is  not  refer-
          enced.

INFO (output)
          = 0: successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
          gal value
```

FURTHER DETAILS

```
The reciprocal of the  condition  number  of  an  eigenvalue
lambda is defined as

        S(lambda) = |v'*u| / (norm(u)*norm(v))

where u and v are the  right  and  left  eigenvectors  of  T
corresponding  to lambda; v' denotes the conjugate transpose
of v, and norm(u) denotes the Euclidean norm. These recipro-
cal  condition  numbers  always lie between zero (very badly
conditioned) and one (very well  conditioned).  If  n  =  1,
S(lambda) is defined to be 1.

An approximate error bound for a computed eigenvalue W(i) is
given by
```

EPS * norm(T) / S(i)

where EPS is the machine precision.

The reciprocal of the condition number of the right eigenvector u corresponding to lambda is defined as follows. Suppose

```
T = ( lambda  c  )
    (   0    T22 )
```

Then the reciprocal condition number is

SEP(lambda, T22) = sigma-min(T22 - lambda*I)

where sigma-min denotes the smallest singular value. We approximate the smallest singular value by the reciprocal of an estimate of the one-norm of the inverse of T22 - lambda*I. If n = 1, SEP(1) is defined to be abs(T(1,1)).

An approximate error bound for a computed right eigenvector VR(i) is given by

EPS * norm(T) / SEP(i)

Contents

NAME

```
ctrsv - solve one of the systems of equations   A*x = b,  or
A'*x = b, or conjg( A' )*x = b
```

SYNOPSIS

```
SUBROUTINE CTRSV(UPLO, TRANSA, DIAG, N, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(LDA,*), Y(*)
INTEGER N, LDA, INCY

SUBROUTINE CTRSV_64(UPLO, TRANSA, DIAG, N, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(LDA,*), Y(*)
INTEGER*8 N, LDA, INCY
```

F95 INTERFACE

```
SUBROUTINE TRSV(UPLO, [TRANSA], DIAG, [N], A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, INCY

SUBROUTINE TRSV_64(UPLO, [TRANSA], DIAG, [N], A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:) :: Y
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INCY
```

C INTERFACE

```
#include <sunperf.h>

void ctrsv(char uplo, char transa, char diag, int n, complex
          *a, int lda, complex *y, int incy);

void ctrsv_64(char uplo, char transa,  char  diag,  long  n,
          complex *a, long lda, complex *y, long incy);
```

PURPOSE

ctrsv solves one of the systems of equations A*x = b, or A'*x = b, or conjg(A')*x = b where b and x are n element vectors and A is an n by n unit, or non-unit, upper or lower triangular matrix.
No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the equations to be solved as follows:

TRANSA = 'N' or 'n'   A*x = b.

TRANSA = 'T' or 't'   A'*x = b.

TRANSA = 'C' or 'c'   conjg(A')*x = b.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)

On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit triangular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

N (input)

On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

A (input)

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular matrix and the strictly lower triangular part of A is not referenced. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular matrix and the strictly upper triangular part of A is not referenced. Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced either, but are assumed to be unity. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

Y (input/output)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element right-hand side vector b. On exit, Y is overwritten with the solution vector x.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
ctrsyl - solve the complex Sylvester matrix equation
```

SYNOPSIS

```
SUBROUTINE CTRSYL(TRANA, TRANB, ISGN, M, N, A, LDA, B, LDB, C, LDC,
      SCALE, INFO)

CHARACTER * 1 TRANA, TRANB
COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER ISGN, M, N, LDA, LDB, LDC, INFO
REAL SCALE

SUBROUTINE CTRSYL_64(TRANA, TRANB, ISGN, M, N, A, LDA, B, LDB, C,
      LDC, SCALE, INFO)

CHARACTER * 1 TRANA, TRANB
COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER*8 ISGN, M, N, LDA, LDB, LDC, INFO
REAL SCALE
```

F95 INTERFACE

```
SUBROUTINE TRSYL(TRANA, TRANB, ISGN, [M], [N], A, [LDA], B, [LDB], C,
       [LDC], SCALE, [INFO])

CHARACTER(LEN=1) :: TRANA, TRANB
COMPLEX, DIMENSION(:,:) :: A, B, C
INTEGER :: ISGN, M, N, LDA, LDB, LDC, INFO
REAL :: SCALE

SUBROUTINE TRSYL_64(TRANA, TRANB, ISGN, [M], [N], A, [LDA], B, [LDB],
       C, [LDC], SCALE, [INFO])
```

```
CHARACTER(LEN=1) :: TRANA, TRANB
COMPLEX, DIMENSION(:,:) :: A, B, C
INTEGER(8) :: ISGN, M, N, LDA, LDB, LDC, INFO
REAL :: SCALE
```

C INTERFACE

```
#include <sunperf.h>

void ctrsyl(char trana, char tranb, int isgn, int m, int  n,
          complex  *a, int lda, complex *b, int ldb, complex
          *c, int ldc, float *scale, int *info);

void ctrsyl_64(char trana, char tranb, long  isgn,  long  m,
          long  n,  complex  *a,  long lda, complex *b, long
          ldb, complex *c,  long  ldc,  float  *scale,  long
          *info);
```

PURPOSE

ctrsyl solves the complex Sylvester matrix equation:

```
op(A)*X + X*op(B) = scale*C or
op(A)*X - X*op(B) = scale*C,
```

where op(A) = A or A**H, and A and B are both upper triangular. A is M-by-M and B is N-by-N; the right hand side C and the solution X are M-by-N; and scale is an output scale factor, set <= 1 to avoid overflow in X.

ARGUMENTS

TRANA (input)
Specifies the option op(A):
= 'N': op(A) = A   (No transpose)
= 'C': op(A) = A**H (Conjugate transpose)

TRANB (input)
Specifies the option op(B):
= 'N': op(B) = B   (No transpose)
= 'C': op(B) = B**H (Conjugate transpose)

ISGN (input)
Specifies the sign in the equation:
= +1: solve op(A)*X + X*op(B) = scale*C
= -1: solve op(A)*X - X*op(B) = scale*C

M (input) The order of the matrix A, and the number of rows

```
          in the matrices X and C. M >= 0.

N (input) The order of the  matrix  B,  and  the  number  of
          columns in the matrices X and C. N >= 0.

A (input) The upper triangular matrix A.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,M).

B (input) The upper triangular matrix B.
LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          max(1,N).

C (input/output)
          On entry, the M-by-N right hand side matrix C.  On
          exit, C is overwritten by the solution matrix X.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M)

SCALE (output)
          The scale factor, scale, set <= 1 to  avoid  over-
          flow in X.

INFO (output)
          = 0: successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
          gal value
          = 1: A and B have  common  or  very  close  eigen-
          values;  perturbed  values  were used to solve the
          equation (but the matrices A and B are unchanged).
```

Contents

NAME

ctrti2 - compute the inverse of a complex upper or lower triangular matrix

SYNOPSIS

```
SUBROUTINE CTRTI2(UPLO, DIAG, N, A, LDA, INFO)

CHARACTER * 1 UPLO, DIAG
COMPLEX A(LDA,*)
INTEGER N, LDA, INFO

SUBROUTINE CTRTI2_64(UPLO, DIAG, N, A, LDA, INFO)

CHARACTER * 1 UPLO, DIAG
COMPLEX A(LDA,*)
INTEGER*8 N, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE TRTI2(UPLO, DIAG, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO

SUBROUTINE TRTI2_64(UPLO, DIAG, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void ctrti2(char uplo, char diag, int  n,  complex  *a,  int
          lda, int *info);

void ctrti2_64(char uplo, char diag,  long  n,  complex  *a,
          long lda, long *info);
```

PURPOSE

ctrti2 computes the inverse of a complex upper or lower triangular matrix.

This is the Level 2 BLAS version of the algorithm.

ARGUMENTS

UPLO (input)
Specifies whether the matrix A is upper or lower triangular. = 'U': Upper triangular
= 'L': Lower triangular

DIAG (input)
Specifies whether or not the matrix A is unit triangular. = 'N': Non-unit triangular
= 'U': Unit triangular

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the triangular matrix A. If UPLO = 'U', the leading n by n upper triangular part of the array A contains the upper triangular matrix, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading n by n lower triangular part of the array A contains the lower triangular matrix, and the strictly upper triangular part of A is not referenced. If DIAG = 'U', the diagonal elements of A are also not referenced and are assumed to be 1.

On exit, the (triangular) inverse of the original matrix, in the same storage format.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

INFO (output)

= 0: successful exit

< 0: if INFO = -k, the k-th argument had an illegal value

Contents

NAME

ctrtri - compute the inverse of a complex upper or lower triangular matrix A

SYNOPSIS

```
SUBROUTINE CTRTRI(UPLO, DIAG, N, A, LDA, INFO)

CHARACTER * 1 UPLO, DIAG
COMPLEX A(LDA,*)
INTEGER N, LDA, INFO

SUBROUTINE CTRTRI_64(UPLO, DIAG, N, A, LDA, INFO)

CHARACTER * 1 UPLO, DIAG
COMPLEX A(LDA,*)
INTEGER*8 N, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE TRTRI(UPLO, DIAG, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO

SUBROUTINE TRTRI_64(UPLO, DIAG, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void ctrtri(char uplo, char diag, int  n,  complex  *a,  int
          lda, int *info);

void ctrtri_64(char uplo, char diag,  long  n,  complex  *a,
          long lda, long *info);
```

PURPOSE

ctrtri computes the inverse of a complex upper or lower triangular matrix A.

This is the Level 3 BLAS version of the algorithm.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the triangular matrix A.  If UPLO = 'U',
          the  leading  N-by-N  upper triangular part of the
          array A contains the upper triangular matrix,  and
          the  strictly  lower  triangular  part of A is not
          referenced.  If UPLO =  'L',  the  leading  N-by-N
          lower  triangular part of the array A contains the
          lower triangular matrix, and  the  strictly  upper
          triangular part of A is not referenced.  If DIAG =
          'U', the diagonal  elements  of  A  are  also  not
          referenced  and are assumed to be 1.  On exit, the
          (triangular) inverse of the  original  matrix,  in
          the same storage format.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

INFO (output)
          = 0: successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
```

gal value
> 0: if INFO = i, A(i,i) is exactly zero. The triangular matrix is singular and its inverse can not be computed.

Contents

NAME

```
ctrtrs - solve a triangular system of the form   A * X =  B,
A**T * X = B, or A**H * X = B,
```

SYNOPSIS

```
SUBROUTINE CTRTRS(UPLO, TRANSA, DIAG, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, NRHS, LDA, LDB, INFO

SUBROUTINE CTRTRS_64(UPLO, TRANSA, DIAG, N, NRHS, A, LDA, B, LDB,
      INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, NRHS, LDA, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE TRTRS(UPLO, [TRANSA], DIAG, [N], [NRHS], A, [LDA], B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER :: N, NRHS, LDA, LDB, INFO

SUBROUTINE TRTRS_64(UPLO, [TRANSA], DIAG, [N], [NRHS], A, [LDA], B,
       [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX, DIMENSION(:,:) :: A, B
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void ctrtrs(char uplo, char transa, char diag,  int  n,  int
          nrhs,  complex  *a,  int lda, complex *b, int ldb,
          int *info);

void ctrtrs_64(char uplo, char transa, char  diag,  long  n,
          long  nrhs, complex *a, long lda, complex *b, long
          ldb, long *info);
```

PURPOSE

ctrtrs solves a triangular system of the form
where A is a triangular matrix of order N, and B is an N-by-NRHS matrix. A check is made to verify that A is non-singular.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

A (input) The triangular matrix A.  If UPLO = 'U', the lead-
          ing  N-by-N  upper  triangular part of the array A
          contains the  upper  triangular  matrix,  and  the
          strictly  lower triangular part of A is not refer-
```

enced. If UPLO = 'L', the leading N-by-N lower triangular part of the array A contains the lower triangular matrix, and the strictly upper triangular part of A is not referenced. If DIAG = 'U', the diagonal elements of A are also not referenced and are assumed to be 1.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the right hand side matrix B. On exit, if INFO = 0, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the i-th diagonal element of A is zero, indicating that the matrix is singular and the solutions X have not been computed.

Contents

NAME

ctzrqf - routine is deprecated and has been replaced by routine CTZRZF

SYNOPSIS

```
SUBROUTINE CTZRQF(M, N, A, LDA, TAU, INFO)

COMPLEX A(LDA,*), TAU(*)
INTEGER M, N, LDA, INFO

SUBROUTINE CTZRQF_64(M, N, A, LDA, TAU, INFO)

COMPLEX A(LDA,*), TAU(*)
INTEGER*8 M, N, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE TZRQF([M], [N], A, [LDA], TAU, [INFO])

COMPLEX, DIMENSION(:) :: TAU
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, INFO

SUBROUTINE TZRQF_64([M], [N], A, [LDA], TAU, [INFO])

COMPLEX, DIMENSION(:) :: TAU
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void ctzrqf(int m, int n, complex *a, int lda, complex *tau,
          int *info);

void ctzrqf_64(long m, long n, complex *a, long lda, complex
          *tau, long *info);
```

PURPOSE

ctzrqf routine is deprecated and has been replaced by routine CTZRZF.

CTZRQF reduces the M-by-N (M<=N) complex upper trapezoidal matrix A to upper triangular form by means of unitary transformations.

The upper trapezoidal matrix A is factored as

```
   A = ( R  0 ) * Z,
```

where Z is an N-by-N unitary matrix and R is an M-by-M upper triangular matrix.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= M.

A (input/output)
On entry, the leading M-by-N upper trapezoidal part of the array A must contain the matrix to be factorized. On exit, the leading M-by-M upper triangular part of A contains the upper triangular matrix R, and elements M+1 to N of the first M rows of A, with the array TAU, represent the unitary matrix Z as a product of M elementary reflectors.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-

gal value

FURTHER DETAILS

```
The  factorization is obtained by Householder's method.  The
kth transformation matrix, Z( k ), whose conjugate transpose
is used to introduce zeros into the (m - k + 1)th row of  A,
is given in the form

   Z( k ) = ( I     0   ),
            ( 0  T( k ) )

where

   T( k ) = I - tau*u( k )*u( k )',   u( k ) = (   1    ),
                                               (   0    )
                                               ( z( k ) )

tau is a scalar and z( k ) is an ( n - m )  element  vector.
tau  and z( k ) are chosen to annihilate the elements of the
kth row of X.

The scalar tau is returned in the kth element of TAU and the
vector u( k ) in the kth row of A, such that the elements of
z( k ) are in  a( k, m + 1 ), ..., a( k, n ).  The  elements
of R are returned in the upper triangular part of A.

Z is given by

   Z =  Z( 1 ) * Z( 2 ) * ... * Z( m ).
```

Contents

NAME

```
ctzrzf - reduce the M-by-N  (  M<=N  )  complex  upper  tra-
pezoidal  matrix A to upper triangular form by means of uni-
tary transformations
```

SYNOPSIS

```
SUBROUTINE CTZRZF(M, N, A, LDA, TAU, WORK, LWORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, LDA, LWORK, INFO

SUBROUTINE CTZRZF_64(M, N, A, LDA, TAU, WORK, LWORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, LDA, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE TZRZF([M], [N], A, [LDA], TAU, [WORK], [LWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, LWORK, INFO

SUBROUTINE TZRZF_64([M], [N], A, [LDA], TAU, [WORK], [LWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void ctzrzf(int m, int n, complex *a, int lda, complex *tau,
          int *info);

void ctzrzf_64(long m, long n, complex *a, long lda, complex
          *tau, long *info);
```

PURPOSE

ctzrzf reduces the M-by-N (M<=N) complex upper trapezoidal matrix A to upper triangular form by means of unitary transformations.

The upper trapezoidal matrix A is factored as

```
   A = ( R  0 ) * Z,
```

where Z is an N-by-N unitary matrix and R is an M-by-M upper triangular matrix.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the leading M-by-N upper trapezoidal part of the array A must contain the matrix to be factorized. On exit, the leading M-by-M upper triangular part of A contains the upper triangular matrix R, and elements M+1 to N of the first M rows of A, with the array TAU, represent the unitary matrix Z as a product of M elementary reflectors.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)

The dimension of the array WORK. LWORK >= max(1,M). For optimum performance LWORK >= M*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

Based on contributions by
A. Petitet, Computer Science Dept., Univ. of Tenn., Knoxville, USA

The factorization is obtained by Householder's method. The kth transformation matrix, Z(k), which is used to introduce zeros into the (m - k + 1)th row of A, is given in the form

```
   Z( k ) = ( I     0   ),
            ( 0  T( k ) )
```

where

```
   T( k ) = I - tau*u( k )*u( k )',   u( k ) = (   1    ),
                                               (   0    )
                                               ( z( k ) )
```

tau is a scalar and z(k) is an (n - m) element vector. tau and z(k) are chosen to annihilate the elements of the kth row of X.

The scalar tau is returned in the kth element of TAU and the vector u(k) in the kth row of A, such that the elements of z(k) are in a(k, m + 1), ..., a(k, n). The elements of R are returned in the upper triangular part of A.

Z is given by

```
   Z =  Z( 1 ) * Z( 2 ) * ... * Z( m ).
```

Contents

NAME

```
cung2l - generate an m by n complex matrix Q with  orthonor-
mal columns,
```

SYNOPSIS

```
SUBROUTINE CUNG2L(M, N, K, A, LDA, TAU, WORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, K, LDA, INFO

SUBROUTINE CUNG2L_64(M, N, K, A, LDA, TAU, WORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, K, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE UNG2L(M, [N], [K], A, [LDA], TAU, [WORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, K, LDA, INFO

SUBROUTINE UNG2L_64(M, [N], [K], A, [LDA], TAU, [WORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, K, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cung2l(int m, int n, int k, complex *a, int  lda,  com-
```

```
          plex *tau, int *info);

void cung2l_64(long m, long n, long k, complex *a, long lda,
          complex *tau, long *info);
```

PURPOSE

cung2l L generates an m by n complex matrix Q with orthonormal columns, which is defined as the last n columns of a product of k elementary reflectors of order m

Q = H(k) . . . H(2) H(1)

as returned by CGEQLF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. M >= N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. N >= K >= 0.

A (input/output)
On entry, the (n-k+i)-th column must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGEQLF in the last k columns of its array argument A. On exit, the m-by-n matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGEQLF.

WORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
cung2r - generate an m by n complex matrix Q with  orthonor-
mal columns,
```

SYNOPSIS

```
SUBROUTINE CUNG2R(M, N, K, A, LDA, TAU, WORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, K, LDA, INFO

SUBROUTINE CUNG2R_64(M, N, K, A, LDA, TAU, WORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, K, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE UNG2R(M, [N], [K], A, [LDA], TAU, [WORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, K, LDA, INFO

SUBROUTINE UNG2R_64(M, [N], [K], A, [LDA], TAU, [WORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, K, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cung2r(int m, int n, int k, complex *a, int  lda,  com-
```

```
          plex *tau, int *info);

void cung2r_64(long m, long n, long k, complex *a, long lda,
          complex *tau, long *info);
```

PURPOSE

cung2r R generates an m by n complex matrix Q with orthonormal columns, which is defined as the first n columns of a product of k elementary reflectors of order m

      Q = H(1) H(2) . . . H(k)

as returned by CGEQRF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. M >= N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. N >= K >= 0.

A (input/output)
On entry, the i-th column must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGEQRF in the first k columns of its array argument A. On exit, the m by n matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGEQRF.

WORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
cungbr - generate one of the complex unitary matrices  Q  or
P**H  determined  by CGEBRD when reducing a complex matrix A
to bidiagonal form
```

SYNOPSIS

```
SUBROUTINE CUNGBR(VECT, M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

CHARACTER * 1 VECT
COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, K, LDA, LWORK, INFO

SUBROUTINE CUNGBR_64(VECT, M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

CHARACTER * 1 VECT
COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, K, LDA, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNGBR(VECT, M, [N], K, A, [LDA], TAU, [WORK], [LWORK],
       [INFO])

CHARACTER(LEN=1) :: VECT
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, K, LDA, LWORK, INFO

SUBROUTINE UNGBR_64(VECT, M, [N], K, A, [LDA], TAU, [WORK], [LWORK],
       [INFO])

CHARACTER(LEN=1) :: VECT
COMPLEX, DIMENSION(:) :: TAU, WORK
```

```
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, K, LDA, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cungbr(char vect, int m, int n, int k, complex *a,  int
          lda, complex *tau, int *info);

void cungbr_64(char vect, long m, long n,  long  k,  complex
          *a, long lda, complex *tau, long *info);
```

PURPOSE

```
cungbr generates one of the complex unitary  matrices  Q  or
P**H  determined  by CGEBRD when reducing a complex matrix A
to bidiagonal form: A = Q *  B  *  P**H.   Q  and  P**H  are
defined  as  products  of elementary reflectors H(i) or G(i)
respectively.

If VECT = 'Q', A is assumed to have been an  M-by-K  matrix,
and Q is of order M:
if m >= k, Q = H(1) H(2) . . . H(k) and CUNGBR  returns  the
first n columns of Q, where m >= n >= k;
if m < k, Q = H(1) H(2) . . . H(m-1) and CUNGBR returns Q as
an M-by-M matrix.

If VECT = 'P', A is assumed to have been  a  K-by-N  matrix,
and P**H is of order N:
if k < n, P**H = G(k) . . . G(2) G(1) and CUNGBR returns the
first m rows of P**H, where n >= m >= k;
if k >= n, P**H = G(n-1) . . . G(2) G(1) and CUNGBR  returns
P**H as an N-by-N matrix.
```

ARGUMENTS

```
VECT (input)
          Specifies whether the matrix Q or the matrix  P**H
          is  required,  as  defined  in  the transformation
          applied by CGEBRD:
          = 'Q':  generate Q;
          = 'P':  generate P**H.

M (input) The number of rows of the matrix Q or P**H  to  be
          returned.  M >= 0.

N (input) The number of columns of the matrix Q or  P**H  to
```

be returned. N >= 0. If VECT = 'Q', M >= N >= min(M,K); if VECT = 'P', N >= M >= min(N,K).

K (input) If VECT = 'Q', the number of columns in the original M-by-K matrix reduced by CGEBRD. If VECT = 'P', the number of rows in the original K-by-N matrix reduced by CGEBRD. K >= 0.

A (input/output)
On entry, the vectors which define the elementary reflectors, as returned by CGEBRD. On exit, the M-by-N matrix Q or P**H.

LDA (input)
The leading dimension of the array A. LDA >= M.

TAU (input)
(min(M,K)) if VECT = 'Q' (min(N,K)) if VECT = 'P' TAU(i) must contain the scalar factor of the elementary reflector H(i) or G(i), which determines Q or P**H, as returned by CGEBRD in its array argument TAUQ or TAUP.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,min(M,N)). For optimum performance LWORK >= min(M,N)*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

cunghr - generate a complex unitary matrix Q which is defined as the product of IHI-ILO elementary reflectors of order N, as returned by CGEHRD

SYNOPSIS

```
SUBROUTINE CUNGHR(N, ILO, IHI, A, LDA, TAU, WORK, LWORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER N, ILO, IHI, LDA, LWORK, INFO

SUBROUTINE CUNGHR_64(N, ILO, IHI, A, LDA, TAU, WORK, LWORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 N, ILO, IHI, LDA, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNGHR([N], ILO, IHI, A, [LDA], TAU, [WORK], [LWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, ILO, IHI, LDA, LWORK, INFO

SUBROUTINE UNGHR_64([N], ILO, IHI, A, [LDA], TAU, [WORK], [LWORK],
       [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, ILO, IHI, LDA, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void cunghr(int n, int ilo, int ihi, complex  *a,  int  lda,
          complex *tau, int *info);

void cunghr_64(long n, long ilo, long ihi, complex *a,  long
          lda, complex *tau, long *info);
```

PURPOSE

cunghr generates a complex unitary matrix Q which is defined as the product of IHI-ILO elementary reflectors of order N, as returned by CGEHRD:

Q = H(ilo) H(ilo+1) . . . H(ihi-1).

ARGUMENTS

N (input) The order of the matrix Q. N >= 0.

ILO (input)
ILO and IHI must have the same values as in the previous call of CGEHRD. Q is equal to the unit matrix except in the submatrix Q(ilo+1:ihi,ilo+1:ihi). 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

IHI (input)
See the description of IHI.

A (input/output)
On entry, the vectors which define the elementary reflectors, as returned by CGEHRD. On exit, the N-by-N unitary matrix Q.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGEHRD.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)

The dimension of the array WORK. LWORK >= IHI-ILO. For optimum performance LWORK >= (IHI-ILO)*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

cungl2 - generate an m-by-n complex matrix Q with orthonormal rows,

SYNOPSIS

```
SUBROUTINE CUNGL2(M, N, K, A, LDA, TAU, WORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, K, LDA, INFO

SUBROUTINE CUNGL2_64(M, N, K, A, LDA, TAU, WORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, K, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE UNGL2([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, K, LDA, INFO

SUBROUTINE UNGL2_64([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, K, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cungl2(int m, int n, int k, complex *a, int  lda,  com-
```

```
          plex *tau, int *info);

void cungl2_64(long m, long n, long k, complex *a, long lda,
          complex *tau, long *info);
```

PURPOSE

cungl2 generates an m-by-n complex matrix Q with orthonormal rows, which is defined as the first m rows of a product of k elementary reflectors of order n

```
      Q  =  H(k)' . . . H(2)' H(1)'
```

as returned by CGELQF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. N >= M.

K (input) The number of elementary reflectors whose product defines the matrix Q. M >= K >= 0.

A (input/output)
On entry, the i-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGELQF in the first k rows of its array argument A. On exit, the m by n matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGELQF.

WORK (workspace)
dimension(M)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
cunglq - generate an M-by-N complex matrix Q with  orthonor-
mal rows,
```

SYNOPSIS

```
SUBROUTINE CUNGLQ(M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, K, LDA, LWORK, INFO

SUBROUTINE CUNGLQ_64(M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, K, LDA, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNGLQ(M, [N], [K], A, [LDA], TAU, [WORK], [LWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, K, LDA, LWORK, INFO

SUBROUTINE UNGLQ_64(M, [N], [K], A, [LDA], TAU, [WORK], [LWORK],
       [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, K, LDA, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void cunglq(int m, int n, int k, complex *a, int  lda,  com-
          plex *tau, int *info);

void cunglq_64(long m, long n, long k, complex *a, long lda,
          complex *tau, long *info);
```

PURPOSE

cunglq generates an M-by-N complex matrix Q with orthonormal rows, which is defined as the first M rows of a product of K elementary reflectors of order N

      Q = H(k)' . . . H(2)' H(1)'

as returned by CGELQF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. N >= M.

K (input) The number of elementary reflectors whose product defines the matrix Q. M >= K >= 0.

A (input/output)
On entry, the i-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGELQF in the first k rows of its array argument A. On exit, the M-by-N matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGELQF.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,M). For optimum performance LWORK >= M*NB,

where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit;
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
cungql - generate an M-by-N complex matrix Q with  orthonor-
mal columns,
```

SYNOPSIS

```
SUBROUTINE CUNGQL(M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, K, LDA, LWORK, INFO

SUBROUTINE CUNGQL_64(M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, K, LDA, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNGQL(M, [N], [K], A, [LDA], TAU, [WORK], [LWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, K, LDA, LWORK, INFO

SUBROUTINE UNGQL_64(M, [N], [K], A, [LDA], TAU, [WORK], [LWORK],
       [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, K, LDA, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void cungql(int m, int n, int k, complex *a, int  lda,  com-
          plex *tau, int *info);

void cungql_64(long m, long n, long k, complex *a, long lda,
          complex *tau, long *info);
```

PURPOSE

cungql generates an M-by-N complex matrix Q with orthonormal columns, which is defined as the last N columns of a product of K elementary reflectors of order M

      Q = H(k) . . . H(2) H(1)

as returned by CGEQLF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. M >= N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. N >= K >= 0.

A (input/output)
On entry, the (n-k+i)-th column must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGEQLF in the last k columns of its array argument A. On exit, the M-by-N matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGEQLF.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >=

```
          max(1,N).   For optimum performance LWORK >= N*NB,
          where NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument has an ille-
          gal value
```

Contents

NAME

```
cungqr - generate an M-by-N complex matrix Q with  orthonor-
mal columns,
```

SYNOPSIS

```
SUBROUTINE CUNGQR(M, N, K, A, LDA, TAU, WORKIN, LWORKIN, INFO)

COMPLEX A(LDA,*), TAU(*), WORKIN(*)
INTEGER M, N, K, LDA, LWORKIN, INFO

SUBROUTINE CUNGQR_64(M, N, K, A, LDA, TAU, WORKIN, LWORKIN, INFO)

COMPLEX A(LDA,*), TAU(*), WORKIN(*)
INTEGER*8 M, N, K, LDA, LWORKIN, INFO
```

F95 INTERFACE

```
SUBROUTINE UNGQR(M, [N], [K], A, [LDA], TAU, [WORKIN], [LWORKIN],
       [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORKIN
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, K, LDA, LWORKIN, INFO

SUBROUTINE UNGQR_64(M, [N], [K], A, [LDA], TAU, [WORKIN], [LWORKIN],
       [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORKIN
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, K, LDA, LWORKIN, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void cungqr(int m, int n, int k, complex *a, int  lda,  com-
          plex *tau, int *info);

void cungqr_64(long m, long n, long k, complex *a, long lda,
          complex *tau, long *info);
```

PURPOSE

cungqr generates an M-by-N complex matrix Q with orthonormal columns, which is defined as the first N columns of a product of K elementary reflectors of order M

      Q = H(1) H(2) . . . H(k)

as returned by CGEQRF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. M >= N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. N >= K >= 0.

A (input/output)
On entry, the i-th column must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGEQRF in the first k columns of its array argument A. On exit, the M-by-N matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGEQRF.

WORKIN (workspace)
On exit, if INFO = 0, WORKIN(1) returns the optimal LWORKIN.

LWORKIN (input)
The dimension of the array WORKIN. LWORKIN >=

max(1,N). For optimum performance LWORKIN >= N*NB, where NB is the optimal blocksize.

If LWORKIN = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORKIN array, returns this value as the first entry of the WORKIN array, and no error message related to LWORKIN is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
cungr2 - generate an m by n complex matrix Q with  orthonor-
mal rows,
```

SYNOPSIS

```
SUBROUTINE CUNGR2(M, N, K, A, LDA, TAU, WORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, K, LDA, INFO

SUBROUTINE CUNGR2_64(M, N, K, A, LDA, TAU, WORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, K, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE UNGR2([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, K, LDA, INFO

SUBROUTINE UNGR2_64([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, K, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cungr2(int m, int n, int k, complex *a, int  lda,  com-
```

```
          plex *tau, int *info);

void cungr2_64(long m, long n, long k, complex *a, long lda,
          complex *tau, long *info);
```

PURPOSE

cungr2 generates an m by n complex matrix Q with orthonormal rows, which is defined as the last m rows of a product of k elementary reflectors of order n

```
      Q  =  H(1)' H(2)' . . . H(k)'
```

as returned by CGERQF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. N >= M.

K (input) The number of elementary reflectors whose product defines the matrix Q. M >= K >= 0.

A (input/output)
On entry, the (m-k+i)-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGERQF in the last k rows of its array argument A. On exit, the m-by-n matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGERQF.

WORK (workspace)
dimension(M)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
cungrq - generate an M-by-N complex matrix Q with  orthonor-
mal rows,
```

SYNOPSIS

```
SUBROUTINE CUNGRQ(M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, K, LDA, LWORK, INFO

SUBROUTINE CUNGRQ_64(M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, K, LDA, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNGRQ(M, [N], [K], A, [LDA], TAU, [WORK], [LWORK], [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: M, N, K, LDA, LWORK, INFO

SUBROUTINE UNGRQ_64(M, [N], [K], A, [LDA], TAU, [WORK], [LWORK],
       [INFO])

COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: M, N, K, LDA, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void cungrq(int m, int n, int k, complex *a, int  lda,  com-
          plex *tau, int *info);

void cungrq_64(long m, long n, long k, complex *a, long lda,
          complex *tau, long *info);
```

PURPOSE

cungrq generates an M-by-N complex matrix Q with orthonormal rows, which is defined as the last M rows of a product of K elementary reflectors of order N

      Q = H(1)' H(2)' . . . H(k)'

as returned by CGERQF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. N >= M.

K (input) The number of elementary reflectors whose product defines the matrix Q. M >= K >= 0.

A (input/output)
On entry, the (m-k+i)-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGERQF in the last k rows of its array argument A. On exit, the M-by-N matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGERQF.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,M). For optimum performance LWORK >= M*NB,

where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
cungtr - generate  a  complex  unitary  matrix  Q  which  is
defined as the product of n-1 elementary reflectors of order
N, as returned by CHETRD
```

SYNOPSIS

```
SUBROUTINE CUNGTR(UPLO, N, A, LDA, TAU, WORK, LWORK, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER N, LDA, LWORK, INFO

SUBROUTINE CUNGTR_64(UPLO, N, A, LDA, TAU, WORK, LWORK, INFO)

CHARACTER * 1 UPLO
COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 N, LDA, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNGTR(UPLO, [N], A, [LDA], TAU, [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER :: N, LDA, LWORK, INFO

SUBROUTINE UNGTR_64(UPLO, [N], A, [LDA], TAU, [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cungtr(char uplo, int n, complex *a, int  lda,  complex
          *tau, int *info);

void cungtr_64(char uplo, long n, complex *a, long lda, com-
          plex *tau, long *info);
```

PURPOSE

cungtr generates a complex unitary matrix Q which is defined as the product of n-1 elementary reflectors of order N, as returned by CHETRD:
if UPLO = 'U', Q = H(n-1) . . . H(2) H(1),

if UPLO = 'L', Q = H(1) H(2) . . . H(n-1).

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A contains elementary reflectors from CHETRD; = 'L': Lower triangle of A contains elementary reflectors from CHETRD.

N (input) The order of the matrix Q. N >= 0.

A (input/output)
On entry, the vectors which define the elementary reflectors, as returned by CHETRD. On exit, the N-by-N unitary matrix Q.

LDA (input)
The leading dimension of the array A. LDA >= N.

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CHETRD.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= N-1. For optimum performance LWORK >= (N-1)*NB, where

NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
cunmbr - VECT = 'Q', CUNMBR overwrites the  general  complex
M-by-N matrix C with  SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE CUNMBR(VECT, SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 VECT, SIDE, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE CUNMBR_64(VECT, SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 VECT, SIDE, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNMBR(VECT, SIDE, [TRANS], [M], [N], K, A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: VECT, SIDE, TRANS
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE UNMBR_64(VECT, SIDE, [TRANS], [M], [N], K, A, [LDA], TAU,
       C, [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: VECT, SIDE, TRANS
```

```
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cunmbr(char vect, char side, char trans, int m, int  n,
          int  k, complex *a, int lda, complex *tau, complex
          *c, int ldc, int *info);

void cunmbr_64(char vect, char side,  char  trans,  long  m,
          long  n,  long  k,  complex  *a, long lda, complex
          *tau, complex *c, long ldc, long *info);
```

PURPOSE

```
cunmbr VECT = 'Q', CUNMBR overwrites the general complex  M-
by-N matrix C with
                SIDE = 'L'      SIDE  =  'R'  TRANS  =  'N':
Q  *  C           C * Q TRANS = 'C':      Q**H * C       C *
Q**H

If VECT = 'P', CUNMBR overwrites the general complex  M-by-N
matrix C with
                SIDE = 'L'     SIDE = 'R'
TRANS = 'N':      P * C          C * P
TRANS = 'C':      P**H * C       C * P**H

Here Q and P**H  are  the  unitary  matrices  determined  by
CGEBRD  when reducing a complex matrix A to bidiagonal form:
A = Q * B * P**H. Q and P**H are defined as products of ele-
mentary reflectors H(i) and G(i) respectively.

Let nq = m if SIDE = 'L' and nq = n if SIDE = 'R'.  Thus  nq
is  the  order  of  the  unitary  matrix  Q  or P**H that is
applied.

If VECT = 'Q', A is assumed to have been an NQ-by-K  matrix:
if nq >= k, Q = H(1) H(2) . . . H(k);
if nq < k, Q = H(1) H(2) . . . H(nq-1).

If VECT = 'P', A is assumed to have been a  K-by-NQ  matrix:
if k < nq, P = G(1) G(2) . . . G(k);
if k >= nq, P = G(1) G(2) . . . G(nq-1).
```

ARGUMENTS

```
VECT (input)
          = 'Q': apply Q or Q**H;
          = 'P': apply P or P**H.

SIDE (input)
          = 'L': apply Q, Q**H, P or P**H from the Left;
          = 'R': apply Q, Q**H, P or P**H from the Right.

TRANS (input)
          = 'N':  No transpose, apply Q or P;
          = 'C':  Conjugate transpose, apply Q**H or P**H.

          TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.
N (input) The number of columns of the matrix C. N >= 0.

K (input) If VECT = 'Q', the number of columns in the origi-
          nal  matrix reduced by CGEBRD.  If VECT = 'P', the
          number of rows in the original matrix  reduced  by
          CGEBRD.  K >= 0.

A (input) (LDA,min(nq,K)) if VECT = 'Q'  (LDA,nq)         if
          VECT = 'P' The vectors which define the elementary
          reflectors H(i) and G(i), whose products determine
          the matrices Q and P, as returned by CGEBRD.

LDA (input)
          The leading dimension of the array A.  If  VECT  =
          'Q',  LDA  >=  max(1,nq);  if  VECT  = 'P', LDA >=
          max(1,min(nq,K)).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary  reflector H(i) or G(i) which determines Q
          or P, as returned by CGEBRD in the array  argument
          TAUQ or TAUP.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten  by  Q*C or Q**H*C or C*Q**H or C*Q or
          P*C or P**H*C or C*P or C*P**H.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
```

LWORK.

LWORK (input)
The dimension of the array WORK. If SIDE = 'L', LWORK >= max(1,N); if SIDE = 'R', LWORK >= max(1,M). For optimum performance LWORK >= N*NB if SIDE = 'L', and LWORK >= M*NB if SIDE = 'R', where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
cunmhr - overwrite the general complex M-by-N matrix C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE CUNMHR(SIDE, TRANS, M, N, ILO, IHI, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, ILO, IHI, LDA, LDC, LWORK, INFO

SUBROUTINE CUNMHR_64(SIDE, TRANS, M, N, ILO, IHI, A, LDA, TAU, C,
      LDC, WORK, LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, ILO, IHI, LDA, LDC, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNMHR(SIDE, [TRANS], [M], [N], ILO, IHI, A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER :: M, N, ILO, IHI, LDA, LDC, LWORK, INFO

SUBROUTINE UNMHR_64(SIDE, [TRANS], [M], [N], ILO, IHI, A, [LDA], TAU,
       C, [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, ILO, IHI, LDA, LDC, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cunmhr(char side, char trans, int m, int  n,  int  ilo,
          int  ihi,  complex *a, int lda, complex *tau, com-
          plex *c, int ldc, int *info);

void cunmhr_64(char side, char trans, long m, long  n,  long
          ilo, long ihi, complex *a, long lda, complex *tau,
          complex *c, long ldc, long *info);
```

PURPOSE

cunmhr overwrites the general complex M-by-N matrix C with
TRANS = 'C':   Q**H * C   C * Q**H

where Q is a complex unitary matrix of order nq, with nq = m if SIDE = 'L' and nq = n if SIDE = 'R'. Q is defined as the product of IHI-ILO elementary reflectors, as returned by CGEHRD:

Q = H(ilo) H(ilo+1) . . . H(ihi-1).

ARGUMENTS

SIDE (input)
= 'L': apply Q or Q**H from the Left;
= 'R': apply Q or Q**H from the Right.

TRANS (input)
= 'N': apply Q (No transpose)
= 'C': apply Q**H (Conjugate transpose)

TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

ILO (input)
ILO and IHI must have the same values as in the previous call of CGEHRD. Q is equal to the unit matrix except in the submatrix

```
          Q(ilo+1:ihi,ilo+1:ihi).   If SIDE = 'L', then 1 <=
          ILO <= IHI <= M, if M > 0, and ILO = 1 and  IHI  =
          0,  if  M = 0; if SIDE = 'R', then 1 <= ILO <= IHI
          <= N, if N > 0, and ILO = 1 and IHI = 0, if N = 0.

IHI (input)
          See the description of ILO.

A (input) (LDA,M) if SIDE = 'L' (LDA,N) if SIDE  =  'R'  The
          vectors which define the elementary reflectors, as
          returned by CGEHRD.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,M)  if SIDE = 'L'; LDA >= max(1,N) if SIDE =
          'R'.

TAU (input)
          (M-1) if SIDE = 'L' (N-1) if  SIDE  =  'R'  TAU(i)
          must  contain  the scalar factor of the elementary
          reflector H(i), as returned by CGEHRD.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten by Q*C or Q**H*C or C*Q**H or C*Q.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK.  If SIDE  =  'L',
          LWORK  >=  max(1,N);  if  SIDE  =  'R',  LWORK  >=
          max(1,M).  For optimum performance LWORK  >=  N*NB
          if  SIDE  =  'L', and LWORK >= M*NB if SIDE = 'R',
          where NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
```

gal value

Contents

NAME

```
cunml2 - overwrite the general complex m-by-n matrix C  with
Q  *  C  if SIDE = 'L' and TRANS = 'N', or   Q'* C if SIDE =
'L' and TRANS = 'C', or   C * Q if SIDE = 'R'  and  TRANS  =
'N', or   C * Q' if SIDE = 'R' and TRANS = 'C',
```

SYNOPSIS

```
SUBROUTINE CUNML2(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      INFO)

CHARACTER * 1 SIDE, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, K, LDA, LDC, INFO

SUBROUTINE CUNML2_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      INFO)

CHARACTER * 1 SIDE, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, K, LDA, LDC, INFO
```

F95 INTERFACE

```
SUBROUTINE UNML2(SIDE, TRANS, [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER :: M, N, K, LDA, LDC, INFO

SUBROUTINE UNML2_64(SIDE, TRANS, [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, K, LDA, LDC, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cunml2(char side, char trans, int m, int n, int k, com-
          plex  *a,  int  lda, complex *tau, complex *c, int
          ldc, int *info);

void cunml2_64(char side, char trans, long m, long  n,  long
          k, complex *a, long lda, complex *tau, complex *c,
          long ldc, long *info);
```

PURPOSE

cunml2 overwrites the general complex m-by-n matrix C with

where Q is a complex unitary matrix defined as the product of k elementary reflectors

      Q = H(k)' . . . H(2)' H(1)'

as returned by CGELQF. Q is of order m if SIDE = 'L' and of order n if SIDE = 'R'.

ARGUMENTS

SIDE (input)
          = 'L': apply Q or Q' from the Left
          = 'R': apply Q or Q' from the Right

TRANS (input)
          = 'N': apply Q (No transpose)
          = 'C': apply Q' (Conjugate transpose)

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. If SIDE = 'L', M >= K >= 0; if SIDE = 'R', N >= K >= 0.

A (input) (LDA,M) if SIDE = 'L', (LDA,N) if SIDE = 'R' The i-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGELQF in the first k rows of its array argument A. A is modified by the routine but restored on exit.

LDA (input)
The leading dimension of the array A. LDA >= max(1,K).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGELQF.

C (input/output)
On entry, the m-by-n matrix C. On exit, C is overwritten by Q*C or Q'*C or C*Q' or C*Q.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
(N) if SIDE = 'L', (M) if SIDE = 'R'

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
cunmlq - overwrite the general complex M-by-N matrix C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE CUNMLQ(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE CUNMLQ_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNMLQ(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE UNMLQ_64(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cunmlq(char side, char trans, int m, int n, int k, com-
          plex  *a,  int  lda, complex *tau, complex *c, int
          ldc, int *info);

void cunmlq_64(char side, char trans, long m, long  n,  long
          k, complex *a, long lda, complex *tau, complex *c,
          long ldc, long *info);
```

PURPOSE

cunmlq overwrites the general complex M-by-N matrix C with
TRANS = 'C':   Q**H * C   C * Q**H

where Q is a complex unitary matrix defined as the product
of k elementary reflectors

      Q = H(k)' . . . H(2)' H(1)'

as returned by CGELQF. Q is of order M if SIDE = 'L' and of
order N if SIDE = 'R'.

ARGUMENTS

SIDE (input)
          = 'L': apply Q or Q**H from the Left;
          = 'R': apply Q or Q**H from the Right.

TRANS (input)
          = 'N': No transpose, apply Q;
          = 'C': Conjugate transpose, apply Q**H.

          TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose product
          defines the matrix Q. If SIDE = 'L', M >= K >= 0;
          if SIDE = 'R', N >= K >= 0.

```
A (input) (LDA,M) if SIDE = 'L', (LDA,N) if SIDE =  'R'  The
          i-th row must contain the vector which defines the
          elementary reflector H(i), for i =  1,2,...,k,  as
          returned  by  CGELQF  in  the  first k rows of its
          array argument A.  A is modified  by  the  routine
          but restored on exit.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,K).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i), as returned by CGELQF.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten by Q*C or Q**H*C or C*Q**H or C*Q.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK.  If SIDE  =  'L',
          LWORK  >=  max(1,N);  if  SIDE  =  'R',  LWORK  >=
          max(1,M).  For optimum performance LWORK  >=  N*NB
          if  SIDE  'L',  and  LWORK  >= M*NB if SIDE = 'R',
          where NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
cunmql - overwrite the general complex M-by-N matrix C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE CUNMQL(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE CUNMQL_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNMQL(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE UNMQL_64(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cunmql(char side, char trans, int m, int n, int k, com-
          plex  *a,  int  lda, complex *tau, complex *c, int
          ldc, int *info);

void cunmql_64(char side, char trans, long m, long  n,  long
          k, complex *a, long lda, complex *tau, complex *c,
          long ldc, long *info);
```

PURPOSE

```
cunmql overwrites the general complex M-by-N matrix  C  with
TRANS = 'C':      Q**H * C       C * Q**H

where Q is a complex unitary matrix defined as  the  product
of k elementary reflectors

      Q = H(k) . . . H(2) H(1)

as returned by CGEQLF. Q is of order M if SIDE = 'L' and  of
order N if SIDE = 'R'.
```

ARGUMENTS

```
SIDE (input)
          = 'L': apply Q or Q**H from the Left;
          = 'R': apply Q or Q**H from the Right.

TRANS (input)
          = 'N':  No transpose, apply Q;
          = 'C':  Transpose, apply Q**H.

          TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose  product
          defines the matrix Q.  If SIDE = 'L', M >= K >= 0;
          if SIDE = 'R', N >= K >= 0.
```

A (input) The i-th column must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGEQLF in the last k columns of its array argument A. A is modified by the routine but restored on exit.

LDA (input)
The leading dimension of the array A. If SIDE = 'L', LDA >= max(1,M); if SIDE = 'R', LDA >= max(1,N).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGEQLF.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by Q*C or Q**H*C or C*Q**H or C*Q.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If SIDE = 'L', LWORK >= max(1,N); if SIDE = 'R', LWORK >= max(1,M). For optimum performance LWORK >= N*NB if SIDE = 'L', and LWORK >= M*NB if SIDE = 'R', where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
cunmqr - overwrite the general complex M-by-N matrix C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE CUNMQR(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE CUNMQR_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNMQR(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE UNMQR_64(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cunmqr(char side, char trans, int m, int n, int k, com-
          plex  *a,  int  lda, complex *tau, complex *c, int
          ldc, int *info);

void cunmqr_64(char side, char trans, long m, long  n,  long
          k, complex *a, long lda, complex *tau, complex *c,
          long ldc, long *info);
```

PURPOSE

```
cunmqr overwrites the general complex M-by-N matrix  C  with
TRANS = 'C':      Q**H * C       C * Q**H

where Q is a complex unitary matrix defined as  the  product
of k elementary reflectors

      Q = H(1) H(2) . . . H(k)

as returned by CGEQRF. Q is of order M if SIDE = 'L' and  of
order N if SIDE = 'R'.
```

ARGUMENTS

```
SIDE (input)
          = 'L': apply Q or Q**H from the Left;
          = 'R': apply Q or Q**H from the Right.

TRANS (input)
          = 'N':  No transpose, apply Q;
          = 'C':  Conjugate transpose, apply Q**H.

          TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose  product
          defines the matrix Q.  If SIDE = 'L', M >= K >= 0;
          if SIDE = 'R', N >= K >= 0.
```

```
A (input) The i-th column  must  contain  the  vector  which
          defines  the  elementary  reflector  H(i), for i =
          1,2,...,k, as returned by CGEQRF in  the  first  k
          columns of its array argument A.  A is modified by
          the routine but restored on exit.

LDA (input)
          The leading dimension of the array A.  If  SIDE  =
          'L',  LDA  >=  max(1,M);  if  SIDE  =  'R', LDA >=
          max(1,N).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i), as returned by CGEQRF.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten by Q*C or Q**H*C or C*Q**H or C*Q.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK.  If SIDE  =  'L',
          LWORK  >=  max(1,N);  if  SIDE  =  'R',  LWORK  >=
          max(1,M).  For optimum performance LWORK  >=  N*NB
          if  SIDE  =  'L', and LWORK >= M*NB if SIDE = 'R',
          where NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
cunmr2 - overwrite the general complex m-by-n matrix C  with
Q  *  C  if SIDE = 'L' and TRANS = 'N', or   Q'* C if SIDE =
'L' and TRANS = 'C', or   C * Q if SIDE = 'R'  and  TRANS  =
'N', or   C * Q' if SIDE = 'R' and TRANS = 'C',
```

SYNOPSIS

```
SUBROUTINE CUNMR2(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      INFO)

CHARACTER * 1 SIDE, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, K, LDA, LDC, INFO

SUBROUTINE CUNMR2_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      INFO)

CHARACTER * 1 SIDE, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, K, LDA, LDC, INFO
```

F95 INTERFACE

```
SUBROUTINE UNMR2(SIDE, TRANS, [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER :: M, N, K, LDA, LDC, INFO

SUBROUTINE UNMR2_64(SIDE, TRANS, [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, K, LDA, LDC, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cunmr2(char side, char trans, int m, int n, int k, com-
          plex  *a,  int  lda, complex *tau, complex *c, int
          ldc, int *info);

void cunmr2_64(char side, char trans, long m, long  n,  long
          k, complex *a, long lda, complex *tau, complex *c,
          long ldc, long *info);
```

PURPOSE

cunmr2 overwrites the general complex m-by-n matrix C with

where Q is a complex unitary matrix defined as the product of k elementary reflectors

Q = H(1)' H(2)' . . . H(k)'

as returned by CGERQF. Q is of order m if SIDE = 'L' and of order n if SIDE = 'R'.

ARGUMENTS

SIDE (input)
= 'L': apply Q or Q' from the Left
= 'R': apply Q or Q' from the Right

TRANS (input)
= 'N': apply Q (No transpose)
= 'C': apply Q' (Conjugate transpose)

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. If SIDE = 'L', M >= K >= 0; if SIDE = 'R', N >= K >= 0.

```
A (input) (LDA,M) if SIDE = 'L', (LDA,N) if SIDE =  'R'  The
          i-th row must contain the vector which defines the
          elementary reflector H(i), for i =  1,2,...,k,  as
          returned by CGERQF in the last k rows of its array
          argument A.  A is  modified  by  the  routine  but
          restored on exit.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,K).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i), as returned by CGERQF.

C (input/output)
          On entry, the m-by-n matrix  C.   On  exit,  C  is
          overwritten by Q*C or Q'*C or C*Q' or C*Q.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          (N) if SIDE = 'L', (M) if SIDE = 'R'

INFO (output)
          = 0: successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
          gal value
```

Contents

NAME

```
cunmrq - overwrite the general complex M-by-N matrix C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE CUNMRQ(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE CUNMRQ_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNMRQ(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE UNMRQ_64(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cunmrq(char side, char trans, int m, int n, int k, com-
          plex  *a,  int  lda, complex *tau, complex *c, int
          ldc, int *info);

void cunmrq_64(char side, char trans, long m, long  n,  long
          k, complex *a, long lda, complex *tau, complex *c,
          long ldc, long *info);
```

PURPOSE

cunmrq overwrites the general complex M-by-N matrix C with
TRANS = 'C':   Q**H * C   C * Q**H

where Q is a complex unitary matrix defined as the product
of k elementary reflectors

      Q = H(1)' H(2)' . . . H(k)'

as returned by CGERQF. Q is of order M if SIDE = 'L' and of
order N if SIDE = 'R'.

ARGUMENTS

SIDE (input)
= 'L': apply Q or Q**H from the Left;
= 'R': apply Q or Q**H from the Right.

TRANS (input)
= 'N': No transpose, apply Q;
= 'C': Transpose, apply Q**H.

TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose product
defines the matrix Q. If SIDE = 'L', M >= K >= 0;
if SIDE = 'R', N >= K >= 0.

```
A (input) (LDA,M) if SIDE = 'L', (LDA,N) if SIDE =  'R'  The
          i-th row must contain the vector which defines the
          elementary reflector H(i), for i =  1,2,...,k,  as
          returned by CGERQF in the last k rows of its array
          argument A.  A is  modified  by  the  routine  but
          restored on exit.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,K).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i), as returned by CGERQF.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten by Q*C or Q**H*C or C*Q**H or C*Q.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK.  If SIDE  =  'L',
          LWORK  >=  max(1,N);  if  SIDE  =  'R',  LWORK  >=
          max(1,M).  For optimum performance LWORK  >=  N*NB
          if  SIDE  =  'L', and LWORK >= M*NB if SIDE = 'R',
          where NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
cunmrz - overwrite the general complex M-by-N matrix C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE CUNMRZ(SIDE, TRANS, M, N, K, L, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, K, L, LDA, LDC, LWORK, INFO

SUBROUTINE CUNMRZ_64(SIDE, TRANS, M, N, K, L, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, K, L, LDA, LDC, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE CUNMRZ(SIDE, TRANS, M, N, K, L, A, LDA, TAU, C, LDC,
       WORK, LWORK, INFO)

CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER :: M, N, K, L, LDA, LDC, LWORK, INFO

SUBROUTINE CUNMRZ_64(SIDE, TRANS, M, N, K, L, A, LDA, TAU, C, LDC,
       WORK, LWORK, INFO)
```

```
CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, K, L, LDA, LDC, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cunmrz(char side, char trans, int m, int n, int k,  int
          l,  complex *a, int lda, complex *tau, complex *c,
          int ldc, int *info);

void cunmrz_64(char side, char trans, long m, long  n,  long
          k,  long  l,  complex  *a, long lda, complex *tau,
          complex *c, long ldc, long *info);
```

PURPOSE

```
cunmrz overwrites the general complex M-by-N matrix  C  with
TRANS = 'C':      Q**H * C       C * Q**H

where Q is a complex unitary matrix defined as  the  product
of k elementary reflectors

      Q = H(1) H(2) . . . H(k)

as returned by CTZRZF. Q is of order M if SIDE = 'L' and  of
order N if SIDE = 'R'.
```

ARGUMENTS

```
SIDE (input)
          = 'L': apply Q or Q**H from the Left;
          = 'R': apply Q or Q**H from the Right.

TRANS (input)
          = 'N':  No transpose, apply Q;
          = 'C':  Conjugate transpose, apply Q**H.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose  product
          defines the matrix Q.  If SIDE = 'L', M >= K >= 0;
          if SIDE = 'R', N >= K >= 0.
```

L (input) The number of columns of the matrix A containing the meaningful part of the Householder reflectors. If SIDE = 'L', M >= L >= 0, if SIDE = 'R', N >= L >= 0.

A (input) (LDA,M) if SIDE = 'L', (LDA,N) if SIDE = 'R' The i-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CTZRZF in the last k rows of its array argument A. A is modified by the routine but restored on exit.

LDA (input)
The leading dimension of the array A. LDA >= max(1,K).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CTZRZF.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by Q*C or Q**H*C or C*Q**H or C*Q.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If SIDE = 'L', LWORK >= max(1,N); if SIDE = 'R', LWORK >= max(1,M). For optimum performance LWORK >= N*NB if SIDE = 'L', and LWORK >= M*NB if SIDE = 'R', where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

```
Based on contributions by
  A. Petitet, Computer Science Dept., Univ. of Tenn.,  Knox-
ville, USA
```

Contents

NAME

```
cunmtr - overwrite the general complex M-by-N matrix C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE CUNMTR(SIDE, UPLO, TRANS, M, N, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, UPLO, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, LDA, LDC, LWORK, INFO

SUBROUTINE CUNMTR_64(SIDE, UPLO, TRANS, M, N, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 SIDE, UPLO, TRANS
COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, LDA, LDC, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNMTR(SIDE, UPLO, [TRANS], [M], [N], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANS
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER :: M, N, LDA, LDC, LWORK, INFO

SUBROUTINE UNMTR_64(SIDE, UPLO, [TRANS], [M], [N], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANS
```

```
COMPLEX, DIMENSION(:) :: TAU, WORK
COMPLEX, DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, LDA, LDC, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cunmtr(char side, char uplo, char trans, int m, int  n,
          complex *a, int lda, complex *tau, complex *c, int
          ldc, int *info);

void cunmtr_64(char side, char uplo,  char  trans,  long  m,
          long  n,  complex *a, long lda, complex *tau, com-
          plex *c, long ldc, long *info);
```

PURPOSE

cunmtr overwrites the general complex M-by-N matrix C with
TRANS = 'C':   Q**H * C   C * Q**H

where Q is a complex unitary matrix of order nq, with nq = m if SIDE = 'L' and nq = n if SIDE = 'R'. Q is defined as the product of nq-1 elementary reflectors, as returned by CHETRD:

if UPLO = 'U', Q = H(nq-1) . . . H(2) H(1);

if UPLO = 'L', Q = H(1) H(2) . . . H(nq-1).

ARGUMENTS

SIDE (input)
= 'L': apply Q or Q**H from the Left;
= 'R': apply Q or Q**H from the Right.

UPLO (input)
= 'U': Upper triangle of A contains elementary reflectors from CHETRD; = 'L': Lower triangle of A contains elementary reflectors from CHETRD.

TRANS (input)
= 'N': No transpose, apply Q;
= 'C': Conjugate transpose, apply Q**H.

TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

A (input) (LDA,M) if SIDE = 'L' (LDA,N) if SIDE = 'R' The vectors which define the elementary reflectors, as returned by CHETRD.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M) if SIDE = 'L'; LDA >= max(1,N) if SIDE = 'R'.

TAU (input)
(M-1) if SIDE = 'L' (N-1) if SIDE = 'R' TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CHETRD.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by Q*C or Q**H*C or C*Q**H or C*Q.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If SIDE = 'L', LWORK >= max(1,N); if SIDE = 'R', LWORK >= max(1,M). For optimum performance LWORK >= N*NB if SIDE = 'L', and LWORK >=M*NB if SIDE = 'R', where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

cupgtr - generate a complex unitary matrix Q which is defined as the product of n-1 elementary reflectors H(i) of order n, as returned by CHPTRD using packed storage

SYNOPSIS

```
SUBROUTINE CUPGTR(UPLO, N, AP, TAU, Q, LDQ, WORK, INFO)

CHARACTER * 1 UPLO
COMPLEX AP(*), TAU(*), Q(LDQ,*), WORK(*)
INTEGER N, LDQ, INFO

SUBROUTINE CUPGTR_64(UPLO, N, AP, TAU, Q, LDQ, WORK, INFO)

CHARACTER * 1 UPLO
COMPLEX AP(*), TAU(*), Q(LDQ,*), WORK(*)
INTEGER*8 N, LDQ, INFO
```

F95 INTERFACE

```
SUBROUTINE UPGTR(UPLO, [N], AP, TAU, Q, [LDQ], [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: AP, TAU, WORK
COMPLEX, DIMENSION(:,:) :: Q
INTEGER :: N, LDQ, INFO

SUBROUTINE UPGTR_64(UPLO, [N], AP, TAU, Q, [LDQ], [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX, DIMENSION(:) :: AP, TAU, WORK
COMPLEX, DIMENSION(:,:) :: Q
INTEGER(8) :: N, LDQ, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cupgtr(char uplo, int n,  complex  *ap,  complex  *tau,
          complex *q, int ldq, int *info);

void cupgtr_64(char uplo, long n, complex *ap, complex *tau,
          complex *q, long ldq, long *info);
```

PURPOSE

cupgtr generates a complex unitary matrix Q which is defined as the product of n-1 elementary reflectors H(i) of order n, as returned by CHPTRD using packed storage:
if UPLO = 'U', Q = H(n-1) . . . H(2) H(1),

if UPLO = 'L', Q = H(1) H(2) . . . H(n-1).

ARGUMENTS

UPLO (input)
= 'U': Upper triangular packed storage used in previous call to CHPTRD; = 'L': Lower triangular packed storage used in previous call to CHPTRD.

N (input) The order of the matrix Q. N >= 0.

AP (input)
The vectors which define the elementary reflectors, as returned by CHPTRD.

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CHPTRD.

Q (output)
The N-by-N unitary matrix Q.

LDQ (input)
The leading dimension of the array Q. LDQ >= max(1,N).

WORK (workspace)
dimension(N-1)

INFO (output)

= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-
gal value

Contents

NAME

```
cupmtr - overwrite the general complex M-by-N matrix C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE CUPMTR(SIDE, UPLO, TRANS, M, N, AP, TAU, C, LDC, WORK,
      INFO)

CHARACTER * 1 SIDE, UPLO, TRANS
COMPLEX AP(*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, LDC, INFO

SUBROUTINE CUPMTR_64(SIDE, UPLO, TRANS, M, N, AP, TAU, C, LDC, WORK,
      INFO)

CHARACTER * 1 SIDE, UPLO, TRANS
COMPLEX AP(*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, LDC, INFO
```

F95 INTERFACE

```
SUBROUTINE UPMTR(SIDE, UPLO, [TRANS], [M], [N], AP, TAU, C, [LDC],
       [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANS
COMPLEX, DIMENSION(:) :: AP, TAU, WORK
COMPLEX, DIMENSION(:,:) :: C
INTEGER :: M, N, LDC, INFO

SUBROUTINE UPMTR_64(SIDE, UPLO, [TRANS], [M], [N], AP, TAU, C, [LDC],
       [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANS
```

```
COMPLEX, DIMENSION(:) :: AP, TAU, WORK
COMPLEX, DIMENSION(:,:) :: C
INTEGER(8) :: M, N, LDC, INFO
```

C INTERFACE

```
#include <sunperf.h>

void cupmtr(char side, char uplo, char trans, int m, int  n,
          complex  *ap,  complex  *tau, complex *c, int ldc,
          int *info);

void cupmtr_64(char side, char uplo,  char  trans,  long  m,
          long  n,  complex  *ap,  complex *tau, complex *c,
          long ldc, long *info);
```

PURPOSE

```
cupmtr overwrites the general complex M-by-N matrix  C  with
TRANS = 'C':      Q**H * C       C * Q**H

where Q is a complex unitary matrix of order nq, with nq = m
if  SIDE = 'L' and nq = n if SIDE = 'R'. Q is defined as the
product of nq-1 elementary reflectors, as returned by CHPTRD
using packed storage:

if UPLO = 'U', Q = H(nq-1) . . . H(2) H(1);

if UPLO = 'L', Q = H(1) H(2) . . . H(nq-1).
```

ARGUMENTS

```
SIDE (input)
          = 'L': apply Q or Q**H from the Left;
          = 'R': apply Q or Q**H from the Right.

UPLO (input)
          = 'U': Upper triangular  packed  storage  used  in
          previous  call  to CHPTRD; = 'L': Lower triangular
          packed storage used in previous call to CHPTRD.

TRANS (input)
          = 'N':  No transpose, apply Q;
          = 'C':  Conjugate transpose, apply Q**H.

          TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.
```

N (input) The number of columns of the matrix C. N >= 0.

AP (input)
(M*(M+1)/2) if SIDE = 'L' (N*(N+1)/2) if SIDE = 'R' The vectors which define the elementary reflectors, as returned by CHPTRD. AP is modified by the routine but restored on exit.

TAU (input)
or (N-1) if SIDE = 'R' TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CHPTRD.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by Q*C or Q**H*C or C*Q**H or C*Q.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
(N) if SIDE = 'L' (M) if SIDE = 'R'

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

cvbrmm - variable block sparse row format matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE CVBRMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, KB, DESCRA(5), LDB, LDC, LWORK
 INTEGER    INDX(*), BINDX(*), RPNTR(MB+1), CPNTR(KB+1),
*           BPNTRB(MB), BPNTRE(MB)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(*), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CVBRMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), LDB, LDC, LWORK
 INTEGER*8  INDX(*), BINDX(*), RPNTR(MB+1), CPNTR(KB+1),
*           BPNTRB(MB), BPNTRE(MB)
 COMPLEX    ALPHA, BETA
 COMPLEX    VAL(*), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE VBRMM(TRANSA, MB, [N], KB, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, KB
 INTEGER, DIMENSION(:) ::  DESCRA, INDX, BINDX
 INTEGER, DIMENSION(:) ::  RPNTR, CPNTR, BPNTRB, BPNTRE
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL
```

```
      COMPLEX, DIMENSION(:, :) ::  B, C

      SUBROUTINE VBRMM_64(TRANSA, MB, [N], KB, ALPHA, DESCRA,
     *           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
     *           B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
      INTEGER*8    TRANSA, MB, KB
      INTEGER*8, DIMENSION(:) ::  DESCRA, INDX, BINDX
      INTEGER*8, DIMENSION(:) ::  RPNTR, CPNTR, BPNTRB, BPNTRE
      COMPLEX    ALPHA, BETA
      COMPLEX, DIMENSION(:) :: VAL
      COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
         C <- alpha op(A) B + beta C
where ALPHA and BETA are scalar, C and B are  matrices,
A is a matrix represented in variable block sparse row  format
and op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if the matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
```

```
                  DESCRA(2) upper/lower triangular indicator
                    1 : lower
                    2 : upper
                  DESCRA(3) main diagonal type
                    0 : non-unit
                    1 : unit
                  DESCRA(4) Array base  (NOT IMPLEMENTED)
                    0 : C/C++ compatible
                    1 : Fortran compatible
                  DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                    0 : unknown
                    1 : no repeated indices

VAL()             scalar array of length NNZ consisting of the block entries
                  of A where each block entry is a dense rectangular matrix
                  stored column by column.
                  NNZ is the total number of point entries in all nonzero
                  block  entries of a matrix A.
INDX()            integer array of length BNNZ+1 where BNNZ is the number of
                  block entries of a matrix A such that the I-th element of
                  INDX[] points to the location in VAL of the (1,1) element
                  of the I-th block entry.

BINDX()           integer array of length BNNZ consisting of the block
                  column indices of the block entries of A where BNNZ is
                  the number block entries of a matrix A.

RPNTR()           integer array of length MB+1 such that RPNTR(I)-RPNTR(1)+1
                  is the row index of the first point row in the I-th block
                  row.
                  RPNTR(MB+1) is set to M+RPNTR(1) where M is the number of
                  rows in matrix A.
                  Thus, the number of point rows in the I-th block row is
                  RPNTR(I+1)-RPNTR(I).

CPNTR()           integer array of length KB+1 such that CPNTR(J)-CPNTR(1)+1
                  is the column index of the first point column in the J-th
                  block column. CPNTR(KB+1) is set to K+CPNTR(1) where K is
                  the number of columns in matrix A.
                  Thus, the number of point columns in the J-th block column
                  is CPNTR(J+1)-CPNTR(J).

BPNTRB()          integer array of length MB such that BPNTRB(I)-BPNTRB(1)+1
                  points to location in BINDX of the first block entry of
                  the I-th block row of A.

BPNTRE()          integer array of length MB such that BPNTRE(I)-BPNTRB(1)
                  points to location in BINDX of the last block entry of
                  the I-th block row of A.
```

B()   rectangular array with first dimension LDB.

LDB   leading dimension of B

BETA   Scalar parameter

C()   rectangular array with first dimension LDC.

LDC   leading dimension of C

WORK()   scratch array of length LWORK. WORK is not referenced in the current version.

LWORK   length of WORK array. LWORK is not referenced in the current version.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. For a general matrix (*DESCRA*(1)=0), array CPNTR can be different from RPNTR. For all other matrix types, RPNTR must equal CPNTR and a single array can be passed for both arguments.

2. It is known that there exists another representation of the variable block sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of six array instead of the seven used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block row in the array BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of variable block sparse row format the following calling sequence should be used

```
  SUBROUTINE SVBRMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *           VAL, INDX, BINDX, RPNTR, CPNTR, IA, IA(2),
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

cvbrsm - variable block sparse row format triangular solve

SYNOPSIS

```
 SUBROUTINE CVBRSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), LDB, LDC, LWORK
 INTEGER    INDX(*), BINDX(*), RPNTR(MB+1), CPNTR(MB+1),
*           BPNTRB(MB), BPNTRE(MB)
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(*), VAL(*), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE CVBRSM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), LDB, LDC, LWORK
 INTEGER*8  INDX(*), BINDX(*), RPNTR(MB+1), CPNTR(MB+1),
*           BPNTRB(MB), BPNTRE(MB)
 COMPLEX    ALPHA, BETA
 COMPLEX    DV(*), VAL(*), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE VBRSM(TRANSA, MB, [N], UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, UNITD
 INTEGER, DIMENSION(:) ::  DESCRA, INDX, BINDX
 INTEGER, DIMENSION(:) ::  RPNTR, CPNTR, BPNTRB, BPNTRE
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL, DV
 COMPLEX, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE VBRSM_64(TRANSA, MB, [N], UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, UNITD
 INTEGER*8, DIMENSION(:) ::  DESCRA, INDX, BINDX
 INTEGER*8, DIMENSION(:) ::  RPNTR, CPNTR, BPNTRB, BPNTRE
 COMPLEX    ALPHA, BETA
 COMPLEX, DIMENSION(:) :: VAL, DV
 COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C
where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in variable block sparse row
format and  op( A )  is one  of

 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row block scaling)
                3 : Scale on right (column block scaling)

DV()          Array containing the block entries of the block
              diagonal matrix D.  The size of the J-th block is
              RPNTR(J+1)-RPNTR(J) and each block contains matrix
              entries stored column-major.  The total length of
              array DV is given by the formula:

              sum over J from 1 to MB:
```

```
                  ((RPNTR(J+1)-RPNTR(J))*(RPNTR(J+1)-RPNTR(J)))

ALPHA             Scalar parameter

DESCRA()          Descriptor argument.  Five element integer array
                  DESCRA(1) matrix structure
                    0 : general
                    1 : symmetric (A=A')
                    2 : Hermitian (A= CONJG(A'))
                    3 : Triangular
                    4 : Skew(Anti)-Symmetric (A=-A')
                    5 : Diagonal
                    6 : Skew-Hermitian (A= -CONJG(A'))
                  Note: For the routine, DESCRA(1)=3 is only supported.
                  DESCRA(2) upper/lower triangular indicator
                    1 : lower
                    2 : upper
                  DESCRA(3) main diagonal type
                    0 : non-identity blocks on the main diagonal
                    1 : identity diagonal block
                    2 : diagonal blocks are dense matrices
                  DESCRA(4) Array base  (NOT IMPLEMENTED)
                    0 : C/C++ compatible
                    1 : Fortran compatible
                  DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                    0 : unknown
                    1 : no repeated indices

VAL()             scalar array of length NNZ consisting of the block entries
                  of A where each block entry is a dense rectangular matrix
                  stored column by column.
                  NNZ is the total number of point entries in all nonzero
                  block  entries of a matrix A.

INDX()            integer array of length BNNZ+1 where BNNZ is the number
                  block entries of a matrix A such that the I-th element of
                  INDX[] points to the location in VAL of the (1,1) element
                  of the I-th block entry.

BINDX()           integer array of length BNNZ consisting of the block
                  column indices of the block entries of A where BNNZ is
                  the number block entries of a matrix A.   Block column
                  indices MUST be sorted in increasing order for each block
                  row.

RPNTR()           integer array of length MB+1 such that RPNTR(I)-RPNTR(1)+1
                  is the row index of the first point row in the I-th block
                  row.
                  RPNTR(MB+1) is set to M+RPNTR(1) where M is the number
                  of rows  in square triangular matrix A.
```

Thus, the number of point rows in the I-th block row is RPNTR(I+1)-RPNTR(I).

NOTE: For the current version CPNTR must equal RPNTR and a single array can be passed for both arguments

CPNTR() integer array of length MB+1 such that CPNTR(J)-CPNTR(1)+1 is the column index of the first point column in the J-th block column. CPNTR(MB+1) is set to M+CPNTR(1). Thus, the number of point columns in the J-th block column is CPNTR(J+1)-CPNTR(J).

NOTE: For the current version CPNTR must equal RPNTR and a single array can be passed for both arguments

BPNTRB() integer array of length MB such that BPNTRB(I)-BPNTRB(1)+1 points to location in BINDX of the first block entry of the I-th block row of A.

BPNTRE() integer array of length MB such that BPNTRE(I)-BPNTRB(1) points to location in BINDX of the last block entry of the I-th block row of A.

B() rectangular array with first dimension LDB.

LDB leading dimension of B

BETA Scalar parameter

C() rectangular array with first dimension LDC.

LDC leading dimension of C

WORK() scratch array of length LWORK. On exit, if LWORK= -1, WORK(1) returns the optimum size of LWORK.

LWORK length of WORK array. LWORK should be at least M = RPNTR(MB+1)-RPNTR(1).

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued

by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If *DESCRA*(3)=0,the lower or upper triangular part of each diagonal block is used by the routine depending on *DESCRA*(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might not be referenced in the VBR representation of a sparse matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense matrices and the LU factorization with partial pivoting is used by the routine. *WORK*(1)=0 on return if the factorization for all diagonal blocks has been completed successfully, otherwise *WORK*(1) = -i where i is the block number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3 in this case. *DESCRA*(2) indicates which triangle will be used.

6. It is known that there exists another representation of the variable block sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of six array instead of the seven used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block row in the array BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of variable block sparse row

format the following calling sequence should be used

```
  SUBROUTINE CVBRSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, INDX, BINDX, RPNTR, CPNTR, IA, IA(2),
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

```
cvmul - compute the scaled product of complex vectors
```

SYNOPSIS

```
SUBROUTINE CVMUL(N, ALPHA, X, INCX, Y, INCY, BETA, Z, INCZ)

COMPLEX ALPHA, BETA
COMPLEX X(*), Y(*), Z(*)
INTEGER N, INCX, INCY, INCZ

SUBROUTINE CVMUL_64(N, ALPHA, X, INCX, Y, INCY, BETA, Z, INCZ)

COMPLEX ALPHA, BETA
COMPLEX X(*), Y(*), Z(*)
INTEGER*8 N, INCX, INCY, INCZ
```

F95 INTERFACE

```
SUBROUTINE VMUL([N], ALPHA, X, [INCX], Y, [INCY], BETA, Z, [INCZ])

COMPLEX :: ALPHA, BETA
COMPLEX, DIMENSION(:) :: X, Y, Z
INTEGER :: N, INCX, INCY, INCZ

SUBROUTINE VMUL_64([N], ALPHA, X, [INCX], Y, [INCY], BETA, Z, [INCZ])

COMPLEX :: ALPHA, BETA
COMPLEX, DIMENSION(:) :: X, Y, Z
INTEGER(8) :: N, INCX, INCY, INCZ
```

C INTERFACE

```
#include <sunperf.h>
```

```
void cvmul(int n, complex *alpha, complex *x, int incx, com-
          plex  *y, int incy, complex *beta, complex *z, int
          incz);

void cvmul_64(long n, complex *alpha, complex *x, long incx,
          complex  *y, long incy, complex *beta, complex *z,
          long incz);
```

PURPOSE

```
cvmul computes the scaled product of complex vectors:
  z(i) = ALPHA * x(i) * y(i) + BETA * z(i)
for 1 <= i <= N.
```

ARGUMENTS

```
N (input)
          Length of the vectors.  N >= 0.   Returns  immedi-
          ately if N = 0.

ALPHA (input)
          Scale factor on the multiplicand vectors.

X (input)  dimension(*)
          Multiplicand vector.

INCX (input)
          Stride between elements of the multiplicand vector
          X.  INCX > 0.

Y (input)  dimension(*)
          Multiplicand vector.

INCY (input)
          Stride between elements of the multiplicand vector
          Y.  INCY > 0.

BETA (input)
          Scale factor on the product vector.

Z (input/output)
           dimension(*)
          Product vector.  On exit, z(i) = ALPHA  *  x(i)  *
          y(i) + BETA * z(i).

INCZ (input)
          Stride between elements of Z.  INCZ > 0.
```

Contents

NAME

```
dasum - Return the sum of the absolute values of a vector x.
```

SYNOPSIS

```
DOUBLE PRECISION FUNCTION DASUM(N, X, INCX)

INTEGER N, INCX
DOUBLE PRECISION X(*)

DOUBLE PRECISION FUNCTION DASUM_64(N, X, INCX)

INTEGER*8 N, INCX
DOUBLE PRECISION X(*)
```

F95 INTERFACE

```
REAL(8) FUNCTION ASUM([N], X, [INCX])

INTEGER :: N, INCX
REAL(8), DIMENSION(:) :: X

REAL(8) FUNCTION ASUM_64([N], X, [INCX])

INTEGER(8) :: N, INCX
REAL(8), DIMENSION(:) :: X
```

C INTERFACE

```
#include <sunperf.h>

double dasum(int n, double *x, int incx);

double dasum_64(long n, double *x, long incx);
```

PURPOSE

dasum Return the sum of the absolute values of x where x is an n-vector.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Contents

NAME

```
daxpy - compute y := alpha * x + y
```

SYNOPSIS

```
SUBROUTINE DAXPY(N, ALPHA, X, INCX, Y, INCY)

INTEGER N, INCX, INCY
DOUBLE PRECISION ALPHA
DOUBLE PRECISION X(*), Y(*)

SUBROUTINE DAXPY_64(N, ALPHA, X, INCX, Y, INCY)

INTEGER*8 N, INCX, INCY
DOUBLE PRECISION ALPHA
DOUBLE PRECISION X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE AXPY([N], ALPHA, X, [INCX], Y, [INCY])

INTEGER :: N, INCX, INCY
REAL(8) :: ALPHA
REAL(8), DIMENSION(:) :: X, Y

SUBROUTINE AXPY_64([N], ALPHA, X, [INCX], Y, [INCY])

INTEGER(8) :: N, INCX, INCY
REAL(8) :: ALPHA
REAL(8), DIMENSION(:) :: X, Y
```

C INTERFACE

```
#include <sunperf.h>
```

```
void daxpy(int n, double alpha, double *x, int incx,  double
          *y, int incy);

void daxpy_64(long n, double alpha, double  *x,  long  incx,
          double *y, long incy);
```

PURPOSE

daxpy compute y := alpha * x + y where alpha is a scalar and x and y are n-vectors.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
daxpyi - Compute y := alpha * x + y
```

SYNOPSIS

```
 SUBROUTINE DAXPYI(NZ, A, X, INDX, Y)

 DOUBLE PRECISION A
 DOUBLE PRECISION X(*), Y(*)
 INTEGER NZ
 INTEGER INDX(*)

 SUBROUTINE DAXPYI_64(NZ, A, X, INDX, Y)

 DOUBLE PRECISION A
 DOUBLE PRECISION X(*), Y(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 SUBROUTINE AXPYI([NZ], [A], X, INDX, Y)

 REAL(8) :: A
 REAL(8), DIMENSION(:) :: X, Y
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 SUBROUTINE AXPYI_64([NZ], [A], X, INDX, Y)

 REAL(8) :: A
 REAL(8), DIMENSION(:) :: X, Y
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
DAXPYI Compute y := alpha * x + y where alpha is a scalar, x
is a sparse vector, and y is a vector in full storage form

 do i = 1, n
   y(indx(i)) = alpha * x(i) + y(indx(i))
 enddo
```

ARGUMENTS

```
NZ (input) - INTEGER
        Number of elements in the compressed form.
        Unchanged on exit.

A (input)
        On entry, A(LPHA) specifies the scaling value.
        Unchanged on exit.  A is defaulted to 1.0D0 for F95
        INTERFACE.
X (input)
        Vector containing the values of the compressed form.
        Unchanged on exit.

INDX (input) - INTEGER
        Vector containing the indices of the compressed
        form.  It is assumed that the elements in INDX are
        distinct and greater than zero.  Unchanged on exit.

Y (output)
        Vector on input which contains the vector Y in full
        storage form.  On exit, only the elements
        corresponding to the indices in INDX have been
        modified.
```

Contents

NAME

dbcomm - block coordinate matrix-matrix multiply

SYNOPSIS

```
  SUBROUTINE DBCOMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *           VAL, BINDX, BJNDX, BNNZ, LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK)
  INTEGER    TRANSA, MB, N, KB, DESCRA(5), BNNZ, LB,
 *           LDB, LDC, LWORK
  INTEGER    BINDX(BNNZ), BJNDX(BNNZ)
  DOUBLE PRECISION ALPHA, BETA
  DOUBLE PRECISION VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

  SUBROUTINE DBCOMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *           VAL, BINDX, BJNDX, BNNZ, LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK)
  INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), BNNZ, LB,
 *           LDB, LDC, LWORK
  INTEGER*8  BINDX(BNNZ), BJNDX(BNNZ)
  DOUBLE PRECISION ALPHA, BETA
  DOUBLE PRECISION VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
   SUBROUTINE BCOMM(TRANSA,MB,N,KB,ALPHA,DESCRA,VAL,BINDX, BJNDX,
 *   BNNZ, LB, B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
  INTEGER    TRANSA, MB, N, KB, BNNZ, LB
  INTEGER, DIMENSION(:) ::   DESCRA, BINDX, BJNDX
  DOUBLE PRECISION    ALPHA, BETA
  DOUBLE PRECISION, DIMENSION(:) :: VAL
  DOUBLE PRECISION, DIMENSION(:, :) ::  B, C

   SUBROUTINE BCOMM_64(TRANSA,MB,N,KB,ALPHA,DESCRA,VAL,BINDX, BJNDX,
```

```
*   BNNZ, LB, B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, N, KB, BNNZ, LB
 INTEGER*8, DIMENSION(:) ::   DESCRA, BINDX, BJNDX
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
         C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block coordinate format and
op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if the matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
```

```
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices

VAL()          scalar array of length LB*LB*BNNZ consisting of
               the non-zero block entries of A, in any order.
               Each block is stored in standard column-major form.

BINDX()        integer array of length BNNZ consisting of the
               block row indices of the block entries of A.

BJNDX()        integer array of length BNNZ consisting of the
               block column indices of the block entries of A.

BNNZ           number of block entries

LB             dimension of dense blocks composing A.

B()            rectangular array with first dimension LDB.
LDB            leading dimension of B

BETA           Scalar parameter

C()            rectangular array with first dimension LDC.

LDC            leading dimension of C

WORK()         scratch array of length LWORK. WORK is not
               referenced in the current version.

LWORK          length of WORK array. LWORK is not referenced
               in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

dbdimm - block diagonal format matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE DBDIMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BLDA, IBDIAG, NBDIAG, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, MB, N, KB, DESCRA(5), BLDA, NBDIAG, LB,
*           LDB, LDC, LWORK
 INTEGER    IBDIAG(NBDIAG)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION VAL(LB*LB*BLDA*NBDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE DBDIMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BLDA, IBDIAG, NBDIAG, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), BLDA, NBDIAG, LB,
*           LDB, LDC, LWORK
 INTEGER*8  IBDIAG(NBDIAG)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION VAL(LB*LB*BLDA*NBDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
  SUBROUTINE BDIMM(TRANSA,MB, [N], KB, ALPHA, DESCRA, VAL, BLDA,
*    IBDIAG, NBDIAG, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, KB, BLDA, NBDIAG, LB
 INTEGER, DIMENSION(:) ::    DESCRA, IBDIAG
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C

  SUBROUTINE BDIMM_64(TRANSA,MB, [N], KB, ALPHA, DESCRA, VAL, BLDA,
```

```
*    IBDIAG, NBDIAG, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, KB, BLDA, NBDIAG, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, IBDIAG
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block diagonal format and
op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix  is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
```

```
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

 VAL()          two-dimensional LB*LB*BLDA-by-NBDIAG scalar array
                consisting of the NBDIAG nonzero block diagonal in
                any order. Each dense block is stored in standard
                column-major form.

 BLDA           leading block dimension of VAL().

 IBDIAG()       integer array of length NBDIAG consisting of the
                corresponding diagonal offsets of the non-zero
                block diagonals of A in VAL.  Lower triangular
                block diagonals have negative offsets, the main
                block diagonal has offset 0, and upper triangular
                block diagonals have positive offset.

 NBDIAG         the number of non-zero block diagonals in A.
 LB             dimension of dense blocks composing A.

 B()            rectangular array with first dimension LDB.

 LDB            leading dimension of B

 BETA           Scalar parameter

 C()            rectangular array with first dimension LDC.

 LDC            leading dimension of C

 WORK()         scratch array of length LWORK. WORK is not
                referenced in the current version.

 LWORK          length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

```
dbdism -  block diagonal format triangular solve
```

SYNOPSIS

```
 SUBROUTINE DBDISM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, BLDA, IBDIAG, NBDIAG, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), BLDA, NBDIAG, LB,
*           LDB, LDC, LWORK
 INTEGER    IBDIAG(NBDIAG)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION DV(MB*LB*LB), VAL(LB*LB*BLDA, NBDIAG), B(LDB,*), C(LDC,*),
*           WORK(LWORK)

 SUBROUTINE DBDISM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, BLDA, IBDIAG, NBDIAG, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), BLDA, NBDIAG, LB,
*           LDB, LDC, LWORK
 INTEGER*8  IBDIAG(NBDIAG)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION DV(MB*LB*LB), VAL(LB*LB*BLDA, NBDIAG), B(LDB,*), C(LDC,*),
*           WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE BDISM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BLDA,
*   IBDIAG, NBDIAG, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, N, UNITD, BLDA, NBDIAG, LB
 INTEGER, DIMENSION(:) ::    DESCRA, IBDIAG
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL, DV
 DOUBLE PRECISION, DIMENSION(:, :) ::   B, C
```

```
 SUBROUTINE BDISM_64(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BLDA,
*   IBDIAG, NBDIAG, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, N, UNITD, BLDA, NBDIAG, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, IBDIAG
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL, DV
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in block diagonal format
and  op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)

DV()          Array of length MB*LB*LB containing the elements of
              the diagonal blocks of the matrix D.  The size of each
              square block is LB-by-LB and each block
              is stored in standard column-major form.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                 0 : general
                 1 : symmetric (A=A')
                 2 : Hermitian (A= CONJG(A'))
                 3 : Triangular
                 4 : Skew(Anti)-Symmetric (A=-A')
                 5 : Diagonal
                 6 : Skew-Hermitian (A= -CONJG(A'))
               Note: For the routine, DESCRA(1)=3 is only supported.

               DESCRA(2) upper/lower triangular indicator
                 1 : lower
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-identity blocks on the main diagonal
                 1 : identity diagonal blocks
                 2 : diagonal blocks are dense matrices
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices
VAL()          Two-dimensional LB*LB*BLDA-by-NBDIAG scalar array
               consisting of the NBDIAG non-zero block diagonal.
               Each dense block is stored in standard column-major form.

BLDA           Leading block dimension of VAL().  Should be greater
               than or equal to MB.

IBDIAG()       integer array of length NBDIAG consisting of the
               corresponding diagonal offsets of the non-zero block
               diagonals of A in VAL.  Lower triangular block diagonals
               have negative offsets, the main block diagonal has offset
               0, and upper triangular block diagonals have positive offset.
               Elements of IBDIAG  MUST be sorted in  increasing order.

NBDIAG         The number of non-zero block diagonals in A.

LB             Dimension of dense blocks composing A.

B()            Rectangular array with first dimension LDB.

LDB            Leading dimension of B.

BETA           Scalar parameter.

C()            Rectangular array with first dimension LDC.

LDC            Leading dimension of C.
```

```
WORK()        scratch array of length LWORK.
              On exit, if LWORK= -1, WORK(1) returns the optimum  size
              of LWORK.

LWORK        length of WORK array. LWORK should be at least
             MB*LB.

             For good performance, LWORK should generally be larger.
             For optimum performance on multiple processors, LWORK
             >=MB*LB*N_CPUS where N_CPUS is the maximum number of
             processors available to the program.

             If LWORK=0, the routine is to allocate workspace needed.

             If LWORK = -1, then a workspace query is assumed; the
             routine only calculates the optimum size of the WORK array,
             returns this value as the first entry of the WORK array,
             and no error message related to LWORK is issued by XERBLA.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If *DESCRA*(3)=0,the lower or upper triangular part of each diagonal block is used by the routine depending on *DESCRA*(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might not be referenced in the BDI representation of a sparse matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense matrices and the LU factorization with partial pivoting is used by the routine. *WORK*(1)=0 on return if the factorization for all diagonal blocks has been completed successfully, otherwise *WORK*(1) = -i where i is the block

number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

```
dbdsdc - compute the singular value decomposition (SVD) of a
real N-by-N (upper or lower) bidiagonal matrix B
```

SYNOPSIS

```
SUBROUTINE DBDSDC(UPLO, COMPQ, N, D, E, U, LDU, VT, LDVT, Q, IQ,
      WORK, IWORK, INFO)

CHARACTER * 1 UPLO, COMPQ
INTEGER N, LDU, LDVT, INFO
INTEGER IQ(*), IWORK(*)
DOUBLE PRECISION D(*),  E(*),  U(LDU,*),  VT(LDVT,*),  Q(*),
WORK(*)

SUBROUTINE DBDSDC_64(UPLO, COMPQ, N, D, E, U, LDU, VT, LDVT, Q, IQ,
      WORK, IWORK, INFO)

CHARACTER * 1 UPLO, COMPQ
INTEGER*8 N, LDU, LDVT, INFO
INTEGER*8 IQ(*), IWORK(*)
DOUBLE PRECISION D(*),  E(*),  U(LDU,*),  VT(LDVT,*),  Q(*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE BDSDC(UPLO, COMPQ, [N], D, E, U, [LDU], VT, [LDVT], Q, IQ,
       [WORK], [IWORK], [INFO])

CHARACTER(LEN=1) :: UPLO, COMPQ
INTEGER :: N, LDU, LDVT, INFO
INTEGER, DIMENSION(:) :: IQ, IWORK
REAL(8), DIMENSION(:) :: D, E, Q, WORK
```

```
REAL(8), DIMENSION(:,:) :: U, VT

SUBROUTINE BDSDC_64(UPLO, COMPQ, [N], D, E, U, [LDU], VT, [LDVT], Q,
       IQ, [WORK], [IWORK], [INFO])

CHARACTER(LEN=1) :: UPLO, COMPQ
INTEGER(8) :: N, LDU, LDVT, INFO
INTEGER(8), DIMENSION(:) :: IQ, IWORK
REAL(8), DIMENSION(:) :: D, E, Q, WORK
REAL(8), DIMENSION(:,:) :: U, VT
```

C INTERFACE

```
#include <sunperf.h>

void dbdsdc(char uplo, char compq, int n, double *d,  double
          *e,  double  *u,  int  ldu,  double *vt, int ldvt,
          double *q, int *iq, int *info);

void dbdsdc_64(char uplo, char compq,  long  n,  double  *d,
          double  *e,  double *u, long ldu, double *vt, long
          ldvt, double *q, long *iq, long *info);
```

PURPOSE

dbdsdc computes the singular value decomposition (SVD) of a real N-by-N (upper or lower) bidiagonal matrix B: B = U * S * VT, using a divide and conquer method, where S is a diagonal matrix with non-negative diagonal elements (the singular values of B), and U and VT are orthogonal matrices of left and right singular vectors, respectively. SBDSDC can be used to compute all singular values, and optionally, singular vectors or singular vectors in compact form.

This code makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none. See SLASD3 for details.

The code currently call SLASDQ if singular values only are desired. However, it can be slightly modified to compute singular values using the divide and conquer method.

ARGUMENTS

```
UPLO (input)
          = 'U':  B is upper bidiagonal.
          = 'L':  B is lower bidiagonal.

COMPQ (input)
          Specifies whether singular vectors are to be  com-
          puted as follows:
          = 'N':  Compute singular values only;
          = 'P':  Compute singular values and compute singu-
          lar  vectors  in  compact  form;  =  'I':  Compute
          singular values and singular vectors.

N (input) The order of the matrix B.  N >= 0.

D (input/output)
          On entry, the n diagonal elements of the  bidiago-
          nal  matrix  B.   On exit, if INFO=0, the singular
          values of B.

E (input/output)
          On entry, the elements of E contain the  offdiago-
          nal elements of the bidiagonal matrix whose SVD is
          desired.  On exit, E has been destroyed.

U (output)
          If  COMPQ = 'I', then:  On exit, if INFO  =  0,  U
          contains the left singular vectors of the bidiago-
          nal matrix.  For other values of COMPQ, U  is  not
          referenced.

LDU (input)
          The leading dimension of the array U.  LDU  >=  1.
          If  singular vectors are desired, then LDU >= max(
          1, N ).

VT (output)
          If  COMPQ = 'I', then:  On exit, if INFO = 0,  VT'
          contains the right singular vectors of the bidiag-
          onal matrix.  For other values of COMPQ, VT is not
          referenced.

LDVT (input)
          The leading dimension of the array VT.  LDVT >= 1.
          If singular vectors are desired, then LDVT >= max(
          1, N ).

Q (input) If  COMPQ = 'P', then:  On exit, if INFO  =  0,  Q
          and IQ contain the left and right singular vectors
          in a compact form,  requiring  O(N  log  N)  space
          instead  of 2*N**2.  In particular, Q contains all
```

the REAL data in LDQ >= N*(11 + 2*SMLSIZ + 8*INT(LOG_2(N/(SMLSIZ+1)))) words of memory, where SMLSIZ is returned by ILAENV and is equal to the maximum size of the subproblems at the bottom of the computation tree (usually about 25). For other values of COMPQ, Q is not referenced.

IQ (output)
If COMPQ = 'P', then: On exit, if INFO = 0, Q and IQ contain the left and right singular vectors in a compact form, requiring O(N log N) space instead of 2*N**2. In particular, IQ contains all INTEGER data in LDIQ >= N*(3 + 3*INT(LOG_2(N/(SMLSIZ+1)))) words of memory, where SMLSIZ is returned by ILAENV and is equal to the maximum size of the subproblems at the bottom of the computation tree (usually about 25). For other values of COMPQ, IQ is not referenced.

WORK (workspace)
If COMPQ = 'N' then LWORK >= (2 * N). If COMPQ = 'P' then LWORK >= (6 * N). If COMPQ = 'I' then LWORK >= (3 * N**2 + 4 * N).

IWORK (workspace)
dimension(8*N)

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: The algorithm failed to compute an singular value. The update process of divide and conquer failed.

FURTHER DETAILS

Based on contributions by
Ming Gu and Huan Ren, Computer Science Division, University of
California at Berkeley, USA

Contents

NAME

```
dbdsqr - compute the singular value decomposition (SVD) of a
real N-by-N (upper or lower) bidiagonal matrix B.
```

SYNOPSIS

```
SUBROUTINE DBDSQR(UPLO, N, NCVT, NRU, NCC, D, E, VT, LDVT, U, LDU, C,
      LDC, WORK, INFO)

CHARACTER * 1 UPLO
INTEGER N, NCVT, NRU, NCC, LDVT, LDU, LDC, INFO
DOUBLE PRECISION D(*), E(*), VT(LDVT,*), U(LDU,*), C(LDC,*),
WORK(*)

SUBROUTINE DBDSQR_64(UPLO, N, NCVT, NRU, NCC, D, E, VT, LDVT, U, LDU,
      C, LDC, WORK, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NCVT, NRU, NCC, LDVT, LDU, LDC, INFO
DOUBLE PRECISION D(*), E(*), VT(LDVT,*), U(LDU,*), C(LDC,*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE BDSQR(UPLO, [N], [NCVT], [NRU], [NCC], D, E, VT, [LDVT],
       U, [LDU], C, [LDC], [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NCVT, NRU, NCC, LDVT, LDU, LDC, INFO
REAL(8), DIMENSION(:) :: D, E, WORK
REAL(8), DIMENSION(:,:) :: VT, U, C

SUBROUTINE BDSQR_64(UPLO, [N], [NCVT], [NRU], [NCC], D, E, VT, [LDVT],
       U, [LDU], C, [LDC], [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NCVT, NRU, NCC, LDVT, LDU, LDC, INFO
REAL(8), DIMENSION(:) :: D, E, WORK
REAL(8), DIMENSION(:,:) :: VT, U, C
```

C INTERFACE

```
#include <sunperf.h>

void dbdsqr(char uplo, int n, int ncvt, int  nru,  int  ncc,
          double *d, double *e, double *vt, int ldvt, double
          *u, int ldu, double *c, int ldc, int *info);

void dbdsqr_64(char uplo, long n, long ncvt, long nru,  long
          ncc,  double *d, double *e, double *vt, long ldvt,
          double *u, long ldu, double  *c,  long  ldc,  long
          *info);
```

PURPOSE

dbdsqr computes the singular value decomposition (SVD) of a real N-by-N (upper or lower) bidiagonal matrix B: B = Q * S * P' (P' denotes the transpose of P), where S is a diagonal matrix with non-negative diagonal elements (the singular values of B), and Q and P are orthogonal matrices.

The routine computes S, and optionally computes U * Q, P' * VT, or Q' * C, for given real input matrices U, VT, and C.

See "Computing Small Singular Values of Bidiagonal Matrices With Guaranteed High Relative Accuracy," by J. Demmel and W. Kahan, LAPACK Working Note #3 (or SIAM J. Sci. Statist. Comput. vol. 11, no. 5, pp. 873-912, Sept 1990) and "Accurate singular values and differential qd algorithms," by B. Parlett and V. Fernando, Technical Report CPAM-554, Mathematics Department, University of California at Berkeley, July 1992 for a detailed description of the algorithm.

ARGUMENTS

UPLO (input)
= 'U': B is upper bidiagonal;
= 'L': B is lower bidiagonal.

N (input) The order of the matrix B. N >= 0.

NCVT (input)

```
          The number of columns of the matrix VT. NCVT >= 0.

NRU (input)
          The number of rows of the matrix U. NRU >= 0.

NCC (input)
          The number of columns of the matrix C. NCC >= 0.

D (input/output)
          On entry, the n diagonal elements of the  bidiago-
          nal  matrix  B.   On exit, if INFO=0, the singular
          values of B in decreasing order.
E (input/output)
          On entry, the elements of E contain the  offdiago-
          nal elements of the bidiagonal matrix whose SVD is
          desired. On normal exit (INFO  =  0),  E  is  des-
          troyed.   If the algorithm does not converge (INFO
          > 0), D and E will contain the diagonal and super-
          diagonal  elements of a bidiagonal matrix orthogo-
          nally equivalent to the one given as  input.  E(N)
          is used for workspace.

VT (input/output)
          On entry, an N-by-NCVT matrix VT.  On exit, VT  is
          overwritten  by  P' * VT.  VT is not referenced if
          NCVT = 0.

LDVT (input)
          The leading dimension of the array  VT.   LDVT  >=
          max(1,N) if NCVT > 0; LDVT >= 1 if NCVT = 0.

U (input/output)
          On entry, an NRU-by-N matrix U.   On  exit,  U  is
          overwritten  by U * Q.  U is not referenced if NRU
          = 0.

LDU (input)
          The leading dimension of  the  array  U.   LDU  >=
          max(1,NRU).

C (input/output)
          On entry, an N-by-NCC matrix C.   On  exit,  C  is
          overwritten by Q' * C.  C is not referenced if NCC
          = 0.

LDC (input)
          The leading dimension of  the  array  C.   LDC  >=
          max(1,N) if NCC > 0; LDC >=1 if NCC = 0.

WORK (workspace)
```

dimension(4*N)

INFO (output)
= 0: successful exit
< 0: If INFO = -i, the i-th argument had an illegal value
> 0: the algorithm did not converge; D and E contain the elements of a bidiagonal matrix which is orthogonally similar to the input matrix B; if INFO = i, i elements of E have not converged to zero.

Contents

NAME

dbelmm - block Ellpack format matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE DBELMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BLDA, MAXBNZ, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, MB, N, KB, DESCRA(5), BLDA, MAXBNZ, LB,
*           LDB, LDC, LWORK
 INTEGER    BINDX(BLDA,MAXBNZ)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION VAL(LB*LB*BLDA*MAXBNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE DBELMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BLDA, MAXBNZ, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), BLDA, MAXBNZ, LB,
*           LDB, LDC, LWORK
 INTEGER*8  BINDX(BLDA,MAXBNZ)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION VAL(LB*LB*BLDA*MAXBNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE BELMM( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
*           BLDA, MAXBNZ, LB, B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, KB, BLDA, MAXBNZ, LB
 INTEGER, DIMENSION(:) ::    DESCRA, BINDX
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C

 SUBROUTINE BELMM_64( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
```

```
*            BLDA, MAXBNZ, LB, B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, KB, BLDA, MAXBNZ, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block Ellpack format and
op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
```

```
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

 VAL()          scalar array of length LB*LB*BLDA*MAXBNZ containing
                matrix entries, stored column-major within each dense
                block.

 BINDX()        two-dimensional integer BLDA-by-MAXBNZ array such
                BINDX(i,:) consists of the block column indices of the
                nonzero blocks in block row i, padded by the integer
                value i if the number of nonzero blocks is less than
                MAXBNZ.

 BLDA           leading dimension of BINDX(:,:).

 MAXBNZ         max number of nonzeros blocks per row.
 LB             row and column dimension of the dense blocks composing
                VAL.

 B()            rectangular array with first dimension LDB.

 LDB            leading dimension of B

 BETA           Scalar parameter

 C()            rectangular array with first dimension LDC.

 LDC            leading dimension of C

 WORK()         scratch array of length LWORK. WORK is not
                referenced in the current version.

 LWORK          length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS)

Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

```
dbelsm - block Ellpack format triangular solve
```

SYNOPSIS

```
 SUBROUTINE DBELSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, BINDX, BLDA, MAXBNZ, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), BLDA, MAXBNZ, LB,
*           LDB, LDC, LWORK
 INTEGER    BINDX(BLDA,MAXBNZ)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION DV(MB*LB*LB), VAL(LB*LB*BLDA*MAXBNZ), B(LDB,*), C(LDC,*),
*           WORK(LWORK)

 SUBROUTINE DBELSM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, BINDX, BLDA, MAXBNZ, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), BLDA, MAXBNZ, LB,
*           LDB, LDC, LWORK
 INTEGER*8  BINDX(BLDA,MAXBNZ)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION DV(MB*LB*LB), VAL(LB*LB*BLDA*MAXBNZ), B(LDB,*), C(LDC,*),
*           WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE BELSM( TRANSA, MB, [N], UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*   BLDA, MAXBNZ, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, UNITD,  BLDA, MAXBNZ, LB
 INTEGER, DIMENSION(:) ::    DESCRA,  BINDX
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL, DV
 DOUBLE PRECISION, DIMENSION(:, :) ::   B, C
```

```
 SUBROUTINE BELSM_64( TRANSA, MB, [N], UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*   BLDA, MAXBNZ, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8  TRANSA, MB, UNITD,  BLDA, MAXBNZ, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA,  BINDX
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL, DV
 DOUBLE PRECISION, DIMENSION(:, :) ::   B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in block Ellpack format and
op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)

DV()          Array of the length MB*LB*LB consisting of the block
              entries of block diagonal matrix D where each
              block is stored in standard column-major form.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
```

```
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))
                Note: For the routine, DESCRA(1)=3 is only supported.

                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-identity blocks on the main diagonal
                  1 : identity diagonal blocks
                  2 : diagonal blocks are dense matrices
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices
VAL()           scalar array of length LB*LB*BLDA*MAXBNZ containing
                matrix entries, stored column-major within each dense
                block.

BINDX()         two-dimensional integer BLDA-by-MAXBNZ array such
                BINDX(i,:) consists of the block column indices of the
                nonzero blocks in block row i, padded by the integer
                value i if the number of nonzero blocks is less than
                MAXBNZ.  The block column indices MUST be sorted
                in increasing order for each block row.

BLDA            leading dimension of BINDX(:,:).

MAXBNZ          max number of nonzeros blocks per row.

LB              row and column dimension of the dense blocks composing A.

B()             rectangular array with first dimension LDB.

LDB             leading dimension of B

BETA            Scalar parameter

C()             rectangular array with first dimension LDC.

LDC             leading dimension of C

WORK()          scratch array of length LWORK.
                On exit, if LWORK= -1, WORK(1) returns the minimum
```

size of LWORK.

LWORK length of WORK array. LWORK should be at least MB*LB.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=MB*LB*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at: http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If *DESCRA*(3)=0,the lower or upper triangular part of each diagonal block is used by the routine depending on *DESCRA*(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might not be referenced in the BEL representation of a sparse matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense matrices and the LU factorization with partial pivoting is used by the routine. *WORK*(1)=0 on return if the factorization for all diagonal blocks has been completed successfully, otherwise *WORK*(1) = -i where i is the block number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

dbscmm - block sparse column matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE DBSCMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BPNTRB, BPNTRE, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, MB, N, KB, DESCRA(5), LB,
*           LDB, LDC, LWORK
 INTEGER    BINDX(BNNZ), BPNTRB(KB), BPNTRE(KB)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE DBSCMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BPNTRB, BPNTRE, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), LB,
*           LDB, LDC, LWORK
 INTEGER*8  BINDX(BNNZ), BPNTRB(KB), BPNTRE(KB)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where: BNNZ = BPNTRE(KB)-BPNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE BSCMM( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, KB, LB
 INTEGER, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE BSCMM_64( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB,  KB, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block sparse column format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
```

```
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

  VAL()         scalar array of length LB*LB*BNNZ consisting of
                the block entries stored column-major within each
                dense block.

  BINDX()       integer array of length BNNZ consisting of the
                block row indices of the block entries of A.

  BPNTRB()      integer array of length KB such that
                BPNTRB(J)-BPNTRB(1)+1 points to location in BINDX
                of the first block entry of the J-th block column
                of A.
  BPNTRE()      integer array of length KB such that
                BPNTRE(J)-BPNTRB(1) points to location in BINDX
                of the last block entry of the J-th block column
                of A.

  LB            dimension of dense blocks composing A.

  B()           rectangular array with first dimension LDB.

  LDB           leading dimension of B

  BETA          Scalar parameter

  C()           rectangular array with first dimension LDC.

  LDC           leading dimension of C

  WORK()        scratch array of length LWORK. WORK is not
                referenced in the current version.

  LWORK         length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

It is known that there exists another representation of the block sparse column format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block column in the arrays VAL and BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of block sparse column format the following calling sequence should be used

```
  CALL SBSCMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *           VAL, BINDX, IA, IA(2), LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

dbscsm - block sparse column format triangular solve

SYNOPSIS

```
       SUBROUTINE DBSCSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
      *            VAL, BINDX, BPNTRB, BPNTRE, LB,
      *            B, LDB, BETA, C, LDC, WORK, LWORK)
       INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), LB,
      *            LDB, LDC, LWORK
       INTEGER    BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
       DOUBLE PRECISION ALPHA, BETA
       DOUBLE PRECISION DV(MB*LB*LB), VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK
(LWORK)

       SUBROUTINE DBSCSM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
      *            VAL, BINDX, BPNTRB, BPNTRE, LB,
      *            B, LDB, BETA, C, LDC, WORK, LWORK)
       INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), LB,
      *            LDB, LDC, LWORK
       INTEGER*8  BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
       DOUBLE PRECISION ALPHA, BETA
       DOUBLE PRECISION DV(MB*LB*LB), VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK
(LWORK)

       where: BNNZ = BPNTRE(MB)- BPNTRB(1)
```

F95 INTERFACE

```
       SUBROUTINE BSCSM(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
      *      BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
       INTEGER    TRANSA, MB, N, UNITD, LB
       INTEGER, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
       DOUBLE PRECISION    ALPHA, BETA
```

```
 DOUBLE PRECISION, DIMENSION(:) :: VAL, DV
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C

 SUBROUTINE BSCSM_64(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, N, UNITD, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL, DV
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in block sparse column format
and  op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row block scaling)
                3 : Scale on right (column block scaling)

DV()          Array of the length MB*LB*LB consisting of the block
              entries of block diagonal matrix D where each
              block is stored in standard column-major form.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
```

```
                DESCRA(1) matrix structure
                  0 : general
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))

                Note: For the routine, DESCRA(1)=3 is only supported.

                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-identity blocks on the main diagonal
                  1 : identity diagonal blocks
                  2 : diagonal blocks are dense matrices
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices
VAL()           scalar array of length LB*LB*BNNZ consisting of the block
                entries stored column-major within each dense block.

BINDX()         integer array of length BNNZ consisting of the
                block row indices of the block entries of A.
                The block row indices MUST be sorted
                in increasing order for each block column.

BPNTRB()        integer array of length MB such that
                BPNTRB(J)-BPNTRB(1)+1 points to location in BINDX
                of the first block entry of the J-th block column of A.

BPNTRE()        integer array of length MB such that
                BPNTRE(J)-BPNTRB(1) points to location in BINDX
                of the last block entry of the J-th block column of A.

LB              dimension of dense blocks composing A.

B()             rectangular array with first dimension LDB.

LDB             leading dimension of B

BETA            Scalar parameter

C()             rectangular array with first dimension LDC.
```

```
LDC           leading dimension of C

WORK()        scratch array of length LWORK.
              On exit, if LWORK= -1, WORK(1) returns the optimum
              size of LWORK.

LWORK         length of WORK array. LWORK should be at least
              MB*LB.

              For good performance, LWORK should generally be larger.
              For optimum performance on multiple processors, LWORK
              >=MB*LB*N_CPUS where N_CPUS is the maximum number of
              processors available to the program.

              If LWORK=0, the routine is to allocate workspace needed.

              If LWORK = -1, then a workspace query is assumed; the
              routine only calculates the optimum size of the WORK
              array, returns this value as the first entry of the WORK
              array, and no error message related to LWORK is issued
              by XERBLA.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If *DESCRA*(3)=0,the lower or upper triangular part of each diagonal block is used by the routine depending on *DESCRA*(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might not be referenced in the BSC representation of a sparse matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense matrices and the LU factorization with partial pivoting is

used by the routine. *WORK*(1)=0 on return if the factorization for all diagonal blocks has been completed successfully, otherwise *WORK*(1) = -i where i is the block number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3 in this case.

6. It is known that there exists another representation of the block sparse column format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block column in the arrays VAL and BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of block sparse column format the following calling sequence should be used

```
  CALL SBSCSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, BINDX, IA, IA(2), LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

```
dbsrmm - block sparse row format matrix-matrix multiply
```

SYNOPSIS

```
  SUBROUTINE DBSRMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *           VAL, BINDX, BPNTRB, BPNTRE, LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK)
  INTEGER    TRANSA, MB, N, KB, DESCRA(5), LB,
 *           LDB, LDC, LWORK
  INTEGER    BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
  DOUBLE PRECISION ALPHA, BETA
  DOUBLE PRECISION VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

  SUBROUTINE DBSRMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *           VAL, BINDX, BPNTRB, BPNTRE, LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK)
  INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), LB,
 *           LDB, LDC, LWORK
  INTEGER*8  BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
  DOUBLE PRECISION ALPHA, BETA
  DOUBLE PRECISION VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

  where: BNNZ = BPNTRE(MB)-BPNTRB(1)
```

F95 INTERFACE

```
  SUBROUTINE BSRMM( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
 *     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
  INTEGER    TRANSA, MB, KB, LB
  INTEGER, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
  DOUBLE PRECISION    ALPHA, BETA
  DOUBLE PRECISION, DIMENSION(:) :: VAL
  DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE BSRMM_64( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB,  KB, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block sparse row format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix A is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
```

```
               DESCRA(3) main diagonal type
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices

VAL()          scalar array of length LB*LB*BNNZ consisting
               of the block  entries stored column-major within
               each dense block.

BINDX()        integer array of length BNNZ consisting of the
               block column indices of the block entries of A.

BPNTRB()       integer array of length MB such that
               BPNTRB(J)-BPNTRB(1)+1 points to location in BINDX
               of the first block entry of the J-th block row of A.
BPNTRE()       integer array of length MB such that
               BPNTRE(J)-BPNTRB(1) points to location in BINDX
               of the last block entry of the J-th block row of A.

LB             dimension of dense blocks composing A.

B()            rectangular array with first dimension LDB.

LDB            leading dimension of B

BETA           Scalar parameter

C()            rectangular array with first dimension LDC.

LDC            leading dimension of C

WORK()         scratch array of length LWORK. WORK is not
               referenced in the current version.

LWORK          length of WORK array. LWORK is not referenced
               in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

It is known that there exists another representation of the block sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block row in the arrays VAL and BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of block sparse row format the following calling sequence should be used

```
  CALL SBSRMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *           VAL, BINDX, IA, IA(2), LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

dbsrsm - block sparse row format triangular solve

SYNOPSIS

```
       SUBROUTINE DBSRSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
      *           VAL, BINDX, BPNTRB, BPNTRE, LB,
      *           B, LDB, BETA, C, LDC, WORK, LWORK)
       INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), LB,
      *           LDB, LDC, LWORK
       INTEGER    BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
       DOUBLE PRECISION ALPHA, BETA
       DOUBLE PRECISION DV(MB*LB*LB), VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK
(LWORK)

       SUBROUTINE DBSRSM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
      *           VAL, BINDX, BPNTRB, BPNTRE, LB,
      *           B, LDB, BETA, C, LDC, WORK, LWORK)
       INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), LB,
      *           LDB, LDC, LWORK
       INTEGER*8  BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
       DOUBLE PRECISION ALPHA, BETA
       DOUBLE PRECISION DV(MB*LB*LB), VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK
(LWORK)

       where: BNNZ = BPNTRE(MB)-BPNTRB(1)
```

F95 INTERFACE

```
       SUBROUTINE BSRSM(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
      *     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
       INTEGER    TRANSA, MB, N, UNITD, LB
       INTEGER, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
       DOUBLE PRECISION    ALPHA, BETA
```

```
 DOUBLE PRECISION, DIMENSION(:) :: VAL, DV
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C

 SUBROUTINE BSRSM_64(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, N, UNITD, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL, DV
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in block sparse row format
format and  op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row block scaling)
                3 : Scale on right (column block scaling)

DV()          Array of the length MB*LB*LB consisting of the block
              entries of block diagonal matrix D where each
              block is stored in standard column-major form.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
```

```
               DESCRA(1) matrix structure
                 0 : general
                 1 : symmetric (A=A')
                 2 : Hermitian (A= CONJG(A'))
                 3 : Triangular
                 4 : Skew(Anti)-Symmetric (A=-A')
                 5 : Diagonal
                 6 : Skew-Hermitian (A= -CONJG(A'))

               Note: For the routine, DESCRA(1)=3 is only supported.

               DESCRA(2) upper/lower triangular indicator
                 1 : lower
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-identity blocks on the main diagonal
                 1 : identity diagonal blocks
                 2 : diagonal blocks are dense matrices
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices
VAL()          scalar array of length LB*LB*BNNZ consisting of the block
               entries stored column-major within each dense block.

BINDX()        integer array of length BNNZ consisting of the
               block column indices of the block entries of A.
               The block column indices MUST be sorted
               in increasing order for each block row.

BPNTRB()       integer array of length MB such that
               BPNTRB(J)-BPNTRB(1)+1 points to location in BINDX
               of the first block entry of the J-th block row of A.

BPNTRE()       integer array of length MB such that
               BPNTRE(J)-BPNTRB(1) points to location in BINDX
               of the last block entry of the J-th block row of A.

LB             dimension of dense blocks composing A.

B()            rectangular array with first dimension LDB.

LDB            leading dimension of B

BETA           Scalar parameter

C()            rectangular array with first dimension LDC.
```

```
 LDC           leading dimension of C

 WORK()        scratch array of length LWORK.
               On exit, if LWORK= -1, WORK(1) returns the optimum size
               of LWORK.

 LWORK         length of WORK array. LWORK should be at least
               MB*LB.

               For good performance, LWORK should generally be larger.
               For optimum performance on multiple processors, LWORK
               >=MB*LB*N_CPUS where N_CPUS is the maximum number of
               processors available to the program.

               If LWORK=0, the routine is to allocate workspace needed.

               If LWORK = -1, then a workspace query is assumed; the
               routine only calculates the optimum size of the WORK
               array, returns this value as the first entry of the WORK
               array, and no error message related to LWORK is issued
               by XERBLA.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If *DESCRA*(3)=0,the lower or upper triangular part of each diagonal block is used by the routine depending on *DESCRA*(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might not be referenced in the BSR representation of a sparse matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense matrices and the LU factorization with partial pivoting is

used by the routine. *WORK*(1)=0 on return if the factorization for all diagonal blocks has been completed successfully, otherwise *WORK*(1) = -i where i is the block number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3 in this case.

6. It is known that there exists another representation of the block sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block row in the arrays VAL and BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of block sparse row format the following calling sequence should be used

```
  CALL DBSRSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, BINDX, IA, IA(2), LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

```
dcnvcor - compute the convolution  or  correlation  of  real
vectors
```

SYNOPSIS

```
SUBROUTINE DCNVCOR(CNVCOR, FOUR, NX, X, IFX, INCX, NY, NPRE, M, Y,
      IFY, INC1Y, INC2Y, NZ, K, Z, IFZ, INC1Z, INC2Z, WORK, LWORK)

CHARACTER * 1 CNVCOR, FOUR
INTEGER NX, IFX, INCX, NY, NPRE, M, IFY, INC1Y,  INC2Y,  NZ,
K, IFZ, INC1Z, INC2Z, LWORK
DOUBLE PRECISION X(*), Y(*), Z(*), WORK(*)

SUBROUTINE DCNVCOR_64(CNVCOR, FOUR, NX, X, IFX, INCX, NY, NPRE, M, Y,
      IFY, INC1Y, INC2Y, NZ, K, Z, IFZ, INC1Z, INC2Z, WORK, LWORK)

CHARACTER * 1 CNVCOR, FOUR
INTEGER*8 NX, IFX, INCX, NY, NPRE, M, IFY, INC1Y, INC2Y, NZ,
K, IFZ, INC1Z, INC2Z, LWORK
DOUBLE PRECISION X(*), Y(*), Z(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE CNVCOR(CNVCOR, FOUR, [NX], X, IFX, [INCX], NY, NPRE, M, Y,
       IFY, INC1Y, INC2Y, NZ, K, Z, IFZ, INC1Z, INC2Z, WORK, [LWORK])

CHARACTER(LEN=1) :: CNVCOR, FOUR
INTEGER :: NX, IFX, INCX, NY, NPRE, M,  IFY,  INC1Y,  INC2Y,
NZ, K, IFZ, INC1Z, INC2Z, LWORK
REAL(8), DIMENSION(:) :: X, Y, Z, WORK

SUBROUTINE CNVCOR_64(CNVCOR, FOUR, [NX], X, IFX, [INCX], NY, NPRE, M,
```

```
       Y, IFY, INC1Y, INC2Y, NZ, K, Z, IFZ, INC1Z, INC2Z, WORK, [LWORK])

CHARACTER(LEN=1) :: CNVCOR, FOUR
INTEGER(8) :: NX, IFX, INCX, NY, NPRE, M, IFY, INC1Y, INC2Y,
NZ, K, IFZ, INC1Z, INC2Z, LWORK
REAL(8), DIMENSION(:) :: X, Y, Z, WORK
```

C INTERFACE

```
#include <sunperf.h>

void dcnvcor(char cnvcor, char four, int nx, double *x,  int
          ifx, int incx, int ny, int npre, int m, double *y,
          int ify, int inc1y, int inc2y, int nz, int k, dou-
          ble  *z,  int  ifz,  int  inc1z, int inc2z, double
          *work, int lwork);
void dcnvcor_64(char cnvcor, char four, long nx, double  *x,
          long  ifx,  long incx, long ny, long npre, long m,
          double *y, long ify, long inc1y, long inc2y,  long
          nz,  long k, double *z, long ifz, long inc1z, long
          inc2z, double *work, long lwork);
```

PURPOSE

dcnvcor computes the convolution or correlation of real vectors.

ARGUMENTS

CNVCOR (input)
'V' or 'v' if convolution is desired, 'R' or 'r' if correlation is desired.

FOUR (input)
'T' or 't' if the Fourier transform method is to be used, 'D' or 'd' if the computation should be done directly from the definition. The Fourier transform method is generally faster, but it may introduce noticeable errors into certain results, notably when both the filter and data vectors consist entirely of integers or vectors where elements of either the filter vector or a given data vector differ significantly in magnitude from the 1-norm of the vector.

NX (input)
Length of the filter vector. NX >= 0. DCNVCOR will return immediately if NX = 0.

X (input)
Filter vector.

IFX (input)
Index of the first element of X. NX >= IFX >= 1.

INCX (input)
Stride between elements of the filter vector in X. INCX > 0.

NY (input)
Length of the input vectors. NY >= 0. DCNVCOR will return immediately if NY = 0.

NPRE (input)
The number of implicit zeros prepended to the Y vectors. NPRE >= 0.

M (input)
Number of input vectors. M >= 0. DCNVCOR will return immediately if M = 0.

Y (input)
Input vectors.

IFY (input)
Index of the first element of Y. NY >= IFY >= 1.

INC1Y (input)
Stride between elements of the input vectors in Y. INC1Y > 0.

INC2Y (input)
Stride between the input vectors in Y. INC2Y > 0.

NZ (input)
Length of the output vectors. NZ >= 0. DCNVCOR will return immediately if NZ = 0. See the Notes section below for information about how this argument interacts with NX and NY to control circular versus end-off shifting.

K (input)
Number of Z vectors. K >= 0. If K = 0 then DCNVCOR will return immediately. If K < M then only the first K input vectors will be processed. If K > M then M input vectors will be processed.

Z (output)
Result vectors.

IFZ (input)
Index of the first element of Z. NZ >= IFZ >= 1.

INC1Z (input)
Stride between elements of the output vectors in Z. INC1Z > 0.

INC2Z (input)
Stride between the output vectors in Z. INC2Z > 0.

WORK (input/output)
Scratch space. Before the first call to DCNVCOR with particular values of the integer arguments the first element of WORK must be set to zero. If WORK is written between calls to DCNVCOR or if DCNVCOR is called with different values of the integer arguments then the first element of WORK must again be set to zero before each call. If WORK has not been written and the same values of the integer arguments are used then the first element of WORK to zero. This can avoid certain initializations that store their results into WORK, and avoiding the initialization can make DCNVCOR run faster.

LWORK (input)
Length of WORK. LWORK >= 4*MAX(NX,NY,NZ)+15.

NOTES

If any vector overlaps a writable vector, either because of argument aliasing or ill-chosen values of the various INC arguments, the results are undefined and may vary from one run to the next.

The most common form of the computation, and the case that executes fastest, is applying a filter vector X to a series of vectors stored in the columns of Y with the result placed into the columns of Z. In that case, INCX = 1, INC1Y = 1, INC2Y >= NY, INC1Z = 1, INC2Z >= NZ. Another common form is applying a filter vector X to a series of vectors stored in the rows of Y and store the result in the row of Z, in which case INCX = 1, INC1Y >= NY, INC2Y = 1, INC1Z >= NZ, and INC2Z = 1.

Contents

NAME

```
dcnvcor2 - compute the convolution or  correlation  of  real
matrices
```

SYNOPSIS

```
SUBROUTINE DCNVCOR2(CNVCOR, METHOD, TRANSX, SCRATCHX, TRANSY,
      SCRATCHY, MX, NX, X, LDX, MY, NY, MPRE, NPRE, Y, LDY, MZ, NZ, Z,
      LDZ, WORKIN, LWORK)

CHARACTER * 1  CNVCOR,  METHOD,  TRANSX,  SCRATCHX,  TRANSY,
SCRATCHY
DOUBLE COMPLEX WORKIN(*)
INTEGER MX, NX, LDX, MY, NY, MPRE, NPRE, LDY, MZ,  NZ,  LDZ,
LWORK
DOUBLE PRECISION X(LDX,*), Y(LDY,*), Z(LDZ,*)

SUBROUTINE DCNVCOR2_64(CNVCOR, METHOD, TRANSX, SCRATCHX, TRANSY,
      SCRATCHY, MX, NX, X, LDX, MY, NY, MPRE, NPRE, Y, LDY, MZ, NZ, Z,
      LDZ, WORKIN, LWORK)

CHARACTER * 1  CNVCOR,  METHOD,  TRANSX,  SCRATCHX,  TRANSY,
SCRATCHY
DOUBLE COMPLEX WORKIN(*)
INTEGER*8 MX, NX, LDX, MY, NY, MPRE, NPRE, LDY, MZ, NZ, LDZ,
LWORK
DOUBLE PRECISION X(LDX,*), Y(LDY,*), Z(LDZ,*)
```

F95 INTERFACE

```
SUBROUTINE CNVCOR2(CNVCOR, METHOD, TRANSX, SCRATCHX, TRANSY,
       SCRATCHY, [MX], [NX], X, [LDX], [MY], [NY], MPRE, NPRE, Y, [LDY],
       [MZ], [NZ], Z, [LDZ], WORKIN, [LWORK])
```

```
CHARACTER(LEN=1)  ::  CNVCOR,  METHOD,   TRANSX,   SCRATCHX,
TRANSY, SCRATCHY
COMPLEX(8), DIMENSION(:) :: WORKIN
INTEGER :: MX, NX, LDX, MY, NY, MPRE,  NPRE,  LDY,  MZ,  NZ,
LDZ, LWORK
REAL(8), DIMENSION(:,:) :: X, Y, Z

SUBROUTINE CNVCOR2_64(CNVCOR, METHOD, TRANSX, SCRATCHX, TRANSY,
       SCRATCHY, [MX], [NX], X, [LDX], [MY], [NY], MPRE, NPRE, Y, [LDY],
       [MZ], [NZ], Z, [LDZ], WORKIN, [LWORK])

CHARACTER(LEN=1)  ::  CNVCOR,  METHOD,   TRANSX,   SCRATCHX,
TRANSY, SCRATCHY
COMPLEX(8), DIMENSION(:) :: WORKIN
INTEGER(8) :: MX, NX, LDX, MY, NY, MPRE, NPRE, LDY, MZ,  NZ,
LDZ, LWORK
REAL(8), DIMENSION(:,:) :: X, Y, Z
```

C INTERFACE

```
#include <sunperf.h>

void dcnvcor2(char cnvcor, char method,  char  transx,  char
          scratchx,  char transy, char scratchy, int mx, int
          nx, double *x, int ldx, int my, int ny, int  mpre,
          int npre, double *y, int ldy, int mz, int nz, dou-
          ble  *z,  int  ldz,  doublecomplex  *workin,   int
          lwork);

void dcnvcor2_64(char cnvcor, char method, char transx, char
          scratchx,  char  transy,  char  scratchy, long mx,
          long nx, double *x, long ldx, long  my,  long  ny,
          long  mpre,  long  npre, double *y, long ldy, long
          mz, long nz, double *z,  long  ldz,  doublecomplex
          *workin, long lwork);
```

PURPOSE

dcnvcor2 computes the convolution or correlation of real matrices.

ARGUMENTS

CNVCOR (input)
'V' or 'v' to compute convolution, 'R' or 'r' to compute correlation.

METHOD (input)
'T' or 't' if the Fourier transform method is to

```
          be  used,  'D' or 'd' to compute directly from the
          definition.

TRANSX (input)
          'N' or 'n' if X is the filter matrix, 'T'  or  't'
          if transpose(X) is the filter matrix.

SCRATCHX (input)
          'N' or 'n' if X must be preserved, 'S' or 's' if X
          can  be  used as scratch space.  The contents of X
          are undefined after returning from a call in which
          X is allowed to be used for scratch.

TRANSY (input)
          'N' or 'n' if Y is the input matrix, 'T' or 't' if
          transpose(Y) is the input matrix.
SCRATCHY (input)
          'N' or 'n' if Y must be preserved, 'S' or 's' if Y
          can  be  used as scratch space.  The contents of Y
          are undefined after returning from a call in which
          Y is allowed to be used for scratch.

MX (input)
          Number of rows in the filter matrix.  MX >= 0.

NX (input)
          Number of columns in the filter matrix.  NX >= 0.

X (input)  dimension(LDX,NX)
          On entry, the filter matrix.  Unchanged on exit if
          SCRATCHX  is  'N'  or  'n',  undefined  on exit if
          SCRATCHX is 'S' or 's'.

LDX (input)
          Leading dimension of the array that  contains  the
          filter matrix.

MY (input)
          Number of rows in the input matrix.  MY >= 0.

NY (input)
          Number of columns in the input matrix.  NY >= 0.

MPRE (input)
          Number of implicit zeros to prepend to each row of
          the input matrix.  MPRE >= 0.

NPRE (input)
          Number of implicit zeros to prepend to each column
          of the input matrix.  NPRE >= 0.
```

```
Y (input)  dimension(LDY,*)
          Input matrix.  Unchanged on exit  if  SCRATCHY  is
          'N'  or  'n', undefined on exit if SCRATCHY is 'S'
          or 's'.

LDY (input)
          Leading dimension of the array that  contains  the
          input matrix.

MZ (input)
          Number of rows in the output  matrix.   MZ  >=  0.
          DCNVCOR2 will return immediately if MZ = 0.

NZ (input)
          Number of columns in the output matrix.  NZ >=  0.
          DCNVCOR2 will return immediately if NZ = 0.

Z (output)
           dimension(LDZ,*)
          Result matrix.

LDZ (input)
          Leading dimension of the array that  contains  the
          result matrix.  LDZ >= MAX(1,MZ).

WORKIN (input/output)
          (input/scratch) dimension(LWORK)
          On entry for the first call to DCNVCOR2, WORKIN(1)
          must contain 0.0.  After the first call, WORKIN(1)
          must be set to 0.0 iff  WORKIN  has  been  altered
          since  the  last call to this subroutine or if the
          sizes of the arrays have changed.

LWORK (input)
          Length of the work vector.  If the FFT  is  to  be
          used  then for best performance LWORK should be at
          least 30 words longer than the  amount  of  memory
          needed to hold the trig tables.  If the FFT is not
          used, the value of LWORK is unimportant.
```

Contents

NAME

dcoomm - coordinate matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE DCOOMM( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, JNDX, NNZ,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, K, DESCRA(5), NNZ
*           LDB, LDC, LWORK
 INTEGER    INDX(NNZ), JNDX(NNZ)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE DCOOMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, JNDX, NNZ,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, K, DESCRA(5), NNZ
*           LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), JNDX(NNZ)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE COOMM( TRANSA, M, [N], K, ALPHA, DESCRA,
*           VAL, INDX, JNDX, NNZ, B, [LDB], BETA, C, [LDC],
*           [WORK], [LWORK] )
 INTEGER TRANSA, M, K,  NNZ
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, JNDX
 DOUBLE PRECISION   ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

```
       SUBROUTINE COOMM_64( TRANSA, M, [N], K, ALPHA, DESCRA,
      *           VAL, INDX, JNDX, NNZ, B, [LDB], BETA, C, [LDC],
      *           [WORK], [LWORK] )
       INTEGER*8 TRANSA, M, K,  NNZ
       INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, JNDX
       DOUBLE PRECISION   ALPHA, BETA
       DOUBLE PRECISION, DIMENSION(:) :: VAL
       DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
              C <- alpha op(A) B + beta C

      where ALPHA and BETA are scalar, C and B are dense matrices,
      A is a matrix represented in coordinate format and
      op( A )  is one  of
      op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                           ( ' indicates matrix transpose)
```

ARGUMENTS

```
      TRANSA        Indicates how to operate with the sparse matrix
                      0 : operate with matrix
                      1 : operate with transpose matrix
                      2 : operate with the conjugate transpose of matrix.
                          2 is equivalent to 1 if matrix is real.

      M             Number of rows in matrix A

      N             Number of columns in matrix C

      K             Number of columns in matrix A

      ALPHA         Scalar parameter

      DESCRA()      Descriptor argument.  Five element integer array
                    DESCRA(1) matrix structure
                      0 : general
                      1 : symmetric (A=A')
                      2 : Hermitian (A= CONJG(A'))
                      3 : Triangular
                      4 : Skew(Anti)-Symmetric (A=-A')
                      5 : Diagonal
                      6 : Skew-Hermitian (A= -CONJG(A'))
                    DESCRA(2) upper/lower triangular indicator
                      1 : lower
                      2 : upper
```

```
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

 VAL()          scalar array of length NNZ consisting of the
                non-zero entries of A, in any order.

 INDX()         integer array of length NNZ consisting of the
                corresponding row indices of the entries of A.

 JNDX()         integer array of length NNZ consisting of the
                corresponding column indices of the entries of A.

 NNZ            number of non-zero elements in A.
 B()            rectangular array with first dimension LDB.

 LDB            leading dimension of B

 BETA           Scalar parameter

 C()            rectangular array with first dimension LDC.

 LDC            leading dimension of C

 WORK()         scratch array of length LWORK. WORK is not
                referenced in the current version.

 LWORK          length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

```
dcopy - Copy x to y
```

SYNOPSIS

```
SUBROUTINE DCOPY(N, X, INCX, Y, INCY)

INTEGER N, INCX, INCY
DOUBLE PRECISION X(*), Y(*)

SUBROUTINE DCOPY_64(N, X, INCX, Y, INCY)

INTEGER*8 N, INCX, INCY
DOUBLE PRECISION X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE COPY([N], X, [INCX], Y, [INCY])

INTEGER :: N, INCX, INCY
REAL(8), DIMENSION(:) :: X, Y

SUBROUTINE COPY_64([N], X, [INCX], Y, [INCY])

INTEGER(8) :: N, INCX, INCY
REAL(8), DIMENSION(:) :: X, Y
```

C INTERFACE

```
#include <sunperf.h>

void dcopy(int n, double *x, int incx, double *y, int incy);

void dcopy_64(long n, double *x, long incx, double *y,  long
          incy);
```

PURPOSE

dcopy Copy x to y where x and y are n-vectors.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (output)
(1 + (m - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the vector x.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
dcosqb - synthesize a Fourier sequence from its  representa-
tion in terms of a cosine series with odd wave numbers.  The
COSQ operations are unnormalized inverses of themselves,  so
a  call  to  COSQF followed by a call to COSQB will multiply
the input sequence by 4 * N.
```

SYNOPSIS

```
SUBROUTINE DCOSQB(N, X, WSAVE)

INTEGER N
DOUBLE PRECISION X(*), WSAVE(*)

SUBROUTINE DCOSQB_64(N, X, WSAVE)

INTEGER*8 N
DOUBLE PRECISION X(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE COSQB([N], X, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: X, WSAVE

SUBROUTINE COSQB_64([N], X, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: X, WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void dcosqb(int n, double *x, double *wsave);
```

```
void dcosqb_64(long n, double *x, double *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence to be  transformed.   These
          subroutines are most efficient when N is a product
          of small primes.  N >= 0.

X (input/output)
          On entry, an array  of  length  N  containing  the
          sequence   to   be   transformed.   On  exit,  the
          quarter-wave cosine synthesis of the input.
WSAVE (input)
          On entry, an array with dimension of at least (3 *
          N + 15) that has been initialized by DCOSQI.
```

Contents

NAME

```
dcosqf - compute the Fourier coefficients in a cosine series
representation  with only odd wave numbers.  The COSQ opera-
tions are unnormalized inverses of themselves, so a call  to
COSQF  followed  by  a call to COSQB will multiply the input
sequence by 4 * N.
```

SYNOPSIS

```
SUBROUTINE DCOSQF(N, X, WSAVE)

INTEGER N
DOUBLE PRECISION X(*), WSAVE(*)

SUBROUTINE DCOSQF_64(N, X, WSAVE)

INTEGER*8 N
DOUBLE PRECISION X(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE COSQF([N], X, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: X, WSAVE

SUBROUTINE COSQF_64([N], X, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: X, WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void dcosqf(int n, double *x, double *wsave);
```

```
void dcosqf_64(long n, double *x, double *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence to be  transformed.   These
          subroutines are most efficient when N is a product
          of small primes.  N >= 0.

X (input/output)
          On entry, an array  of  length  N  containing  the
          sequence   to   be   transformed.   On  exit,  the
          quarter-wave cosine transform of the input.
WSAVE (input)
          On entry, an array with dimension of at least   (3
          * N + 15) that has been initialized by DCOSQI.
```

Contents

NAME

```
dcosqi - initialize the array WSAVE, which is used  in  both
COSQF and COSQB.
```

SYNOPSIS

```
SUBROUTINE DCOSQI(N, WSAVE)

INTEGER N
DOUBLE PRECISION WSAVE(*)

SUBROUTINE DCOSQI_64(N, WSAVE)

INTEGER*8 N
DOUBLE PRECISION WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE COSQI(N, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: WSAVE

SUBROUTINE COSQI_64(N, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void dcosqi(int n, double *wsave);

void dcosqi_64(long n, double *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. The method is most efficient when N is a product of small primes.

WSAVE (input)
On entry, an array of dimension (3 * N + 15) or greater. DCOSQI needs to be called only once to initialize WSAVE before calling DCOSQF and/or DCOSQB if N and WSAVE remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

dcost - compute the discrete Fourier cosine transform of an even sequence. The COST transforms are unnormalized inverses of themselves, so a call of COST followed by another call of COST will multiply the input sequence by 2 * (N-1).

SYNOPSIS

```
SUBROUTINE DCOST(N, X, WSAVE)

INTEGER N
DOUBLE PRECISION X(*), WSAVE(*)

SUBROUTINE DCOST_64(N, X, WSAVE)

INTEGER*8 N
DOUBLE PRECISION X(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE COST([N], X, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: X, WSAVE

SUBROUTINE COST_64([N], X, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: X, WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void dcost(int n, double *x, double *wsave);
```

```
void dcost_64(long n, double *x, double *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence to be  transformed.   These
          subroutines  are  most  efficient  when N - 1 is a
          product of small primes.  N >= 2.

X (input/output)
          On entry, an array  of  length  N  containing  the
          sequence  to  be transformed.  On exit, the cosine
          transform of the input.
WSAVE (input)
          On entry, an array with dimension of at least   (3
          * N + 15), initialized by DCOSTI.
```

Contents

NAME

```
dcosti - initialize the array WSAVE, which is used in COST.
```

SYNOPSIS

```
SUBROUTINE DCOSTI(N, WSAVE)

INTEGER N
DOUBLE PRECISION WSAVE(*)

SUBROUTINE DCOSTI_64(N, WSAVE)

INTEGER*8 N
DOUBLE PRECISION WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE COSTI(N, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: WSAVE

SUBROUTINE COSTI_64(N, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void dcosti(int n, double *wsave);

void dcosti_64(long n, double *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. The method is most efficient when N - 1 is a product of small primes. N >= 2.

WSAVE (input)
On entry, an array of dimension (3 * N + 15) or greater. DCOSTI is called once to initialize WSAVE before calling DCOST and need not be called again between calls to DCOST if N and WSAVE remain unchanged. Thus, subsequent transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

dcscmm - compressed sparse column format matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE DCSCMM( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, K, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTRB(K), PNTRE(K)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE DCSCMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, K, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTRB(K), PNTRE(K)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTRE(K)-PNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE CSCMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER TRANSA, M, K
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL
```

```
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C

 SUBROUTINE CSCMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER*8 TRANSA, M, K
 INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in compressed sparse column format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
```

```
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices

 VAL()         scalar array of length NNZ consisting of nonzero
               entries of A.

 INDX()        integer array of length NNZ consisting of the row
               indices of nonzero entries of A.

 PNTRB()       integer array of length K such that PNTRB(J)-PNTRB(1)+1
               points to location in VAL of the first nonzero element
               in column J.
 PNTRE()       integer array of length K such that PNTRE(J)-PNTRB(1)
               points to location in VAL of the last nonzero element
               in column J.

 B()           rectangular array with first dimension LDB.

 LDB           leading dimension of B

 BETA          Scalar parameter

 C()           rectangular array with first dimension LDC.

 LDC           leading dimension of C

 WORK()        scratch array of length LWORK. WORK is not
               referenced in the current version.

 LWORK         length of WORK array. LWORK is not referenced
               in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS)
Standard", University of Tennessee, Knoxville, Tennessee,

1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

It is known that there exists another representation of the compressed sparse column format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each column in the arrays VAL and INDX is used instead of two arrays PNTRB and PNTRE. To use the routine with this kind of sparse column format the following calling sequence should be used

```
  SUBROUTINE SCSCMM( TRANSA, M, N, K, ALPHA, DESCRA,
 *           VAL, INDX, IA, IA(2), B, LDB, BETA,
 *           C, LDC, WORK, LWORK )
```

Contents

NAME

```
dcscsm - compressed sparse column format triangular solve
```

SYNOPSIS

```
 SUBROUTINE DCSCSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTRB(M), PNTRE(M)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE DCSCSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTRB(M), PNTRE(M)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTRE(M)-PNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE CSCSM( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER TRANSA, M, UNITD
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL, DV
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE CSCSM_64(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER*8 TRANSA, M, UNITD
 INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL, DV
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a diagonal scaling matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in compressed sparse column
format and op( A )  is one  of
op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic column scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                  0 : general
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))

                Note: For the routine, DESCRA(1)=3 is only supported.

                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices
VAL()           scalar array of length NNZ consisting of nonzero entries
                of A.

INDX()          integer array of length NNZ consisting of the row indices
                of nonzero entries of A. (Row indices MUST be sorted in
                increasing order for each column).

PNTRB()         integer array of length M such that PNTRB(J)-PNTRB(1)+1
                points to location in VAL of the first nonzero element
                in column J.

PNTRE()         integer array of length M such that PNTRE(J)-PNTRB(1)
                points to location in VAL of the last nonzero element
                in column J.

B()             rectangular array with first dimension LDB.

LDB             leading dimension of B

BETA            Scalar parameter

C()             rectangular array with first dimension LDC.

LDC             leading dimension of C

WORK()          scratch array of length LWORK.

                On exit,  if LWORK = -1, WORK(1) returns the optimum  LWORK.
```

LWORK length of WORK array. LWORK should be at least M.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the columns of A such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the columns have been scaled. UNITD=3 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the column number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the CSC representation

of a sparse matrix. They are not used anyway in these cases.
But if UNITD=4, the unit diagonal elements MUST be
referenced in the CSC representation.

4. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

5. It is known that there exists another representation of the compressed sparse column format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each column in the arrays VAL and INDX is used instead of two arrays PNTRB and PNTRE. To use the routine with this kind of sparse column format the following calling sequence should be used

```
  SUBROUTINE SCSCSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, INDX, IA, IA(2), B, LDB, BETA,
 *           C, LDC, WORK, LWORK )
```

Contents

NAME

```
dcsrmm - compressed sparse row format matrix-matrix multiply
```

SYNOPSIS

```
 SUBROUTINE DCSRMM( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, K, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTRB(M), PNTRE(M)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE DCSRMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, K, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTRB(M), PNTRE(M)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTRE(M)-PNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE CSRMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER TRANSA, M, K
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE CSRMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER*8 TRANSA, M, K
 INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in compressed sparse row format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
```

```
               DESCRA(3) main diagonal type
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices

 VAL()         scalar array of length NNZ consisting of nonzero
               entries of A.

 INDX()        integer array of length NNZ consisting of the
               column indices of nonzero entries of A.

 PNTRB()       integer array of length M such that PNTRB(J)-PNTRB(1)+1
               points to location in VAL of the first nonzero element
               in row J.
 PNTRE()       integer array of length M such that PNTRE(J)-PNTRB(1)
               points to location in VAL of the last nonzero element
               in row J.

 B()           rectangular array with first dimension LDB.

 LDB           leading dimension of B

 BETA          Scalar parameter

 C()           rectangular array with first dimension LDC.

 LDC           leading dimension of C

 WORK()        scratch array of length LWORK. WORK is not
               referenced in the current version.

 LWORK         length of WORK array. LWORK is not referenced
               in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

It is known that there exists another representation of the compressed sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each row in the arrays VAL and INDX is used instead of two arrays PNTRB and PNTRE. To use the routine with this kind of compressed sparse row format the following calling sequence should be used

```
  SUBROUTINE SCSRMM( TRANSA, M, N, K, ALPHA, DESCRA,
 *           VAL, INDX, IA, IA(2), B, LDB, BETA,
 *           C, LDC, WORK, LWORK )
```

Contents

NAME

dcsrsm - compressed sparse row format triangular solve

SYNOPSIS

```
 SUBROUTINE DCSRSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTRB(M), PNTRE(M)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE DCSRSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTRB(M), PNTRE(M)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTRE(M)-PNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE CSRSM( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER TRANSA, M, UNITD
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL, DV
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE CSRSM_64(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER*8 TRANSA, M, UNITD
 INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL, DV
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a diagonal scaling matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in compressed sparse row
format and op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of
              the scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                  0 : general
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))

                Note: For the routine, only DESCRA(1)=3 is supported.

                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices
VAL()           scalar array of length NNZ consisting of nonzero entries
                of A.

INDX()          integer array of length NNZ consisting of the column
                indices of nonzero entries of A (column indices MUST be
                sorted in increasing order for each row)

PNTRB()         integer array of length M such that PNTRB(J)-PNTRB(1)+1
                points to location in VAL of the first nonzero element
                in row J.

PNTRE()         integer array of length M such that PNTRE(J)-PNTRB(1)
                points to location in VAL of the last nonzero element
                in row J.

B()             rectangular array with first dimension LDB.

LDB             leading dimension of B

BETA            Scalar parameter

C()             rectangular array with first dimension LDC.

LDC             leading dimension of C

WORK()          scratch array of length LWORK.

                On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.
```

LWORK length of WORK array. LWORK should be at least M.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the rows of A such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows have been scaled. UNITD=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the CSR representation of a sparse matrix. They are not used anyway in these cases.

But if UNITD=4, the unit diagonal elements MUST be referenced in the CSR representation.

4. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

5. It is known that there exists another representation of the compressed sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each row in the arrays VAL and INDX is used instead of two arrays PNTRB and PNTRE. To use the routine with this kind of compressed sparse row format the following calling sequence should be used

```
  SUBROUTINE SCSRSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, INDX, IA, IA(2), B, LDB, BETA, C,
 *            LDC, WORK, LWORK )
```

Contents

NAME

ddiamm - diagonal format matrix-matrix multiply

SYNOPSIS

```
  SUBROUTINE DDIAMM( TRANSA, M, N, K, ALPHA, DESCRA,
 *           VAL, LDA, IDIAG, NDIAG,
 *           B, LDB, BETA, C, LDC, WORK, LWORK)
  INTEGER    TRANSA, M, N, K, DESCRA(5), LDA, NDIAG,
 *           LDB, LDC, LWORK
  INTEGER    IDIAG(NDIAG)
  DOUBLE PRECISION ALPHA, BETA
  DOUBLE PRECISION VAL(LDA,NDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)

  SUBROUTINE DDIAMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
 *           VAL, LDA, IDIAG, NDIAG,
 *           B, LDB, BETA, C, LDC, WORK, LWORK)
  INTEGER*8  TRANSA, M, N, K, DESCRA(5), LDA, NDIAG,
 *           LDB, LDC, LWORK
  INTEGER*8  IDIAG(NDIAG)
  DOUBLE PRECISION ALPHA, BETA
  DOUBLE PRECISION VAL(LDA,NDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
   SUBROUTINE DIAMM(TRANSA, M, [N], K, ALPHA, DESCRA, VAL, [LDA],
 *    IDIAG, NDIAG, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
  INTEGER    TRANSA, M, K,  NDIAG
  INTEGER, DIMENSION(:) ::    DESCRA, IDIAG
  DOUBLE PRECISION    ALPHA, BETA
  DOUBLE PRECISION, DIMENSION(:, :) ::  VAL, B, C

   SUBROUTINE DIAMM_64(TRANSA, M, [N], K, ALPHA, DESCRA, VAL, [LDA],
 *    IDIAG, NDIAG, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
```

```
 INTEGER*8    TRANSA, M, K,  NDIAG
 INTEGER*8, DIMENSION(:) ::    DESCRA, IDIAG
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:, :) ::  VAL, B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in diagonal format and op( A ) is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)

TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
                0 : non-unit
                1 : unit
              DESCRA(4) Array base  (NOT IMPLEMENTED)
                0 : C/C++ compatible
                1 : Fortran compatible
              DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                0 : unknown
                1 : no repeated indices
```

```
VAL()          two-dimensional LDA-by-NDIAG array such that VAL(:,I)
               consists of non-zero elements on diagonal IDIAG(I)
               of A.  Diagonals in the lower triangular part of A
               are padded from the top, and those in the upper
               triangular part are padded from the bottom.

LDA            leading dimension of VAL, must be .GE. MIN(M,K)

IDIAG()        integer array of length NDIAG consisting of the
               corresponding diagonal offsets of the non-zero
               diagonals of A in VAL.  Lower triangular diagonals
               have negative offsets, the main diagonal has offset
               0, and upper triangular diagonals have positive offset.

NDIAG          number of non-zero diagonals in A.

B()            rectangular array with first dimension LDB.
LDB            leading dimension of B

BETA           Scalar parameter

C()            rectangular array with first dimension LDC.

LDC            leading dimension of C

WORK()         scratch array of length LWORK. WORK is not
               referenced in the current version.

LWORK          length of WORK array. LWORK is not referenced
               in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS)
Standard", University of Tennessee, Knoxville, Tennessee,
1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

ddiasm - diagonal format triangular solve

SYNOPSIS

```
 SUBROUTINE DDIASM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, LDA, IDIAG, NDIAG,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5), LDA, NDIAG,
*           LDB, LDC, LWORK
 INTEGER    IDIAG(NDIAG)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION DV(M), VAL(LDA,NDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE DDIASM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, LDA, IDIAG, NDIAG,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5), LDA, NDIAG,
*           LDB, LDC, LWORK
 INTEGER*8  IDIAG(NDIAG)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION DV(M), VAL(LDA,NDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
  SUBROUTINE DIASM(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
*   [LDA], IDIAG, NDIAG, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, NDIAG
 INTEGER, DIMENSION(:) ::    DESCRA, IDIAG
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) ::  DV
 DOUBLE PRECISION, DIMENSION(:, :) ::  VAL, B, C

  SUBROUTINE DIASM_64(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
```

```
*   [LDA], IDIAG, NDIAG, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, NDIAG
 INTEGER*8, DIMENSION(:) ::    DESCRA, IDIAG
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) ::  DV
 DOUBLE PRECISION, DIMENSION(:, :) ::  VAL, B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a diagonal scaling matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in diagonal format and
op( A )  is one  of

 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
```

```
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))
                Note: For the routine, only DESCRA(1)=3 is supported.

                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices
VAL()           two-dimensional LDA-by-NDIAG array such that VAL(:,I)
                consists of non-zero elements on diagonal IDIAG(I)
                of A.  Diagonals in the lower triangular part of A
                are padded from the top, and those in the upper
                triangular part are padded from the bottom.

LDA             leading dimension of VAL, must be .GE. MIN(M,K)

IDIAG()         integer array of length NDIAG consisting of the
                corresponding diagonal offsets of the non-zero
                diagonals of A in VAL.  Lower triangular diagonals
                have negative offsets, the main diagonal has offset
                0, and upper triangular diagonals have positive offset.
                Elements of IDIAG of MUST be sorted in  increasing order.

NDIAG           number of non-zero diagonals in A.

B()             rectangular array with first dimension LDB.

LDB             leading dimension of B

BETA            Scalar parameter

C()             rectangular array with first dimension LDC.

LDC             leading dimension of C

WORK()          scratch array of length LWORK.
                On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.
```

LWORK length of WORK array. LWORK should be at least M.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/
"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the rows of A such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows have been scaled. UNITD=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the DIA representation of a sparse matrix. They are not used anyway in these cases. But if UNITD=4, the unit diagonal elements MUST be referenced in the DIA representation.

4. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

```
ddisna - compute the reciprocal condition  numbers  for  the
eigenvectors of a real symmetric or complex Hermitian matrix
or for the left or right singular vectors of  a  general  m-
by-n matrix
```

SYNOPSIS

```
SUBROUTINE DDISNA(JOB, M, N, D, SEP, INFO)

CHARACTER * 1 JOB
INTEGER M, N, INFO
DOUBLE PRECISION D(*), SEP(*)

SUBROUTINE DDISNA_64(JOB, M, N, D, SEP, INFO)

CHARACTER * 1 JOB
INTEGER*8 M, N, INFO
DOUBLE PRECISION D(*), SEP(*)
```

F95 INTERFACE

```
SUBROUTINE DISNA(JOB, M, N, D, SEP, [INFO])

CHARACTER(LEN=1) :: JOB
INTEGER :: M, N, INFO
REAL(8), DIMENSION(:) :: D, SEP

SUBROUTINE DISNA_64(JOB, M, N, D, SEP, [INFO])

CHARACTER(LEN=1) :: JOB
INTEGER(8) :: M, N, INFO
REAL(8), DIMENSION(:) :: D, SEP
```

C INTERFACE

```
#include <sunperf.h>

void ddisna(char job, int m, int n, double *d, double  *sep,
          int *info);

void ddisna_64(char job, long m, long n, double  *d,  double
          *sep, long *info);
```

PURPOSE

ddisna computes the reciprocal condition numbers for the eigenvectors of a real symmetric or complex Hermitian matrix or for the left or right singular vectors of a general m-by-n matrix. The reciprocal condition number is the 'gap' between the corresponding eigenvalue or singular value and the nearest other one.

The bound on the error, measured by angle in radians, in the I-th computed vector is given by

```
       SLAMCH( 'E' ) * ( ANORM / SEP( I ) )
```

where ANORM = 2-norm(A) = max(abs(D(j))). SEP(I) is not allowed to be smaller than SLAMCH('E')*ANORM in order to limit the size of the error bound.

SDISNA may also be used to compute error bounds for eigenvectors of the generalized symmetric definite eigenproblem.

ARGUMENTS

```
JOB (input)
          Specifies for which problem the reciprocal  condi-
          tion numbers should be computed:
          = 'E':  the eigenvectors of a  symmetric/Hermitian
          matrix;
          = 'L':  the left singular  vectors  of  a  general
          matrix;
          = 'R':  the right singular vectors  of  a  general
          matrix.

M (input) The number of rows of the matrix. M >= 0.

N (input) If JOB = 'L' or 'R', the number of columns of  the
          matrix,  in  which  case  N >= 0. Ignored if JOB =
          'E'.
```

D (input) dimension (min(M,N)) if JOB = 'L' or 'R' The eigenvalues (if JOB = 'E') or singular values (if JOB = 'L' or 'R') of the matrix, in either increasing or decreasing order. If singular values, they must be non-negative.

SEP (output)
dimension (min(M,N)) if JOB = 'L' or 'R' The reciprocal condition numbers of the vectors.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

```
ddot - compute the dot product of two vectors x and y.
```

SYNOPSIS

```
DOUBLE PRECISION FUNCTION DDOT(N, X, INCX, Y, INCY)

INTEGER N, INCX, INCY
DOUBLE PRECISION X(*), Y(*)

DOUBLE PRECISION FUNCTION DDOT_64(N, X, INCX, Y, INCY)

INTEGER*8 N, INCX, INCY
DOUBLE PRECISION X(*), Y(*)
```

F95 INTERFACE

```
REAL(8) FUNCTION DOT([N], X, [INCX], Y, [INCY])

INTEGER :: N, INCX, INCY
REAL(8), DIMENSION(:) :: X, Y

REAL(8) FUNCTION DOT_64([N], X, [INCX], Y, [INCY])

INTEGER(8) :: N, INCX, INCY
REAL(8), DIMENSION(:) :: X, Y
```

C INTERFACE

```
#include <sunperf.h>

double ddot(int n, double  *x,  int  incx,  double  *y,  int
          incy);

double ddot_64(long n, double *x, long incx, double *y, long
```

```
           incy);
```

PURPOSE

ddot compute the dot product of x and y where x and y are n-vectors.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. If N is not positive then the function returns the value 0.0. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input)
(1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. Unchanged on exit.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
ddoti - Compute the indexed dot product.
```

SYNOPSIS

```
 DOUBLE PRECISION FUNCTION DDOTI(NZ, X, INDX, Y)

 DOUBLE PRECISION X(*), Y(*)
 INTEGER NZ
 INTEGER INDX(*)

 DOUBLE PRECISION FUNCTION DDOTI_64(NZ, X, INDX, Y)

 DOUBLE PRECISION X(*), Y(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 DOUBLE PRECISION FUNCTION DOTI([NZ], X, INDX, Y)

 REAL(8), DIMENSION(:) :: X, Y
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 DOUBLE PRECISION FUNCTION DOTI_64([NZ], X, INDX, Y)

 REAL(8), DIMENSION(:) :: X, Y
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
DDOTI Compute the indexed dot product of a real sparse
vector x stored in compressed form with a real vector y in
```

full storage form.

```
dot = 0
do i = 1, n
  dot = dot + x(i) * y(indx(i))
enddo
```

ARGUMENTS

NZ (input)
Number of elements in the compressed form. Unchanged on exit.

X (input)
Vector in compressed form. Unchanged on exit.

INDX (input)
Vector containing the indices of the compressed form. It is assumed that the elements in INDX are distinct and greater than zero. Unchanged on exit.

Y (input)
Vector in full storage form. Only the elements corresponding to the indices in INDX will be accessed.

Contents

NAME

dellmm - Ellpack format matrix-matrix multiply

SYNOPSIS

```
  SUBROUTINE DELLMM( TRANSA, M, N, K, ALPHA, DESCRA,
 *           VAL, INDX, LDA, MAXNZ,
 *           B, LDB, BETA, C, LDC, WORK, LWORK)
  INTEGER    TRANSA, M, N, K, DESCRA(5), LDA, MAXNZ,
 *           LDB, LDC, LWORK
  INTEGER    INDX(LDA,MAXNZ)
  DOUBLE PRECISION ALPHA, BETA
  DOUBLE PRECISION VAL(LDA,MAXNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

  SUBROUTINE DELLMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
 *           VAL, INDX, LDA, MAXNZ,
 *           B, LDB, BETA, C, LDC, WORK, LWORK)
  INTEGER*8  TRANSA, M, N, K, DESCRA(5), LDA, MAXNZ,
 *           LDB, LDC, LWORK
  INTEGER*8  INDX(LDA,MAXNZ)
  DOUBLE PRECISION ALPHA, BETA
  DOUBLE PRECISION VAL(LDA,MAXNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
  SUBROUTINE ELLMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
 *        [LDA], MAXNZ, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
  INTEGER    TRANSA, M, K, MAXNZ
  INTEGER, DIMENSION(:) ::  DESCRA
  INTEGER, DIMENSION(:, :) ::    INDX
  DOUBLE PRECISION    ALPHA, BETA
  DOUBLE PRECISION, DIMENSION(:, :) ::  VAL, B, C

  SUBROUTINE ELLMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
```

```
*        [LDA], MAXNZ, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, K, MAXNZ
 INTEGER*8, DIMENSION(:) ::  DESCRA
 INTEGER*8, DIMENSION(:, :) ::    INDX
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:, :) ::  VAL, B, C
```

DESCRIPTION

```
where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in  Ellpack format format and
op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
                0 : non-unit
                1 : unit
```

```
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

 VAL()          two-dimensional LDA-by-MAXNZ array such that VAL(I,:)
                consists of non-zero elements in row I of A, padded by
                zero values if the row contains less than MAXNZ.

 INDX()         two-dimensional integer LDA-by-MAXNZ array such
                INDX(I,:) consists of the column indices of the
                nonzero elements in row I, padded by the integer
                value I if the number of nonzeros is less than MAXNZ.

 LDA            leading dimension of VAL and INDX.

 MAXNZ          max number of nonzeros elements per row.

 B()            rectangular array with first dimension LDB.
 LDB            leading dimension of B

 BETA           Scalar parameter

 C()            rectangular array with first dimension LDC.

 LDC            leading dimension of C

 WORK()         scratch array of length LWORK. WORK is not
                referenced in the current version.

 LWORK          length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS)
Standard", University of Tennessee, Knoxville, Tennessee,
1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

dellsm - Ellpack format triangular solve

SYNOPSIS

```
 SUBROUTINE DELLSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, LDA, MAXNZ,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5), LDA, MAXNZ,
*           LDB, LDC, LWORK
 INTEGER    INDX(LDA,MAXNZ)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION DV(M), VAL(LDA,MAXNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE DELLSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, LDA, MAXNZ,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5), LDA, MAXNZ,
*           LDB, LDC, LWORK
 INTEGER*8  INDX(LDA,MAXNZ)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION DV(M), VAL(LDA,MAXNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE ELLSM( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
*    INDX, [LDA], MAXNZ, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M,  MAXNZ
 INTEGER, DIMENSION(:) ::  DESCRA
 INTEGER, DIMENSION(:, :) ::    INDX
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) ::  DV
 DOUBLE PRECISION, DIMENSION(:, :) ::  VAL, B, C
```

```
 SUBROUTINE ELLSM_64( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
*    INDX, [LDA], MAXNZ, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M,  MAXNZ
 INTEGER*8, DIMENSION(:) ::  DESCRA
 INTEGER*8, DIMENSION(:, :) ::    INDX
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) ::  DV
 DOUBLE PRECISION, DIMENSION(:, :) ::  VAL, B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a diagonal scaling matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in Ellpack format and
op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                  0 : general
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))

                Note: For the routine, only DESCRA(1)=3 is supported.

                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices
VAL()           two-dimensional LDA-by-MAXNZ array such that VAL(I,:)
                consists of non-zero elements in row I of A, padded by
                zero values if the row contains less than MAXNZ.

INDX()          two-dimensional integer LDA-by-MAXNZ array such
                INDX(I,:) consists of the column indices of the
                nonzero elements in row I, padded by the integer
                value I if the number of nonzeros is less than MAXNZ.
                The column indices MUST be sorted in increasing order
                for each row.

LDA             leading dimension of VAL and INDX.

MAXNZ           max number of nonzeros elements per row.

B()             rectangular array with first dimension LDB.

LDB             leading dimension of B

BETA            Scalar parameter

C()             rectangular array with first dimension LDC.

LDC             leading dimension of C

WORK()          scratch array of length LWORK.

                On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.
```

```
LWORK         length of WORK array.  LWORK should be at least M.

              For good performance, LWORK should generally be larger.
              For optimum performance on multiple processors, LWORK
              >=M*N_CPUS where N_CPUS is the maximum number of
              processors available to the program.

              If LWORK=0, the routine is to allocate workspace needed.

              If LWORK = -1, then a workspace query is assumed; the
              routine only calculates the optimum size of the WORK
              array, returns this value as the first entry of the WORK
              array, and no error message related to LWORK is issued
              by XERBLA.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the rows of A such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows have been scaled. UNITD=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the ELL representation of a sparse matrix. They are not used anyway in these cases.

But if UNITD=4, the unit diagonal elements MUST be referenced in the ELL representation.

4. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

```
dezftb - computes a periodic sequence from its Fourier coef-
ficients.   DEZFTB  is  a  simplified  but slower version of
DFFTB.
```

SYNOPSIS

```
SUBROUTINE DEZFTB(N, R, AZERO, A, B, WSAVE)

INTEGER N
DOUBLE PRECISION AZERO
DOUBLE PRECISION R(*), A(*), B(*), WSAVE(*)

SUBROUTINE DEZFTB_64(N, R, AZERO, A, B, WSAVE)

INTEGER*8 N
DOUBLE PRECISION AZERO
DOUBLE PRECISION R(*), A(*), B(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE DEZFTB(N, R, AZERO, A, B, WSAVE)

INTEGER :: N
REAL(8) :: AZERO
REAL(8), DIMENSION(:) :: R, A, B, WSAVE

SUBROUTINE DEZFTB_64(N, R, AZERO, A, B, WSAVE)

INTEGER(8) :: N
REAL(8) :: AZERO
REAL(8), DIMENSION(:) :: R, A, B, WSAVE
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dezftb(int n, double *r, double azero, double *a,  dou-
          ble *b, double *wsave);

void dezftb_64(long n, double *r, double azero,  double  *a,
          double *b, double *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be synthesized. The method is most efficient when N is the product of small primes. N >= 0.

R (output)
On exit, the Fourier synthesis of the inputs.

AZERO (input)
On entry, the constant Fourier coefficient A0. Unchanged on exit.

A (input/output)
On entry, array that contains the remaining Fourier coefficients. On exit, these arrays are unchanged.

B (input/output)
On entry, array that contains the remaining Fourier coefficients. On exit, these arrays are unchanged.

WSAVE (input)
On entry, an array with dimension of at least (3 * N + 15), initialized by DEZFTI.

Contents

NAME

dezftf - computes the Fourier coefficients of a periodic sequence. DEZFTF is a simplified but slower version of DFFTF.

SYNOPSIS

```
SUBROUTINE DEZFTF(N, R, AZERO, A, B, WSAVE)

INTEGER N
DOUBLE PRECISION AZERO
DOUBLE PRECISION R(*), A(*), B(*), WSAVE(*)

SUBROUTINE DEZFTF_64(N, R, AZERO, A, B, WSAVE)

INTEGER*8 N
DOUBLE PRECISION AZERO
DOUBLE PRECISION R(*), A(*), B(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE DEZFTF(N, R, AZERO, A, B, WSAVE)

INTEGER :: N
REAL(8) :: AZERO
REAL(8), DIMENSION(:) :: R, A, B, WSAVE

SUBROUTINE DEZFTF_64(N, R, AZERO, A, B, WSAVE)

INTEGER(8) :: N
REAL(8) :: AZERO
REAL(8), DIMENSION(:) :: R, A, B, WSAVE
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dezftf(int n, double *r, double azero, double *a,  dou-
          ble *b, double *wsave);

void dezftf_64(long n, double *r, double azero,  double  *a,
          double *b, double *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. The method is most efficient when N is the product of small primes. N >= 0.

R (input/output)
On entry, a real array of length N containing the sequence to be transformed. On exit, R is unchanged.

AZERO (output)
On exit, the sum from i=1 to i=n of r(i)/n.

A (input/output)
On entry, array that contains the remaining Fourier coefficients. On exit, these arrays are unchanged.

B (input/output)
On entry, array that contains the remaining Fourier coefficients. On exit, these arrays are unchanged.

WSAVE (input)
On entry, an array with dimension of at least (3 * N + 15), initialized by DEZFTI.

Contents

NAME

```
dezfti - initializes the array WSAVE, which is used in  both
DEZFTF and DEZFTB.
```

SYNOPSIS

```
SUBROUTINE DEZFTI(N, WSAVE)

INTEGER N
DOUBLE PRECISION WSAVE(*)

SUBROUTINE DEZFTI_64(N, WSAVE)

INTEGER*8 N
DOUBLE PRECISION WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE DEZFTI(N, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: WSAVE

SUBROUTINE DEZFTI_64(N, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void dezfti(int n, double *wsave);

void dezfti_64(long n, double *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. N >= 0.

WSAVE (input)
On entry, an array with a dimension of at least (3 * N + 15). The same work array can be used for both DEZFTF and DEZFTB as long as N remains unchanged. Different WSAVE arrays are required for different values of N. This initialization does not have to be repeated between calls to DEZFTF or DEZFTB as long as N and WSAVE remain unchanged, thus subsequent transforms can be obtained faster than the first.

Contents

NAME

```
dfft2b - compute a periodic sequence from its Fourier  coef-
ficients.   The  DFFT operations are unnormalized, so a call
of DFFT2F followed by a call of  DFFT2B  will  multiply  the
input sequence by M*N.
```

SYNOPSIS

```
SUBROUTINE DFFT2B(PLACE, M, N, A, LDA, B, LDB, WORK, LWORK)

CHARACTER * 1 PLACE
INTEGER M, N, LDA, LDB, LWORK
DOUBLE PRECISION A(LDA,*), B(LDB,*), WORK(*)

SUBROUTINE DFFT2B_64(PLACE, M, N, A, LDA, B, LDB, WORK, LWORK)

CHARACTER * 1 PLACE
INTEGER*8 M, N, LDA, LDB, LWORK
DOUBLE PRECISION A(LDA,*), B(LDB,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT2B(PLACE, [M], [N], A, [LDA], B, [LDB], WORK, LWORK)

CHARACTER(LEN=1) :: PLACE
INTEGER :: M, N, LDA, LDB, LWORK
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE FFT2B_64(PLACE, [M], [N], A, [LDA], B, [LDB], WORK, LWORK)

CHARACTER(LEN=1) :: PLACE
INTEGER(8) :: M, N, LDA, LDB, LWORK
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dfft2b(char place, int m, int n, double  *a,  int  lda,
          double *b, int ldb, double *work, int lwork);

void dfft2b_64(char place, long m, long n, double  *a,  long
          lda,  double  *b,  long  ldb,  double  *work, long
          lwork);
```

ARGUMENTS

PLACE (input)
Character. If PLACE = 'I' or 'i' (for in-place) , the input and output data are stored in array A. If PLACE = 'O' or 'o' (for out-of-place), the input data is stored in array B while the output is stored in A.

M (input) Integer specifying the number of rows to be transformed. It is most efficient when M is a product of small primes. M >= 0; when M = 0, the subroutine returns immediately without changing any data.

N (input) Integer specifying the number of columns to be transformed. It is most most efficient when N is a product of small primes. N >= 0; when N = 0, the subroutine returns immediately without changing any data.

A (input/output)
Real array of dimension (LDA,N). On entry, the two-dimensional array A(LDA,N) contains the input data to be transformed if an in-place transform is requested. Otherwise, it is not referenced. Upon exit, results are stored in A(1:M,1:N).

LDA (input)
Integer specifying the leading dimension of A. If an out-of-place transform is desired LDA >= M. Else if an in-place transform is desired LDA >= 2*(M/2+1).

B (input/output)
Real array of dimension (2*LDB, N). On entry, if an out-of-place transform is requested B contains the input data. Otherwise, B is not referenced. B is unchanged upon exit.

LDB (input)

Integer. If an out-of-place transform is desired, 2*LDB is the leading dimension of the array B which contains the data to be transformed and 2*LDB >= 2*(M/2+1). Otherwise it is not referenced.

WORK (input/output)

One-dimensional real array of length at least LWORK. On input, WORK must have been initialized by DFFT2I.

LWORK (input)

Integer. LWORK >= (M + 2*N + MAX(M, 2*N) + 30)

Contents

NAME

```
dfft2f - compute the  Fourier  coefficients  of  a  periodic
sequence.   The  DFFT operations are unnormalized, so a call
of DFFT2F followed by a call of  DFFT2B  will  multiply  the
input sequence by M*N.
```

SYNOPSIS

```
SUBROUTINE DFFT2F(PLACE, FULL, M, N, A, LDA, B, LDB, WORK, LWORK)

CHARACTER * 1 PLACE, FULL
INTEGER M, N, LDA, LDB, LWORK
DOUBLE PRECISION A(LDA,*), B(LDB,*), WORK(*)

SUBROUTINE DFFT2F_64(PLACE, FULL, M, N, A, LDA, B, LDB, WORK, LWORK)

CHARACTER * 1 PLACE, FULL
INTEGER*8 M, N, LDA, LDB, LWORK
DOUBLE PRECISION A(LDA,*), B(LDB,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT2F(PLACE, FULL, [M], [N], A, [LDA], B, [LDB], WORK,
       LWORK)

CHARACTER(LEN=1) :: PLACE, FULL
INTEGER :: M, N, LDA, LDB, LWORK
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE FFT2F_64(PLACE, FULL, [M], [N], A, [LDA], B, [LDB], WORK,
       LWORK)

CHARACTER(LEN=1) :: PLACE, FULL
INTEGER(8) :: M, N, LDA, LDB, LWORK
REAL(8), DIMENSION(:) :: WORK
```

```
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dfft2f(char place, char full, int m, int n, double  *a,
          int  lda,  double  *b,  int ldb, double *work, int
          lwork);

void dfft2f_64(char place, char full, long m, long n, double
          *a,  long  lda, double *b, long ldb, double *work,
          long lwork);
```

ARGUMENTS

PLACE (input)
Character. If PLACE = 'I' or 'i' (for in-place) , the input and output data are stored in array A. If PLACE = 'O' or 'o' (for out-of-place), the input data is stored in array B while the output is stored in A.

FULL (input)
Indicates whether or not to generate the full result matrix. 'F' or 'f' will cause DFFT2F to generate the full result matrix. Otherwise only a partial matrix that takes advantage of symmetry will be generated.

M (input) Integer specifying the number of rows to be transformed. It is most efficient when M is a product of small primes. M >= 0; when M = 0, the subroutine returns immediately without changing any data.

N (input) Integer specifying the number of columns to be transformed. It is most most efficient when N is a product of small primes. N >= 0; when N = 0, the subroutine returns immediately without changing any data.

A (input/output)
On entry, a two-dimensional array A(LDA,N) that contains the data to be transformed. Upon exit, A is unchanged if an out-of-place transform is done. If an in-place transform with partial result is requested, A(1:(M/2+1)*2,1:N) will contain the transformed results. If an in-place transform with full result is requested, A(1:2*M,1:N) will

contain complete transformed results.

LDA (input)
Leading dimension of the array containing the data to be transformed. LDA must be even if the transformed sequences are to be stored in A.

If PLACE = ('O' or 'o') LDA >= M

If PLACE = ('I' or 'i') LDA must be even. If FULL = ('F' or 'f'), LDA >= 2*M

FULL is not ('F' or 'f'), LDA >= (M/2+1)*2

B (input/output)
Upon exit, a two-dimensional array B(2*LDB,N) that contains the transformed results if an out-of-place transform is done. Otherwise, B is not used.

If an out-of-place transform is done and FULL is not 'F' or 'f', B(1:(M/2+1)*2,1:N) will contain the partial transformed results. If FULL = 'F' or 'f', B(1:2*M,1:N) will contain the complete transformed results.

LDB (input)
2*LDB is the leading dimension of the array B. If an in-place transform is desired LDB is ignored.

If PLACE is ('O' or 'o') and

FULL is ('F' or 'f'), LDB >= M

FULL is not ('F' or 'f'), LDB >= M/2+1

Note that even though LDB is used in the argument list, 2*LDB is the actual leading dimension of B.

WORK (input/output)
One-dimensional real array of length at least LWORK. On input, WORK must have been initialized by DFFT2I.

LWORK (input)
Integer. LWORK >= (M + 2*N + MAX(M, 2*N) + 30)

Contents

NAME

```
dfft2i - initialize the array WSAVE, which is used  in  both
the forward and backward transforms.
```

SYNOPSIS

```
SUBROUTINE DFFT2I(M, N, WORK)

INTEGER M, N
DOUBLE PRECISION WORK(*)

SUBROUTINE DFFT2I_64(M, N, WORK)

INTEGER*8 M, N
DOUBLE PRECISION WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT2I(M, N, WORK)

INTEGER :: M, N
REAL(8), DIMENSION(:) :: WORK

SUBROUTINE FFT2I_64(M, N, WORK)

INTEGER(8) :: M, N
REAL(8), DIMENSION(:) :: WORK
```

C INTERFACE

```
#include <sunperf.h>

void dfft2i(int m, int n, double *work);

void dfft2i_64(long m, long n, double *work);
```

ARGUMENTS

M (input) Number of rows to be transformed. M >= 0.

N (input) Number of columns to be transformed. N >= 0.

WORK (input/output)
On entry, an array of dimension (M + 2*N + MAX(M, 2*N) + 30) or greater. DFFT2I needs to be called only once to initialize array WORK before calling DFFT2F and/or DFFT2B if M, N and WORK remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
dfft3b - compute a periodic sequence from its Fourier  coef-
ficients. The DFFT operations are unnormalized, so a call of
DFFT3F followed by a call of DFFT3B will multiply the  input
sequence by M*N*K.
```

SYNOPSIS

```
SUBROUTINE DFFT3B(PLACE, M, N, K, A, LDA, B, LDB, WORK, LWORK)

CHARACTER * 1 PLACE
INTEGER M, N, K, LDA, LDB, LWORK
DOUBLE PRECISION A(LDA,N,*), B(LDB,N,*), WORK(*)

SUBROUTINE DFFT3B_64(PLACE, M, N, K, A, LDA, B, LDB, WORK, LWORK)

CHARACTER * 1 PLACE
INTEGER*8 M, N, K, LDA, LDB, LWORK
DOUBLE PRECISION A(LDA,N,*), B(LDB,N,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT3B(PLACE, [M], [N], [K], A, [LDA], B, [LDB], WORK,
       LWORK)

CHARACTER(LEN=1) :: PLACE
INTEGER :: M, N, K, LDA, LDB, LWORK
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:,:) :: A, B

SUBROUTINE FFT3B_64(PLACE, [M], [N], [K], A, [LDA], B, [LDB], WORK,
       LWORK)

CHARACTER(LEN=1) :: PLACE
INTEGER(8) :: M, N, K, LDA, LDB, LWORK
REAL(8), DIMENSION(:) :: WORK
```

```
REAL(8), DIMENSION(:,:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dfft3b(char place, int m, int n, int k, double *a,  int
          lda, double *b, int ldb, double *work, int lwork);

void dfft3b_64(char place, long m, long n,  long  k,  double
          *a,  long  lda, double *b, long ldb, double *work,
          long lwork);
```

ARGUMENTS

PLACE (input)
Select an in-place ('I' or 'i') or out-of-place ('O' or 'o') transform.

M (input) Integer specifying the number of rows to be transformed. It is most efficient when M is a product of small primes. M >= 0; when M = 0, the subroutine returns immediately without changing any data.

N (input) Integer specifying the number of columns to be transformed. It is most efficient when N is a product of small primes. N >= 0; when N = 0, the subroutine returns immediately without changing any data.

K (input) Integer specifying the number of planes to be transformed. It is most efficient when K is a product of small primes. K >= 0; when K = 0, the subroutine returns immediately without changing any data.

A (input/output)
On entry, the three-dimensional array A(LDA,N,K) contains the data to be transformed if an in-place transform is requested. Otherwise, it is not referenced. Upon exit, results are stored in A(1:M,1:N,1:K).

LDA (input)
Integer specifying the leading dimension of A. If an out-of-place transform is desired LDA >= M. Else if an in-place transform is desired LDA >= 2*(M/2+1).

B (input/output)
Real array of dimension B(2*LDB,N,K). On entry, if an out-of-place transform is requested B(1:2*(M/2+1),1:N,1:K) contains the input data. Otherwise, B is not referenced. B is unchanged upon exit.

LDB (input)
If an out-of-place transform is desired, 2*LDB is the leading dimension of the array B which contains the data to be transformed and 2*LDB >= 2*(M/2+1). Otherwise it is not referenced.

WORK (input/output)
One-dimensional real array of length at least LWORK. On input, WORK must have been initialized by DFFT3I.

LWORK (input)
Integer. LWORK >= (M + 2*(N + K) + 4*K + 45).

Contents

NAME

```
dfft3f - compute the Fourier coefficients of a real periodic
sequence. The DFFT operations are unnormalized, so a call of
DFFT3F followed by a call of DFFT3B will multiply the  input
sequence by M*N*K.
```

SYNOPSIS

```
SUBROUTINE DFFT3F(PLACE, FULL, M, N, K, A, LDA, B, LDB, WORK, LWORK)

CHARACTER * 1 PLACE, FULL
INTEGER M, N, K, LDA, LDB, LWORK
DOUBLE PRECISION A(LDA,N,*), B(LDB,N,*), WORK(*)

SUBROUTINE DFFT3F_64(PLACE, FULL, M, N, K, A, LDA, B, LDB, WORK,
      LWORK)

CHARACTER * 1 PLACE, FULL
INTEGER*8 M, N, K, LDA, LDB, LWORK
DOUBLE PRECISION A(LDA,N,*), B(LDB,N,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT3F(PLACE, FULL, [M], [N], [K], A, [LDA], B, [LDB],
       WORK, LWORK)

CHARACTER(LEN=1) :: PLACE, FULL
INTEGER :: M, N, K, LDA, LDB, LWORK
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:,:) :: A, B

SUBROUTINE FFT3F_64(PLACE, FULL, [M], [N], [K], A, [LDA], B, [LDB],
       WORK, LWORK)

CHARACTER(LEN=1) :: PLACE, FULL
INTEGER(8) :: M, N, K, LDA, LDB, LWORK
```

```
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dfft3f(char place, char full, int m, int n, int k, dou-
          ble *a, int lda, double *b, int ldb, double *work,
          int lwork);

void dfft3f_64(char place, char full, long m, long  n,  long
          k,  double *a, long lda, double *b, long ldb, dou-
          ble *work, long lwork);
```

ARGUMENTS

PLACE (input)
Select an in-place ('I' or 'i') or out-of-place ('O' or 'o') transform.

FULL (input)
Select a full ('F' or 'f') or partial (' ') representation of the results. If the caller selects full representation then an MxNxK real array will transform to produce an MxNxK complex array. If the caller does not select full representation then an MxNxK real array will transform to a (M/2+1)xNxK complex array that takes advantage of the symmetry properties of a transformed real sequence.

M (input) Integer specifying the number of rows to be transformed. It is most efficient when M is a product of small primes. M >= 0; when M = 0, the subroutine returns immediately without changing any data.

N (input) Integer specifying the number of columns to be transformed. It is most efficient when N is a product of small primes. N >= 0; when N = 0, the subroutine returns immediately without changing any data.

K (input) Integer specifying the number of planes to be transformed. It is most efficient when K is a product of small primes. K >= 0; when K = 0, the subroutine returns immediately without changing any data.

A (input/output)

On entry, a three-dimensional array A(LDA,N,K) that contains input data to be transformed. On exit, if an in-place transform is done and FULL is not 'F' or 'f', A(1:2*(M/2+1),1:N,1:K) will contain the partial transformed results. If FULL = 'F' or 'f', A(1:2*M,1:N,1:K) will contain the complete transformed results.

LDA (input)

Leading dimension of the array containing the data to be transformed. LDA must be even if the transformed sequences are to be stored in A.

If PLACE = ('O' or 'o') LDA >= M

If PLACE = ('I' or 'i') LDA must be even. If

FULL = ('F' or 'f'), LDA >= 2*M

FULL is not ('F' or 'f'), LDA >= 2*(M/2+1)

B (input/output)

Upon exit, a three-dimensional array B(2*LDB,N,K) that contains the transformed results if an out-of-place transform is done. Otherwise, B is not used.

If an out-of-place transform is done and FULL is not 'F' or 'f', B(1:2*(M/2+1),1:N,1:K) will contain the partial transformed results. If FULL = 'F' or 'f', B(1:2*M,1:N,1:K) will contain the complete transformed results.

LDB (input)

2*LDB is the leading dimension of the array B. If an in-place transform is desired LDB is ignored.

If PLACE is ('O' or 'o') and

FULL is ('F' or 'f'), then LDB >= M

FULL is not ('F' or 'f'), then LDB >= M/2 + 1

Note that even though LDB is used in the argument list, 2*LDB is the actual leading dimension of B.

WORK (input/output)

One-dimensional real array of length at least LWORK. WORK must have been initialized by DFFT3I.

```
LWORK (input)
          Integer.  LWORK >= (M + 2*(N + K) + 4*K + 45).
```

Contents

NAME

```
dfft3i - initialize the array WSAVE, which is used  in  both
DFFT3F and DFFT3B.
```

SYNOPSIS

```
SUBROUTINE DFFT3I(M, N, K, WORK)

INTEGER M, N, K
DOUBLE PRECISION WORK(*)

SUBROUTINE DFFT3I_64(M, N, K, WORK)

INTEGER*8 M, N, K
DOUBLE PRECISION WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT3I(M, N, K, WORK)

INTEGER :: M, N, K
REAL(8), DIMENSION(:) :: WORK

SUBROUTINE FFT3I_64(M, N, K, WORK)

INTEGER(8) :: M, N, K
REAL(8), DIMENSION(:) :: WORK
```

C INTERFACE

```
#include <sunperf.h>

void dfft3i(int m, int n, int k, double *work);

void dfft3i_64(long m, long n, long k, double *work);
```

ARGUMENTS

M (input) Number of rows to be transformed. M >= 0.

N (input) Number of columns to be transformed. N >= 0.

K (input) Number of planes to be transformed. K >= 0.

WORK (input/output)
On entry, an array of dimension (M + 2*(N + K) + 30) or greater. DFFT3I needs to be called only once to initialize array WORK before calling DFFT3F and/or DFFT3B if M, N, K and WORK remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

dfftb - compute a periodic sequence from its Fourier coefficients. The DFFT operations are unnormalized, so a call of DFFTF followed by a call of DFFTB will multiply the input sequence by N.

SYNOPSIS

```
SUBROUTINE DFFTB(N, X, WSAVE)

INTEGER N
DOUBLE PRECISION X(*), WSAVE(*)

SUBROUTINE DFFTB_64(N, X, WSAVE)

INTEGER*8 N
DOUBLE PRECISION X(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE FFTB([N], X, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: X, WSAVE

SUBROUTINE FFTB_64([N], X, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: X, WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void dfftb(int n, double *x, double *wsave);
```

```
void dfftb_64(long n, double *x, double *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence to be  transformed.   These
          subroutines are most efficient when N is a product
          of small primes.  N >= 0.

X (input) On entry, an array  of  length  N  containing  the
          sequence to be transformed.

WSAVE (input)
          On entry, WSAVE must be an array of dimension (2 *
          N  + 15) or greater and must have been initialized
          by DFFTI.
```

Contents

NAME

```
dfftf - compute  the  Fourier  coefficients  of  a  periodic
sequence.  The FFT operations are unnormalized, so a call of
DFFTF followed by a call of DFFTB will  multiply  the  input
sequence by N.
```

SYNOPSIS

```
SUBROUTINE DFFTF(N, X, WSAVE)

INTEGER N
DOUBLE PRECISION X(*), WSAVE(*)

SUBROUTINE DFFTF_64(N, X, WSAVE)

INTEGER*8 N
DOUBLE PRECISION X(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE FFTF([N], X, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: X, WSAVE

SUBROUTINE FFTF_64([N], X, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: X, WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void dfftf(int n, double *x, double *wsave);
```

```
void dfftf_64(long n, double *x, double *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence to be  transformed.   These
          subroutines are most efficient when N is a product
          of small primes.  N >= 0.

X (input) On entry, an array  of  length  N  containing  the
          sequence to be transformed.

WSAVE (input)
          On entry, WSAVE must be an array of dimension (2 *
          N  + 15) or greater and must have been initialized
          by DFFTI.
```

Contents

NAME

```
dffti - initialize the array WSAVE, which is  used  in  both
DFFTF and DFFTB.
```

SYNOPSIS

```
SUBROUTINE DFFTI(N, WSAVE)

INTEGER N
DOUBLE PRECISION WSAVE(*)

SUBROUTINE DFFTI_64(N, WSAVE)

INTEGER*8 N
DOUBLE PRECISION WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE FFTI(N, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: WSAVE

SUBROUTINE FFTI_64(N, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void dffti(int n, double *wsave);

void dffti_64(long n, double *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. N >= 0.

WSAVE (input)
On entry, an array of dimension (2 * N + 15) or greater. DFFTI needs to be called only once to initialize array WORK before calling DFFTF and/or DFFTB if N and WSAVE remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
dfftopt - compute the length of the closest fast FFT
```

SYNOPSIS

```
INTEGER FUNCTION DFFTOPT(LEN)

INTEGER LEN

INTEGER*8 FUNCTION DFFTOPT_64(LEN)

INTEGER*8 LEN
```

F95 INTERFACE

```
INTEGER FUNCTION DFFTOPT(LEN)

INTEGER :: LEN

INTEGER(8) FUNCTION DFFTOPT_64(LEN)

INTEGER(8) :: LEN
```

C INTERFACE

```
#include <sunperf.h>

int dfftopt(int len);

long dfftopt_64(long len);
```

PURPOSE

```
dfftopt computes the length of the closest fast  FFT.   Fast
```

Fourier transform algorithms, including those used in Performance Library, work best with vector lengths that are products of small primes. For example, an FFT of length 32=2**5 will run faster than an FFT of prime length 31 because 32 is a product of small primes and 31 is not. If your application is such that you can taper or zero pad your vector to a larger length then this function may help you select a better length and run your FFT faster.

DFFTOPT will return an integer no smaller than the input argument N that is the closest number that is the product of small primes. DFFTOPT will return 16 for an input of N=16 and return 18=2*3*3 for an input of N=17.

Note that the length computed here is not guaranteed to be optimal, only to be a
product of small primes. Also, the value returned may change as the underlying
FFTs become capable of handling larger primes. For example, passing in N=51 to day will return 52=2*2*13 rather than 51=3*17 because the FFTs in Performance Li brary do not have fast radix 17 code. In the future, radix 17 code may be added
and then N=51 will return 51.

Contents

NAME

```
dfftz -  initialize  the  trigonometric  weight  and  factor
tables  or  compute  the forward Fast Fourier Transform of a
double precision sequence.
```

SYNOPSIS

```
SUBROUTINE DFFTZ(IOPT, N, SCALE, X, Y, TRIGS, IFAC, WORK, LWORK, IERR)

INTEGER IOPT, N, IFAC(*), LWORK, IERR
DOUBLE COMPLEX Y(*)
DOUBLE PRECISION X(*), SCALE, TRIGS(*), WORK(*)

SUBROUTINE DFFTZ_64(IOPT, N, SCALE, X, Y, TRIGS, IFAC, WORK, LWORK, IERR)

INTEGER*8 IOPT, N, IFAC(*), LWORK, IERR
DOUBLE COMPLEX Y(*)
DOUBLE PRECISION X(*), SCALE, TRIGS(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT(IOPT, N, SCALE, X, Y, TRIGS, IFAC, WORK, [LWORK], IERR)

INTEGER, INTENT(IN) :: IOPT
INTEGER, INTENT(IN), OPTIONAL :: N, LWORK
REAL(8), INTENT(IN), OPTIONAL :: SCALE
REAL(8), INTENT(IN), DIMENSION(:) :: X
COMPLEX(8), INTENT(OUT), DIMENSION(:) :: Y
REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
INTEGER, INTENT(INOUT), DIMENSION(:) :: IFAC
REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
INTEGER, INTENT(OUT) :: IERR

SUBROUTINE FFT_64(IOPT, [N], [SCALE], X, Y, TRIGS, IFAC, WORK, [LWORK], IERR)
```

```
INTEGER(8), INTENT(IN) :: IOPT
INTEGER(8), INTENT(IN), OPTIONAL :: N, LWORK
REAL(8), INTENT(IN), OPTIONAL :: SCALE
REAL(8), INTENT(IN), DIMENSION(:) :: X
COMPLEX(8), INTENT(OUT), DIMENSION(:) :: Y
REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
#include <sunperf.h>

void dfftz_ (int *iopt, int *n, double  *scale,  double  *x,
          doublecomplex *y, double *trigs, int *ifac, double
          *work, int *lwork, int *ierr);

void dfftz_64_ (long *iopt, long *n, double  *scale,  double
          *x,  doublecomplex  *y, double *trigs, long *ifac,
          double *work, long *lwork, long *ierr);
```

PURPOSE

dfftz initializes the trigonometric weight and factor tables or computes the forward Fast Fourier Transform of a double precision sequence as follows:

```
               N-1
Y(k) = scale * SUM  W*X(j)
               j=0
```

where
k ranges from 0 to N-1
i = sqrt(-1)
isign = -1 for forward transform
W = exp(isign*i*j*k*2*pi/N)
In real-to-complex transform of length N, the (N/2+1) complex output data points stored are the positive-frequency half of the spectrum of the Discrete Fourier Transform. The other half can be obtained through complex conjugation and therefore is not stored.

ARGUMENTS

IOPT (input)
Integer specifying the operation to be performed:

IOPT = 0 computes the trigonometric weight table and factor table
IOPT = -1 computes forward FFT

N (input)
Integer specifying length of the input sequence X. N is most efficient when it is a product of small primes. N >= 0. Unchanged on exit.

SCALE (input)
Double precision scalar by which transform results are scaled. Unchanged on exit. SCALE is defaulted to 1.0D0 for F95 INTERFACE.

X (input) On entry, X is a real array whose first N elements contain the sequence to be transformed.

Y (output)
Double complex array whose first (N/2+1) elements contain the transform results. X and Y may be the same array starting at the same memory location, in which case the dimension of X must be at least 2*(N/2+1). Otherwise, it is assumed that there is no overlap between X and Y in memory.

TRIGS (input/output)
Double precision array of length 2*N that contains the trigonometric weights. The weights are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = -1. Unchanged on exit.

IFAC (input/output)
Integer array of dimension at least 128 that contains the factors of N. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls where IOPT = -1. Unchanged on exit.

WORK (workspace)
Double precision array of dimension at least N. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)
Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)
On exit, integer IERR has one of the following

```
values:
0 = normal return
-1 = IOPT is not 0 or -1
-2 = N < 0
-3 = (LWORK is not 0) and (LWORK is less than N)
-4 = memory allocation for workspace failed
```

SEE ALSO

fft

Contents

NAME

```
dfftz2 - initialize  the  trigonometric  weight  and  factor
tables  or  compute the two-dimensional forward Fast Fourier
Transform of a two-dimensional double precision array.
```

SYNOPSIS

```
     SUBROUTINE DFFTZ2(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     DOUBLE COMPLEX Y(LDY, *)
     DOUBLE PRECISION X(LDX, *), SCALE, TRIGS(*), WORK(*)

     SUBROUTINE DFFTZ2_64(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER*8 IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     DOUBLE COMPLEX Y(LDY, *)
     DOUBLE PRECISION X(LDX, *), SCALE, TRIGS(*), WORK(*)
```

F95 INTERFACE

```
     SUBROUTINE FFT2(IOPT, [N1], [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
               IFAC, WORK, [LWORK], IERR)

     INTEGER, INTENT(IN) :: IOPT
     INTEGER, INTENT(IN), OPTIONAL :: N1, N2, LDX, LDY, LWORK
     REAL(8), INTENT(IN), OPTIONAL :: SCALE
     REAL(8), INTENT(IN), DIMENSION(:,:) :: X
     COMPLEX(8), INTENT(OUT), DIMENSION(:,:) :: Y
     REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER, INTENT(INOUT), DIMENSION(:) :: IFAC
```

```
     REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER, INTENT(OUT) :: IERR

     SUBROUTINE FFT2_64(IOPT, [N1], [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
IFAC, WORK, [LWORK], IERR)

     INTEGER(8), INTENT(IN) :: IOPT
     INTEGER(8), INTENT(IN), OPTIONAL :: N1, N2, LDX, LDY, LWORK
     REAL(8), INTENT(IN), OPTIONAL :: SCALE
     REAL(8), INTENT(IN), DIMENSION(:,:) :: X
     COMPLEX(8), INTENT(OUT), DIMENSION(:,:) :: Y
     REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
     #include <sunperf.h>
     void dfftz2_ (int *iopt, int *n1, int  *n2,  double  *scale,
               double  *x,  int *ldx, doublecomplex *y, int *ldy,
               double  *trigs,  int  *ifac,  double  *work,   int
               *lwork, int *ierr);

     void dfftz2_64_ (long *iopt,  long  *n1,  long  *n2,  double
               *scale,  double  *x,  long *ldx, doublecomplex *y,
               long  *ldy,  double  *trigs,  long  *ifac,  double
               *work, long *lwork, long *ierr);
```

PURPOSE

dfftz2 initializes the trigonometric weight and factor tables or computes the two-dimensional forward Fast Fourier Transform of a two-dimensional double precision array. In computing the two-dimensional FFT, one-dimensional FFTs are computed along the columns of the input array. One-dimensional FFTs are then computed along the rows of the intermediate results.

```
                   N2-1  N1-1
Y(k1,k2) = scale * SUM   SUM   W2*W1*X(j1,j2)
                   j2=0  j1=0
```

where
k1 ranges from 0 to N1-1 and k2 ranges from 0 to N2-1
i = sqrt(-1)
isign = -1 for forward transform
W1 = exp(isign*i*j1*k1*2*pi/N1)
W2 = exp(isign*i*j2*k2*2*pi/N2)
In real-to-complex transform of length N1, the (N1/2+1) com-

plex output data points stored are the positive-frequency half of the spectrum of the Discrete Fourier Transform. The other half can be obtained through complex conjugation and therefore is not stored.

ARGUMENTS

IOPT (input)
Integer specifying the operation to be performed:
IOPT = 0 computes the trigonometric weight table and factor table
IOPT = -1 computes forward FFT

N1 (input)
Integer specifying length of the transform in the first dimension. N1 is most efficient when it is a product of small primes. N1 >= 0. Unchanged on exit.

N2 (input)
Integer specifying length of the transform in the second dimension. N2 is most efficient when it is a product of small primes N2 >= 0. Unchanged on exit.

SCALE (input)
Double precision scalar by which transform results are scaled. Unchanged on exit. SCALE is defaulted to 1.0D0 for F95 INTERFACE.

X (input) X is a double complex array of dimensions (LDX, N2) that contains input data to be transformed. X and Y can be the same array.

LDX (input)
Leading dimension of X. LDX >= N1 if X is not the same array as Y. Else, LDX = 2*LDY. Unchanged on exit.

Y (output)
Y is a double complex array of dimensions (LDY, N2) that contains the transform results. X and Y can be the same array starting at the same memory location, in which case the input data are overwritten by their transform results. Otherwise, it is assumed that there is no overlap between X and Y in memory.

LDY (input)
Leading dimension of Y. LDY >= N1/2+1 Unchanged on exit.

TRIGS (input/output)
Double precision array of length 2*(N1+N2) that contains the trigonometric weights. The weights are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = -1. Unchanged on exit.

IFAC (input/output)
Integer array of dimension at least 2*128 that contains the factors of N1 and N2. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = -1. Unchanged on exit.

WORK (workspace)
Double precision array of dimension at least MAX(N1, 2*N2) where NCPUS is the number of threads used to execute the routine. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)
Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)
On exit, integer IERR has one of the following values:
0 = normal return
-1 = IOPT is not 0 or -1
-2 = N1 < 0
-3 = N2 < 0
-4 = (LDX < N1) or (LDX not equal 2*LDY when X and Y are same array)
-5 = (LDY < N1/2+1)
-6 = (LWORK not equal 0) and (LWORK < MAX(N1,2*N2))
-7 = memory allocation failed

SEE ALSO

fft

CAUTIONS

On exit, output array Y(1:LDY, 1:N2) is overwritten.

Contents

NAME

```
     dfftz3 - initialize  the  trigonometric  weight  and  factor
     tables or compute the three-dimensional forward Fast Fourier
     Transform of a three-dimensional double complex array.
```

SYNOPSIS

```
     SUBROUTINE DFFTZ3(IOPT, N1, N2, N3, SCALE, X, LDX1, LDX2, Y, LDY1, LDY2,
TRIGS, IFAC, WORK, LWORK, IERR)

     INTEGER IOPT, N1, N2, N3, LDX1, LDX2, LDY1,  LDY2,  IFAC(*),
     LWORK, IERR
     DOUBLE COMPLEX Y(LDY1, LDY2, *)
     DOUBLE PRECISION X(LDX1, LDX2, *), SCALE, TRIGS(*), WORK(*)

     SUBROUTINE DFFTZ3_64(IOPT, N1, N2, N3, SCALE, X, LDX1, LDX2, Y, LDY1, LDY2,
TRIGS, IFAC, WORK, LWORK, IERR)

     INTEGER*8 IOPT, N1, N2, N3, LDX1, LDX2, LDY1, LDY2, IFAC(*),
     LWORK, IERR
     DOUBLE COMPLEX Y(LDY1, LDY2, *)
     DOUBLE PRECISION X(LDX1, LDX2, *), SCALE, TRIGS(*), WORK(*)
```

F95 INTERFACE

```
     SUBROUTINE FFT3(IOPT, [N1], [N2], [N3], [SCALE], X, [LDX1], LDX2, Y, [LDY1],
LDY2, TRIGS, IFAC, WORK, [LWORK], IERR)

     INTEGER, INTENT(IN) :: IOPT, LDX2, LDY2
     INTEGER, INTENT(IN), OPTIONAL :: N1,  N2,  N3,  LDX1,  LDY1,
     LWORK
     REAL(8), INTENT(IN), OPTIONAL :: SCALE
     REAL(8), INTENT(IN), DIMENSION(:,:) :: X
```

```
     COMPLEX(8), INTENT(OUT), DIMENSION(:,:) :: Y
     REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER, INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER, INTENT(OUT) :: IERR

     SUBROUTINE FFT3_64(IOPT, [N1], [N2], [N3], [SCALE], X, [LDX1], LDX2, Y,
[LDY1], LDY2, TRIGS,
               IFAC, WORK, [LWORK], IERR)

     INTEGER(8), INTENT(IN) :: IOPT, LDX2, LDY2
     INTEGER(8), INTENT(IN), OPTIONAL :: N1, N2, N3, LDX1,  LDY1,
     LWORK
     REAL(8), INTENT(IN), OPTIONAL :: SCALE
     REAL(8), INTENT(IN), DIMENSION(:,:) :: X
     COMPLEX(8), INTENT(OUT), DIMENSION(:,:) :: Y
     REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
     #include <sunperf.h>

     void dfftz3_ (int *iopt, int *n1, int *n2, int  *n3,  double
               *scale,  double  *x,  int  *ldx1, int *ldx2, doub-
               lecomplex *y, int *ldy1, int *ldy2, double *trigs,
               int *ifac, double *work, int *lwork, int *ierr);

     void dfftz3_64_ (long *iopt, long *n1, long *n2,  long  *n3,
               double  *scale, double *x, long *ldx1, long *ldx2,
               doublecomplex *y, long *ldy1, long  *ldy2,  double
               *trigs,  long  *ifac,  double  *work, long *lwork,
               long *ierr);
```

PURPOSE

dfftz3 initializes the trigonometric weight and factor tables or computes the three-dimensional forward Fast Fourier Transform of a three-dimensional double complex array.

```
                          N3-1  N2-1  N1-1
     Y(k1,k2,k3) = scale * SUM   SUM   SUM   W3*W2*W1*X(j1,j2,j3)
                          j3=0  j2=0  j1=0
```

where
k1 ranges from 0 to N1-1; k2 ranges from 0 to N2-1 and k3 ranges from 0 to N3-1
i = sqrt(-1)

```
isign = -1 for forward transform
W1 = exp(isign*i*j1*k1*2*pi/N1)
W2 = exp(isign*i*j2*k2*2*pi/N2)
W3 = exp(isign*i*j3*k3*2*pi/N3)
```

ARGUMENTS

```
IOPT (input)
          Integer specifying the operation to be performed:
          IOPT = 0 computes the trigonometric  weight  table
          and factor table
          IOPT = -1 computes forward FFT

N1 (input)
          Integer specifying length of the transform in  the
          first  dimension.  N1 is most efficient when it is
          a product of small primes.  N1 >= 0.  Unchanged on
          exit.

N2 (input)
          Integer specifying length of the transform in  the
          second dimension.  N2 is most efficient when it is
          a product of small primes.  N2 >= 0.  Unchanged on
          exit.

N3 (input)
          Integer specifying length of the transform in  the
          third  dimension.  N3 is most efficient when it is
          a product of small primes.  N3 >= 0.  Unchanged on
          exit.

SCALE (input)
          Double precision scalar by which transform results
          are   scaled.   Unchanged   on  exit.   SCALE  is
          defaulted to 1.0D0 for F95 INTERFACE.

X (input) X is a double precision array of dimensions (LDX1,
          LDX2,   N3)   that   contains  input  data  to  be
          transformed.  X can be same array as Y.

LDX1 (input)
          first dimension of X.  If X is not same  array  as
          Y,  LDX1  >=  N1  Else, LDX1 = 2*LDY1 Unchanged on
          exit.

LDX2 (input)
          second dimension of X.  LDX2 >=  N2  Unchanged  on
          exit.
```

Y (output)
Y is a double complex array of dimensions (LDY1, LDY2, N3) that contains the transform results. X and Y can be the same array starting at the same memory location, in which case the input data are overwritten by their transform results. Otherwise, it is assumed that there is no overlap between X and Y in memory.

LDY1 (input)
first dimension of Y. LDY1 >= N1/2+1 Unchanged on exit.

LDY2 (input)
second dimension of Y. If X and Y are the same array, LDY2 = LDX2 Else LDY2 >= N2 Unchanged on exit.

TRIGS (input/output)
Double precision array of length 2*(N1+N2+N3) that contains the trigonometric weights. The weights are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = -1. Unchanged on exit.

IFAC (input/output)
Integer array of dimension at least 3*128 that contains the factors of N1, N2 and N3. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = -1. Unchanged on exit.

WORK (workspace)
Double precision array of dimension at least (MAX(N,2*N2,2*N3) + 16*N3) * NCPUS where NCPUS is the number of threads used to execute the routine. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)
Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)
On exit, integer IERR has one of the following values:
0 = normal return
-1 = IOPT is not 0 or -1
-2 = N1 < 0

-3 = N2 < 0
-4 = N3 < 0
-5 = (LDX1 < N1) or (LDX not equal 2*LDY when X and Y are same array)
-6 = (LDX2 < N2)
-7 = (LDY1 < N1/2+1)
-8 = (LDY2 < N2) or (LDY2 not equal LDX2 when X and Y are same array)
-9 = (LWORK not equal 0) and (LWORK < (MAX(N,2*N2,2*N3) + 16*N3))
-10 = memory allocation failed

SEE ALSO

fft

CAUTIONS

On exit, output subarray Y(1:LDY1, 1:N2, 1:N3) is overwritten.

Contents

NAME

```
     dfftzm - initialize  the  trigonometric  weight  and  factor
     tables  or  compute the one-dimensional forward Fast Fourier
     Transform of a set of double precision data sequences stored
     in a two-dimensional array.
```

SYNOPSIS

```
     SUBROUTINE DFFTZM(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     DOUBLE PRECISION X(LDX, *), SCALE, TRIGS(*), WORK(*)
     DOUBLE COMPLEX Y(LDY, *)

     SUBROUTINE DFFTZM_64(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER*8 IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     DOUBLE PRECISION X(LDX, *), SCALE, TRIGS(*), WORK(*)
     DOUBLE COMPLEX Y(LDY, *)
```

F95 INTERFACE

```
     SUBROUTINE FFTM(IOPT, [N1], [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
               IFAC, WORK, [LWORK], IERR)

     INTEGER, INTENT(IN) :: IOPT
     INTEGER, INTENT(IN), OPTIONAL :: N1, N2, LDX, LDY, LWORK
     REAL(8), INTENT(IN), OPTIONAL :: SCALE
     REAL(8), INTENT(IN), DIMENSION(:,:) :: X
     COMPLEX(8), INTENT(OUT), DIMENSION(:,:) :: Y
     REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER, INTENT(INOUT), DIMENSION(:) :: IFAC
```

```
     REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER, INTENT(OUT) :: IERR

     SUBROUTINE FFTM_64(IOPT, [N1], [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
IFAC, WORK, [LWORK], IERR)

     INTEGER(8), INTENT(IN) :: IOPT
     INTEGER(8), INTENT(IN), OPTIONAL :: N1, N2, LDX, LDY, LWORK
     REAL(8), INTENT(IN), OPTIONAL :: SCALE
     REAL(8), INTENT(IN), DIMENSION(:,:) :: X
     COMPLEX(8), INTENT(OUT), DIMENSION(:,:) :: Y
     REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
     #include <sunperf.h>
     void dfftzm_ (int *iopt, int *m, int *n, double *scale, dou-
               ble *x, int *ldx, doublecomplex *y, int *ldy, dou-
               ble *trigs, int *ifac, double *work,  int  *lwork,
               int *ierr);

     void dfftzm_64_  (long  *iopt,  long  *m,  long  *n,  double
               *scale,  double  *x,  long *ldx, doublecomplex *y,
               long  *ldy,  double  *trigs,  long  *ifac,  double
               *work, long *lwork, long *ierr);
```

PURPOSE

dfftzm initializes the trigonometric weight and factor tables or computes the one-dimensional forward Fast Fourier Transform of a set of double precision data sequences stored in a two-dimensional array:

```
                     N1-1
     Y(k,l) = scale * SUM  W*X(j,l)
                     j=0
```

where
k ranges from 0 to N1-1 and l ranges from 0 to N2-1
i = sqrt(-1)
isign = -1 for forward transform
W = exp(isign*i*j*k*2*pi/N1)
In real-to-complex transform of length N1, the (N1/2+1) complex output data points stored are the positive-frequency half of the spectrum of the discrete Fourier transform. The other half can be obtained through complex conjugation and therefore is not stored.

ARGUMENTS

IOPT (input)
Integer specifying the operation to be performed:
IOPT = 0 computes the trigonometric weight table and factor table
IOPT = -1 computes forward FFT

N1 (input)
Integer specifying length of the input sequences. N1 is most efficient when it is a product of small primes. N1 >= 0. Unchanged on exit.

N2 (input)
Integer specifying number of input sequences. N2 >= 0. Unchanged on exit.

SCALE (input)
Double precision scalar by which transform results are scaled. Unchanged on exit. SCALE is defaulted to 1.0D0 for F95 INTERFACE.

X (input) X is a double precision array of dimensions (LDX, N2) that contains the sequences to be transformed stored in its columns.

LDX (input)
Leading dimension of X. If X and Y are the same array, LDX = 2*LDY Else LDX >= N1 Unchanged on exit.

Y (output)
Y is a double complex array of dimensions (LDY, N2) that contains the transform results of the input sequences. X and Y can be the same array starting at the same memory location, in which case the input sequences are overwritten by their transform results. Otherwise, it is assumed that there is no overlap between X and Y in memory.

LDY (input)
Leading dimension of Y. LDY >= N1/2 + 1 Unchanged on exit.

TRIGS (input/output)
Double precision array of length 2*N1 that contains the trigonometric weights. The weights are

computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = -1. Unchanged on exit.

IFAC (input/output)
Integer array of dimension at least 128 that contains the factors of N1. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = -1. Unchanged on exit.

WORK (workspace)
Double precision array of dimension at least N1. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)
Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)
On exit, integer IERR has one of the following values:
0 = normal return
-1 = IOPT is not 0 or -1
-2 = N1 < 0
-3 = N2 < 0
-4 = (LDX < N1) or (LDX not equal 2*LDY when X and Y are same array)
-4 = (LDY < N1/2 + 1)
-6 = (LWORK not equal 0) and (LWORK < N1)
-7 = memory allocation failed

SEE ALSO

fft

Contents

NAME

```
dgbbrd - reduce a real general m-by-n band matrix A to upper
bidiagonal form B by an orthogonal transformation
```

SYNOPSIS

```
SUBROUTINE DGBBRD(VECT, M, N, NCC, KL, KU, AB, LDAB, D, E, Q, LDQ,
      PT, LDPT, C, LDC, WORK, INFO)

CHARACTER * 1 VECT
INTEGER M, N, NCC, KL, KU, LDAB, LDQ, LDPT, LDC, INFO
DOUBLE   PRECISION   AB(LDAB,*),   D(*),   E(*),   Q(LDQ,*),
PT(LDPT,*), C(LDC,*), WORK(*)

SUBROUTINE DGBBRD_64(VECT, M, N, NCC, KL, KU, AB, LDAB, D, E, Q, LDQ,
      PT, LDPT, C, LDC, WORK, INFO)

CHARACTER * 1 VECT
INTEGER*8 M, N, NCC, KL, KU, LDAB, LDQ, LDPT, LDC, INFO
DOUBLE   PRECISION   AB(LDAB,*),   D(*),   E(*),   Q(LDQ,*),
PT(LDPT,*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GBBRD(VECT, M, [N], [NCC], KL, KU, AB, [LDAB], D, E, Q,
       [LDQ], PT, [LDPT], C, [LDC], [WORK], [INFO])

CHARACTER(LEN=1) :: VECT
INTEGER :: M, N, NCC, KL, KU, LDAB, LDQ, LDPT, LDC, INFO
REAL(8), DIMENSION(:) :: D, E, WORK
REAL(8), DIMENSION(:,:) :: AB, Q, PT, C

SUBROUTINE GBBRD_64(VECT, M, [N], [NCC], KL, KU, AB, [LDAB], D, E,
       Q, [LDQ], PT, [LDPT], C, [LDC], [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: VECT
INTEGER(8) :: M, N, NCC, KL, KU, LDAB, LDQ, LDPT, LDC, INFO
REAL(8), DIMENSION(:) :: D, E, WORK
REAL(8), DIMENSION(:,:) :: AB, Q, PT, C
```

C INTERFACE

```
#include <sunperf.h>

void dgbbrd(char vect, int m, int n, int ncc,  int  kl,  int
          ku,  double  *ab,  int ldab, double *d, double *e,
          double *q, int ldq, double *pt, int  ldpt,  double
          *c, int ldc, int *info);
void dgbbrd_64(char vect, long m, long n, long ncc, long kl,
          long  ku, double *ab, long ldab, double *d, double
          *e, double *q, long ldq, double  *pt,  long  ldpt,
          double *c, long ldc, long *info);
```

PURPOSE

dgbbrd reduces a real general m-by-n band matrix A to upper bidiagonal form B by an orthogonal transformation: Q' * A * P = B.

The routine computes B, and optionally forms Q or P', or computes Q'*C for a given matrix C.

ARGUMENTS

VECT (input)
Specifies whether or not the matrices Q and P' are to be formed. = 'N': do not form Q or P';
= 'Q': form Q only;
= 'P': form P' only;
= 'B': form both.

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

NCC (input)
The number of columns of the matrix C. NCC >= 0.

KL (input)
The number of subdiagonals of the matrix A. KL >= 0.

KU (input)
The number of superdiagonals of the matrix A. KU >= 0.

AB (input/output)
DOUBLE PRECISION array, dimension(LDAB,N) On entry, the m-by-n band matrix A, stored in rows 1 to KL+KU+1. The j-th column of A is stored in the j-th column of the array AB as follows: AB(ku+1+i-j,j) = A(i,j) for max(1,j-ku)<=i<=min(m,j+kl). On exit, A is overwritten by values generated during the reduction.

LDAB (input)
The leading dimension of the array A. LDAB >= KL+KU+1.

D (output)
DOUBLE PRECISION array, dimension(min(M,N)) The diagonal elements of the bidiagonal matrix B.

E (output)
DOUBLE PRECISION array, dimension(min(M,N)-1) The superdiagonal elements of the bidiagonal matrix B.

Q (output)
DOUBLE PRECISION array, dimension(LDQ,M) If VECT = 'Q' or 'B', the m-by-m orthogonal matrix Q. If VECT = 'N' or 'P', the array Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= max(1,M) if VECT = 'Q' or 'B'; LDQ >= 1 otherwise.

PT (output)
DOUBLE PRECISION array, dimension(LDPT,N) If VECT = 'P' or 'B', the n-by-n orthogonal matrix P'. If VECT = 'N' or 'Q', the array PT is not referenced.

LDPT (input)
The leading dimension of the array PT. LDPT >= max(1,N) if VECT = 'P' or 'B'; LDPT >= 1 otherwise.

C (input/output)
DOUBLE PRECISION array, dimension(LDC,NCC) On entry, an m-by-ncc matrix C. On exit, C is overwritten by Q'*C. C is not referenced if NCC = 0.

```
LDC (input)
          The leading dimension of  the  array  C.   LDC  >=
          max(1,M) if NCC > 0; LDC >= 1 if NCC = 0.
WORK (workspace)
          DOUBLE PRECISION array, dimension(2*MAX(M,N))

INFO (output)
          = 0:  successful exit.
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value.
```

Contents

NAME

```
dgbcon - estimate the reciprocal of the condition number  of
a  real  general  band matrix A, in either the 1-norm or the
infinity-norm,
```

SYNOPSIS

```
SUBROUTINE DGBCON(NORM, N, NSUB, NSUPER, A, LDA, IPIVOT, ANORM,
      RCOND, WORK, WORK2, INFO)

CHARACTER * 1 NORM
INTEGER N, NSUB, NSUPER, LDA, INFO
INTEGER IPIVOT(*), WORK2(*)
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION A(LDA,*), WORK(*)

SUBROUTINE DGBCON_64(NORM, N, NSUB, NSUPER, A, LDA, IPIVOT, ANORM,
      RCOND, WORK, WORK2, INFO)

CHARACTER * 1 NORM
INTEGER*8 N, NSUB, NSUPER, LDA, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GBCON(NORM, [N], NSUB, NSUPER, A, [LDA], IPIVOT, ANORM,
       RCOND, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM
INTEGER :: N, NSUB, NSUPER, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
REAL(8) :: ANORM, RCOND
```

```
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE GBCON_64(NORM, [N], NSUB, NSUPER, A, [LDA], IPIVOT, ANORM,
       RCOND, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM
INTEGER(8) :: N, NSUB, NSUPER, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
void dgbcon(char norm, int n, int nsub, int  nsuper,  double
          *a,  int  lda,  int  *ipivot, double anorm, double
          *rcond, int *info);

void dgbcon_64(char norm, long n, long  nsub,  long  nsuper,
          double  *a,  long lda, long *ipivot, double anorm,
          double *rcond, long *info);
```

PURPOSE

dgbcon estimates the reciprocal of the condition number of a real general band matrix A, in either the 1-norm or the infinity-norm, using the LU factorization computed by SGBTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as

RCOND = 1 / (norm(A) * norm(inv(A))).

ARGUMENTS

NORM (input)
Specifies whether the 1-norm condition number or the infinity-norm condition number is required:
= '1' or 'O': 1-norm;
= 'I':   Infinity-norm.

N (input) The order of the matrix A. N >= 0.

NSUB (input)
The number of subdiagonals within the band of A. NSUB >= 0.

```
NSUPER (input)
          The number of superdiagonals within the band of A.
          NSUPER >= 0.

A (input) Details of the LU factorization of the band matrix
          A, as computed by SGBTRF.  U is stored as an upper
          triangular band matrix with NSUB+NSUPER superdiag-
          onals  in  rows 1 to NSUB+NSUPER+1, and the multi-
          pliers used during the factorization are stored in
          rows NSUB+NSUPER+2 to 2*NSUB+NSUPER+1.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          2*NSUB+NSUPER+1.

IPIVOT (input)
          The pivot indices; for 1 <= i <= N, row i  of  the
          matrix was interchanged with row IPIVOT(i).

ANORM (input)
          If NORM = '1' or 'O', the 1-norm of  the  original
          matrix A.  If NORM = 'I', the infinity-norm of the
          original matrix A.

RCOND (output)
          The reciprocal of  the  condition  number  of  the
          matrix   A,  computed  as  RCOND  =  1/(norm(A)  *
          norm(inv(A))).

WORK (workspace)
          dimension(3*N)

WORK2 (workspace)
          dimension (N)

INFO (output)
          = 0:  successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
          gal value
```

Contents

NAME

```
dgbequ - compute row and column scalings intended to equili-
brate  an  M-by-N  band  matrix  A  and reduce its condition
number
```

SYNOPSIS

```
SUBROUTINE DGBEQU(M, N, KL, KU, A, LDA, R, C, ROWCN,
      COLCN, AMAX, INFO)

INTEGER M, N, KL, KU, LDA, INFO
DOUBLE PRECISION ROWCN, COLCN, AMAX
DOUBLE PRECISION A(LDA,*), R(*), C(*)

SUBROUTINE DGBEQU_64(M, N, KL, KU, A, LDA, R, C, ROWCN,
      COLCN, AMAX, INFO)

INTEGER*8 M, N, KL, KU, LDA, INFO
DOUBLE PRECISION ROWCN, COLCN, AMAX
DOUBLE PRECISION A(LDA,*), R(*), C(*)
```

F95 INTERFACE

```
SUBROUTINE GBEQU([M], [N], KL, KU, A, [LDA], R, C,
       ROWCN, COLCN, AMAX, [INFO])

INTEGER :: M, N, KL, KU, LDA, INFO
REAL(8) :: ROWCN, COLCN, AMAX
REAL(8), DIMENSION(:) :: R, C
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE GBEQU_64([M], [N], KL, KU, A, [LDA], R, C,
       ROWCN, COLCN, AMAX, [INFO])
```

```
INTEGER(8) :: M, N, KL, KU, LDA, INFO
REAL(8) :: ROWCN, COLCN, AMAX
REAL(8), DIMENSION(:) :: R, C
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dgbequ(int m, int n, int kl, int  ku,  double  *a,  int
          lda,  double  *r, double *c, double *rowcn, double
          *colcn, double *amax, int *info);

void dgbequ_64(long m, long n, long kl, long ku, double  *a,
          long  lda,  double  *r,  double *c, double *rowcn,
          double *colcn, double *amax, long *info);
```

PURPOSE

dgbequ computes row and column scalings intended to equilibrate an M-by-N band matrix A and reduce its condition number. R returns the row scale factors and C the column scale factors, chosen to try to make the largest element in each row and column of the matrix B with elements B(i,j)=R(i)*A(i,j)*C(j) have absolute value 1.

R(i) and C(j) are restricted to be between SMLNUM = smallest safe number and BIGNUM = largest safe number. Use of these scaling factors is not guaranteed to reduce the condition number of A but works well in practice.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

KL (input)
The number of subdiagonals within the band of A. KL >= 0.

KU (input)
The number of superdiagonals within the band of A. KU >= 0.

A (input) The band matrix A, stored in rows 1 to KL+KU+1. The j-th column of A is stored in the j-th column of the array A as follows: A(ku+1+i-j,j) = A(i,j)

```
          for max(1,j-ku)<=i<=min(m,j+kl).

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          KL+KU+1.

R (output)
          If INFO = 0, or INFO > M, R contains the row scale
          factors for A.

C (output)
          If INFO = 0, C contains the column  scale  factors
          for A.
ROWCN (output)
          If INFO = 0 or INFO > M, ROWCN contains the  ratio
          of  the  smallest  R(i)  to  the largest R(i).  If
          ROWCN >= 0.1 and AMAX is neither too large nor too
          small, it is not worth scaling by R.

COLCN (output)
          If INFO = 0, COLCN contains the ratio of the smal-
          lest  C(i)  to the largest C(i).  If COLCN >= 0.1,
          it is not worth scaling by C.

AMAX (output)
          Absolute value of largest matrix element.  If AMAX
          is  very close to overflow or very close to under-
          flow, the matrix should be scaled.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, and i is
          <= M:  the i-th row of A is exactly zero
          >  M:  the (i-M)-th column of A is exactly zero
```

Contents

NAME

```
dgbmv - perform one of the matrix-vector operations    y  :=
alpha*A*x + beta*y or y := alpha*A'*x + beta*y
```

SYNOPSIS

```
SUBROUTINE DGBMV(TRANSA, M, N, NSUB, NSUPER, ALPHA, A, LDA, X, INCX,
      BETA, Y, INCY)

CHARACTER * 1 TRANSA
INTEGER M, N, NSUB, NSUPER, LDA, INCX, INCY
DOUBLE PRECISION ALPHA, BETA
DOUBLE PRECISION A(LDA,*), X(*), Y(*)

SUBROUTINE DGBMV_64(TRANSA, M, N, NSUB, NSUPER, ALPHA, A, LDA, X,
      INCX, BETA, Y, INCY)

CHARACTER * 1 TRANSA
INTEGER*8 M, N, NSUB, NSUPER, LDA, INCX, INCY
DOUBLE PRECISION ALPHA, BETA
DOUBLE PRECISION A(LDA,*), X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE GBMV([TRANSA], [M], [N], NSUB, NSUPER, ALPHA, A, [LDA], X,
       [INCX], BETA, Y, [INCY])

CHARACTER(LEN=1) :: TRANSA
INTEGER :: M, N, NSUB, NSUPER, LDA, INCX, INCY
REAL(8) :: ALPHA, BETA
REAL(8), DIMENSION(:) :: X, Y
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE GBMV_64([TRANSA], [M], [N], NSUB, NSUPER, ALPHA, A, [LDA],
```

```
          X, [INCX], BETA, Y, [INCY])

   CHARACTER(LEN=1) :: TRANSA
   INTEGER(8) :: M, N, NSUB, NSUPER, LDA, INCX, INCY
   REAL(8) :: ALPHA, BETA
   REAL(8), DIMENSION(:) :: X, Y
   REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dgbmv(char transa, int m, int n, int nsub, int  nsuper,
          double  alpha,  double *a, int lda, double *x, int
          incx, double beta, double *y, int incy);
void dgbmv_64(char transa, long m, long n, long  nsub,  long
          nsuper,  double alpha, double *a, long lda, double
          *x, long incx, double beta, double *y, long incy);
```

PURPOSE

dgbmv performs one of the matrix-vector operations y := alpha*A*x + beta*y or y := alpha*A'*x + beta*y, where alpha and beta are scalars, x and y are vectors and A is an m by n band matrix, with nsub sub-diagonals and nsuper super-diagonals.

ARGUMENTS

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   y := alpha*A*x + beta*y.

TRANSA = 'T' or 't'   y := alpha*A'*x + beta*y.

TRANSA = 'C' or 'c'   y := alpha*A'*x + beta*y.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

M (input)
On entry, M specifies the number of rows of the matrix A. M >= 0. Unchanged on exit.

N (input)

On entry, N specifies the number of columns of the matrix A. N >= 0. Unchanged on exit.

NSUB (input)
On entry, NSUB specifies the number of sub-diagonals of the matrix A. NSUB >= 0. Unchanged on exit.

NSUPER (input)
On entry, NSUPER specifies the number of super-diagonals of the matrix A. NSUPER >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
Before entry, the leading (nsub + nsuper + 1) by n part of the array A must contain the matrix of coefficients, supplied column by column, with the leading diagonal of the matrix in row (nsuper + 1) of the array, the first super-diagonal starting at position 2 in row nsuper, the first sub-diagonal starting at position 1 in row (nsuper + 2), and so on. Elements in the array A that do not correspond to elements in the band matrix (such as the top left nsuper by nsuper triangle) are not referenced. The following program segment will transfer a band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     K = NSUPER + 1 - J
     DO 10, I = MAX( 1, J - NSUPER ), MIN( M, J  +
NSUB )
       A( K + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= (nsub + nsuper + 1). Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)) when TRANSA = 'N' or 'n' and at least (1 + (m - 1)*abs(INCX)) otherwise. Before entry, the incremented array X

must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. When BETA is supplied as zero then Y need not be set on input. Unchanged on exit.

Y (input/output)
(1 + (m - 1)*abs(INCY)) when TRANSA = 'N' or 'n' and at least (1 + (n - 1)*abs(INCY)) otherwise. Before entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
dgbrfs - improve the computed solution to a system of linear
equations  when  the  coefficient matrix is banded, and pro-
vides error bounds and  backward  error  estimates  for  the
solution
```

SYNOPSIS

```
SUBROUTINE DGBRFS(TRANSA, N, KL, KU, NRHS, A, LDA, AF, LDAF,
      IPIVOT, B, LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
INTEGER N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*), WORK2(*)
DOUBLE PRECISION A(LDA,*), AF(LDAF,*),  B(LDB,*),  X(LDX,*),
FERR(*), BERR(*), WORK(*)

SUBROUTINE DGBRFS_64(TRANSA, N, KL, KU, NRHS, A, LDA, AF, LDAF,
      IPIVOT, B, LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
INTEGER*8 N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
DOUBLE PRECISION A(LDA,*), AF(LDAF,*),  B(LDB,*),  X(LDX,*),
FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GBRFS([TRANSA], [N], KL, KU, [NRHS], A, [LDA], AF,
       [LDAF], IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: TRANSA
INTEGER :: N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
```

```
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
REAL(8), DIMENSION(:) :: FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: A, AF, B, X

SUBROUTINE GBRFS_64([TRANSA], [N], KL, KU, [NRHS], A, [LDA],
       AF, [LDAF], IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: TRANSA
INTEGER(8) :: N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL(8), DIMENSION(:) :: FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
#include <sunperf.h>

void dgbrfs(char transa, int n, int kl, int  ku,  int  nrhs,
          double  *a,  int  lda,  double  *af, int ldaf, int
          *ipivot, double *b, int ldb, double *x,  int  ldx,
          double *ferr, double *berr, int *info);

void dgbrfs_64(char transa, long n, long kl, long  ku,  long
          nrhs,  double *a, long lda, double *af, long ldaf,
          long *ipivot, double *b, long ldb, double *x, long
          ldx, double *ferr, double *berr, long *info);
```

PURPOSE

dgbrfs improves the computed solution to a system of linear equations when the coefficient matrix is banded, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

```
TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose = Tran-
          spose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A.  N >= 0.

KL (input)
```

The number of subdiagonals within the band of A. KL >= 0.

KU (input)
The number of superdiagonals within the band of A. KU >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The original band matrix A, stored in rows 1 to KL+KU+1. The j-th column of A is stored in the j-th column of the array A as follows: A(ku+1+i-j,j) = A(i,j) for max(1,j-ku)<=i<=min(n,j+kl).

LDA (input)
The leading dimension of the array A. LDA >= KL+KU+1.

AF (input)
Details of the LU factorization of the band matrix A, as computed by SGBTRF. U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1.

LDAF (input)
The leading dimension of the array AF. LDAF >= 2*KL*KU+1.

IPIVOT (input)
The pivot indices from SGBTRF; for 1<=i<=N, row i of the matrix was interchanged with row IPIVOT(i).

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by SGBTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

```
FERR (output)
          The estimated forward error bound for  each  solu-
          tion  vector X(j) (the j-th column of the solution
          matrix  X).   If  XTRUE  is  the   true   solution
          corresponding  to  X(j),  FERR(j)  is an estimated
          upper bound for the magnitude of the largest  ele-
          ment in (X(j) - XTRUE) divided by the magnitude of
          the largest element in X(j).  The estimate  is  as
          reliable  as the estimate for RCOND, and is almost
          always a slight overestimate of the true error.

BERR (output)
          The componentwise relative backward error of  each
          solution  vector X(j) (i.e., the smallest relative
          change in any element of A or B that makes X(j) an
          exact solution).

WORK (workspace)
          dimension(3*N)

WORK2 (workspace)
          dimension(N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
dgbsv - compute the solution to  a  real  system  of  linear
equations  A  *  X  = B, where A is a band matrix of order N
with KL subdiagonals and KU superdiagonals, and X and B  are
N-by-NRHS matrices
```

SYNOPSIS

```
SUBROUTINE DGBSV(N, KL, KU, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

INTEGER N, KL, KU, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*)

SUBROUTINE DGBSV_64(N, KL, KU, NRHS, A, LDA, IPIVOT, B, LDB,
      INFO)

INTEGER*8 N, KL, KU, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE GBSV([N], KL, KU, [NRHS], A, [LDA], IPIVOT, B, [LDB],
       [INFO])

INTEGER :: N, KL, KU, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE GBSV_64([N], KL, KU, [NRHS], A, [LDA], IPIVOT, B,
       [LDB], [INFO])
```

```
INTEGER(8) :: N, KL, KU, NRHS, LDA, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dgbsv(int n, int kl, int ku, int nrhs, double  *a,  int
          lda, int *ipivot, double *b, int ldb, int *info);

void dgbsv_64(long n, long kl, long ku,  long  nrhs,  double
          *a,  long  lda, long *ipivot, double *b, long ldb,
          long *info);
```

PURPOSE

dgbsv computes the solution to a real system of linear equations A * X = B, where A is a band matrix of order N with KL subdiagonals and KU superdiagonals, and X and B are N-by-NRHS matrices.

The LU decomposition with partial pivoting and row interchanges is used to factor A as A = L * U, where L is a product of permutation and unit lower triangular matrices with KL subdiagonals, and U is upper triangular with KL+KU superdiagonals. The factored form of A is then used to solve the system of equations A * X = B.

ARGUMENTS

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

KL (input)
The number of subdiagonals within the band of A. KL >= 0.

KU (input)
The number of superdiagonals within the band of A. KU >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input/output)
On entry, the matrix A in band storage, in rows

KL+1 to 2*KL+KU+1; rows 1 to KL of the array need not be set. The j-th column of A is stored in the j-th column of the array A as follows: A(KL+KU+1+i-j,j) = A(i,j) for max(1,j-KU)<=i<=min(N,j+KL) On exit, details of the factorization: U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1. See below for further details.

LDA (input)
The leading dimension of the array A. LDA >= 2*KL+KU+1.

IPIVOT (output)
The pivot indices that define the permutation matrix P; row i of the matrix was interchanged with row IPIVOT(i).

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly singular, and the solution has not been computed.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when M = N = 6, KL = 2, KU = 1:

```
On entry:                       On exit:

    *    *    *    +    +    +       *    *    *   u14   u25
u36
    *    *    +    +    +    +       *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
```

```
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
   a21  a32  a43  a54  a65   *      m21  m32  m43  m54   m65
*
   a31  a42  a53  a64   *    *      m31  m42  m53   m64    *
*
```

Array elements marked * are not used by the routine; elements marked + need not be set on entry, but are required by the routine to store elements of U because of fill-in resulting from the row interchanges.

Contents

NAME

```
dgbsvx - use the LU factorization to compute the solution to
a  real  system of linear equations A * X = B, A**T * X = B,
or A**H * X = B,
```

SYNOPSIS

```
SUBROUTINE DGBSVX(FACT, TRANSA, N, KL, KU, NRHS, A, LDA, AF,
      LDAF, IPIVOT, EQUED, R, C, B, LDB, X, LDX, RCOND, FERR,
      BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, TRANSA, EQUED
INTEGER N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*), WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(LDA,*), AF(LDAF,*), R(*), C(*), B(LDB,*),
X(LDX,*), FERR(*), BERR(*), WORK(*)

SUBROUTINE DGBSVX_64(FACT, TRANSA, N, KL, KU, NRHS, A, LDA, AF,
      LDAF, IPIVOT, EQUED, R, C, B, LDB, X, LDX, RCOND, FERR,
      BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, TRANSA, EQUED
INTEGER*8 N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(LDA,*), AF(LDAF,*), R(*), C(*), B(LDB,*),
X(LDX,*), FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GBSVX(FACT, [TRANSA], [N], KL, KU, [NRHS], A, [LDA],
       AF, [LDAF], IPIVOT, EQUED, R, C, B, [LDB], X, [LDX],
       RCOND, FERR, BERR, [WORK], [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: FACT, TRANSA, EQUED
INTEGER :: N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: R, C, FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: A, AF, B, X

SUBROUTINE GBSVX_64(FACT, [TRANSA], [N], KL, KU, [NRHS], A,
       [LDA], AF, [LDAF], IPIVOT, EQUED, R, C, B, [LDB], X, [LDX],
       RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, TRANSA, EQUED
INTEGER(8) :: N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: R, C, FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
#include <sunperf.h>

void dgbsvx(char fact, char transa, int n, int kl,  int  ku,
          int  nrhs,  double  *a,  int  lda, double *af, int
          ldaf, int *ipivot, char equed, double  *r,  double
          *c, double *b, int ldb, double *x, int ldx, double
          *rcond, double *ferr, double *berr, int *info);

void dgbsvx_64(char fact, char transa, long n, long kl, long
          ku,  long  nrhs,  double *a, long lda, double *af,
          long ldaf, long *ipivot, char  equed,  double  *r,
          double  *c,  double  *b, long ldb, double *x, long
          ldx, double *rcond, double  *ferr,  double  *berr,
          long *info);
```

PURPOSE

dgbsvx uses the LU factorization to compute the solution to a real system of linear equations A * X = B, A**T * X = B, or A**H * X = B, where A is a band matrix of order N with KL subdiagonals and KU superdiagonals, and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed by this subroutine:

1. If FACT = 'E', real scaling factors are computed to

equilibrate
   the system:
      TRANS = 'N': diag(R)*A*diag(C)   *inv(diag(C))*X =
diag(R)*B
      TRANS = 'T': (diag(R)*A*diag(C))**T *inv(diag(R))*X =
diag(C)*B
      TRANS = 'C': (diag(R)*A*diag(C))**H *inv(diag(R))*X =
diag(C)*B
   Whether or not the system will be equilibrated depends on
the
   scaling of the matrix A, but if equilibration is used, A
is
   overwritten by diag(R)*A*diag(C) and B by diag(R)*B (if
TRANS='N')
   or diag(C)*B (if TRANS = 'T' or 'C').

2. If FACT = 'N' or 'E', the LU decomposition is used to
factor the
   matrix A (after equilibration if FACT = 'E') as
      A = L * U,
   where L is a product of permutation and unit lower tri-
angular
   matrices with KL subdiagonals, and U is upper triangular
with
   KL+KU superdiagonals.

3. If some U(i,i)=0, so that U is exactly singular, then the
routine
   returns with INFO = i. Otherwise, the factored form of A
is used
   to estimate the condition number of the matrix A. If the
   reciprocal of the condition number is less than machine
precision,
   INFO = N+1 is returned as a warning, but the routine
still goes on
   to solve for X and compute error bounds as described
below.

4. The system of equations is solved for X using the fac-
tored form
   of A.

5. Iterative refinement is applied to improve the computed
solution
   matrix and calculate error bounds and backward error
estimates
   for it.

6. If equilibration was used, the matrix X is premultiplied
by

```
   diag(C) (if TRANS = 'N') or diag(R) (if TRANS  =  'T'  or
'C') so
   that it solves the original system before equilibration.
```

ARGUMENTS

```
FACT (input)
          Specifies whether or not the factored form of  the
          matrix A is supplied on entry, and if not, whether
          the matrix A should be equilibrated before  it  is
          factored.  = 'F':  On entry, AF and IPIVOT contain
          the factored form of A.  If EQUED is not 'N',  the
          matrix  A  has been equilibrated with scaling fac-
          tors given by R and C.  A, AF, and IPIVOT are  not
          modified.   = 'N':  The matrix A will be copied to
          AF and factored.
          = 'E':  The  matrix  A  will  be  equilibrated  if
          necessary, then copied to AF and factored.
TRANSA (input)
          Specifies the form of the system of equations.   =
          'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Transpose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

KL (input)
          The number of subdiagonals within the band  of  A.
          KL >= 0.

KU (input)
          The number of superdiagonals within the band of A.
          KU >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input/output)
          On entry, the matrix A in band storage, in rows  1
          to KL+KU+1.  The j-th column of A is stored in the
          j-th column of the array A as follows:   A(KU+1+i-
          j,j) = A(i,j) for max(1,j-KU)<=i<=min(N,j+kl)

          If FACT = 'F' and EQUED is not 'N',  then  A  must
          have been equilibrated by the scaling factors in R
```

and/or C. A is not modified if FACT = 'F' or 'N', or if FACT = 'E' and EQUED = 'N' on exit.

On exit, if EQUED .ne. 'N', A is scaled as follows: EQUED = 'R': A := diag(R) * A
EQUED = 'C': A := A * diag(C)
EQUED = 'B': A := diag(R) * A * diag(C).

LDA (input)
The leading dimension of the array A. LDA >= KL+KU+1.

AF (input/output)
If FACT = 'F', then AF is an input argument and on entry contains details of the LU factorization of the band matrix A, as computed by SGBTRF. U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1. If EQUED .ne. 'N', then AF is the factored form of the equilibrated matrix A.

If FACT = 'N', then AF is an output argument and on exit returns details of the LU factorization of A.

If FACT = 'E', then AF is an output argument and on exit returns details of the LU factorization of the equilibrated matrix A (see the description of A for the form of the equilibrated matrix).

LDAF (input)
The leading dimension of the array AF. LDAF >= 2*KL+KU+1.

IPIVOT (input)
If FACT = 'F', then IPIVOT is an input argument and on entry contains the pivot indices from the factorization A = L*U as computed by SGBTRF; row i of the matrix was interchanged with row IPIVOT(i).

If FACT = 'N', then IPIVOT is an output argument and on exit contains the pivot indices from the factorization A = L*U of the original matrix A.

If FACT = 'E', then IPIVOT is an output argument and on exit contains the pivot indices from the factorization A = L*U of the equilibrated matrix A.

EQUED (input)
Specifies the form of equilibration that was done. = 'N': No equilibration (always true if FACT = 'N').
= 'R': Row equilibration, i.e., A has been premultiplied by diag(R). = 'C': Column equilibration, i.e., A has been postmultiplied by diag(C). = 'B': Both row and column equilibration, i.e., A has been replaced by diag(R) * A * diag(C). EQUED is an input argument if FACT = 'F'; otherwise, it is an output argument.

R (input/output)
The row scale factors for A. If EQUED = 'R' or 'B', A is multiplied on the left by diag(R); if EQUED = 'N' or 'C', R is not accessed. R is an input argument if FACT = 'F'; otherwise, R is an output argument. If FACT = 'F' and EQUED = 'R' or 'B', each element of R must be positive.

C (input/output)
The column scale factors for A. If EQUED = 'C' or 'B', A is multiplied on the right by diag(C); if EQUED = 'N' or 'R', C is not accessed. C is an input argument if FACT = 'F'; otherwise, C is an output argument. If FACT = 'F' and EQUED = 'C' or 'B', each element of C must be positive.

B (input/output)
On entry, the right hand side matrix B. On exit, if EQUED = 'N', B is not modified; if TRANSA = 'N' and EQUED = 'R' or 'B', B is overwritten by diag(R)*B; if TRANSA = 'T' or 'C' and EQUED = 'C' or 'B', B is overwritten by diag(C)*B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X to the original system of equations. Note that A and B are modified on exit if EQUED .ne. 'N', and the solution to the equilibrated system is inv(diag(C))*X if TRANSA = 'N' and EQUED = 'C' or 'B', or inv(diag(R))*X if TRANSA = 'T' or 'C' and EQUED = 'R' or 'B'.

LDX (input)
The leading dimension of the array X. LDX >=

max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A after equilibration (if done). If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N) On exit, WORK(1) contains the reciprocal pivot growth factor norm(A)/norm(U). The "max absolute element" norm is used. If WORK(1) is much less than 1, then the stability of the LU factorization of the (equilibrated) matrix A could be poor. This also means that the solution X, condition estimator RCOND, and forward error bound FERR could be unreliable. If factorization fails with 0<INFO<=N, then WORK(1) contains the reciprocal pivot growth factor for the leading INFO columns of A.

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly

singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
dgbtf2 - compute an LU factorization of a real  m-by-n  band
matrix A using partial pivoting with row interchanges
```

SYNOPSIS

```
SUBROUTINE DGBTF2(M, N, KL, KU, AB, LDAB, IPIV, INFO)

INTEGER M, N, KL, KU, LDAB, INFO
INTEGER IPIV(*)
DOUBLE PRECISION AB(LDAB,*)

SUBROUTINE DGBTF2_64(M, N, KL, KU, AB, LDAB, IPIV, INFO)

INTEGER*8 M, N, KL, KU, LDAB, INFO
INTEGER*8 IPIV(*)
DOUBLE PRECISION AB(LDAB,*)
```

F95 INTERFACE

```
SUBROUTINE GBTF2([M], [N], KL, KU, AB, [LDAB], IPIV, [INFO])

INTEGER :: M, N, KL, KU, LDAB, INFO
INTEGER, DIMENSION(:) :: IPIV
REAL(8), DIMENSION(:,:) :: AB

SUBROUTINE GBTF2_64([M], [N], KL, KU, AB, [LDAB], IPIV, [INFO])

INTEGER(8) :: M, N, KL, KU, LDAB, INFO
INTEGER(8), DIMENSION(:) :: IPIV
REAL(8), DIMENSION(:,:) :: AB
```

C INTERFACE

```
#include <sunperf.h>

void dgbtf2(int m, int n, int kl, int ku,  double  *ab,  int
          ldab, int *ipiv, int *info);

void dgbtf2_64(long m, long n, long kl, long ku, double *ab,
          long ldab, long *ipiv, long *info);
```

PURPOSE

dgbtf2 computes an LU factorization of a real m-by-n band matrix A using partial pivoting with row interchanges.

This is the unblocked version of the algorithm, calling Level 2 BLAS.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

KL (input)
The number of subdiagonals within the band of A. KL >= 0.

KU (input)
The number of superdiagonals within the band of A. KU >= 0.

AB (input/output)
On entry, the matrix A in band storage, in rows KL+1 to 2*KL+KU+1; rows 1 to KL of the array need not be set. The j-th column of A is stored in the j-th column of the array AB as follows: AB(kl+ku+1+i-j,j) = A(i,j) for max(1,j-ku)<=i<=min(m,j+kl)

On exit, details of the factorization: U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1. See below for further details.

LDAB (input)
The leading dimension of the array AB. LDAB >=

2*KL+KU+1.

IPIV (output)
The pivot indices; for 1 <= i <= min(M,N), row i of the matrix was interchanged with row IPIV(i).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = +i, U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when M = N = 6, KL = 2, KU = 1:

```
On entry:                       On exit:

    *    *    *    +    +    +       *    *    *   u14   u25
u36
    *    *    +    +    +    +       *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
   a21  a32  a43  a54  a65   *      m21  m32  m43  m54   m65
*
   a31  a42  a53  a64   *    *      m31  m42  m53   m64    *
*
```

Array elements marked * are not used by the routine; elements marked + need not be set on entry, but are required by the routine to store elements of U, because of fill-in resulting from the row interchanges.

Contents

NAME

```
dgbtrf - compute an LU factorization of a real  m-by-n  band
matrix A using partial pivoting with row interchanges
```

SYNOPSIS

```
SUBROUTINE DGBTRF(M, N, KL, KU, AB, LDAB, IPIVOT, INFO)

INTEGER M, N, KL, KU, LDAB, INFO
INTEGER IPIVOT(MIN(M,N))
DOUBLE PRECISION AB(LDAB,N)

SUBROUTINE DGBTRF_64(M, N, KL, KU, AB, LDAB, IPIVOT, INFO)

INTEGER*8 M, N, KL, KU, LDAB, INFO
INTEGER*8 IPIVOT(N)
DOUBLE PRECISION AB(LDAB,N)
```

F95 INTERFACE

```
SUBROUTINE GBTRF(M, [N], KL, KU, AB, [LDAB], IPIVOT, [INFO])

INTEGER :: M, N, KL, KU, LDAB, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:,:) :: AB

SUBROUTINE GBTRF_64(M, [N], KL, KU, AB, [LDAB], IPIVOT, [INFO])

INTEGER(8) :: M, N, KL, KU, LDAB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:,:) :: AB
```

C INTERFACE

```
#include <sunperf.h>

void dgbtrf(int m, int n, int kl, int ku,  double  *ab,  int
          ldab, int *ipivot, int *info);

void dgbtrf_64(long m, long n, long kl, long ku, double *ab,
          long ldab, long *ipivot, long *info);
```

PURPOSE

dgbtrf computes an LU factorization of a real m-by-n band matrix A using partial pivoting with row interchanges.

This is the blocked version of the algorithm, calling Level 3 BLAS.

ARGUMENTS

M (input) Integer
The number of rows of the matrix A. M >= 0.

N (input) Integer
The number of columns of the matrix A. N >= 0.

KL (input) Integer
The number of subdiagonals within the band of A. KL >= 0.

KU (input) Integer
The number of superdiagonals within the band of A. KU >= 0.

AB (input/output) Double precision array of dimension (LDAB,N).
On entry, the matrix A in band storage, in rows KL+1 to 2*KL+KU+1; rows 1 to KL of the array need not be set. The j-th column of A is stored in the j-th column of the array A as follows: AB(KL+KU+1+I-J,J) = A(I,J) for MAX(1,J-KU)<=I<=MIN(M,J+KL)

On exit, details of the factorization: U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1. See below for further details.

```
LDAB (input) Integer array of dimension MIN(M,N)
          The leading dimension of the array  AB.   LDAB  >=
          2*KL+KU+1.

IPIVOT (output) Integer array of dimension MIN(M,N)
          The pivot indices; for 1 <= I <= MIN(M,N),  row  I
          of the matrix was interchanged with row IPIVOT(I).

INFO (output) Integer
          = 0: successful exit
          < 0: if INFO = -I, the I-th argument had an  ille-
          gal value
          > 0: if INFO = +I, U(I,I)  is  exactly  zero.  The
          factorization has been completed, but the factor U
          is exactly singular, and  division  by  zero  will
          occur  if  it  is  used to solve a system of equa-
          tions.
```

FURTHER DETAILS

```
The band storage scheme  is  illustrated  by  the  following
example, when M = N = 6, KL = 2, KU = 1:

On entry:                       On exit:

    *    *    *    +    +    +       *    *    *   u14   u25
u36
    *    *    +    +    +    +       *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
   a21  a32  a43  a54  a65   *      m21  m32  m43  m54   m65
*
   a31  a42  a53  a64   *    *      m31  m42  m53   m64    *
*

Array elements marked * are not used by  the  routine;  ele-
ments marked + need not be set on entry, but are required by
the routine to  store  elements  of  U  because  of  fill-in
resulting from the row interchanges.
```

Contents

NAME

```
dgbtrs - solve a system of linear equations  A * X = B or A'
* X = B with a general band matrix A using the LU factoriza-
tion computed by SGBTRF
```

SYNOPSIS

```
SUBROUTINE DGBTRS(TRANSA, N, NSUB, NSUPER, NRHS, A, LDA, IPIVOT, B,
      LDB, INFO)

CHARACTER * 1 TRANSA
INTEGER N, NSUB, NSUPER, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*)

SUBROUTINE DGBTRS_64(TRANSA, N, NSUB, NSUPER, NRHS, A, LDA, IPIVOT,
      B, LDB, INFO)

CHARACTER * 1 TRANSA
INTEGER*8 N, NSUB, NSUPER, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE GBTRS([TRANSA], [N], NSUB, NSUPER, [NRHS], A, [LDA],
       IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: TRANSA
INTEGER :: N, NSUB, NSUPER, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE GBTRS_64([TRANSA], [N], NSUB, NSUPER, [NRHS], A, [LDA],
```

```
          IPIVOT, B, [LDB], [INFO])

   CHARACTER(LEN=1) :: TRANSA
   INTEGER(8) :: N, NSUB, NSUPER, NRHS, LDA, LDB, INFO
   INTEGER(8), DIMENSION(:) :: IPIVOT
   REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
   #include <sunperf.h>

   void dgbtrs(char transa, int n, int nsub,  int  nsuper,  int
             nrhs,  double *a, int lda, int *ipivot, double *b,
             int ldb, int *info);
   void dgbtrs_64(char transa, long n, long nsub, long  nsuper,
             long nrhs, double *a, long lda, long *ipivot, dou-
             ble *b, long ldb, long *info);
```

PURPOSE

```
   dgbtrs solves a system of linear equations
      A * X = B  or  A' * X = B with a general  band  matrix  A
   using the LU factorization computed by SGBTRF.
```

ARGUMENTS

```
   TRANSA (input)
             Specifies the form of the system of equations.   =
             'N':  A * X = B  (No transpose)
             = 'T':  A'* X = B  (Transpose)
             = 'C':  A'* X = B  (Conjugate  transpose  =  Tran-
             spose)

             TRANSA is defaulted to 'N' for F95 INTERFACE.

   N (input) The order of the matrix A.  N >= 0.

   NSUB (input)
             The number of subdiagonals within the band  of  A.
             NSUB >= 0.

   NSUPER (input)
             The number of superdiagonals within the band of A.
             NSUPER >= 0.

   NRHS (input)
             The number of right hand sides, i.e.,  the  number
             of columns of the matrix B.  NRHS >= 0.
```

A (input) Details of the LU factorization of the band matrix A, as computed by SGBTRF. U is stored as an upper triangular band matrix with NSUB+NSUPER superdiagonals in rows 1 to NSUB+NSUPER+1, and the multipliers used during the factorization are stored in rows NSUB+NSUPER+2 to 2*NSUB+NSUPER+1.

LDA (input)
The leading dimension of the array A. LDA >= 2*NSUB+NSUPER+1.

IPIVOT (input)
The pivot indices; for 1 <= i <= N, row i of the matrix was interchanged with row IPIVOT(i).

B (input/output)
On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dgebak - form the right or left eigenvectors of a real  gen-
eral  matrix  by  backward  transformation  on  the computed
eigenvectors of the balanced matrix output by SGEBAL
```

SYNOPSIS

```
SUBROUTINE DGEBAK(JOB, SIDE, N, ILO, IHI, SCALE, M, V, LDV, INFO)

CHARACTER * 1 JOB, SIDE
INTEGER N, ILO, IHI, M, LDV, INFO
DOUBLE PRECISION SCALE(*), V(LDV,*)

SUBROUTINE DGEBAK_64(JOB, SIDE, N, ILO, IHI, SCALE, M, V, LDV, INFO)

CHARACTER * 1 JOB, SIDE
INTEGER*8 N, ILO, IHI, M, LDV, INFO
DOUBLE PRECISION SCALE(*), V(LDV,*)
```

F95 INTERFACE

```
SUBROUTINE GEBAK(JOB, SIDE, [N], ILO, IHI, SCALE, [M], V, [LDV],
       [INFO])

CHARACTER(LEN=1) :: JOB, SIDE
INTEGER :: N, ILO, IHI, M, LDV, INFO
REAL(8), DIMENSION(:) :: SCALE
REAL(8), DIMENSION(:,:) :: V

SUBROUTINE GEBAK_64(JOB, SIDE, [N], ILO, IHI, SCALE, [M], V, [LDV],
       [INFO])

CHARACTER(LEN=1) :: JOB, SIDE
INTEGER(8) :: N, ILO, IHI, M, LDV, INFO
```

```
REAL(8), DIMENSION(:) :: SCALE
REAL(8), DIMENSION(:,:) :: V
```

C INTERFACE

```
#include <sunperf.h>

void dgebak(char job, char side, int n, int  ilo,  int  ihi,
          double  *scale,  int  m,  double  *v, int ldv, int
          *info);

void dgebak_64(char job, char side, long n, long  ilo,  long
          ihi,  double  *scale, long m, double *v, long ldv,
          long *info);
```

PURPOSE

dgebak forms the right or left eigenvectors of a real general matrix by backward transformation on the computed eigenvectors of the balanced matrix output by SGEBAL.

ARGUMENTS

JOB (input)
Specifies the type of backward transformation required: = 'N', do nothing, return immediately; = 'P', do backward transformation for permutation only; = 'S', do backward transformation for scaling only; = 'B', do backward transformations for both permutation and scaling. JOB must be the same as the argument JOB supplied to SGEBAL.

SIDE (input)
= 'R': V contains right eigenvectors;
= 'L': V contains left eigenvectors.

N (input) The number of rows of the matrix V. N >= 0.

ILO (input)
The integers ILO and IHI determined by SGEBAL. 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

IHI (input)
See the description for ILO.

SCALE (input)
Details of the permutation and scaling factors, as

returned by SGEBAL.

M (input) The number of columns of the matrix V. M >= 0.

V (input/output)
On entry, the matrix of right or left eigenvectors to be transformed, as returned by SHSEIN or STREVC. On exit, V is overwritten by the transformed eigenvectors.

LDV (input)
The leading dimension of the array V. LDV >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

```
dgebal - balance a general real matrix A
```

SYNOPSIS

```
SUBROUTINE DGEBAL(JOB, N, A, LDA, ILO, IHI, SCALE, INFO)

CHARACTER * 1 JOB
INTEGER N, LDA, ILO, IHI, INFO
DOUBLE PRECISION A(LDA,*), SCALE(*)

SUBROUTINE DGEBAL_64(JOB, N, A, LDA, ILO, IHI, SCALE, INFO)

CHARACTER * 1 JOB
INTEGER*8 N, LDA, ILO, IHI, INFO
DOUBLE PRECISION A(LDA,*), SCALE(*)
```

F95 INTERFACE

```
SUBROUTINE GEBAL(JOB, [N], A, [LDA], ILO, IHI, SCALE, [INFO])

CHARACTER(LEN=1) :: JOB
INTEGER :: N, LDA, ILO, IHI, INFO
REAL(8), DIMENSION(:) :: SCALE
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE GEBAL_64(JOB, [N], A, [LDA], ILO, IHI, SCALE, [INFO])

CHARACTER(LEN=1) :: JOB
INTEGER(8) :: N, LDA, ILO, IHI, INFO
REAL(8), DIMENSION(:) :: SCALE
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dgebal(char job, int n, double *a, int lda,  int  *ilo,
          int *ihi, double *scale, int *info);

void dgebal_64(char job, long n, double *a, long  lda,  long
          *ilo, long *ihi, double *scale, long *info);
```

PURPOSE

dgebal balances a general real matrix A. This involves, first, permuting A by a similarity transformation to isolate eigenvalues in the first 1 to ILO-1 and last IHI+1 to N elements on the diagonal; and second, applying a diagonal similarity transformation to rows and columns ILO to IHI to make the rows and columns as close in norm as possible. Both steps are optional.

Balancing may reduce the 1-norm of the matrix, and improve the accuracy of the computed eigenvalues and/or eigenvectors.

ARGUMENTS

JOB (input)
Specifies the operations to be performed on A:
= 'N': none: simply set ILO = 1, IHI = N, SCALE(I) = 1.0 for i = 1,...,N; = 'P': permute only;
= 'S': scale only;
= 'B': both permute and scale.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the input matrix A. On exit, A is overwritten by the balanced matrix. If JOB = 'N', A is not referenced. See Further Details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

ILO (output)
ILO and IHI are set to integers such that on exit A(i,j) = 0 if i > j and j = 1,...,ILO-1 or I =

IHI+1,...,N. If JOB = 'N' or 'S', ILO = 1 and IHI = N.

IHI (output)
See the description for ILO.

SCALE (output)
Details of the permutations and scaling factors applied to A. If P(j) is the index of the row and column interchanged with row and column j and D(j) is the scaling factor applied to row and column j, then SCALE(j) = P(j) for j = 1,...,ILO-1 = D(j) for j = ILO,...,IHI = P(j) for j = IHI+1,...,N. The order in which the interchanges are made is N to IHI+1, then 1 to ILO-1.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

The permutations consist of row and column interchanges which put the matrix in the form

```
           ( T1   X   Y  )
   P A P = (  0   B   Z  )
           (  0   0   T2 )
```

where T1 and T2 are upper triangular matrices whose eigenvalues lie along the diagonal. The column indices ILO and IHI mark the starting and ending columns of the submatrix B. Balancing consists of applying a diagonal similarity transformation inv(D) * B * D to make the 1-norms of each row of B and its corresponding column nearly equal. The output matrix is

```
   ( T1     X*D          Y    )
   (  0  inv(D)*B*D  inv(D)*Z ).
   (  0      0           T2   )
```

Information about the permutations P and the diagonal matrix D is returned in the vector SCALE.

This subroutine is based on the EISPACK routine BALANC.

Modified by Tzu-Yi Chen, Computer Science Division, University of
California at Berkeley, USA

Contents

NAME

```
dgebrd - reduce a general real M-by-N matrix A to  upper  or
lower bidiagonal form B by an orthogonal transformation
```

SYNOPSIS

```
SUBROUTINE DGEBRD(M, N, A, LDA, D, E, TAUQ, TAUP, WORK, LWORK, INFO)

INTEGER M, N, LDA, LWORK, INFO
DOUBLE PRECISION A(LDA,*),  D(*),  E(*),  TAUQ(*),  TAUP(*),
WORK(*)

SUBROUTINE DGEBRD_64(M, N, A, LDA, D, E, TAUQ, TAUP, WORK, LWORK,
      INFO)

INTEGER*8 M, N, LDA, LWORK, INFO
DOUBLE PRECISION A(LDA,*),  D(*),  E(*),  TAUQ(*),  TAUP(*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEBRD([M], [N], A, [LDA], D, E, TAUQ, TAUP, [WORK], [LWORK],
       [INFO])

INTEGER :: M, N, LDA, LWORK, INFO
REAL(8), DIMENSION(:) :: D, E, TAUQ, TAUP, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE GEBRD_64([M], [N], A, [LDA], D, E, TAUQ, TAUP, [WORK],
       [LWORK], [INFO])

INTEGER(8) :: M, N, LDA, LWORK, INFO
REAL(8), DIMENSION(:) :: D, E, TAUQ, TAUP, WORK
```

```
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dgebrd(int m, int n, double *a,  int  lda,  double  *d,
          double *e, double *tauq, double *taup, int *info);

void dgebrd_64(long m, long n, double *a, long  lda,  double
          *d,  double  *e,  double *tauq, double *taup, long
          *info);
```

PURPOSE

dgebrd reduces a general real M-by-N matrix A to upper or lower bidiagonal form B by an orthogonal transformation: Q**T * A * P = B.

If m >= n, B is upper bidiagonal; if m < n, B is lower bidiagonal.

ARGUMENTS

M (input) The number of rows in the matrix A. M >= 0.

N (input) The number of columns in the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N general matrix to be reduced. On exit, if m >= n, the diagonal and the first superdiagonal are overwritten with the upper bidiagonal matrix B; the elements below the diagonal, with the array TAUQ, represent the orthogonal matrix Q as a product of elementary reflectors, and the elements above the first superdiagonal, with the array TAUP, represent the orthogonal matrix P as a product of elementary reflectors; if m < n, the diagonal and the first subdiagonal are overwritten with the lower bidiagonal matrix B; the elements below the first subdiagonal, with the array TAUQ, represent the orthogonal matrix Q as a product of elementary reflectors, and the elements above the diagonal, with the array TAUP, represent the orthogonal matrix P as a product of elementary reflectors. See Further Details.

LDA (input)

```
            The leading dimension of  the  array  A.   LDA  >=
            max(1,M).

D (output)
            The diagonal elements of the bidiagonal matrix  B:
            D(i) = A(i,i).

E (output)
            The off-diagonal elements of the bidiagonal matrix
            B:   if m >= n, E(i) = A(i,i+1) for i = 1,2,...,n-
            1; if m < n, E(i) = A(i+1,i) for i = 1,2,...,m-1.

TAUQ (output)
            The scalar factors of  the  elementary  reflectors
            which  represent  the  orthogonal  matrix  Q.  See
            Further Details.

TAUP (output)
            The scalar factors of  the  elementary  reflectors
            which  represent  the  orthogonal  matrix  P.  See
            Further Details.

WORK (workspace)
            On exit, if INFO = 0, WORK(1) returns the  optimal
            LWORK.

LWORK (input)
            The  length  of  the   array   WORK.    LWORK   >=
            max(1,M,N).   For  optimum  performance  LWORK  >=
            (M+N)*NB, where NB is the optimal blocksize.

            If LWORK = -1, then a workspace query is  assumed;
            the  routine  only  calculates the optimal size of
            the WORK array, returns this value  as  the  first
            entry  of  the  WORK  array,  and no error message
            related to LWORK is issued by XERBLA.

INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value.
```

FURTHER DETAILS

```
The matrices Q and P are represented as products of  elemen-
tary reflectors:

If m >= n,

   Q = H(1) H(2) . . . H(n)  and  P = G(1) G(2) . . . G(n-1)
```

Each H(i) and G(i) has the form:

```
   H(i) = I - tauq * v * v'  and G(i) = I - taup * u * u'
```

where tauq and taup are real scalars, and v and u are real vectors; v(1:i-1) = 0, v(i) = 1, and v(i+1:m) is stored on exit in A(i+1:m,i); u(1:i) = 0, u(i+1) = 1, and u(i+2:n) is stored on exit in A(i,i+2:n); tauq is stored in TAUQ(i) and taup in TAUP(i).

If m < n,

```
   Q = H(1) H(2) . . . H(m-1)  and  P = G(1) G(2) . . . G(m)
```

Each H(i) and G(i) has the form:

```
   H(i) = I - tauq * v * v'  and G(i) = I - taup * u * u'
```

where tauq and taup are real scalars, and v and u are real vectors; v(1:i) = 0, v(i+1) = 1, and v(i+2:m) is stored on exit in A(i+2:m,i); u(1:i-1) = 0, u(i) = 1, and u(i+1:n) is stored on exit in A(i,i+1:n); tauq is stored in TAUQ(i) and taup in TAUP(i).

The contents of A on exit are illustrated by the following examples:

```
m = 6 and n = 5 (m > n):          m = 5 and n = 6 (m < n):

  (  d   e   u1  u1  u1 )           (  d   u1   u1   u1   u1
u1 )
  (  v1  d   e   u2  u2 )           (  e   d    u2   u2   u2
u2 )
  (  v1  v2  d   e   u3 )           (  v1  e    d    u3   u3
u3 )
  (  v1  v2  v3  d   e  )           (  v1  v2   e    d    u4
u4 )
  (  v1  v2  v3  v4  d  )           (  v1   v2   v3   e    d
u5 )
  (  v1  v2  v3  v4  v5 )
```

where d and e denote diagonal and off-diagonal elements of B, vi denotes an element of the vector defining H(i), and ui an element of the vector defining G(i).

Contents

NAME

```
dgecon - estimate the reciprocal of the condition number  of
a  general  real  matrix  A,  in  either  the  1-norm or the
infinity-norm, using the LU factorization computed by SGETRF
```

SYNOPSIS

```
SUBROUTINE DGECON(NORM, N, A, LDA, ANORM, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 NORM
INTEGER N, LDA, INFO
INTEGER WORK2(*)
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION A(LDA,*), WORK(*)

SUBROUTINE DGECON_64(NORM, N, A, LDA, ANORM, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 NORM
INTEGER*8 N, LDA, INFO
INTEGER*8 WORK2(*)
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GECON(NORM, [N], A, [LDA], ANORM, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: NORM
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: WORK
```

```
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE GECON_64(NORM, [N], A, [LDA], ANORM, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: NORM
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
void dgecon(char norm, int n, double  *a,  int  lda,  double
          anorm, double *rcond, int *info);

void dgecon_64(char norm, long n, double *a, long lda,  dou-
          ble anorm, double *rcond, long *info);
```

PURPOSE

dgecon estimates the reciprocal of the condition number of a general real matrix A, in either the 1-norm or the infinity-norm, using the LU factorization computed by SGETRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as
   RCOND = 1 / (norm(A) * norm(inv(A))).

ARGUMENTS

NORM (input)
Specifies whether the 1-norm condition number or the infinity-norm condition number is required:
= '1' or 'O': 1-norm;
= 'I':   Infinity-norm.

N (input) The order of the matrix A. N >= 0.

A (input) The factors L and U from the factorization A = P*L*U as computed by SGETRF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

ANORM (input)
If NORM = '1' or 'O', the 1-norm of the original matrix A. If NORM = 'I', the infinity-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(norm(A) * norm(inv(A))).

WORK (workspace)
dimension(4*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

dgeequ - compute row and column scalings intended to equilibrate an M-by-N matrix A and reduce its condition number

SYNOPSIS

```
SUBROUTINE DGEEQU(M, N, A, LDA, R, C, ROWCN, COLCN, AMAX,
      INFO)

INTEGER M, N, LDA, INFO
DOUBLE PRECISION ROWCN, COLCN, AMAX
DOUBLE PRECISION A(LDA,*), R(*), C(*)

SUBROUTINE DGEEQU_64(M, N, A, LDA, R, C, ROWCN, COLCN, AMAX,
      INFO)

INTEGER*8 M, N, LDA, INFO
DOUBLE PRECISION ROWCN, COLCN, AMAX
DOUBLE PRECISION A(LDA,*), R(*), C(*)
```

F95 INTERFACE

```
SUBROUTINE GEEQU([M], [N], A, [LDA], R, C, ROWCN, COLCN,
       AMAX, [INFO])

INTEGER :: M, N, LDA, INFO
REAL(8) :: ROWCN, COLCN, AMAX
REAL(8), DIMENSION(:) :: R, C
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE GEEQU_64([M], [N], A, [LDA], R, C, ROWCN, COLCN,
       AMAX, [INFO])

INTEGER(8) :: M, N, LDA, INFO
```

```
REAL(8) :: ROWCN, COLCN, AMAX
REAL(8), DIMENSION(:) :: R, C
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dgeequ(int m, int n, double *a,  int  lda,  double  *r,
          double  *c,  double  *rowcn, double *colcn, double
          *amax, int *info);

void dgeequ_64(long m, long n, double *a, long  lda,  double
          *r,  double *c, double *rowcn, double *colcn, dou-
          ble *amax, long *info);
```

PURPOSE

dgeequ computes row and column scalings intended to equilibrate an M-by-N matrix A and reduce its condition number. R returns the row scale factors and C the column scale factors, chosen to try to make the largest element in each row and column of the matrix B with elements B(i,j)=R(i)*A(i,j)*C(j) have absolute value 1.

R(i) and C(j) are restricted to be between SMLNUM = smallest safe number and BIGNUM = largest safe number. Use of these scaling factors is not guaranteed to reduce the condition number of A but works well in practice.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input) The M-by-N matrix whose equilibration factors are to be computed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

R (output)
If INFO = 0 or INFO > M, R contains the row scale factors for A.

C (output)

If INFO = 0, C contains the column scale factors for A.

ROWCN (output)
If INFO = 0 or INFO > M, ROWCN contains the ratio of the smallest R(i) to the largest R(i). If ROWCN >= 0.1 and AMAX is neither too large nor too small, it is not worth scaling by R.

COLCN (output)
If INFO = 0, COLCN contains the ratio of the smallest C(i) to the largest C(i). If COLCN >= 0.1, it is not worth scaling by C.

AMAX (output)
Absolute value of largest matrix element. If AMAX is very close to overflow or very close to underflow, the matrix should be scaled.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= M: the i-th row of A is exactly zero
> M: the (i-M)-th column of A is exactly zero

Contents

NAME

```
dgees - compute for an N-by-N real  nonsymmetric  matrix  A,
the eigenvalues, the real Schur form T, and, optionally, the
matrix of Schur vectors Z
```

SYNOPSIS

```
SUBROUTINE DGEES(JOBZ, SORTEV, SELECT, N, A, LDA, NOUT, WR, WI, Z,
      LDZ, WORK, LDWORK, WORK3, INFO)

CHARACTER * 1 JOBZ, SORTEV
INTEGER N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL SELECT
LOGICAL WORK3(*)
DOUBLE PRECISION A(LDA,*), WR(*), WI(*), Z(LDZ,*), WORK(*)

SUBROUTINE DGEES_64(JOBZ, SORTEV, SELECT, N, A, LDA, NOUT, WR, WI, Z,
      LDZ, WORK, LDWORK, WORK3, INFO)

CHARACTER * 1 JOBZ, SORTEV
INTEGER*8 N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL*8 SELECT
LOGICAL*8 WORK3(*)
DOUBLE PRECISION A(LDA,*), WR(*), WI(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEES(JOBZ, SORTEV, SELECT, [N], A, [LDA], NOUT, WR, WI, Z,
       [LDZ], [WORK], [LDWORK], [WORK3], [INFO])

CHARACTER(LEN=1) :: JOBZ, SORTEV
INTEGER :: N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL :: SELECT
LOGICAL, DIMENSION(:) :: WORK3
```

```
REAL(8), DIMENSION(:) :: WR, WI, WORK
REAL(8), DIMENSION(:,:) :: A, Z

SUBROUTINE GEES_64(JOBZ, SORTEV, SELECT, [N], A, [LDA], NOUT, WR, WI,
       Z, [LDZ], [WORK], [LDWORK], [WORK3], [INFO])

CHARACTER(LEN=1) :: JOBZ, SORTEV
INTEGER(8) :: N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL(8) :: SELECT
LOGICAL(8), DIMENSION(:) :: WORK3
REAL(8), DIMENSION(:) :: WR, WI, WORK
REAL(8), DIMENSION(:,:) :: A, Z
```

C INTERFACE

```
#include <sunperf.h>
void       dgees(char       jobz,        char        sortev,
          int(*select)(double,double), int n, double *a, int
          lda, int *nout, double *wr, double *wi, double *z,
          int ldz, int *info);

void      dgees_64(char       jobz,       char       sortev,
          long(*select)(double,double),  long  n, double *a,
          long lda, long *nout, double *wr, double *wi, dou-
          ble *z, long ldz, long *info);
```

PURPOSE

dgees computes for an N-by-N real nonsymmetric matrix A, the eigenvalues, the real Schur form T, and, optionally, the matrix of Schur vectors Z. This gives the Schur factorization A = Z*T*(Z**T).

Optionally, it also orders the eigenvalues on the diagonal of the real Schur form so that selected eigenvalues are at the top left. The leading columns of Z then form an orthonormal basis for the invariant subspace corresponding to the selected eigenvalues.

A matrix is in real Schur form if it is upper quasi-triangular with 1-by-1 and 2-by-2 blocks. 2-by-2 blocks will be standardized in the form

```
        [  a  b  ]
        [  c  a  ]
```

where b*c < 0. The eigenvalues of such a block are a +- sqrt(bc).

ARGUMENTS

```
JOBZ (input)
          = 'N': Schur vectors are not computed;
          = 'V': Schur vectors are computed.

SORTEV (input)
          Specifies whether or not to order the  eigenvalues
          on  the diagonal of the Schur form.  = 'N': Eigen-
          values are not ordered;
          = 'S': Eigenvalues are ordered (see SELECT).

SELECT (input)
          SELECT must be declared EXTERNAL  in  the  calling
          subroutine.   If  SORTEV  = 'S', SELECT is used to
          select eigenvalues to sort to the top left of  the
          Schur  form.   If  SORTEV  =  'N',  SELECT  is not
          referenced.  An eigenvalue WR(j)+sqrt(-1)*WI(j) is
          selected  if SELECT(WR(j),WI(j)) is true; i.e., if
          either one of a complex conjugate pair  of  eigen-
          values  is selected, then both complex eigenvalues
          are selected.  Note that a selected complex eigen-
          value  may no longer satisfy SELECT(WR(j),WI(j)) =
          .TRUE. after ordering, since ordering  may  change
          the  value  of  complex eigenvalues (especially if
          the eigenvalue is ill-conditioned); in  this  case
          INFO is set to N+2 (see INFO below).

N (input) The order of the matrix A. N >= 0.

A (input/output)
          On entry, the N-by-N matrix A.   On  exit,  A  has
          been overwritten by its real Schur form T.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

NOUT (output)
          If SORTEV = 'N', NOUT = 0.  If SORTEV = 'S',  NOUT
          =  number of eigenvalues (after sorting) for which
          SELECT is true. (Complex conjugate pairs for which
          SELECT is true for either eigenvalue count as 2.)

WR (output)
          WR and WI contain the real  and  imaginary  parts,
          respectively,  of  the computed eigenvalues in the
          same order that they appear on the diagonal of the
          output  Schur  form T.  Complex conjugate pairs of
          eigenvalues will  appear  consecutively  with  the
```

eigenvalue having the positive imaginary part first.

WI (output)
See the description for WR.

Z (output)
If JOBZ = 'V', Z contains the orthogonal matrix Z of Schur vectors. If JOBZ = 'N', Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1; if JOBZ = 'V', LDZ >= N.

WORK (workspace)
On exit, if INFO = 0, WORK(1) contains the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,3*N). For good performance, LDWORK must generally be larger.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK3 (workspace)
dimension(N) Not referenced if SORTEV = 'N'.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, and i is
<= N: the QR algorithm failed to compute all the eigenvalues; elements 1:ILO-1 and i+1:N of WR and WI contain those eigenvalues which have converged; if JOBZ = 'V', Z contains the matrix which reduces A to its partially converged Schur form. = N+1: the eigenvalues could not be reordered because some eigenvalues were too close to separate (the problem is very ill-conditioned); = N+2: after reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the Schur form no longer satisfy SELECT=.TRUE. This could also be caused by underflow due to scaling.

Contents

NAME

```
dgeesx - compute for an N-by-N real nonsymmetric  matrix  A,
the eigenvalues, the real Schur form T, and, optionally, the
matrix of Schur vectors Z
```

SYNOPSIS

```
SUBROUTINE DGEESX(JOBZ, SORTEV, SELECT, SENSE, N, A, LDA, NOUT, WR,
      WI, Z, LDZ, SRCONE, RCONV, WORK, LDWORK, IWORK2, LDWRK2, BWORK3,
      INFO)

CHARACTER * 1 JOBZ, SORTEV, SENSE
INTEGER N, LDA, NOUT, LDZ, LDWORK, LDWRK2, INFO
INTEGER IWORK2(*)
LOGICAL SELECT
LOGICAL BWORK3(*)
DOUBLE PRECISION SRCONE, RCONV
DOUBLE PRECISION A(LDA,*), WR(*), WI(*), Z(LDZ,*), WORK(*)

SUBROUTINE DGEESX_64(JOBZ, SORTEV, SELECT, SENSE, N, A, LDA, NOUT,
      WR, WI, Z, LDZ, SRCONE, RCONV, WORK, LDWORK, IWORK2, LDWRK2,
      BWORK3, INFO)

CHARACTER * 1 JOBZ, SORTEV, SENSE
INTEGER*8 N, LDA, NOUT, LDZ, LDWORK, LDWRK2, INFO
INTEGER*8 IWORK2(*)
LOGICAL*8 SELECT
LOGICAL*8 BWORK3(*)
DOUBLE PRECISION SRCONE, RCONV
DOUBLE PRECISION A(LDA,*), WR(*), WI(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEESX(JOBZ, SORTEV, SELECT, SENSE, [N], A, [LDA], NOUT,
```

```
       WR, WI, Z, [LDZ], SRCONE, RCONV, [WORK], [LDWORK], [IWORK2],
       [LDWRK2], [BWORK3], [INFO])

CHARACTER(LEN=1) :: JOBZ, SORTEV, SENSE
INTEGER :: N, LDA, NOUT, LDZ, LDWORK, LDWRK2, INFO
INTEGER, DIMENSION(:) :: IWORK2
LOGICAL :: SELECT
LOGICAL, DIMENSION(:) :: BWORK3
REAL(8) :: SRCONE, RCONV
REAL(8), DIMENSION(:) :: WR, WI, WORK
REAL(8), DIMENSION(:,:) :: A, Z

SUBROUTINE GEESX_64(JOBZ, SORTEV, SELECT, SENSE, [N], A, [LDA], NOUT,
       WR, WI, Z, [LDZ], SRCONE, RCONV, [WORK], [LDWORK], [IWORK2],
       [LDWRK2], [BWORK3], [INFO])

CHARACTER(LEN=1) :: JOBZ, SORTEV, SENSE
INTEGER(8) :: N, LDA, NOUT, LDZ, LDWORK, LDWRK2, INFO
INTEGER(8), DIMENSION(:) :: IWORK2
LOGICAL(8) :: SELECT
LOGICAL(8), DIMENSION(:) :: BWORK3
REAL(8) :: SRCONE, RCONV
REAL(8), DIMENSION(:) :: WR, WI, WORK
REAL(8), DIMENSION(:,:) :: A, Z
```

C INTERFACE

```
#include <sunperf.h>

void       dgeesx(char       jobz,       char        sortev,
          int(*select)(double,double),  char  sense,  int n,
          double *a, int lda, int *nout, double *wr,  double
          *wi,  double  *z,  int ldz, double *srcone, double
          *rconv, int *info);

void      dgeesx_64(char      jobz,       char       sortev,
          long(*select)(double,double),  char sense, long n,
          double *a, long lda, long *nout, double *wr,  dou-
          ble *wi, double *z, long ldz, double *srcone, dou-
          ble *rconv, long *info);
```

PURPOSE

dgeesx computes for an N-by-N real nonsymmetric matrix A, the eigenvalues, the real Schur form T, and, optionally, the matrix of Schur vectors Z. This gives the Schur factorization A = Z*T*(Z**T).

Optionally, it also orders the eigenvalues on the diagonal of the real Schur form so that selected eigenvalues are at the top left; computes a reciprocal condition number for the average of the selected eigenvalues (RCONDE); and computes a reciprocal condition number for the right invariant subspace corresponding to the selected eigenvalues (RCONDV). The leading columns of Z form an orthonormal basis for this invariant subspace.

For further explanation of the reciprocal condition numbers RCONDE and RCONDV, see Section 4.10 of the LAPACK Users' Guide (where these quantities are called s and sep respectively).

A real matrix is in real Schur form if it is upper quasi-triangular with 1-by-1 and 2-by-2 blocks. 2-by-2 blocks will be standardized in the form

```
          [  a  b  ]
          [  c  a  ]
```

where b*c < 0. The eigenvalues of such a block are a +- sqrt(bc).

ARGUMENTS

JOBZ (input)
= 'N': Schur vectors are not computed;
= 'V': Schur vectors are computed.

SORTEV (input)
Specifies whether or not to order the eigenvalues on the diagonal of the Schur form. = 'N': Eigenvalues are not ordered;
= 'S': Eigenvalues are ordered (see SELECT).

SELECT (input)
SELECT must be declared EXTERNAL in the calling subroutine. If SORTEV = 'S', SELECT is used to select eigenvalues to sort to the top left of the Schur form. If SORTEV = 'N', SELECT is not referenced. An eigenvalue WR(j)+sqrt(-1)*WI(j) is selected if SELECT(WR(j),WI(j)) is true; i.e., if either one of a complex conjugate pair of eigenvalues is selected, then both are. Note that a selected complex eigenvalue may no longer satisfy SELECT(WR(j),WI(j)) = .TRUE. after ordering, since ordering may change the value of complex eigenvalues (especially if the eigenvalue is ill-conditioned); in this case INFO may be set to N+3 (see INFO below).

SENSE (input)
Determines which reciprocal condition numbers are computed. = 'N': None are computed;
= 'E': Computed for average of selected eigenvalues only;
= 'V': Computed for selected right invariant subspace only;
= 'B': Computed for both. If SENSE = 'E', 'V' or 'B', SORTEV must equal 'S'.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the N-by-N matrix A. On exit, A is overwritten by its real Schur form T.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

NOUT (output)
If SORTEV = 'N', NOUT = 0. If SORTEV = 'S', NOUT = number of eigenvalues (after sorting) for which SELECT is true. (Complex conjugate pairs for which SELECT is true for either eigenvalue count as 2.)

WR (output)
WR and WI contain the real and imaginary parts, respectively, of the computed eigenvalues, in the same order that they appear on the diagonal of the output Schur form T. Complex conjugate pairs of eigenvalues appear consecutively with the eigenvalue having the positive imaginary part first.

WI (output)
See the description for WR.

Z (output)
If JOBZ = 'V', Z contains the orthogonal matrix Z of Schur vectors. If JOBZ = 'N', Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= N.

SRCONE (output)
If SENSE = 'E' or 'B', SRCONE contains the reciprocal condition number for the average of the selected eigenvalues. Not referenced if SENSE = 'N' or 'V'.

RCONV (output)
If SENSE = 'V' or 'B', RCONV contains the reciprocal condition number for the selected right invariant subspace. Not referenced if SENSE = 'N' or 'E'.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,3*N). Also, if SENSE = 'E' or 'V' or 'B', LDWORK >= N+2*NOUT*(N-NOUT), where NOUT is the number of selected eigenvalues computed by this routine. Note that N+2*NOUT*(N-NOUT) <= N+N*N/2. For good performance, LDWORK must generally be larger.

IWORK2 (workspace/output)
Not referenced if SENSE = 'N' or 'E'. On exit, if INFO = 0, IWORK2(1) returns the optimal LDWRK2.

LDWRK2 (input)
The dimension of the array IWORK2. LDWRK2 >= 1; if SENSE = 'V' or 'B', LDWRK2 >= NOUT*(N-NOUT).

BWORK3 (workspace)
dimension(N) Not referenced if SORTEV = 'N'.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, and i is
<= N: the QR algorithm failed to compute all the eigenvalues; elements 1:ILO-1 and i+1:N of WR and WI contain those eigenvalues which have converged; if JOBZ = 'V', Z contains the transformation which reduces A to its partially converged Schur form.
= N+1: the eigenvalues could not be reordered because some eigenvalues were too close to separate (the problem is very ill-conditioned); = N+2: after reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the Schur form no longer satisfy SELECT=.TRUE. This could also be caused by underflow due to scaling.

Contents

NAME

```
dgeev - compute for an N-by-N real  nonsymmetric  matrix  A,
the  eigenvalues  and,  optionally,  the  left  and/or right
eigenvectors
```

SYNOPSIS

```
SUBROUTINE DGEEV(JOBVL, JOBVR, N, A, LDA, WR, WI, VL, LDVL, VR, LDVR,
      WORK, LDWORK, INFO)

CHARACTER * 1 JOBVL, JOBVR
INTEGER N, LDA, LDVL, LDVR, LDWORK, INFO
DOUBLE  PRECISION  A(LDA,*),   WR(*),   WI(*),   VL(LDVL,*),
VR(LDVR,*), WORK(*)

SUBROUTINE DGEEV_64(JOBVL, JOBVR, N, A, LDA, WR, WI, VL, LDVL, VR,
      LDVR, WORK, LDWORK, INFO)

CHARACTER * 1 JOBVL, JOBVR
INTEGER*8 N, LDA, LDVL, LDVR, LDWORK, INFO
DOUBLE  PRECISION  A(LDA,*),   WR(*),   WI(*),   VL(LDVL,*),
VR(LDVR,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEEV(JOBVL, JOBVR, [N], A, [LDA], WR, WI, VL, [LDVL], VR,
       [LDVR], [WORK], [LDWORK], [INFO])

CHARACTER(LEN=1) :: JOBVL, JOBVR
INTEGER :: N, LDA, LDVL, LDVR, LDWORK, INFO
REAL(8), DIMENSION(:) :: WR, WI, WORK
REAL(8), DIMENSION(:,:) :: A, VL, VR

SUBROUTINE GEEV_64(JOBVL, JOBVR, [N], A, [LDA], WR, WI, VL, [LDVL],
```

```
          VR, [LDVR], [WORK], [LDWORK], [INFO])

   CHARACTER(LEN=1) :: JOBVL, JOBVR
   INTEGER(8) :: N, LDA, LDVL, LDVR, LDWORK, INFO
   REAL(8), DIMENSION(:) :: WR, WI, WORK
   REAL(8), DIMENSION(:,:) :: A, VL, VR
```

C INTERFACE

```
   #include <sunperf.h>

   void dgeev(char jobvl, char jobvr, int  n,  double  *a,  int
             lda, double *wr, double *wi, double *vl, int ldvl,
             double *vr, int ldvr, int *info);
   void dgeev_64(char jobvl, char jobvr,  long  n,  double  *a,
             long lda, double *wr, double *wi, double *vl, long
             ldvl, double *vr, long ldvr, long *info);
```

PURPOSE

```
   dgeev computes for an N-by-N real nonsymmetric matrix A, the
   eigenvalues and, optionally, the left and/or right eigenvec-
   tors.

   The right eigenvector v(j) of A satisfies
                    A * v(j) = lambda(j) * v(j)
   where lambda(j) is its eigenvalue.
   The left eigenvector u(j) of A satisfies
                 u(j)**H * A = lambda(j) * u(j)**H
   where u(j)**H denotes the conjugate transpose of u(j).

   The computed eigenvectors are normalized to  have  Euclidean
   norm equal to 1 and largest component real.
```

ARGUMENTS

```
   JOBVL (input)
             = 'N': left eigenvectors of A are not computed;
             = 'V': left eigenvectors of A are computed.

   JOBVR (input)
             = 'N': right eigenvectors of A are not computed;
             = 'V': right eigenvectors of A are computed.

   N (input) The order of the matrix A. N >= 0.

   A (input/output)
             On entry, the N-by-N matrix A.   On  exit,  A  has
```

been overwritten.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

WR (output)
WR and WI contain the real and imaginary parts, respectively, of the computed eigenvalues. Complex conjugate pairs of eigenvalues appear consecutively with the eigenvalue having the positive imaginary part first.

WI (output)
See the description for WR.

VL (output)
If JOBVL = 'V', the left eigenvectors u(j) are stored one after another in the columns of VL, in the same order as their eigenvalues. If JOBVL = 'N', VL is not referenced. If the j-th eigenvalue is real, then u(j) = VL(:,j), the j-th column of VL. If the j-th and (j+1)-st eigenvalues form a complex conjugate pair, then u(j) = VL(:,j) + i*VL(:,j+1) and
u(j+1) = VL(:,j) - i*VL(:,j+1).

LDVL (input)
The leading dimension of the array VL. LDVL >= 1; if JOBVL = 'V', LDVL >= N.

VR (input)
If JOBVR = 'V', the right eigenvectors v(j) are stored one after another in the columns of VR, in the same order as their eigenvalues. If JOBVR = 'N', VR is not referenced. If the j-th eigenvalue is real, then v(j) = VR(:,j), the j-th column of VR. If the j-th and (j+1)-st eigenvalues form a complex conjugate pair, then v(j) = VR(:,j) + i*VR(:,j+1) and
v(j+1) = VR(:,j) - i*VR(:,j+1).

LDVR (input)
The leading dimension of the array VR. LDVR >= 1; if JOBVR = 'V', LDVR >= N.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)

The dimension of the array WORK. LDWORK >= max(1,3*N), and if JOBVL = 'V' or JOBVR = 'V', LDWORK >= 4*N. For good performance, LDWORK must generally be larger.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, the QR algorithm failed to compute all the eigenvalues, and no eigenvectors have been computed; elements i+1:N of WR and WI contain eigenvalues which have converged.

Contents

NAME

dgeevx - compute for an N-by-N real nonsymmetric matrix A, the eigenvalues and, optionally, the left and/or right eigenvectors

SYNOPSIS

```
SUBROUTINE DGEEVX(BALANC, JOBVL, JOBVR, SENSE, N, A, LDA, WR, WI, VL,
      LDVL, VR, LDVR, ILO, IHI, SCALE, ABNRM, RCONE, RCONV, WORK,
      LDWORK, IWORK2, INFO)

CHARACTER * 1 BALANC, JOBVL, JOBVR, SENSE
INTEGER N, LDA, LDVL, LDVR, ILO, IHI, LDWORK, INFO
INTEGER IWORK2(*)
DOUBLE PRECISION ABNRM
DOUBLE  PRECISION  A(LDA,*),   WR(*),   WI(*),   VL(LDVL,*),
VR(LDVR,*), SCALE(*), RCONE(*), RCONV(*), WORK(*)

SUBROUTINE DGEEVX_64(BALANC, JOBVL, JOBVR, SENSE, N, A, LDA, WR, WI,
      VL, LDVL, VR, LDVR, ILO, IHI, SCALE, ABNRM, RCONE, RCONV, WORK,
      LDWORK, IWORK2, INFO)

CHARACTER * 1 BALANC, JOBVL, JOBVR, SENSE
INTEGER*8 N, LDA, LDVL, LDVR, ILO, IHI, LDWORK, INFO
INTEGER*8 IWORK2(*)
DOUBLE PRECISION ABNRM
DOUBLE  PRECISION  A(LDA,*),   WR(*),   WI(*),   VL(LDVL,*),
VR(LDVR,*), SCALE(*), RCONE(*), RCONV(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEEVX(BALANC, JOBVL, JOBVR, SENSE, [N], A, [LDA], WR, WI,
       VL, [LDVL], VR, [LDVR], ILO, IHI, SCALE, ABNRM, RCONE, RCONV,
       [WORK], [LDWORK], [IWORK2], [INFO])
```

```
CHARACTER(LEN=1) :: BALANC, JOBVL, JOBVR, SENSE
INTEGER :: N, LDA, LDVL, LDVR, ILO, IHI, LDWORK, INFO
INTEGER, DIMENSION(:) :: IWORK2
REAL(8) :: ABNRM
REAL(8), DIMENSION(:) :: WR, WI, SCALE, RCONE, RCONV, WORK
REAL(8), DIMENSION(:,:) :: A, VL, VR

SUBROUTINE GEEVX_64(BALANC, JOBVL, JOBVR, SENSE, [N], A, [LDA], WR,
       WI, VL, [LDVL], VR, [LDVR], ILO, IHI, SCALE, ABNRM, RCONE, RCONV,
       [WORK], [LDWORK], [IWORK2], [INFO])

CHARACTER(LEN=1) :: BALANC, JOBVL, JOBVR, SENSE
INTEGER(8) :: N, LDA, LDVL, LDVR, ILO, IHI, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK2
REAL(8) :: ABNRM
REAL(8), DIMENSION(:) :: WR, WI, SCALE, RCONE, RCONV, WORK
REAL(8), DIMENSION(:,:) :: A, VL, VR
```

C INTERFACE

```
#include <sunperf.h>

void dgeevx(char balanc, char jobvl, char jobvr, char sense,
          int n, double *a, int lda, double *wr, double *wi,
          double *vl, int ldvl, double *vr,  int  ldvr,  int
          *ilo, int *ihi, double *scale, double *abnrm, dou-
          ble *rcone, double *rconv, int *info);

void dgeevx_64(char balanc, char  jobvl,  char  jobvr,  char
          sense,  long  n,  double *a, long lda, double *wr,
          double *wi, double *vl,  long  ldvl,  double  *vr,
          long  ldvr,  long  *ilo, long *ihi, double *scale,
          double *abnrm, double *rcone, double *rconv,  long
          *info);
```

PURPOSE

dgeevx computes for an N-by-N real nonsymmetric matrix A, the eigenvalues and, optionally, the left and/or right eigenvectors.

Optionally also, it computes a balancing transformation to improve the conditioning of the eigenvalues and eigenvectors (ILO, IHI, SCALE, and ABNRM), reciprocal condition numbers for the eigenvalues (RCONDE), and reciprocal condition numbers for the right
eigenvectors (RCONDV).

The right eigenvector v(j) of A satisfies

```
                 A * v(j) = lambda(j) * v(j)
where lambda(j) is its eigenvalue.
The left eigenvector u(j) of A satisfies
              u(j)**H * A = lambda(j) * u(j)**H
where u(j)**H denotes the conjugate transpose of u(j).
```

The computed eigenvectors are normalized to have Euclidean norm equal to 1 and largest component real.

Balancing a matrix means permuting the rows and columns to make it more nearly upper triangular, and applying a diagonal similarity transformation D * A * D**(-1), where D is a diagonal matrix, to make its rows and columns closer in norm and the condition numbers of its eigenvalues and eigenvectors smaller. The computed reciprocal condition numbers correspond to the balanced matrix. Permuting rows and columns will not change the condition numbers (in exact arithmetic) but diagonal scaling will. For further explanation of balancing, see section 4.10.2 of the LAPACK Users' Guide.

ARGUMENTS

BALANC (input)
Indicates how the input matrix should be diagonally scaled and/or permuted to improve the conditioning of its eigenvalues. = 'N': Do not diagonally scale or permute;
= 'P': Perform permutations to make the matrix more nearly upper triangular. Do not diagonally scale; = 'S': Diagonally scale the matrix, i.e. replace A by D*A*D**(-1), where D is a diagonal matrix chosen to make the rows and columns of A more equal in norm. Do not permute; = 'B': Both diagonally scale and permute A.

Computed reciprocal condition numbers will be for the matrix after balancing and/or permuting. Permuting does not change condition numbers (in exact arithmetic), but balancing does.

JOBVL (input)
= 'N': left eigenvectors of A are not computed;
= 'V': left eigenvectors of A are computed. If SENSE = 'E' or 'B', JOBVL must = 'V'.

JOBVR (input)
= 'N': right eigenvectors of A are not computed;

= 'V': right eigenvectors of A are computed. If SENSE = 'E' or 'B', JOBVR must = 'V'.

SENSE (input)
Determines which reciprocal condition numbers are computed. = 'N': None are computed;
= 'E': Computed for eigenvalues only;
= 'V': Computed for right eigenvectors only;
= 'B': Computed for eigenvalues and right eigenvectors.

If SENSE = 'E' or 'B', both left and right eigenvectors must also be computed (JOBVL = 'V' and JOBVR = 'V').

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the N-by-N matrix A. On exit, A has been overwritten. If JOBVL = 'V' or JOBVR = 'V', A contains the real Schur form of the balanced version of the input matrix A.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

WR (output)
WR and WI contain the real and imaginary parts, respectively, of the computed eigenvalues. Complex conjugate pairs of eigenvalues will appear consecutively with the eigenvalue having the positive imaginary part first.

WI (output)
See the description for WR.

VL (output)
If JOBVL = 'V', the left eigenvectors u(j) are stored one after another in the columns of VL, in the same order as their eigenvalues. If JOBVL = 'N', VL is not referenced. If the j-th eigenvalue is real, then u(j) = VL(:,j), the j-th column of VL. If the j-th and (j+1)-st eigenvalues form a complex conjugate pair, then u(j) = VL(:,j) + i*VL(:,j+1) and
u(j+1) = VL(:,j) - i*VL(:,j+1).

LDVL (input)
The leading dimension of the array VL. LDVL >= 1; if JOBVL = 'V', LDVL >= N.

VR (output)
If JOBVR = 'V', the right eigenvectors v(j) are stored one after another in the columns of VR, in the same order as their eigenvalues. If JOBVR = 'N', VR is not referenced. If the j-th eigenvalue is real, then v(j) = VR(:,j), the j-th column of VR. If the j-th and (j+1)-st eigenvalues form a complex conjugate pair, then v(j) = VR(:,j) + i*VR(:,j+1) and
v(j+1) = VR(:,j) - i*VR(:,j+1).

LDVR (input)
The leading dimension of the array VR. LDVR >= 1, and if JOBVR = 'V', LDVR >= N.

ILO (output)
ILO and IHI are integer values determined when A was balanced. The balanced A(i,j) = 0 if I > J and J = 1,...,ILO-1 or I = IHI+1,...,N.

IHI (output)
See the description of ILO.

SCALE (output)
Details of the permutations and scaling factors applied when balancing A. If P(j) is the index of the row and column interchanged with row and column j, and D(j) is the scaling factor applied to row and column j, then SCALE(J) = P(J), for J = 1,...,ILO-1 = D(J), for J = ILO,...,IHI = P(J) for J = IHI+1,...,N. The order in which the interchanges are made is N to IHI+1, then 1 to ILO-1.

ABNRM (output)
The one-norm of the balanced matrix (the maximum of the sum of absolute values of elements of any column).

RCONE (output)
RCONE(j) is the reciprocal condition number of the j-th eigenvalue.

RCONV (output)
RCONV(j) is the reciprocal condition number of the j-th right eigenvector.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. If SENSE = 'N' or 'E', LDWORK >= max(1,2*N), and if JOBVL = 'V' or JOBVR = 'V', LDWORK >= 3*N. If SENSE = 'V' or 'B', LDWORK >= N*(N+6). For good performance, LDWORK must generally be larger.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

IWORK2 (workspace)
dimension(2*N-2) If SENSE = 'N' or 'E', not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, the QR algorithm failed to compute all the eigenvalues, and no eigenvectors or condition numbers have been computed; elements 1:ILO-1 and i+1:N of WR and WI contain eigenvalues which have converged.

Contents

NAME

```
dgegs - routine is deprecated and has been replaced by  rou-
tine SGGES
```

SYNOPSIS

```
SUBROUTINE DGEGS(JOBVSL, JOBVSR, N, A, LDA, B, LDB, ALPHAR, ALPHAI,
      BETA, VSL, LDVSL, VSR, LDVSR, WORK, LDWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR
INTEGER N, LDA, LDB, LDVSL, LDVSR, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*),  ALPHAR(*),  ALPHAI(*),
BETA(*), VSL(LDVSL,*), VSR(LDVSR,*), WORK(*)

SUBROUTINE DGEGS_64(JOBVSL, JOBVSR, N, A, LDA, B, LDB, ALPHAR,
      ALPHAI, BETA, VSL, LDVSL, VSR, LDVSR, WORK, LDWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR
INTEGER*8 N, LDA, LDB, LDVSL, LDVSR, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*),  ALPHAR(*),  ALPHAI(*),
BETA(*), VSL(LDVSL,*), VSR(LDVSR,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEGS(JOBVSL, JOBVSR, [N], A, [LDA], B, [LDB], ALPHAR,
       ALPHAI, BETA, VSL, [LDVSL], VSR, [LDVSR], [WORK], [LDWORK], [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR
INTEGER :: N, LDA, LDB, LDVSL, LDVSR, LDWORK, INFO
REAL(8), DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
REAL(8), DIMENSION(:,:) :: A, B, VSL, VSR

SUBROUTINE GEGS_64(JOBVSL, JOBVSR, [N], A, [LDA], B, [LDB], ALPHAR,
       ALPHAI, BETA, VSL, [LDVSL], VSR, [LDVSR], [WORK], [LDWORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOBVSL, JOBVSR
INTEGER(8) :: N, LDA, LDB, LDVSL, LDVSR, LDWORK, INFO
REAL(8), DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
REAL(8), DIMENSION(:,:) :: A, B, VSL, VSR
```

C INTERFACE

```
#include <sunperf.h>

void dgegs(char jobvsl, char jobvsr, int n, double  *a,  int
          lda,  double  *b,  int ldb, double *alphar, double
          *alphai, double *beta,  double  *vsl,  int  ldvsl,
          double *vsr, int ldvsr, int *info);
void dgegs_64(char jobvsl, char jobvsr, long n,  double  *a,
          long  lda,  double  *b,  long ldb, double *alphar,
          double *alphai, double *beta,  double  *vsl,  long
          ldvsl, double *vsr, long ldvsr, long *info);
```

PURPOSE

dgegs routine is deprecated and has been replaced by routine SGGES.

SGEGS computes for a pair of N-by-N real nonsymmetric matrices A, B: the generalized eigenvalues (alphar +/- alphai*i, beta), the real Schur form (A, B), and optionally left and/or right Schur vectors (VSL and VSR).

(If only the generalized eigenvalues are needed, use the driver SGEGV instead.)

A generalized eigenvalue for a pair of matrices (A,B) is, roughly speaking, a scalar w or a ratio alpha/beta = w, such that A - w*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0, and even for both being zero. A good beginning reference is the book, "Matrix Computations", by G. Golub & C. van Loan (Johns Hopkins U. Press)

The (generalized) Schur form of a pair of matrices is the result of multiplying both matrices on the left by one orthogonal matrix and both on the right by another orthogonal matrix, these two orthogonal matrices being chosen so as to bring the pair of matrices into (real) Schur form.

A pair of matrices A, B is in generalized real Schur form if B is upper triangular with non-negative diagonal and A is block upper triangular with 1-by-1 and 2-by-2 blocks. 1-by-1 blocks correspond to real generalized eigenvalues,

```
while 2-by-2 blocks of A will be  "standardized"  by  making
the corresponding elements of B have the form:
        [  a  0  ]
        [  0  b  ]

and the pair of corresponding 2-by-2 blocks in A and B  will
have a complex conjugate pair of generalized eigenvalues.

The left and right Schur vectors are the columns of VSL  and
VSR,  respectively,  where  VSL  and  VSR are the orthogonal
matrices which reduce A and B to Schur form:

Schur form of (A,B) = ( (VSL)**T A (VSR), (VSL)**T B (VSR) )
```

ARGUMENTS

```
JOBVSL (input)
          = 'N':  do not compute the left Schur vectors;
          = 'V':  compute the left Schur vectors.

JOBVSR (input)
          = 'N':  do not compute the right Schur vectors;
          = 'V':  compute the right Schur vectors.

N (input) The order of the matrices A, B, VSL, and  VSR.   N
          >= 0.

A (input/output)
          On entry, the first of the pair of matrices  whose
          generalized  eigenvalues  and  (optionally)  Schur
          vectors are to be computed.  On exit, the general-
          ized  Schur  form  of A.  Note: to avoid overflow,
          the Frobenius norm of the matrix A should be  less
          than the overflow threshold.

LDA (input)
          The leading dimension of A.  LDA >= max(1,N).

B (input/output)
          On entry, the second of the pair of matrices whose
          generalized  eigenvalues  and  (optionally)  Schur
          vectors are to be computed.  On exit, the general-
          ized  Schur  form  of B.  Note: to avoid overflow,
          the Frobenius norm of the matrix B should be  less
          than the overflow threshold.

LDB (input)
          The leading dimension of B.  LDB >= max(1,N).
```

ALPHAR (output)
On exit, (ALPHAR(j) + ALPHAI(j)*i)/BETA(j), j=1,...,N, will be the generalized eigenvalues. ALPHAR(j) + ALPHAI(j)*i, j=1,...,N and BETA(j),j=1,...,N are the diagonals of the complex Schur form (A,B) that would result if the 2-by-2 diagonal blocks of the real Schur form of (A,B) were further reduced to triangular form using 2-by-2 complex unitary transformations. If ALPHAI(j) is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-st eigenvalues are a complex conjugate pair, with ALPHAI(j+1) negative.

Note: the quotients ALPHAR(j)/BETA(j) and ALPHAI(j)/BETA(j) may easily over- or underflow, and BETA(j) may even be zero. Thus, the user should avoid naively computing the ratio alpha/beta. However, ALPHAR and ALPHAI will be always less than and usually comparable with norm(A) in magnitude, and BETA always less than and usually comparable with norm(B).

ALPHAI (output)
See the description for ALPHAR.

BETA (output)
See the description for ALPHAR.

VSL (input)
If JOBVSL = 'V', VSL will contain the left Schur vectors. (See "Purpose", above.) Not referenced if JOBVSL = 'N'.

LDVSL (input)
The leading dimension of the matrix VSL. LDVSL >=1, and if JOBVSL = 'V', LDVSL >= N.

VSR (input)
If JOBVSR = 'V', VSR will contain the right Schur vectors. (See "Purpose", above.) Not referenced if JOBVSR = 'N'.

LDVSR (input)
The leading dimension of the matrix VSR. LDVSR >= 1, and if JOBVSR = 'V', LDVSR >= N.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal

LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,4*N). For good performance, LDWORK must generally be larger. To compute the optimal value of LDWORK, call ILAENV to get blocksizes (for SGEQRF, SORMQR, and SORGQR.) Then compute: NB -- MAX of the blocksizes for SGEQRF, SORMQR, and SORGQR The optimal LDWORK is 2*N + N*(NB+1).

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
= 1,...,N: The QZ iteration failed. (A,B) are not in Schur form, but ALPHAR(j), ALPHAI(j), and BETA(j) should be correct for j=INFO+1,...,N. > N: errors that usually indicate LAPACK problems:
=N+1: error return from SGGBAL
=N+2: error return from SGEQRF
=N+3: error return from SORMQR
=N+4: error return from SORGQR
=N+5: error return from SGGHRD
=N+6: error return from SHGEQZ (other than failed iteration) =N+7: error return from SGGBAK (computing VSL)
=N+8: error return from SGGBAK (computing VSR)
=N+9: error return from SLASCL (various places)

Contents

NAME

dgegv - routine is deprecated and has been replaced by rou-
tine SGGEV

SYNOPSIS

```
SUBROUTINE DGEGV(JOBVL, JOBVR, N, A, LDA, B, LDB, ALPHAR, ALPHAI,
      BETA, VL, LDVL, VR, LDVR, WORK, LDWORK, INFO)

CHARACTER * 1 JOBVL, JOBVR
INTEGER N, LDA, LDB, LDVL, LDVR, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*),  ALPHAR(*),  ALPHAI(*),
BETA(*), VL(LDVL,*), VR(LDVR,*), WORK(*)

SUBROUTINE DGEGV_64(JOBVL, JOBVR, N, A, LDA, B, LDB, ALPHAR, ALPHAI,
      BETA, VL, LDVL, VR, LDVR, WORK, LDWORK, INFO)

CHARACTER * 1 JOBVL, JOBVR
INTEGER*8 N, LDA, LDB, LDVL, LDVR, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*),  ALPHAR(*),  ALPHAI(*),
BETA(*), VL(LDVL,*), VR(LDVR,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEGV(JOBVL, JOBVR, [N], A, [LDA], B, [LDB], ALPHAR,
       ALPHAI, BETA, VL, [LDVL], VR, [LDVR], [WORK], [LDWORK], [INFO])

CHARACTER(LEN=1) :: JOBVL, JOBVR
INTEGER :: N, LDA, LDB, LDVL, LDVR, LDWORK, INFO
REAL(8), DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
REAL(8), DIMENSION(:,:) :: A, B, VL, VR

SUBROUTINE GEGV_64(JOBVL, JOBVR, [N], A, [LDA], B, [LDB], ALPHAR,
```

```
        ALPHAI, BETA, VL, [LDVL], VR, [LDVR], [WORK], [LDWORK], [INFO])

CHARACTER(LEN=1) :: JOBVL, JOBVR
INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, LDWORK, INFO
REAL(8), DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
REAL(8), DIMENSION(:,:) :: A, B, VL, VR
```

C INTERFACE

```
#include <sunperf.h>

void dgegv(char jobvl, char jobvr, int  n,  double  *a,  int
          lda,  double  *b,  int ldb, double *alphar, double
          *alphai, double *beta, double *vl, int ldvl,  dou-
          ble *vr, int ldvr, int *info);
void dgegv_64(char jobvl, char jobvr,  long  n,  double  *a,
          long  lda,  double  *b,  long ldb, double *alphar,
          double *alphai, double  *beta,  double  *vl,  long
          ldvl, double *vr, long ldvr, long *info);
```

PURPOSE

dgegv routine is deprecated and has been replaced by routine SGGEV.

SGEGV computes for a pair of n-by-n real nonsymmetric matrices A and B, the generalized eigenvalues (alphar +/- alphai*i, beta), and optionally, the left and/or right generalized eigenvectors (VL and VR).

A generalized eigenvalue for a pair of matrices (A,B) is, roughly speaking, a scalar w or a ratio alpha/beta = w, such that A - w*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0, and even for both being zero. A good beginning reference is the book, "Matrix Computations", by G. Golub & C. van Loan (Johns Hopkins U. Press)

A right generalized eigenvector corresponding to a generalized eigenvalue w for a pair of matrices (A,B) is a vector r such that (A - w B) r = 0 . A left generalized eigenvector is a vector l such that l**H * (A - w B) = 0, where l**H is the
conjugate-transpose of l.

Note: this routine performs "full balancing" on A and B -- see "Further Details", below.

ARGUMENTS

JOBVL (input)
= 'N': do not compute the left generalized eigenvectors;
= 'V': compute the left generalized eigenvectors.

JOBVR (input)
= 'N': do not compute the right generalized eigenvectors;
= 'V': compute the right generalized eigenvectors.

N (input) The order of the matrices A, B, VL, and VR. N >= 0.

A (input/output)
On entry, the first of the pair of matrices whose generalized eigenvalues and (optionally) generalized eigenvectors are to be computed. On exit, the contents will have been destroyed. (For a description of the contents of A on exit, see "Further Details", below.)

LDA (input)
The leading dimension of A. LDA >= max(1,N).

B (input/output)
On entry, the second of the pair of matrices whose generalized eigenvalues and (optionally) generalized eigenvectors are to be computed. On exit, the contents will have been destroyed. (For a description of the contents of B on exit, see "Further Details", below.)

LDB (input)
The leading dimension of B. LDB >= max(1,N).

ALPHAR (output)
On exit, (ALPHAR(j) + ALPHAI(j)*i)/BETA(j), j=1,...,N, will be the generalized eigenvalues. If ALPHAI(j) is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-st eigenvalues are a complex conjugate pair, with ALPHAI(j+1) negative.

Note: the quotients ALPHAR(j)/BETA(j) and ALPHAI(j)/BETA(j) may easily over- or underflow, and BETA(j) may even be zero. Thus, the user should avoid naively computing the ratio alpha/beta. However, ALPHAR and ALPHAI will be always less than and usually comparable with norm(A) in magnitude, and BETA always less than and usually comparable with norm(B).

ALPHAI (output)
See the description of ALPHAR.

BETA (output)
See the description of ALPHAR.

VL (output)
If JOBVL = 'V', the left generalized eigenvectors. (See "Purpose", above.) Real eigenvectors take one column, complex take two columns, the first for the real part and the second for the imaginary part. Complex eigenvectors correspond to an eigenvalue with positive imaginary part. Each eigenvector will be scaled so the largest component will have abs(real part) + abs(imag. part) = 1, *except* that for eigenvalues with alpha=beta=0, a zero vector will be returned as the corresponding eigenvector. Not referenced if JOBVL = 'N'.

LDVL (input)
The leading dimension of the matrix VL. LDVL >= 1, and if JOBVL = 'V', LDVL >= N.

VR (output)
If JOBVR = 'V', the right generalized eigenvectors. (See "Purpose", above.) Real eigenvectors take one column, complex take two columns, the first for the real part and the second for the imaginary part. Complex eigenvectors correspond to an eigenvalue with positive imaginary part. Each eigenvector will be scaled so the largest component will have abs(real part) + abs(imag. part) = 1, *except* that for eigenvalues with alpha=beta=0, a zero vector will be returned as the corresponding eigenvector. Not referenced if JOBVR = 'N'.

LDVR (input)
The leading dimension of the matrix VR. LDVR >= 1, and if JOBVR = 'V', LDVR >= N.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)

The dimension of the array WORK. LDWORK >= max(1,8*N). For good performance, LDWORK must generally be larger. To compute the optimal value of LDWORK, call ILAENV to get blocksizes (for SGEQRF, SORMQR, and SORGQR.) Then compute: NB -- MAX of the blocksizes for SGEQRF, SORMQR, and SORGQR; The optimal LDWORK is: 2*N + MAX(6*N, N*(NB+1)).

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
= 1,...,N: The QZ iteration failed. No eigenvectors have been calculated, but ALPHAR(j), ALPHAI(j), and BETA(j) should be correct for j=INFO+1,...,N. > N: errors that usually indicate LAPACK problems:
=N+1: error return from SGGBAL
=N+2: error return from SGEQRF
=N+3: error return from SORMQR
=N+4: error return from SORGQR
=N+5: error return from SGGHRD
=N+6: error return from SHGEQZ (other than failed iteration) =N+7: error return from STGEVC
=N+8: error return from SGGBAK (computing VL)
=N+9: error return from SGGBAK (computing VR)
=N+10: error return from SLASCL (various calls)

FURTHER DETAILS

Balancing

This driver calls SGGBAL to both permute and scale rows and columns of A and B. The permutations PL and PR are chosen so that PL*A*PR and PL*B*R will be upper triangular except for the diagonal blocks A(i:j,i:j) and B(i:j,i:j), with i and j as close together as possible. The diagonal scaling matrices DL and DR are chosen so that the pair DL*PL*A*PR*DR, DL*PL*B*PR*DR have elements close to one (except for the elements that start out zero.)

After the eigenvalues and eigenvectors of the balanced matrices have been computed, SGGBAK transforms the eigenvectors back to what they would have been (in perfect arithmetic) if they had not been balanced.

Contents of A and B on Exit
-------- -- - --- - -- ----

If any eigenvectors are computed (either JOBVL='V' or JOBVR='V' or both), then on exit the arrays A and B will contain the real Schur form[*] of the "balanced" versions of A and B. If no eigenvectors are computed, then only the diagonal blocks will be correct.

[*] See SHGEQZ, SGEGS, or read the book "Matrix Computations",
by Golub & van Loan, pub. by Johns Hopkins U. Press.

Contents

NAME

```
dgehrd - reduce a real general matrix A to upper  Hessenberg
form H by an orthogonal similarity transformation
```

SYNOPSIS

```
SUBROUTINE DGEHRD(N, ILO, IHI, A, LDA, TAU, WORKIN, LWORKIN, INFO)

INTEGER N, ILO, IHI, LDA, LWORKIN, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORKIN(*)

SUBROUTINE DGEHRD_64(N, ILO, IHI, A, LDA, TAU, WORKIN, LWORKIN, INFO)

INTEGER*8 N, ILO, IHI, LDA, LWORKIN, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORKIN(*)
```

F95 INTERFACE

```
SUBROUTINE GEHRD([N], ILO, IHI, A, [LDA], TAU, [WORKIN], [LWORKIN],
       [INFO])

INTEGER :: N, ILO, IHI, LDA, LWORKIN, INFO
REAL(8), DIMENSION(:) :: TAU, WORKIN
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE GEHRD_64([N], ILO, IHI, A, [LDA], TAU, [WORKIN], [LWORKIN],
       [INFO])

INTEGER(8) :: N, ILO, IHI, LDA, LWORKIN, INFO
REAL(8), DIMENSION(:) :: TAU, WORKIN
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dgehrd(int n, int ilo, int ihi,  double  *a,  int  lda,
          double *tau, int *info);

void dgehrd_64(long n, long ilo, long ihi, double  *a,  long
          lda, double *tau, long *info);
```

PURPOSE

dgehrd reduces a real general matrix A to upper Hessenberg form H by an orthogonal similarity transformation: Q' * A * Q = H .

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

ILO (input)
It is assumed that A is already upper triangular in rows and columns 1:ILO-1 and IHI+1:N. ILO and IHI are normally set by a previous call to SGEBAL; otherwise they should be set to 1 and N respectively. See Further Details.

IHI (input)
See the description of ILO.

A (input/output)
On entry, the N-by-N general matrix to be reduced. On exit, the upper triangle and the first subdiagonal of A are overwritten with the upper Hessenberg matrix H, and the elements below the first subdiagonal, with the array TAU, represent the orthogonal matrix Q as a product of elementary reflectors. See Further Details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

TAU (output)
The scalar factors of the elementary reflectors (see Further Details). Elements 1:ILO-1 and IHI:N-1 of TAU are set to zero.

WORKIN (workspace)

On exit, if INFO = 0, WORKIN(1) returns the optimal LWORKIN.

LWORKIN (input)
The length of the array WORKIN. LWORKIN >= max(1,N). For optimum performance LWORKIN >= N*NB, where NB is the optimal blocksize.

If LWORKIN = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORKIN array, returns this value as the first entry of the WORKIN array, and no error message related to LWORKIN is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

The matrix Q is represented as a product of (ihi-ilo) elementary reflectors

```
   Q = H(ilo) H(ilo+1) . . . H(ihi-1).
```

Each H(i) has the form

```
   H(i) = I - tau * v * v'
```

where tau is a real scalar, and v is a real vector with v(1:i) = 0, v(i+1) = 1 and v(ihi+1:n) = 0; v(i+2:ihi) is stored on exit in A(i+2:ihi,i), and tau in TAU(i).

The contents of A are illustrated by the following example, with n = 7, ilo = 2 and ihi = 6:

```
on entry,                        on exit,

( a   a   a   a   a   a   a )    (  a   a   h    h    h    h
a  )  (      a   a   a   a   a   a )    (      a   h   h   h
h   a ) (     a   a   a   a   a   a )    (       h    h    h
h    h    h  ) (     a   a   a   a   a   a )    (      v2  h
h   h   h   h ) (     a   a   a   a   a   a )     (       v2
v3   h    h    h    h  )  (     a   a   a   a   a   a )    (
v2  v3  v4  h   h   h ) (                         a  )     (
a )
```

where a denotes an element of the original matrix A, h denotes a modified element of the upper Hessenberg matrix H, and vi denotes an element of the vector defining H(i).

Contents

NAME

```
dgelqf - compute an LQ factorization of a real M-by-N matrix
A
```

SYNOPSIS

```
SUBROUTINE DGELQF(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER M, N, LDA, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)

SUBROUTINE DGELQF_64(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER*8 M, N, LDA, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GELQF([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER :: M, N, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE GELQF_64([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER(8) :: M, N, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dgelqf(int m, int n, double *a, int lda,  double  *tau,
          int *info);

void dgelqf_64(long m, long n, double *a, long  lda,  double
          *tau, long *info);
```

PURPOSE

dgelqf computes an LQ factorization of a real M-by-N matrix A: A = L * Q.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, the elements on and below the diagonal of the array contain the m-by-min(m,n) lower trapezoidal matrix L (L is lower triangular if m <= n); the elements above the diagonal, with the array TAU, represent the orthogonal matrix Q as a product of elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,M). For optimum performance LDWORK >= M*NB, where NB is the optimal blocksize.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

```
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

FURTHER DETAILS

```
The matrix Q is  represented  as  a  product  of  elementary
reflectors

   Q = H(k) . . . H(2) H(1), where k = min(m,n).

Each H(i) has the form
   H(i) = I - tau * v * v'

where tau is a real scalar, and v is a real vector with
v(1:i-1) = 0 and v(i) = 1; v(i+1:n) is  stored  on  exit  in
A(i,i+1:n), and tau in TAU(i).
```

Contents

NAME

```
dgels - solve overdetermined or underdetermined real  linear
systems  involving  an  M-by-N  matrix  A, or its transpose,
using a QR or LQ factorization of A
```

SYNOPSIS

```
SUBROUTINE DGELS(TRANSA, M, N, NRHS, A, LDA, B, LDB, WORK, LDWORK,
      INFO)

CHARACTER * 1 TRANSA
INTEGER M, N, NRHS, LDA, LDB, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*), WORK(*)

SUBROUTINE DGELS_64(TRANSA, M, N, NRHS, A, LDA, B, LDB, WORK, LDWORK,
      INFO)

CHARACTER * 1 TRANSA
INTEGER*8 M, N, NRHS, LDA, LDB, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GELS([TRANSA], [M], [N], [NRHS], A, [LDA], B, [LDB], [WORK],
       LDWORK, [INFO])

CHARACTER(LEN=1) :: TRANSA
INTEGER :: M, N, NRHS, LDA, LDB, LDWORK, INFO
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE GELS_64([TRANSA], [M], [N], [NRHS], A, [LDA], B, [LDB],
       [WORK], LDWORK, [INFO])
```

```
CHARACTER(LEN=1) :: TRANSA
INTEGER(8) :: M, N, NRHS, LDA, LDB, LDWORK, INFO
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dgels (char, int, int, int, double*, int, double*, int,
          int*);

void dgels_64 (char, long, long, long, double*,  long,  dou-
          ble*, long, long*);
```

PURPOSE

dgels solves overdetermined or underdetermined real linear systems involving an M-by-N matrix A, or its transpose, using a QR or LQ factorization of A. It is assumed that A has full rank.

The following options are provided:

```
1. If TRANS = 'N' and m >= n:  find the least squares  solu-
tion of
   an overdetermined system, i.e., solve the  least  squares
problem
                minimize || B - A*X ||.

2. If TRANS = 'N' and m < n:  find the minimum norm solution
of
   an underdetermined system A * X = B.

3. If TRANS = 'T' and m >= n:  find the minimum  norm  solu-
tion of
   an undetermined system A**T * X = B.

4. If TRANS = 'T' and m < n:  find the least  squares  solu-
tion of
   an overdetermined system, i.e., solve the  least  squares
problem
                minimize || B - A**T * X ||.
```

Several right hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the M-by-NRHS right hand side matrix B and the N-by-NRHS solution matrix X.

ARGUMENTS

```
TRANSA (input)
          = 'N': the linear system involves A;
          = 'T': the linear system involves A**T.

          TRANSA is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix A.  M >= 0.

N (input) The number of columns of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X. NRHS >=0.
A (input/output)
          On entry, the M-by-N matrix A.  On exit, if  M  >=
          N,  A is overwritten by details of its QR factori-
          zation as returned by SGEQRF; if  M  <   N,  A  is
          overwritten  by details of its LQ factorization as
          returned by SGELQF.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,M).

B (input/output)
          On entry, the matrix B of right hand side vectors,
          stored columnwise; B is M-by-NRHS if TRANSA = 'N',
          or N-by-NRHS if TRANSA  =  'T'.   On  exit,  B  is
          overwritten   by   the  solution  vectors,  stored
          columnwise:  if TRANSA = 'N' and m >= n, rows 1 to
          n of B contain the least squares solution vectors;
          the residual sum of squares for  the  solution  in
          each column is given by the sum of squares of ele-
          ments N+1 to M in that column; if TRANSA = 'N' and
          m  <  n, rows 1 to N of B contain the minimum norm
          solution vectors; if TRANSA = 'T' and m >= n, rows
          1 to M of B contain the minimum norm solution vec-
          tors; if TRANSA = 'T' and m < n, rows 1 to M of  B
          contain  the  least  squares solution vectors; the
          residual sum of squares for the solution  in  each
          column  is given by the sum of squares of elements
          M+1 to N in that column.

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          MAX(1,M,N).
```

```
WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LDWORK.

LDWORK (output)
          The dimension of the array WORK.  LDWORK  >=  max(
          1,  MN  +  max( MN, NRHS ) ).  For optimal perfor-
          mance, LDWORK >= max( 1, MN + max( MN,  NRHS  )*NB
          ).   where  MN  =  min(M,N)  and NB is the optimum
          block size.

          If LDWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LDWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
dgelsd - compute the minimum-norm solution to a real  linear
least squares problem
```

SYNOPSIS

```
SUBROUTINE DGELSD(M, N, NRHS, A, LDA, B, LDB, S, RCOND, RANK, WORK,
      LWORK, IWORK, INFO)

INTEGER M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER IWORK(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(LDA,*), B(LDB,*), S(*), WORK(*)

SUBROUTINE DGELSD_64(M, N, NRHS, A, LDA, B, LDB, S, RCOND, RANK,
      WORK, LWORK, IWORK, INFO)

INTEGER*8 M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(LDA,*), B(LDB,*), S(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GELSD([M], [N], [NRHS], A, [LDA], B, [LDB], S, RCOND,
       RANK, [WORK], [LWORK], [IWORK], [INFO])

INTEGER :: M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: S, WORK
REAL(8), DIMENSION(:,:) :: A, B
```

```
SUBROUTINE GELSD_64([M], [N], [NRHS], A, [LDA], B, [LDB], S, RCOND,
       RANK, [WORK], [LWORK], [IWORK], [INFO])

INTEGER(8) :: M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: S, WORK
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dgelsd(int m, int n, int nrhs, double *a, int lda, dou-
          ble  *b,  int  ldb,  double  *s, double rcond, int
          *rank, int *info);
void dgelsd_64(long m, long n, long nrhs,  double  *a,  long
          lda, double *b, long ldb, double *s, double rcond,
          long *rank, long *info);
```

PURPOSE

```
dgelsd computes the minimum-norm solution to a  real  linear
least squares problem:
    minimize 2-norm(| b - A*x |)
using the singular value decomposition (SVD) of A. A  is  an
M-by-N matrix which may be rank-deficient.

Several right hand side vectors b and solution vectors x can
be  handled in a single call; they are stored as the columns
of the M-by-NRHS right hand side matrix B and the  N-by-NRHS
solution matrix X.

The problem is solved in three steps:
(1) Reduce the coefficient matrix A to bidiagonal form with
    Householder transformations, reducing the original prob-
lem
    into a "bidiagonal least squares problem" (BLS)
(2) Solve the BLS using a divide and conquer approach.
(3) Apply back all the Householder tranformations to solve
    the original least squares problem.

The effective rank of A is determined by  treating  as  zero
those  singular  values  which are less than RCOND times the
largest singular value.

The divide and conquer algorithm makes very mild assumptions
about  floating  point  arithmetic. It will work on machines
with a guard digit  in  add/subtract,  or  on  those  binary
machines  without  guard digits which subtract like the Cray
```

X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

M (input) The number of rows of A. M >= 0.

N (input) The number of columns of A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, A has been destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the M-by-NRHS right hand side matrix B. On exit, B is overwritten by the N-by-NRHS solution matrix X. If m >= n and RANK = n, the residual sum-of-squares for the solution in the i-th column is given by the sum of squares of elements n+1:m in that column.

LDB (input)
The leading dimension of the array B. LDB >= max(1,max(M,N)).

S (output)
The singular values of A in decreasing order. The condition number of A in the 2-norm = S(1)/S(min(m,n)).

RCOND (input)
RCOND is used to determine the effective rank of A. Singular values S(i) <= RCOND*S(1) are treated as zero. If RCOND < 0, machine precision is used instead.

RANK (output)
The effective rank of A, i.e., the number of singular values which are greater than RCOND*S(1).

```
WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK. LWORK >= 1.   The
          exact  minimum  amount of workspace needed depends
          on M, N and NRHS.  As long as LWORK  is  at  least
          12*N   +   2*N*SMLSIZ   +   8*N*NLVL  +  N*NRHS  *
          (SMLSIZ+1)**2, if M is greater than or equal to  N
          or  12*M  +  2*M*SMLSIZ  +  8*M*NLVL  +  M*NRHS  +
          (SMLSIZ+1)**2, if M is less than N, the code  will
          execute  correctly.   SMLSIZ is returned by ILAENV
          and is equal to the maximum size of  the  subprob-
          lems  at  the bottom of the computation tree (usu-
          ally about 25), and NLVL = INT(  LOG_2(  MIN(  M,N
          )/(SMLSIZ+1)  )  ) + 1 For good performance, LWORK
          should generally be larger.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

IWORK (workspace)
          LIWORK >= 3 * MINMN * NLVL +  11  *  MINMN,  where
          MINMN = MIN( M,N ).

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value.
          > 0:  the algorithm for computing the  SVD  failed
          to  converge; if INFO = i, i off-diagonal elements
          of an intermediate bidiagonal form  did  not  con-
          verge to zero.
```

FURTHER DETAILS

```
Based on contributions by
   Ming Gu  and  Ren-Cang  Li,  Computer  Science  Division,
University of California at Berkeley, USA
   Osni Marques, LBNL/NERSC, USA
```

Contents

NAME

```
dgelss - compute the minimum norm solution to a real  linear
least squares problem
```

SYNOPSIS

```
SUBROUTINE DGELSS(M, N, NRHS, A, LDA, B, LDB, SING, RCOND, IRANK,
      WORK, LDWORK, INFO)

INTEGER M, N, NRHS, LDA, LDB, IRANK, LDWORK, INFO
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(LDA,*), B(LDB,*), SING(*), WORK(*)

SUBROUTINE DGELSS_64(M, N, NRHS, A, LDA, B, LDB, SING, RCOND, IRANK,
      WORK, LDWORK, INFO)

INTEGER*8 M, N, NRHS, LDA, LDB, IRANK, LDWORK, INFO
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(LDA,*), B(LDB,*), SING(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GELSS([M], [N], [NRHS], A, [LDA], B, [LDB], SING, RCOND,
       IRANK, [WORK], [LDWORK], [INFO])

INTEGER :: M, N, NRHS, LDA, LDB, IRANK, LDWORK, INFO
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: SING, WORK
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE GELSS_64([M], [N], [NRHS], A, [LDA], B, [LDB], SING,
       RCOND, IRANK, [WORK], [LDWORK], [INFO])

INTEGER(8) :: M, N, NRHS, LDA, LDB, IRANK, LDWORK, INFO
```

```
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: SING, WORK
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dgelss(int m, int n, int nrhs, double *a, int lda, dou-
          ble  *b,  int ldb, double *sing, double rcond, int
          *irank, int *info);

void dgelss_64(long m, long n, long nrhs,  double  *a,  long
          lda,  double  *b,  long  ldb, double *sing, double
          rcond, long *irank, long *info);
```

PURPOSE

dgelss computes the minimum norm solution to a real linear least squares problem:

Minimize 2-norm(| b - A*x |).

using the singular value decomposition (SVD) of A. A is an M-by-N matrix which may be rank-deficient.

Several right hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the M-by-NRHS right hand side matrix B and the N-by-NRHS solution matrix X.

The effective rank of A is determined by treating as zero those singular values which are less than RCOND times the largest singular value.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, the first min(m,n) rows of A are overwritten with its right

singular vectors, stored rowwise.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the M-by-NRHS right hand side matrix B. On exit, B is overwritten by the N-by-NRHS solution matrix X. If m >= n and IRANK = n, the residual sum-of-squares for the solution in the i-th column is given by the sum of squares of elements n+1:m in that column.

LDB (input)
The leading dimension of the array B. LDB >= max(1,max(M,N)).

SING (output)
The singular values of A in decreasing order. The condition number of A in the 2-norm = SING(1)/SING(min(m,n)).

RCOND (input)
RCOND is used to determine the effective rank of A. Singular values SING(i) <= RCOND*SING(1) are treated as zero. If RCOND < 0, machine precision is used instead.

IRANK (output)
The effective rank of A, i.e., the number of singular values which are greater than RCOND*SING(1).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= 1, and also: LDWORK >= 3*min(M,N) + max(2*min(M,N), max(M,N), NRHS) For good performance, LDWORK should generally be larger.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)

= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: the algorithm for computing the SVD failed to converge; if INFO = i, i off-diagonal elements of an intermediate bidiagonal form did not converge to zero.

Contents

NAME

```
dgelsx - routine is deprecated and has been replaced by rou-
tine SGELSY
```

SYNOPSIS

```
SUBROUTINE DGELSX(M, N, NRHS, A, LDA, B, LDB, JPIVOT, RCOND, IRANK,
      WORK, INFO)

INTEGER M, N, NRHS, LDA, LDB, IRANK, INFO
INTEGER JPIVOT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(LDA,*), B(LDB,*), WORK(*)

SUBROUTINE DGELSX_64(M, N, NRHS, A, LDA, B, LDB, JPIVOT, RCOND,
      IRANK, WORK, INFO)

INTEGER*8 M, N, NRHS, LDA, LDB, IRANK, INFO
INTEGER*8 JPIVOT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(LDA,*), B(LDB,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GELSX([M], [N], [NRHS], A, [LDA], B, [LDB], JPIVOT, RCOND,
       IRANK, [WORK], [INFO])

INTEGER :: M, N, NRHS, LDA, LDB, IRANK, INFO
INTEGER, DIMENSION(:) :: JPIVOT
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE GELSX_64([M], [N], [NRHS], A, [LDA], B, [LDB], JPIVOT,
```

```
          RCOND, IRANK, [WORK], [INFO])

     INTEGER(8) :: M, N, NRHS, LDA, LDB, IRANK, INFO
     INTEGER(8), DIMENSION(:) :: JPIVOT
     REAL(8) :: RCOND
     REAL(8), DIMENSION(:) :: WORK
     REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dgelsx(int m, int n, int nrhs, double *a, int lda, dou-
          ble  *b,  int  ldb, int *jpivot, double rcond, int
          *irank, int *info);
void dgelsx_64(long m, long n, long nrhs,  double  *a,  long
          lda,  double  *b,  long  ldb, long *jpivot, double
          rcond, long *irank, long *info);
```

PURPOSE

```
dgelsx routine is deprecated and has been replaced  by  rou-
tine SGELSY.

SGELSX computes the minimum-norm solution to a  real  linear
least squares problem:
    minimize || A * X - B ||
using a complete orthogonal factorization of A.  A is an  M-
by-N matrix which may be rank-deficient.

Several right hand side vectors b and solution vectors x can
be  handled in a single call; they are stored as the columns
of the M-by-NRHS right hand side matrix B and the  N-by-NRHS
solution matrix X.

The routine first computes a QR  factorization  with  column
pivoting:
    A * P = Q * [ R11 R12 ]
                [  0  R22 ]
with R11 defined as  the  largest  leading  submatrix  whose
estimated  condition number is less than 1/RCOND.  The order
of R11, RANK, is the effective rank of A.

Then, R22 is considered to be negligible, and R12 is annihi-
lated by orthogonal transformations from the right, arriving
at the complete orthogonal factorization:
   A * P = Q * [ T11 0 ] * Z
               [  0  0 ]
The minimum-norm solution is then
   X = P * Z' [ inv(T11)*Q1'*B ]
```

```
                [        0       ]
     where Q1 consists of the first RANK columns of Q.
```

ARGUMENTS

```
     M (input) The number of rows of the matrix A.  M >= 0.

     N (input) The number of columns of the matrix A.  N >= 0.

     NRHS (input)
               The number of right hand sides, i.e.,  the  number
               of columns of matrices B and X. NRHS >= 0.
     A (input/output)
               On entry, the M-by-N matrix A.   On  exit,  A  has
               been   overwritten  by  details  of  its  complete
               orthogonal factorization.

     LDA (input)
               The leading dimension of  the  array  A.   LDA  >=
               max(1,M).

     B (input/output)
               On entry, the M-by-NRHS right hand side matrix  B.
               On exit, the N-by-NRHS solution matrix X.  If m >=
               n and IRANK = n, the residual  sum-of-squares  for
               the  solution  in  the i-th column is given by the
               sum of squares of elements N+1:M in that column.

     LDB (input)
               The leading dimension  of  the  array  B.  LDB  >=
               max(1,M,N).

     JPIVOT (input/output)
               On entry, if JPIVOT(i) .ne. 0, the i-th column  of
               A  is  an  initial  column, otherwise it is a free
               column.  Before the QR  factorization  of  A,  all
               initial  columns are permuted to the leading posi-
               tions; only the remaining free columns  are  moved
               as a result of column pivoting during the factori-
               zation.  On exit, if JPIVOT(i) = k, then the  i-th
               column of A*P was the k-th column of A.

     RCOND (input)
               RCOND is used to determine the effective  rank  of
               A,  which  is  defined as the order of the largest
               leading triangular submatrix R11 in the QR factor-
               ization with pivoting of A, whose estimated condi-
               tion number < 1/RCOND.
```

IRANK (output)
The effective rank of A, i.e., the order of the submatrix R11. This is the same as the order of the submatrix T11 in the complete orthogonal factorization of A.

WORK (workspace)
(max(min(M,N)+3*N, 2*min(M,N)+NRHS)),

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dgelsy - compute the minimum-norm solution to a real  linear
least squares problem
```

SYNOPSIS

```
SUBROUTINE DGELSY(M, N, NRHS, A, LDA, B, LDB, JPVT, RCOND, RANK,
      WORK, LWORK, INFO)

INTEGER M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER JPVT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(LDA,*), B(LDB,*), WORK(*)

SUBROUTINE DGELSY_64(M, N, NRHS, A, LDA, B, LDB, JPVT, RCOND, RANK,
      WORK, LWORK, INFO)

INTEGER*8 M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER*8 JPVT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(LDA,*), B(LDB,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GELSY([M], [N], [NRHS], A, [LDA], B, [LDB], JPVT, RCOND,
       RANK, [WORK], [LWORK], [INFO])

INTEGER :: M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER, DIMENSION(:) :: JPVT
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A, B
```

```
SUBROUTINE GELSY_64([M], [N], [NRHS], A, [LDA], B, [LDB], JPVT,
       RCOND, RANK, [WORK], [LWORK], [INFO])

INTEGER(8) :: M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER(8), DIMENSION(:) :: JPVT
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dgelsy(int m, int n, int nrhs, double *a, int lda, dou-
          ble  *b,  int  ldb,  int  *jpvt, double rcond, int
          *rank, int *info);
void dgelsy_64(long m, long n, long nrhs,  double  *a,  long
          lda,  double  *b,  long  ldb,  long  *jpvt, double
          rcond, long *rank, long *info);
```

PURPOSE

```
dgelsy computes the minimum-norm solution to a  real  linear
least squares problem:
    minimize || A * X - B ||
using a complete orthogonal factorization of A.  A is an  M-
by-N matrix which may be rank-deficient.

Several right hand side vectors b and solution vectors x can
be  handled in a single call; they are stored as the columns
of the M-by-NRHS right hand side matrix B and the  N-by-NRHS
solution matrix X.

The routine first computes a QR  factorization  with  column
pivoting:
    A * P = Q * [ R11 R12 ]
                [  0  R22 ]
with R11 defined as  the  largest  leading  submatrix  whose
estimated  condition number is less than 1/RCOND.  The order
of R11, RANK, is the effective rank of A.

Then, R22 is considered to be negligible, and R12 is annihi-
lated by orthogonal transformations from the right, arriving
at the complete orthogonal factorization:
   A * P = Q * [ T11 0 ] * Z
               [  0  0 ]
The minimum-norm solution is then
   X = P * Z' [ inv(T11)*Q1'*B ]
              [        0       ]
where Q1 consists of the first RANK columns of Q.
```

```
This routine is basically identical to the  original  xGELSX
except three differences:
  o The call to the subroutine xGEQPF has  been  substituted
by the
    the call to the subroutine xGEQP3. This subroutine is  a
Blas-3
    version of the QR factorization with column pivoting.
  o Matrix B (the right hand side) is updated with Blas-3.
  o The permutation of matrix B (the  right  hand  side)  is
faster and
    more simple.
```

ARGUMENTS

```
M (input) The number of rows of the matrix A.  M >= 0.
N (input) The number of columns of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of matrices B and X. NRHS >= 0.

A (input/output)
          On entry, the M-by-N matrix A.   On  exit,  A  has
          been   overwritten  by  details  of  its  complete
          orthogonal factorization.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,M).

B (input/output)
          On entry, the M-by-NRHS right hand side matrix  B.
          On exit, the N-by-NRHS solution matrix X.

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          max(1,M,N).

JPVT (input/output)
          On entry, if JPVT(i) .ne. 0, the i-th column of  A
          is permuted to the front of AP, otherwise column i
          is a free column.  On exit, if JPVT(i) =  k,  then
          the i-th column of AP was the k-th column of A.

RCOND (input)
          RCOND is used to determine the effective  rank  of
          A,  which  is  defined as the order of the largest
```

leading triangular submatrix R11 in the QR factorization with pivoting of A, whose estimated condition number < 1/RCOND.

RANK (output)
The effective rank of A, i.e., the order of the submatrix R11. This is the same as the order of the submatrix T11 in the complete orthogonal factorization of A.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. The unblocked strategy requires that: LWORK >= MAX(MN+3*N+1, 2*MN+NRHS), where MN = min(M, N). The block algorithm requires that: LWORK >= MAX(MN+2*N+NB*(N+1), 2*MN+NB*NRHS), where NB is an upper bound on the blocksize returned by ILAENV for the routines SGEQP3, STZRZF, STZRQF, SORMQR, and SORMRZ.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: If INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

Based on contributions by
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E. Quintana-Orti, Depto. de Informatica, Universidad Jaime I, Spain
G. Quintana-Orti, Depto. de Informatica, Universidad Jaime I, Spain

Contents

NAME

dgemm - perform one of the matrix-matrix operations C := alpha*op(A)*op(B) + beta*C

SYNOPSIS

```
SUBROUTINE DGEMM(TRANSA, TRANSB, M, N, K, ALPHA, A, LDA, B, LDB,
      BETA, C, LDC)

CHARACTER * 1 TRANSA, TRANSB
INTEGER M, N, K, LDA, LDB, LDC
DOUBLE PRECISION ALPHA, BETA
DOUBLE PRECISION A(LDA,*), B(LDB,*), C(LDC,*)

SUBROUTINE DGEMM_64(TRANSA, TRANSB, M, N, K, ALPHA, A, LDA, B, LDB,
      BETA, C, LDC)

CHARACTER * 1 TRANSA, TRANSB
INTEGER*8 M, N, K, LDA, LDB, LDC
DOUBLE PRECISION ALPHA, BETA
DOUBLE PRECISION A(LDA,*), B(LDB,*), C(LDC,*)
```

F95 INTERFACE

```
SUBROUTINE GEMM([TRANSA], [TRANSB], [M], [N], [K], ALPHA, A, [LDA],
       B, [LDB], BETA, C, [LDC])

CHARACTER(LEN=1) :: TRANSA, TRANSB
INTEGER :: M, N, K, LDA, LDB, LDC
REAL(8) :: ALPHA, BETA
REAL(8), DIMENSION(:,:) :: A, B, C

SUBROUTINE GEMM_64([TRANSA], [TRANSB], [M], [N], [K], ALPHA, A, [LDA],
       B, [LDB], BETA, C, [LDC])
```

```
CHARACTER(LEN=1) :: TRANSA, TRANSB
INTEGER(8) :: M, N, K, LDA, LDB, LDC
REAL(8) :: ALPHA, BETA
REAL(8), DIMENSION(:,:) :: A, B, C
```

C INTERFACE

```
#include <sunperf.h>

void dgemm(char transa, char transb, int m, int  n,  int  k,
          double  alpha,  double *a, int lda, double *b, int
          ldb, double beta, double *c, int ldc);

void dgemm_64(char transa, char transb, long m, long n, long
          k,  double  alpha, double *a, long lda, double *b,
          long ldb, double beta, double *c, long ldc);
```

PURPOSE

dgemm performs one of the matrix-matrix operations C := alpha*op(A)*op(B) + beta*C where op(X) is one of

op(X) = X   or   op(X) = X',

alpha and beta are scalars, and A, B and C are matrices, with op(A) an m by k matrix, op(B) a k by n matrix and C an m by n matrix.

ARGUMENTS

TRANSA (input)
On entry, TRANSA specifies the form of op(A) to be used in the matrix multiplication as follows:

TRANSA = 'N' or 'n', op(A) = A.

TRANSA = 'T' or 't', op(A) = A'.

TRANSA = 'C' or 'c', op(A) = A'.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

TRANSB (input)
On entry, TRANSB specifies the form of op(B) to be used in the matrix multiplication as follows:

TRANSB = 'N' or 'n', op(B) = B.

TRANSB = 'T' or 't', op(B) = B'.

TRANSB = 'C' or 'c', op(B) = B'.

Unchanged on exit.

TRANSB is defaulted to 'N' for F95 INTERFACE.

M (input)
On entry, M specifies the number of rows of the matrix op(A) and of the matrix C. M must be at least zero. Unchanged on exit.

N (input)
On entry, N specifies the number of columns of the matrix op(B) and the number of columns of the matrix C. N must be at least zero. Unchanged on exit.

K (input)
On entry, K specifies the number of columns of the matrix op(A) and the number of rows of the matrix op(B). K must be at least zero. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
DOUBLE PRECISION array of DIMENSION (LDA, ka), where ka is k when TRANSA = 'N' or 'n', and is m otherwise. Before entry with TRANSA = 'N' or 'n', the leading m by k part of the array A must contain the matrix A, otherwise the leading k by m part of the array A must contain the matrix A. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDA >= max(1, m), otherwise LDA >= max(1, k). Unchanged on exit.

B (input)
DOUBLE PRECISION array of DIMENSION (LDB, kb), where kb is n when TRANSB = 'N' or 'n', and is k otherwise. Before entry with TRANSB = 'N' or

'n', the leading k by n part of the array B must contain the matrix B, otherwise the leading n by k part of the array B must contain the matrix B. Unchanged on exit.

LDB (input)

On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. When TRANSB = 'N' or 'n' then LDB >= max(1, k), otherwise LDB >= max(1, n). Unchanged on exit.

BETA (input)

On entry, BETA specifies the scalar beta. When BETA is supplied as zero then C need not be set on input. Unchanged on exit.

C (input/output)

DOUBLE PRECISION array of DIMENSION (LDC, n). Before entry, the leading m by n part of the array C must contain the matrix C, except when beta is zero, in which case C need not be set on entry. On exit, the array C is overwritten by the m by n matrix (alpha*op(A)*op(B) + beta*C).

LDC (input)

On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC >= max(1, m). Unchanged on exit.

Contents

NAME

```
dgemv - perform one of the matrix-vector operations    y  :=
alpha*A*x + beta*y or y := alpha*A'*x + beta*y
```

SYNOPSIS

```
SUBROUTINE DGEMV(TRANSA, M, N, ALPHA, A, LDA, X, INCX, BETA, Y, INCY)

CHARACTER * 1 TRANSA
INTEGER M, N, LDA, INCX, INCY
DOUBLE PRECISION ALPHA, BETA
DOUBLE PRECISION A(LDA,*), X(*), Y(*)

SUBROUTINE DGEMV_64(TRANSA, M, N, ALPHA, A, LDA, X, INCX, BETA, Y,
      INCY)

CHARACTER * 1 TRANSA
INTEGER*8 M, N, LDA, INCX, INCY
DOUBLE PRECISION ALPHA, BETA
DOUBLE PRECISION A(LDA,*), X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE GEMV([TRANSA], [M], [N], ALPHA, A, [LDA], X, [INCX], BETA,
       Y, [INCY])

CHARACTER(LEN=1) :: TRANSA
INTEGER :: M, N, LDA, INCX, INCY
REAL(8) :: ALPHA, BETA
REAL(8), DIMENSION(:) :: X, Y
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE GEMV_64([TRANSA], [M], [N], ALPHA, A, [LDA], X, [INCX],
       BETA, Y, [INCY])
```

```
CHARACTER(LEN=1) :: TRANSA
INTEGER(8) :: M, N, LDA, INCX, INCY
REAL(8) :: ALPHA, BETA
REAL(8), DIMENSION(:) :: X, Y
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dgemv(char transa, int m, int n, double  alpha,  double
          *a,  int  lda,  double  *x, int incx, double beta,
          double *y, int incy);
void dgemv_64(char transa, long m,  long  n,  double  alpha,
          double  *a, long lda, double *x, long incx, double
          beta, double *y, long incy);
```

PURPOSE

dgemv performs one of the matrix-vector operations y := alpha*A*x + beta*y, or y := alpha*A'*x + beta*y, where alpha and beta are scalars, x and y are vectors and A is an m by n matrix.

ARGUMENTS

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   y := alpha*A*x + beta*y.

TRANSA = 'T' or 't'   y := alpha*A'*x + beta*y.

TRANSA = 'C' or 'c'   y := alpha*A'*x + beta*y.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

M (input)
On entry, M specifies the number of rows of the matrix A. M >= 0. Unchanged on exit.

N (input)
On entry, N specifies the number of columns of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
Before entry, the leading m by n part of the array A must contain the matrix of coefficients. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, m). Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)) when TRANSA = 'N' or 'n' and at least (1 + (m - 1)*abs(INCX)) otherwise. Before entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. When BETA is supplied as zero then Y need not be set on input. Unchanged on exit.

Y (input/output)
(1 + (m - 1)*abs(INCY)) when TRANSA = 'N' or 'n' and at least (1 + (n - 1)*abs(INCY)) otherwise. Before entry with BETA non-zero, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
dgeqlf - compute a QL factorization of a real M-by-N  matrix
A
```

SYNOPSIS

```
SUBROUTINE DGEQLF(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER M, N, LDA, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)

SUBROUTINE DGEQLF_64(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER*8 M, N, LDA, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEQLF([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER :: M, N, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE GEQLF_64([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER(8) :: M, N, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dgeqlf(int m, int n, double *a, int lda,  double  *tau,
          int *info);

void dgeqlf_64(long m, long n, double *a, long  lda,  double
          *tau, long *info);
```

PURPOSE

dgeqlf computes a QL factorization of a real M-by-N matrix A: A = Q * L.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, if m >= n,   the lower triangle of the subarray A(m-n+1:m,1:n) contains the N-by-N lower triangular matrix L; if m <= n, the elements on and below the (n-m)-th superdiagonal contain the M-by-N lower trapezoidal matrix L; the remaining elements, with the array TAU, represent the orthogonal matrix Q as a product of elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A.   LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK.   LDWORK >= max(1,N). For optimum performance LDWORK >= N*NB, where NB is the optimal blocksize.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first

entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The matrix Q is represented as a product of elementary reflectors

```
   Q = H(k) . . . H(2) H(1), where k = min(m,n).
```

Each H(i) has the form

```
   H(i) = I - tau * v * v'
```

where tau is a real scalar, and v is a real vector with v(m-k+i+1:m) = 0 and v(m-k+i) = 1; v(1:m-k+i-1) is stored on exit in A(1:m-k+i-1,n-k+i), and tau in TAU(i).

Contents

NAME

```
dgeqp3 - compute a QR factorization with column pivoting  of
a matrix A
```

SYNOPSIS

```
SUBROUTINE DGEQP3(M, N, A, LDA, JPVT, TAU, WORK, LWORK, INFO)

INTEGER M, N, LDA, LWORK, INFO
INTEGER JPVT(*)
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)

SUBROUTINE DGEQP3_64(M, N, A, LDA, JPVT, TAU, WORK, LWORK, INFO)

INTEGER*8 M, N, LDA, LWORK, INFO
INTEGER*8 JPVT(*)
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEQP3([M], [N], A, [LDA], JPVT, TAU, [WORK], [LWORK],
       [INFO])

INTEGER :: M, N, LDA, LWORK, INFO
INTEGER, DIMENSION(:) :: JPVT
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE GEQP3_64([M], [N], A, [LDA], JPVT, TAU, [WORK], [LWORK],
       [INFO])

INTEGER(8) :: M, N, LDA, LWORK, INFO
INTEGER(8), DIMENSION(:) :: JPVT
```

```
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dgeqp3(int m, int n, double *a,  int  lda,  int  *jpvt,
          double *tau, int *info);

void dgeqp3_64(long m, long n, double  *a,  long  lda,  long
          *jpvt, double *tau, long *info);
```

PURPOSE

dgeqp3 computes a QR factorization with column pivoting of a matrix A: A*P = Q*R using Level 3 BLAS.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, the upper triangle of the array contains the min(M,N)-by-N upper trapezoidal matrix R; the elements below the diagonal, together with the array TAU, represent the orthogonal matrix Q as a product of min(M,N) elementary reflectors.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

JPVT (input/output)
On entry, if JPVT(J).ne.0, the J-th column of A is permuted to the front of A*P (a leading column); if JPVT(J)=0, the J-th column of A is a free column. On exit, if JPVT(J)=K, then the J-th column of A*P was the the K-th column of A.

TAU (output)
The scalar factors of the elementary reflectors.

WORK (workspace)
On exit, if INFO=0, WORK(1) returns the optimal

LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 3*N+1. For optimal performance LWORK >= 2*N+(N+1)*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

The matrix Q is represented as a product of elementary reflectors

Q = H(1) H(2) . . . H(k), where k = min(m,n).

Each H(i) has the form

H(i) = I - tau * v * v'

where tau is a real/complex scalar, and v is a real/complex vector with v(1:i-1) = 0 and v(i) = 1; v(i+1:m) is stored on exit in A(i+1:m,i), and tau in TAU(i).

Based on contributions by
G. Quintana-Orti, Depto. de Informatica, Universidad Jaime I, Spain
X. Sun, Computer Science Dept., Duke University, USA

Contents

NAME

dgeqpf - routine is deprecated and has been replaced by routine SGEQP3

SYNOPSIS

```
SUBROUTINE DGEQPF(M, N, A, LDA, JPIVOT, TAU, WORK, INFO)

INTEGER M, N, LDA, INFO
INTEGER JPIVOT(*)
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)

SUBROUTINE DGEQPF_64(M, N, A, LDA, JPIVOT, TAU, WORK, INFO)

INTEGER*8 M, N, LDA, INFO
INTEGER*8 JPIVOT(*)
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEQPF([M], [N], A, [LDA], JPIVOT, TAU, [WORK], [INFO])

INTEGER :: M, N, LDA, INFO
INTEGER, DIMENSION(:) :: JPIVOT
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE GEQPF_64([M], [N], A, [LDA], JPIVOT, TAU, [WORK], [INFO])

INTEGER(8) :: M, N, LDA, INFO
INTEGER(8), DIMENSION(:) :: JPIVOT
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dgeqpf(int m, int n, double *a, int lda,  int  *jpivot,
          double *tau, int *info);

void dgeqpf_64(long m, long n, double  *a,  long  lda,  long
          *jpivot, double *tau, long *info);
```

PURPOSE

dgeqpf routine is deprecated and has been replaced by routine SGEQP3.

SGEQPF computes a QR factorization with column pivoting of a real M-by-N matrix A: A*P = Q*R.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0

A (input/output)
On entry, the M-by-N matrix A. On exit, the upper triangle of the array contains the min(M,N)-by-N upper triangular matrix R; the elements below the diagonal, together with the array TAU, represent the orthogonal matrix Q as a product of min(m,n) elementary reflectors.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

JPIVOT (input/output)
On entry, if JPIVOT(i) .ne. 0, the i-th column of A is permuted to the front of A*P (a leading column); if JPIVOT(i) = 0, the i-th column of A is a free column. On exit, if JPIVOT(i) = k, then the i-th column of A*P was the k-th column of A.

TAU (output)
The scalar factors of the elementary reflectors.

WORK (workspace)

dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The matrix Q is represented as a product of elementary reflectors

```
Q = H(1) H(2) . . . H(n)
```

Each H(i) has the form

```
H = I - tau * v * v'
```

where tau is a real scalar, and v is a real vector with v(1:i-1) = 0 and v(i) = 1; v(i+1:m) is stored on exit in A(i+1:m,i).

The matrix P is represented in jpvt as follows: If

```
jpvt(j) = i
```

then the jth column of P is the ith canonical unit vector.

Contents

NAME

```
dgeqrf - compute a QR factorization of a real M-by-N  matrix
A
```

SYNOPSIS

```
SUBROUTINE DGEQRF(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER M, N, LDA, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)

SUBROUTINE DGEQRF_64(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER*8 M, N, LDA, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEQRF([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER :: M, N, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE GEQRF_64([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER(8) :: M, N, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dgeqrf(int m, int n, double *a, int lda,  double  *tau,
          int *info);

void dgeqrf_64(long m, long n, double *a, long  lda,  double
          *tau, long *info);
```

PURPOSE

dgeqrf computes a QR factorization of a real M-by-N matrix A: A = Q * R.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.
N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, the elements on and above the diagonal of the array contain the min(M,N)-by-N upper trapezoidal matrix R (R is upper triangular if m >= n); the elements below the diagonal, with the array TAU, represent the orthogonal matrix Q as a product of min(m,n) elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A.   LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK.   LDWORK >= max(1,N). For optimum performance LDWORK >= N*NB, where NB is the optimal blocksize.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

```
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

FURTHER DETAILS

```
The matrix Q is  represented  as  a  product  of  elementary
reflectors

   Q = H(1) H(2) . . . H(k), where k = min(m,n).

Each H(i) has the form
   H(i) = I - tau * v * v'

where tau is a real scalar, and v is a real vector with
v(1:i-1) = 0 and v(i) = 1; v(i+1:m) is  stored  on  exit  in
A(i+1:m,i), and tau in TAU(i).
```

Contents

NAME

```
dger - perform the rank 1 operation   A := alpha*x*y' + A
```

SYNOPSIS

```
SUBROUTINE DGER(M, N, ALPHA, X, INCX, Y, INCY, A, LDA)

INTEGER M, N, INCX, INCY, LDA
DOUBLE PRECISION ALPHA
DOUBLE PRECISION X(*), Y(*), A(LDA,*)

SUBROUTINE DGER_64(M, N, ALPHA, X, INCX, Y, INCY, A, LDA)

INTEGER*8 M, N, INCX, INCY, LDA
DOUBLE PRECISION ALPHA
DOUBLE PRECISION X(*), Y(*), A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE GER([M], [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

INTEGER :: M, N, INCX, INCY, LDA
REAL(8) :: ALPHA
REAL(8), DIMENSION(:) :: X, Y
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE GER_64([M], [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

INTEGER(8) :: M, N, INCX, INCY, LDA
REAL(8) :: ALPHA
REAL(8), DIMENSION(:) :: X, Y
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dger(int m, int n, double alpha, double *x,  int  incx,
          double *y, int incy, double *a, int lda);

void dger_64(long m, long n, double alpha, double  *x,  long
          incx, double *y, long incy, double *a, long lda);
```

PURPOSE

dger performs the rank 1 operation A := alpha*x*y' + A, where alpha is a scalar, x is an m element vector, y is an n element vector and A is an m by n matrix.

ARGUMENTS

M (input)
On entry, M specifies the number of rows of the matrix A. M >= 0. Unchanged on exit.

N (input)
On entry, N specifies the number of columns of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (m - 1)*abs(INCX)). Before entry, the incremented array X must contain the m element vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

Y (input)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. Unchanged on exit.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

A (input/output)

Before entry, the leading m by n part of the array A must contain the matrix of coefficients. On exit, A is overwritten by the updated matrix.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, m). Unchanged on exit.

Contents

NAME

```
dgerfs - improve the computed solution to a system of linear
equations and provides error bounds and backward error esti-
mates for the solution
```

SYNOPSIS

```
SUBROUTINE DGERFS(TRANSA, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*), WORK2(*)
DOUBLE PRECISION A(LDA,*), AF(LDAF,*),  B(LDB,*),  X(LDX,*),
FERR(*), BERR(*), WORK(*)

SUBROUTINE DGERFS_64(TRANSA, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B,
      LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
DOUBLE PRECISION A(LDA,*), AF(LDAF,*),  B(LDB,*),  X(LDX,*),
FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GERFS([TRANSA], [N], [NRHS], A, [LDA], AF, [LDAF], IPIVOT,
       B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: TRANSA
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
REAL(8), DIMENSION(:) :: FERR, BERR, WORK
```

```
REAL(8), DIMENSION(:,:) :: A, AF, B, X

SUBROUTINE GERFS_64([TRANSA], [N], [NRHS], A, [LDA], AF, [LDAF],
       IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: TRANSA
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL(8), DIMENSION(:) :: FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
#include <sunperf.h>
void dgerfs(char transa, int n, int  nrhs,  double  *a,  int
          lda, double *af, int ldaf, int *ipivot, double *b,
          int ldb, double *x, int ldx, double *ferr,  double
          *berr, int *info);

void dgerfs_64(char transa, long n, long  nrhs,  double  *a,
          long  lda,  double  *af,  long ldaf, long *ipivot,
          double *b, long ldb, double *x, long  ldx,  double
          *ferr, double *berr, long *info);
```

PURPOSE

dgerfs improves the computed solution to a system of linear equations and provides error bounds and backward error estimates for the solution.

ARGUMENTS

```
TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose = Tran-
          spose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input) The original N-by-N matrix A.
```

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input)
The factors L and U from the factorization A = P*L*U as computed by SGETRF.

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

IPIVOT (input)
The pivot indices from SGETRF; for 1<=i<=N, row i of the matrix was interchanged with row IPIVOT(i).

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by SGETRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-
gal value

Contents

NAME

```
dgerqf - compute an RQ factorization of a real M-by-N matrix
A
```

SYNOPSIS

```
SUBROUTINE DGERQF(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER M, N, LDA, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)

SUBROUTINE DGERQF_64(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER*8 M, N, LDA, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GERQF([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER :: M, N, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE GERQF_64([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER(8) :: M, N, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dgerqf(int m, int n, double *a, int lda,  double  *tau,
          int *info);

void dgerqf_64(long m, long n, double *a, long  lda,  double
          *tau, long *info);
```

PURPOSE

dgerqf computes an RQ factorization of a real M-by-N matrix A: A = R * Q.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, if m <= n, the upper triangle of the subarray A(1:m,n-m+1:n) contains the M-by-M upper triangular matrix R; if m >= n, the elements on and above the (m-n)-th subdiagonal contain the M-by-N upper trapezoidal matrix R; the remaining elements, with the array TAU, represent the orthogonal matrix Q as a product of min(m,n) elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,M). For optimum performance LDWORK >= M*NB, where NB is the optimal blocksize.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first

entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The matrix Q is represented as a product of elementary reflectors

```
   Q = H(1) H(2) . . . H(k), where k = min(m,n).
Each H(i) has the form

   H(i) = I - tau * v * v'

where tau is a real scalar, and v is a real vector with
v(n-k+i+1:n) = 0 and v(n-k+i) = 1; v(1:n-k+i-1) is stored on
exit in A(m-k+i,1:n-k+i-1), and tau in TAU(i).
```

Contents

NAME

```
dgesdd - compute the singular value decomposition (SVD) of a
real  M-by-N  matrix  A,  optionally  computing the left and
right singular vectors
```

SYNOPSIS

```
SUBROUTINE DGESDD(JOBZ, M, N, A, LDA, S, U, LDU, VT, LDVT, WORK,
      LWORK, IWORK, INFO)

CHARACTER * 1 JOBZ
INTEGER M, N, LDA, LDU, LDVT, LWORK, INFO
INTEGER IWORK(*)
DOUBLE  PRECISION  A(LDA,*),  S(*),  U(LDU,*),   VT(LDVT,*),
WORK(*)

SUBROUTINE DGESDD_64(JOBZ, M, N, A, LDA, S, U, LDU, VT, LDVT, WORK,
      LWORK, IWORK, INFO)

CHARACTER * 1 JOBZ
INTEGER*8 M, N, LDA, LDU, LDVT, LWORK, INFO
INTEGER*8 IWORK(*)
DOUBLE  PRECISION  A(LDA,*),  S(*),  U(LDU,*),   VT(LDVT,*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GESDD(JOBZ, [M], [N], A, [LDA], S, U, [LDU], VT, [LDVT],
       [WORK], [LWORK], [IWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ
INTEGER :: M, N, LDA, LDU, LDVT, LWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
```

```
REAL(8), DIMENSION(:) :: S, WORK
REAL(8), DIMENSION(:,:) :: A, U, VT

SUBROUTINE GESDD_64(JOBZ, [M], [N], A, [LDA], S, U, [LDU], VT, [LDVT],
       [WORK], [LWORK], [IWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ
INTEGER(8) :: M, N, LDA, LDU, LDVT, LWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: S, WORK
REAL(8), DIMENSION(:,:) :: A, U, VT
```

C INTERFACE

```
#include <sunperf.h>
void dgesdd(char jobz, int m, int n,  double  *a,  int  lda,
          double  *s,  double  *u,  int ldu, double *vt, int
          ldvt, int *info);

void dgesdd_64(char jobz, long m, long n,  double  *a,  long
          lda,  double  *s, double *u, long ldu, double *vt,
          long ldvt, long *info);
```

PURPOSE

dgesdd computes the singular value decomposition (SVD) of a real M-by-N matrix A, optionally computing the left and right singular vectors. If singular vectors are desired, it uses a divide-and-conquer algorithm.

The SVD is written
 = U * SIGMA * transpose(V)

where SIGMA is an M-by-N matrix which is zero except for its min(m,n) diagonal elements, U is an M-by-M orthogonal matrix, and V is an N-by-N orthogonal matrix. The diagonal elements of SIGMA are the singular values of A; they are real and non-negative, and are returned in descending order. The first min(m,n) columns of U and V are the left and right singular vectors of A.

Note that the routine returns VT = V**T, not V.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

```
JOBZ (input)
          Specifies options for computing all or part of the
          matrix U:
          = 'A':  all M columns of U and all N rows of  V**T
          are  returned  in the arrays U and VT; = 'S':  the
          first min(M,N) columns of U and the first min(M,N)
          rows  of V**T are returned in the arrays U and VT;
          = 'O':  If M >= N, the first N columns  of  U  are
          overwritten  on  the  array A and all rows of V**T
          are returned  in  the  array  VT;  otherwise,  all
          columns  of  U are returned in the array U and the
          first M rows of V**T are overwritten in the  array
          VT;  =  'N':   no columns of U or rows of V**T are
          computed.

M (input) The number of rows of the input matrix A.  M >= 0.

N (input) The number of columns of the input matrix A.  N >=
          0.

A (input/output)
          On entry, the M-by-N matrix A.  On exit, if JOBZ =
          'O',  A is overwritten with the first N columns of
          U (the left singular vectors,  stored  columnwise)
          if  M >= N; A is overwritten with the first M rows
          of V**T (the right singular vectors,  stored  row-
          wise)  otherwise.   if JOBZ .ne. 'O', the contents
          of A are destroyed.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,M).

S (output)
          The singular values of A, sorted so that  S(i)  >=
          S(i+1).

U (output)
          UCOL = M if JOBZ = 'A' or JOBZ = 'O' and  M  <  N;
          UCOL  =  min(M,N) if JOBZ = 'S'.  If JOBZ = 'A' or
          JOBZ = 'O' and  M  <  N,  U  contains  the  M-by-M
          orthogonal matrix U; if JOBZ = 'S', U contains the
          first min(M,N) columns of  U  (the  left  singular
          vectors,  stored  columnwise); if JOBZ = 'O' and M
          >= N, or JOBZ = 'N', U is not referenced.
```

```
LDU (input)
          The leading dimension of the array U.  LDU  >=  1;
          if  JOBZ = 'S' or 'A' or JOBZ = 'O' and M < N, LDU
          >= M.

VT (output)
          If JOBZ = 'A' or JOBZ = 'O' and M >=  N,  VT  con-
          tains the N-by-N orthogonal matrix V**T; if JOBZ =
          'S', VT contains the first min(M,N) rows  of  V**T
          (the  right  singular vectors, stored rowwise); if
          JOBZ = 'O' and M < N, or JOBZ =  'N',  VT  is  not
          referenced.

LDVT (input)
          The leading dimension of the array VT.  LDVT >= 1;
          if JOBZ = 'A' or JOBZ = 'O' and M >= N, LDVT >= N;
          if JOBZ = 'S', LDVT >= min(M,N).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK;

LWORK (input)
          The dimension of the array WORK. LWORK >=  1.   If
          JOBZ    =    'N',    LWORK    >=    3*min(M,N)   +
          max(max(M,N),6*min(M,N)).  If JOBZ = 'O', LWORK >=
          3*min(M,N)*min(M,N)   +   max(max(M,N),5*min(M,N)*
          min(M,N)+4*min(M,N)).  If JOBZ = 'S' or 'A'  LWORK
          >=  3*min(M,N)*min(M,N) + max(max(M,N),4*min(M,N)*
          min(M,N)+4*min(M,N)).  For good performance, LWORK
          should  generally  be  larger.   If  LWORK < 0 but
          other input arguments are legal,  WORK(1)  returns
          optimal LWORK.

IWORK (workspace)
          dimension(8*MIN(M,N))

INFO (output)
          = 0:  successful exit.
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value.
          > 0:  SBDSDC did not  converge,  updating  process
          failed.
```

FURTHER DETAILS

```
Based on contributions by
   Ming Gu and Huan Ren, Computer Science Division,  Univer-
sity of
```

California at Berkeley, USA

Contents

NAME

```
dgesv - compute the solution to  a  real  system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE DGESV(N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

INTEGER N, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*)

SUBROUTINE DGESV_64(N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

INTEGER*8 N, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE GESV([N], [NRHS], A, [LDA], IPIVOT, B, [LDB], [INFO])

INTEGER :: N, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE GESV_64([N], [NRHS], A, [LDA], IPIVOT, B, [LDB], [INFO])

INTEGER(8) :: N, NRHS, LDA, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dgesv(int n, int nrhs, double *a, int lda, int *ipivot,
          double *b, int ldb, int *info);

void dgesv_64(long n, long nrhs, double *a, long  lda,  long
          *ipivot, double *b, long ldb, long *info);
```

PURPOSE

dgesv computes the solution to a real system of linear equations
   A * X = B, where A is an N-by-N matrix and X and B are N-by-NRHS matrices.

The LU decomposition with partial pivoting and row interchanges is used to factor A as
   A = P * L * U,
where P is a permutation matrix, L is unit lower triangular, and U is upper triangular. The factored form of A is then used to solve the system of equations A * X = B.

ARGUMENTS

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input/output)
On entry, the N-by-N coefficient matrix A. On exit, the factors L and U from the factorization A = P*L*U; the unit diagonal elements of L are not stored.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (output)
The pivot indices that define the permutation matrix P; row i of the matrix was interchanged with row IPIVOT(i).

B (input/output)
On entry, the N-by-NRHS matrix of right hand side

matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution could not be computed.

Contents

NAME

```
dgesvd - compute the singular value decomposition (SVD) of a
real  M-by-N  matrix A, optionally computing the left and/or
right singular vectors
```

SYNOPSIS

```
SUBROUTINE DGESVD(JOBU, JOBVT, M, N, A, LDA, SING, U, LDU, VT, LDVT,
      WORK, LDWORK, INFO)

CHARACTER * 1 JOBU, JOBVT
INTEGER M, N, LDA, LDU, LDVT, LDWORK, INFO
DOUBLE PRECISION A(LDA,*),  SING(*),  U(LDU,*),  VT(LDVT,*),
WORK(*)

SUBROUTINE DGESVD_64(JOBU, JOBVT, M, N, A, LDA, SING, U, LDU, VT,
      LDVT, WORK, LDWORK, INFO)

CHARACTER * 1 JOBU, JOBVT
INTEGER*8 M, N, LDA, LDU, LDVT, LDWORK, INFO
DOUBLE PRECISION A(LDA,*),  SING(*),  U(LDU,*),  VT(LDVT,*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GESVD(JOBU, JOBVT, [M], [N], A, [LDA], SING, U, [LDU], VT,
       [LDVT], [WORK], [LDWORK], [INFO])

CHARACTER(LEN=1) :: JOBU, JOBVT
INTEGER :: M, N, LDA, LDU, LDVT, LDWORK, INFO
REAL(8), DIMENSION(:) :: SING, WORK
REAL(8), DIMENSION(:,:) :: A, U, VT

SUBROUTINE GESVD_64(JOBU, JOBVT, [M], [N], A, [LDA], SING, U, [LDU],
```

```
           VT, [LDVT], [WORK], [LDWORK], [INFO])

    CHARACTER(LEN=1) :: JOBU, JOBVT
    INTEGER(8) :: M, N, LDA, LDU, LDVT, LDWORK, INFO
    REAL(8), DIMENSION(:) :: SING, WORK
    REAL(8), DIMENSION(:,:) :: A, U, VT
```

C INTERFACE

```
    #include <sunperf.h>

    void dgesvd(char jobu, char jobvt, int m, int n, double  *a,
              int  lda, double *sing, double *u, int ldu, double
              *vt, int ldvt, int *info);
    void dgesvd_64(char jobu, char jobvt, long m, long n, double
              *a,  long  lda, double *sing, double *u, long ldu,
              double *vt, long ldvt, long *info);
```

PURPOSE

dgesvd computes the singular value decomposition (SVD) of a real M-by-N matrix A, optionally computing the left and/or right singular vectors. The SVD is written
= U * SIGMA * transpose(V)

where SIGMA is an M-by-N matrix which is zero except for its min(m,n) diagonal elements, U is an M-by-M orthogonal matrix, and V is an N-by-N orthogonal matrix. The diagonal elements of SIGMA are the singular values of A; they are real and non-negative, and are returned in descending order. The first min(m,n) columns of U and V are the left and right singular vectors of A.

Note that the routine returns V**T, not V.

ARGUMENTS

JOBU (input)
Specifies options for computing all or part of the matrix U:
= 'A': all M columns of U are returned in array U:
= 'S': the first min(m,n) columns of U (the left singular vectors) are returned in the array U; = 'O': the first min(m,n) columns of U (the left singular vectors) are overwritten on the array A;
= 'N': no columns of U (no left singular vectors) are computed.

JOBVT (input)
Specifies options for computing all or part of the matrix V**T:
= 'A': all N rows of V**T are returned in the array VT;
= 'S': the first min(m,n) rows of V**T (the right singular vectors) are returned in the array VT; = 'O': the first min(m,n) rows of V**T (the right singular vectors) are overwritten on the array A;
= 'N': no rows of V**T (no right singular vectors) are computed.

JOBVT and JOBU cannot both be 'O'.

M (input) The number of rows of the input matrix A. M >= 0.

N (input) The number of columns of the input matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, if JOBU = 'O', A is overwritten with the first min(m,n) columns of U (the left singular vectors, stored columnwise); if JOBVT = 'O', A is overwritten with the first min(m,n) rows of V**T (the right singular vectors, stored rowwise); if JOBU .ne. 'O' and JOBVT .ne. 'O', the contents of A are destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

SING (output)
The singular values of A, sorted so that SING(i) >= SING(i+1).

U (input) (LDU,M) if JOBU = 'A' or (LDU,min(M,N)) if JOBU = 'S'. If JOBU = 'A', U contains the M-by-M orthogonal matrix U; if JOBU = 'S', U contains the first min(m,n) columns of U (the left singular vectors, stored columnwise); if JOBU = 'N' or 'O', U is not referenced.

LDU (input)
The leading dimension of the array U. LDU >= 1; if JOBU = 'S' or 'A', LDU >= M.

VT (input)
If JOBVT = 'A', VT contains the N-by-N orthogonal matrix V**T; if JOBVT = 'S', VT contains the first

min(m,n) rows of V**T (the right singular vectors, stored rowwise); if JOBVT = 'N' or 'O', VT is not referenced.

LDVT (input)
The leading dimension of the array VT. LDVT >= 1; if JOBVT = 'A', LDVT >= N; if JOBVT = 'S', LDVT >= min(M,N).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK; if INFO > 0, WORK(2:MIN(M,N)) contains the unconverged superdiagonal elements of an upper bidiagonal matrix B whose diagonal is in SING (not necessarily sorted). B satisfies A = U * B * VT, so it has the same singular values as A, and singular vectors related by U and VT.

LDWORK (input)
The dimension of the array WORK. LDWORK >= 1. LDWORK >= MAX(3*MIN(M,N)+MAX(M,N),5*MIN(M,N)). For good performance, LDWORK should generally be larger.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if SBDSQR did not converge, INFO specifies how many superdiagonals of an intermediate bidiagonal form B did not converge to zero. See the description of WORK above for details.

Contents

NAME

```
dgesvx - use the LU factorization to compute the solution to
a real system of linear equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE DGESVX(FACT, TRANSA, N, NRHS, A, LDA, AF, LDAF, IPIVOT,
      EQUED, R, C, B, LDB, X, LDX, RCOND, FERR, BERR, WORK,
      WORK2, INFO)

CHARACTER * 1 FACT, TRANSA, EQUED
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*), WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(LDA,*), AF(LDAF,*), R(*), C(*), B(LDB,*),
X(LDX,*), FERR(*), BERR(*), WORK(*)

SUBROUTINE DGESVX_64(FACT, TRANSA, N, NRHS, A, LDA, AF, LDAF, IPIVOT,
      EQUED, R, C, B, LDB, X, LDX, RCOND, FERR, BERR, WORK,
      WORK2, INFO)

CHARACTER * 1 FACT, TRANSA, EQUED
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(LDA,*), AF(LDAF,*), R(*), C(*), B(LDB,*),
X(LDX,*), FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GESVX(FACT, [TRANSA], [N], [NRHS], A, [LDA], AF, [LDAF],
       IPIVOT, EQUED, R, C, B, [LDB], X, [LDX], RCOND, FERR,
       BERR, [WORK], [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: FACT, TRANSA, EQUED
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: R, C, FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: A, AF, B, X

SUBROUTINE GESVX_64(FACT, [TRANSA], [N], [NRHS], A, [LDA], AF, [LDAF],
       IPIVOT, EQUED, R, C, B, [LDB], X, [LDX], RCOND, FERR,
       BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, TRANSA, EQUED
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: R, C, FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
#include <sunperf.h>

void dgesvx(char fact, char transa, int n, int nrhs,  double
          *a,  int  lda,  double *af, int ldaf, int *ipivot,
          char equed, double *r, double *c, double  *b,  int
          ldb,  double  *x,  int  ldx, double *rcond, double
          *ferr, double *berr, int *info);

void dgesvx_64(char fact, char transa, long  n,  long  nrhs,
          double  *a,  long lda, double *af, long ldaf, long
          *ipivot, char equed, double *r, double *c,  double
          *b,  long ldb, double *x, long ldx, double *rcond,
          double *ferr, double *berr, long *info);
```

PURPOSE

```
dgesvx uses the LU factorization to compute the solution  to
a real system of linear equations
   A * X = B, where A is an N-by-N matrix and X  and  B  are
N-by-NRHS matrices.

Error bounds on the solution and a  condition  estimate  are
also provided.

The following steps are performed:

1. If FACT = 'E',  real  scaling  factors  are  computed  to
equilibrate
   the system:
      TRANS = 'N':  diag(R)*A*diag(C)     *inv(diag(C))*X  =
diag(R)*B
```

```
      TRANS = 'T': (diag(R)*A*diag(C))**T *inv(diag(R))*X  =
diag(C)*B
      TRANS = 'C': (diag(R)*A*diag(C))**H *inv(diag(R))*X  =
diag(C)*B
   Whether or not the system will be equilibrated depends on
the
   scaling of the matrix A, but if equilibration is used,  A
is
   overwritten by diag(R)*A*diag(C) and B by  diag(R)*B  (if
TRANS='N')
   or diag(C)*B (if TRANS = 'T' or 'C').

2. If FACT = 'N' or 'E', the LU  decomposition  is  used  to
factor the
   matrix A (after equilibration if FACT = 'E') as
      A = P * L * U,
   where P is a permutation matrix, L is a unit  lower  tri-
angular
   matrix, and U is upper triangular.
3. If some U(i,i)=0, so that U is exactly singular, then the
routine
   returns with INFO = i. Otherwise, the factored form of  A
is used
   to estimate the condition number of the matrix A.  If the
   reciprocal of the condition number is less  than  machine
precision,
   INFO = N+1 is returned as  a  warning,  but  the  routine
still goes on
   to solve for X and  compute  error  bounds  as  described
below.

4. The system of equations is solved for X  using  the  fac-
tored form
   of A.

5. Iterative refinement is applied to improve  the  computed
solution
   matrix and calculate  error  bounds  and  backward  error
estimates
   for it.

6. If equilibration was used, the matrix X is  premultiplied
by
   diag(C) (if TRANS = 'N') or diag(R) (if TRANS  =  'T'  or
'C') so
   that it solves the original system before equilibration.
```

ARGUMENTS

```
FACT (input)
          Specifies whether or not the factored form of  the
          matrix A is supplied on entry, and if not, whether
          the matrix A should be equilibrated before  it  is
          factored.  = 'F':  On entry, AF and IPIVOT contain
          the factored form of A.  If EQUED is not 'N',  the
          matrix  A  has been equilibrated with scaling fac-
          tors given by R and C.  A, AF, and IPIVOT are  not
          modified.   = 'N':  The matrix A will be copied to
          AF and factored.
          = 'E':  The  matrix  A  will  be  equilibrated  if
          necessary, then copied to AF and factored.

TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Transpose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.
N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input/output)
          On entry, the N-by-N matrix A.  If FACT = 'F'  and
          EQUED  is  not  'N', then A must have been equili-
          brated by the scaling factors in R and/or C.  A is
          not  modified  if  FACT = 'F' or 'N', or if FACT =
          'E' and EQUED = 'N' on exit.

          On exit, if EQUED .ne. 'N', A is  scaled  as  fol-
          lows:  EQUED = 'R':  A := diag(R) * A
          EQUED = 'C':  A := A * diag(C)
          EQUED = 'B':  A := diag(R) * A * diag(C).

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

AF (input/output)
          If FACT = 'F', then AF is an input argument and on
          entry  contains  the factors L and U from the fac-
          torization A = P*L*U as computed  by  SGETRF.   If
          EQUED  .ne.  'N',  then AF is the factored form of
          the equilibrated matrix A.
```

If FACT = 'N', then AF is an output argument and on exit returns the factors L and U from the factorization A = P*L*U of the original matrix A.

If FACT = 'E', then AF is an output argument and on exit returns the factors L and U from the factorization A = P*L*U of the equilibrated matrix A (see the description of A for the form of the equilibrated matrix).

LDAF (input)

The leading dimension of the array AF. LDAF >= max(1,N).

IPIVOT (input or output)

If FACT = 'F', then IPIVOT is an input argument and on entry contains the pivot indices from the factorization A = P*L*U as computed by SGETRF; row i of the matrix was interchanged with row IPIVOT(i).

If FACT = 'N', then IPIVOT is an output argument and on exit contains the pivot indices from the factorization A = P*L*U of the original matrix A.

If FACT = 'E', then IPIVOT is an output argument and on exit contains the pivot indices from the factorization A = P*L*U of the equilibrated matrix A.

EQUED (input/output)

Specifies the form of equilibration that was done.
= 'N': No equilibration (always true if FACT = 'N').
= 'R': Row equilibration, i.e., A has been premultiplied by diag(R). = 'C': Column equilibration, i.e., A has been postmultiplied by diag(C). = 'B': Both row and column equilibration, i.e., A has been replaced by diag(R) * A * diag(C). EQUED is an input argument if FACT = 'F'; otherwise, it is an output argument.

R (input/output)

The row scale factors for A. If EQUED = 'R' or 'B', A is multiplied on the left by diag(R); if EQUED = 'N' or 'C', R is not accessed. R is an input argument if FACT = 'F'; otherwise, R is an output argument. If FACT = 'F' and EQUED = 'R' or 'B', each element of R must be positive.

C (input/output)

The column scale factors for A. If EQUED = 'C' or 'B', A is multiplied on the right by diag(C); if EQUED = 'N' or 'R', C is not accessed. C is an input argument if FACT = 'F'; otherwise, C is an output argument. If FACT = 'F' and EQUED = 'C' or 'B', each element of C must be positive.

B (input/output)

On entry, the N-by-NRHS right hand side matrix B. On exit, if EQUED = 'N', B is not modified; if TRANSA = 'N' and EQUED = 'R' or 'B', B is overwritten by diag(R)*B; if TRANSA = 'T' or 'C' and EQUED = 'C' or 'B', B is overwritten by diag(C)*B.

LDB (input)

The leading dimension of the array B. LDB >= max(1,N).

X (output)

If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X to the original system of equations. Note that A and B are modified on exit if EQUED .ne. 'N', and the solution to the equilibrated system is inv(diag(C))*X if TRANSA = 'N' and EQUED = 'C' or 'B', or inv(diag(R))*X if TRANSA = 'T' or 'C' and EQUED = 'R' or 'B'.

LDX (input)

The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)

The estimate of the reciprocal condition number of the matrix A after equilibration (if done). If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)

The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(4*N) On exit, WORK(1) contains the reciprocal pivot growth factor norm(A)/norm(U). The "max absolute element" norm is used. If WORK(1) is much less than 1, then the stability of the LU factorization of the (equilibrated) matrix A could be poor. This also means that the solution X, condition estimator RCOND, and forward error bound FERR could be unreliable. If factorization fails with 0<INFO<=N, then WORK(1) contains the reciprocal pivot growth factor for the leading INFO columns of A.

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

dgetf2 - compute an LU factorization of a general m-by-n matrix A using partial pivoting with row interchanges

SYNOPSIS

```
SUBROUTINE DGETF2(M, N, A, LDA, IPIV, INFO)

INTEGER M, N, LDA, INFO
INTEGER IPIV(*)
DOUBLE PRECISION A(LDA,*)

SUBROUTINE DGETF2_64(M, N, A, LDA, IPIV, INFO)

INTEGER*8 M, N, LDA, INFO
INTEGER*8 IPIV(*)
DOUBLE PRECISION A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE GETF2([M], [N], A, [LDA], IPIV, [INFO])

INTEGER :: M, N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIV
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE GETF2_64([M], [N], A, [LDA], IPIV, [INFO])

INTEGER(8) :: M, N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIV
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dgetf2(int m, int n, double *a, int lda, int *ipiv, int
          *info);

void dgetf2_64(long m, long n, double  *a,  long  lda,  long
          *ipiv, long *info);
```

PURPOSE

dgetf2 computes an LU factorization of a general m-by-n matrix A using partial pivoting with row interchanges.

The factorization has the form
   A = P * L * U
where P is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if m > n), and U is upper triangular (upper trapezoidal if m < n).

This is the right-looking Level 2 BLAS version of the algorithm.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the m by n matrix to be factored. On exit, the factors L and U from the factorization A = P*L*U; the unit diagonal elements of L are not stored.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

IPIV (output)
The pivot indices; for 1 <= i <= min(M,N), row i of the matrix was interchanged with row IPIV(i).

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, U(k,k) is exactly zero. The factorization has been completed, but the factor U is

exactly singular, and division by zero will occur if it is used to solve a system of equations.

Contents

NAME

```
dgetrf - compute an LU factorization  of  a  general  M-by-N
matrix A using partial pivoting with row interchanges
```

SYNOPSIS

```
SUBROUTINE DGETRF(M, N, A, LDA, IPIVOT, INFO)

INTEGER M, N, LDA, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION A(LDA,*)

SUBROUTINE DGETRF_64(M, N, A, LDA, IPIVOT, INFO)

INTEGER*8 M, N, LDA, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE GETRF([M], [N], A, [LDA], IPIVOT, [INFO])

INTEGER :: M, N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE GETRF_64([M], [N], A, [LDA], IPIVOT, [INFO])

INTEGER(8) :: M, N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dgetrf(int m, int n, double *a, int lda,  int  *ipivot,
          int *info);

void dgetrf_64(long m, long n, double  *a,  long  lda,  long
          *ipivot, long *info);
```

PURPOSE

dgetrf computes an LU factorization of a general M-by-N matrix A using partial pivoting with row interchanges.

The factorization has the form

```
   A = P * L * U
```

where P is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if m > n), and U is upper triangular (upper trapezoidal if m < n).

This is the right-looking Level 3 BLAS version of the algorithm.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix to be factored.   On exit, the factors L and U from the factorization A = P*L*U; the unit diagonal elements of L are not stored.

LDA (input)
The leading dimension of the array A.   LDA >= max(1,M).

IPIVOT (output)
The pivot indices; for 1 <= i <= min(M,N), row i of the matrix was interchanged with row IPIVOT(i).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, U(i,i) is exactly zero. The factorization has been completed, but the factor U

is exactly singular, and division by zero will occur if it is used to solve a system of equations.

Contents

NAME

```
dgetri - compute the inverse of a matrix using the  LU  fac-
torization computed by SGETRF
```

SYNOPSIS

```
SUBROUTINE DGETRI(N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

INTEGER N, LDA, LDWORK, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION A(LDA,*), WORK(*)

SUBROUTINE DGETRI_64(N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

INTEGER*8 N, LDA, LDWORK, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GETRI([N], A, [LDA], IPIVOT, [WORK], [LDWORK], [INFO])

INTEGER :: N, LDA, LDWORK, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE GETRI_64([N], A, [LDA], IPIVOT, [WORK], [LDWORK], [INFO])

INTEGER(8) :: N, LDA, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dgetri(int n, double *a,  int  lda,  int  *ipivot,  int
          *info);

void dgetri_64(long n, double *a, long  lda,  long  *ipivot,
          long *info);
```

PURPOSE

dgetri computes the inverse of a matrix using the LU factorization computed by SGETRF.

This method inverts U and then computes inv(A) by solving the system inv(A)*L = inv(U) for inv(A).

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the factors L and U from the factorization A = P*L*U as computed by SGETRF. On exit, if INFO = 0, the inverse of the original matrix A.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (input)
The pivot indices from SGETRF; for 1<=i<=N, row i of the matrix was interchanged with row IPIVOT(i).

WORK (workspace)
On exit, if INFO=0, then WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,N). For optimal performance LDWORK >= N*NB, where NB is the optimal blocksize returned by ILAENV.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first

entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, U(i,i) is exactly zero; the matrix is singular and its inverse could not be computed.

Contents

NAME

```
dgetrs - solve a system of linear equations  A * X = B or A'
* X = B with a general N-by-N matrix A using the LU factori-
zation computed by SGETRF
```

SYNOPSIS

```
SUBROUTINE DGETRS(TRANSA, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 TRANSA
INTEGER N, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*)

SUBROUTINE DGETRS_64(TRANSA, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 TRANSA
INTEGER*8 N, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE GETRS([TRANSA], [N], [NRHS], A, [LDA], IPIVOT, B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: TRANSA
INTEGER :: N, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE GETRS_64([TRANSA], [N], [NRHS], A, [LDA], IPIVOT, B, [LDB],
       [INFO])
```

```
CHARACTER(LEN=1) :: TRANSA
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dgetrs(char transa, int n, int  nrhs,  double  *a,  int
          lda, int *ipivot, double *b, int ldb, int *info);

void dgetrs_64(char transa, long n, long  nrhs,  double  *a,
          long  lda, long *ipivot, double *b, long ldb, long
          *info);
```

PURPOSE

dgetrs solves a system of linear equations
   A * X = B or A' * X = B with a general N-by-N matrix A
using the LU factorization computed by SGETRF.

ARGUMENTS

TRANSA (input)
Specifies the form of the system of equations:
= 'N': A * X = B (No transpose)
= 'T': A'* X = B (Transpose)
= 'C': A'* X = B (Conjugate transpose = Transpose)

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The factors L and U from the factorization A = P*L*U as computed by SGETRF.

LDA (input)
The leading dimension of the array A.   LDA >= max(1,N).

IPIVOT (input)
The pivot indices from SGETRF; for 1<=i<=N, row i

```
          of the matrix was interchanged with row IPIVOT(i).

B (input/output)
          On entry, the right hand side matrix B.  On  exit,
          the solution matrix X.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
dggbak - form the right or left eigenvectors of a real  gen-
eralized  eigenvalue  problem  A*x = lambda*B*x, by backward
transformation on the computed eigenvectors of the  balanced
pair of matrices output by SGGBAL
```

SYNOPSIS

```
SUBROUTINE DGGBAK(JOB, SIDE, N, ILO, IHI, LSCALE, RSCALE, M, V, LDV,
      INFO)

CHARACTER * 1 JOB, SIDE
INTEGER N, ILO, IHI, M, LDV, INFO
DOUBLE PRECISION LSCALE(*), RSCALE(*), V(LDV,*)

SUBROUTINE DGGBAK_64(JOB, SIDE, N, ILO, IHI, LSCALE, RSCALE, M, V,
      LDV, INFO)

CHARACTER * 1 JOB, SIDE
INTEGER*8 N, ILO, IHI, M, LDV, INFO
DOUBLE PRECISION LSCALE(*), RSCALE(*), V(LDV,*)
```

F95 INTERFACE

```
SUBROUTINE GGBAK(JOB, SIDE, [N], ILO, IHI, LSCALE, RSCALE, [M], V,
       [LDV], [INFO])

CHARACTER(LEN=1) :: JOB, SIDE
INTEGER :: N, ILO, IHI, M, LDV, INFO
REAL(8), DIMENSION(:) :: LSCALE, RSCALE
REAL(8), DIMENSION(:,:) :: V

SUBROUTINE GGBAK_64(JOB, SIDE, [N], ILO, IHI, LSCALE, RSCALE, [M], V,
```

```
          [LDV], [INFO])

   CHARACTER(LEN=1) :: JOB, SIDE
   INTEGER(8) :: N, ILO, IHI, M, LDV, INFO
   REAL(8), DIMENSION(:) :: LSCALE, RSCALE
   REAL(8), DIMENSION(:,:) :: V
```

C INTERFACE

```
#include <sunperf.h>

void dggbak(char job, char side, int n, int  ilo,  int  ihi,
          double  *lscale, double *rscale, int m, double *v,
          int ldv, int *info);

void dggbak_64(char job, char side, long n, long  ilo,  long
          ihi,  double  *lscale,  double  *rscale,  long  m,
          double *v, long ldv, long *info);
```

PURPOSE

dggbak forms the right or left eigenvectors of a real generalized eigenvalue problem A*x = lambda*B*x, by backward transformation on the computed eigenvectors of the balanced pair of matrices output by SGGBAL.

ARGUMENTS

```
JOB (input)
          Specifies  the  type  of  backward  transformation
          required:
          = 'N':  do nothing, return immediately;
          = 'P':  do backward transformation for permutation
          only;
          = 'S':  do  backward  transformation  for  scaling
          only;
          = 'B':  do backward transformations for both  per-
          mutation and scaling.  JOB must be the same as the
          argument JOB supplied to SGGBAL.

SIDE (input)
          = 'R':  V contains right eigenvectors;
          = 'L':  V contains left eigenvectors.

N (input) The number of rows of the matrix V.  N >= 0.

ILO (input)
          The integers ILO and IHI determined by SGGBAL.   1
```

<= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

IHI (input)
See the description for ILO.

LSCALE (input)
Details of the permutations and/or scaling factors applied to the left side of A and B, as returned by SGGBAL.

RSCALE (input)
Details of the permutations and/or scaling factors applied to the right side of A and B, as returned by SGGBAL.

M (input) The number of columns of the matrix V. M >= 0.

V (input/output)
On entry, the matrix of right or left eigenvectors to be transformed, as returned by STGEVC. On exit, V is overwritten by the transformed eigen- vectors.

LDV (input)
The leading dimension of the matrix V. LDV >= max(1,N).

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an ille- gal value.

FURTHER DETAILS

See R.C. Ward, Balancing the generalized eigenvalue problem,
SIAM J. Sci. Stat. Comp. 2 (1981), 141-152.

Contents

NAME

```
dggbal - balance a pair of general real matrices (A,B)
```

SYNOPSIS

```
SUBROUTINE DGGBAL(JOB, N, A, LDA, B, LDB, ILO, IHI, LSCALE, RSCALE,
      WORK, INFO)

CHARACTER * 1 JOB
INTEGER N, LDA, LDB, ILO, IHI, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*),  LSCALE(*),  RSCALE(*),
WORK(*)

SUBROUTINE DGGBAL_64(JOB, N, A, LDA, B, LDB, ILO, IHI, LSCALE,
      RSCALE, WORK, INFO)

CHARACTER * 1 JOB
INTEGER*8 N, LDA, LDB, ILO, IHI, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*),  LSCALE(*),  RSCALE(*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGBAL(JOB, [N], A, [LDA], B, [LDB], ILO, IHI, LSCALE,
       RSCALE, [WORK], [INFO])

CHARACTER(LEN=1) :: JOB
INTEGER :: N, LDA, LDB, ILO, IHI, INFO
REAL(8), DIMENSION(:) :: LSCALE, RSCALE, WORK
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE GGBAL_64(JOB, [N], A, [LDA], B, [LDB], ILO, IHI, LSCALE,
       RSCALE, [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOB
INTEGER(8) :: N, LDA, LDB, ILO, IHI, INFO
REAL(8), DIMENSION(:) :: LSCALE, RSCALE, WORK
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dggbal(char job, int n, double *a, int lda, double  *b,
          int  ldb, int *ilo, int *ihi, double *lscale, dou-
          ble *rscale, int *info);

void dggbal_64(char job, long n, double *a, long lda, double
          *b,   long  ldb,  long  *ilo,  long  *ihi,  double
          *lscale, double *rscale, long *info);
```

PURPOSE

dggbal balances a pair of general real matrices (A,B). This involves, first, permuting A and B by similarity transformations to isolate eigenvalues in the first 1 to ILO$-$1 and last IHI+1 to N elements on the diagonal; and second, applying a diagonal similarity transformation to rows and columns ILO to IHI to make the rows and columns as close in norm as possible. Both steps are optional.

Balancing may reduce the 1-norm of the matrices, and improve the accuracy of the computed eigenvalues and/or eigenvectors in the generalized eigenvalue problem A*x = lambda*B*x.

ARGUMENTS

JOB (input)
Specifies the operations to be performed on A and B:
= 'N': none: simply set ILO = 1, IHI = N, LSCALE(I) = 1.0 and RSCALE(I) = 1.0 for i = 1,...,N. = 'P': permute only;
= 'S': scale only;
= 'B': both permute and scale.

N (input) The order of the matrices A and B. N >= 0.

A (input/output)
On entry, the input matrix A. On exit, A is overwritten by the balanced matrix. If JOB = 'N',

```
          A is not referenced.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,N).

B (input) On entry, the input matrix  B.   On  exit,   B  is
          overwritten by the balanced matrix.  If JOB = 'N',
          B is not referenced.

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          max(1,N).

ILO (output)
          ILO and IHI are set to integers such that on  exit
          A(i,j)  =  0  and  B(i,j)  =  0  if  i > j and j =
          1,...,ILO-1 or i = IHI+1,...,N.  If JOB =  'N'  or
          'S', ILO = 1 and IHI = N.

IHI (output)
          See the description for ILO.

LSCALE (input)
          Details of the permutations  and  scaling  factors
          applied  to  the left side of A and B.  If P(j) is
          the index of the row interchanged with row j,  and
          D(j)  is the scaling factor applied to row j, then
          LSCALE(j) = P(j)    for J  =  1,...,ILO-1  =  D(j)
          for J = ILO,...,IHI = P(j)    for J = IHI+1,...,N.
          The order in which the interchanges are made is  N
          to IHI+1, then 1 to ILO-1.

RSCALE (input)
          Details of the permutations  and  scaling  factors
          applied  to the right side of A and B.  If P(j) is
          the index of the column interchanged  with  column
          j,  and  D(j)  is  the  scaling  factor applied to
          column  j,  then  LSCALE(j)  =  P(j)     for  J  =
          1,...,ILO-1  =  D(j)    for J = ILO,...,IHI = P(j)
          for J =  IHI+1,...,N.   The  order  in  which  the
          interchanges  are  made  is  N to IHI+1, then 1 to
          ILO-1.

WORK (workspace)
          dimension(6*N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
```

gal value.

FURTHER DETAILS

```
See R.C. WARD, Balancing the generalized eigenvalue problem,
               SIAM J. Sci. Stat. Comp. 2 (1981), 141-152.
```

Contents

NAME

```
dgges - compute for  a  pair  of  N-by-N  real  nonsymmetric
matrices (A,B),
```

SYNOPSIS

```
SUBROUTINE DGGES(JOBVSL, JOBVSR, SORT, DELCTG, N, A, LDA, B, LDB,
      SDIM, ALPHAR, ALPHAI, BETA, VSL, LDVSL, VSR, LDVSR, WORK, LWORK,
      BWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR, SORT
INTEGER N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, INFO
LOGICAL DELCTG
LOGICAL BWORK(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*),  ALPHAR(*),  ALPHAI(*),
BETA(*), VSL(LDVSL,*), VSR(LDVSR,*), WORK(*)

SUBROUTINE DGGES_64(JOBVSL, JOBVSR, SORT, DELCTG, N, A, LDA, B, LDB,
      SDIM, ALPHAR, ALPHAI, BETA, VSL, LDVSL, VSR, LDVSR, WORK, LWORK,
      BWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR, SORT
INTEGER*8 N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, INFO
LOGICAL*8 DELCTG
LOGICAL*8 BWORK(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*),  ALPHAR(*),  ALPHAI(*),
BETA(*), VSL(LDVSL,*), VSR(LDVSR,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGES(JOBVSL, JOBVSR, SORT, [DELCTG], [N], A, [LDA], B, [LDB],
       SDIM, ALPHAR, ALPHAI, BETA, VSL, [LDVSL], VSR, [LDVSR], [WORK],
       [LWORK], [BWORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOBVSL, JOBVSR, SORT
INTEGER :: N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, INFO
LOGICAL :: DELCTG
LOGICAL, DIMENSION(:) :: BWORK
REAL(8), DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
REAL(8), DIMENSION(:,:) :: A, B, VSL, VSR

SUBROUTINE GGES_64(JOBVSL, JOBVSR, SORT, [DELCTG], [N], A, [LDA], B,
       [LDB], SDIM, ALPHAR, ALPHAI, BETA, VSL, [LDVSL], VSR, [LDVSR],
       [WORK], [LWORK], [BWORK], [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR, SORT
INTEGER(8) :: N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, INFO
LOGICAL(8) :: DELCTG
LOGICAL(8), DIMENSION(:) :: BWORK
REAL(8), DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
REAL(8), DIMENSION(:,:) :: A, B, VSL, VSR
```

C INTERFACE

```
#include <sunperf.h>

void dgges(char jobvsl, char jobvsr, char sort,
          int(*delctg)(double,double,double), int n, double
          *a, int lda, double *b, int ldb, int *sdim, double
          *alphar, double *alphai, double *beta, double
          *vsl, int ldvsl, double *vsr, int ldvsr, int
          *info);

void dgges_64(char jobvsl, char jobvsr, char sort,
          long(*delctg)(double,double,double), long n, dou-
          ble *a, long lda, double *b, long ldb, long *sdim,
          double *alphar, double *alphai, double *beta, dou-
          ble *vsl, long ldvsl, double *vsr, long ldvsr,
          long *info);
```

PURPOSE

dgges computes for a pair of N-by-N real nonsymmetric matrices (A,B), the generalized eigenvalues, the generalized real Schur form (S,T), optionally, the left and/or right matrices of Schur vectors (VSL and VSR). This gives the generalized Schur factorization

(A,B) = ((VSL)*S*(VSR)**T, (VSL)*T*(VSR)**T)

Optionally, it also orders the eigenvalues so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper quasi-triangular matrix S and the upper triangular matrix T.The leading columns of VSL and VSR then form an orthonormal basis for the corresponding left and right eigenspaces (deflating subspaces).

(If only the generalized eigenvalues are needed, use the driver SGGEV instead, which is faster.)

A generalized eigenvalue for a pair of matrices (A,B) is a scalar w or a ratio alpha/beta = w, such that A - w*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0 or both being zero.

A pair of matrices (S,T) is in generalized real Schur form if T is upper triangular with non-negative diagonal and S is block upper triangular with 1-by-1 and 2-by-2 blocks. 1-by-1 blocks correspond to real generalized eigenvalues, while 2-by-2 blocks of S will be "standardized" by making the corresponding elements of T have the form:

```
        [  a  0  ]
        [  0  b  ]
```

and the pair of corresponding 2-by-2 blocks in S and T will have a complex conjugate pair of generalized eigenvalues.

ARGUMENTS

JOBVSL (input)
- = 'N': do not compute the left Schur vectors;
- = 'V': compute the left Schur vectors.

JOBVSR (input)
- = 'N': do not compute the right Schur vectors;
- = 'V': compute the right Schur vectors.

SORT (input)
Specifies whether or not to order the eigenvalues on the diagonal of the generalized Schur form. = 'N': Eigenvalues are not ordered;
= 'S': Eigenvalues are ordered (see DELCTG);

DELCTG (input)
DELCTG must be declared EXTERNAL in the calling subroutine. If SORT = 'N', DELCTG is not referenced. If SORT = 'S', DELCTG is used to select eigenvalues to sort to the top left of the Schur form. An eigenvalue (ALPHAR(j)+ALPHAI(j))/BETA(j) is selected if DELCTG(ALPHAR(j),ALPHAI(j),BETA(j)) is true; i.e. if either one of a complex conjugate pair of eigenvalues is selected, then both complex eigenvalues are selected.

Note that in the ill-conditioned case, a selected complex eigenvalue may no longer satisfy DELCTG(ALPHAR(j),ALPHAI(j), BETA(j)) = .TRUE. after ordering. INFO is to be set to N+2 in this case.

N (input) The order of the matrices A, B, VSL, and VSR. N >= 0.

A (input/output)
On entry, the first of the pair of matrices. On exit, A has been overwritten by its generalized Schur form S.

LDA (input)
The leading dimension of A. LDA >= max(1,N).

B (input/output)
On entry, the second of the pair of matrices. On exit, B has been overwritten by its generalized Schur form T.

LDB (input)
The leading dimension of B. LDB >= max(1,N).

SDIM (output)
If SORT = 'N', SDIM = 0. If SORT = 'S', SDIM = number of eigenvalues (after sorting) for which DELCTG is true. (Complex conjugate pairs for which DELCTG is true for either eigenvalue count as 2.)

ALPHAR (output)
On exit, (ALPHAR(j) + ALPHAI(j)*i)/BETA(j), j=1,...,N, will be the generalized eigenvalues. ALPHAR(j) + ALPHAI(j)*i, and BETA(j),j=1,...,N are the diagonals of the complex Schur form (S,T) that would result if the 2-by-2 diagonal blocks of the real Schur form of (A,B) were further reduced to triangular form using 2-by-2 complex unitary transformations. If ALPHAI(j) is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-st eigenvalues are a complex conjugate pair, with ALPHAI(j+1) negative.

Note: the quotients ALPHAR(j)/BETA(j) and ALPHAI(j)/BETA(j) may easily over- or underflow, and BETA(j) may even be zero. Thus, the user should avoid naively computing the ratio. However, ALPHAR and ALPHAI will be always less than and usually comparable with norm(A) in magnitude, and BETA always less than and usually comparable with norm(B).

ALPHAI (output)
See the description for ALPHAR.

BETA (output)
See the description for ALPHAR.

VSL (input)
If JOBVSL = 'V', VSL will contain the left Schur vectors. Not referenced if JOBVSL = 'N'.

LDVSL (input)
The leading dimension of the matrix VSL. LDVSL >=1, and if JOBVSL = 'V', LDVSL >= N.

VSR (input)
If JOBVSR = 'V', VSR will contain the right Schur vectors. Not referenced if JOBVSR = 'N'.

LDVSR (input)
The leading dimension of the matrix VSR. LDVSR >= 1, and if JOBVSR = 'V', LDVSR >= N.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 8*N+16.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

BWORK (workspace)
dimension(N) Not referenced if SORT = 'N'.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
= 1,...,N: The QZ iteration failed. (A,B) are not in Schur form, but ALPHAR(j), ALPHAI(j), and BETA(j) should be correct for j=INFO+1,...,N. > N: =N+1: other than QZ iteration failed in SHGEQZ.

=N+2: after reordering, roundoff changed values of some complex eigenvalues so that leading eigen- values in the Generalized Schur form no longer satisfy DELCTG=.TRUE. This could also be caused due to scaling. =N+3: reordering failed in STGSEN.

Contents

NAME

```
dggesx - compute for a  pair  of  N-by-N  real  nonsymmetric
matrices  (A,B), the generalized eigenvalues, the real Schur
form (S,T), and,
```

SYNOPSIS

```
SUBROUTINE DGGESX(JOBVSL, JOBVSR, SORT, DELCTG, SENSE, N, A, LDA, B,
      LDB, SDIM, ALPHAR, ALPHAI, BETA, VSL, LDVSL, VSR, LDVSR, RCONDE,
      RCONDV, WORK, LWORK, IWORK, LIWORK, BWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR, SORT, SENSE
INTEGER N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, LIWORK, INFO
INTEGER IWORK(*)
LOGICAL DELCTG
LOGICAL BWORK(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*),  ALPHAR(*),  ALPHAI(*),
BETA(*),  VSL(LDVSL,*),  VSR(LDVSR,*), RCONDE(*), RCONDV(*),
WORK(*)

SUBROUTINE DGGESX_64(JOBVSL, JOBVSR, SORT, DELCTG, SENSE, N, A, LDA,
      B, LDB, SDIM, ALPHAR, ALPHAI, BETA, VSL, LDVSL, VSR, LDVSR,
      RCONDE, RCONDV, WORK, LWORK, IWORK, LIWORK, BWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR, SORT, SENSE
INTEGER*8 N, LDA, LDB, SDIM, LDVSL,  LDVSR,  LWORK,  LIWORK,
INFO
INTEGER*8 IWORK(*)
LOGICAL*8 DELCTG
LOGICAL*8 BWORK(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*),  ALPHAR(*),  ALPHAI(*),
BETA(*),  VSL(LDVSL,*),  VSR(LDVSR,*), RCONDE(*), RCONDV(*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGESX(JOBVSL, JOBVSR, SORT, [DELCTG], SENSE, [N], A, [LDA],
       B, [LDB], SDIM, ALPHAR, ALPHAI, BETA, VSL, [LDVSL], VSR, [LDVSR],
       RCONDE, RCONDV, [WORK], [LWORK], [IWORK], [LIWORK], [BWORK],
       [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR, SORT, SENSE
INTEGER :: N, LDA, LDB, SDIM, LDVSL, LDVSR,  LWORK,  LIWORK,
INFO
INTEGER, DIMENSION(:) :: IWORK
LOGICAL :: DELCTG
LOGICAL, DIMENSION(:) :: BWORK
REAL(8),  DIMENSION(:)  ::  ALPHAR,  ALPHAI,  BETA,  RCONDE,
RCONDV, WORK
REAL(8), DIMENSION(:,:) :: A, B, VSL, VSR
SUBROUTINE GGESX_64(JOBVSL, JOBVSR, SORT, [DELCTG], SENSE, [N], A, [LDA],
       B, [LDB], SDIM, ALPHAR, ALPHAI, BETA, VSL, [LDVSL], VSR, [LDVSR],
       RCONDE, RCONDV, [WORK], [LWORK], [IWORK], [LIWORK], [BWORK],
       [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR, SORT, SENSE
INTEGER(8) ::  N,  LDA,  LDB,  SDIM,  LDVSL,  LDVSR,  LWORK,
LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
LOGICAL(8) :: DELCTG
LOGICAL(8), DIMENSION(:) :: BWORK
REAL(8),  DIMENSION(:)  ::  ALPHAR,  ALPHAI,  BETA,  RCONDE,
RCONDV, WORK
REAL(8), DIMENSION(:,:) :: A, B, VSL, VSR
```

C INTERFACE

```
#include <sunperf.h>

void   dggesx(char   jobvsl,   char   jobvsr,   char   sort,
          int(*delctg)(double,double,double),   char  sense,
          int n, double *a, int lda, double *b, int ldb, int
          *sdim,  double  *alphar,  double  *alphai,  double
          *beta, double *vsl, int ldvsl,  double  *vsr,  int
          ldvsr, double *rconde, double *rcondv, int *info);

void  dggesx_64(char  jobvsl,  char   jobvsr,   char   sort,
          long(*delctg)(double,double,double),  char  sense,
          long n, double *a, long lda, double *b, long  ldb,
          long *sdim, double *alphar, double *alphai, double
          *beta, double *vsl, long ldvsl, double *vsr,  long
          ldvsr,   double   *rconde,  double  *rcondv,  long
          *info);
```

PURPOSE

dggesx computes for a pair of N-by-N real nonsymmetric matrices (A,B), the generalized eigenvalues, the real Schur form (S,T), and, optionally, the left and/or right matrices of Schur vectors (VSL and VSR). This gives the generalized Schur factorization
A,B) = ((VSL) S (VSR)**T, (VSL) T (VSR)**T)

Optionally, it also orders the eigenvalues so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper quasi-triangular matrix S and the upper triangular matrix T; computes a reciprocal condition number for the average of the selected eigenvalues (RCONDE); and computes a reciprocal condition number for the right and left deflating subspaces corresponding to the selected eigenvalues (RCONDV). The leading columns of VSL and VSR then form an orthonormal basis for the corresponding left and right eigenspaces (deflating subspaces).

A generalized eigenvalue for a pair of matrices (A,B) is a scalar w or a ratio alpha/beta = w, such that A - w*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0 or for both being zero.

A pair of matrices (S,T) is in generalized real Schur form if T is upper triangular with non-negative diagonal and S is block upper triangular with 1-by-1 and 2-by-2 blocks. 1-by-1 blocks correspond to real generalized eigenvalues, while 2-by-2 blocks of S will be "standardized" by making the corresponding elements of T have the form:

```
        [  a  0  ]
        [  0  b  ]
```

and the pair of corresponding 2-by-2 blocks in S and T will have a complex conjugate pair of generalized eigenvalues.

ARGUMENTS

JOBVSL (input)
= 'N': do not compute the left Schur vectors;
= 'V': compute the left Schur vectors.

JOBVSR (input)
= 'N': do not compute the right Schur vectors;
= 'V': compute the right Schur vectors.

SORT (input)
Specifies whether or not to order the eigenvalues on the diagonal of the generalized Schur form. = 'N': Eigenvalues are not ordered;
= 'S': Eigenvalues are ordered (see DELCTG).

DELCTG (input)
DELCTG must be declared EXTERNAL in the calling subroutine. If SORT = 'N', DELCTG is not referenced. If SORT = 'S', DELCTG is used to select eigenvalues to sort to the top left of the Schur form. An eigenvalue (ALPHAR(j)+ALPHAI(j))/BETA(j) is selected if DELCTG(ALPHAR(j),ALPHAI(j),BETA(j)) is true; i.e. if either one of a complex conjugate pair of eigenvalues is selected, then both complex eigenvalues are selected. Note that a selected complex eigenvalue may no longer satisfy DELCTG(ALPHAR(j),ALPHAI(j),BETA(j)) = .TRUE. after ordering, since ordering may change the value of complex eigenvalues (especially if the eigenvalue is ill-conditioned), in this case INFO is set to N+3.

SENSE (input)
Determines which reciprocal condition numbers are computed. = 'N' : None are computed;
= 'E' : Computed for average of selected eigenvalues only;
= 'V' : Computed for selected deflating subspaces only;
= 'B' : Computed for both. If SENSE = 'E', 'V', or 'B', SORT must equal 'S'.

N (input) The order of the matrices A, B, VSL, and VSR. N >= 0.

A (input/output)
On entry, the first of the pair of matrices. On exit, A has been overwritten by its generalized Schur form S.

LDA (input)
The leading dimension of A. LDA >= max(1,N).

B (input/output)
On entry, the second of the pair of matrices. On exit, B has been overwritten by its generalized Schur form T.

LDB (input)

The leading dimension of B. LDB >= max(1,N).

SDIM (output)
If SORT = 'N', SDIM = 0. If SORT = 'S', SDIM = number of eigenvalues (after sorting) for which DELCTG is true. (Complex conjugate pairs for which DELCTG is true for either eigenvalue count as 2.)

ALPHAR (output)
On exit, (ALPHAR(j) + ALPHAI(j)*i)/BETA(j), j=1,...,N, will be the generalized eigenvalues. ALPHAR(j) + ALPHAI(j)*i and BETA(j),j=1,...,N are the diagonals of the complex Schur form (S,T) that would result if the 2-by-2 diagonal blocks of the real Schur form of (A,B) were further reduced to triangular form using 2-by-2 complex unitary transformations. If ALPHAI(j) is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-st eigenvalues are a complex conjugate pair, with ALPHAI(j+1) negative.

Note: the quotients ALPHAR(j)/BETA(j) and ALPHAI(j)/BETA(j) may easily over- or underflow, and BETA(j) may even be zero. Thus, the user should avoid naively computing the ratio. However, ALPHAR and ALPHAI will be always less than and usually comparable with norm(A) in magnitude, and BETA always less than and usually comparable with norm(B).

ALPHAI (output)
See the description for ALPHAR.

BETA (output)
See the description for ALPHAR.

VSL (input)
If JOBVSL = 'V', VSL will contain the left Schur vectors. Not referenced if JOBVSL = 'N'.

LDVSL (input)
The leading dimension of the matrix VSL. LDVSL >=1, and if JOBVSL = 'V', LDVSL >= N.

VSR (input)
If JOBVSR = 'V', VSR will contain the right Schur vectors. Not referenced if JOBVSR = 'N'.

LDVSR (input)

The leading dimension of the matrix VSR. LDVSR >= 1, and if JOBVSR = 'V', LDVSR >= N.

RCONDE (output)

If SENSE = 'E' or 'B', RCONDE(1) and RCONDE(2) contain the reciprocal condition numbers for the average of the selected eigenvalues. Not referenced if SENSE = 'N' or 'V'.

RCONDV (output)

If SENSE = 'V' or 'B', RCONDV(1) and RCONDV(2) contain the reciprocal condition numbers for the selected deflating subspaces. Not referenced if SENSE = 'N' or 'E'.

WORK (workspace)

On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)

The dimension of the array WORK. LWORK >= 8*(N+1)+16. If SENSE = 'E', 'V', or 'B', LWORK >= MAX(8*(N+1)+16, 2*SDIM*(N-SDIM)).

IWORK (workspace)

Not referenced if SENSE = 'N'.

LIWORK (input)

The dimension of the array WORK. LIWORK >= N+6.

BWORK (workspace)

dimension(N) Not referenced if SORT = 'N'.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value.

= 1,...,N: The QZ iteration failed. (A,B) are not in Schur form, but ALPHAR(j), ALPHAI(j), and BETA(j) should be correct for j=INFO+1,...,N. > N: =N+1: other than QZ iteration failed in SHGEQZ =N+2: after reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the Generalized Schur form no longer satisfy DELCTG=.TRUE. This could also be caused due to scaling. =N+3: reordering failed in STGSEN.

Further details ===============

An approximate (asymptotic) bound on the average absolute error of the selected eigenvalues is

EPS * norm((A, B)) / RCONDE(1).

An approximate (asymptotic) bound on the maximum angular error in the computed deflating subspaces is

EPS * norm((A, B)) / RCONDV(2).

See LAPACK User's Guide, section 4.11 for more information.

Contents

NAME

```
dggev - compute for  a  pair  of  N-by-N  real  nonsymmetric
matrices (A,B)
```

SYNOPSIS

```
SUBROUTINE DGGEV(JOBVL, JOBVR, N, A, LDA, B, LDB, ALPHAR, ALPHAI,
      BETA, VL, LDVL, VR, LDVR, WORK, LWORK, INFO)

CHARACTER * 1 JOBVL, JOBVR
INTEGER N, LDA, LDB, LDVL, LDVR, LWORK, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*),  ALPHAR(*),  ALPHAI(*),
BETA(*), VL(LDVL,*), VR(LDVR,*), WORK(*)

SUBROUTINE DGGEV_64(JOBVL, JOBVR, N, A, LDA, B, LDB, ALPHAR, ALPHAI,
      BETA, VL, LDVL, VR, LDVR, WORK, LWORK, INFO)

CHARACTER * 1 JOBVL, JOBVR
INTEGER*8 N, LDA, LDB, LDVL, LDVR, LWORK, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*),  ALPHAR(*),  ALPHAI(*),
BETA(*), VL(LDVL,*), VR(LDVR,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGEV(JOBVL, JOBVR, [N], A, [LDA], B, [LDB], ALPHAR,
       ALPHAI, BETA, VL, [LDVL], VR, [LDVR], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: JOBVL, JOBVR
INTEGER :: N, LDA, LDB, LDVL, LDVR, LWORK, INFO
REAL(8), DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
REAL(8), DIMENSION(:,:) :: A, B, VL, VR

SUBROUTINE GGEV_64(JOBVL, JOBVR, [N], A, [LDA], B, [LDB], ALPHAR,
       ALPHAI, BETA, VL, [LDVL], VR, [LDVR], [WORK], [LWORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOBVL, JOBVR
INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, LWORK, INFO
REAL(8), DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
REAL(8), DIMENSION(:,:) :: A, B, VL, VR
```

C INTERFACE

```
#include <sunperf.h>

void dggev(char jobvl, char jobvr, int  n,  double  *a,  int
          lda,  double  *b,  int ldb, double *alphar, double
          *alphai, double *beta, double *vl, int ldvl,  dou-
          ble *vr, int ldvr, int *info);
void dggev_64(char jobvl, char jobvr,  long  n,  double  *a,
          long  lda,  double  *b,  long ldb, double *alphar,
          double *alphai, double  *beta,  double  *vl,  long
          ldvl, double *vr, long ldvr, long *info);
```

PURPOSE

dggev computes for a pair of N-by-N real nonsymmetric matrices (A,B) the generalized eigenvalues, and optionally, the left and/or right generalized eigenvectors.

A generalized eigenvalue for a pair of matrices (A,B) is a scalar lambda or a ratio alpha/beta = lambda, such that A - lambda*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0, and even for both being zero.

The right eigenvector v(j) corresponding to the eigenvalue lambda(j) of (A,B) satisfies

A * v(j) = lambda(j) * B * v(j).

The left eigenvector u(j) corresponding to the eigenvalue lambda(j) of (A,B) satisfies

u(j)**H * A = lambda(j) * u(j)**H * B .

where u(j)**H is the conjugate-transpose of u(j).

ARGUMENTS

JOBVL (input)
= 'N': do not compute the left generalized eigenvectors;

= 'V': compute the left generalized eigenvectors.

JOBVR (input)
= 'N': do not compute the right generalized eigenvectors;
= 'V': compute the right generalized eigenvectors.

N (input) The order of the matrices A, B, VL, and VR. N >= 0.

A (input/output)
On entry, the matrix A in the pair (A,B). On exit, A has been overwritten.

LDA (input)
The leading dimension of A. LDA >= max(1,N).

B (input/output)
On entry, the matrix B in the pair (A,B). On exit, B has been overwritten.

LDB (input)
The leading dimension of B. LDB >= max(1,N).

ALPHAR (output)
On exit, (ALPHAR(j) + ALPHAI(j)*i)/BETA(j), j=1,...,N, will be the generalized eigenvalues. If ALPHAI(j) is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-st eigenvalues are a complex conjugate pair, with ALPHAI(j+1) negative.

Note: the quotients ALPHAR(j)/BETA(j) and ALPHAI(j)/BETA(j) may easily over- or underflow, and BETA(j) may even be zero. Thus, the user should avoid naively computing the ratio alpha/beta. However, ALPHAR and ALPHAI will be always less than and usually comparable with norm(A) in magnitude, and BETA always less than and usually comparable with norm(B).

ALPHAI (output)
See the description for ALPHAR.

BETA (output)
See the description for ALPHAR.

VL (input)
If JOBVL = 'V', the left eigenvectors u(j) are

stored one after another in the columns of VL, in the same order as their eigenvalues. If the j-th eigenvalue is real, then u(j) = VL(:,j), the j-th column of VL. If the j-th and (j+1)-th eigenvalues form a complex conjugate pair, then u(j) = VL(:,j)+i*VL(:,j+1) and u(j+1) = VL(:,j)-i*VL(:,j+1). Each eigenvector will be scaled so the largest component have abs(real part)+abs(imag. part)=1. Not referenced if JOBVL = 'N'.

LDVL (input)
The leading dimension of the matrix VL. LDVL >= 1, and if JOBVL = 'V', LDVL >= N.

VR (input)
If JOBVR = 'V', the right eigenvectors v(j) are stored one after another in the columns of VR, in the same order as their eigenvalues. If the j-th eigenvalue is real, then v(j) = VR(:,j), the j-th column of VR. If the j-th and (j+1)-th eigenvalues form a complex conjugate pair, then v(j) = VR(:,j)+i*VR(:,j+1) and v(j+1) = VR(:,j)-i*VR(:,j+1). Each eigenvector will be scaled so the largest component have abs(real part)+abs(imag. part)=1. Not referenced if JOBVR = 'N'.

LDVR (input)
The leading dimension of the matrix VR. LDVR >= 1, and if JOBVR = 'V', LDVR >= N.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,8*N). For good performance, LWORK must generally be larger.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-

gal value.
= 1,...,N: The QZ iteration failed. No eigenvectors have been calculated, but ALPHAR(j), ALPHAI(j), and BETA(j) should be correct for j=INFO+1,...,N. > N: =N+1: other than QZ iteration failed in SHGEQZ.
=N+2: error return from STGEVC.

Contents

NAME

dggevx - compute for a pair of N-by-N real nonsymmetric matrices (A,B)

SYNOPSIS

```
SUBROUTINE DGGEVX(BALANC, JOBVL, JOBVR, SENSE, N, A, LDA, B, LDB,
      ALPHAR, ALPHAI, BETA, VL, LDVL, VR, LDVR, ILO, IHI, LSCALE,
      RSCALE, ABNRM, BBNRM, RCONDE, RCONDV, WORK, LWORK, IWORK, BWORK,
      INFO)

CHARACTER * 1 BALANC, JOBVL, JOBVR, SENSE
INTEGER N, LDA, LDB, LDVL, LDVR, ILO, IHI, LWORK, INFO
INTEGER IWORK(*)
LOGICAL BWORK(*)
DOUBLE PRECISION ABNRM, BBNRM
DOUBLE PRECISION A(LDA,*), B(LDB,*),  ALPHAR(*),  ALPHAI(*),
BETA(*),   VL(LDVL,*),   VR(LDVR,*),  LSCALE(*),  RSCALE(*),
RCONDE(*), RCONDV(*), WORK(*)

SUBROUTINE DGGEVX_64(BALANC, JOBVL, JOBVR, SENSE, N, A, LDA, B, LDB,
      ALPHAR, ALPHAI, BETA, VL, LDVL, VR, LDVR, ILO, IHI, LSCALE,
      RSCALE, ABNRM, BBNRM, RCONDE, RCONDV, WORK, LWORK, IWORK, BWORK,
      INFO)

CHARACTER * 1 BALANC, JOBVL, JOBVR, SENSE
INTEGER*8 N, LDA, LDB, LDVL, LDVR, ILO, IHI, LWORK, INFO
INTEGER*8 IWORK(*)
LOGICAL*8 BWORK(*)
DOUBLE PRECISION ABNRM, BBNRM
DOUBLE PRECISION A(LDA,*), B(LDB,*),  ALPHAR(*),  ALPHAI(*),
BETA(*),   VL(LDVL,*),   VR(LDVR,*),  LSCALE(*),  RSCALE(*),
```

```
   RCONDE(*), RCONDV(*), WORK(*)
```

F95 INTERFACE

```
   SUBROUTINE GGEVX(BALANC, JOBVL, JOBVR, SENSE, [N], A, [LDA], B, [LDB],
          ALPHAR, ALPHAI, BETA, VL, [LDVL], VR, [LDVR], ILO, IHI, LSCALE,
          RSCALE, ABNRM, BBNRM, RCONDE, RCONDV, [WORK], [LWORK], [IWORK],
          [BWORK], [INFO])

   CHARACTER(LEN=1) :: BALANC, JOBVL, JOBVR, SENSE
   INTEGER :: N, LDA, LDB, LDVL, LDVR, ILO, IHI, LWORK, INFO
   INTEGER, DIMENSION(:) :: IWORK
   LOGICAL, DIMENSION(:) :: BWORK
   REAL(8) :: ABNRM, BBNRM
   REAL(8),  DIMENSION(:)  ::  ALPHAR,  ALPHAI,  BETA,  LSCALE,
   RSCALE, RCONDE, RCONDV, WORK
   REAL(8), DIMENSION(:,:) :: A, B, VL, VR
   SUBROUTINE GGEVX_64(BALANC, JOBVL, JOBVR, SENSE, [N], A, [LDA], B,
          [LDB], ALPHAR, ALPHAI, BETA, VL, [LDVL], VR, [LDVR], ILO, IHI,
          LSCALE, RSCALE, ABNRM, BBNRM, RCONDE, RCONDV, [WORK], [LWORK],
          [IWORK], [BWORK], [INFO])

   CHARACTER(LEN=1) :: BALANC, JOBVL, JOBVR, SENSE
   INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, ILO, IHI, LWORK, INFO
   INTEGER(8), DIMENSION(:) :: IWORK
   LOGICAL(8), DIMENSION(:) :: BWORK
   REAL(8) :: ABNRM, BBNRM
   REAL(8),  DIMENSION(:)  ::  ALPHAR,  ALPHAI,  BETA,  LSCALE,
   RSCALE, RCONDE, RCONDV, WORK
   REAL(8), DIMENSION(:,:) :: A, B, VL, VR
```

C INTERFACE

```
   #include <sunperf.h>

   void dggevx(char balanc, char jobvl, char jobvr, char sense,
             int  n,  double  *a,  int lda, double *b, int ldb,
             double *alphar, double *alphai, double *beta, dou-
             ble *vl, int ldvl, double *vr, int ldvr, int *ilo,
             int *ihi, double *lscale, double  *rscale,  double
             *abnrm,  double  *bbnrm,  double  *rconde,  double
             *rcondv, int *info);

   void dggevx_64(char balanc, char  jobvl,  char  jobvr,  char
             sense,  long  n,  double  *a, long lda, double *b,
             long ldb, double *alphar, double  *alphai,  double
             *beta,  double  *vl,  long  ldvl, double *vr, long
             ldvr, long *ilo, long *ihi, double *lscale, double
             *rscale,  double  *abnrm,  double  *bbnrm,  double
             *rconde, double *rcondv, long *info);
```

PURPOSE

dggevx computes for a pair of N-by-N real nonsymmetric matrices (A,B) the generalized eigenvalues, and optionally, the left and/or right generalized eigenvectors.

Optionally also, it computes a balancing transformation to improve the conditioning of the eigenvalues and eigenvectors (ILO, IHI, LSCALE, RSCALE, ABNRM, and BBNRM), reciprocal condition numbers for the eigenvalues (RCONDE), and reciprocal condition numbers for the right eigenvectors (RCONDV).

A generalized eigenvalue for a pair of matrices (A,B) is a scalar lambda or a ratio alpha/beta = lambda, such that A - lambda*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0, and even for both being zero.
The right eigenvector v(j) corresponding to the eigenvalue lambda(j) of (A,B) satisfies

```
A * v(j) = lambda(j) * B * v(j) .
```

The left eigenvector u(j) corresponding to the eigenvalue lambda(j) of (A,B) satisfies

```
u(j)**H * A  = lambda(j) * u(j)**H * B.
```

where u(j)**H is the conjugate-transpose of u(j).

ARGUMENTS

BALANC (input)
Specifies the balance option to be performed. = 'N': do not diagonally scale or permute;
= 'P': permute only;
= 'S': scale only;
= 'B': both permute and scale. Computed reciprocal condition numbers will be for the matrices after permuting and/or balancing. Permuting does not change condition numbers (in exact arithmetic), but balancing does.

JOBVL (input)
= 'N': do not compute the left generalized eigenvectors;
= 'V': compute the left generalized eigenvectors.

JOBVR (input)

```
          = 'N':   do  not  compute  the  right  generalized
          eigenvectors;
          = 'V':  compute the  right  generalized  eigenvec-
          tors.

SENSE (input)
          Determines which reciprocal condition numbers  are
          computed.  = 'N': none are computed;
          = 'E': computed for eigenvalues only;
          = 'V': computed for eigenvectors only;
          = 'B': computed for eigenvalues and eigenvectors.

N (input) The order of the matrices A, B, VL, and VR.  N  >=
          0.
A (input/output)
          On entry, the matrix A  in  the  pair  (A,B).   On
          exit,  A  has  been  overwritten.  If JOBVL='V' or
          JOBVR='V' or both, then A contains the first  part
          of  the real Schur form of the "balanced" versions
          of the input A and B.

LDA (input)
          The leading dimension of A.  LDA >= max(1,N).

B (input/output)
          On entry, the matrix B  in  the  pair  (A,B).   On
          exit,  B  has  been  overwritten.  If JOBVL='V' or
          JOBVR='V' or both, then B contains the second part
          of  the real Schur form of the "balanced" versions
          of the input A and B.

LDB (input)
          The leading dimension of B.  LDB >= max(1,N).

ALPHAR (output)
          On  exit,   (ALPHAR(j)   +   ALPHAI(j)*i)/BETA(j),
          j=1,...,N,  will  be  the generalized eigenvalues.
          If ALPHAI(j) is zero, then the j-th eigenvalue  is
          real;  if  positive,  then  the  j-th and (j+1)-st
          eigenvalues are a  complex  conjugate  pair,  with
          ALPHAI(j+1) negative.

          Note:   the   quotients   ALPHAR(j)/BETA(j)    and
          ALPHAI(j)/BETA(j)  may  easily over- or underflow,
          and BETA(j) may even  be  zero.   Thus,  the  user
          should   avoid   naively   computing   the   ratio
          ALPHA/BETA. However, ALPHAR  and  ALPHAI  will  be
          always  less  than  and  usually  comparable  with
          norm(A) in magnitude, and BETA  always  less  than
          and usually comparable with norm(B).
```

ALPHAI (output)
See the description of ALPHAR.

BETA (output)
See the description of ALPHAR.

VL (output)
If JOBVL = 'V', the left eigenvectors u(j) are stored one after another in the columns of VL, in the same order as their eigenvalues. If the j-th eigenvalue is real, then u(j) = VL(:,j), the j-th column of VL. If the j-th and (j+1)-th eigenvalues form a complex conjugate pair, then u(j) = VL(:,j)+i*VL(:,j+1) and u(j+1) = VL(:,j)-i*VL(:,j+1). Each eigenvector will be scaled so the largest component have abs(real part) + abs(imag. part) = 1. Not referenced if JOBVL = 'N'.

LDVL (input)
The leading dimension of the matrix VL. LDVL >= 1, and if JOBVL = 'V', LDVL >= N.

VR (output)
If JOBVR = 'V', the right eigenvectors v(j) are stored one after another in the columns of VR, in the same order as their eigenvalues. If the j-th eigenvalue is real, then v(j) = VR(:,j), the j-th column of VR. If the j-th and (j+1)-th eigenvalues form a complex conjugate pair, then v(j) = VR(:,j)+i*VR(:,j+1) and v(j+1) = VR(:,j)-i*VR(:,j+1). Each eigenvector will be scaled so the largest component have abs(real part) + abs(imag. part) = 1. Not referenced if JOBVR = 'N'.

LDVR (input)
The leading dimension of the matrix VR. LDVR >= 1, and if JOBVR = 'V', LDVR >= N.

ILO (output)
ILO and IHI are integer values such that on exit A(i,j) = 0 and B(i,j) = 0 if i > j and j = 1,...,ILO-1 or i = IHI+1,...,N. If BALANC = 'N' or 'S', ILO = 1 and IHI = N.

IHI (output)
See the description of ILO.

LSCALE (output)
Details of the permutations and scaling factors applied to the left side of A and B. If PL(j) is the index of the row interchanged with row j, and DL(j) is the scaling factor applied to row j, then LSCALE(j) = PL(j) for j = 1,...,ILO-1 = DL(j) for j = ILO,...,IHI = PL(j) for j = IHI+1,...,N. The order in which the interchanges are made is N to IHI+1, then 1 to ILO-1.

RSCALE (output)
Details of the permutations and scaling factors applied to the right side of A and B. If PR(j) is the index of the column interchanged with column j, and DR(j) is the scaling factor applied to column j, then RSCALE(j) = PR(j) for j = 1,...,ILO-1 = DR(j) for j = ILO,...,IHI = PR(j) for j = IHI+1,...,N The order in which the interchanges are made is N to IHI+1, then 1 to ILO-1.

ABNRM (output)
The one-norm of the balanced matrix A.

BBNRM (output)
The one-norm of the balanced matrix B.

RCONDE (output)
If SENSE = 'E' or 'B', the reciprocal condition numbers of the selected eigenvalues, stored in consecutive elements of the array. For a complex conjugate pair of eigenvalues two consecutive elements of RCONDE are set to the same value. Thus RCONDE(j), RCONDV(j), and the j-th columns of VL and VR all correspond to the same eigenpair (but not in general the j-th eigenpair, unless all eigenpairs are selected). If SENSE = 'V', RCONDE is not referenced.

RCONDV (output)
If SENSE = 'V' or 'B', the estimated reciprocal condition numbers of the selected eigenvectors, stored in consecutive elements of the array. For a complex eigenvector two consecutive elements of RCONDV are set to the same value. If the eigenvalues cannot be reordered to compute RCONDV(j), RCONDV(j) is set to 0; this can only occur when the true value would be very small anyway. If SENSE = 'E', RCONDV is not referenced.

WORK (workspace)

On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,6*N). If SENSE = 'E', LWORK >= 12*N. If SENSE = 'V' or 'B', LWORK >= 2*N*N+12*N+16.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace)
dimension(N+6) If SENSE = 'E', IWORK is not referenced.

BWORK (workspace)
dimension(N) If SENSE = 'N', BWORK is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
= 1,...,N: The QZ iteration failed. No eigenvectors have been calculated, but ALPHAR(j), ALPHAI(j), and BETA(j) should be correct for j=INFO+1,...,N. > N: =N+1: other than QZ iteration failed in SHGEQZ.
=N+2: error return from STGEVC.

FURTHER DETAILS

Balancing a matrix pair (A,B) includes, first, permuting rows and columns to isolate eigenvalues, second, applying diagonal similarity transformation to the rows and columns to make the rows and columns as close in norm as possible. The computed reciprocal condition numbers correspond to the balanced matrix. Permuting rows and columns will not change the condition numbers (in exact arithmetic) but diagonal scaling will. For further explanation of balancing, see section 4.11.1.2 of LAPACK Users' Guide.

An approximate error bound on the chordal distance between the i-th computed generalized eigenvalue w and the corresponding exact eigenvalue lambda is
hord(w, lambda) <= EPS * norm(ABNRM, BBNRM) / RCONDE(I)
An approximate error bound for the angle between the i-th

computed eigenvector VL(i) or VR(i) is given by
PS * norm(ABNRM, BBNRM) / DIF(i).

For further explanation of the reciprocal condition numbers
RCONDE and RCONDV, see section 4.11 of LAPACK User's Guide.

Contents

NAME

```
dggglm - solve a general  Gauss-Markov  linear  model  (GLM)
problem
```

SYNOPSIS

```
SUBROUTINE DGGGLM(N, M, P, A, LDA, B, LDB, D, X, Y, WORK, LDWORK,
      INFO)

INTEGER N, M, P, LDA, LDB, LDWORK, INFO
DOUBLE  PRECISION  A(LDA,*),  B(LDB,*),  D(*),  X(*),  Y(*),
WORK(*)

SUBROUTINE DGGGLM_64(N, M, P, A, LDA, B, LDB, D, X, Y, WORK, LDWORK,
      INFO)

INTEGER*8 N, M, P, LDA, LDB, LDWORK, INFO
DOUBLE  PRECISION  A(LDA,*),  B(LDB,*),  D(*),  X(*),  Y(*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGGLM([N], [M], [P], A, [LDA], B, [LDB], D, X, Y, [WORK],
       [LDWORK], [INFO])

INTEGER :: N, M, P, LDA, LDB, LDWORK, INFO
REAL(8), DIMENSION(:) :: D, X, Y, WORK
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE GGGLM_64([N], [M], [P], A, [LDA], B, [LDB], D, X, Y, [WORK],
       [LDWORK], [INFO])

INTEGER(8) :: N, M, P, LDA, LDB, LDWORK, INFO
REAL(8), DIMENSION(:) :: D, X, Y, WORK
```

```
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dggglm(int n, int m, int p, double *a, int lda,  double
          *b,  int ldb, double *d, double *x, double *y, int
          *info);

void dggglm_64(long n, long m, long p, double *a, long  lda,
          double  *b, long ldb, double *d, double *x, double
          *y, long *info);
```

PURPOSE

```
dggglm solves a  general  Gauss-Markov  linear  model  (GLM)
problem:

        minimize || y ||_2   subject to   d = A*x + B*y
            x

where A is an N-by-M matrix, B is an N-by-P matrix, and d is
a given N-vector. It is assumed that M <= N <= M+P, and

           rank(A) = M    and    rank( A B ) = N.

Under these assumptions, the constrained equation is  always
consistent,  and  there is a unique solution x and a minimal
2-norm solution y, which is obtained using a generalized  QR
factorization of A and B.

In particular, if matrix B is square nonsingular,  then  the
problem  GLM  is equivalent to the following weighted linear
least squares problem

             minimize || inv(B)*(d-A*x) ||_2
                 x

where inv(B) denotes the inverse of B.
```

ARGUMENTS

```
N (input) The number of rows of the matrices A and B.  N  >=
          0.

M (input) The number of columns of the matrix A.  0 <= M  <=
          N.
```

P (input) The number of columns of the matrix B. P >= N-M.

A (input/output)
On entry, the N-by-M matrix A. On exit, A is destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the N-by-P matrix B. On exit, B is destroyed.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

D (input/output)
On entry, D is the left hand side of the GLM equation. On exit, D is destroyed.

X (output)
On exit, X and Y are the solutions of the GLM problem.

Y (output)
See the description of X.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,N+M+P). For optimum performance, LDWORK >= M+min(N,P)+max(N,P)*NB, where NB is an upper bound for the optimal blocksizes for SGEQRF, SGERQF, SORMQR and SORMRQ.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an ille-

gal value.

Contents

NAME

dgghrd - reduce a pair of real matrices (A,B) to generalized upper Hessenberg form using orthogonal transformations, where A is a general matrix and B is upper triangular

SYNOPSIS

```
SUBROUTINE DGGHRD(COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB, Q, LDQ,
      Z, LDZ, INFO)

CHARACTER * 1 COMPQ, COMPZ
INTEGER N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*)

SUBROUTINE DGGHRD_64(COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB, Q,
      LDQ, Z, LDZ, INFO)

CHARACTER * 1 COMPQ, COMPZ
INTEGER*8 N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*)
```

F95 INTERFACE

```
SUBROUTINE GGHRD(COMPQ, COMPZ, [N], ILO, IHI, A, [LDA], B, [LDB], Q,
       [LDQ], Z, [LDZ], [INFO])

CHARACTER(LEN=1) :: COMPQ, COMPZ
INTEGER :: N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO
REAL(8), DIMENSION(:,:) :: A, B, Q, Z

SUBROUTINE GGHRD_64(COMPQ, COMPZ, [N], ILO, IHI, A, [LDA], B, [LDB],
       Q, [LDQ], Z, [LDZ], [INFO])
```

```
CHARACTER(LEN=1) :: COMPQ, COMPZ
INTEGER(8) :: N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO
REAL(8), DIMENSION(:,:) :: A, B, Q, Z
```

C INTERFACE

```
#include <sunperf.h>

void dgghrd(char compq, char compz, int n, int ilo, int ihi,
          double *a, int lda, double *b, int ldb, double *q,
          int ldq, double *z, int ldz, int *info);

void dgghrd_64(char compq, char compz,  long  n,  long  ilo,
          long  ihi,  double  *a,  long lda, double *b, long
          ldb, double *q, long ldq,  double  *z,  long  ldz,
          long *info);
```

PURPOSE

dgghrd reduces a pair of real matrices (A,B) to generalized upper Hessenberg form using orthogonal transformations, where A is a general matrix and B is upper triangular: Q' * A * Z = H and Q' * B * Z = T, where H is upper Hessenberg, T is upper triangular, and Q and Z are orthogonal, and ' means transpose.

The orthogonal matrices Q and Z are determined as products of Givens rotations. They may either be formed explicitly, or they may be postmultiplied into input matrices Q1 and Z1, so that
1 * A * Z1' = (Q1*Q) * H * (Z1*Z)'

ARGUMENTS

COMPQ (input)
= 'N': do not compute Q;
= 'I': Q is initialized to the unit matrix, and the orthogonal matrix Q is returned; = 'V': Q must contain an orthogonal matrix Q1 on entry, and the product Q1*Q is returned.

COMPZ (input)
= 'N': do not compute Z;
= 'I': Z is initialized to the unit matrix, and the orthogonal matrix Z is returned; = 'V': Z must contain an orthogonal matrix Z1 on entry, and the product Z1*Z is returned.

```
N (input) The order of the matrices A and B.  N >= 0.

ILO (input)
          It is assumed that A is already  upper  triangular
          in  rows and columns 1:ILO-1 and IHI+1:N.  ILO and
          IHI are normally set by a previous call to SGGBAL;
          otherwise  they  should  be set to 1 and N respec-
          tively.  1 <= ILO <= IHI <= N, if N > 0; ILO=1 and
          IHI=0, if N=0.

IHI (input)
          See the description of ILO.

A (input/output)
          On entry, the N-by-N general matrix to be reduced.
          On   exit,   the  upper  triangle  and  the  first
          subdiagonal of A are overwritten  with  the  upper
          Hessenberg matrix H, and the rest is set to zero.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

B (input/output)
          On entry, the N-by-N upper  triangular  matrix  B.
          On  exit,  the upper triangular matrix T = Q' B Z.
          The elements below the diagonal are set to zero.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

Q (input/output)
          If COMPQ='N':  Q is not referenced.
          If COMPQ='I':  on entry, Q need not be set, and on
          exit it contains the orthogonal matrix Q, where Q'
          is the product of the Givens transformations which
          are applied to A and B on the left.  If COMPQ='V':
          on entry, Q must contain an orthogonal matrix  Q1,
          and on exit this is overwritten by Q1*Q.

LDQ (input)
          The leading dimension of the array Q.  LDQ >= N if
          COMPQ='V' or 'I'; LDQ >= 1 otherwise.

Z (input/output)
          If COMPZ='N':  Z is not referenced.
          If COMPZ='I':  on entry, Z need not be set, and on
          exit it contains the orthogonal matrix Z, which is
          the product of the  Givens  transformations  which
```

are applied to A and B on the right. If COMPZ='V': on entry, Z must contain an orthogonal matrix Z1, and on exit this is overwritten by Z1*Z.

LDZ (input)
The leading dimension of the array Z. LDZ >= N if COMPZ='V' or 'I'; LDZ >= 1 otherwise.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

This routine reduces A to Hessenberg and B to triangular form by an unblocked reduction, as described in _Matrix_Computations_, by Golub and Van Loan (Johns Hopkins Press.)

Contents

NAME

```
dgglse - solve the linear equality-constrained least squares
(LSE) problem
```

SYNOPSIS

```
SUBROUTINE DGGLSE(M, N, P, A, LDA, B, LDB, C, D, X, WORK, LDWORK,
      INFO)

INTEGER M, N, P, LDA, LDB, LDWORK, INFO
DOUBLE  PRECISION  A(LDA,*),  B(LDB,*),  C(*),  D(*),  X(*),
WORK(*)

SUBROUTINE DGGLSE_64(M, N, P, A, LDA, B, LDB, C, D, X, WORK, LDWORK,
      INFO)

INTEGER*8 M, N, P, LDA, LDB, LDWORK, INFO
DOUBLE  PRECISION  A(LDA,*),  B(LDB,*),  C(*),  D(*),  X(*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGLSE([M], [N], [P], A, [LDA], B, [LDB], C, D, X, [WORK],
       [LDWORK], [INFO])

INTEGER :: M, N, P, LDA, LDB, LDWORK, INFO
REAL(8), DIMENSION(:) :: C, D, X, WORK
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE GGLSE_64([M], [N], [P], A, [LDA], B, [LDB], C, D, X, [WORK],
       [LDWORK], [INFO])

INTEGER(8) :: M, N, P, LDA, LDB, LDWORK, INFO
REAL(8), DIMENSION(:) :: C, D, X, WORK
```

```
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dgglse(int m, int n, int p, double *a, int lda,  double
          *b,  int ldb, double *c, double *d, double *x, int
          *info);

void dgglse_64(long m, long n, long p, double *a, long  lda,
          double  *b, long ldb, double *c, double *d, double
          *x, long *info);
```

PURPOSE

```
dgglse solves the linear equality-constrained least  squares
(LSE) problem:

        minimize || c - A*x ||_2   subject to   B*x = d

where A is an M-by-N matrix, B is a P-by-N matrix,  c  is  a
given  M-vector,  and  d  is a given P-vector. It is assumed
that
P <= N <= M+P, and

         rank(B) = P and  rank( ( A ) ) = N.
                                ( ( B ) )

These conditions ensure that the LSE problem  has  a  unique
solution, which is obtained using a GRQ factorization of the
matrices B and A.
```

ARGUMENTS

```
M (input) The number of rows of the matrix A.  M >= 0.

N (input) The number of columns of the matrices A and  B.  N
          >= 0.

P (input) The number of rows of the matrix B. 0 <= P <= N <=
          M+P.

A (input/output)
          On entry, the M-by-N matrix A.  On exit, A is des-
          troyed.

LDA (input)
```

The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the P-by-N matrix B. On exit, B is destroyed.

LDB (input)
The leading dimension of the array B. LDB >= max(1,P).

C (input/output)
On entry, C contains the right hand side vector for the least squares part of the LSE problem. On exit, the residual sum of squares for the solution is given by the sum of squares of elements N-P+1 to M of vector C.

D (input/output)
On entry, D contains the right hand side vector for the constrained equation. On exit, D is destroyed.

X (output)
On exit, X is the solution of the LSE problem.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,M+N+P). For optimum performance LDWORK >= P+min(M,N)+max(M,N)*NB, where NB is an upper bound for the optimal blocksizes for SGEQRF, SGERQF, SORMQR and SORMRQ.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

```
dggqrf - compute a generalized QR factorization of an N-by-M
matrix A and an N-by-P matrix B.
```

SYNOPSIS

```
SUBROUTINE DGGQRF(N, M, P, A, LDA, TAUA, B, LDB, TAUB, WORK, LWORK,
      INFO)

INTEGER N, M, P, LDA, LDB, LWORK, INFO
DOUBLE  PRECISION  A(LDA,*),  TAUA(*),  B(LDB,*),   TAUB(*),
WORK(*)

SUBROUTINE DGGQRF_64(N, M, P, A, LDA, TAUA, B, LDB, TAUB, WORK,
      LWORK, INFO)

INTEGER*8 N, M, P, LDA, LDB, LWORK, INFO
DOUBLE  PRECISION  A(LDA,*),  TAUA(*),  B(LDB,*),   TAUB(*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGQRF([N], [M], [P], A, [LDA], TAUA, B, [LDB], TAUB, [WORK],
       [LWORK], [INFO])

INTEGER :: N, M, P, LDA, LDB, LWORK, INFO
REAL(8), DIMENSION(:) :: TAUA, TAUB, WORK
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE GGQRF_64([N], [M], [P], A, [LDA], TAUA, B, [LDB], TAUB,
       [WORK], [LWORK], [INFO])

INTEGER(8) :: N, M, P, LDA, LDB, LWORK, INFO
```

```
REAL(8), DIMENSION(:) :: TAUA, TAUB, WORK
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dggqrf(int n, int m, int p, double *a, int lda,  double
          *taua,  double  *b,  int  ldb,  double  *taub, int
          *info);

void dggqrf_64(long n, long m, long p, double *a, long  lda,
          double  *taua,  double *b, long ldb, double *taub,
          long *info);
```

PURPOSE

```
dggqrf computes a generalized QR factorization of an  N-by-M
matrix A and an N-by-P matrix B:

            A = Q*R,        B = Q*T*Z,

where Q is an  N-by-N  orthogonal  matrix,  Z  is  a  P-by-P
orthogonal matrix, and R and T assume one of the forms:

if N >= M,  R = ( R11 ) M  ,   or if N < M,  R = ( R11   R12
) N,
                (  0  ) N-M                         N   M-N
                   M

where R11 is upper triangular, and

if N <= P,  T = ( 0  T12 ) N,   or if N > P,  T =  (  T11  )
N-P,
                 P-N  N                           ( T21 ) P
                                                     P

where T12 or T21 is upper triangular.

In particular, if B is square and nonsingular, the GQR  fac-
torization  of A and B implicitly gives the QR factorization
of inv(B)*A:

             inv(B)*A = Z'*(inv(T)*R)

where inv(B) denotes the inverse of the  matrix  B,  and  Z'
denotes the transpose of the matrix Z.
```

ARGUMENTS

N (input) The number of rows of the matrices A and B. N >= 0.

M (input) The number of columns of the matrix A. M >= 0.

P (input) The number of columns of the matrix B. P >= 0.

A (input/output)
On entry, the N-by-M matrix A. On exit, the elements on and above the diagonal of the array contain the min(N,M)-by-M upper trapezoidal matrix R (R is upper triangular if N >= M); the elements below the diagonal, with the array TAUA, represent the orthogonal matrix Q as a product of min(N,M) elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

TAUA (output)
The scalar factors of the elementary reflectors which represent the orthogonal matrix Q (see Further Details).

B (input/output)
On entry, the N-by-P matrix B. On exit, if N <= P, the upper triangle of the subarray B(1:N,P-N+1:P) contains the N-by-N upper triangular matrix T; if N > P, the elements on and above the (N-P)-th subdiagonal contain the N-by-P upper trapezoidal matrix T; the remaining elements, with the array TAUB, represent the orthogonal matrix Z as a product of elementary reflectors (see Further Details).

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

TAUB (output)
The scalar factors of the elementary reflectors which represent the orthogonal matrix Z (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

```
LWORK (input)
          The  dimension  of  the  array  WORK.   LWORK   >=
          max(1,N,M,P).   For  optimum  performance LWORK >=
          max(N,M,P)*max(NB1,NB2,NB3),  where  NB1  is   the
          optimal  blocksize  for the QR factorization of an
          N-by-M matrix, NB2 is the  optimal  blocksize  for
          the  RQ factorization of an N-by-P matrix, and NB3
          is the optimal blocksize for a call of SORMQR.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value.
```

FURTHER DETAILS

```
The matrix Q is  represented  as  a  product  of  elementary
reflectors

   Q = H(1) H(2) . . . H(k), where k = min(n,m).

Each H(i) has the form

   H(i) = I - taua * v * v'

where taua is a real scalar, and v is a real vector with
v(1:i-1) = 0 and v(i) = 1; v(i+1:n) is  stored  on  exit  in
A(i+1:n,i), and taua in TAUA(i).
To form Q explicitly, use LAPACK subroutine SORGQR.
To use Q to update another  matrix,  use  LAPACK  subroutine
SORMQR.

The matrix Z is  represented  as  a  product  of  elementary
reflectors

   Z = H(1) H(2) . . . H(k), where k = min(n,p).

Each H(i) has the form

   H(i) = I - taub * v * v'

where taub is a real scalar, and v is a real vector with
v(p-k+i+1:p) = 0 and v(p-k+i) = 1; v(1:p-k+i-1) is stored on
```

exit in B(n-k+i,1:p-k+i-1), and taub in TAUB(i).
To form Z explicitly, use LAPACK subroutine SORGRQ.
To use Z to update another matrix, use LAPACK subroutine SORMRQ.

Contents

NAME

dggrqf - compute a generalized RQ factorization of an M-by-N matrix A and a P-by-N matrix B

SYNOPSIS

```
SUBROUTINE DGGRQF(M, P, N, A, LDA, TAUA, B, LDB, TAUB, WORK, LWORK,
      INFO)

INTEGER M, P, N, LDA, LDB, LWORK, INFO
DOUBLE  PRECISION  A(LDA,*),  TAUA(*),  B(LDB,*),   TAUB(*),
WORK(*)

SUBROUTINE DGGRQF_64(M, P, N, A, LDA, TAUA, B, LDB, TAUB, WORK,
      LWORK, INFO)

INTEGER*8 M, P, N, LDA, LDB, LWORK, INFO
DOUBLE  PRECISION  A(LDA,*),  TAUA(*),  B(LDB,*),   TAUB(*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGRQF([M], [P], [N], A, [LDA], TAUA, B, [LDB], TAUB, [WORK],
       [LWORK], [INFO])

INTEGER :: M, P, N, LDA, LDB, LWORK, INFO
REAL(8), DIMENSION(:) :: TAUA, TAUB, WORK
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE GGRQF_64([M], [P], [N], A, [LDA], TAUA, B, [LDB], TAUB,
       [WORK], [LWORK], [INFO])

INTEGER(8) :: M, P, N, LDA, LDB, LWORK, INFO
```

```
REAL(8), DIMENSION(:) :: TAUA, TAUB, WORK
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dggrqf(int m, int p, int n, double *a, int lda,  double
          *taua,  double  *b,  int  ldb,  double  *taub, int
          *info);

void dggrqf_64(long m, long p, long n, double *a, long  lda,
          double  *taua,  double *b, long ldb, double *taub,
          long *info);
```

PURPOSE

```
dggrqf computes a generalized RQ factorization of an  M-by-N
matrix A and a P-by-N matrix B:

            A = R*Q,        B = Z*T*Q,

where Q is an  N-by-N  orthogonal  matrix,  Z  is  a  P-by-P
orthogonal matrix, and R and T assume one of the forms:

if M <= N,  R = ( 0  R12 ) M,   or if M > N,  R =  (  R11  )
M-N,
                 N-M  M                           ( R21 ) N
                                                     N

where R12 or R21 is upper triangular, and

if P >= N,  T = ( T11 ) N  ,   or if P < N,  T = ( T11   T12
) P,
                (  0  ) P-N                         P   N-P
                   N

where T11 is upper triangular.

In particular, if B is square and nonsingular, the GRQ  fac-
torization  of A and B implicitly gives the RQ factorization
of A*inv(B):

             A*inv(B) = (R*inv(T))*Z'

where inv(B) denotes the inverse of the  matrix  B,  and  Z'
denotes the transpose of the matrix Z.
```

ARGUMENTS

```
M (input) The number of rows of the matrix A.  M >= 0.

P (input) The number of rows of the matrix B.  P >= 0.

N (input) The number of columns of the matrices A and  B.  N
          >= 0.

A (input/output)
          On entry, the M-by-N matrix A.  On exit, if  M  <=
          N,  the  upper  triangle  of the subarray A(1:M,N-
          M+1:N) contains the M-by-M upper triangular matrix
          R;  if M > N, the elements on and above the (M-N)-
          th  subdiagonal  contain  the  M-by-N  upper  tra-
          pezoidal  matrix  R;  the remaining elements, with
          the array TAUA, represent the orthogonal matrix  Q
          as a product of elementary reflectors (see Further
          Details).

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,M).

TAUA (output)
          The scalar factors of  the  elementary  reflectors
          which  represent  the  orthogonal  matrix  Q  (see
          Further Details).

B (input/output)
          On entry, the P-by-N matrix B.  On exit, the  ele-
          ments  on and above the diagonal of the array con-
          tain the min(P,N)-by-N upper trapezoidal matrix  T
          (T  is  upper  triangular if P >= N); the elements
          below the diagonal, with the array TAUB, represent
          the orthogonal matrix Z as a product of elementary
          reflectors (see Further Details).

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          max(1,P).

TAUB (output)
          The scalar factors of  the  elementary  reflectors
          which  represent  the  orthogonal  matrix  Z  (see
          Further Details).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.
```

```
LWORK (input)
          The  dimension  of  the  array  WORK.   LWORK   >=
          max(1,N,M,P).   For  optimum  performance LWORK >=
          max(N,M,P)*max(NB1,NB2,NB3),  where  NB1  is   the
          optimal  blocksize  for the RQ factorization of an
          M-by-N matrix, NB2 is the  optimal  blocksize  for
          the  QR  factorization of a P-by-N matrix, and NB3
          is the optimal blocksize for a call of SORMRQ.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INF0= -i, the i-th argument had an  ille-
          gal value.
```

FURTHER DETAILS

```
The matrix Q is  represented  as  a  product  of  elementary
reflectors

   Q = H(1) H(2) . . . H(k), where k = min(m,n).

Each H(i) has the form

   H(i) = I - taua * v * v'

where taua is a real scalar, and v is a real vector with
v(n-k+i+1:n) = 0 and v(n-k+i) = 1; v(1:n-k+i-1) is stored on
exit in A(m-k+i,1:n-k+i-1), and taua in TAUA(i).
To form Q explicitly, use LAPACK subroutine SORGRQ.
To use Q to update another  matrix,  use  LAPACK  subroutine
SORMRQ.

The matrix Z is  represented  as  a  product  of  elementary
reflectors

   Z = H(1) H(2) . . . H(k), where k = min(p,n).

Each H(i) has the form

   H(i) = I - taub * v * v'

where taub is a real scalar, and v is a real vector with
v(1:i-1) = 0 and v(i) = 1; v(i+1:p) is  stored  on  exit  in
```

```
B(i+1:p,i), and taub in TAUB(i).
To form Z explicitly, use LAPACK subroutine SORGQR.
To use Z to update another  matrix,  use  LAPACK  subroutine
SORMQR.
```

Contents

NAME

```
dggsvd - compute the generalized singular  value  decomposi-
tion  (GSVD)  of  an  M-by-N  real  matrix A and P-by-N real
matrix B
```

SYNOPSIS

```
SUBROUTINE DGGSVD(JOBU, JOBV, JOBQ, M, N, P, K, L, A, LDA, B, LDB,
      ALPHA, BETA, U, LDU, V, LDV, Q, LDQ, WORK, IWORK3, INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
INTEGER M, N, P, K, L, LDA, LDB, LDU, LDV, LDQ, INFO
INTEGER IWORK3(*)
DOUBLE  PRECISION  A(LDA,*),  B(LDB,*),  ALPHA(*),  BETA(*),
U(LDU,*), V(LDV,*), Q(LDQ,*), WORK(*)

SUBROUTINE DGGSVD_64(JOBU, JOBV, JOBQ, M, N, P, K, L, A, LDA, B, LDB,
      ALPHA, BETA, U, LDU, V, LDV, Q, LDQ, WORK, IWORK3, INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
INTEGER*8 M, N, P, K, L, LDA, LDB, LDU, LDV, LDQ, INFO
INTEGER*8 IWORK3(*)
DOUBLE  PRECISION  A(LDA,*),  B(LDB,*),  ALPHA(*),  BETA(*),
U(LDU,*), V(LDV,*), Q(LDQ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGSVD(JOBU, JOBV, JOBQ, [M], [N], [P], K, L, A, [LDA], B,
       [LDB], ALPHA, BETA, U, [LDU], V, [LDV], Q, [LDQ], [WORK], IWORK3,
       [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
INTEGER :: M, N, P, K, L, LDA, LDB, LDU, LDV, LDQ, INFO
INTEGER, DIMENSION(:) :: IWORK3
```

```
REAL(8), DIMENSION(:) :: ALPHA, BETA, WORK
REAL(8), DIMENSION(:,:) :: A, B, U, V, Q

SUBROUTINE GGSVD_64(JOBU, JOBV, JOBQ, [M], [N], [P], K, L, A, [LDA],
       B, [LDB], ALPHA, BETA, U, [LDU], V, [LDV], Q, [LDQ], [WORK],
       IWORK3, [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
INTEGER(8) :: M, N, P, K, L, LDA, LDB, LDU, LDV, LDQ, INFO
INTEGER(8), DIMENSION(:) :: IWORK3
REAL(8), DIMENSION(:) :: ALPHA, BETA, WORK
REAL(8), DIMENSION(:,:) :: A, B, U, V, Q
```

C INTERFACE

```
#include <sunperf.h>
void dggsvd(char jobu, char jobv, char jobq, int m,  int  n,
          int  p, int *k, int *l, double *a, int lda, double
          *b, int ldb, double *alpha, double  *beta,  double
          *u,  int  ldu,  double *v, int ldv, double *q, int
          ldq, int *iwork3, int *info);

void dggsvd_64(char jobu, char jobv, char jobq, long m, long
          n,  long p, long *k, long *l, double *a, long lda,
          double *b, long ldb, double *alpha, double  *beta,
          double  *u,  long ldu, double *v, long ldv, double
          *q, long ldq, long *iwork3, long *info);
```

PURPOSE

```
dggsvd computes the generalized singular value decomposition
(GSVD) of an M-by-N real matrix A and P-by-N real matrix B:

    U'*A*Q = D1*( 0 R ),    V'*B*Q = D2*( 0 R )

where U, V and Q are orthogonal  matrices,  and  Z'  is  the
transpose  of  Z.  Let K+L = the effective numerical rank of
the matrix (A',B')', then  R  is  a  K+L-by-K+L  nonsingular
upper  triangular matrix, D1 and D2 are M-by-(K+L) and P-by-
(K+L) "diagonal" matrices and of the  following  structures,
respectively:

If M-K-L >= 0,

                    K  L
       D1 =     K ( I  0 )
                L ( 0  C )
            M-K-L ( 0  0 )

                  K  L
```

```
       D2 =   L ( 0  S )
            P-L ( 0  0 )

                N-K-L  K    L
  ( 0 R ) = K (  0   R11  R12 )
            L (  0    0   R22 )

where

  C = diag( ALPHA(K+1), ... , ALPHA(K+L) ),
  S = diag( BETA(K+1),  ... , BETA(K+L) ),
  C**2 + S**2 = I.

  R is stored in A(1:K+L,N-K-L+1:N) on exit.

If M-K-L < 0,

                  K M-K K+L-M
       D1 =   K ( I  0    0   )
            M-K ( 0  C    0   )

                    K M-K K+L-M
       D2 =   M-K ( 0  S    0  )
            K+L-M ( 0  0    I  )
              P-L ( 0  0    0  )

                   N-K-L  K   M-K  K+L-M
  ( 0 R ) =     K ( 0    R11  R12  R13  )
              M-K ( 0     0   R22  R23  )
            K+L-M ( 0     0    0   R33  )

where

  C = diag( ALPHA(K+1), ... , ALPHA(M) ),
  S = diag( BETA(K+1),  ... , BETA(M) ),
  C**2 + S**2 = I.

  (R11 R12 R13 ) is stored in A(1:M, N-K-L+1:N), and R33  is
stored
  ( 0  R22 R23 )
  in B(M-K+1:L,N+M-K-L+1:N) on exit.

The routine computes C, S, R, and optionally the  orthogonal
transformation matrices U, V and Q.

In particular, if B is an N-by-N  nonsingular  matrix,  then
the GSVD of A and B implicitly gives the SVD of A*inv(B):
                     A*inv(B) = U*(D1*inv(D2))*V'.
If ( A',B')' has orthonormal columns, then the GSVD of A and
B is also equal to the CS decomposition of A and B. Further-
```

more, the GSVD can be used to derive the solution of the eigenvalue problem:

```
A'*A x = lambda* B'*B x.
```

In some literature, the GSVD of A and B is presented in the form

```
U'*A*X = ( 0 D1 ),   V'*B*X = ( 0 D2 )
```

where U and V are orthogonal and X is nonsingular, D1 and D2 are ``diagonal''. The former GSVD form can be converted to the latter form by taking the nonsingular matrix X as

```
X = Q*( I   0    )
      ( 0 inv(R) ).
```

ARGUMENTS

JOBU (input)
= 'U': Orthogonal matrix U is computed;
= 'N': U is not computed.

JOBV (input)
= 'V': Orthogonal matrix V is computed;
= 'N': V is not computed.

JOBQ (input)
= 'Q': Orthogonal matrix Q is computed;
= 'N': Q is not computed.

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrices A and B. N >= 0.

P (input) The number of rows of the matrix B. P >= 0.

K (output)
On exit, K and L specify the dimension of the sub-blocks described in the Purpose section. K + L = effective numerical rank of (A',B')'.

L (output)
See the description of K.

A (input/output)
On entry, the M-by-N matrix A. On exit, A contains the triangular matrix R, or part of R. See Purpose for details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the P-by-N matrix B. On exit, B contains the triangular matrix R if M-K-L < 0. See Purpose for details.

LDB (input)
The leading dimension of the array B. LDA >= max(1,P).

ALPHA (output)
On exit, ALPHA and BETA contain the generalized singular value pairs of A and B; ALPHA(1:K) = 1,
BETA(1:K) = 0, and if M-K-L >= 0, ALPHA(K+1:K+L) = C,
BETA(K+1:K+L) = S, or if M-K-L < 0,
ALPHA(K+1:M)=C, ALPHA(M+1:K+L)=0
BETA(K+1:M) =S, BETA(M+1:K+L) =1 and
ALPHA(K+L+1:N) = 0
BETA(K+L+1:N) = 0

BETA (output)
See the description of ALPHA.

U (output)
If JOBU = 'U', U contains the M-by-M orthogonal matrix U. If JOBU = 'N', U is not referenced.

LDU (input)
The leading dimension of the array U. LDU >= max(1,M) if JOBU = 'U'; LDU >= 1 otherwise.

V (output)
If JOBV = 'V', V contains the P-by-P orthogonal matrix V. If JOBV = 'N', V is not referenced.

LDV (input)
The leading dimension of the array V. LDV >= max(1,P) if JOBV = 'V'; LDV >= 1 otherwise.

Q (output)
If JOBQ = 'Q', Q contains the N-by-N orthogonal matrix Q. If JOBQ = 'N', Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= max(1,N) if JOBQ = 'Q'; LDQ >= 1 otherwise.

WORK (workspace)
dimension (max(3*N,M,P)+N)

IWORK3 (output)
dimension(N) On exit, IWORK3 stores the sorting information. More precisely, the following loop will sort ALPHA for I = K+1, min(M,K+L) swap ALPHA(I) and ALPHA(IWORK3(I)) endfor such that ALPHA(1) >= ALPHA(2) >= ... >= ALPHA(N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = 1, the Jacobi-type procedure failed to converge. For further details, see subroutine STGSJA.

Contents

NAME

```
dggsvp - compute orthogonal matrices U, V and  Q  such  that
N-K-L K L  U'*A*Q = K ( 0 A12 A13 ) if M-K-L >= 0
```

SYNOPSIS

```
SUBROUTINE DGGSVP(JOBU, JOBV, JOBQ, M, P, N, A, LDA, B, LDB, TOLA,
      TOLB, K, L, U, LDU, V, LDV, Q, LDQ, IWORK, TAU, WORK, INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
INTEGER M, P, N, LDA, LDB, K, L, LDU, LDV, LDQ, INFO
INTEGER IWORK(*)
DOUBLE PRECISION TOLA, TOLB
DOUBLE PRECISION  A(LDA,*),  B(LDB,*),  U(LDU,*),  V(LDV,*),
Q(LDQ,*), TAU(*), WORK(*)

SUBROUTINE DGGSVP_64(JOBU, JOBV, JOBQ, M, P, N, A, LDA, B, LDB, TOLA,
      TOLB, K, L, U, LDU, V, LDV, Q, LDQ, IWORK, TAU, WORK, INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
INTEGER*8 M, P, N, LDA, LDB, K, L, LDU, LDV, LDQ, INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION TOLA, TOLB
DOUBLE PRECISION  A(LDA,*),  B(LDB,*),  U(LDU,*),  V(LDV,*),
Q(LDQ,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGSVP(JOBU, JOBV, JOBQ, [M], [P], [N], A, [LDA], B, [LDB],
       TOLA, TOLB, K, L, U, [LDU], V, [LDV], Q, [LDQ], [IWORK], [TAU],
       [WORK], [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
```

```
INTEGER :: M, P, N, LDA, LDB, K, L, LDU, LDV, LDQ, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL(8) :: TOLA, TOLB
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A, B, U, V, Q

SUBROUTINE GGSVP_64(JOBU, JOBV, JOBQ, [M], [P], [N], A, [LDA], B,
       [LDB], TOLA, TOLB, K, L, U, [LDU], V, [LDV], Q, [LDQ], [IWORK],
       [TAU], [WORK], [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
INTEGER(8) :: M, P, N, LDA, LDB, K, L, LDU, LDV, LDQ, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8) :: TOLA, TOLB
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A, B, U, V, Q
```

C INTERFACE

```
#include <sunperf.h>

void dggsvp(char jobu, char jobv, char jobq, int m,  int  p,
          int  n,  double  *a,  int lda, double *b, int ldb,
          double tola, double tolb, int *k, int  *l,  double
          *u,  int  ldu,  double *v, int ldv, double *q, int
          ldq, int *info);

void dggsvp_64(char jobu, char jobv, char jobq, long m, long
          p,  long  n,  double *a, long lda, double *b, long
          ldb, double tola, double tolb, long *k,  long  *l,
          double  *u,  long ldu, double *v, long ldv, double
          *q, long ldq, long *info);
```

PURPOSE

```
dggsvp computes orthogonal matrices U, V and Q such that
              L ( 0     0   A23 )
          M-K-L ( 0     0    0  )

                 N-K-L  K    L
        =     K ( 0    A12  A13 )  if M-K-L < 0;
            M-K ( 0     0   A23 )

               N-K-L  K    L
 V'*B*Q =   L ( 0     0   B13 )
          P-L ( 0     0    0  )

where the K-by-K matrix A12 and L-by-L matrix B13  are  non-
singular upper triangular; A23 is L-by-L upper triangular if
M-K-L >= 0, otherwise A23 is (M-K)-by-L  upper  trapezoidal.
K+L  = the effective numerical rank of the (M+P)-by-N matrix
```

(A',B')'. Z' denotes the transpose of Z.

This decomposition is the preprocessing step for computing the Generalized Singular Value Decomposition (GSVD), see subroutine SGGSVD.

ARGUMENTS

JOBU (input)
= 'U': Orthogonal matrix U is computed;
= 'N': U is not computed.

JOBV (input)
= 'V': Orthogonal matrix V is computed;
= 'N': V is not computed.

JOBQ (input)
= 'Q': Orthogonal matrix Q is computed;
= 'N': Q is not computed.

M (input) The number of rows of the matrix A. M >= 0.

P (input) The number of rows of the matrix B. P >= 0.

N (input) The number of columns of the matrices A and B. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, A contains the triangular (or trapezoidal) matrix described in the Purpose section.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the P-by-N matrix B. On exit, B contains the triangular matrix described in the Purpose section.

LDB (input)
The leading dimension of the array B. LDB >= max(1,P).

TOLA (input)
TOLA and TOLB are the thresholds to determine the effective numerical rank of matrix B and a subblock of A. Generally, they are set to TOLA =

MAX(M,N)*norm(A)*MACHEPS, TOLB = MAX(P,N)*norm(B)*MACHEPS. The size of TOLA and TOLB may affect the size of backward errors of the decomposition.

TOLB (input)
See the description of TOLA.

K (output)
On exit, K and L specify the dimension of the sub-blocks described in Purpose. K + L = effective numerical rank of (A',B')'.

L (output)
See the description of K.

U (input) If JOBU = 'U', U contains the orthogonal matrix U. If JOBU = 'N', U is not referenced.

LDU (input)
The leading dimension of the array U. LDU >= max(1,M) if JOBU = 'U'; LDU >= 1 otherwise.

V (input) If JOBV = 'V', V contains the orthogonal matrix V. If JOBV = 'N', V is not referenced.

LDV (input)
The leading dimension of the array V. LDV >= max(1,P) if JOBV = 'V'; LDV >= 1 otherwise.

Q (input) If JOBQ = 'Q', Q contains the orthogonal matrix Q. If JOBQ = 'N', Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= max(1,N) if JOBQ = 'Q'; LDQ >= 1 otherwise.

IWORK (workspace)
dimension(N)

TAU (workspace)
dimension(N)

WORK (workspace)
dimension(MAX(3*N,M,P))

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

The subroutine uses LAPACK subroutine SGEQPF for the QR factorization with column pivoting to detect the effective numerical rank of the a matrix. It may be replaced by a better rank determination strategy.

Contents

NAME

```
dgssco - General sparse solver condition number estimate.
```

SYNOPSIS

```
 SUBROUTINE DGSSCO ( COND, HANDLE, IER )

 INTEGER            IER
 DOUBLE PRECISION   COND
 DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

DGSSCO - Condition number estimate.

PARAMETERS

```
COND      - DOUBLE PRECISION
        On exit, an estimate of the condition number of the
        factored matrix.  Must be called after the numerical
        factorization subroutine, DGSSFA().

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -700 : Invalid calling sequence - need to call DGSSFA first.
```

```
-710 : Condition number estimate not available (not implemented
       for this HANDLE's matix type).
```

Contents

NAME

```
dgssda - Deallocate working storage for the general sparse
solver.
```

SYNOPSIS

```
 SUBROUTINE ZGSSDA ( HANDLE, IER )

 INTEGER           IER
 DOUBLE PRECISION  HANDLE(150)
```

PURPOSE

ZGSSDA - Deallocate dynamically allocated working storage.

PARAMETERS

```
HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         none
```

Contents

NAME

dgssfa - General sparse solver numeric factorization.

SYNOPSIS

```
SUBROUTINE DGSSFA ( NEQNS, COLSTR, ROWIND, VALUES, HANDLE, IER )

INTEGER            NEQNS, COLSTR(*), ROWIND(*), IER
DOUBLE PRECISION   VALUES(*)
DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

DGSSFA - Numeric factorization of a sparse matrix.

PARAMETERS

```
NEQNS     - INTEGER
        On entry, NEQNS specifies the number of equations in
        coefficient matrix.  Unchanged on exit.

COLSTR(*) - INTEGER array
        On entry, COLSTR(*) is an array of size (NEQNS+1),
        containing the pointers of the matrix structure.
        Unchanged on exit.

ROWIND(*) - INTEGER array
        On entry, ROWIND(*) is an array of size
        COLSTR(NEQNS+1)-1, containing the indices of the
        matrix structure.  Unchanged on exit.

VALUES(*) - DOUBLE PRECISION array
        On entry, VALUES(*) is an array of size
        COLSTR(NEQNS+1)-1, containing the numeric values of
```

```
        the sparse matrix to be factored.  Unchanged on
        exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER        - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -300 : Invalid calling sequence - need to call DGSSOR first.
         -301 : Failure to dynamically allocate memory.
         -666 : Internal error.
```

Contents

NAME

dgssfs - General sparse solver one call interface.

SYNOPSIS

```
SUBROUTINE DGSSFS ( MTXTYP, PIVOT , NEQNS, COLSTR, ROWIND,
                    VALUES, NRHS  , RHS   , LDRHS , ORDMTHD,
                    OUTUNT, MSGLVL, HANDLE, IER )

CHARACTER*2       MTXTYP
CHARACTER*1       PIVOT
INTEGER           NEQNS, COLSTR(*), ROWIND(*), NRHS, LDRHS,
                  OUTUNT, MSGLVL, IER
CHARACTER*3       ORDMTHD
DOUBLE PRECISION  VALUES(*), RHS(*)
DOUBLE PRECISION  HANDLE(150)
```

PURPOSE

DGSSFS - General sparse solver one call interface.

PARAMETERS

```
MTXTYP    - CHARACTER*2
        On entry, MTXTYP specifies the coefficient matrix
        type.  Specifically, the valid options are:

         'sp' or 'SP' - symmetric structure, positive-definite values
         'ss' or 'SS' - symmetric structure, symmetric values
         'su' or 'SU' - symmetric structure, unsymmetric values
         'uu' or 'UU' - unsymmetric structure, unsymmetric values

        Unchanged on exit.
```

```
PIVOT     - CHARACTER*1
        On entry, pivot specifies whether or not pivoting is
        used in the course of the numeric factorization.
        The valid options are:

         'n' or 'N' - no pivoting is used
         (Pivoting is not supported for this release).

        Unchanged on exit.

NEQNS     - INTEGER
        On entry, NEQNS specifies the number of equations in
        the coefficient matrix.  NEQNS must be at least one.
        Unchanged on exit.

COLSTR(*) - INTEGER array
        On entry, COLSTR(*) is an array of size (NEQNS+1),
        containing the pointers of the matrix structure.
        Unchanged on exit.
ROWIND(*) - INTEGER array
        On entry, ROWIND(*) is an array of size
        COLSTR(NEQNS+1)-1, containing the indices of the
        matrix structure.  Unchanged on exit.

VALUES(*) - DOUBLE PRECISION array
        On entry, VALUES(*) is an array of size
        COLSTR(NEQNS+1)-1, containing the non-zero numeric
        values of the sparse matrix to be factored.
        Unchanged on exit.

NRHS      - INTEGER
        On entry, NRHS specifies the number of right hand
        sides to solve for.  Unchanged on exit.

RHS(*)    - DOUBLE PRECISION array
        On entry, RHS(LDRHS,NRHS) contains the NRHS right
        hand sides.  On exit, it contains the solutions.

LDRHS     - INTEGER
        On entry, LDRHS specifies the leading dimension of
        the RHS array. Unchanged on exit.

ORDMTHD   - CHARACTER*3
        On entry, ORDMTHD specifies the fill-reducing
        ordering to be used by the sparse solver.
        Specifically, the valid options are:

         'nat' or 'NAT' - natural ordering (no ordering)
         'mmd' or 'MMD' - multiple minimum degree
         'gnd' or 'GND' - general nested dissection
```

```
          'uso' or 'USO' - user specified ordering (see DGSSUO)

         Unchanged on exit.

 OUTUNT    - INTEGER
         Output unit.  Unchanged on exit.

 MSGLVL    - INTEGER
         Message level.

          0 - no output from solver.
          (No messages supported for this release.)

         Unchanged on exit.

 HANDLE(150)  - DOUBLE PRECISION array
         On entry, HANDLE(*) is an array of containing
         information needed by the solver, and must be passed
         unchanged to each sparse solver subroutine.
         Modified on exit.
 IER       - INTEGER
         Error number.  If no error encountered, unchanged on
         exit.  If error encountered, it is set to a non-zero
         integer.  Error numbers set by this subroutine:

          -101 : Failure to dynamically allocate memory.
          -102 : Invalid matrix type.
          -103 : Invalid pivot option.
          -104 : Number of nonzeros is less than NEQNS.
          -105 : NEQNS < 1
          -201 : Failure to dynamically allocate memory.
          -301 : Failure to dynamically allocate memory.
          -401 : Failure to dynamically allocate memory.
          -402 : NRHS < 1
          -403 : NEQNS > LDRHS
          -666 : Internal error.
```

Contents

NAME

dgssin - Initialize the general sparse solver.

SYNOPSIS

```
SUBROUTINE DGSSIN ( MTXTYP, PIVOT, NEQNS, COLSTR, ROWIND, OUTUNT,
                    MSGLVL, HANDLE, IER )

CHARACTER*2       MTXTYP
CHARACTER*1       PIVOT
INTEGER           NEQNS, COLSTR(*), ROWIND(*), OUTUNT, MSGLVL, IER
DOUBLE PRECISION  HANDLE(150)
```

PURPOSE

DGSSIN - Initialize the sparse solver and input the matrix structure.

PARAMETERS

```
MTXTYP    - CHARACTER*2
        On entry, MTXTYP specifies the coefficient matrix
        type.  Specifically, the valid options are:

         'sp' or 'SP' - symmetric structure, positive-definite values
         'ss' or 'SS' - symmetric structure, symmetric values
         'su' or 'SU' - symmetric structure, unsymmetric values
         'uu' or 'UU' - unsymmetric structure, unsymmetric values

        Unchanged on exit.

PIVOT     - CHARACTER*1
        On entry, PIVOT specifies whether or not pivoting is
        used in the course of the numeric factorization.
```

The valid options are:

'n' or 'N' - no pivoting is used
(Pivoting is not supported for this release).

Unchanged on exit.

NEQNS - INTEGER
On entry, NEQNS specifies the number of equations in the coefficient matrix. NEQNS must be at least one. Unchanged on exit.

COLSTR(*) - INTEGER array
On entry, *COLSTR*(*) is an array of size (NEQNS+1), containing the pointers of the matrix structure. Unchanged on exit.

ROWIND(*) - INTEGER array
On entry, *ROWIND*(*) is an array of size COLSTR(NEQNS+1)-1, containing the indices of the matrix structure. Unchanged on exit.

HANDLE(150) - DOUBLE PRECISION array
On entry, *HANDLE*(*) is an array containing information needed by the solver, and must be passed unchanged to each sparse solver subroutine. Modified on exit.

OUTUNT - INTEGER
Output unit. Unchanged on exit.

MSGLVL - INTEGER
Message level.

0 - no output from solver.
(No messages supported for this release.)

Unchanged on exit.

IER - INTEGER
Error number. If no error encountered, unchanged on exit. If error encountered, it is set to a non-zero integer. Error numbers set by this subroutine:

-101 : Failure to dynamically allocate memory.
-102 : Invalid matrix type.
-103 : Invalid pivot option.
-104 : Number of nonzeros less than NEQNS.
-105 : NEQNS < 1

Contents

NAME

dgssor - General sparse solver ordering and symbolic factorization.

SYNOPSIS

```
 SUBROUTINE DGSSOR ( ORDMTHD, HANDLE, IER )

 CHARACTER*3       ORDMTHD
 INTEGER           IER
 DOUBLE PRECISION  HANDLE(150)
```

PURPOSE

DGSSOR - Orders and symbolically factors a sparse matrix.

PARAMETERS

```
ORDMTHD   - CHARACTER*3
        On entry, ORDMTHD specifies the fill-reducing
        ordering to be used by the sparse solver.
        Specifically, the valid options are:

         'nat' or 'NAT' - natural ordering (no ordering)
         'mmd' or 'MMD' - multiple minimum degree
         'gnd' or 'GND' - general nested dissection
         'uso' or 'USO' - user specified ordering (see DGSSUO)

        Unchanged on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
```

Modified on exit.

IER - INTEGER
Error number. If no error encountered, unchanged on exit. If error encountered, it is set to a non-zero integer. Error numbers set by this subroutine:

- -200 : Invalid calling sequence - need to call DGSSIN first.
- -201 : Failure to dynamically allocate memory.
- -666 : Internal error.

Contents

NAME

```
dgssps - Print general sparse solver statics.
```

SYNOPSIS

```
 SUBROUTINE DGSSPS ( HANDLE, IER )

 INTEGER           IER
 DOUBLE PRECISION  HANDLE(150)
```

PURPOSE

DGSSPS - Print solver statistics.

PARAMETERS

```
HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER        - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -800 : Invalid calling sequence - need to call DGSSSL first.
         -899 : Printed solver statistics not supported this release.
```

Contents

NAME

dgssrp - Return permutation used by the general sparse solver.

SYNOPSIS

```
SUBROUTINE DGSSRP ( PERM, HANDLE, IER )

INTEGER           PERM(*), IER
DOUBLE PRECISION  HANDLE(150)
```

PURPOSE

DGSSRP - Returns the permutation used by the solver for the fill-reducing ordering.

PARAMETERS

```
PERM(NEQNS)  - INTEGER array
        Undefined on entry.  PERM(NEQNS) is the permutation
        array used by the sparse solver for the fill-
        reducing ordering.  Modified on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER        - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:
```

-600 : Invalid calling sequence - need to call DGSSOR first.

Contents

NAME

```
dgsssl - Solve routine for the general sparse solver.
```

SYNOPSIS

```
 SUBROUTINE DGSSSL ( NRHS, RHS, LDRHS, HANDLE, IER )

 INTEGER            NRHS, LDRHS, IER
 DOUBLE PRECISION   RHS(LDRHS,NRHS)
 DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

DGSSSL - Triangular solve of a factored sparse matrix.

PARAMETERS

```
NRHS      - INTEGER
        On entry, NRHS specifies the number of right hand
        sides to solve for.  Unchanged on exit.

RHS(LDRHS,*) - DOUBLE PRECISION array
        On entry, RHS(LDRHS,NRHS) contains the NRHS right
        hand sides.  On exit, it contains the solutions.

LDRHS     - INTEGER
        On entry, LDRHS specifies the leading dimension of
        the RHS array. Unchanged on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.
```

```
IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -400 : Invalid calling sequence - need to call DGSSFA first.
         -401 : Failure to dynamically allocate memory.
         -402 : NRHS < 1
         -403 : NEQNS > LDRHS
```

Contents

NAME

```
dgssuo - User supplied permutation for ordering used in the
general sparse solver.
```

SYNOPSIS

```
 SUBROUTINE DGSSUO ( PERM, HANDLE, IER )

 INTEGER            PERM(*), IER
 DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

DGSSUO - User supplied permutation for ordering. Must be
called after *DGSSIN*() (sparse solver initialization) and
before *DGSSOR*() (sparse solver ordering).

PARAMETERS

```
PERM(NEQNS)  - INTEGER array
        On entry, PERM(NEQNS) is a permutation array
        supplied by the user for the fill-reducing ordering.
        Unchanged on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER          - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:
```

-500 : Invalid calling sequence - need to call DGSSIN first.

Contents

NAME

```
dgtcon - estimate the reciprocal of the condition number  of
a  real  tridiagonal  matrix A using the LU factorization as
computed by SGTTRF
```

SYNOPSIS

```
SUBROUTINE DGTCON(NORM, N, LOW, DIAG, UP1, UP2, IPIVOT, ANORM, RCOND,
      WORK, IWORK2, INFO)

CHARACTER * 1 NORM
INTEGER N, INFO
INTEGER IPIVOT(*), IWORK2(*)
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION LOW(*), DIAG(*), UP1(*), UP2(*), WORK(*)

SUBROUTINE DGTCON_64(NORM, N, LOW, DIAG, UP1, UP2, IPIVOT, ANORM,
      RCOND, WORK, IWORK2, INFO)

CHARACTER * 1 NORM
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*), IWORK2(*)
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION LOW(*), DIAG(*), UP1(*), UP2(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GTCON(NORM, [N], LOW, DIAG, UP1, UP2, IPIVOT, ANORM,
       RCOND, [WORK], [IWORK2], [INFO])

CHARACTER(LEN=1) :: NORM
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT, IWORK2
REAL(8) :: ANORM, RCOND
```

```
REAL(8), DIMENSION(:) :: LOW, DIAG, UP1, UP2, WORK

SUBROUTINE GTCON_64(NORM, [N], LOW, DIAG, UP1, UP2, IPIVOT, ANORM,
       RCOND, [WORK], [IWORK2], [INFO])

CHARACTER(LEN=1) :: NORM
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, IWORK2
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: LOW, DIAG, UP1, UP2, WORK
```

C INTERFACE

```
#include <sunperf.h>
void dgtcon(char norm, int n,  double  *low,  double  *diag,
          double  *up1,  double  *up2,  int  *ipivot, double
          anorm, double *rcond, int *info);

void dgtcon_64(char norm, long n, double *low, double *diag,
          double  *up1,  double  *up2,  long *ipivot, double
          anorm, double *rcond, long *info);
```

PURPOSE

dgtcon estimates the reciprocal of the condition number of a real tridiagonal matrix A using the LU factorization as computed by SGTTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

NORM (input)
Specifies whether the 1-norm condition number or the infinity-norm condition number is required:
= '1' or 'O': 1-norm;
= 'I': Infinity-norm.

N (input) The order of the matrix A. N >= 0.

LOW (input)
The (n-1) multipliers that define the matrix L from the LU factorization of A as computed by SGTTRF.

DIAG (input)

The n diagonal elements of the upper triangular matrix U from the LU factorization of A.

UP1 (input)

The (n-1) elements of the first superdiagonal of U.

UP2 (input)

The (n-2) elements of the second superdiagonal of U.

IPIVOT (input)

The pivot indices; for 1 <= i <= n, row i of the matrix was interchanged with row IPIVOT(i). IPIVOT(i) will always be either i or i+1; IPIVOT(i) = i indicates a row interchange was not required.

ANORM (input)

If NORM = '1' or 'O', the 1-norm of the original matrix A. If NORM = 'I', the infinity-norm of the original matrix A.

RCOND (output)

The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)

dimension(2*N)

IWORK2 (workspace)

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dgthr - Gathers specified elements from y into x.
```

SYNOPSIS

```
 SUBROUTINE DGTHR(NZ, Y, X, INDX)

 DOUBLE PRECISION Y(*), X(*)
 INTEGER NZ
 INTEGER INDX(*)

 SUBROUTINE DGTHR_64(NZ, Y, X, INDX)

 DOUBLE PRECISION Y(*), X(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 SUBROUTINE GTHR([NZ], Y, X, INDX)

 REAL(8), DIMENSION(:) :: Y, X
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 SUBROUTINE GTHR_64([NZ], Y, X, INDX)

 REAL(8), DIMENSION(:) :: Y, X
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
DGTHR - Gathers the specified elements from a vector y in
full storage form into a vector x in compressed form.  Only
```

the elements of y whose indices are listed in indx are referenced.

```
do i = 1, n
  x(i) = y(indx(i))
enddo
```

ARGUMENTS

NZ (input) - INTEGER
Number of elements in the compressed form. Unchanged on exit.

Y (input)
Vector in full storage form. Unchanged on exit.

X (output)
Vector in compressed form. Contains elements of y whose indices are listed in indx on exit.

INDX (input) - INTEGER
Vector containing the indices of the compressed form. It is assumed that the elements in INDX are distinct and greater than zero. Unchanged on exit.

Contents

NAME

```
dgthrz - Gather and zero.
```

SYNOPSIS

```
 SUBROUTINE DGTHRZ(NZ, Y, X, INDX)

 DOUBLE PRECISION Y(*), X(*)
 INTEGER NZ
 INTEGER INDX(*)

 SUBROUTINE DGTHRZ_64(NZ, Y, X, INDX)

 DOUBLE PRECISION Y(*), X(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 SUBROUTINE GTHRZ([NZ], Y, X, INDX)

 REAL(8), DIMENSION(:) :: Y, X
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 SUBROUTINE GTHRZ_64([NZ], Y, X, INDX)

 REAL(8), DIMENSION(:) :: Y, X
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
DGTHRZ - Gathers the specified elements from a vector y in
full storage form into a vector x in compressed form.  The
```

gathered elements of y are set to zero. Only the elements
of y whose indices are listed in indx are referenced.

```
 do i = 1, n
   x(i) = y(indx(i))
   y(indx(i)) = 0
 enddo
```

ARGUMENTS

NZ (input) - INTEGER
Number of elements in the compressed form.
Unchanged on exit.

Y (input/output)
Vector in full storage form. Gathered elements are
set to zero.

X (output)
Vector in compressed form. Contains elements of y
whose indices are listed in indx on exit.

INDX (input) - INTEGER
Vector containing the indices of the compressed
form. It is assumed that the elements in INDX are
distinct and greater than zero. Unchanged on exit.

Contents

NAME

dgtrfs - improve the computed solution to a system of linear equations when the coefficient matrix is tridiagonal, and provides error bounds and backward error estimates for the solution

SYNOPSIS

```
SUBROUTINE DGTRFS(TRANSA, N, NRHS, LOW, DIAG, UP, LOWF, DIAGF, UPF1,
      UPF2, IPIVOT, B, LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*), WORK2(*)
DOUBLE PRECISION LOW(*), DIAG(*), UP(*), LOWF(*),  DIAGF(*),
UPF1(*),  UPF2(*),  B(LDB,*),  X(LDX,*),  FERR(*),  BERR(*),
WORK(*)

SUBROUTINE DGTRFS_64(TRANSA, N, NRHS, LOW, DIAG, UP, LOWF, DIAGF,
      UPF1, UPF2, IPIVOT, B, LDB, X, LDX, FERR, BERR, WORK, WORK2,
      INFO)

CHARACTER * 1 TRANSA
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
DOUBLE PRECISION LOW(*), DIAG(*), UP(*), LOWF(*),  DIAGF(*),
UPF1(*),  UPF2(*),  B(LDB,*),  X(LDX,*),  FERR(*),  BERR(*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GTRFS([TRANSA], [N], [NRHS], LOW, DIAG, UP, LOWF, DIAGF,
       UPF1, UPF2, IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK],
       [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: TRANSA
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
REAL(8), DIMENSION(:) :: LOW, DIAG, UP, LOWF,  DIAGF,  UPF1,
UPF2, FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: B, X

SUBROUTINE GTRFS_64([TRANSA], [N], [NRHS], LOW, DIAG, UP, LOWF,
       DIAGF, UPF1, UPF2, IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: TRANSA
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL(8), DIMENSION(:) :: LOW, DIAG, UP, LOWF,  DIAGF,  UPF1,
UPF2, FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: B, X
```

C INTERFACE

```
#include <sunperf.h>

void dgtrfs(char transa, int n, int nrhs, double *low,  dou-
          ble   *diag,  double  *up,  double  *lowf,  double
          *diagf, double *upf1, double *upf2,  int  *ipivot,
          double  *b,  int  ldb,  double *x, int ldx, double
          *ferr, double *berr, int *info);

void dgtrfs_64(char transa, long n, long nrhs, double  *low,
          double  *diag,  double  *up,  double *lowf, double
          *diagf, double *upf1, double *upf2, long  *ipivot,
          double  *b,  long ldb, double *x, long ldx, double
          *ferr, double *berr, long *info);
```

PURPOSE

dgtrfs improves the computed solution to a system of linear equations when the coefficient matrix is tridiagonal, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

```
TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
```

= 'C': A**H * X = B (Conjugate transpose = Transpose)

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

LOW (input)
The (n-1) subdiagonal elements of A.

DIAG (input)
The diagonal elements of A.

UP (input)
The (n-1) superdiagonal elements of A.

LOWF (input)
The (n-1) multipliers that define the matrix L from the LU factorization of A as computed by SGTTRF.

DIAGF (input)
The n diagonal elements of the upper triangular matrix U from the LU factorization of A.

UPF1 (input)
The (n-1) elements of the first superdiagonal of U.

UPF2 (input)
The (n-2) elements of the second superdiagonal of U.

IPIVOT (input)
The pivot indices; for 1 <= i <= n, row i of the matrix was interchanged with row IPIVOT(i). IPIVOT(i) will always be either i or i+1; IPIVOT(i) = i indicates a row interchange was not required.

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by SGTTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dgtsv - solve the equation   A*X = B,
```

SYNOPSIS

```
SUBROUTINE DGTSV(N, NRHS, LOW, DIAG, UP, B, LDB, INFO)

INTEGER N, NRHS, LDB, INFO
DOUBLE PRECISION LOW(*), DIAG(*), UP(*), B(LDB,*)

SUBROUTINE DGTSV_64(N, NRHS, LOW, DIAG, UP, B, LDB, INFO)

INTEGER*8 N, NRHS, LDB, INFO
DOUBLE PRECISION LOW(*), DIAG(*), UP(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE GTSV([N], [NRHS], LOW, DIAG, UP, B, [LDB], [INFO])

INTEGER :: N, NRHS, LDB, INFO
REAL(8), DIMENSION(:) :: LOW, DIAG, UP
REAL(8), DIMENSION(:,:) :: B

SUBROUTINE GTSV_64([N], [NRHS], LOW, DIAG, UP, B, [LDB], [INFO])

INTEGER(8) :: N, NRHS, LDB, INFO
REAL(8), DIMENSION(:) :: LOW, DIAG, UP
REAL(8), DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void dgtsv(int n, int nrhs, double *low, double *diag,  dou-
          ble *up, double *b, int ldb, int *info);
```

```
void dgtsv_64(long n, long nrhs, double *low, double  *diag,
          double *up, double *b, long ldb, long *info);
```

PURPOSE

dgtsv solves the equation

where A is an n by n tridiagonal matrix, by Gaussian elimination with partial pivoting.

Note that the equation A'*X = B may be solved by interchanging the order of the arguments DU and DL.

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

LOW (input/output)
On entry, LOW must contain the (n-1) sub-diagonal elements of A.

On exit, LOW is overwritten by the (n-2) elements of the second super-diagonal of the upper triangular matrix U from the LU factorization of A, in LOW(1), ..., LOW(n-2).

DIAG (input/output)
On entry, DIAG must contain the diagonal elements of A.

On exit, DIAG is overwritten by the n diagonal elements of U.

UP (input/output)
On entry, UP must contain the (n-1) super-diagonal elements of A.

On exit, UP is overwritten by the (n-1) elements of the first super-diagonal of U.

B (input/output)
On entry, the N by NRHS matrix of right hand side

matrix B. On exit, if INFO = 0, the N by NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, U(i,i) is exactly zero, and the solution has not been computed. The factorization has not been completed unless i = N.

Contents

NAME

```
dgtsvx - use the LU factorization to compute the solution to
a real system of linear equations A * X = B or A**T * X = B,
```

SYNOPSIS

```
SUBROUTINE DGTSVX(FACT, TRANSA, N, NRHS, LOW, DIAG, UP, LOWF, DIAGF,
      UPF1, UPF2, IPIVOT, B, LDB, X, LDX, RCOND, FERR, BERR, WORK,
      WORK2, INFO)

CHARACTER * 1 FACT, TRANSA
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*), WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION LOW(*), DIAG(*), UP(*), LOWF(*),  DIAGF(*),
UPF1(*),  UPF2(*),  B(LDB,*),  X(LDX,*),  FERR(*),  BERR(*),
WORK(*)

SUBROUTINE DGTSVX_64(FACT, TRANSA, N, NRHS, LOW, DIAG, UP, LOWF,
      DIAGF, UPF1, UPF2, IPIVOT, B, LDB, X, LDX, RCOND, FERR, BERR,
      WORK, WORK2, INFO)

CHARACTER * 1 FACT, TRANSA
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION LOW(*), DIAG(*), UP(*), LOWF(*),  DIAGF(*),
UPF1(*),  UPF2(*),  B(LDB,*),  X(LDX,*),  FERR(*),  BERR(*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GTSVX(FACT, [TRANSA], [N], [NRHS], LOW, DIAG, UP, LOWF,
       DIAGF, UPF1, UPF2, IPIVOT, B, [LDB], X, [LDX], RCOND, FERR, BERR,
```

```
       [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, TRANSA
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: LOW, DIAG, UP, LOWF,  DIAGF,  UPF1,
UPF2, FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: B, X

SUBROUTINE GTSVX_64(FACT, [TRANSA], [N], [NRHS], LOW, DIAG, UP, LOWF,
       DIAGF, UPF1, UPF2, IPIVOT, B, [LDB], X, [LDX], RCOND, FERR, BERR,
       [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, TRANSA
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: LOW, DIAG, UP, LOWF,  DIAGF,  UPF1,
UPF2, FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: B, X
```

C INTERFACE

```
#include <sunperf.h>

void dgtsvx(char fact, char transa, int n, int nrhs,  double
          *low, double *diag, double *up, double *lowf, dou-
          ble  *diagf,  double  *upf1,  double  *upf2,   int
          *ipivot,  double  *b, int ldb, double *x, int ldx,
          double *rcond, double  *ferr,  double  *berr,  int
          *info);

void dgtsvx_64(char fact, char transa, long  n,  long  nrhs,
          double  *low,  double  *diag,  double  *up, double
          *lowf, double *diagf, double *upf1, double  *upf2,
          long *ipivot, double *b, long ldb, double *x, long
          ldx, double *rcond, double  *ferr,  double  *berr,
          long *info);
```

PURPOSE

dgtsvx uses the LU factorization to compute the solution to a real system of linear equations A * X = B or A**T * X = B, where A is a tridiagonal matrix of order N and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'N', the LU decomposition is used to factor the matrix A
   as A = L * U, where L is a product of permutation and unit lower
   bidiagonal matrices and U is upper triangular with nonzeros in
   only the main diagonal and first two superdiagonals.

2. If some U(i,i)=0, so that U is exactly singular, then the routine
   returns with INFO = i. Otherwise, the factored form of A is used
   to estimate the condition number of the matrix A. If the
   reciprocal of the condition number is less than machine precision,
   INFO = N+1 is returned as a warning, but the routine still goes on
   to solve for X and compute error bounds as described below.

3. The system of equations is solved for X using the factored form
   of A.

4. Iterative refinement is applied to improve the computed solution
   matrix and calculate error bounds and backward error estimates
   for it.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of A has been supplied on entry. = 'F': LOWF, DIAGF, UPF1, UPF2, and IPIVOT contain the factored form of A; LOW, DIAG, UP, LOWF, DIAGF, UPF1, UPF2 and IPIVOT will not be modified. = 'N': The matrix will be copied to LOWF, DIAGF, and UPF1 and factored.

TRANSA (input)
Specifies the form of the system of equations:
= 'N': A * X = B (No transpose)
= 'T': A**T * X = B (Transpose)
= 'C': A**H * X = B (Conjugate transpose = Tran-

spose)

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

LOW (input)
The (n-1) subdiagonal elements of A.

DIAG (input)
The n diagonal elements of A.

UP (input/output)
The (n-1) superdiagonal elements of A.

LOWF (input/output)
If FACT = 'F', then LOWF is an input argument and on entry contains the (n-1) multipliers that define the matrix L from the LU factorization of A as computed by SGTTRF.

If FACT = 'N', then LOWF is an output argument and on exit contains the (n-1) multipliers that define the matrix L from the LU factorization of A.

DIAGF (input/output)
If FACT = 'F', then DIAGF is an input argument and on entry contains the n diagonal elements of the upper triangular matrix U from the LU factorization of A.

If FACT = 'N', then DIAGF is an output argument and on exit contains the n diagonal elements of the upper triangular matrix U from the LU factorization of A.

UPF1 (input/output)
If FACT = 'F', then UPF1 is an input argument and on entry contains the (n-1) elements of the first superdiagonal of U.

If FACT = 'N', then UPF1 is an output argument and on exit contains the (n-1) elements of the first superdiagonal of U.

UPF2 (input/output)
If FACT = 'F', then UPF2 is an input argument and

on entry contains the (n-2) elements of the second superdiagonal of U.

If FACT = 'N', then UPF2 is an output argument and on exit contains the (n-2) elements of the second superdiagonal of U.

IPIVOT (input/output)
If FACT = 'F', then IPIVOT is an input argument and on entry contains the pivot indices from the LU factorization of A as computed by SGTTRF.
If FACT = 'N', then IPIVOT is an output argument and on exit contains the pivot indices from the LU factorization of A; row i of the matrix was interchanged with row IPIVOT(i). IPIVOT(i) will always be either i or i+1; IPIVOT(i) = i indicates a row interchange was not required.

B (input) The N-by-NRHS right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A. If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: U(i,i) is exactly zero. The factorization has not been completed unless i = N, but the factor U is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
dgttrf - compute an LU factorization of a  real  tridiagonal
matrix  A  using  elimination  with partial pivoting and row
interchanges
```

SYNOPSIS

```
SUBROUTINE DGTTRF(N, LOW, DIAG, UP1, UP2, IPIVOT, INFO)

INTEGER N, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION LOW(*), DIAG(*), UP1(*), UP2(*)

SUBROUTINE DGTTRF_64(N, LOW, DIAG, UP1, UP2, IPIVOT, INFO)

INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION LOW(*), DIAG(*), UP1(*), UP2(*)
```

F95 INTERFACE

```
SUBROUTINE GTTRF([N], LOW, DIAG, UP1, UP2, IPIVOT, [INFO])

INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: LOW, DIAG, UP1, UP2

SUBROUTINE GTTRF_64([N], LOW, DIAG, UP1, UP2, IPIVOT, [INFO])

INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: LOW, DIAG, UP1, UP2
```

C INTERFACE

```
#include <sunperf.h>

void dgttrf(int n, double *low, double *diag,  double  *up1,
          double *up2, int *ipivot, int *info);

void dgttrf_64(long n, double  *low,  double  *diag,  double
          *up1, double *up2, long *ipivot, long *info);
```

PURPOSE

dgttrf computes an LU factorization of a real tridiagonal matrix A using elimination with partial pivoting and row interchanges.

The factorization has the form

   A = L * U

where L is a product of permutation and unit lower bidiagonal matrices and U is upper triangular with nonzeros in only the main diagonal and first two superdiagonals.

ARGUMENTS

N (input) The order of the matrix A.

LOW (input/output)
On entry, LOW must contain the (n-1) sub-diagonal elements of A.

On exit, LOW is overwritten by the (n-1) multipliers that define the matrix L from the LU factorization of A.

DIAG (input/output)
On entry, DIAG must contain the diagonal elements of A.

On exit, DIAG is overwritten by the n diagonal elements of the upper triangular matrix U from the LU factorization of A.

UP1 (input/output)
On entry, UP1 must contain the (n-1) super-diagonal elements of A.

On exit, UP1 is overwritten by the (n-1) elements of the first super-diagonal of U.

UP2 (output)
On exit, UP2 is overwritten by the (n-2) elements of the second super-diagonal of U.

IPIVOT (output)
The pivot indices; for 1 <= i <= n, row i of the matrix was interchanged with row IPIVOT(i). IPIVOT(i) will always be either i or i+1; IPIVOT(i) = i indicates a row interchange was not required.

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, U(k,k) is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

Contents

NAME

```
dgttrs - solve one of the systems of equations  A*X =  B  or
A'*X = B,
```

SYNOPSIS

```
SUBROUTINE DGTTRS(TRANSA, N, NRHS, LOW, DIAG, UP1, UP2, IPIVOT, B,
      LDB, INFO)

CHARACTER * 1 TRANSA
INTEGER N, NRHS, LDB, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION LOW(*), DIAG(*), UP1(*), UP2(*), B(LDB,*)

SUBROUTINE DGTTRS_64(TRANSA, N, NRHS, LOW, DIAG, UP1, UP2, IPIVOT, B,
      LDB, INFO)

CHARACTER * 1 TRANSA
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION LOW(*), DIAG(*), UP1(*), UP2(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE GTTRS([TRANSA], [N], [NRHS], LOW, DIAG, UP1, UP2, IPIVOT,
       B, [LDB], [INFO])

CHARACTER(LEN=1) :: TRANSA
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: LOW, DIAG, UP1, UP2
REAL(8), DIMENSION(:,:) :: B

SUBROUTINE GTTRS_64([TRANSA], [N], [NRHS], LOW, DIAG, UP1, UP2,
```

```
          IPIVOT, B, [LDB], [INFO])

   CHARACTER(LEN=1) :: TRANSA
   INTEGER(8) :: N, NRHS, LDB, INFO
   INTEGER(8), DIMENSION(:) :: IPIVOT
   REAL(8), DIMENSION(:) :: LOW, DIAG, UP1, UP2
   REAL(8), DIMENSION(:,:) :: B
```

C INTERFACE

```
   #include <sunperf.h>

   void dgttrs(char transa, int n, int nrhs, double *low,  dou-
             ble  *diag, double *up1, double *up2, int *ipivot,
             double *b, int ldb, int *info);
   void dgttrs_64(char transa, long n, long nrhs, double  *low,
             double  *diag,  double  *up1,  double  *up2,  long
             *ipivot, double *b, long ldb, long *info);
```

PURPOSE

```
   dgttrs solves one of the systems of equations
      A*X = B  or  A'*X = B, with a tridiagonal matrix A  using
   the LU factorization computed by SGTTRF.
```

ARGUMENTS

```
   TRANSA (input)
             Specifies the form of the system of equations.   =
             'N':  A * X = B  (No transpose)
             = 'T':  A'* X = B  (Transpose)
             = 'C':  A'* X = B  (Conjugate  transpose  =  Tran-
             spose)

             TRANSA is defaulted to 'N' for F95 INTERFACE.

   N (input) The order of the matrix A.

   NRHS (input)
             The number of right hand sides, i.e.,  the  number
             of columns of the matrix B.  NRHS >= 0.

   LOW (input)
             The (n-1) multipliers that  define  the  matrix  L
             from the LU factorization of A.

   DIAG (input)
             The n diagonal elements of  the  upper  triangular
```

matrix U from the LU factorization of A.

UP1 (input)
The (n-1) elements of the first super-diagonal of U.

UP2 (input)
The (n-2) elements of the second super-diagonal of U.

IPIVOT (input)
The pivot indices; for 1 <= i <= n, row i of the matrix was interchanged with row IPIVOT(i). IPIVOT(i) will always be either i or i+1; IPIVOT(i) = i indicates a row interchange was not required.

B (input/output)
On entry, the matrix of right hand side vectors B. On exit, B is overwritten by the solution vectors X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dhgeqz - implement a single-/double-shift version of the  QZ
method    for    finding    the    generalized   eigenvalues
w(j)=(ALPHAR(j)  +  i*ALPHAI(j))/BETAR(j)  of  the  equation
det(  A-w(i)  B  )  =  0   In  addition, the pair A,B may be
reduced to generalized Schur form
```

SYNOPSIS

```
SUBROUTINE DHGEQZ(JOB, COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB,
      ALPHAR, ALPHAI, BETA, Q, LDQ, Z, LDZ, WORK, LWORK, INFO)

CHARACTER * 1 JOB, COMPQ, COMPZ
INTEGER N, ILO, IHI, LDA, LDB, LDQ, LDZ, LWORK, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*),  ALPHAR(*),  ALPHAI(*),
BETA(*), Q(LDQ,*), Z(LDZ,*), WORK(*)

SUBROUTINE DHGEQZ_64(JOB, COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB,
      ALPHAR, ALPHAI, BETA, Q, LDQ, Z, LDZ, WORK, LWORK, INFO)

CHARACTER * 1 JOB, COMPQ, COMPZ
INTEGER*8 N, ILO, IHI, LDA, LDB, LDQ, LDZ, LWORK, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*),  ALPHAR(*),  ALPHAI(*),
BETA(*), Q(LDQ,*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE HGEQZ(JOB, COMPQ, COMPZ, [N], ILO, IHI, A, [LDA], B, [LDB],
       ALPHAR, ALPHAI, BETA, Q, [LDQ], Z, [LDZ], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: JOB, COMPQ, COMPZ
INTEGER :: N, ILO, IHI, LDA, LDB, LDQ, LDZ, LWORK, INFO
REAL(8), DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
```

```
REAL(8), DIMENSION(:,:) :: A, B, Q, Z

SUBROUTINE HGEQZ_64(JOB, COMPQ, COMPZ, [N], ILO, IHI, A, [LDA], B,
       [LDB], ALPHAR, ALPHAI, BETA, Q, [LDQ], Z, [LDZ], [WORK], [LWORK],
       [INFO])

CHARACTER(LEN=1) :: JOB, COMPQ, COMPZ
INTEGER(8) :: N, ILO, IHI, LDA, LDB, LDQ, LDZ, LWORK, INFO
REAL(8), DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
REAL(8), DIMENSION(:,:) :: A, B, Q, Z
```

C INTERFACE

```
#include <sunperf.h>

void dhgeqz(char job, char compq, char  compz,  int  n,  int
          ilo,  int  ihi, double *a, int lda, double *b, int
          ldb, double *alphar, double *alphai, double *beta,
          double  *q,  int  ldq,  double  *z,  int  ldz, int
          *info);

void dhgeqz_64(char job, char compq,  char  compz,  long  n,
          long  ilo,  long  ihi, double *a, long lda, double
          *b, long ldb, double *alphar, double *alphai, dou-
          ble  *beta,  double  *q, long ldq, double *z, long
          ldz, long *info);
```

PURPOSE

dhgeqz implements a single-/double-shift version of the QZ method for finding the generalized eigenvalues B is upper triangular, and A is block upper triangular, where the diagonal blocks are either 1-by-1 or 2-by-2, the 2-by-2 blocks having complex generalized eigenvalues (see the description of the argument JOB.)

If JOB='S', then the pair (A,B) is simultaneously reduced to Schur form by applying one orthogonal tranformation (usually called Q) on the left and another (usually called Z) on the right. The 2-by-2 upper-triangular diagonal blocks of B corresponding to 2-by-2 blocks of A will be reduced to positive diagonal matrices. (I.e., if A(j+1,j) is non-zero, then B(j+1,j)=B(j,j+1)=0 and B(j,j) and B(j+1,j+1) will be positive.)

If JOB='E', then at each iteration, the same transformations are computed, but they are only applied to those parts of A and B which are needed to compute ALPHAR, ALPHAI, and BETAR.

If JOB='S' and COMPQ and COMPZ are 'V' or 'I', then the

orthogonal transformations used to reduce (A,B) are accumulated into the arrays Q and Z s.t.:
(in) A(in) Z(in)* = Q(out) A(out) Z(out)*

Ref: C.B. Moler & G.W. Stewart, "An Algorithm for Generalized Matrixigenvalue Problems", SIAM J. Numer. Anal., 10(1973),p. 241--256.

ARGUMENTS

JOB (input)
= 'E': compute only ALPHAR, ALPHAI, and BETA. A and B will not necessarily be put into generalized Schur form. = 'S': put A and B into generalized Schur form, as well as computing ALPHAR, ALPHAI, and BETA.

COMPQ (input)
= 'N': do not modify Q.
= 'V': multiply the array Q on the right by the transpose of the orthogonal tranformation that is applied to the left side of A and B to reduce them to Schur form. = 'I': like COMPQ='V', except that Q will be initialized to the identity first.

COMPZ (input)
= 'N': do not modify Z.
= 'V': multiply the array Z on the right by the orthogonal tranformation that is applied to the right side of A and B to reduce them to Schur form. = 'I': like COMPZ='V', except that Z will be initialized to the identity first.

N (input) The order of the matrices A, B, Q, and Z. N >= 0.

ILO (input)
It is assumed that A is already upper triangular in rows and columns 1:ILO-1 and IHI+1:N. 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

IHI (input)
See the description of ILO.

A (input) On entry, the N-by-N upper Hessenberg matrix A. Elements below the subdiagonal must be zero. If JOB='S', then on exit A and B will have been simultaneously reduced to generalized Schur form. If JOB='E', then on exit A will have been destroyed. The diagonal blocks will be correct, but the off-diagonal portion will be meaningless.

LDA (input)
The leading dimension of the array A. LDA >= max(1, N).

B (input) On entry, the N-by-N upper triangular matrix B. Elements below the diagonal must be zero. 2-by-2 blocks in B corresponding to 2-by-2 blocks in A will be reduced to positive diagonal form. (I.e., if A(j+1,j) is non-zero, then B(j+1,j)=B(j,j+1)=0 and B(j,j) and B(j+1,j+1) will be positive.) If JOB='S', then on exit A and B will have been simultaneously reduced to Schur form. If JOB='E', then on exit B will have been destroyed. Elements corresponding to diagonal blocks of A will be correct, but the off-diagonal portion will be meaningless.

LDB (input)
The leading dimension of the array B. LDB >= max(1, N).

ALPHAR (output)
ALPHAR(1:N) will be set to real parts of the diagonal elements of A that would result from reducing A and B to Schur form and then further reducing them both to triangular form using unitary transformations s.t. the diagonal of B was non-negative real. Thus, if A(j,j) is in a 1-by-1 block (i.e., A(j+1,j)=A(j,j+1)=0), then ALPHAR(j)=A(j,j). Note that the (real or complex) values (ALPHAR(j) + i*ALPHAI(j))/BETA(j), j=1,...,N, are the generalized eigenvalues of the matrix pencil A - wB.

ALPHAI (output)
ALPHAI(1:N) will be set to imaginary parts of the diagonal elements of A that would result from reducing A and B to Schur form and then further reducing them both to triangular form using unitary transformations s.t. the diagonal of B was non-negative real. Thus, if A(j,j) is in a 1-by-1 block (i.e., A(j+1,j)=A(j,j+1)=0), then ALPHAR(j)=0. Note that the (real or complex) values (ALPHAR(j) + i*ALPHAI(j))/BETA(j), j=1,...,N, are the generalized eigenvalues of the matrix pencil A - wB.

BETA (output)

BETA(1:N) will be set to the (real) diagonal ele-
ments of B that would result from reducing A and B
to Schur form and then further reducing them both
to triangular form using unitary transformations
s.t. the diagonal of B was non-negative real.
Thus, if A(j,j) is in a 1-by-1 block (i.e.,
A(j+1,j)=A(j,j+1)=0), then BETA(j)=B(j,j). Note
that the (real or complex) values (ALPHAR(j) +
i*ALPHAI(j))/BETA(j), j=1,...,N, are the general-
ized eigenvalues of the matrix pencil A - wB.
(Note that BETA(1:N) will always be non-negative,
and no BETAI is necessary.)

Q (input/output)
If COMPQ='N', then Q will not be referenced. If
COMPQ='V' or 'I', then the transpose of the
orthogonal transformations which are applied to A
and B on the left will be applied to the array Q
on the right.

LDQ (input)
The leading dimension of the array Q. LDQ >= 1.
If COMPQ='V' or 'I', then LDQ >= N.

Z (input/output)
If COMPZ='N', then Z will not be referenced. If
COMPZ='V' or 'I', then the orthogonal transforma-
tions which are applied to A and B on the right
will be applied to the array Z on the right.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1.
If COMPZ='V' or 'I', then LDZ >= N.

WORK (workspace)
On exit, if INFO >= 0, WORK(1) returns the optimal
LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >=
max(1,N).

If LWORK = -1, then a workspace query is assumed;
the routine only calculates the optimal size of
the WORK array, returns this value as the first
entry of the WORK array, and no error message
related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value
= 1,...,N: the QZ iteration did not converge. (A,B) is not in Schur form, but ALPHAR(i), ALPHAI(i), and BETA(i), i=INFO+1,...,N should be correct. = N+1,...,2*N: the shift calculation failed. (A,B) is not in Schur form, but ALPHAR(i), ALPHAI(i), and BETA(i), i=INFO-N+1,...,N should be correct. > 2*N: various "impossible" errors.

FURTHER DETAILS

```
Iteration counters:

JITER  -- counts iterations.
IITER  -- counts iterations run since ILAST was last
          changed.  This is therefore reset only when  a  1-
by-1 or
          2-by-2 block deflates off the bottom.
```

Contents

NAME

dhsein - use inverse iteration to find specified right and/or left eigenvectors of a real upper Hessenberg matrix H

SYNOPSIS

```
SUBROUTINE DHSEIN(SIDE, EIGSRC, INITV, SELECT, N, H, LDH, WR, WI, VL,
      LDVL, VR, LDVR, MM, M, WORK, IFAILL, IFAILR, INFO)

CHARACTER * 1 SIDE, EIGSRC, INITV
INTEGER N, LDH, LDVL, LDVR, MM, M, INFO
INTEGER IFAILL(*), IFAILR(*)
LOGICAL SELECT(*)
DOUBLE  PRECISION  H(LDH,*),   WR(*),   WI(*),   VL(LDVL,*),
VR(LDVR,*), WORK(*)

SUBROUTINE DHSEIN_64(SIDE, EIGSRC, INITV, SELECT, N, H, LDH, WR, WI,
      VL, LDVL, VR, LDVR, MM, M, WORK, IFAILL, IFAILR, INFO)

CHARACTER * 1 SIDE, EIGSRC, INITV
INTEGER*8 N, LDH, LDVL, LDVR, MM, M, INFO
INTEGER*8 IFAILL(*), IFAILR(*)
LOGICAL*8 SELECT(*)
DOUBLE  PRECISION  H(LDH,*),   WR(*),   WI(*),   VL(LDVL,*),
VR(LDVR,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE HSEIN(SIDE, EIGSRC, INITV, SELECT, [N], H, [LDH], WR, WI,
       VL, [LDVL], VR, [LDVR], MM, M, [WORK], IFAILL, IFAILR, [INFO])

CHARACTER(LEN=1) :: SIDE, EIGSRC, INITV
INTEGER :: N, LDH, LDVL, LDVR, MM, M, INFO
```

```
INTEGER, DIMENSION(:) :: IFAILL, IFAILR
LOGICAL, DIMENSION(:) :: SELECT
REAL(8), DIMENSION(:) :: WR, WI, WORK
REAL(8), DIMENSION(:,:) :: H, VL, VR

SUBROUTINE HSEIN_64(SIDE, EIGSRC, INITV, SELECT, [N], H, [LDH], WR,
       WI, VL, [LDVL], VR, [LDVR], MM, M, [WORK], IFAILL, IFAILR, [INFO])

CHARACTER(LEN=1) :: SIDE, EIGSRC, INITV
INTEGER(8) :: N, LDH, LDVL, LDVR, MM, M, INFO
INTEGER(8), DIMENSION(:) :: IFAILL, IFAILR
LOGICAL(8), DIMENSION(:) :: SELECT
REAL(8), DIMENSION(:) :: WR, WI, WORK
REAL(8), DIMENSION(:,:) :: H, VL, VR
```

C INTERFACE

```
#include <sunperf.h>

void dhsein(char side, char eigsrc, char initv, int *select,
          int n, double *h, int ldh, double *wr, double *wi,
          double *vl, int ldvl, double *vr,  int  ldvr,  int
          mm, int *m, int *ifaill, int *ifailr, int *info);

void dhsein_64(char side,  char  eigsrc,  char  initv,  long
          *select,  long n, double *h, long ldh, double *wr,
          double *wi, double *vl,  long  ldvl,  double  *vr,
          long  ldvr,  long  mm, long *m, long *ifaill, long
          *ifailr, long *info);
```

PURPOSE

dhsein uses inverse iteration to find specified right and/or left eigenvectors of a real upper Hessenberg matrix H.

The right eigenvector x and the left eigenvector y of the matrix H corresponding to an eigenvalue w are defined by:

```
H * x = w * x,     y**h * H = w * y**h
```

where y**h denotes the conjugate transpose of the vector y.

ARGUMENTS

```
SIDE (input)
          = 'R': compute right eigenvectors only;
          = 'L': compute left eigenvectors only;
          = 'B': compute both right and left eigenvectors.
```

EIGSRC (input)
Specifies the source of eigenvalues supplied in (WR,WI):
= 'Q': the eigenvalues were found using SHSEQR; thus, if H has zero subdiagonal elements, and so is block-triangular, then the j-th eigenvalue can be assumed to be an eigenvalue of the block containing the j-th row/column. This property allows SHSEIN to perform inverse iteration on just one diagonal block. = 'N': no assumptions are made on the correspondence between eigenvalues and diagonal blocks. In this case, SHSEIN must always perform inverse iteration using the whole matrix H.

INITV (input)
= 'N': no initial vectors are supplied;
= 'U': user-supplied initial vectors are stored in the arrays VL and/or VR.

SELECT (input/output)
Specifies the eigenvectors to be computed. To select the real eigenvector corresponding to a real eigenvalue WR(j), SELECT(j) must be set to .TRUE.. To select the complex eigenvector corresponding to a complex eigenvalue (WR(j),WI(j)), with complex conjugate (WR(j+1),WI(j+1)), either SELECT(j) or SELECT(j+1) or both must be set to

N (input) The order of the matrix H. N >= 0.

H (input) The upper Hessenberg matrix H.

LDH (input)
The leading dimension of the array H. LDH >= max(1,N).

WR (input/output)
On entry, the real and imaginary parts of the eigenvalues of H; a complex conjugate pair of eigenvalues must be stored in consecutive elements of WR and WI. On exit, WR may have been altered since close eigenvalues are perturbed slightly in searching for independent eigenvectors.

WI (input)
See the description of WR.

VL (input/output)
On entry, if INITV = 'U' and SIDE = 'L' or 'B', VL

must contain starting vectors for the inverse iteration for the left eigenvectors; the starting vector for each eigenvector must be in the same column(s) in which the eigenvector will be stored. On exit, if SIDE = 'L' or 'B', the left eigenvectors specified by SELECT will be stored consecutively in the columns of VL, in the same order as their eigenvalues. A complex eigenvector corresponding to a complex eigenvalue is stored in two consecutive columns, the first holding the real part and the second the imaginary part. If SIDE = 'R', VL is not referenced.

LDVL (input)
The leading dimension of the array VL. LDVL >= max(1,N) if SIDE = 'L' or 'B'; LDVL >= 1 otherwise.

VR (input/output)
On entry, if INITV = 'U' and SIDE = 'R' or 'B', VR must contain starting vectors for the inverse iteration for the right eigenvectors; the starting vector for each eigenvector must be in the same column(s) in which the eigenvector will be stored. On exit, if SIDE = 'R' or 'B', the right eigenvectors specified by SELECT will be stored consecutively in the columns of VR, in the same order as their eigenvalues. A complex eigenvector corresponding to a complex eigenvalue is stored in two consecutive columns, the first holding the real part and the second the imaginary part. If SIDE = 'L', VR is not referenced.

LDVR (input)
The leading dimension of the array VR. LDVR >= max(1,N) if SIDE = 'R' or 'B'; LDVR >= 1 otherwise.

MM (input)
The number of columns in the arrays VL and/or VR. MM >= M.

M (output)
The number of columns in the arrays VL and/or VR required to store the eigenvectors; each selected real eigenvector occupies one column and each selected complex eigenvector occupies two columns.

WORK (workspace)
dimension((N+2)*N)

IFAILL (output)

If SIDE = 'L' or 'B', IFAILL(i) = j > 0 if the left eigenvector in the i-th column of VL (corresponding to the eigenvalue w(j)) failed to converge; IFAILL(i) = 0 if the eigenvector converged satisfactorily. If the i-th and (i+1)th columns of VL hold a complex eigenvector, then IFAILL(i) and IFAILL(i+1) are set to the same value. If SIDE = 'R', IFAILL is not referenced.

IFAILR (output)

If SIDE = 'R' or 'B', IFAILR(i) = j > 0 if the right eigenvector in the i-th column of VR (corresponding to the eigenvalue w(j)) failed to converge; IFAILR(i) = 0 if the eigenvector converged satisfactorily. If the i-th and (i+1)th columns of VR hold a complex eigenvector, then IFAILR(i) and IFAILR(i+1) are set to the same value. If SIDE = 'L', IFAILR is not referenced.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, i is the number of eigenvectors which failed to converge; see IFAILL and IFAILR for further details.

FURTHER DETAILS

Each eigenvector is normalized so that the element of largest magnitude has magnitude 1; here the magnitude of a complex number (x,y) is taken to be |x|+|y|.

Contents

NAME

```
dhseqr - compute the eigenvalues of a real upper  Hessenberg
matrix  H  and,  optionally,  the  matrices T and Z from the
Schur decomposition H = Z  T  Z**T,  where  T  is  an  upper
quasi-triangular  matrix  (the  Schur  form),  and  Z is the
orthogonal matrix of Schur vectors
```

SYNOPSIS

```
SUBROUTINE DHSEQR(JOB, COMPZ, N, ILO, IHI, H, LDH, WR, WI, Z, LDZ,
      WORK, LWORK, INFO)

CHARACTER * 1 JOB, COMPZ
INTEGER N, ILO, IHI, LDH, LDZ, LWORK, INFO
DOUBLE PRECISION H(LDH,*), WR(*), WI(*), Z(LDZ,*), WORK(*)

SUBROUTINE DHSEQR_64(JOB, COMPZ, N, ILO, IHI, H, LDH, WR, WI, Z, LDZ,
      WORK, LWORK, INFO)

CHARACTER * 1 JOB, COMPZ
INTEGER*8 N, ILO, IHI, LDH, LDZ, LWORK, INFO
DOUBLE PRECISION H(LDH,*), WR(*), WI(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE HSEQR(JOB, COMPZ, N, ILO, IHI, H, [LDH], WR, WI, Z, [LDZ],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: JOB, COMPZ
INTEGER :: N, ILO, IHI, LDH, LDZ, LWORK, INFO
REAL(8), DIMENSION(:) :: WR, WI, WORK
REAL(8), DIMENSION(:,:) :: H, Z

SUBROUTINE HSEQR_64(JOB, COMPZ, N, ILO, IHI, H, [LDH], WR, WI, Z,
```

```
          [LDZ], [WORK], [LWORK], [INFO])

     CHARACTER(LEN=1) :: JOB, COMPZ
     INTEGER(8) :: N, ILO, IHI, LDH, LDZ, LWORK, INFO
     REAL(8), DIMENSION(:) :: WR, WI, WORK
     REAL(8), DIMENSION(:,:) :: H, Z
```

C INTERFACE

```
     #include <sunperf.h>

     void dhseqr(char job, char compz, int n, int ilo,  int  ihi,
               double *h, int ldh, double *wr, double *wi, double
               *z, int ldz, int *info);
     void dhseqr_64(char job, char compz, long n, long ilo,  long
               ihi,  double *h, long ldh, double *wr, double *wi,
               double *z, long ldz, long *info);
```

PURPOSE

dhseqr computes the eigenvalues of a real upper Hessenberg matrix H and, optionally, the matrices T and Z from the Schur decomposition H = Z T Z**T, where T is an upper quasi-triangular matrix (the Schur form), and Z is the orthogonal matrix of Schur vectors.

Optionally Z may be postmultiplied into an input orthogonal matrix Q, so that this routine can give the Schur factorization of a matrix A which has been reduced to the Hessenberg form H by the orthogonal matrix Q: A = Q*H*Q**T = (QZ)*T*(QZ)**T.

ARGUMENTS

JOB (input)
= 'E': compute eigenvalues only;
= 'S': compute eigenvalues and the Schur form T.

COMPZ (input)
= 'N': no Schur vectors are computed;
= 'I': Z is initialized to the unit matrix and the matrix Z of Schur vectors of H is returned; = 'V': Z must contain an orthogonal matrix Q on entry, and the product Q*Z is returned.

N (input) The order of the matrix H. N >= 0.

ILO (input)

```
            It is assumed that H is already  upper  triangular
            in  rows  and columns 1:ILO-1 and IHI+1:N. ILO and
            IHI are normally set by a previous call to SGEBAL,
            and  then  passed to SGEHRD when the matrix output
            by SGEBAL is reduced to Hessenberg form. Otherwise
            ILO and IHI should be set to 1 and N respectively.
            1 <= ILO <= IHI <= N, if N > 0; ILO=1  and  IHI=0,
            if N=0.

IHI (input)
            See the description of ILO.
H (input/output)
            On entry, the upper Hessenberg matrix H.  On exit,
            if   JOB  =  'S',  H  contains  the  upper  quasi-
            triangular matrix T from the  Schur  decomposition
            (the   Schur   form);   2-by-2   diagonal   blocks
            (corresponding  to  complex  conjugate  pairs   of
            eigenvalues)  are  returned in standard form, with
            H(i,i) = H(i+1,i+1) and H(i+1,i)*H(i,i+1) < 0.  If
            JOB  =  'E',  the contents of H are unspecified on
            exit.

LDH (input)
            The leading dimension  of  the  array  H.  LDH  >=
            max(1,N).

WR (output)
            The real and imaginary parts, respectively, of the
            computed  eigenvalues. If two eigenvalues are com-
            puted as a complex conjugate pair, they are stored
            in consecutive elements of WR and WI, say the i-th
            and (i+1)th, with WI(i) > 0 and WI(i+1)  <  0.  If
            JOB  = 'S', the eigenvalues are stored in the same
            order  as  on  the  diagonal  of  the  Schur  form
            returned  in  H,  with  WR(i)  =  H(i,i)  and,  if
            H(i:i+1,i:i+1) is a 2-by-2 diagonal block, WI(i) =
            sqrt(H(i+1,i)*H(i,i+1)) and WI(i+1) = -WI(i).

WI (output)
            See the description of WR.

Z (input)   If COMPZ = 'N': Z is not referenced.
            If COMPZ = 'I': on entry, Z need not be  set,  and
            on exit, Z contains the orthogonal matrix Z of the
            Schur vectors of H.  If COMPZ = 'V':  on  entry  Z
            must  contain an N-by-N matrix Q, which is assumed
            to be equal to the unit matrix except for the sub-
            matrix Z(ILO:IHI,ILO:IHI); on exit Z contains Q*Z.
            Normally Q is the orthogonal matrix  generated  by
            SORGHR  after  the call to SGEHRD which formed the
```

Hessenberg matrix H.

LDZ (input)
The leading dimension of the array Z. LDZ >= max(1,N) if COMPZ = 'I' or 'V'; LDZ >= 1 otherwise.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,N).

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, SHSEQR failed to compute all of the eigenvalues in a total of 30*(IHI-ILO+1) iterations; elements 1:ilo-1 and i+1:n of WR and WI contain those eigenvalues which have been successfully computed.

Contents

NAME

djadmm - Jagged diagonal matrix-matrix multiply (modified Ellpack)

SYNOPSIS

```
 SUBROUTINE DJADMM( TRANSA, M, N, K, ALPHA, DESCRA,
*            VAL, INDX, PNTR, MAXNZ, IPERM,
*            B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, K, DESCRA(5), MAXNZ,
*            LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTR(MAXNZ+1), IPERM(M)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE DJADMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*            VAL, INDX, PNTR, MAXNZ, IPERM,
*            B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, K, DESCRA(5), MAXNZ,
*            LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTR(MAXNZ+1), IPERM(M)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

where NNZ=PNTR(MAXNZ+1)-PNTR(1)+1 is the number of non-zero elements
```

F95 INTERFACE

```
 SUBROUTINE JADMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*    PNTR, MAXNZ, IPERM, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, K, MAXNZ
 INTEGER, DIMENSION(:) ::  DESCRA, INDX, PNTR, IPERM
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) ::  VAL
```

```
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C

 SUBROUTINE JADMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*    PNTR, MAXNZ, IPERM, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8     TRANSA, M, K, MAXNZ
 INTEGER*8, DIMENSION(:) ::  DESCRA, INDX, PNTR, IPERM
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) ::  VAL
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in jagged-diagonal format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
```

```
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

  VAL()         array of length NNZ consisting of entries of A.
                VAL can be viewed as a column major ordering of a
                row permutation of the Ellpack representation of A,
                where the Ellpack representation is permuted so that
                the rows are non-increasing in the number of nonzero
                entries.  Values added for padding in Ellpack are
                not included in the Jagged-Diagonal format.

  INDX()        array of length NNZ consisting of the column indices
                of the corresponding entries in VAL.
  PNTR()        array of length MAXNZ+1, where PNTR(I)-PNTR(1)+1
                points to the location in VAL of the first element
                in the row-permuted Ellpack represenation of A.

  MAXNZ         max number of nonzeros elements per row.

  IPERM()       integer array of length M such that I = IPERM(I'),
                where row I in the original Ellpack representation
                corresponds to row I' in the permuted representation.
                If IPERM(1) = 0, it is assumed by convention that
                IPERM(I) = I. IPERM is used to determine the order
                in which rows of C are updated.

  B()           rectangular array with first dimension LDB.

  LDB           leading dimension of B

  BETA          Scalar parameter

  C()           rectangular array with first dimension LDC.

  LDC           leading dimension of C

  WORK()        scratch array of length LWORK. WORK is not
                referenced in the current version.

  LWORK         length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

djadrp - right permutation of a jagged diagonal matrix

SYNOPSIS

```
 SUBROUTINE DJADRP( TRANSP, M, K, VAL, INDX, PNTR, MAXNZ,
*                     IPERM,WORK,LWORK)
 INTEGER    TRANSP, M, K, MAXNZ, LWORK
 INTEGER    INDX(*), PNTR(MAXNZ+1), IPERM(K), WORK(LWORK)
 DOUBLE PRECISION VAL(*)

 SUBROUTINE DJADRP_64( TRANSP, M, K, VAL, INDX, PNTR, MAXNZ,
*                     IPERM,WORK,LWORK)
 INTEGER*8  TRANSP, M, K, MAXNZ, LWORK
 INTEGER*8  INDX(*), PNTR(MAXNZ+1), IPERM(K), WORK(LWORK)
 DOUBLE PRECISION VAL(*)
```

F95 INTERFACE

```
 SUBROUTINE JADRP( TRANSP, M, K, VAL, INDX, PNTR, MAXNZ,
*                   IPERM, [WORK], [LWORK] )
 INTEGER TRANSP, M, K,  MAXNZ
 INTEGER, DIMENSION(:) :: INDX, PNTR, IPERM
 DOUBLE PRECISION, DIMENSION(:) :: VAL

 SUBROUTINE JADRP_64( TRANSP, M, K, VAL, INDX, PNTR, MAXNZ,
*                   IPERM, [WORK], [LWORK] )
 INTEGER*8 TRANSP, M, K,  MAXNZ
 INTEGER*8, DIMENSION(:) :: INDX, PNTR, IPERM
 DOUBLE PRECISION, DIMENSION(:) :: VAL
```

DESCRIPTION

```
A <- A P
A <- A P'
                                   ( ' indicates matrix transpose)
```

where permutation P is represented by an integer vector IPERM, such that IPERM(I) is equal to the position of the only nonzero element in row I of permutation matrix P.

NOTE: In order to get a symetrically permuted jagged diagonal matrix P A P', one can explicitly permute the columns P A by calling

```
   SJADRP(0, M, M, VAL, INDX, PNTR, MAXNZ, IPERM, WORK, LWORK)
```

where parameters VAL, INDX, PNTR, MAXNZ, IPERM are the representation of A in the jagged diagonal format. The operation makes sense if the original matrix A is square.

ARGUMENTS

TRANSP — Indicates how to operate with the permutation matrix
  0 : operate with matrix
  1 : operate with transpose matrix

M — Number of rows in matrix A

K — Number of columns in matrix A

VAL() — array of length PNTR(MAXNZ+1)-PNTR(1) consisting of entries of A. VAL can be viewed as a column major ordering of a row permutation of the Ellpack representation of A, where the Ellpack representation is permuted so that the rows are non-increasing in the number of nonzero entries. Values added for padding in Ellpack are not included in the Jagged-Diagonal format.

INDX() — array of length PNTR(MAXNZ+1)-PNTR(1) consisting of the column indices of the corresponding entries in VAL.

PNTR() — array of length MAXNZ+1, where PNTR(I)-PNTR(1)+1 points to the location in VAL of the first element in the row-permuted Ellpack represenation of A.

MAXNZ — max number of nonzeros elements per row.

IPERM() — integer array of length K such that I = IPERM(I').

```
               Array IPERM represents a permutation P, such that
               IPERM(I) is equal to the position of the only nonzero
               element in row I of permutation matrix P.
               For example, if
                              | 0 0 1 |
                          P  =| 1 0 0 |
                              | 0 1 0 |
               then IPERM = (3, 1, 2).

 WORK()        scratch array of length LWORK.  LWORK should be at
               least K.

 LWORK         length of WORK array

               If LWORK = -1, then a workspace query is assumed;
               the routine only calculates the optimal size of the
               WORK array, returns this value as the first entry of
               the WORK array, and no error message related to LWORK
               is issued by XERBLA.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

djadsm - Jagged-diagonal format triangular solve

SYNOPSIS

```
 SUBROUTINE DJADSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, INDX, PNTR, MAXNZ, IPERM,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5), MAXNZ,
*            LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTR(MAXNZ+1), IPERM(M)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE DJADSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, INDX, PNTR, MAXNZ, IPERM,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5), MAXNZ,
*            LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTR(MAXNZ+1), IPERM(M)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

where NNZ=PNTR(MAXNZ+1)-PNTR(1)+1 is the number of non-zero elements
```

F95 INTERFACE

```
 SUBROUTINE JADSM(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*    PNTR, MAXNZ, IPERM, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, MAXNZ
 INTEGER, DIMENSION(:) ::  DESCRA, INDX, PNTR, IPERM
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) ::  VAL, DV
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE JADSM_64(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*    PNTR, MAXNZ, IPERM, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, MAXNZ
 INTEGER*8, DIMENSION(:) ::  DESCRA, INDX, PNTR, IPERM
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) ::  VAL, DV
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a diagonal scaling matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in jagged-diagonal format and
op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                 0 : general
                 1 : symmetric (A=A')
                 2 : Hermitian (A= CONJG(A'))
                 3 : Triangular
                 4 : Skew(Anti)-Symmetric (A=-A')
                 5 : Diagonal
                 6 : Skew-Hermitian (A= -CONJG(A'))
               Note: For the routine, DESCRA(1)=3 is only supported.

               DESCRA(2) upper/lower triangular indicator
                 1 : lower
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices
VAL()          array of length NNZ consisting of entries of A.
               VAL can be viewed as a column major ordering of a
               row permutation of the Ellpack representation of A,
               where the Ellpack representation is permuted so that
               the rows are non-increasing in the number of nonzero
               entries.  Values added for padding in Ellpack are
               not included in the Jagged-Diagonal format.

INDX()         array of length NNZ consisting of the column indices
               of the corresponding entries in VAL.

PNTR()         array of length MAXNZ+1, where PNTR(I)-PNTR(1)+1
               points to the location in VAL of the first element
               in the row-permuted Ellpack represenation of A.

MAXNZ          max number of nonzeros elements per row.

IPERM()        integer array of length M such that I = IPERM(I'),
               where row I in the original Ellpack representation
               corresponds to row I' in the permuted representation.
               If IPERM(1)=0,   it's assumed by convention that
               IPERM(I)=I. IPERM is used to determine the order
               in which rows of C are updated.

B()            rectangular array with first dimension LDB.

LDB            leading dimension of B

BETA           Scalar parameter
```

```
C()            rectangular array with first dimension LDC.

LDC            leading dimension of C

WORK()         scratch array of length LWORK.
               On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.

LWORK          length of WORK array.  LWORK should be at least 2*M.

               For good performance, LWORK should generally be larger.
               For optimum performance on multiple processors, LWORK
               >=2*M*N_CPUS where N_CPUS is the maximum number of
               processors available to the program.

               If LWORK=0, the routine is to allocate workspace needed.

               If LWORK = -1, then a workspace query is assumed; the
               routine only calculates the optimum size of the WORK
               array, returns this value as the first entry of the WORK
               array, and no error message related to LWORK is issued
               by XERBLA.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the rows of A such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows have been scaled. UNITD=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the JAD representation of a sparse matrix. They are not used anyway in these cases. But if UNITD=4, the unit diagonal elements MUST be referenced in the JAD representation.

4. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

```
dlagtf - factorize the matrix (T-lambda*I), where T is an  n
by  n  tridiagonal  matrix  and  lambda  is  a scalar, as T-
lambda*I = PLU
```

SYNOPSIS

```
SUBROUTINE DLAGTF(N, A, LAMBDA, B, C, TOL, D, IN, INFO)

INTEGER N, INFO
INTEGER IN(*)
DOUBLE PRECISION LAMBDA, TOL
DOUBLE PRECISION A(*), B(*), C(*), D(*)

SUBROUTINE DLAGTF_64(N, A, LAMBDA, B, C, TOL, D, IN, INFO)

INTEGER*8 N, INFO
INTEGER*8 IN(*)
DOUBLE PRECISION LAMBDA, TOL
DOUBLE PRECISION A(*), B(*), C(*), D(*)
```

F95 INTERFACE

```
SUBROUTINE LAGTF([N], A, LAMBDA, B, C, TOL, D, IN, [INFO])

INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IN
REAL(8) :: LAMBDA, TOL
REAL(8), DIMENSION(:) :: A, B, C, D

SUBROUTINE LAGTF_64([N], A, LAMBDA, B, C, TOL, D, IN, [INFO])

INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IN
```

```
REAL(8) :: LAMBDA, TOL
REAL(8), DIMENSION(:) :: A, B, C, D
```

C INTERFACE

```
#include <sunperf.h>

void dlagtf(int n, double *a, double lambda, double *b, dou-
          ble  *c,  double  tol,  double  *d,  int  *in, int
          *info);

void dlagtf_64(long n, double *a, double lambda, double  *b,
          double  *c,  double tol, double *d, long *in, long
          *info);
```

PURPOSE

dlagtf factorizes the matrix (T - lambda*I), where T is an n by n tridiagonal matrix and lambda is a scalar, as where P is a permutation matrix, L is a unit lower tridiagonal matrix with at most one non-zero sub-diagonal elements per column and U is an upper triangular matrix with at most two non-zero super-diagonal elements per column.

The factorization is obtained by Gaussian elimination with partial pivoting and implicit row scaling.

The parameter LAMBDA is included in the routine so that SLAGTF may be used, in conjunction with SLAGTS, to obtain eigenvectors of T by inverse iteration.

ARGUMENTS

N (input) The order of the matrix T.

A (input/output)
On entry, A must contain the diagonal elements of T.

On exit, A is overwritten by the n diagonal elements of the upper triangular matrix U of the factorization of T.

LAMBDA (input)
On entry, the scalar lambda.

B (input/output)
On entry, B must contain the (n-1) super-diagonal

elements of T.

On exit, B is overwritten by the (n-1) super-diagonal elements of the matrix U of the factorization of T.

C (input/output)

On entry, C must contain the (n-1) sub-diagonal elements of T.

On exit, C is overwritten by the (n-1) sub-diagonal elements of the matrix L of the factorization of T.

TOL (input/output)

On entry, a relative tolerance used to indicate whether or not the matrix (T - lambda*I) is nearly singular. TOL should normally be chose as approximately the largest relative error in the elements of T. For example, if the elements of T are correct to about 4 significant figures, then TOL should be set to about 5*10**(-4). If TOL is supplied as less than eps, where eps is the relative machine precision, then the value eps is used in place of TOL.

D (output)

On exit, D is overwritten by the (n-2) second super-diagonal elements of the matrix U of the factorization of T.

IN (output)

On exit, IN contains details of the permutation matrix P. If an interchange occurred at the kth step of the elimination, then IN(k) = 1, otherwise IN(k) = 0. The element IN(n) returns the smallest positive integer j such that

abs(u(j,j)).le. norm((T - lambda*I)(j))*TOL,

where norm(A(j)) denotes the sum of the absolute values of the jth row of the matrix A. If no such j exists then IN(n) is returned as zero. If IN(n) is returned as positive, then a diagonal element of U is small, indicating that (T - lambda*I) is singular or nearly singular,

INFO (output)

= 0 : successful exit

Contents

NAME

```
dlamrg - will create a permutation list which will merge the
elements of A (which is composed of two independently sorted
sets) into a single set which is sorted in ascending order
```

SYNOPSIS

```
SUBROUTINE DLAMRG(N1, N2, A, TRD1, TRD2, INDEX)

INTEGER N1, N2, TRD1, TRD2
INTEGER INDEX(*)
DOUBLE PRECISION A(*)

SUBROUTINE DLAMRG_64(N1, N2, A, TRD1, TRD2, INDEX)

INTEGER*8 N1, N2, TRD1, TRD2
INTEGER*8 INDEX(*)
DOUBLE PRECISION A(*)
```

F95 INTERFACE

```
SUBROUTINE LAMRG(N1, N2, A, TRD1, TRD2, INDEX)

INTEGER :: N1, N2, TRD1, TRD2
INTEGER, DIMENSION(:) :: INDEX
REAL(8), DIMENSION(:) :: A

SUBROUTINE LAMRG_64(N1, N2, A, TRD1, TRD2, INDEX)

INTEGER(8) :: N1, N2, TRD1, TRD2
INTEGER(8), DIMENSION(:) :: INDEX
REAL(8), DIMENSION(:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dlamrg(int n1, int n2, double *a, int trd1,  int  trd2,
          int *index);

void dlamrg_64(long n1, long n2, double *a, long trd1,  long
          trd2, long *index);
```

PURPOSE

dlamrg will create a permutation list which will merge the elements of A (which is composed of two independently sorted sets) into a single set which is sorted in ascending order.

ARGUMENTS

N1 (input)
Length of the first sequence to be merged.

N2 (input)
Length of the second sequence to be merged.

A (input) On entry, the first N1 elements of A contain a list of numbers which are sorted in either ascending or descending order. Likewise for the final N2 elements.

TRD1 (input)
Describes the stride to be taken through the array A for the first N1 elements.
= -1 subset is sorted in descending order.
= 1 subset is sorted in ascending order.

TRD2 (input)
Describes the stride to be taken through the array A for the first N1 elements.
= -1 subset is sorted in descending order.
= 1 subset is sorted in ascending order.

INDEX (output)
On exit this array will contain a permutation such that if B(I) = A(INDEX(I)) for I=1,N1+N2, then B will be sorted in ascending order.

Contents

NAME

dlarz - applies a real elementary reflector H to a real M-by-N matrix C, from either the left or the right

SYNOPSIS

```
SUBROUTINE DLARZ(SIDE, M, N, L, V, INCV, TAU, C, LDC, WORK)

CHARACTER * 1 SIDE
INTEGER M, N, L, INCV, LDC
DOUBLE PRECISION TAU
DOUBLE PRECISION V(*), C(LDC,*), WORK(*)

SUBROUTINE DLARZ_64(SIDE, M, N, L, V, INCV, TAU, C, LDC, WORK)

CHARACTER * 1 SIDE
INTEGER*8 M, N, L, INCV, LDC
DOUBLE PRECISION TAU
DOUBLE PRECISION V(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE LARZ(SIDE, [M], [N], L, V, [INCV], TAU, C, [LDC], [WORK])

CHARACTER(LEN=1) :: SIDE
INTEGER :: M, N, L, INCV, LDC
REAL(8) :: TAU
REAL(8), DIMENSION(:) :: V, WORK
REAL(8), DIMENSION(:,:) :: C

SUBROUTINE LARZ_64(SIDE, [M], [N], L, V, [INCV], TAU, C, [LDC], [WORK])

CHARACTER(LEN=1) :: SIDE
```

```
INTEGER(8) :: M, N, L, INCV, LDC
REAL(8) :: TAU
REAL(8), DIMENSION(:) :: V, WORK
REAL(8), DIMENSION(:,:) :: C
```

C INTERFACE

```
#include <sunperf.h>

void dlarz(char side, int m, int n, int l,  double  *v,  int
          incv, double tau, double *c, int ldc);

void dlarz_64(char side, long m, long n, long l, double  *v,
          long incv, double tau, double *c, long ldc);
```

PURPOSE

dlarz applies a real elementary reflector H to a real M-by-N matrix C, from either the left or the right. H is represented in the form

```
      H = I - tau * v * v'
```

where tau is a real scalar and v is a real vector.

If tau = 0, then H is taken to be the unit matrix.

H is a product of k elementary reflectors as returned by STZRZF.

ARGUMENTS

```
SIDE (input)
          = 'L': form  H * C
          = 'R': form  C * H

M (input) The number of rows of the matrix C.

N (input) The number of columns of the matrix C.

L (input) The number of entries of the vector  V  containing
          the  meaningful  part  of the Householder vectors.
          If SIDE = 'L', M >= L >= 0, if SIDE = 'R', N >=  L
          >= 0.

V (input) The  vector  v  in  the  representation  of  H  as
          returned by STZRZF. V is not used if TAU = 0.
```

```
INCV (input)
          The increment between elements of v. INCV <> 0.

TAU (input)
          The value tau in the representation of H.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten  by the matrix H * C if SIDE = 'L', or
          C * H if SIDE = 'R'.
LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          (N) if SIDE = 'L' or (M) if SIDE = 'R'
```

FURTHER DETAILS

```
Based on contributions by
  A. Petitet, Computer Science Dept., Univ. of Tenn.,  Knox-
ville, USA
```

Contents

NAME

```
dlarzb - applies a real block reflector H or  its  transpose
H**T  to  a  real  distributed M-by-N C from the left or the
right
```

SYNOPSIS

```
SUBROUTINE DLARZB(SIDE, TRANS, DIRECT, STOREV, M, N, K, L, V, LDV, T,
      LDT, C, LDC, WORK, LDWORK)

CHARACTER * 1 SIDE, TRANS, DIRECT, STOREV
INTEGER M, N, K, L, LDV, LDT, LDC, LDWORK
DOUBLE    PRECISION    V(LDV,*),     T(LDT,*),     C(LDC,*),
WORK(LDWORK,*)

SUBROUTINE DLARZB_64(SIDE, TRANS, DIRECT, STOREV, M, N, K, L, V, LDV,
      T, LDT, C, LDC, WORK, LDWORK)

CHARACTER * 1 SIDE, TRANS, DIRECT, STOREV
INTEGER*8 M, N, K, L, LDV, LDT, LDC, LDWORK
DOUBLE    PRECISION    V(LDV,*),     T(LDT,*),     C(LDC,*),
WORK(LDWORK,*)
```

F95 INTERFACE

```
SUBROUTINE LARZB(SIDE, TRANS, DIRECT, STOREV, [M], [N], K, L, V, [LDV],
       T, [LDT], C, [LDC], [WORK], [LDWORK])

CHARACTER(LEN=1) :: SIDE, TRANS, DIRECT, STOREV
INTEGER :: M, N, K, L, LDV, LDT, LDC, LDWORK
REAL(8), DIMENSION(:,:) :: V, T, C, WORK

SUBROUTINE LARZB_64(SIDE, TRANS, DIRECT, STOREV, [M], [N], K, L, V,
```

```
       [LDV], T, [LDT], C, [LDC], [WORK], [LDWORK])

CHARACTER(LEN=1) :: SIDE, TRANS, DIRECT, STOREV
INTEGER(8) :: M, N, K, L, LDV, LDT, LDC, LDWORK
REAL(8), DIMENSION(:,:) :: V, T, C, WORK
```

C INTERFACE

```
#include <sunperf.h>

void dlarzb(char side, char trans, char direct, char storev,
          int  m,  int  n, int k, int l, double *v, int ldv,
          double *t,  int  ldt,  double  *c,  int  ldc,  int
          ldwork);

void dlarzb_64(char side,  char  trans,  char  direct,  char
          storev, long m, long n, long k, long l, double *v,
          long ldv, double *t, long  ldt,  double  *c,  long
          ldc, long ldwork);
```

PURPOSE

dlarzb applies a real block reflector H or its transpose H**T to a real distributed M-by-N C from the left or the right.

Currently, only STOREV = 'R' and DIRECT = 'B' are supported.

ARGUMENTS

SIDE (input)
= 'L': apply H or H' from the Left
= 'R': apply H or H' from the Right

TRANS (input)
= 'N': apply H (No transpose)
= 'C': apply H' (Transpose)

DIRECT (input)
Indicates how H is formed from a product of elementary reflectors = 'F': H = H(1) H(2) . . . H(k) (Forward, not supported yet)
= 'B': H = H(k) . . . H(2) H(1) (Backward)

STOREV (input)
Indicates how the vectors which define the elementary reflectors are stored:
= 'C': Columnwise (not sup-

ported yet)
= 'R': Rowwise

M (input) The number of rows of the matrix C.

N (input) The number of columns of the matrix C.

K (input) The order of the matrix T (= the number of elementary reflectors whose product defines the block reflector).

L (input) The number of columns of the matrix V containing the meaningful part of the Householder reflectors. If SIDE = 'L', M >= L >= 0, if SIDE = 'R', N >= L >= 0.

V (input) If STOREV = 'C', NV = K; if STOREV = 'R', NV = L.

LDV (input)
The leading dimension of the array V. If STOREV = 'C', LDV >= L; if STOREV = 'R', LDV >= K.

T (input) The triangular K-by-K matrix T in the representation of the block reflector.

LDT (input)
The leading dimension of the array T. LDT >= K.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by H*C or H'*C or C*H or C*H'.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
dimension(MAX(M,N),K)

LDWORK (input)
The leading dimension of the array WORK. If SIDE = 'L', LDWORK >= max(1,N); if SIDE = 'R', LDWORK >= max(1,M).

FURTHER DETAILS

Based on contributions by
A. Petitet, Computer Science Dept., Univ. of Tenn., Knoxville, USA

Contents

NAME

dlarzt - form the triangular factor T of a real block reflector H of order > n, which is defined as a product of k elementary reflectors

SYNOPSIS

```
SUBROUTINE DLARZT(DIRECT, STOREV, N, K, V, LDV, TAU, T, LDT)

CHARACTER * 1 DIRECT, STOREV
INTEGER N, K, LDV, LDT
DOUBLE PRECISION V(LDV,*), TAU(*), T(LDT,*)

SUBROUTINE DLARZT_64(DIRECT, STOREV, N, K, V, LDV, TAU, T, LDT)

CHARACTER * 1 DIRECT, STOREV
INTEGER*8 N, K, LDV, LDT
DOUBLE PRECISION V(LDV,*), TAU(*), T(LDT,*)
```

F95 INTERFACE

```
SUBROUTINE LARZT(DIRECT, STOREV, N, K, V, [LDV], TAU, T, [LDT])

CHARACTER(LEN=1) :: DIRECT, STOREV
INTEGER :: N, K, LDV, LDT
REAL(8), DIMENSION(:) :: TAU
REAL(8), DIMENSION(:,:) :: V, T

SUBROUTINE LARZT_64(DIRECT, STOREV, N, K, V, [LDV], TAU, T, [LDT])

CHARACTER(LEN=1) :: DIRECT, STOREV
INTEGER(8) :: N, K, LDV, LDT
REAL(8), DIMENSION(:) :: TAU
```

```
REAL(8), DIMENSION(:,:) :: V, T
```

C INTERFACE

```
#include <sunperf.h>

void dlarzt(char direct, char storev, int n, int  k,  double
          *v, int ldv, double *tau, double *t, int ldt);

void dlarzt_64(char direct, char storev,  long  n,  long  k,
          double  *v, long ldv, double *tau, double *t, long
          ldt);
```

PURPOSE

dlarzt forms the triangular factor T of a real block reflector H of order > n, which is defined as a product of k elementary reflectors.

If DIRECT = 'F', H = H(1) H(2) . . . H(k) and T is upper triangular;

If DIRECT = 'B', H = H(k) . . . H(2) H(1) and T is lower triangular.

If STOREV = 'C', the vector which defines the elementary reflector H(i) is stored in the i-th column of the array V, and

```
H  =  I - V * T * V'
```

If STOREV = 'R', the vector which defines the elementary reflector H(i) is stored in the i-th row of the array V, and

```
H  =  I - V' * T * V
```

Currently, only STOREV = 'R' and DIRECT = 'B' are supported.

ARGUMENTS

DIRECT (input)
Specifies the order in which the elementary reflectors are multiplied to form the block reflector:
= 'F': H = H(1) H(2) . . . H(k) (Forward, not supported yet)
= 'B': H = H(k) . . . H(2) H(1) (Backward)

```
STOREV (input)
          Specifies how the vectors which define the elemen-
          tary  reflectors  are  stored  (see  also  Further
          Details):
          = 'R': rowwise

N (input) The order of the block reflector H. N >= 0.

K (input) The order of the triangular factor T (= the number
          of elementary reflectors). K >= 1.

V (input) (LDV,K) if STOREV = 'C' (LDV,N) if  STOREV  =  'R'
          The matrix V. See further details.

LDV (input)
          The leading dimension of the array V.  If STOREV =
          'C', LDV >= max(1,N); if STOREV = 'R', LDV >= K.

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i).

T (input) The k by  k  triangular  factor  T  of  the  block
          reflector.   If  DIRECT = 'F', T is upper triangu-
          lar; if DIRECT = 'B', T is lower  triangular.  The
          rest of the array is not used.

LDT (input)
          The leading dimension of the array T. LDT >= K.
```

FURTHER DETAILS

```
Based on contributions by
  A. Petitet, Computer Science Dept., Univ. of Tenn.,  Knox-
ville, USA

The shape of the matrix V and the  storage  of  the  vectors
which  define  the H(i) is best illustrated by the following
example with n = 5 and k = 3. The elements equal  to  1  are
not  stored;  the  corresponding array elements are modified
but restored on exit. The rest of the array is not used.

DIRECT = 'F' and STOREV =  'C':          DIRECT  =  'F'  and
STOREV = 'R':

                                                ______V_____
       (    v1    v2    v3    )                            /
( v1 v2 v3 )                        ( v1 v1 v1 v1 v1 . . . . 1
)
   V = ( v1 v2 v3 )                        ( v2 v2 v2 v2 v2  .
```

```
. . 1   )
       ( v1 v2 v3 )                      ( v3 v3 v3 v3 v3  .
. 1     )
       ( v1 v2 v3 )
          .  .  .
          1  .  .
             1  .
                1

DIRECT = 'B' and STOREV =  'C':          DIRECT  =  'B'  and
STOREV = 'R':

                                                      ______V_____
          1                                                /
.  1                           ( 1 . . . . v1 v1 v1 v1 v1 )
          .  .  1                        ( . 1 . . .  v2  v2
v2 v2 v2 )
          .  .  .                        ( . . 1 . .  v3  v3
v3 v3 v3 )
          .  .  .
       ( v1 v2 v3 )
   V = ( v1 v2 v3 )
       ( v1 v2 v3 )
```

Contents

NAME

```
dlasrt - the numbers in D in increasing order (if ID =  'I')
or in decreasing order (if ID = 'D' )
```

SYNOPSIS

```
SUBROUTINE DLASRT(ID, N, D, INFO)

CHARACTER * 1 ID
INTEGER N, INFO
DOUBLE PRECISION D(*)

SUBROUTINE DLASRT_64(ID, N, D, INFO)

CHARACTER * 1 ID
INTEGER*8 N, INFO
DOUBLE PRECISION D(*)
```

F95 INTERFACE

```
SUBROUTINE LASRT(ID, [N], D, [INFO])

CHARACTER(LEN=1) :: ID
INTEGER :: N, INFO
REAL(8), DIMENSION(:) :: D

SUBROUTINE LASRT_64(ID, [N], D, [INFO])

CHARACTER(LEN=1) :: ID
INTEGER(8) :: N, INFO
REAL(8), DIMENSION(:) :: D
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dlasrt(char id, int n, double *d, int *info);

void dlasrt_64(char id, long n, double *d, long *info);
```

PURPOSE

```
dlasrt the numbers in D in increasing order (if ID = 'I') or
in decreasing order (if ID = 'D' ).

Use Quick Sort, reverting to Insertion sort on arrays of
size <= 20. Dimension of STACK limits N to about 2**32.
```

ARGUMENTS

```
ID (input)
          = 'I': sort D in increasing order;
          = 'D': sort D in decreasing order.

N (input) The length of the array D.

D (input/output)
          On entry, the array to be sorted.  On exit, D  has
          been  sorted into increasing order (D(1) <= ... <=
          D(N) ) or into decreasing order (D(1)  >=  ...  >=
          D(N) ), depending on ID.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

dlatzm - routine is deprecated and has been replaced by rou- tine SORMRZ

SYNOPSIS

```
SUBROUTINE DLATZM(SIDE, M, N, V, INCV, TAU, C1, C2, LDC, WORK)

CHARACTER * 1 SIDE
INTEGER M, N, INCV, LDC
DOUBLE PRECISION TAU
DOUBLE PRECISION V(*), C1(LDC,*), C2(LDC,*), WORK(*)

SUBROUTINE DLATZM_64(SIDE, M, N, V, INCV, TAU, C1, C2, LDC, WORK)

CHARACTER * 1 SIDE
INTEGER*8 M, N, INCV, LDC
DOUBLE PRECISION TAU
DOUBLE PRECISION V(*), C1(LDC,*), C2(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE LATZM(SIDE, [M], [N], V, [INCV], TAU, C1, C2, [LDC], [WORK])

CHARACTER(LEN=1) :: SIDE
INTEGER :: M, N, INCV, LDC
REAL(8) :: TAU
REAL(8), DIMENSION(:) :: V, WORK
REAL(8), DIMENSION(:,:) :: C1, C2

SUBROUTINE LATZM_64(SIDE, [M], [N], V, [INCV], TAU, C1, C2, [LDC],
       [WORK])

CHARACTER(LEN=1) :: SIDE
```

```
INTEGER(8) :: M, N, INCV, LDC
REAL(8) :: TAU
REAL(8), DIMENSION(:) :: V, WORK
REAL(8), DIMENSION(:,:) :: C1, C2
```

C INTERFACE

```
#include <sunperf.h>

void dlatzm(char side, int m, int n, double  *v,  int  incv,
          double tau, double *c1, double *c2, int ldc);

void dlatzm_64(char side, long m, long n,  double  *v,  long
          incv,  double  tau,  double  *c1, double *c2, long
          ldc);
```

PURPOSE

dlatzm routine is deprecated and has been replaced by routine SORMRZ.

SLATZM applies a Householder matrix generated by STZRQF to a matrix.

```
Let P = I - tau*u*u',   u = ( 1 ),
                            ( v )
where v is an (m-1) vector if SIDE = 'L', or a (n-1)  vector
if SIDE = 'R'.

If SIDE equals 'L', let
       C = [ C1 ] 1
           [ C2 ] m-1
             n
Then C is overwritten by P*C.

If SIDE equals 'R', let
       C = [ C1, C2 ] m
              1  n-1
Then C is overwritten by C*P.
```

ARGUMENTS

```
SIDE (input)
          = 'L': form P * C
          = 'R': form C * P
```

M (input) The number of rows of the matrix C.

N (input) The number of columns of the matrix C.

V (input) (1 + (M-1)*abs(INCV)) if SIDE = 'L' (1 + (N-1)*abs(INCV)) if SIDE = 'R' The vector v in the representation of P. V is not used if TAU = 0.

INCV (input)
The increment between elements of v. INCV <> 0

TAU (input)
The value tau in the representation of P.

C1 (input/output)
(LDC,N) if SIDE = 'L' (M,1) if SIDE = 'R' On entry, the n-vector C1 if SIDE = 'L', or the m-vector C1 if SIDE = 'R'.

On exit, the first row of P*C if SIDE = 'L', or the first column of C*P if SIDE = 'R'.

C2 (input/output)
(LDC, N) if SIDE = 'L' (LDC, N-1) if SIDE = 'R' On entry, the (m - 1) x n matrix C2 if SIDE = 'L', or the m x (n - 1) matrix C2 if SIDE = 'R'.

On exit, rows 2:m of P*C if SIDE = 'L', or columns 2:m of C*P if SIDE = 'R'.

LDC (input)
The leading dimension of the arrays C1 and C2. LDC >= (1,M).

WORK (workspace)
(N) if SIDE = 'L' (M) if SIDE = 'R'

Contents

NAME

```
dnrm2 - Return the Euclidian norm of a vector.
```

SYNOPSIS

```
DOUBLE PRECISION FUNCTION DNRM2(N, X, INCX)

INTEGER N, INCX
DOUBLE PRECISION X(*)

DOUBLE PRECISION FUNCTION DNRM2_64(N, X, INCX)

INTEGER*8 N, INCX
DOUBLE PRECISION X(*)
```

F95 INTERFACE

```
REAL(8) FUNCTION NRM2([N], X, [INCX])

INTEGER :: N, INCX
REAL(8), DIMENSION(:) :: X

REAL(8) FUNCTION NRM2_64([N], X, [INCX])

INTEGER(8) :: N, INCX
REAL(8), DIMENSION(:) :: X
```

C INTERFACE

```
#include <sunperf.h>

double dnrm2(int n, double *x, int incx);

double dnrm2_64(long n, double *x, long incx);
```

PURPOSE

dnrm2 Return the Euclidian norm of a vector x where x is an n-vector.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must be positive. Unchanged on exit.

Contents

NAME

```
dopgtr - generate  a  real  orthogonal  matrix  Q  which  is
defined  as the product of n-1 elementary reflectors H(i) of
order n, as returned by SSPTRD using packed storage
```

SYNOPSIS

```
SUBROUTINE DOPGTR(UPLO, N, AP, TAU, Q, LDQ, WORK, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDQ, INFO
DOUBLE PRECISION AP(*), TAU(*), Q(LDQ,*), WORK(*)

SUBROUTINE DOPGTR_64(UPLO, N, AP, TAU, Q, LDQ, WORK, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDQ, INFO
DOUBLE PRECISION AP(*), TAU(*), Q(LDQ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE OPGTR(UPLO, [N], AP, TAU, Q, [LDQ], [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDQ, INFO
REAL(8), DIMENSION(:) :: AP, TAU, WORK
REAL(8), DIMENSION(:,:) :: Q

SUBROUTINE OPGTR_64(UPLO, [N], AP, TAU, Q, [LDQ], [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDQ, INFO
REAL(8), DIMENSION(:) :: AP, TAU, WORK
REAL(8), DIMENSION(:,:) :: Q
```

C INTERFACE

```
#include <sunperf.h>

void dopgtr(char uplo, int n, double *ap, double *tau,  dou-
          ble *q, int ldq, int *info);

void dopgtr_64(char uplo, long n, double *ap,  double  *tau,
          double *q, long ldq, long *info);
```

PURPOSE

dopgtr generates a real orthogonal matrix Q which is defined as the product of n-1 elementary reflectors H(i) of order n, as returned by SSPTRD using packed storage:
if UPLO = 'U', Q = H(n-1) . . . H(2) H(1),

if UPLO = 'L', Q = H(1) H(2) . . . H(n-1).

ARGUMENTS

UPLO (input)
= 'U': Upper triangular packed storage used in previous call to SSPTRD; = 'L': Lower triangular packed storage used in previous call to SSPTRD.

N (input) The order of the matrix Q. N >= 0.

AP (input)
The vectors which define the elementary reflectors, as returned by SSPTRD.

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SSPTRD.

Q (output)
The N-by-N orthogonal matrix Q.

LDQ (input)
The leading dimension of the array Q. LDQ >= max(1,N).

WORK (workspace)
dimension(N-1)

INFO (output)

= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-
gal value

Contents

NAME

```
dopmtr - overwrite the general real  M-by-N  matrix  C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE DOPMTR(SIDE, UPLO, TRANS, M, N, AP, TAU, C, LDC, WORK,
      INFO)

CHARACTER * 1 SIDE, UPLO, TRANS
INTEGER M, N, LDC, INFO
DOUBLE PRECISION AP(*), TAU(*), C(LDC,*), WORK(*)

SUBROUTINE DOPMTR_64(SIDE, UPLO, TRANS, M, N, AP, TAU, C, LDC, WORK,
      INFO)

CHARACTER * 1 SIDE, UPLO, TRANS
INTEGER*8 M, N, LDC, INFO
DOUBLE PRECISION AP(*), TAU(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE OPMTR(SIDE, UPLO, [TRANS], [M], [N], AP, TAU, C, [LDC],
       [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANS
INTEGER :: M, N, LDC, INFO
REAL(8), DIMENSION(:) :: AP, TAU, WORK
REAL(8), DIMENSION(:,:) :: C

SUBROUTINE OPMTR_64(SIDE, UPLO, [TRANS], [M], [N], AP, TAU, C, [LDC],
       [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANS
```

```
INTEGER(8) :: M, N, LDC, INFO
REAL(8), DIMENSION(:) :: AP, TAU, WORK
REAL(8), DIMENSION(:,:) :: C
```

C INTERFACE

```
#include <sunperf.h>

void dopmtr(char side, char uplo, char trans, int m, int  n,
          double  *ap,  double *tau, double *c, int ldc, int
          *info);

void dopmtr_64(char side, char uplo,  char  trans,  long  m,
          long  n,  double *ap, double *tau, double *c, long
          ldc, long *info);
```

PURPOSE

dopmtr overwrites the general real M-by-N matrix C with
TRANS = 'T':   Q**T * C   C * Q**T

where Q is a real orthogonal matrix of order nq, with nq = m if SIDE = 'L' and nq = n if SIDE = 'R'. Q is defined as the product of nq-1 elementary reflectors, as returned by SSPTRD using packed storage:

if UPLO = 'U', Q = H(nq-1) . . . H(2) H(1);

if UPLO = 'L', Q = H(1) H(2) . . . H(nq-1).

ARGUMENTS

SIDE (input)
          = 'L': apply Q or Q**T from the Left;
          = 'R': apply Q or Q**T from the Right.

UPLO (input)
          = 'U': Upper triangular packed storage used in previous call to SSPTRD; = 'L': Lower triangular packed storage used in previous call to SSPTRD.

TRANS (input)
          = 'N': No transpose, apply Q;
          = 'T': Transpose, apply Q**T.

          TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

AP (input)
(M*(M+1)/2) if SIDE = 'L' (N*(N+1)/2) if SIDE = 'R' The vectors which define the elementary reflectors, as returned by SSPTRD. AP is modified by the routine but restored on exit.

TAU (input)
or (N-1) if SIDE = 'R' TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SSPTRD.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by Q*C or Q**T*C or C*Q**T or C*Q.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
(N) if SIDE = 'L' (M) if SIDE = 'R'

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dorg2l - generate an m by n real matrix Q  with  orthonormal
columns,
```

SYNOPSIS

```
SUBROUTINE DORG2L(M, N, K, A, LDA, TAU, WORK, INFO)

INTEGER M, N, K, LDA, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)

SUBROUTINE DORG2L_64(M, N, K, A, LDA, TAU, WORK, INFO)

INTEGER*8 M, N, K, LDA, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORG2L([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

INTEGER :: M, N, K, LDA, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE ORG2L_64([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

INTEGER(8) :: M, N, K, LDA, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dorg2l(int m, int n, int k, double *a, int lda,  double
```

```
          *tau, int *info);

void dorg2l_64(long m, long n, long k, double *a, long  lda,
          double *tau, long *info);
```

PURPOSE

dorg2l L generates an m by n real matrix Q with orthonormal columns, which is defined as the last n columns of a product of k elementary reflectors of order m

```
      Q  =  H(k) . . . H(2) H(1)
```

as returned by SGEQLF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. M >= N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. N >= K >= 0.

A (input/output)
On entry, the (n-k+i)-th column must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by SGEQLF in the last k columns of its array argument A. On exit, the m by n matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGEQLF.

WORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
dorg2r - generate an m by n real matrix Q  with  orthonormal
columns,
```

SYNOPSIS

```
SUBROUTINE DORG2R(M, N, K, A, LDA, TAU, WORK, INFO)

INTEGER M, N, K, LDA, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)

SUBROUTINE DORG2R_64(M, N, K, A, LDA, TAU, WORK, INFO)

INTEGER*8 M, N, K, LDA, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORG2R([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

INTEGER :: M, N, K, LDA, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE ORG2R_64([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

INTEGER(8) :: M, N, K, LDA, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dorg2r(int m, int n, int k, double *a, int lda,  double
```

```
          *tau, int *info);

void dorg2r_64(long m, long n, long k, double *a, long  lda,
          double *tau, long *info);
```

PURPOSE

dorg2r R generates an m by n real matrix Q with orthonormal columns, which is defined as the first n columns of a product of k elementary reflectors of order m

```
      Q  =  H(1) H(2) . . . H(k)
```

as returned by SGEQRF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. M >= N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. N >= K >= 0.

A (input/output)
On entry, the i-th column must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by SGEQRF in the first k columns of its array argument A. On exit, the m-by-n matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGEQRF.

WORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
dorgbr - generate one of the real orthogonal matrices  Q  or
P**T  determined  by SGEBRD when reducing a real matrix A to
bidiagonal form
```

SYNOPSIS

```
SUBROUTINE DORGBR(VECT, M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

CHARACTER * 1 VECT
INTEGER M, N, K, LDA, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)

SUBROUTINE DORGBR_64(VECT, M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

CHARACTER * 1 VECT
INTEGER*8 M, N, K, LDA, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORGBR(VECT, M, [N], K, A, [LDA], TAU, [WORK], [LWORK],
       [INFO])

CHARACTER(LEN=1) :: VECT
INTEGER :: M, N, K, LDA, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE ORGBR_64(VECT, M, [N], K, A, [LDA], TAU, [WORK], [LWORK],
       [INFO])

CHARACTER(LEN=1) :: VECT
INTEGER(8) :: M, N, K, LDA, LWORK, INFO
```

```
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dorgbr(char vect, int m, int n, int k, double  *a,  int
          lda, double *tau, int *info);

void dorgbr_64(char vect, long m, long n, long k, double *a,
          long lda, double *tau, long *info);
```

PURPOSE

dorgbr generates one of the real orthogonal matrices Q or P**T determined by SGEBRD when reducing a real matrix A to bidiagonal form: A = Q * B * P**T. Q and P**T are defined as products of elementary reflectors H(i) or G(i) respectively.

If VECT = 'Q', A is assumed to have been an M-by-K matrix, and Q is of order M:
if m >= k, Q = H(1) H(2) . . . H(k) and SORGBR returns the first n columns of Q, where m >= n >= k;
if m < k, Q = H(1) H(2) . . . H(m-1) and SORGBR returns Q as an M-by-M matrix.

If VECT = 'P', A is assumed to have been a K-by-N matrix, and P**T is of order N:
if k < n, P**T = G(k) . . . G(2) G(1) and SORGBR returns the first m rows of P**T, where n >= m >= k;
if k >= n, P**T = G(n-1) . . . G(2) G(1) and SORGBR returns P**T as an N-by-N matrix.

ARGUMENTS

VECT (input)
Specifies whether the matrix Q or the matrix P**T is required, as defined in the transformation applied by SGEBRD:
= 'Q': generate Q;
= 'P': generate P**T.

M (input) The number of rows of the matrix Q or P**T to be returned. M >= 0.

N (input) The number of columns of the matrix Q or P**T to

```
          be  returned.   N  >= 0.  If VECT = 'Q', M >= N >=
          min(M,K); if VECT = 'P', N >= M >= min(N,K).

K (input) If VECT = 'Q', the number of columns in the origi-
          nal  M-by-K  matrix  reduced by SGEBRD.  If VECT =
          'P', the number of rows  in  the  original  K-by-N
          matrix reduced by SGEBRD.  K >= 0.

A (input/output)
          On entry, the vectors which define the  elementary
          reflectors,  as  returned by SGEBRD.  On exit, the
          M-by-N matrix Q or P**T.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,M).

TAU (input)
          (min(M,K)) if VECT = 'Q' (min(N,K)) if VECT =  'P'
          TAU(i)  must contain the scalar factor of the ele-
          mentary reflector H(i) or G(i), which determines Q
          or  P**T, as returned by SGEBRD in its array argu-
          ment TAUQ or TAUP.

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The  dimension  of  the  array  WORK.   LWORK   >=
          max(1,min(M,N)).  For optimum performance LWORK >=
          min(M,N)*NB, where NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
dorghr - generate  a  real  orthogonal  matrix  Q  which  is
defined  as  the product of IHI-ILO elementary reflectors of
order N, as returned by SGEHRD
```

SYNOPSIS

```
SUBROUTINE DORGHR(N, ILO, IHI, A, LDA, TAU, WORK, LWORK, INFO)

INTEGER N, ILO, IHI, LDA, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)

SUBROUTINE DORGHR_64(N, ILO, IHI, A, LDA, TAU, WORK, LWORK, INFO)

INTEGER*8 N, ILO, IHI, LDA, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORGHR([N], ILO, IHI, A, [LDA], TAU, [WORK], [LWORK], [INFO])

INTEGER :: N, ILO, IHI, LDA, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE ORGHR_64([N], ILO, IHI, A, [LDA], TAU, [WORK], [LWORK],
       [INFO])

INTEGER(8) :: N, ILO, IHI, LDA, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dorghr(int n, int ilo, int ihi,  double  *a,  int  lda,
          double *tau, int *info);

void dorghr_64(long n, long ilo, long ihi, double  *a,  long
          lda, double *tau, long *info);
```

PURPOSE

dorghr generates a real orthogonal matrix Q which is defined as the product of IHI-ILO elementary reflectors of order N, as returned by SGEHRD:

Q = H(ilo) H(ilo+1) . . . H(ihi-1).

ARGUMENTS

N (input) The order of the matrix Q. N >= 0.

ILO (input)
ILO and IHI must have the same values as in the previous call of SGEHRD. Q is equal to the unit matrix except in the submatrix Q(ilo+1:ihi,ilo+1:ihi). 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

IHI (input)
See the description of ILO.

A (input/output)
On entry, the vectors which define the elementary reflectors, as returned by SGEHRD. On exit, the N-by-N orthogonal matrix Q.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGEHRD.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)

The dimension of the array WORK. LWORK >= IHI-ILO. For optimum performance LWORK >= (IHI-ILO)*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dorgl2 - generate an m by n real matrix Q  with  orthonormal
rows,
```

SYNOPSIS

```
SUBROUTINE DORGL2(M, N, K, A, LDA, TAU, WORK, INFO)

INTEGER M, N, K, LDA, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)

SUBROUTINE DORGL2_64(M, N, K, A, LDA, TAU, WORK, INFO)

INTEGER*8 M, N, K, LDA, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORGL2([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

INTEGER :: M, N, K, LDA, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE ORGL2_64([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

INTEGER(8) :: M, N, K, LDA, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dorgl2(int m, int n, int k, double *a, int lda,  double
```

```
          *tau, int *info);

void dorgl2_64(long m, long n, long k, double *a, long  lda,
          double *tau, long *info);
```

PURPOSE

dorgl2 generates an m by n real matrix Q with orthonormal rows, which is defined as the first m rows of a product of k elementary reflectors of order n

```
      Q  =  H(k) . . . H(2) H(1)
```

as returned by SGELQF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. N >= M.

K (input) The number of elementary reflectors whose product defines the matrix Q. M >= K >= 0.

A (input/output)
On entry, the i-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by SGELQF in the first k rows of its array argument A. On exit, the m-by-n matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGELQF.

WORK (workspace)
dimension(M)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
dorglq - generate an M-by-N real matrix Q  with  orthonormal
rows,
```

SYNOPSIS

```
SUBROUTINE DORGLQ(M, N, K, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER M, N, K, LDA, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)

SUBROUTINE DORGLQ_64(M, N, K, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER*8 M, N, K, LDA, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORGLQ(M, [N], [K], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER :: M, N, K, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE ORGLQ_64(M, [N], [K], A, [LDA], TAU, [WORK], [LDWORK],
       [INFO])

INTEGER(8) :: M, N, K, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dorglq(int m, int n, int k, double *a, int lda,  double
          *tau, int *info);

void dorglq_64(long m, long n, long k, double *a, long  lda,
          double *tau, long *info);
```

PURPOSE

dorglq generates an M-by-N real matrix Q with orthonormal rows, which is defined as the first M rows of a product of K elementary reflectors of order N

```
      Q  =  H(k) . . . H(2) H(1)
```

as returned by SGELQF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. N >= M.

K (input) The number of elementary reflectors whose product defines the matrix Q. M >= K >= 0.

A (input/output)
On entry, the i-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by SGELQF in the first k rows of its array argument A. On exit, the M-by-N matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGELQF.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK   >= max(1,M). For optimum performance LDWORK >= M*NB,

where NB is the optimal blocksize.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
dorgql - generate an M-by-N real matrix Q  with  orthonormal
columns,
```

SYNOPSIS

```
SUBROUTINE DORGQL(M, N, K, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER M, N, K, LDA, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)

SUBROUTINE DORGQL_64(M, N, K, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER*8 M, N, K, LDA, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORGQL(M, [N], [K], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER :: M, N, K, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE ORGQL_64(M, [N], [K], A, [LDA], TAU, [WORK], [LDWORK],
       [INFO])

INTEGER(8) :: M, N, K, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dorgql(int m, int n, int k, double *a, int lda,  double
          *tau, int *info);

void dorgql_64(long m, long n, long k, double *a, long  lda,
          double *tau, long *info);
```

PURPOSE

dorgql generates an M-by-N real matrix Q with orthonormal columns, which is defined as the last N columns of a product of K elementary reflectors of order M

```
      Q  =  H(k) . . . H(2) H(1)
```

as returned by SGEQLF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. M >= N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. N >= K >= 0.

A (input/output)
On entry, the (n-k+i)-th column must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by SGEQLF in the last k columns of its array argument A. On exit, the M-by-N matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGEQLF.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >=

```
             max(1,N).  For optimum performance LDWORK >= N*NB,
             where NB is the optimal blocksize.

             If LDWORK = -1, then a workspace query is assumed;
             the  routine  only  calculates the optimal size of
             the WORK array, returns this value  as  the  first
             entry  of  the  WORK  array,  and no error message
             related to LDWORK is issued by XERBLA.

   INFO (output)
             = 0:  successful exit
             < 0:  if INFO = -i, the i-th argument has an ille-
             gal value
```

Contents

NAME

```
dorgqr - generate an M-by-N real matrix Q  with  orthonormal
columns,
```

SYNOPSIS

```
SUBROUTINE DORGQR(M, N, K, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER M, N, K, LDA, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)

SUBROUTINE DORGQR_64(M, N, K, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER*8 M, N, K, LDA, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORGQR(M, [N], [K], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER :: M, N, K, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE ORGQR_64(M, [N], [K], A, [LDA], TAU, [WORK], [LDWORK],
       [INFO])

INTEGER(8) :: M, N, K, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dorgqr(int m, int n, int k, double *a, int lda,  double
          *tau, int *info);

void dorgqr_64(long m, long n, long k, double *a, long  lda,
          double *tau, long *info);
```

PURPOSE

```
dorgqr generates an M-by-N real matrix  Q  with  orthonormal
columns,  which  is defined as the first N columns of a pro-
duct of K elementary reflectors of order M

      Q  =  H(1) H(2) . . . H(k)

as returned by SGEQRF.
```

ARGUMENTS

```
M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. M >=  N  >=
          0.

K (input) The number of elementary reflectors whose  product
          defines the matrix Q. N >= K >= 0.

A (input/output)
          On entry, the i-th column must contain the  vector
          which defines the elementary reflector H(i), for i
          = 1,2,...,k, as returned by SGEQRF in the first  k
          columns  of its array argument A.  On exit, the M-
          by-N matrix Q.

LDA (input)
          The  first  dimension  of  the  array  A.  LDA  >=
          max(1,M).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i), as returned by SGEQRF.

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LDWORK.

LDWORK (input)
          The  dimension  of  the  array  WORK.  LDWORK   >=
```

max(1,N). For optimum performance LDWORK >= N*NB, where NB is the optimal blocksize.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
dorgr2 - generate an m by n real matrix Q  with  orthonormal
rows,
```

SYNOPSIS

```
SUBROUTINE DORGR2(M, N, K, A, LDA, TAU, WORK, INFO)

INTEGER M, N, K, LDA, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)

SUBROUTINE DORGR2_64(M, N, K, A, LDA, TAU, WORK, INFO)

INTEGER*8 M, N, K, LDA, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORGR2([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

INTEGER :: M, N, K, LDA, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE ORGR2_64([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

INTEGER(8) :: M, N, K, LDA, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dorgr2(int m, int n, int k, double *a, int lda,  double
```

```
          *tau, int *info);

void dorgr2_64(long m, long n, long k, double *a, long  lda,
          double *tau, long *info);
```

PURPOSE

dorgr2 generates an m by n real matrix Q with orthonormal rows, which is defined as the last m rows of a product of k elementary reflectors of order n

```
      Q  =  H(1) H(2) . . . H(k)
```

as returned by SGERQF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. N >= M.

K (input) The number of elementary reflectors whose product defines the matrix Q. M >= K >= 0.

A (input/output)
On entry, the (m-k+i)-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by SGERQF in the last k rows of its array argument A. On exit, the m by n matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGERQF.

WORK (workspace)
dimension(M)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
dorgrq - generate an M-by-N real matrix Q  with  orthonormal
rows,
```

SYNOPSIS

```
SUBROUTINE DORGRQ(M, N, K, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER M, N, K, LDA, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)

SUBROUTINE DORGRQ_64(M, N, K, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER*8 M, N, K, LDA, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORGRQ(M, [N], [K], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER :: M, N, K, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE ORGRQ_64(M, [N], [K], A, [LDA], TAU, [WORK], [LDWORK],
       [INFO])

INTEGER(8) :: M, N, K, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dorgrq(int m, int n, int k, double *a, int lda,  double
          *tau, int *info);

void dorgrq_64(long m, long n, long k, double *a, long  lda,
          double *tau, long *info);
```

PURPOSE

dorgrq generates an M-by-N real matrix Q with orthonormal rows, which is defined as the last M rows of a product of K elementary reflectors of order N

```
      Q  =  H(1) H(2) . . . H(k)
```

as returned by SGERQF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. N >= M.

K (input) The number of elementary reflectors whose product defines the matrix Q. M >= K >= 0.

A (input/output)
On entry, the (m-k+i)-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by SGERQF in the last k rows of its array argument A. On exit, the M-by-N matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGERQF.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,M). For optimum performance LDWORK >= M*NB,

```
            where NB is the optimal blocksize.

            If LDWORK = -1, then a workspace query is assumed;
            the  routine  only  calculates the optimal size of
            the WORK array, returns this value  as  the  first
            entry  of  the  WORK  array,  and no error message
            related to LDWORK is issued by XERBLA.

INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument has an ille-
            gal value
```

Contents

NAME

```
dorgtr - generate  a  real  orthogonal  matrix  Q  which  is
defined as the product of n-1 elementary reflectors of order
N, as returned by SSYTRD
```

SYNOPSIS

```
SUBROUTINE DORGTR(UPLO, N, A, LDA, TAU, WORK, LWORK, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)

SUBROUTINE DORGTR_64(UPLO, N, A, LDA, TAU, WORK, LWORK, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORGTR(UPLO, [N], A, [LDA], TAU, [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE ORGTR_64(UPLO, [N], A, [LDA], TAU, [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dorgtr(char uplo, int n, double  *a,  int  lda,  double
          *tau, int *info);

void dorgtr_64(char uplo, long n, double *a, long lda,  dou-
          ble *tau, long *info);
```

PURPOSE

dorgtr generates a real orthogonal matrix Q which is defined as the product of n-1 elementary reflectors of order N, as returned by SSYTRD:
if UPLO = 'U', Q = H(n-1) . . . H(2) H(1),

if UPLO = 'L', Q = H(1) H(2) . . . H(n-1).

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A contains elementary reflectors from SSYTRD; = 'L': Lower triangle of A contains elementary reflectors from SSYTRD.

N (input) The order of the matrix Q. N >= 0.

A (input/output)
On entry, the vectors which define the elementary reflectors, as returned by SSYTRD. On exit, the N-by-N orthogonal matrix Q.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SSYTRD.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >=

max(1,N-1). For optimum performance LWORK >= (N-1)*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dormbr - VECT = 'Q', SORMBR overwrites the general  real  M-
by-N matrix C with  SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE DORMBR(VECT, SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 VECT, SIDE, TRANS
INTEGER M, N, K, LDA, LDC, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), C(LDC,*), WORK(*)

SUBROUTINE DORMBR_64(VECT, SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 VECT, SIDE, TRANS
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORMBR(VECT, SIDE, [TRANS], [M], [N], K, A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: VECT, SIDE, TRANS
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A, C

SUBROUTINE ORMBR_64(VECT, SIDE, [TRANS], [M], [N], K, A, [LDA], TAU,
       C, [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: VECT, SIDE, TRANS
```

```
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A, C
```

C INTERFACE

```
#include <sunperf.h>

void dormbr(char vect, char side, char trans, int m, int  n,
          int k, double *a, int lda, double *tau, double *c,
          int ldc, int *info);

void dormbr_64(char vect, char side,  char  trans,  long  m,
          long  n, long k, double *a, long lda, double *tau,
          double *c, long ldc, long *info);
```

PURPOSE

```
dormbr VECT = 'Q', SORMBR overwrites the general real M-by-N
matrix C with
                SIDE = 'L'      SIDE  =  'R'  TRANS  =  'N':
Q  *  C           C * Q TRANS = 'T':      Q**T * C       C *
Q**T

If VECT = 'P', SORMBR overwrites  the  general  real  M-by-N
matrix C with
                SIDE = 'L'     SIDE = 'R'
TRANS = 'N':      P * C          C * P
TRANS = 'T':      P**T * C       C * P**T
```

Here Q and P**T are the orthogonal matrices determined by SGEBRD when reducing a real matrix A to bidiagonal form: A = Q * B * P**T. Q and P**T are defined as products of elementary reflectors H(i) and G(i) respectively.

Let nq = m if SIDE = 'L' and nq = n if SIDE = 'R'. Thus nq is the order of the orthogonal matrix Q or P**T that is applied.

```
If VECT = 'Q', A is assumed to have been an NQ-by-K  matrix:
if nq >= k, Q = H(1) H(2) . . . H(k);
if nq < k, Q = H(1) H(2) . . . H(nq-1).

If VECT = 'P', A is assumed to have been a  K-by-NQ  matrix:
if k < nq, P = G(1) G(2) . . . G(k);
if k >= nq, P = G(1) G(2) . . . G(nq-1).
```

ARGUMENTS

```
VECT (input)
          = 'Q': apply Q or Q**T;
          = 'P': apply P or P**T.

SIDE (input)
          = 'L': apply Q, Q**T, P or P**T from the Left;
          = 'R': apply Q, Q**T, P or P**T from the Right.

TRANS (input)
          = 'N':  No transpose, apply Q  or P;
          = 'T':  Transpose, apply Q**T or P**T.

          TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.
N (input) The number of columns of the matrix C. N >= 0.

K (input) If VECT = 'Q', the number of columns in the origi-
          nal  matrix reduced by SGEBRD.  If VECT = 'P', the
          number of rows in the original matrix  reduced  by
          SGEBRD.  K >= 0.

A (input) (LDA,min(nq,K)) if VECT = 'Q'  (LDA,nq)         if
          VECT = 'P' The vectors which define the elementary
          reflectors H(i) and G(i), whose products determine
          the matrices Q and P, as returned by SGEBRD.

LDA (input)
          The leading dimension of the array A.  If  VECT  =
          'Q',  LDA  >=  max(1,nq);  if  VECT  = 'P', LDA >=
          max(1,min(nq,K)).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary  reflector H(i) or G(i) which determines Q
          or P, as returned by SGEBRD in the array  argument
          TAUQ or TAUP.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten  by  Q*C or Q**T*C or C*Q**T or C*Q or
          P*C or P**T*C or C*P or C*P**T.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
```

LWORK.

LWORK (input)

The dimension of the array WORK. If SIDE = 'L', LWORK >= max(1,N); if SIDE = 'R', LWORK >= max(1,M). For optimum performance LWORK >= N*NB if SIDE = 'L', and LWORK >= M*NB if SIDE = 'R', where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dormhr - overwrite the general real  M-by-N  matrix  C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE DORMHR(SIDE, TRANS, M, N, ILO, IHI, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER M, N, ILO, IHI, LDA, LDC, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), C(LDC,*), WORK(*)

SUBROUTINE DORMHR_64(SIDE, TRANS, M, N, ILO, IHI, A, LDA, TAU, C,
      LDC, WORK, LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER*8 M, N, ILO, IHI, LDA, LDC, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORMHR(SIDE, [TRANS], [M], [N], ILO, IHI, A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
INTEGER :: M, N, ILO, IHI, LDA, LDC, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A, C

SUBROUTINE ORMHR_64(SIDE, [TRANS], [M], [N], ILO, IHI, A, [LDA], TAU,
       C, [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
INTEGER(8) :: M, N, ILO, IHI, LDA, LDC, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A, C
```

C INTERFACE

```
#include <sunperf.h>

void dormhr(char side, char trans, int m, int  n,  int  ilo,
          int  ihi,  double *a, int lda, double *tau, double
          *c, int ldc, int *info);

void dormhr_64(char side, char trans, long m, long  n,  long
          ilo,  long  ihi, double *a, long lda, double *tau,
          double *c, long ldc, long *info);
```

PURPOSE

dormhr overwrites the general real M-by-N matrix C with
TRANS = 'T':   Q**T * C   C * Q**T

where Q is a real orthogonal matrix of order nq, with nq = m if SIDE = 'L' and nq = n if SIDE = 'R'. Q is defined as the product of IHI-ILO elementary reflectors, as returned by SGEHRD:

Q = H(ilo) H(ilo+1) . . . H(ihi-1).

ARGUMENTS

SIDE (input)
= 'L': apply Q or Q**T from the Left;
= 'R': apply Q or Q**T from the Right.

TRANS (input)
= 'N': No transpose, apply Q;
= 'T': Transpose, apply Q**T.

TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

ILO (input)
ILO and IHI must have the same values as in the previous call of SGEHRD. Q is equal to the unit matrix   except   in   the   submatrix

```
          Q(ilo+1:ihi,ilo+1:ihi).   If SIDE = 'L', then 1 <=
          ILO <= IHI <= M, if M > 0, and ILO = 1 and  IHI  =
          0,  if  M = 0; if SIDE = 'R', then 1 <= ILO <= IHI
          <= N, if N > 0, and ILO = 1 and IHI = 0, if N = 0.

IHI (input)
          See the description of ILO.

A (input) (LDA,M) if SIDE = 'L' (LDA,N) if SIDE  =  'R'  The
          vectors which define the elementary reflectors, as
          returned by SGEHRD.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,M)  if SIDE = 'L'; LDA >= max(1,N) if SIDE =
          'R'.

TAU (input)
          (M-1) if SIDE = 'L' (N-1) if  SIDE  =  'R'  TAU(i)
          must  contain  the scalar factor of the elementary
          reflector H(i), as returned by SGEHRD.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten by Q*C or Q**T*C or C*Q**T or C*Q.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK.  If SIDE  =  'L',
          LWORK  >=  max(1,N);  if  SIDE  =  'R',  LWORK  >=
          max(1,M).  For optimum performance LWORK  >=  N*NB
          if  SIDE  =  'L', and LWORK >= M*NB if SIDE = 'R',
          where NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
```

gal value

Contents

NAME

```
dormlq - overwrite the general real  M-by-N  matrix  C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE DORMLQ(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER M, N, K, LDA, LDC, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), C(LDC,*), WORK(*)

SUBROUTINE DORMLQ_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORMLQ(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A, C

SUBROUTINE ORMLQ_64(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A, C
```

C INTERFACE

```
#include <sunperf.h>

void dormlq(char side, char trans, int m, int n, int k, dou-
          ble  *a, int lda, double *tau, double *c, int ldc,
          int *info);

void dormlq_64(char side, char trans, long m, long  n,  long
          k,  double  *a,  long lda, double *tau, double *c,
          long ldc, long *info);
```

PURPOSE

```
dormlq overwrites the general  real  M-by-N  matrix  C  with
TRANS = 'T':      Q**T * C       C * Q**T

where Q is a real orthogonal matrix defined as  the  product
of k elementary reflectors

      Q = H(k) . . . H(2) H(1)

as returned by SGELQF. Q is of order M if SIDE = 'L' and  of
order N if SIDE = 'R'.
```

ARGUMENTS

```
SIDE (input)
          = 'L': apply Q or Q**T from the Left;
          = 'R': apply Q or Q**T from the Right.

TRANS (input)
          = 'N':  No transpose, apply Q;
          = 'T':  Transpose, apply Q**T.

          TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose  product
          defines the matrix Q.  If SIDE = 'L', M >= K >= 0;
          if SIDE = 'R', N >= K >= 0.
```

```
A (input) (LDA,M) if SIDE = 'L', (LDA,N) if SIDE =  'R'  The
          i-th row must contain the vector which defines the
          elementary reflector H(i), for i =  1,2,...,k,  as
          returned  by  SGELQF  in  the  first k rows of its
          array argument A.  A is modified  by  the  routine
          but restored on exit.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,K).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i), as returned by SGELQF.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten by Q*C or Q**T*C or C*Q**T or C*Q.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK.  If SIDE  =  'L',
          LWORK  >=  max(1,N);  if  SIDE  =  'R',  LWORK  >=
          max(1,M).  For optimum performance LWORK  >=  N*NB
          if  SIDE  =  'L', and LWORK >= M*NB if SIDE = 'R',
          where NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
dormql - overwrite the general real  M-by-N  matrix  C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE DORMQL(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER M, N, K, LDA, LDC, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), C(LDC,*), WORK(*)

SUBROUTINE DORMQL_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORMQL(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A, C

SUBROUTINE ORMQL_64(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A, C
```

C INTERFACE

```
#include <sunperf.h>

void dormql(char side, char trans, int m, int n, int k, dou-
          ble  *a, int lda, double *tau, double *c, int ldc,
          int *info);

void dormql_64(char side, char trans, long m, long  n,  long
          k,  double  *a,  long lda, double *tau, double *c,
          long ldc, long *info);
```

PURPOSE

```
dormql overwrites the general  real  M-by-N  matrix  C  with
TRANS = 'T':      Q**T * C       C * Q**T

where Q is a real orthogonal matrix defined as  the  product
of k elementary reflectors

      Q = H(k) . . . H(2) H(1)

as returned by SGEQLF. Q is of order M if SIDE = 'L' and  of
order N if SIDE = 'R'.
```

ARGUMENTS

```
SIDE (input)
          = 'L': apply Q or Q**T from the Left;
          = 'R': apply Q or Q**T from the Right.

TRANS (input)
          = 'N':  No transpose, apply Q;
          = 'T':  Transpose, apply Q**T.

          TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose  product
          defines the matrix Q.  If SIDE = 'L', M >= K >= 0;
          if SIDE = 'R', N >= K >= 0.
```

```
A (input) The i-th column  must  contain  the  vector  which
          defines  the  elementary  reflector  H(i), for i =
          1,2,...,k, as returned by SGEQLF  in  the  last  k
          columns of its array argument A.  A is modified by
          the routine but restored on exit.

LDA (input)
          The leading dimension of the array A.  If  SIDE  =
          'L',  LDA  >=  max(1,M);  if  SIDE  =  'R', LDA >=
          max(1,N).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i), as returned by SGEQLF.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten by Q*C or Q**T*C or C*Q**T or C*Q.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK.  If SIDE  =  'L',
          LWORK  >=  max(1,N);  if  SIDE  =  'R',  LWORK  >=
          max(1,M).  For optimum performance LWORK  >=  N*NB
          if  SIDE  =  'L', and LWORK >= M*NB if SIDE = 'R',
          where NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
dormqr - overwrite the general real  M-by-N  matrix  C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE DORMQR(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER M, N, K, LDA, LDC, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), C(LDC,*), WORK(*)

SUBROUTINE DORMQR_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORMQR(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A, C

SUBROUTINE ORMQR_64(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A, C
```

C INTERFACE

```
#include <sunperf.h>

void dormqr(char side, char trans, int m, int n, int k, dou-
          ble  *a, int lda, double *tau, double *c, int ldc,
          int *info);

void dormqr_64(char side, char trans, long m, long  n,  long
          k,  double  *a,  long lda, double *tau, double *c,
          long ldc, long *info);
```

PURPOSE

dormqr overwrites the general real M-by-N matrix C with
TRANS = 'T': Q**T * C C * Q**T

where Q is a real orthogonal matrix defined as the product of k elementary reflectors

Q = H(1) H(2) . . . H(k)

as returned by SGEQRF. Q is of order M if SIDE = 'L' and of order N if SIDE = 'R'.

ARGUMENTS

SIDE (input)
= 'L': apply Q or Q**T from the Left;
= 'R': apply Q or Q**T from the Right.

TRANS (input)
= 'N': No transpose, apply Q;
= 'T': Transpose, apply Q**T.

TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. If SIDE = 'L', M >= K >= 0; if SIDE = 'R', N >= K >= 0.

```
A (input) The i-th column  must  contain  the  vector  which
          defines  the  elementary  reflector  H(i), for i =
          1,2,...,k, as returned by SGEQRF in  the  first  k
          columns of its array argument A.  A is modified by
          the routine but restored on exit.

LDA (input)
          The leading dimension of the array A.  If  SIDE  =
          'L',  LDA  >=  max(1,M);  if  SIDE  =  'R', LDA >=
          max(1,N).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i), as returned by SGEQRF.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten by Q*C or Q**T*C or C*Q**T or C*Q.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK.  If SIDE  =  'L',
          LWORK  >=  max(1,N);  if  SIDE  =  'R',  LWORK  >=
          max(1,M).  For optimum performance LWORK  >=  N*NB
          if  SIDE  =  'L', and LWORK >= M*NB if SIDE = 'R',
          where NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
dormrq - overwrite the general real  M-by-N  matrix  C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE DORMRQ(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER M, N, K, LDA, LDC, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), C(LDC,*), WORK(*)

SUBROUTINE DORMRQ_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORMRQ(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A, C

SUBROUTINE ORMRQ_64(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A, C
```

C INTERFACE

```
#include <sunperf.h>

void dormrq(char side, char trans, int m, int n, int k, dou-
          ble  *a, int lda, double *tau, double *c, int ldc,
          int *info);

void dormrq_64(char side, char trans, long m, long  n,  long
          k,  double  *a,  long lda, double *tau, double *c,
          long ldc, long *info);
```

PURPOSE

dormrq overwrites the general real M-by-N matrix C with
TRANS = 'T':   Q**T * C   C * Q**T

where Q is a real orthogonal matrix defined as the product of k elementary reflectors

      Q = H(1) H(2) . . . H(k)

as returned by SGERQF. Q is of order M if SIDE = 'L' and of order N if SIDE = 'R'.

ARGUMENTS

SIDE (input)
= 'L': apply Q or Q**T from the Left;
= 'R': apply Q or Q**T from the Right.

TRANS (input)
= 'N': No transpose, apply Q;
= 'T': Transpose, apply Q**T.

TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. If SIDE = 'L', M >= K >= 0; if SIDE = 'R', N >= K >= 0.

```
A (input) (LDA,M) if SIDE = 'L', (LDA,N) if SIDE =  'R'  The
          i-th row must contain the vector which defines the
          elementary reflector H(i), for i =  1,2,...,k,  as
          returned by SGERQF in the last k rows of its array
          argument A.  A is  modified  by  the  routine  but
          restored on exit.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,K).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i), as returned by SGERQF.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten by Q*C or Q**T*C or C*Q**T or C*Q.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK.  If SIDE  =  'L',
          LWORK  >=  max(1,N);  if  SIDE  =  'R',  LWORK  >=
          max(1,M).  For optimum performance LWORK  >=  N*NB
          if  SIDE  =  'L', and LWORK >= M*NB if SIDE = 'R',
          where NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
dormrz - overwrite the general real  M-by-N  matrix  C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE DORMRZ(SIDE, TRANS, M, N, K, L, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER M, N, K, L, LDA, LDC, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), C(LDC,*), WORK(*)

SUBROUTINE DORMRZ_64(SIDE, TRANS, M, N, K, L, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER*8 M, N, K, L, LDA, LDC, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORMRZ(SIDE, TRANS, [M], [N], K, L, A, [LDA], TAU, C, [LDC],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
INTEGER :: M, N, K, L, LDA, LDC, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A, C

SUBROUTINE ORMRZ_64(SIDE, TRANS, [M], [N], K, L, A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])
```

```
CHARACTER(LEN=1) :: SIDE, TRANS
INTEGER(8) :: M, N, K, L, LDA, LDC, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A, C
```

C INTERFACE

```
#include <sunperf.h>

void dormrz(char side, char trans, int m, int n, int k,  int
          l, double *a, int lda, double *tau, double *c, int
          ldc, int *info);

void dormrz_64(char side, char trans, long m, long  n,  long
          k,  long l, double *a, long lda, double *tau, dou-
          ble *c, long ldc, long *info);
```

PURPOSE

dormrz overwrites the general real M-by-N matrix C with
TRANS = 'T':   Q**T * C   C * Q**T

where Q is a real orthogonal matrix defined as the product of k elementary reflectors

      Q = H(1) H(2) . . . H(k)

as returned by STZRZF. Q is of order M if SIDE = 'L' and of order N if SIDE = 'R'.

ARGUMENTS

SIDE (input)
= 'L': apply Q or Q**T from the Left;
= 'R': apply Q or Q**T from the Right.

TRANS (input)
= 'N': No transpose, apply Q;
= 'T': Transpose, apply Q**T.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. If SIDE = 'L', M >= K >= 0; if SIDE = 'R', N >= K >= 0.

```
L (input) The number of columns of the matrix  A  containing
          the meaningful part of the Householder reflectors.
          If SIDE = 'L', M >= L >= 0, if SIDE = 'R', N >=  L
          >= 0.

A (input) (LDA,M) if SIDE = 'L', (LDA,N) if SIDE =  'R'  The
          i-th row must contain the vector which defines the
          elementary reflector H(i), for i =  1,2,...,k,  as
          returned by STZRZF in the last k rows of its array
          argument A.  A is  modified  by  the  routine  but
          restored on exit.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,K).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i), as returned by STZRZF.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten by Q*C or Q**H*C or C*Q**H or C*Q.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK.  If SIDE  =  'L',
          LWORK  >=  max(1,N);  if  SIDE  =  'R',  LWORK  >=
          max(1,M).  For optimum performance LWORK  >=  N*NB
          if  SIDE  =  'L', and LWORK >= M*NB if SIDE = 'R',
          where NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

FURTHER DETAILS

```
Based on contributions by
  A. Petitet, Computer Science Dept., Univ. of Tenn.,  Knox-
ville, USA
```

Contents

NAME

```
dormtr - overwrite the general real  M-by-N  matrix  C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE DORMTR(SIDE, UPLO, TRANS, M, N, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, UPLO, TRANS
INTEGER M, N, LDA, LDC, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), C(LDC,*), WORK(*)

SUBROUTINE DORMTR_64(SIDE, UPLO, TRANS, M, N, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 SIDE, UPLO, TRANS
INTEGER*8 M, N, LDA, LDC, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORMTR(SIDE, UPLO, [TRANS], [M], [N], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANS
INTEGER :: M, N, LDA, LDC, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A, C

SUBROUTINE ORMTR_64(SIDE, UPLO, [TRANS], [M], [N], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANS
```

```
INTEGER(8) :: M, N, LDA, LDC, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A, C
```

C INTERFACE

```
#include <sunperf.h>

void dormtr(char side, char uplo, char trans, int m, int  n,
          double  *a,  int  lda, double *tau, double *c, int
          ldc, int *info);

void dormtr_64(char side, char uplo,  char  trans,  long  m,
          long  n,  double *a, long lda, double *tau, double
          *c, long ldc, long *info);
```

PURPOSE

```
dormtr overwrites the general  real  M-by-N  matrix  C  with
TRANS = 'T':      Q**T * C       C * Q**T

where Q is a real orthogonal matrix of order nq, with nq = m
if  SIDE = 'L' and nq = n if SIDE = 'R'. Q is defined as the
product  of  nq-1  elementary  reflectors,  as  returned  by
SSYTRD:

if UPLO = 'U', Q = H(nq-1) . . . H(2) H(1);

if UPLO = 'L', Q = H(1) H(2) . . . H(nq-1).
```

ARGUMENTS

```
SIDE (input)
          = 'L': apply Q or Q**T from the Left;
          = 'R': apply Q or Q**T from the Right.

UPLO (input)
          = 'U': Upper triangle  of  A  contains  elementary
          reflectors from SSYTRD; = 'L': Lower triangle of A
          contains elementary reflectors from SSYTRD.

TRANS (input)
          = 'N':  No transpose, apply Q;
          = 'T':  Transpose, apply Q**T.

          TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.
```

```
N (input) The number of columns of the matrix C. N >= 0.

A (input) (LDA,M) if SIDE = 'L' (LDA,N) if SIDE  =  'R'  The
          vectors which define the elementary reflectors, as
          returned by SSYTRD.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,M)  if SIDE = 'L'; LDA >= max(1,N) if SIDE =
          'R'.

TAU (input)
          (M-1) if SIDE = 'L' (N-1) if  SIDE  =  'R'  TAU(i)
          must  contain  the scalar factor of the elementary
          reflector H(i), as returned by SSYTRD.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten by Q*C or Q**T*C or C*Q**T or C*Q.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK.  If SIDE  =  'L',
          LWORK  >=  max(1,N);  if  SIDE  =  'R',  LWORK  >=
          max(1,M).  For optimum performance LWORK  >=  N*NB
          if  SIDE  =  'L', and LWORK >= M*NB if SIDE = 'R',
          where NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
dpbcon - estimate the reciprocal of the condition number (in
the  1-norm)  of  a  real  symmetric  positive definite band
matrix using the Cholesky factorization A = U**T*U  or  A  =
L*L**T computed by SPBTRF
```

SYNOPSIS

```
SUBROUTINE DPBCON(UPLO, N, KD, A, LDA, ANORM, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 UPLO
INTEGER N, KD, LDA, INFO
INTEGER WORK2(*)
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION A(LDA,*), WORK(*)

SUBROUTINE DPBCON_64(UPLO, N, KD, A, LDA, ANORM, RCOND, WORK,
      WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, KD, LDA, INFO
INTEGER*8 WORK2(*)
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PBCON(UPLO, [N], KD, A, [LDA], ANORM, RCOND, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, KD, LDA, INFO
INTEGER, DIMENSION(:) :: WORK2
```

```
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE PBCON_64(UPLO, [N], KD, A, [LDA], ANORM, RCOND, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, KD, LDA, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dpbcon(char uplo, int n, int kd, double  *a,  int  lda,
          double anorm, double *rcond, int *info);

void dpbcon_64(char uplo, long n, long kd, double  *a,  long
          lda, double anorm, double *rcond, long *info);
```

PURPOSE

dpbcon estimates the reciprocal of the condition number (in the 1-norm) of a real symmetric positive definite band matrix using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPBTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangular factor stored in A;
          = 'L':  Lower triangular factor stored in A.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  KD >= 0.

A (input) The triangular factor U or  L  from  the  Cholesky
```

factorization A = U**T*U or A = L*L**T of the band matrix A, stored in the first KD+1 rows of the array. The j-th column of U or L is stored in the j-th column of the array A as follows: if UPLO ='U', A(kd+1+i-j,j) = U(i,j) for max(1,j-kd)<=i<=j; if UPLO ='L', A(1+i-j,j) = L(i,j) for j<=i<=min(n,j+kd).

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

ANORM (input)
The 1-norm (or infinity-norm) of the symmetric band matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

dpbequ - compute row and column scalings intended to equilibrate a symmetric positive definite band matrix A and reduce its condition number (with respect to the two-norm)

SYNOPSIS

```
SUBROUTINE DPBEQU(UPLO, N, KD, A, LDA, SCALE, SCOND, AMAX, INFO)

CHARACTER * 1 UPLO
INTEGER N, KD, LDA, INFO
DOUBLE PRECISION SCOND, AMAX
DOUBLE PRECISION A(LDA,*), SCALE(*)

SUBROUTINE DPBEQU_64(UPLO, N, KD, A, LDA, SCALE, SCOND, AMAX,
      INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, KD, LDA, INFO
DOUBLE PRECISION SCOND, AMAX
DOUBLE PRECISION A(LDA,*), SCALE(*)
```

F95 INTERFACE

```
SUBROUTINE PBEQU(UPLO, [N], KD, A, [LDA], SCALE, SCOND, AMAX,
       [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, KD, LDA, INFO
REAL(8) :: SCOND, AMAX
REAL(8), DIMENSION(:) :: SCALE
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE PBEQU_64(UPLO, [N], KD, A, [LDA], SCALE, SCOND, AMAX,
```

```
       [INFO])

    CHARACTER(LEN=1) :: UPLO
    INTEGER(8) :: N, KD, LDA, INFO
    REAL(8) :: SCOND, AMAX
    REAL(8), DIMENSION(:) :: SCALE
    REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
    #include <sunperf.h>

    void dpbequ(char uplo, int n, int kd, double  *a,  int  lda,
              double  *scale,  double  *scond, double *amax, int
              *info);
    void dpbequ_64(char uplo, long n, long kd, double  *a,  long
              lda,  double  *scale, double *scond, double *amax,
              long *info);
```

PURPOSE

dpbequ computes row and column scalings intended to equilibrate a symmetric positive definite band matrix A and reduce its condition number (with respect to the two-norm). S contains the scale factors, S(i) = 1/sqrt(A(i,i)), chosen so that the scaled matrix B with elements B(i,j) = S(i)*A(i,j)*S(j) has ones on the diagonal. This choice of S puts the condition number of B within a factor N of the smallest possible condition number over all possible diagonal scalings.

ARGUMENTS

UPLO (input)
= 'U': Upper triangular of A is stored;
= 'L': Lower triangular of A is stored.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

A (input) The upper or lower triangle of the symmetric band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

SCALE (output)
If INFO = 0, SCALE contains the scale factors for A.

SCOND (output)
If INFO = 0, SCALE contains the ratio of the smallest SCALE(i) to the largest SCALE(i). If SCOND >= 0.1 and AMAX is neither too large nor too small, it is not worth scaling by SCALE.

AMAX (output)
Absolute value of largest matrix element. If AMAX is very close to overflow or very close to underflow, the matrix should be scaled.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, the i-th diagonal element is nonpositive.

Contents

NAME

```
dpbrfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is symmetric positive
definite and banded, and provides error bounds and  backward
error estimates for the solution
```

SYNOPSIS

```
SUBROUTINE DPBRFS(UPLO, N, KD, NRHS, A, LDA, AF, LDAF, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER N, KD, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER WORK2(*)
DOUBLE PRECISION A(LDA,*), AF(LDAF,*),  B(LDB,*),  X(LDX,*),
FERR(*), BERR(*), WORK(*)

SUBROUTINE DPBRFS_64(UPLO, N, KD, NRHS, A, LDA, AF, LDAF, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, KD, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 WORK2(*)
DOUBLE PRECISION A(LDA,*), AF(LDAF,*),  B(LDB,*),  X(LDX,*),
FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PBRFS(UPLO, [N], KD, [NRHS], A, [LDA], AF, [LDAF], B,
       [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, KD, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: WORK2
```

```
REAL(8), DIMENSION(:) :: FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: A, AF, B, X

SUBROUTINE PBRFS_64(UPLO, [N], KD, [NRHS], A, [LDA], AF, [LDAF],
       B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, KD, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL(8), DIMENSION(:) :: FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
#include <sunperf.h>
void dpbrfs(char uplo, int n, int kd, int nrhs,  double  *a,
          int lda, double *af, int ldaf, double *b, int ldb,
          double *x, int ldx, double  *ferr,  double  *berr,
          int *info);

void dpbrfs_64(char uplo, long n, long kd, long nrhs, double
          *a,  long  lda,  double *af, long ldaf, double *b,
          long ldb, double *x, long ldx, double *ferr,  dou-
          ble *berr, long *info);
```

PURPOSE

dpbrfs improves the computed solution to a system of linear equations when the coefficient matrix is symmetric positive definite and banded, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The upper or lower triangle of the symmetric band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

AF (input)
The triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T of the band matrix A as computed by SPBTRF, in the same storage format as A (see A).

LDAF (input)
The leading dimension of the array AF. LDAF >= KD+1.

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by SPBTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative

```
            change in any element of A or B that makes X(j) an
            exact solution).

WORK (workspace)
            dimension(3*N)
WORK2 (workspace)
            dimension(N)

INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value
```

Contents

NAME

dpbstf - compute a split Cholesky factorization of a real symmetric positive definite band matrix A

SYNOPSIS

```
SUBROUTINE DPBSTF(UPLO, N, KD, AB, LDAB, INFO)

CHARACTER * 1 UPLO
INTEGER N, KD, LDAB, INFO
DOUBLE PRECISION AB(LDAB,*)

SUBROUTINE DPBSTF_64(UPLO, N, KD, AB, LDAB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, KD, LDAB, INFO
DOUBLE PRECISION AB(LDAB,*)
```

F95 INTERFACE

```
SUBROUTINE PBSTF(UPLO, [N], KD, AB, [LDAB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, KD, LDAB, INFO
REAL(8), DIMENSION(:,:) :: AB

SUBROUTINE PBSTF_64(UPLO, [N], KD, AB, [LDAB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, KD, LDAB, INFO
REAL(8), DIMENSION(:,:) :: AB
```

C INTERFACE

```
#include <sunperf.h>

void dpbstf(char uplo, int n, int kd, double *ab, int  ldab,
          int *info);

void dpbstf_64(char uplo, long n, long kd, double *ab,  long
          ldab, long *info);
```

PURPOSE

dpbstf computes a split Cholesky factorization of a real symmetric positive definite band matrix A.

This routine is designed to be used in conjunction with SSBGST.
The factorization has the form A = S**T*S   where S is a band matrix of the same bandwidth as A and the following structure:

```
  S = ( U    )
      ( M  L )
```

where U is upper triangular of order m = (n+kd)/2, and L is lower triangular of order n-m.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

AB (input/output)
On entry, the upper or lower triangle of the symmetric band matrix A, stored in the first kd+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', AB(1+i-j,j)   = A(i,j) for j<=i<=min(n,j+kd).

On exit, if INFO = 0, the factor S from the split Cholesky factorization A = S**T*S. See Further Details.

LDAB (input)
The leading dimension of the array AB. LDAB >= KD+1.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the factorization could not be completed, because the updated element a(i,i) was negative; the matrix A is not positive definite.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when N = 7, KD = 2:

```
S = ( s11  s12  s13                     )
    (      s22  s23  s24                )
    (           s33  s34                )
    (                s44                )
    (           s53  s54  s55           )
    (                s64  s65  s66      )
    (                     s75  s76  s77 )
```

If UPLO = 'U', the array AB holds:

```
on entry:                          on exit:

 *    *   a13  a24  a35  a46  a57   *    *   s13   s24   s53
s64  s75
 *   a12  a23  a34  a45  a56  a67   *   s12  s23   s34   s54
s65   s76  a11   a22  a33  a44  a55  a66  a77  s11  s22  s33
s44  s55  s66  s77
```

If UPLO = 'L', the array AB holds:

```
on entry:                          on exit:

a11  a22  a33  a44  a55  a66  a77  s11  s22  s33   s44   s55
s66   s77  a21   a32  a43  a54  a65  a76   *   s12  s23  s34
s54  s65  s76   * a31  a42  a53  a64   a64    *     *    s13
s24  s53  s64  s75   *    *
```

Array elements marked * are not used by the routine.

Contents

NAME

```
dpbsv - compute the solution to  a  real  system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE DPBSV(UPLO, N, NDIAG, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER N, NDIAG, NRHS, LDA, LDB, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*)

SUBROUTINE DPBSV_64(UPLO, N, NDIAG, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NDIAG, NRHS, LDA, LDB, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE PBSV(UPLO, [N], NDIAG, [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NDIAG, NRHS, LDA, LDB, INFO
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE PBSV_64(UPLO, [N], NDIAG, [NRHS], A, [LDA], B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NDIAG, NRHS, LDA, LDB, INFO
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dpbsv(char uplo, int n, int ndiag, int nrhs, double *a,
          int lda, double *b, int ldb, int *info);

void dpbsv_64(char uplo, long n, long ndiag, long nrhs, dou-
          ble  *a,  long  lda,  double  *b,  long  ldb, long
          *info);
```

PURPOSE

```
dpbsv computes the solution to a real system of linear equa-
tions
   A * X = B, where A is an N-by-N symmetric positive defin-
ite band matrix and X and B are N-by-NRHS matrices.
The Cholesky decomposition is used to factor A as
   A = U**T * U,  if UPLO = 'U', or
   A = L * L**T,  if UPLO = 'L',
where U is an upper triangular band matrix, and L is a lower
triangular  band matrix, with the same number of superdiago-
nals or subdiagonals as A.  The factored form of A  is  then
used to solve the system of equations A * X = B.
```

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NDIAG (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  NDIAG >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

A (input/output)
          On entry, the upper or lower triangle of the  sym-
          metric  band matrix A, stored in the first NDIAG+1
          rows of the array.  The j-th column of A is stored
          in  the j-th column of the array A as follows:  if
```

```
UPLO  =  'U',  A(NDIAG+1+i-j,j)   =   A(i,j)   for
max(1,j-NDIAG)<=i<=j;  if  UPLO  = 'L', A(1+i-j,j)
= A(i,j) for j<=i<=min(N,j+NDIAG).  See below  for
further details.

On exit, if INFO = 0, the triangular factor U or L
from  the Cholesky factorization A = U**T*U or A =
L*L**T of the band matrix A, in the  same  storage
format as A.
```

```
LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          NDIAG+1.
B (input/output)
          On entry, the N-by-NRHS right hand side matrix  B.
          On  exit,  if  INFO  =  0,  the N-by-NRHS solution
          matrix X.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, the leading minor of order i of
          A  is  not positive definite, so the factorization
          could not be completed, and the solution  has  not
          been computed.
```

FURTHER DETAILS

```
The band storage scheme  is  illustrated  by  the  following
example, when N = 6, NDIAG = 2, and UPLO = 'U':

On entry:                       On exit:

    *    *   a13  a24  a35  a46      *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66

Similarly, if UPLO = 'L' the format of A is as follows:

On entry:                       On exit:

   a11  a22  a33  a44  a55  a66     l11  l22  l33  l44   l55
```

```
l66
   a21  a32  a43  a54  a65   *      l21  l32  l43  l54   l65
*
   a31  a42  a53  a64   *    *      l31  l42  l53   l64    *
*
```

Array elements marked * are not used by the routine.

Contents

NAME

```
dpbsvx - use the Cholesky factorization A = U**T*U  or  A  =
L*L**T  to  compute  the solution to a real system of linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE DPBSVX(FACT, UPLO, N, NDIAG, NRHS, A, LDA, AF, LDAF,
      EQUED, S, B, LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2,
      INFO)

CHARACTER * 1 FACT, UPLO, EQUED
INTEGER N, NDIAG, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE  PRECISION  A(LDA,*),  AF(LDAF,*),  S(*),   B(LDB,*),
X(LDX,*), FERR(*), BERR(*), WORK(*)

SUBROUTINE DPBSVX_64(FACT, UPLO, N, NDIAG, NRHS, A, LDA, AF, LDAF,
      EQUED, S, B, LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2,
      INFO)

CHARACTER * 1 FACT, UPLO, EQUED
INTEGER*8 N, NDIAG, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE  PRECISION  A(LDA,*),  AF(LDAF,*),  S(*),   B(LDB,*),
X(LDX,*), FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PBSVX(FACT, UPLO, [N], NDIAG, [NRHS], A, [LDA], AF, [LDAF],
       EQUED, S, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK],
```

```
       [WORK2], [INFO])

    CHARACTER(LEN=1) :: FACT, UPLO, EQUED
    INTEGER :: N, NDIAG, NRHS, LDA, LDAF, LDB, LDX, INFO
    INTEGER, DIMENSION(:) :: WORK2
    REAL(8) :: RCOND
    REAL(8), DIMENSION(:) :: S, FERR, BERR, WORK
    REAL(8), DIMENSION(:,:) :: A, AF, B, X

    SUBROUTINE PBSVX_64(FACT, UPLO, [N], NDIAG, [NRHS], A, [LDA], AF,
           [LDAF], EQUED, S, B, [LDB], X, [LDX], RCOND, FERR, BERR,
           [WORK], [WORK2], [INFO])

    CHARACTER(LEN=1) :: FACT, UPLO, EQUED
    INTEGER(8) :: N, NDIAG, NRHS, LDA, LDAF, LDB, LDX, INFO
    INTEGER(8), DIMENSION(:) :: WORK2
    REAL(8) :: RCOND
    REAL(8), DIMENSION(:) :: S, FERR, BERR, WORK
    REAL(8), DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
#include <sunperf.h>

void dpbsvx(char fact, char uplo,  int  n,  int  ndiag,  int
          nrhs,  double  *a,  int lda, double *af, int ldaf,
          char equed, double *s, double *b, int ldb,  double
          *x,  int  ldx, double *rcond, double *ferr, double
          *berr, int *info);

void dpbsvx_64(char fact, char uplo,  long  n,  long  ndiag,
          long  nrhs,  double *a, long lda, double *af, long
          ldaf, char equed, double *s, double *b, long  ldb,
          double  *x, long ldx, double *rcond, double *ferr,
          double *berr, long *info);
```

PURPOSE

dpbsvx uses the Cholesky factorization A = U**T*U or A = L*L**T to compute the solution to a real system of linear equations
   A * X = B, where A is an N-by-N symmetric positive defin- ite band matrix and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'E', real scaling factors are computed to

```
equilibrate
   the system:
      diag(S) * A * diag(S) * inv(diag(S)) * X = diag(S) * B
   Whether or not the system will be equilibrated depends on
the
   scaling of the matrix A, but if equilibration is used,  A
is
   overwritten by diag(S)*A*diag(S) and B by diag(S)*B.

2. If FACT = 'N' or 'E', the Cholesky decomposition is  used
to
   factor the matrix A (after equilibration if FACT  =  'E')
as
      A = U**T * U,  if UPLO = 'U', or
      A = L * L**T,  if UPLO = 'L',
   where U is an upper triangular band matrix, and  L  is  a
lower
   triangular band matrix.
3. If the leading i-by-i principal  minor  is  not  positive
definite,
   then the routine returns with INFO =  i.  Otherwise,  the
factored
   form of A is used to estimate the condition number of the
matrix
   A.  If the reciprocal of the  condition  number  is  less
than machine
   precision, INFO = N+1 is returned as a warning,  but  the
routine
   still goes on to solve for X and compute error bounds as
   described below.

4. The system of equations is solved for X  using  the  fac-
tored form
   of A.

5. Iterative refinement is applied to improve  the  computed
solution
   matrix and calculate  error  bounds  and  backward  error
estimates
   for it.

6. If equilibration was used, the matrix X is  premultiplied
by
   diag(S) so that it solves the original system before
   equilibration.
```

ARGUMENTS

```
FACT (input)
          Specifies whether or not the factored form of  the
          matrix A is supplied on entry, and if not, whether
          the matrix A should be equilibrated before  it  is
          factored.   = 'F':  On entry, AF contains the fac-
          tored form of A.  If EQUED = 'Y', the matrix A has
          been equilibrated with scaling factors given by S.
          A and AF will not be modified.  = 'N':  The matrix
          A will be copied to AF and factored.
          = 'E':  The  matrix  A  will  be  equilibrated  if
          necessary, then copied to AF and factored.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.
NDIAG (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  NDIAG >= 0.

NRHS (input)
          The number of right-hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input/output)
          On entry, the upper or lower triangle of the  sym-
          metric  band matrix A, stored in the first NDIAG+1
          rows of the array, except if FACT = 'F' and  EQUED
          = 'Y', then A must contain the equilibrated matrix
          diag(S)*A*diag(S).  The j-th column of A is stored
          in  the j-th column of the array A as follows:  if
          UPLO  =  'U',  A(NDIAG+1+i-j,j)   =   A(i,j)   for
          max(1,j-NDIAG)<=i<=j;  if  UPLO  = 'L', A(1+i-j,j)
          = A(i,j) for j<=i<=min(N,j+NDIAG).  See below  for
          further details.

          On exit, if FACT = 'E'  and  EQUED  =  'Y',  A  is
          overwritten by diag(S)*A*diag(S).

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          NDIAG+1.

AF (input/output)
          If FACT = 'F', then AF is an input argument and on
          entry  contains  the triangular factor U or L from
          the Cholesky factorization  A  =  U**T*U  or  A  =
```

L*L**T of the band matrix A, in the same storage format as A (see A). If EQUED = 'Y', then AF is the factored form of the equilibrated matrix A.

If FACT = 'N', then AF is an output argument and on exit returns the triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T.

If FACT = 'E', then AF is an output argument and on exit returns the triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T of the equilibrated matrix A (see the description of A for the form of the equilibrated matrix).

LDAF (input)

The leading dimension of the array AF. LDAF >= NDIAG+1.

EQUED (input)

Specifies the form of equilibration that was done.
= 'N': No equilibration (always true if FACT = 'N').
= 'Y': Equilibration was done, i.e., A has been replaced by diag(S) * A * diag(S). EQUED is an input argument if FACT = 'F'; otherwise, it is an output argument.

S (input/output)

The scale factors for A; not accessed if EQUED = 'N'. S is an input argument if FACT = 'F'; otherwise, S is an output argument. If FACT = 'F' and EQUED = 'Y', each element of S must be positive.

B (input/output)

On entry, the N-by-NRHS right hand side matrix B. On exit, if EQUED = 'N', B is not modified; if EQUED = 'Y', B is overwritten by diag(S) * B.

LDB (input)

The leading dimension of the array B. LDB >= max(1,N).

X (output)

If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X to the original system of equations. Note that if EQUED = 'Y', A and B are modified on exit, and the solution to the equilibrated system is inv(diag(S))*X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A after equilibration (if done). If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would

suggest.

FURTHER DETAILS

```
The band storage scheme  is  illustrated  by  the  following
example, when N = 6, NDIAG = 2, and UPLO = 'U':

Two-dimensional storage of the symmetric matrix A:
   a11  a12  a13
        a22  a23  a24
             a33  a34  a35
                  a44  a45  a46
                       a55  a56
   (aij=conjg(aji))         a66

Band storage of the upper triangle of A:

    *    *   a13  a24  a35  a46
    *   a12  a23  a34  a45  a56
   a11  a22  a33  a44  a55  a66

Similarly, if UPLO = 'L' the format of A is as follows:

   a11  a22  a33  a44  a55  a66
   a21  a32  a43  a54  a65   *
   a31  a42  a53  a64   *    *

Array elements marked * are not used by the routine.
```

Contents

NAME

```
dpbtf2 - compute the Cholesky factorization of a  real  sym-
metric positive definite band matrix A
```

SYNOPSIS

```
SUBROUTINE DPBTF2(UPLO, N, KD, AB, LDAB, INFO)

CHARACTER * 1 UPLO
INTEGER N, KD, LDAB, INFO
DOUBLE PRECISION AB(LDAB,*)

SUBROUTINE DPBTF2_64(UPLO, N, KD, AB, LDAB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, KD, LDAB, INFO
DOUBLE PRECISION AB(LDAB,*)
```

F95 INTERFACE

```
SUBROUTINE PBTF2(UPLO, [N], KD, AB, [LDAB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, KD, LDAB, INFO
REAL(8), DIMENSION(:,:) :: AB

SUBROUTINE PBTF2_64(UPLO, [N], KD, AB, [LDAB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, KD, LDAB, INFO
REAL(8), DIMENSION(:,:) :: AB
```

C INTERFACE

```
#include <sunperf.h>

void dpbtf2(char uplo, int n, int kd, double *ab, int  ldab,
          int *info);

void dpbtf2_64(char uplo, long n, long kd, double *ab,  long
          ldab, long *info);
```

PURPOSE

dpbtf2 computes the Cholesky factorization of a real symmetric positive definite band matrix A.

The factorization has the form
```
   A = U' * U ,  if UPLO = 'U', or
   A = L  * L',  if UPLO = 'L',
```
where U is an upper triangular matrix, U' is the transpose of U, and L is lower triangular.

This is the unblocked version of the algorithm, calling Level 2 BLAS.

ARGUMENTS

UPLO (input)
Specifies whether the upper or lower triangular part of the symmetric matrix A is stored:
= 'U': Upper triangular
= 'L': Lower triangular

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of super-diagonals of the matrix A if UPLO = 'U', or the number of sub-diagonals if UPLO = 'L'. KD >= 0.

AB (input/output)
On entry, the upper or lower triangle of the symmetric band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', AB(1+i-j,j)   = A(i,j) for j<=i<=min(n,j+kd).

On exit, if INFO = 0, the triangular factor U or L

from the Cholesky factorization A = U'*U or A = L*L' of the band matrix A, in the same storage format as A.

LDAB (input)
The leading dimension of the array AB. LDAB >= KD+1.

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, the leading minor of order k is not positive definite, and the factorization could not be completed.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when N = 6, KD = 2, and UPLO = 'U':

```
On entry:                       On exit:

    *    *   a13  a24  a35  a46      *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
```

Similarly, if UPLO = 'L' the format of A is as follows:

```
On entry:                       On exit:

   a11  a22  a33  a44  a55  a66     l11  l22  l33  l44   l55
l66
   a21  a32  a43  a54  a65   *      l21  l32  l43  l54   l65
*
   a31  a42  a53  a64   *    *      l31  l42  l53   l64    *
*
```

Array elements marked * are not used by the routine.

Contents

NAME

```
dpbtrf - compute the Cholesky factorization of a  real  sym-
metric positive definite band matrix A
```

SYNOPSIS

```
SUBROUTINE DPBTRF(UPLO, N, KD, A, LDA, INFO)

CHARACTER * 1 UPLO
INTEGER N, KD, LDA, INFO
DOUBLE PRECISION A(LDA,*)

SUBROUTINE DPBTRF_64(UPLO, N, KD, A, LDA, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, KD, LDA, INFO
DOUBLE PRECISION A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE PBTRF(UPLO, [N], KD, A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, KD, LDA, INFO
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE PBTRF_64(UPLO, [N], KD, A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, KD, LDA, INFO
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dpbtrf(char uplo, int n, int kd, double  *a,  int  lda,
          int *info);

void dpbtrf_64(char uplo, long n, long kd, double  *a,  long
          lda, long *info);
```

PURPOSE

dpbtrf computes the Cholesky factorization of a real symmetric positive definite band matrix A.

The factorization has the form

```
   A = U**T * U,  if UPLO = 'U', or
   A = L  * L**T,  if UPLO = 'L',
```

where U is an upper triangular matrix and L is lower triangular.

ARGUMENTS

UPLO (input)
- = 'U': Upper triangle of A is stored;
- = 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

A (input/output)
On entry, the upper or lower triangle of the symmetric band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).

On exit, if INFO = 0, the triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T of the band matrix A, in the same storage format as A.

LDA (input)

The leading dimension of the array A. LDA >= KD+1.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i is not positive definite, and the factorization could not be completed.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when N = 6, KD = 2, and UPLO = 'U':

```
On entry:                       On exit:

    *    *   a13  a24  a35  a46      *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
```

Similarly, if UPLO = 'L' the format of A is as follows:

```
On entry:                       On exit:

   a11  a22  a33  a44  a55  a66     l11  l22  l33  l44   l55
l66
   a21  a32  a43  a54  a65   *      l21  l32  l43  l54   l65
*
   a31  a42  a53  a64   *    *      l31  l42  l53   l64    *
*
```

Array elements marked * are not used by the routine.

Contributed by
Peter Mayes and Giuseppe Radicati, IBM ECSEC, Rome, March 23, 1989

Contents

NAME

```
dpbtrs - solve a system of linear equations A*X = B  with  a
symmetric positive definite band matrix A using the Cholesky
factorization A = U**T*U or A = L*L**T computed by SPBTRF
```

SYNOPSIS

```
SUBROUTINE DPBTRS(UPLO, N, KD, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER N, KD, NRHS, LDA, LDB, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*)

SUBROUTINE DPBTRS_64(UPLO, N, KD, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, KD, NRHS, LDA, LDB, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE PBTRS(UPLO, [N], KD, [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, KD, NRHS, LDA, LDB, INFO
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE PBTRS_64(UPLO, [N], KD, [NRHS], A, [LDA], B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, KD, NRHS, LDA, LDB, INFO
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dpbtrs(char uplo, int n, int kd, int nrhs,  double  *a,
          int lda, double *b, int ldb, int *info);

void dpbtrs_64(char uplo, long n, long kd, long nrhs, double
          *a, long lda, double *b, long ldb, long *info);
```

PURPOSE

dpbtrs solves a system of linear equations A*X = B with a symmetric positive definite band matrix A using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPBTRF.

ARGUMENTS

UPLO (input)
- = 'U': Upper triangular factor stored in A;
- = 'L': Lower triangular factor stored in A.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T of the band matrix A, stored in the first KD+1 rows of the array. The j-th column of U or L is stored in the j-th column of the array A as follows: if UPLO ='U', A(kd+1+i-j,j) = U(i,j) for max(1,j-kd)<=i<=j; if UPLO ='L', A(1+i-j,j) = L(i,j) for j<=i<=min(n,j+kd).

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

B (input/output)
On entry, the right hand side matrix B. On exit,

the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dpocon - estimate the reciprocal of the condition number (in
the  1-norm)  of  a  real symmetric positive definite matrix
using the Cholesky factorization A = U**T*U or  A  =  L*L**T
computed by SPOTRF
```

SYNOPSIS

```
SUBROUTINE DPOCON(UPLO, N, A, LDA, ANORM, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, INFO
INTEGER WORK2(*)
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION A(LDA,*), WORK(*)

SUBROUTINE DPOCON_64(UPLO, N, A, LDA, ANORM, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, INFO
INTEGER*8 WORK2(*)
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE POCON(UPLO, [N], A, [LDA], ANORM, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL(8) :: ANORM, RCOND
```

```
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE POCON_64(UPLO, [N], A, [LDA], ANORM, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
void dpocon(char uplo, int n, double  *a,  int  lda,  double
          anorm, double *rcond, int *info);

void dpocon_64(char uplo, long n, double *a, long lda,  dou-
          ble anorm, double *rcond, long *info);
```

PURPOSE

dpocon estimates the reciprocal of the condition number (in the 1-norm) of a real symmetric positive definite matrix using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPOTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T, as computed by SPOTRF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

ANORM (input)
The 1-norm (or infinity-norm) of the symmetric matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dpoequ - compute row and column scalings intended to equili-
brate  a symmetric positive definite matrix A and reduce its
condition number (with respect to the two-norm)
```

SYNOPSIS

```
SUBROUTINE DPOEQU(N, A, LDA, SCALE, SCOND, AMAX, INFO)

INTEGER N, LDA, INFO
DOUBLE PRECISION SCOND, AMAX
DOUBLE PRECISION A(LDA,*), SCALE(*)

SUBROUTINE DPOEQU_64(N, A, LDA, SCALE, SCOND, AMAX, INFO)

INTEGER*8 N, LDA, INFO
DOUBLE PRECISION SCOND, AMAX
DOUBLE PRECISION A(LDA,*), SCALE(*)
```

F95 INTERFACE

```
SUBROUTINE POEQU([N], A, [LDA], SCALE, SCOND, AMAX, [INFO])

INTEGER :: N, LDA, INFO
REAL(8) :: SCOND, AMAX
REAL(8), DIMENSION(:) :: SCALE
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE POEQU_64([N], A, [LDA], SCALE, SCOND, AMAX, [INFO])

INTEGER(8) :: N, LDA, INFO
REAL(8) :: SCOND, AMAX
REAL(8), DIMENSION(:) :: SCALE
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dpoequ(int n, double *a, int lda, double *scale, double
          *scond, double *amax, int *info);

void dpoequ_64(long n, double *a, long lda,  double  *scale,
          double *scond, double *amax, long *info);
```

PURPOSE

dpoequ computes row and column scalings intended to equilibrate a symmetric positive definite matrix A and reduce its condition number (with respect to the two-norm). S contains the scale factors, S(i) = 1/sqrt(A(i,i)), chosen so that the scaled matrix B with elements B(i,j) = S(i)*A(i,j)*S(j) has ones on the diagonal. This choice of S puts the condition number of B within a factor N of the smallest possible condition number over all possible diagonal scalings.

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

A (input) The N-by-N symmetric positive definite matrix whose scaling factors are to be computed. Only the diagonal elements of A are referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

SCALE (output)
If INFO = 0, SCALE contains the scale factors for A.

SCOND (output)
If INFO = 0, SCALE contains the ratio of the smallest SCALE(i) to the largest SCALE(i). If SCOND >= 0.1 and AMAX is neither too large nor too small, it is not worth scaling by SCALE.

AMAX (output)
Absolute value of largest matrix element. If AMAX is very close to overflow or very close to underflow, the matrix should be scaled.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the i-th diagonal element is nonpositive.

Contents

NAME

```
dporfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is symmetric positive
definite,
```

SYNOPSIS

```
SUBROUTINE DPORFS(UPLO, N, NRHS, A, LDA, AF, LDAF, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER WORK2(*)
DOUBLE PRECISION A(LDA,*), AF(LDAF,*),  B(LDB,*),  X(LDX,*),
FERR(*), BERR(*), WORK(*)

SUBROUTINE DPORFS_64(UPLO, N, NRHS, A, LDA, AF, LDAF, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 WORK2(*)
DOUBLE PRECISION A(LDA,*), AF(LDAF,*),  B(LDB,*),  X(LDX,*),
FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PORFS(UPLO, [N], [NRHS], A, [LDA], AF, [LDAF], B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL(8), DIMENSION(:) :: FERR, BERR, WORK
```

```
REAL(8), DIMENSION(:,:) :: A, AF, B, X

SUBROUTINE PORFS_64(UPLO, [N], [NRHS], A, [LDA], AF, [LDAF], B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL(8), DIMENSION(:) :: FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
#include <sunperf.h>
void dporfs(char uplo, int n, int nrhs, double *a, int  lda,
          double  *af,  int ldaf, double *b, int ldb, double
          *x, int  ldx,  double  *ferr,  double  *berr,  int
          *info);

void dporfs_64(char uplo, long n, long nrhs, double *a, long
          lda,  double  *af, long ldaf, double *b, long ldb,
          double *x, long ldx, double *ferr,  double  *berr,
          long *info);
```

PURPOSE

dporfs improves the computed solution to a system of linear equations when the coefficient matrix is symmetric positive definite, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input) The symmetric matrix A.  If UPLO = 'U', the  lead-
          ing N-by-N upper triangular part of A contains the
          upper triangular part of the  matrix  A,  and  the
          strictly  lower triangular part of A is not refer-
          enced.  If UPLO = 'L', the  leading  N-by-N  lower
```

triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input)
The triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T, as computed by SPOTRF.

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by SPOTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N)

```
WORK2 (workspace)
          dimension(N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO =  -i,  the  i-th  argument  had  an
          illegal value
```

Contents

NAME

```
dposv - compute the solution to  a  real  system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE DPOSV(UPLO, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDA, LDB, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*)

SUBROUTINE DPOSV_64(UPLO, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDA, LDB, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE POSV(UPLO, [N], [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDA, LDB, INFO
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE POSV_64(UPLO, [N], [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dposv(char uplo, int n, int nrhs, double *a,  int  lda,
          double *b, int ldb, int *info);

void dposv_64(char uplo, long n, long nrhs, double *a,  long
          lda, double *b, long ldb, long *info);
```

PURPOSE

```
dposv computes the solution to a real system of linear equa-
tions
   A * X = B, where A is an N-by-N symmetric positive defin-
ite matrix and X and B are N-by-NRHS matrices.

The Cholesky decomposition is used to factor A as
   A = U**T* U,  if UPLO = 'U', or
   A = L * L**T,  if UPLO = 'L',
where U is an upper triangular matrix and L is a lower  tri-
angular  matrix.   The  factored  form  of A is then used to
solve the system of equations A * X = B.
```

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

A (input/output)
          On entry, the symmetric matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A,
          and the strictly lower triangular part of A is not
          referenced.  If UPLO =  'L',  the  leading  N-by-N
          lower triangular part of A contains the lower tri-
          angular part of the matrix  A,  and  the  strictly
          upper triangular part of A is not referenced.

          On exit, if INFO = 0, the factor U or L  from  the
          Cholesky factorization A = U**T*U or A = L*L**T.
```

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed.

Contents

NAME

dposvx - use the Cholesky factorization A = U**T*U or A = L*L**T to compute the solution to a real system of linear equations A * X = B,

SYNOPSIS

```
SUBROUTINE DPOSVX(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, EQUED,
      S, B, LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO, EQUED
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE  PRECISION  A(LDA,*),  AF(LDAF,*),  S(*),   B(LDB,*),
X(LDX,*), FERR(*), BERR(*), WORK(*)

SUBROUTINE DPOSVX_64(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, EQUED,
      S, B, LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO, EQUED
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE  PRECISION  A(LDA,*),  AF(LDAF,*),  S(*),   B(LDB,*),
X(LDX,*), FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE POSVX(FACT, UPLO, [N], [NRHS], A, [LDA], AF, [LDAF],
       EQUED, S, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
```

```
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: S, FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: A, AF, B, X

SUBROUTINE POSVX_64(FACT, UPLO, [N], [NRHS], A, [LDA], AF, [LDAF],
       EQUED, S, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: S, FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
#include <sunperf.h>

void dposvx(char fact, char uplo, int n,  int  nrhs,  double
          *a,  int  lda,  double  *af, int ldaf, char equed,
          double *s, double *b, int ldb, double *x, int ldx,
          double  *rcond,  double  *ferr,  double *berr, int
          *info);

void dposvx_64(char fact, char uplo, long n, long nrhs, dou-
          ble  *a,  long  lda,  double  *af, long ldaf, char
          equed, double *s, double *b, long ldb, double  *x,
          long  ldx,  double  *rcond,  double  *ferr, double
          *berr, long *info);
```

PURPOSE

```
dposvx uses the Cholesky factorization A =  U**T*U  or  A  =
L*L**T  to  compute  the solution to a real system of linear
equations
   A * X = B, where A is an N-by-N symmetric positive defin-
ite matrix and X and B are N-by-NRHS matrices.

Error bounds on the solution and a  condition  estimate  are
also provided.

The following steps are performed:

1. If FACT = 'E',  real  scaling  factors  are  computed  to
equilibrate
   the system:
      diag(S) * A * diag(S) * inv(diag(S)) * X = diag(S) * B
   Whether or not the system will be equilibrated depends on
```

the
   scaling of the matrix A, but if equilibration is used, A
is
   overwritten by diag(S)*A*diag(S) and B by diag(S)*B.

2. If FACT = 'N' or 'E', the Cholesky decomposition is used
to
   factor the matrix A (after equilibration if FACT = 'E')
as
      A = U**T* U, if UPLO = 'U', or
      A = L * L**T, if UPLO = 'L',
   where U is an upper triangular matrix and L is a lower
triangular
   matrix.

3. If the leading i-by-i principal minor is not positive
definite,
   then the routine returns with INFO = i. Otherwise, the
factored
   form of A is used to estimate the condition number of the
matrix
   A. If the reciprocal of the condition number is less
than machine
   precision, INFO = N+1 is returned as a warning, but the
routine
   still goes on to solve for X and compute error bounds as
   described below.

4. The system of equations is solved for X using the fac-
tored form
   of A.

5. Iterative refinement is applied to improve the computed
solution
   matrix and calculate error bounds and backward error
estimates
   for it.

6. If equilibration was used, the matrix X is premultiplied
by
   diag(S) so that it solves the original system before
   equilibration.

ARGUMENTS

FACT (input)
          Specifies whether or not the factored form of the
          matrix A is supplied on entry, and if not, whether

```
          the matrix A should be equilibrated before  it  is
          factored.   = 'F':  On entry, AF contains the fac-
          tored form of A.  If EQUED = 'Y', the matrix A has
          been equilibrated with scaling factors given by S.
          A and AF will not be modified.  = 'N':  The matrix
          A will be copied to AF and factored.
          = 'E':  The  matrix  A  will  be  equilibrated  if
          necessary, then copied to AF and factored.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.
A (input/output)
          On entry, the symmetric matrix A, except if FACT =
          'F'  and  EQUED  =  'Y',  then  A must contain the
          equilibrated matrix diag(S)*A*diag(S).  If UPLO  =
          'U', the leading N-by-N upper triangular part of A
          contains the upper triangular part of  the  matrix
          A,  and the strictly lower triangular part of A is
          not referenced.  If UPLO = 'L', the leading N-by-N
          lower triangular part of A contains the lower tri-
          angular part of the matrix  A,  and  the  strictly
          upper  triangular  part of A is not referenced.  A
          is not modified if FACT = 'F' or 'N', or if FACT =
          'E' and EQUED = 'N' on exit.

          On exit, if FACT = 'E'  and  EQUED  =  'Y',  A  is
          overwritten by diag(S)*A*diag(S).

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

AF (input/output)
          If FACT = 'F', then AF is an input argument and on
          entry  contains  the triangular factor U or L from
          the Cholesky factorization  A  =  U**T*U  or  A  =
          L*L**T, in the same storage format as A.  If EQUED
          .ne. 'N', then AF is  the  factored  form  of  the
          equilibrated matrix diag(S)*A*diag(S).

          If FACT = 'N', then AF is an output  argument  and
          on  exit returns the triangular factor U or L from
```

the Cholesky factorization A = U**T*U or A = L*L**T of the original matrix A.

If FACT = 'E', then AF is an output argument and on exit returns the triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T of the equilibrated matrix A (see the description of A for the form of the equilibrated matrix).

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

EQUED (input)
Specifies the form of equilibration that was done.
= 'N': No equilibration (always true if FACT = 'N').
= 'Y': Equilibration was done, i.e., A has been replaced by diag(S) * A * diag(S). EQUED is an input argument if FACT = 'F'; otherwise, it is an output argument.

S (input/output)
The scale factors for A; not accessed if EQUED = 'N'. S is an input argument if FACT = 'F'; otherwise, S is an output argument. If FACT = 'F' and EQUED = 'Y', each element of S must be positive.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if EQUED = 'N', B is not modified; if EQUED = 'Y', B is overwritten by diag(S) * B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X to the original system of equations. Note that if EQUED = 'Y', A and B are modified on exit, and the solution to the equilibrated system is inv(diag(S))*X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)

The estimate of the reciprocal condition number of the matrix A after equilibration (if done). If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)

The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)

The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)

dimension(3*N)

WORK2 (workspace)

dimension(N)

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, and i is

<= N: the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
dpotf2 - compute the Cholesky factorization of a  real  sym-
metric positive definite matrix A
```

SYNOPSIS

```
SUBROUTINE DPOTF2(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, INFO
DOUBLE PRECISION A(LDA,*)

SUBROUTINE DPOTF2_64(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, INFO
DOUBLE PRECISION A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE POTF2(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, INFO
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE POTF2_64(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, INFO
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dpotf2(char uplo,  int  n,  double  *a,  int  lda,  int
          *info);

void dpotf2_64(char uplo, long n, double *a, long lda,  long
          *info);
```

PURPOSE

dpotf2 computes the Cholesky factorization of a real symmetric positive definite matrix A.

The factorization has the form

```
   A = U' * U ,  if UPLO = 'U', or
   A = L  * L',  if UPLO = 'L',
```

where U is an upper triangular matrix and L is lower triangular.

This is the unblocked version of the algorithm, calling Level 2 BLAS.

ARGUMENTS

UPLO (input)
Specifies whether the upper or lower triangular part of the symmetric matrix A is stored. = 'U': Upper triangular
= 'L': Lower triangular

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the symmetric matrix A. If UPLO = 'U', the leading n by n upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading n by n lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, if INFO = 0, the factor U or L from the Cholesky factorization A = U'*U or A = L*L'.

LDA (input)
The leading dimension of the array A.   LDA >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, the leading minor of order k is not positive definite, and the factorization could not be completed.

Contents

NAME

```
dpotrf - compute the Cholesky factorization of a  real  sym-
metric positive definite matrix A
```

SYNOPSIS

```
SUBROUTINE DPOTRF(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, INFO
DOUBLE PRECISION A(LDA,*)

SUBROUTINE DPOTRF_64(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, INFO
DOUBLE PRECISION A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE POTRF(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, INFO
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE POTRF_64(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, INFO
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dpotrf(char uplo,  int  n,  double  *a,  int  lda,  int
          *info);

void dpotrf_64(char uplo, long n, double *a, long lda,  long
          *info);
```

PURPOSE

dpotrf computes the Cholesky factorization of a real symmetric positive definite matrix A.

The factorization has the form

```
   A = U**T * U,  if UPLO = 'U', or
   A = L  * L**T,  if UPLO = 'L',
```

where U is an upper triangular matrix and L is lower triangular.

This is the block version of the algorithm, calling Level 3 BLAS.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, if INFO = 0, the factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T.

LDA (input)
The leading dimension of the array A.   LDA >= max(1,N).

INFO (output)

= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i is not positive definite, and the factorization could not be completed.

Contents

NAME

```
dpotri - compute the inverse of a  real  symmetric  positive
definite  matrix  A  using  the  Cholesky  factorization A =
U**T*U or A = L*L**T computed by SPOTRF
```

SYNOPSIS

```
SUBROUTINE DPOTRI(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, INFO
DOUBLE PRECISION A(LDA,*)

SUBROUTINE DPOTRI_64(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, INFO
DOUBLE PRECISION A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE POTRI(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, INFO
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE POTRI_64(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, INFO
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dpotri(char uplo,  int  n,  double  *a,  int  lda,  int
          *info);

void dpotri_64(char uplo, long n, double *a, long lda,  long
          *info);
```

PURPOSE

dpotri computes the inverse of a real symmetric positive definite matrix A using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPOTRF.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T, as computed by SPOTRF. On exit, the upper or lower triangle of the (symmetric) inverse of A, overwriting the input factor U or L.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the (i,i) element of the factor U or L is zero, and the inverse could not be computed.

Contents

NAME

```
dpotrs - solve a system of linear equations A*X = B  with  a
symmetric positive definite matrix A using the Cholesky fac-
torization A = U**T*U or A = L*L**T computed by SPOTRF
```

SYNOPSIS

```
SUBROUTINE DPOTRS(UPLO, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDA, LDB, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*)

SUBROUTINE DPOTRS_64(UPLO, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDA, LDB, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE POTRS(UPLO, [N], [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDA, LDB, INFO
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE POTRS_64(UPLO, [N], [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dpotrs(char uplo, int n, int nrhs, double *a, int  lda,
          double *b, int ldb, int *info);

void dpotrs_64(char uplo, long n, long nrhs, double *a, long
          lda, double *b, long ldb, long *info);
```

PURPOSE

dpotrs solves a system of linear equations A*X = B with a symmetric positive definite matrix A using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPOTRF.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T, as computed by SPOTRF.

LDA (input)
The leading dimension of the array A.   LDA >= max(1,N).

B (input/output)
On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B.   LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

dppcon - estimate the reciprocal of the condition number (in the 1-norm) of a real symmetric positive definite packed matrix using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPPTRF

SYNOPSIS

```
SUBROUTINE DPPCON(UPLO, N, A, ANORM, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER N, INFO
INTEGER WORK2(*)
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION A(*), WORK(*)

SUBROUTINE DPPCON_64(UPLO, N, A, ANORM, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, INFO
INTEGER*8 WORK2(*)
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION A(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PPCON(UPLO, N, A, ANORM, RCOND, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: A, WORK
```

```
SUBROUTINE PPCON_64(UPLO, N, A, ANORM, RCOND, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: A, WORK
```

C INTERFACE

```
#include <sunperf.h>

void dppcon(char uplo, int n, double *a, double anorm,  dou-
          ble *rcond, int *info);
void dppcon_64(char uplo, long n, double *a,  double  anorm,
          double *rcond, long *info);
```

PURPOSE

dppcon estimates the reciprocal of the condition number (in the 1-norm) of a real symmetric positive definite packed matrix using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPPTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T, packed columnwise in a linear array. The j-th column of U or L is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = U(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = L(i,j) for j<=i<=n.

ANORM (input)
The 1-norm (or infinity-norm) of the symmetric matrix A.

```
RCOND (output)
          The reciprocal of  the  condition  number  of  the
          matrix  A, computed as RCOND = 1/(ANORM * AINVNM),
          where AINVNM is  an  estimate  of  the  1-norm  of
          inv(A) computed in this routine.

WORK (workspace)
          dimension(3*N)

WORK2 (workspace)
          dimension(N)
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

dppequ - compute row and column scalings intended to equilibrate a symmetric positive definite matrix A in packed storage and reduce its condition number (with respect to the two-norm)

SYNOPSIS

```
SUBROUTINE DPPEQU(UPLO, N, A, SCALE, SCOND, AMAX, INFO)

CHARACTER * 1 UPLO
INTEGER N, INFO
DOUBLE PRECISION SCOND, AMAX
DOUBLE PRECISION A(*), SCALE(*)

SUBROUTINE DPPEQU_64(UPLO, N, A, SCALE, SCOND, AMAX, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, INFO
DOUBLE PRECISION SCOND, AMAX
DOUBLE PRECISION A(*), SCALE(*)
```

F95 INTERFACE

```
SUBROUTINE PPEQU(UPLO, [N], A, SCALE, SCOND, AMAX, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INFO
REAL(8) :: SCOND, AMAX
REAL(8), DIMENSION(:) :: A, SCALE

SUBROUTINE PPEQU_64(UPLO, [N], A, SCALE, SCOND, AMAX, [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
INTEGER(8) :: N, INFO
REAL(8) :: SCOND, AMAX
REAL(8), DIMENSION(:) :: A, SCALE
```

C INTERFACE

```
#include <sunperf.h>

void dppequ(char uplo, int n, double *a, double *scale, dou-
          ble *scond, double *amax, int *info);

void dppequ_64(char uplo, long n, double *a, double  *scale,
          double *scond, double *amax, long *info);
```

PURPOSE

dppequ computes row and column scalings intended to equilibrate a symmetric positive definite matrix A in packed storage and reduce its condition number (with respect to the two-norm). S contains the scale factors, S(i)=1/sqrt(A(i,i)), chosen so that the scaled matrix B with elements B(i,j)=S(i)*A(i,j)*S(j) has ones on the diagonal. This choice of S puts the condition number of B within a factor N of the smallest possible condition number over all possible diagonal scalings.

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

A (input) The upper  or  lower  triangle  of  the  symmetric
          matrix  A,  packed  columnwise  in a linear array.
          The j-th column of A is stored in the array  A  as
          follows:  if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j)
          for 1<=i<=j; if UPLO = 'L', A(i +  (j-1)*(2n-j)/2)
          = A(i,j) for j<=i<=n.

SCALE (output)
          If INFO = 0, SCALE contains the scale factors  for
          A.

SCOND (output)
          If INFO = 0, SCALE contains the ratio of the smal-
          lest  SCALE(i)  to the largest SCALE(i).  If SCOND
```

>= 0.1 and AMAX is neither too large nor too small, it is not worth scaling by SCALE.

AMAX (output)
Absolute value of largest matrix element. If AMAX is very close to overflow or very close to underflow, the matrix should be scaled.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the i-th diagonal element is nonpositive.

Contents

NAME

```
dpprfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is symmetric positive
definite and packed, and provides error bounds and  backward
error estimates for the solution
```

SYNOPSIS

```
SUBROUTINE DPPRFS(UPLO, N, NRHS, A, AF, B, LDB, X, LDX, FERR, BERR,
      WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER WORK2(*)
DOUBLE PRECISION A(*), AF(*), B(LDB,*),  X(LDX,*),  FERR(*),
BERR(*), WORK(*)

SUBROUTINE DPPRFS_64(UPLO, N, NRHS, A, AF, B, LDB, X, LDX, FERR,
      BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 WORK2(*)
DOUBLE PRECISION A(*), AF(*), B(LDB,*),  X(LDX,*),  FERR(*),
BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PPRFS(UPLO, N, [NRHS], A, AF, B, [LDB], X, [LDX], FERR,
       BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: WORK2
```

```
REAL(8), DIMENSION(:) :: A, AF, FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: B, X

SUBROUTINE PPRFS_64(UPLO, N, [NRHS], A, AF, B, [LDB], X, [LDX], FERR,
       BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL(8), DIMENSION(:) :: A, AF, FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: B, X
```

C INTERFACE

```
#include <sunperf.h>
void dpprfs(char uplo, int n, int nrhs,  double  *a,  double
          *af,  double *b, int ldb, double *x, int ldx, dou-
          ble *ferr, double *berr, int *info);

void dpprfs_64(char uplo, long n, long nrhs, double *a, dou-
          ble *af, double *b, long ldb, double *x, long ldx,
          double *ferr, double *berr, long *info);
```

PURPOSE

dpprfs improves the computed solution to a system of linear equations when the coefficient matrix is symmetric positive definite and packed, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input) The upper  or  lower  triangle  of  the  symmetric
          matrix  A,  packed  columnwise  in a linear array.
          The j-th column of A is stored in the array  A  as
          follows:  if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j)
          for 1<=i<=j; if UPLO = 'L', A(i +  (j-1)*(2n-j)/2)
          = A(i,j) for j<=i<=n.
```

AF (input)
The triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T, as computed by SPPTRF/CPPTRF, packed columnwise in a linear array in the same format as A (see A).

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by SPPTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dppsv - compute the solution to  a  real  system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE DPPSV(UPLO, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDB, INFO
DOUBLE PRECISION A(*), B(LDB,*)

SUBROUTINE DPPSV_64(UPLO, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDB, INFO
DOUBLE PRECISION A(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE PPSV(UPLO, N, [NRHS], A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDB, INFO
REAL(8), DIMENSION(:) :: A
REAL(8), DIMENSION(:,:) :: B

SUBROUTINE PPSV_64(UPLO, N, [NRHS], A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDB, INFO
REAL(8), DIMENSION(:) :: A
REAL(8), DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void dppsv(char uplo, int n, int nrhs, double *a, double *b,
          int ldb, int *info);

void dppsv_64(char uplo, long n, long nrhs, double *a,  dou-
          ble *b, long ldb, long *info);
```

PURPOSE

```
dppsv computes the solution to a real system of linear equa-
tions
   A * X = B, where A is an N-by-N symmetric positive defin-
ite matrix stored in packed format and X and B are N-by-NRHS
matrices.

The Cholesky decomposition is used to factor A as
   A = U**T* U,  if UPLO = 'U', or
   A = L * L**T,  if UPLO = 'L',
where U is an upper triangular matrix and L is a lower  tri-
angular  matrix.   The  factored  form  of A is then used to
solve the system of equations A * X = B.
```

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

A (input/output)
          On entry, the upper or lower triangle of the  sym-
          metric  matrix  A,  packed  columnwise in a linear
          array.  The j-th column of  A  is  stored  in  the
          array  A  as  follows:   if  UPLO = 'U', A(i + (j-
          1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i +
          (j-1)*(2n-j)/2)  =  A(i,j) for j<=i<=n.  See below
          for further details.
```

```
            On exit, if INFO = 0, the factor U or L  from  the
            Cholesky  factorization  A = U**T*U or A = L*L**T,
            in the same storage format as A.

  B (input/output)
            On entry, the N-by-NRHS right hand side matrix  B.
            On  exit,  if  INFO  =  0,  the N-by-NRHS solution
            matrix X.

  LDB (input)
            The leading dimension of  the  array  B.   LDB  >=
            max(1,N).

  INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value
            > 0:  if INFO = i, the leading minor of order i of
            A  is  not positive definite, so the factorization
            could not be completed, and the solution  has  not
            been computed.
```

FURTHER DETAILS

```
  The packed storage scheme is illustrated  by  the  following
  example when N = 4, UPLO = 'U':

  Two-dimensional storage of the symmetric matrix A:

     a11 a12 a13 a14
         a22 a23 a24
             a33 a34     (aij = conjg(aji))
                 a44

  Packed storage of the upper triangle of A:

  A = [ a11, a12, a22, a13, a23, a33, a14, a24, a34, a44 ]
```

Contents

NAME

```
dppsvx - use the Cholesky factorization A = U**T*U  or  A  =
L*L**T  to  compute  the solution to a real system of linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE DPPSVX(FACT, UPLO, N, NRHS, A, AF, EQUED, S, B, LDB,
      X, LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO, EQUED
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION  A(*),  AF(*),  S(*),  B(LDB,*),  X(LDX,*),
FERR(*), BERR(*), WORK(*)

SUBROUTINE DPPSVX_64(FACT, UPLO, N, NRHS, A, AF, EQUED, S, B,
      LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO, EQUED
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION  A(*),  AF(*),  S(*),  B(LDB,*),  X(LDX,*),
FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PPSVX(FACT, UPLO, [N], [NRHS], A, AF, EQUED, S, B,
       [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
```

```
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: A, AF, S, FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: B, X

SUBROUTINE PPSVX_64(FACT, UPLO, [N], [NRHS], A, AF, EQUED, S, B,
       [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: A, AF, S, FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: B, X
```

C INTERFACE

```
#include <sunperf.h>

void dppsvx(char fact, char uplo, int n,  int  nrhs,  double
          *a,  double *af, char equed, double *s, double *b,
          int ldb, double *x, int ldx, double *rcond, double
          *ferr, double *berr, int *info);

void dppsvx_64(char fact, char uplo, long n, long nrhs, dou-
          ble  *a, double *af, char equed, double *s, double
          *b, long ldb, double *x, long ldx, double  *rcond,
          double *ferr, double *berr, long *info);
```

PURPOSE

```
dppsvx uses the Cholesky factorization A =  U**T*U  or  A  =
L*L**T  to  compute  the solution to a real system of linear
equations
   A * X = B, where A is an N-by-N symmetric positive defin-
ite matrix stored in packed format and X and B are N-by-NRHS
matrices.

Error bounds on the solution and a  condition  estimate  are
also provided.

The following steps are performed:

1. If FACT = 'E',  real  scaling  factors  are  computed  to
equilibrate
   the system:
      diag(S) * A * diag(S) * inv(diag(S)) * X = diag(S) * B
   Whether or not the system will be equilibrated depends on
the
   scaling of the matrix A, but if equilibration is used,  A
```

is
overwritten by diag(S)*A*diag(S) and B by diag(S)*B.

2. If FACT = 'N' or 'E', the Cholesky decomposition is used to
factor the matrix A (after equilibration if FACT = 'E') as
A = U**T* U, if UPLO = 'U', or
A = L * L**T, if UPLO = 'L',
where U is an upper triangular matrix and L is a lower triangular
matrix.

3. If the leading i-by-i principal minor is not positive definite,
then the routine returns with INFO = i. Otherwise, the factored
form of A is used to estimate the condition number of the matrix
A. If the reciprocal of the condition number is less than machine
precision, INFO = N+1 is returned as a warning, but the routine
still goes on to solve for X and compute error bounds as
described below.

4. The system of equations is solved for X using the factored form
of A.

5. Iterative refinement is applied to improve the computed solution
matrix and calculate error bounds and backward error estimates
for it.

6. If equilibration was used, the matrix X is premultiplied by
diag(S) so that it solves the original system before
equilibration.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored. = 'F': On entry, AF contains the fac-

```
          tored form of A.  If EQUED = 'Y', the matrix A has
          been equilibrated with scaling factors given by S.
          A and AF will not be modified.  = 'N':  The matrix
          A will be copied to AF and factored.
          = 'E':  The  matrix  A  will  be  equilibrated  if
          necessary, then copied to AF and factored.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.
A (input/output)
          On entry, the upper or lower triangle of the  sym-
          metric  matrix  A,  packed  columnwise in a linear
          array, except if FACT = 'F' and EQUED = 'Y',  then
          A    must    contain    the    equilibrated    matrix
          diag(S)*A*diag(S).  The j-th column of A is stored
          in  the  array A as follows:  if UPLO = 'U', A(i +
          (j-1)*j/2) = A(i,j) for 1<=i<=j; if  UPLO  =  'L',
          A(i  +  (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n.  See
          below for further details.  A is not  modified  if
          FACT  =  'F'  or 'N', or if FACT = 'E' and EQUED =
          'N' on exit.

          On exit, if FACT = 'E'  and  EQUED  =  'Y',  A  is
          overwritten by diag(S)*A*diag(S).

AF (input/output)
          (N*(N+1)/2) If FACT = 'F', then  AF  is  an  input
          argument and on entry contains the triangular fac-
          tor U or L from the  Cholesky  factorization  A  =
          U'*U or A = L*L', in the same storage format as A.
          If EQUED .ne. 'N', then AF is the factored form of
          the equilibrated matrix A.

          If FACT = 'N', then AF is an output  argument  and
          on  exit returns the triangular factor U or L from
          the Cholesky factorization A = U'*U or A = L*L' of
          the original matrix A.

          If FACT = 'E', then AF is an output  argument  and
          on  exit returns the triangular factor U or L from
          the Cholesky factorization A = U'*U or A = L*L' of
          the  equilibrated matrix A (see the description of
```

A for the form of the equilibrated matrix).

EQUED (input)
Specifies the form of equilibration that was done.
= 'N': No equilibration (always true if FACT = 'N').
= 'Y': Equilibration was done, i.e., A has been replaced by diag(S) * A * diag(S). EQUED is an input argument if FACT = 'F'; otherwise, it is an output argument.

S (output)
The scale factors for A; not accessed if EQUED = 'N'. S is an input argument if FACT = 'F'; otherwise, S is an output argument. If FACT = 'F' and EQUED = 'Y', each element of S must be positive.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if EQUED = 'N', B is not modified; if EQUED = 'Y', B is overwritten by diag(S) * B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X to the original system of equations. Note that if EQUED = 'Y', A and B are modified on exit, and the solution to the equilibrated system is inv(diag(S))*X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A after equilibration (if done). If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest ele-

ment in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

FURTHER DETAILS

The packed storage scheme is illustrated by the following example when N = 4, UPLO = 'U':

Two-dimensional storage of the symmetric matrix A:

```
a11 a12 a13 a14
    a22 a23 a24
        a33 a34     (aij = conjg(aji))
            a44
```

Packed storage of the upper triangle of A:

A = [a11, a12, a22, a13, a23, a33, a14, a24, a34, a44]

Contents

NAME

```
dpptrf - compute the Cholesky factorization of a  real  sym-
metric positive definite matrix A stored in packed format
```

SYNOPSIS

```
SUBROUTINE DPPTRF(UPLO, N, A, INFO)

CHARACTER * 1 UPLO
INTEGER N, INFO
DOUBLE PRECISION A(*)

SUBROUTINE DPPTRF_64(UPLO, N, A, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, INFO
DOUBLE PRECISION A(*)
```

F95 INTERFACE

```
SUBROUTINE PPTRF(UPLO, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INFO
REAL(8), DIMENSION(:) :: A

SUBROUTINE PPTRF_64(UPLO, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INFO
REAL(8), DIMENSION(:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dpptrf(char uplo, int n, double *a, int *info);

void dpptrf_64(char uplo, long n, double *a, long *info);
```

PURPOSE

dpptrf computes the Cholesky factorization of a real symmetric positive definite matrix A stored in packed format.

The factorization has the form

```
   A = U**T * U,  if UPLO = 'U', or
   A = L  * L**T,  if UPLO = 'L',
```

where U is an upper triangular matrix and L is lower triangular.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n. See below for further details.

On exit, if INFO = 0, the triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T, in the same storage format as A.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i is not positive definite, and the factorization could not be completed.

FURTHER DETAILS

The packed storage scheme is illustrated by the following example when N = 4, UPLO = 'U':

Two-dimensional storage of the symmetric matrix A:

```
   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = aji)
               a44
```

Packed storage of the upper triangle of A:

A = [a11, a12, a22, a13, a23, a33, a14, a24, a34, a44]

Contents

NAME

```
dpptri - compute the inverse of a  real  symmetric  positive
definite  matrix  A  using  the  Cholesky  factorization A =
U**T*U or A = L*L**T computed by SPPTRF
```

SYNOPSIS

```
SUBROUTINE DPPTRI(UPLO, N, A, INFO)

CHARACTER * 1 UPLO
INTEGER N, INFO
DOUBLE PRECISION A(*)

SUBROUTINE DPPTRI_64(UPLO, N, A, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, INFO
DOUBLE PRECISION A(*)
```

F95 INTERFACE

```
SUBROUTINE PPTRI(UPLO, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INFO
REAL(8), DIMENSION(:) :: A

SUBROUTINE PPTRI_64(UPLO, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INFO
REAL(8), DIMENSION(:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dpptri(char uplo, int n, double *a, int *info);

void dpptri_64(char uplo, long n, double *a, long *info);
```

PURPOSE

```
dpptri computes the inverse of  a  real  symmetric  positive
definite  matrix  A  using  the  Cholesky  factorization A =
U**T*U or A = L*L**T computed by SPPTRF.
```

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangular factor is stored in A;
          = 'L':  Lower triangular factor is stored in A.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the triangular factor U or  L  from  the
          Cholesky  factorization  A = U**T*U or A = L*L**T,
          packed columnwise as a  linear  array.   The  j-th
          column  of U or L is stored in the array A as fol-
          lows:  if UPLO = 'U', A(i +  (j-1)*j/2)  =  U(i,j)
          for  1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2)
          = L(i,j) for j<=i<=n.

          On exit, the upper or lower triangle of the  (sym-
          metric) inverse of A, overwriting the input factor
          U or L.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, the (i,i) element of the factor
          U  or L is zero, and the inverse could not be com-
          puted.
```

Contents

NAME

dpptrs - solve a system of linear equations A*X = B with a symmetric positive definite matrix A in packed storage using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPPTRF

SYNOPSIS

```
SUBROUTINE DPPTRS(UPLO, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDB, INFO
DOUBLE PRECISION A(*), B(LDB,*)

SUBROUTINE DPPTRS_64(UPLO, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDB, INFO
DOUBLE PRECISION A(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE PPTRS(UPLO, N, [NRHS], A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDB, INFO
REAL(8), DIMENSION(:) :: A
REAL(8), DIMENSION(:,:) :: B

SUBROUTINE PPTRS_64(UPLO, N, [NRHS], A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDB, INFO
REAL(8), DIMENSION(:) :: A
```

```
REAL(8), DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void dpptrs(char uplo, int n, int nrhs,  double  *a,  double
          *b, int ldb, int *info);

void dpptrs_64(char uplo, long n, long nrhs, double *a, dou-
          ble *b, long ldb, long *info);
```

PURPOSE

dpptrs solves a system of linear equations A*X = B with a symmetric positive definite matrix A in packed storage using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPPTRF.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T, packed columnwise in a linear array. The j-th column of U or L is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = U(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = L(i,j) for j<=i<=n.

B (input/output)
On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)

```
= 0:  successful exit
< 0:  if INFO = -i, the i-th argument had an ille-
gal value
```

Contents

NAME

```
dptcon - compute the reciprocal of the condition number  (in
the  1-norm) of a real symmetric positive definite tridiago-
nal matrix using the factorization  A  =  L*D*L**T  or  A  =
U**T*D*U computed by SPTTRF
```

SYNOPSIS

```
SUBROUTINE DPTCON(N, DIAG, OFFD, ANORM, RCOND, WORK, INFO)

INTEGER N, INFO
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION DIAG(*), OFFD(*), WORK(*)

SUBROUTINE DPTCON_64(N, DIAG, OFFD, ANORM, RCOND, WORK, INFO)

INTEGER*8 N, INFO
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION DIAG(*), OFFD(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PTCON([N], DIAG, OFFD, ANORM, RCOND, [WORK], [INFO])

INTEGER :: N, INFO
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: DIAG, OFFD, WORK

SUBROUTINE PTCON_64([N], DIAG, OFFD, ANORM, RCOND, [WORK], [INFO])

INTEGER(8) :: N, INFO
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: DIAG, OFFD, WORK
```

C INTERFACE

```
#include <sunperf.h>

void dptcon(int n, double *diag, double *offd, double anorm,
          double *rcond, int *info);

void dptcon_64(long n, double *diag,  double  *offd,  double
          anorm, double *rcond, long *info);
```

PURPOSE

dptcon computes the reciprocal of the condition number (in the 1-norm) of a real symmetric positive definite tridiagonal matrix using the factorization A = L*D*L**T or A = U**T*D*U computed by SPTTRF.
Norm(inv(A)) is computed by a direct method, and the reciprocal of the condition number is computed as
RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

DIAG (input)
The n diagonal elements of the diagonal matrix DIAG from the factorization of A, as computed by SPTTRF.

OFFD (input)
The (n-1) off-diagonal elements of the unit bidiagonal factor U or L from the factorization of A, as computed by SPTTRF.

ANORM (input)
The 1-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(N)

INFO (output)

= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The method used is described in Nicholas J. Higham, "Efficient Algorithms for Computing the Condition Number of a Tridiagonal Matrix", SIAM J. Sci. Stat. Comput., Vol. 7, No. 1, January 1986.

Contents

NAME

```
dpteqr - compute all eigenvalues and, optionally,  eigenvec-
tors  of a symmetric positive definite tridiagonal matrix by
first factoring the matrix using SPTTRF,  and  then  calling
SBDSQR to compute the singular values of the bidiagonal fac-
tor
```

SYNOPSIS

```
SUBROUTINE DPTEQR(COMPZ, N, D, E, Z, LDZ, WORK, INFO)

CHARACTER * 1 COMPZ
INTEGER N, LDZ, INFO
DOUBLE PRECISION D(*), E(*), Z(LDZ,*), WORK(*)

SUBROUTINE DPTEQR_64(COMPZ, N, D, E, Z, LDZ, WORK, INFO)

CHARACTER * 1 COMPZ
INTEGER*8 N, LDZ, INFO
DOUBLE PRECISION D(*), E(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PTEQR(COMPZ, [N], D, E, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
INTEGER :: N, LDZ, INFO
REAL(8), DIMENSION(:) :: D, E, WORK
REAL(8), DIMENSION(:,:) :: Z

SUBROUTINE PTEQR_64(COMPZ, [N], D, E, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
INTEGER(8) :: N, LDZ, INFO
```

```
REAL(8), DIMENSION(:) :: D, E, WORK
REAL(8), DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void dpteqr(char compz, int n, double *d, double *e,  double
          *z, int ldz, int *info);

void dpteqr_64(char compz, long n,  double  *d,  double  *e,
          double *z, long ldz, long *info);
```

PURPOSE

dpteqr computes all eigenvalues and, optionally, eigenvectors of a symmetric positive definite tridiagonal matrix by first factoring the matrix using SPTTRF, and then calling SBDSQR to compute the singular values of the bidiagonal factor.

This routine computes the eigenvalues of the positive definite tridiagonal matrix to high relative accuracy. This means that if the eigenvalues range over many orders of magnitude in size, then the small eigenvalues and corresponding eigenvectors will be computed more accurately than, for example, with the standard QR method.

The eigenvectors of a full or band symmetric positive definite matrix can also be found if SSYTRD, SSPTRD, or SSBTRD has been used to reduce this matrix to tridiagonal form. (The reduction to tridiagonal form, however, may preclude the possibility of obtaining high relative accuracy in the small eigenvalues of the original matrix, if these eigenvalues range over many orders of magnitude.)

ARGUMENTS

COMPZ (input)
= 'N': Compute eigenvalues only.
= 'V': Compute eigenvectors of original symmetric matrix also. Array Z contains the orthogonal matrix used to reduce the original matrix to tridiagonal form. = 'I': Compute eigenvectors of tridiagonal matrix also.

N (input) The order of the matrix. N >= 0.

D (input/output)
On entry, the n diagonal elements of the tridiagonal matrix. On normal exit, D contains the eigenvalues, in descending order.

E (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix. On exit, E has been destroyed.

Z (input) On entry, if COMPZ = 'V', the orthogonal matrix used in the reduction to tridiagonal form. On exit, if COMPZ = 'V', the orthonormal eigenvectors of the original symmetric matrix; if COMPZ = 'I', the orthonormal eigenvectors of the tridiagonal matrix. If INFO > 0 on exit, Z contains the eigenvectors associated with only the stored eigenvalues. If COMPZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if COMPZ = 'V' or 'I', LDZ >= max(1,N).

WORK (workspace)
dimension(4*N)

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, and i is: <= N the Cholesky factorization of the matrix could not be performed because the i-th principal minor was not positive definite. > N the SVD algorithm failed to converge; if INFO = N+i, i off-diagonal elements of the bidiagonal factor did not converge to zero.

Contents

NAME

```
dptrfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is symmetric positive
definite and tridiagonal,  and  provides  error  bounds  and
backward error estimates for the solution
```

SYNOPSIS

```
SUBROUTINE DPTRFS(N, NRHS, DIAG, OFFD, DIAGF, OFFDF, B, LDB, X, LDX,
      FERR, BERR, WORK, INFO)

INTEGER N, NRHS, LDB, LDX, INFO
DOUBLE  PRECISION  DIAG(*),  OFFD(*),  DIAGF(*),   OFFDF(*),
B(LDB,*), X(LDX,*), FERR(*), BERR(*), WORK(*)

SUBROUTINE DPTRFS_64(N, NRHS, DIAG, OFFD, DIAGF, OFFDF, B, LDB, X,
      LDX, FERR, BERR, WORK, INFO)

INTEGER*8 N, NRHS, LDB, LDX, INFO
DOUBLE  PRECISION  DIAG(*),  OFFD(*),  DIAGF(*),   OFFDF(*),
B(LDB,*), X(LDX,*), FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PTRFS([N], [NRHS], DIAG, OFFD, DIAGF, OFFDF, B, [LDB], X,
       [LDX], FERR, BERR, [WORK], [INFO])

INTEGER :: N, NRHS, LDB, LDX, INFO
REAL(8), DIMENSION(:) :: DIAG,  OFFD,  DIAGF,  OFFDF,  FERR,
BERR, WORK
REAL(8), DIMENSION(:,:) :: B, X

SUBROUTINE PTRFS_64([N], [NRHS], DIAG, OFFD, DIAGF, OFFDF, B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [INFO])
```

```
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
REAL(8), DIMENSION(:) :: DIAG,  OFFD,  DIAGF,  OFFDF,  FERR,
BERR, WORK
REAL(8), DIMENSION(:,:) :: B, X
```

C INTERFACE

```
#include <sunperf.h>

void dptrfs(int n, int nrhs,  double  *diag,  double  *offd,
          double  *diagf, double *offdf, double *b, int ldb,
          double *x, int ldx, double  *ferr,  double  *berr,
          int *info);
void dptrfs_64(long  n,  long  nrhs,  double  *diag,  double
          *offd,  double  *diagf,  double *offdf, double *b,
          long ldb, double *x, long ldx, double *ferr,  dou-
          ble *berr, long *info);
```

PURPOSE

dptrfs improves the computed solution to a system of linear equations when the coefficient matrix is symmetric positive definite and tridiagonal, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

DIAG (input)
The n diagonal elements of the tridiagonal matrix A.

OFFD (input)
The (n-1) subdiagonal elements of the tridiagonal matrix A.

DIAGF (input)
The n diagonal elements of the diagonal matrix DIAG from the factorization computed by SPTTRF.

OFFDF (input)
The (n-1) subdiagonal elements of the unit bidiag-

```
          onal  factor  L from the factorization computed by
          SPTTRF.

B (input) The right hand side matrix B.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).
X (input/output)
          On entry, the solution matrix X,  as  computed  by
          SPTTRS.  On exit, the improved solution matrix X.

LDX (input)
          The leading dimension of  the  array  X.   LDX  >=
          max(1,N).

FERR (output)
          The forward error bound for each  solution  vector
          X(j)  (the  j-th column of the solution matrix X).
          If XTRUE is the  true  solution  corresponding  to
          X(j),  FERR(j) is an estimated upper bound for the
          magnitude of the largest element in (X(j) - XTRUE)
          divided by the magnitude of the largest element in
          X(j).

BERR (output)
          The componentwise relative backward error of  each
          solution  vector X(j) (i.e., the smallest relative
          change in any element of A or B that makes X(j) an
          exact solution).

WORK (workspace)
          dimension(2*N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
dptsv - compute the solution to  a  real  system  of  linear
equations  A*X  = B, where A is an N-by-N symmetric positive
definite tridiagonal matrix,  and  X  and  B  are  N-by-NRHS
matrices.
```

SYNOPSIS

```
SUBROUTINE DPTSV(N, NRHS, DIAG, SUB, B, LDB, INFO)

INTEGER N, NRHS, LDB, INFO
DOUBLE PRECISION DIAG(*), SUB(*), B(LDB,*)

SUBROUTINE DPTSV_64(N, NRHS, DIAG, SUB, B, LDB, INFO)

INTEGER*8 N, NRHS, LDB, INFO
DOUBLE PRECISION DIAG(*), SUB(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE PTSV([N], [NRHS], DIAG, SUB, B, [LDB], [INFO])

INTEGER :: N, NRHS, LDB, INFO
REAL(8), DIMENSION(:) :: DIAG, SUB
REAL(8), DIMENSION(:,:) :: B

SUBROUTINE PTSV_64([N], [NRHS], DIAG, SUB, B, [LDB], [INFO])

INTEGER(8) :: N, NRHS, LDB, INFO
REAL(8), DIMENSION(:) :: DIAG, SUB
REAL(8), DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dptsv(int n, int nrhs, double *diag, double *sub,  dou-
          ble *b, int ldb, int *info);

void dptsv_64(long n, long nrhs, double *diag, double  *sub,
          double *b, long ldb, long *info);
```

PURPOSE

dptsv computes the solution to a real system of linear equations A*X = B, where A is an N-by-N symmetric positive definite tridiagonal matrix, and X and B are N-by-NRHS matrices.

A is factored as A = L*D*L**T, and the factored form of A is then used to solve the system of equations.

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

DIAG (input/output)
On entry, the n diagonal elements of the tridiagonal matrix A. On exit, the n diagonal elements of the diagonal matrix DIAG from the factorization A = L*DIAG*L**T.

SUB (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix A. On exit, the (n-1) subdiagonal elements of the unit bidiagonal factor L from the L*DIAG*L**T factorization of A. (SUB can also be regarded as the superdiagonal of the unit bidiagonal factor U from the U**T*DIAG*U factorization of A.)

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >=

max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i is not positive definite, and the solution has not been computed. The factorization has not been completed unless i = N.

Contents

NAME

```
dptsvx - use the factorization A = L*D*L**T to  compute  the
solution to a real system of linear equations A*X = B, where
A is  an  N-by-N  symmetric  positive  definite  tridiagonal
matrix and X and B are N-by-NRHS matrices
```

SYNOPSIS

```
SUBROUTINE DPTSVX(FACT, N, NRHS, DIAG, SUB, DIAGF, SUBF, B, LDB, X,
      LDX, RCOND, FERR, BERR, WORK, INFO)

CHARACTER * 1 FACT
INTEGER N, NRHS, LDB, LDX, INFO
DOUBLE PRECISION RCOND
DOUBLE  PRECISION  DIAG(*),   SUB(*),   DIAGF(*),   SUBF(*),
B(LDB,*), X(LDX,*), FERR(*), BERR(*), WORK(*)

SUBROUTINE DPTSVX_64(FACT, N, NRHS, DIAG, SUB, DIAGF, SUBF, B, LDB,
      X, LDX, RCOND, FERR, BERR, WORK, INFO)

CHARACTER * 1 FACT
INTEGER*8 N, NRHS, LDB, LDX, INFO
DOUBLE PRECISION RCOND
DOUBLE  PRECISION  DIAG(*),   SUB(*),   DIAGF(*),   SUBF(*),
B(LDB,*), X(LDX,*), FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PTSVX(FACT, [N], [NRHS], DIAG, SUB, DIAGF, SUBF, B, [LDB],
       X, [LDX], RCOND, FERR, BERR, [WORK], [INFO])

CHARACTER(LEN=1) :: FACT
INTEGER :: N, NRHS, LDB, LDX, INFO
REAL(8) :: RCOND
```

```
REAL(8), DIMENSION(:) :: DIAG, SUB, DIAGF, SUBF, FERR, BERR,
WORK
REAL(8), DIMENSION(:,:) :: B, X

SUBROUTINE PTSVX_64(FACT, [N], [NRHS], DIAG, SUB, DIAGF, SUBF, B,
       [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [INFO])

CHARACTER(LEN=1) :: FACT
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: DIAG, SUB, DIAGF, SUBF, FERR, BERR,
WORK
REAL(8), DIMENSION(:,:) :: B, X
```

C INTERFACE

```
#include <sunperf.h>

void dptsvx(char fact, int n, int nrhs, double *diag, double
          *sub,  double *diagf, double *subf, double *b, int
          ldb, double *x, int  ldx,  double  *rcond,  double
          *ferr, double *berr, int *info);

void dptsvx_64(char fact, long n, long nrhs,  double  *diag,
          double  *sub,  double *diagf, double *subf, double
          *b, long ldb, double *x, long ldx, double  *rcond,
          double *ferr, double *berr, long *info);
```

PURPOSE

```
dptsvx uses the factorization A = L*D*L**T  to  compute  the
solution to a real system of linear equations A*X = B, where
A is  an  N-by-N  symmetric  positive  definite  tridiagonal
matrix and X and B are N-by-NRHS matrices.

Error bounds on the solution and a  condition  estimate  are
also provided.

The following steps are performed:

1. If FACT = 'N', the matrix A is factored as A =  L*D*L**T,
where L
   is a unit lower bidiagonal matrix and D is diagonal.  The
   factorization can also be regarded as having the form
   A = U**T*D*U.

2. If the leading i-by-i principal  minor  is  not  positive
definite,
   then the routine returns with INFO =  i.  Otherwise,  the
factored
   form of A is used to estimate the condition number of the
```

matrix
A. If the reciprocal of the condition number is less than machine
precision, INFO = N+1 is returned as a warning, but the routine
still goes on to solve for X and compute error bounds as described below.

3. The system of equations is solved for X using the factored form
of A.

4. Iterative refinement is applied to improve the computed solution
matrix and calculate error bounds and backward error estimates
for it.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of A has been supplied on entry. = 'F': On entry, DIAGF and SUBF contain the factored form of A. DIAG, SUB, DIAGF, and SUBF will not be modified. = 'N': The matrix A will be copied to DIAGF and SUBF and factored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

DIAG (input)
The n diagonal elements of the tridiagonal matrix A.

SUB (input)
The (n-1) subdiagonal elements of the tridiagonal matrix A.

DIAGF (input/output)
If FACT = 'F', then DIAGF is an input argument and on entry contains the n diagonal elements of the diagonal matrix DIAG from the L*DIAG*L**T factorization of A. If FACT = 'N', then DIAGF is an output argument and on exit contains the n diagonal

elements of the diagonal matrix DIAG from the L*DIAG*L**T factorization of A.

SUBF (input/output)
If FACT = 'F', then SUBF is an input argument and on entry contains the (n-1) subdiagonal elements of the unit bidiagonal factor L from the L*DIAG*L**T factorization of A. If FACT = 'N', then SUBF is an output argument and on exit contains the (n-1) subdiagonal elements of the unit bidiagonal factor L from the L*DIAG*L**T factorization of A.

B (input) The N-by-NRHS right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 of INFO = N+1, the N-by-NRHS solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The reciprocal condition number of the matrix A. If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j).

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, and i is

<= N: the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
dpttrf - compute the L*D*L' factorization  of  a  real  sym-
metric positive definite tridiagonal matrix A
```

SYNOPSIS

```
SUBROUTINE DPTTRF(N, DIAG, OFFD, INFO)

INTEGER N, INFO
DOUBLE PRECISION DIAG(*), OFFD(*)

SUBROUTINE DPTTRF_64(N, DIAG, OFFD, INFO)

INTEGER*8 N, INFO
DOUBLE PRECISION DIAG(*), OFFD(*)
```

F95 INTERFACE

```
SUBROUTINE PTTRF([N], DIAG, OFFD, [INFO])

INTEGER :: N, INFO
REAL(8), DIMENSION(:) :: DIAG, OFFD

SUBROUTINE PTTRF_64([N], DIAG, OFFD, [INFO])

INTEGER(8) :: N, INFO
REAL(8), DIMENSION(:) :: DIAG, OFFD
```

C INTERFACE

```
#include <sunperf.h>

void dpttrf(int n, double *diag, double *offd, int *info);

void dpttrf_64(long n,  double  *diag,  double  *offd,  long
```

```
          *info);
```

PURPOSE

```
dpttrf computes the L*D*L' factorization of a real symmetric
positive  definite  tridiagonal matrix A.  The factorization
may also be regarded as having the form A = U'*D*U.
```

ARGUMENTS

```
N (input) The order of the matrix A.  N >= 0.

DIAG (input/output)
          On entry, the n diagonal elements of the tridiago-
          nal matrix A.  On exit, the n diagonal elements of
          the diagonal matrix DIAG from the  L*DIAG*L'  fac-
          torization of A.

OFFD (input/output)
          On entry, the (n-1) subdiagonal  elements  of  the
          tridiagonal matrix A.  On exit, the (n-1) subdiag-
          onal elements of the unit bidiagonal factor L from
          the  L*DIAG*L'  factorization of A.  OFFD can also
          be regarded as the superdiagonal of the unit bidi-
          agonal  factor  U from the U'*DIAG*U factorization
          of A.

INFO (output)
          = 0: successful exit
          < 0: if INFO = -k, the k-th argument had an  ille-
          gal value
          > 0: if INFO = k, the leading minor of order k  is
          not positive definite; if k < N, the factorization
          could not be completed, while if k = N,  the  fac-
          torization was completed, but DIAG(N) = 0.
```

Contents

NAME

```
dpttrs - solve a tridiagonal system of the form  A * X  =  B
using the L*D*L' factorization of A computed by SPTTRF
```

SYNOPSIS

```
SUBROUTINE DPTTRS(N, NRHS, DIAG, OFFD, B, LDB, INFO)

INTEGER N, NRHS, LDB, INFO
DOUBLE PRECISION DIAG(*), OFFD(*), B(LDB,*)

SUBROUTINE DPTTRS_64(N, NRHS, DIAG, OFFD, B, LDB, INFO)

INTEGER*8 N, NRHS, LDB, INFO
DOUBLE PRECISION DIAG(*), OFFD(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE PTTRS([N], [NRHS], DIAG, OFFD, B, [LDB], [INFO])

INTEGER :: N, NRHS, LDB, INFO
REAL(8), DIMENSION(:) :: DIAG, OFFD
REAL(8), DIMENSION(:,:) :: B

SUBROUTINE PTTRS_64([N], [NRHS], DIAG, OFFD, B, [LDB], [INFO])

INTEGER(8) :: N, NRHS, LDB, INFO
REAL(8), DIMENSION(:) :: DIAG, OFFD
REAL(8), DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void dpttrs(int n, int nrhs,  double  *diag,  double  *offd,
```

```
          double *b, int ldb, int *info);

void dpttrs_64(long  n,  long  nrhs,  double  *diag,  double
          *offd, double *b, long ldb, long *info);
```

PURPOSE

```
dpttrs solves a tridiagonal system of the form
   A * X = B using the L*D*L' factorization of A computed by
SPTTRF.  D is a diagonal matrix specified in the vector D, L
is a unit bidiagonal matrix whose subdiagonal  is  specified
in the vector E, and X and B are N by NRHS matrices.
```

ARGUMENTS

```
N (input) The order of the tridiagonal matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

DIAG (input)
          The n diagonal elements  of  the  diagonal  matrix
          DIAG from the L*DIAG*L' factorization of A.

OFFD (input/output)
          The (n-1) subdiagonal elements of the unit bidiag-
          onal  factor L from the L*DIAG*L' factorization of
          A.  OFFD can also be regarded as the superdiagonal
          of  the unit bidiagonal factor U from the factori-
          zation A = U'*DIAG*U.

B (input/output)
          On entry, the right hand side vectors  B  for  the
          system of linear equations.  On exit, the solution
          vectors, X.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

INFO (output)
          = 0: successful exit
          < 0: if INFO = -k, the k-th argument had an  ille-
          gal value
```

Contents

NAME

```
dptts2 - solve a tridiagonal system of the form  A * X  =  B
using the L*D*L' factorization of A computed by SPTTRF
```

SYNOPSIS

```
SUBROUTINE DPTTS2(N, NRHS, D, E, B, LDB)

INTEGER N, NRHS, LDB
DOUBLE PRECISION D(*), E(*), B(LDB,*)

SUBROUTINE DPTTS2_64(N, NRHS, D, E, B, LDB)

INTEGER*8 N, NRHS, LDB
DOUBLE PRECISION D(*), E(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE DPTTS2(N, NRHS, D, E, B, LDB)

INTEGER :: N, NRHS, LDB
REAL(8), DIMENSION(:) :: D, E
REAL(8), DIMENSION(:,:) :: B

SUBROUTINE DPTTS2_64(N, NRHS, D, E, B, LDB)

INTEGER(8) :: N, NRHS, LDB
REAL(8), DIMENSION(:) :: D, E
REAL(8), DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void dptts2(int n, int nrhs, double *d,  double  *e,  double
```

```
          *b, int ldb);

void dptts2_64(long n, long nrhs, double *d, double *e, dou-
          ble *b, long ldb);
```

PURPOSE

dptts2 solves a tridiagonal system of the form
A * X = B using the L*D*L' factorization of A computed by SPTTRF. D is a diagonal matrix specified in the vector D, L is a unit bidiagonal matrix whose subdiagonal is specified in the vector E, and X and B are N by NRHS matrices.

ARGUMENTS

N (input) The order of the tridiagonal matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

D (input) The n diagonal elements of the diagonal matrix D from the L*D*L' factorization of A.

E (input) The (n-1) subdiagonal elements of the unit bidiagonal factor L from the L*D*L' factorization of A. E can also be regarded as the superdiagonal of the unit bidiagonal factor U from the factorization A = U'*D*U.

B (input/output)
On entry, the right hand side vectors B for the system of linear equations. On exit, the solution vectors, X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

Contents

NAME

dqdota - compute a double precision constant plus an extended precision constant plus the extended precision dot product of two double precision vectors x and y.

SYNOPSIS

```
DOUBLE PRECISION FUNCTION DQDOTA(N, DB, QC, DX, INCX, DY, INCY)

INTEGER N, INCX, INCY
REAL * 16 QC
DOUBLE PRECISION DB
DOUBLE PRECISION DX(*), DY(*)

DOUBLE PRECISION FUNCTION DQDOTA_64(N, DB, QC, DX, INCX, DY, INCY)

INTEGER*8 N, INCX, INCY
REAL * 16 QC
DOUBLE PRECISION DB
DOUBLE PRECISION DX(*), DY(*)
```

F95 INTERFACE

```
REAL(8) FUNCTION DQDOTA(N, DB, QC, DX, INCX, DY, INCY)

INTEGER :: N, INCX, INCY
REAL(16) :: QC
REAL(8) :: DB
REAL(8), DIMENSION(:) :: DX, DY

REAL(8) FUNCTION DQDOTA_64(N, DB, QC, DX, INCX, DY, INCY)

INTEGER(8) :: N, INCX, INCY
REAL(16) :: QC
```

```
REAL(8) :: DB
REAL(8), DIMENSION(:) :: DX, DY
```

C INTERFACE

```
#include <sunperf.h>

double dqdota(int n, double db, long double *qc, double *dx,
          int incx, double *dy, int incy);

double dqdota_64(long n, double db, long double *qc,  double
          *dx, long incx, double *dy, long incy);
```

PURPOSE

dqdota compute a double precision constant plus an extended precision constant plus the extended precision dot product of two double precision vectors x and y.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. If N <= 0 then the function returns the value DB+QC. Unchanged on exit.

DB (input)
On entry, the constant that is added to the dot product before the result is returned. Unchanged on exit.

QC (input/output)
On entry, the extended precision constant to be added to the dot product. On exit, the extended precision result.

DX (input)
On entry, the incremented array DX must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of DX. INCX must not be zero. Unchanged on exit.

DY (input)
On entry, the incremented array DY must contain the vector y. Unchanged on exit.

INCY (input)

On entry, INCY specifies the increment for the elements of DY. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
dqdoti - compute a constant plus the extended precision  dot
product of two double precision vectors x and y.
```

SYNOPSIS

```
DOUBLE PRECISION FUNCTION DQDOTI(N, DB, QC, DX, INCX, DY, INCY)

INTEGER N, INCX, INCY
REAL * 16 QC
DOUBLE PRECISION DB
DOUBLE PRECISION DX(*), DY(*)

DOUBLE PRECISION FUNCTION DQDOTI_64(N, DB, QC, DX, INCX, DY, INCY)

INTEGER*8 N, INCX, INCY
REAL * 16 QC
DOUBLE PRECISION DB
DOUBLE PRECISION DX(*), DY(*)
```

F95 INTERFACE

```
REAL(8) FUNCTION DQDOTI(N, DB, QC, DX, INCX, DY, INCY)

INTEGER :: N, INCX, INCY
REAL(16) :: QC
REAL(8) :: DB
REAL(8), DIMENSION(:) :: DX, DY

REAL(8) FUNCTION DQDOTI_64(N, DB, QC, DX, INCX, DY, INCY)

INTEGER(8) :: N, INCX, INCY
REAL(16) :: QC
REAL(8) :: DB
```

```
REAL(8), DIMENSION(:) :: DX, DY
```

C INTERFACE

```
#include <sunperf.h>

double dqdoti(int n, double db, long double *qc, double *dx,
          int incx, double *dy, int incy);

double dqdoti_64(long n, double db, long double *qc,  double
          *dx, long incx, double *dy, long incy);
```

PURPOSE

dqdoti computes a constant plus the double precision dot product of x and y where x and y are double precision n-vectors.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. If N <= 0 then the function returns the value DB. Unchanged on exit.

DB (input)
On entry, the constant that is added to the dot product before the result is returned. Unchanged on exit.

QC (output)
On exit, the extended precision result.

DX (input)
On entry, the incremented array DX must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of DX. INCX must not be zero. Unchanged on exit.

DY (input)
On entry, the incremented array DY must contain the vector y. Unchanged on exit.

INCY (input)
On entry, INCY specifies the increment for the

elements of DY. INCY must not be zero. Unchanged
on exit.

Contents

NAME

```
drot - Apply a Given's rotation constructed by SROTG.
```

SYNOPSIS

```
SUBROUTINE DROT(N, X, INCX, Y, INCY, C, S)

INTEGER N, INCX, INCY
DOUBLE PRECISION C, S
DOUBLE PRECISION X(*), Y(*)

SUBROUTINE DROT_64(N, X, INCX, Y, INCY, C, S)

INTEGER*8 N, INCX, INCY
DOUBLE PRECISION C, S
DOUBLE PRECISION X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE ROT([N], X, [INCX], Y, [INCY], C, S)

INTEGER :: N, INCX, INCY
REAL(8) :: C, S
REAL(8), DIMENSION(:) :: X, Y

SUBROUTINE ROT_64([N], X, [INCX], Y, [INCY], C, S)

INTEGER(8) :: N, INCX, INCY
REAL(8) :: C, S
REAL(8), DIMENSION(:) :: X, Y
```

C INTERFACE

```
#include <sunperf.h>
```

```
void drot(int n, double *x, int incx, double *y,  int  incy,
          double c, double s);

void drot_64(long n, double *x, long incx, double  *y,  long
          incy, double c, double s);
```

PURPOSE

drot Apply a Given's rotation constructed by SROTG.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input/output)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

C (input) On entry, the C rotation value constructed by SROTG. Unchanged on exit.

S (input) On entry, the S rotation value constructed by SROTG. Unchanged on exit.

Contents

NAME

```
drotg - Construct a Given's plane rotation
```

SYNOPSIS

```
SUBROUTINE DROTG(A, B, C, S)

DOUBLE PRECISION A, B, C, S

SUBROUTINE DROTG_64(A, B, C, S)

DOUBLE PRECISION A, B, C, S
```

F95 INTERFACE

```
SUBROUTINE ROTG(A, B, C, S)

REAL(8) :: A, B, C, S

SUBROUTINE ROTG_64(A, B, C, S)

REAL(8) :: A, B, C, S
```

C INTERFACE

```
#include <sunperf.h>

void drotg(double *a, double *b, double *c, double *s);

void drotg_64(double *a, double *b, double *c, double *s);
```

PURPOSE

drotg Construct a Given's plane rotation that will annihilate an element of a vector.

ARGUMENTS

A (input/output)
: On entry, A contains the entry in the first vector that corresponds to the element to be annihilated in the second vector. On exit, contains the nonzero element of the rotated vector.

B (input/output)
: On entry, B contains the entry to be annihilated in the second vector. On exit, contains either S or 1/C depending on which of the input values of A and B is larger.

C (output)
: On exit, C and S are the elements of the rotation matrix that will be applied to annihilate B.

S (output)
: See the description of C.

Contents

NAME

```
droti -  Apply an indexed Givens rotation.
```

SYNOPSIS

```
 SUBROUTINE DROTI(NZ, X, INDX, Y, C, S)

 INTEGER NZ
 INTEGER INDX(*)
 DOUBLE PRECISION C, S
 DOUBLE PRECISION X(*), Y(*)

 SUBROUTINE DROTI_64(NZ, X, INDX, Y, C, S)

 INTEGER*8 NZ
 INTEGER*8 INDX(*)
 DOUBLE PRECISION C, S
 DOUBLE PRECISION X(*), Y(*)

F95 INTERFACE
 SUBROUTINE ROTI([NZ], X, INDX, Y, C, S)

 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX
 REAL(8) :: C, S
 REAL(8), DIMENSION(:) :: X, Y

 SUBROUTINE ROTI_64([NZ], X, INDX, Y, C, S)

 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
 REAL(8) :: C, S
 REAL(8), DIMENSION(:) :: X, Y
```

PURPOSE

```
DROTI - Applies a Givens rotation to a sparse vector x
stored in compressed form and another vector y in full
storage form

 do i = 1, n
   temp = -s * x(i) + c * y(indx(i))
   x(i) = c * x(i) + s * y(indx(i))
   y(indx(i)) = temp
 enddo
```

ARGUMENTS

```
NZ (input) - INTEGER
        Number of elements in the compressed form.
        Unchanged on exit.
X (input)
        Vector containing the values of the compressed form.

INDX (input) - INTEGER
        Vector containing the indices of the compressed
        form.  It is assumed that the elements in INDX are
        distinct and greater than zero.  Unchanged on exit.

Y (input/output)
        Vector on input which contains the vector Y in full
        storage form.  On exit, only the elements
        corresponding to the indices in INDX have been
        modified.

C (input)
        Scalar defining the Givens rotation

S (input)
        Scalar defining the Givens rotation
```

Contents

NAME

```
drotm - Apply a Gentleman's modified Given's  rotation  con-
structed by SROTMG.
```

SYNOPSIS

```
SUBROUTINE DROTM(N, X, INCX, Y, INCY, PARAM)

INTEGER N, INCX, INCY
DOUBLE PRECISION X(*), Y(*), PARAM(*)

SUBROUTINE DROTM_64(N, X, INCX, Y, INCY, PARAM)

INTEGER*8 N, INCX, INCY
DOUBLE PRECISION X(*), Y(*), PARAM(*)
```

F95 INTERFACE

```
SUBROUTINE ROTM([N], X, [INCX], Y, [INCY], PARAM)

INTEGER :: N, INCX, INCY
REAL(8), DIMENSION(:) :: X, Y, PARAM

SUBROUTINE ROTM_64([N], X, [INCX], Y, [INCY], PARAM)

INTEGER(8) :: N, INCX, INCY
REAL(8), DIMENSION(:) :: X, Y, PARAM
```

C INTERFACE

```
#include <sunperf.h>

void drotm(int n, double *x, int incx, double *y, int  incy,
          double *param);
```

```
void drotm_64(long n, double *x, long incx, double *y,  long
          incy, double *param);
```

PURPOSE

drotm Apply a Given's rotation constructed by SROTMG.

ARGUMENTS

```
N (input)
          On entry, N specifies the number  of  elements  in
          the  vector.   N must be at least one for the sub-
          routine to have any visible effect.  Unchanged  on
          exit.
X (input/output)
          ( 1 + ( n - 1 )*abs( INCX  )  ).   On  entry,  the
          incremented  array X must contain the vector x. On
          exit, X is overwritten by the updated vector x.

INCX (input)
          On entry, INCX specifies  the  increment  for  the
          elements  of  X. INCX must not be zero.  Unchanged
          on exit.

Y (input/output)
          ( 1 + ( n - 1 )*abs( INCY  )  ).   On  entry,  the
          incremented  array Y must contain the vector y. On
          exit, Y is overwritten by the updated vector y.

INCY (input)
          On entry, INCY specifies  the  increment  for  the
          elements  of  Y. INCY must not be zero.  Unchanged
          on exit.

PARAM (input)
          On  entry,  the  rotation  values  constructed  by
          SROTMG.  Unchanged on exit.
```

Contents

NAME

```
drotmg - Construct  a  Gentleman's  modified  Given's  plane
rotation
```

SYNOPSIS

```
SUBROUTINE DROTMG(D1, D2, B1, B2, PARAM)

DOUBLE PRECISION D1, D2, B1, B2
DOUBLE PRECISION PARAM(*)

SUBROUTINE DROTMG_64(D1, D2, B1, B2, PARAM)

DOUBLE PRECISION D1, D2, B1, B2
DOUBLE PRECISION PARAM(*)
```

F95 INTERFACE

```
SUBROUTINE ROTMG(D1, D2, B1, B2, PARAM)

REAL(8) :: D1, D2, B1, B2
REAL(8), DIMENSION(:) :: PARAM

SUBROUTINE ROTMG_64(D1, D2, B1, B2, PARAM)

REAL(8) :: D1, D2, B1, B2
REAL(8), DIMENSION(:) :: PARAM
```

C INTERFACE

```
#include <sunperf.h>

void drotmg(double d1, double d2, double b1, double b2, dou-
          ble *param);
```

```
void drotmg_64(double d1, double d2, double b1,  double  b2,
          double *param);
```

PURPOSE

drotmg Construct Gentleman's modified a Given's plane rotation that will annihilate an element of a vector.

ARGUMENTS

D1 (input/output)
On entry, the first diagonal entry in the H matrix. On exit, changed to reflect the effect of the transformation.

D2 (input/output)
On entry, the second diagonal entry in the H matrix. On exit, changed to reflect the effect of the transformation.

B1 (input/output)
On entry, the first element of the vector to which the H matrix is applied. On exit, changed to reflect the effect of the transformation.

B2 (input)
On entry, the second element of the vector to which the H matrix is applied. Unchanged on exit.

PARAM (output)
On exit, PARAM(1) describes the form of the rotation matrix H, and PARAM(2..5) contain the H matrix.

If PARAM(1) = -2 then H = I and no elements of PARAM are modified.

If PARAM(1) = -1 then PARAM(2) = h11, PARAM(3) = h21, PARAM(4) = h12, and PARAM(5) = h22.

If PARAM(1) = 0 then h11 = h22 = 1, PARAM(3) = h21, and PARAM(4) = h12.

If PARAM(1) = 1 then h12 = 1, h21 = -1, PARAM(2) = h11, and PARAM(5) = h22.

Contents

NAME

```
dsbev - compute all the eigenvalues and, optionally,  eigen-
vectors of a real symmetric band matrix A
```

SYNOPSIS

```
SUBROUTINE DSBEV(JOBZ, UPLO, N, KD, A, LDA, W, Z, LDZ, WORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER N, KD, LDA, LDZ, INFO
DOUBLE PRECISION A(LDA,*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSBEV_64(JOBZ, UPLO, N, KD, A, LDA, W, Z, LDZ, WORK,
      INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 N, KD, LDA, LDZ, INFO
DOUBLE PRECISION A(LDA,*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SBEV(JOBZ, UPLO, [N], KD, A, [LDA], W, Z, [LDZ], [WORK],
       [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: N, KD, LDA, LDZ, INFO
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: A, Z

SUBROUTINE SBEV_64(JOBZ, UPLO, [N], KD, A, [LDA], W, Z, [LDZ],
       [WORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: N, KD, LDA, LDZ, INFO
```

```
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: A, Z
```

C INTERFACE

```
#include <sunperf.h>

void dsbev(char jobz, char uplo, int n, int kd,  double  *a,
          int  lda,  double  *w,  double  *z,  int  ldz, int
          *info);

void dsbev_64(char jobz, char uplo, long n, long kd,  double
          *a, long lda, double *w, double *z, long ldz, long
          *info);
```

PURPOSE

dsbev computes all the eigenvalues and, optionally, eigenvectors of a real symmetric band matrix A.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  KD >= 0.

A (input/output)
          On entry, the upper or lower triangle of the  sym-
          metric  band  matrix  A,  stored in the first KD+1
          rows of the array.  The j-th column of A is stored
          in  the j-th column of the array A as follows:  if
          UPLO = 'U', A(kd+1+i-j,j) =  A(i,j)  for  max(1,j-
          kd)<=i<=j;  if  UPLO = 'L', A(1+i-j,j)    = A(i,j)
          for j<=i<=min(n,j+kd).

          On exit, A is overwritten by values generated dur-
          ing  the reduction to tridiagonal form.  If UPLO =
```

'U', the first superdiagonal and the diagonal of the tridiagonal matrix T are returned in rows KD and KD+1 of A, and if UPLO = 'L', the diagonal and first subdiagonal of T are returned in the first two rows of A.

LDA (input)
The leading dimension of the array A. LDA >= KD + 1.

W (output)
If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, Z contains the orthonormal eigenvectors of the matrix A, with the i-th column of Z holding the eigenvector associated with W(i). If JOBZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
dimension(MAX(1,3*N-2))

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

Contents

NAME

```
dsbevd - compute all the eigenvalues and, optionally, eigen-
vectors of a real symmetric band matrix A
```

SYNOPSIS

```
SUBROUTINE DSBEVD(JOBZ, UPLO, N, KD, AB, LDAB, W, Z, LDZ, WORK,
      LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER N, KD, LDAB, LDZ, LWORK, LIWORK, INFO
INTEGER IWORK(*)
DOUBLE PRECISION AB(LDAB,*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSBEVD_64(JOBZ, UPLO, N, KD, AB, LDAB, W, Z, LDZ, WORK,
      LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 N, KD, LDAB, LDZ, LWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION AB(LDAB,*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SBEVD(JOBZ, UPLO, [N], KD, AB, [LDAB], W, Z, [LDZ], [WORK],
       [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: N, KD, LDAB, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: AB, Z

SUBROUTINE SBEVD_64(JOBZ, UPLO, [N], KD, AB, [LDAB], W, Z, [LDZ],
```

```
          [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

   CHARACTER(LEN=1) :: JOBZ, UPLO
   INTEGER(8) :: N, KD, LDAB, LDZ, LWORK, LIWORK, INFO
   INTEGER(8), DIMENSION(:) :: IWORK
   REAL(8), DIMENSION(:) :: W, WORK
   REAL(8), DIMENSION(:,:) :: AB, Z
```

C INTERFACE

```
   #include <sunperf.h>

   void dsbevd(char jobz, char uplo, int n, int kd, double *ab,
             int  ldab,  double  *w,  double  *z,  int ldz, int
             *info);
   void dsbevd_64(char jobz, char uplo, long n, long kd, double
             *ab,  long  ldab,  double *w, double *z, long ldz,
             long *info);
```

PURPOSE

dsbevd computes all the eigenvalues and, optionally, eigenvectors of a real symmetric band matrix A. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO

= 'L'. KD >= 0.

AB (input/output)
On entry, the upper or lower triangle of the symmetric band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).

On exit, AB is overwritten by values generated during the reduction to tridiagonal form. If UPLO = 'U', the first superdiagonal and the diagonal of the tridiagonal matrix T are returned in rows KD and KD+1 of AB, and if UPLO = 'L', the diagonal and first subdiagonal of T are returned in the first two rows of AB.

LDAB (input)
The leading dimension of the array AB. LDAB >= KD + 1.

W (output)
If INFO = 0, the eigenvalues in ascending order.

Z (output)
If JOBZ = 'V', then if INFO = 0, Z contains the orthonormal eigenvectors of the matrix A, with the i-th column of Z holding the eigenvector associated with W(i). If JOBZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
dimension (LWORK) On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If N <= 1, LWORK must be at least 1. If JOBZ = 'N' and N > 2, LWORK must be at least 2*N. If JOBZ = 'V' and N > 2, LWORK must be at least (1 + 5*N + 2*N**2).

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of

```
            the WORK array, returns this value  as  the  first
            entry  of  the  WORK  array,  and no error message
            related to LWORK is issued by XERBLA.

  IWORK (workspace/output)
            On exit, if INFO = 0, IWORK(1) returns the optimal
            LIWORK.
  LIWORK (input)
            The dimension of the array LIWORK.  If JOBZ  = 'N'
            or  N <= 1, LIWORK must be at least 1.  If JOBZ  =
            'V' and N > 2, LIWORK must be at least 3 + 5*N.

            If LIWORK = -1, then a workspace query is assumed;
            the  routine  only  calculates the optimal size of
            the IWORK array, returns this value as  the  first
            entry  of  the  IWORK  array, and no error message
            related to LIWORK is issued by XERBLA.

  INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value
            > 0:  if INFO = i, the algorithm  failed  to  con-
            verge;  i off-diagonal elements of an intermediate
            tridiagonal form did not converge to zero.
```

Contents

NAME

dsbevx - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric band matrix A

SYNOPSIS

```
SUBROUTINE DSBEVX(JOBZ, RANGE, UPLO, N, KD, A, LDA, Q, LDQ, VL,
      VU, IL, IU, ABTOL, NFOUND, W, Z, LDZ, WORK, IWORK2, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER N, KD, LDA, LDQ, IL, IU, NFOUND, LDZ, INFO
INTEGER IWORK2(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABTOL
DOUBLE PRECISION A(LDA,*), Q(LDQ,*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSBEVX_64(JOBZ, RANGE, UPLO, N, KD, A, LDA, Q, LDQ, VL,
      VU, IL, IU, ABTOL, NFOUND, W, Z, LDZ, WORK, IWORK2, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER*8 N, KD, LDA, LDQ, IL, IU, NFOUND, LDZ, INFO
INTEGER*8 IWORK2(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABTOL
DOUBLE PRECISION A(LDA,*), Q(LDQ,*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SBEVX(JOBZ, RANGE, UPLO, [N], KD, A, [LDA], Q, [LDQ],
       VL, VU, IL, IU, ABTOL, NFOUND, W, Z, [LDZ], [WORK], [IWORK2],
       IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER :: N, KD, LDA, LDQ, IL, IU, NFOUND, LDZ, INFO
INTEGER, DIMENSION(:) :: IWORK2, IFAIL
REAL(8) :: VL, VU, ABTOL
```

```
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: A, Q, Z

SUBROUTINE SBEVX_64(JOBZ, RANGE, UPLO, [N], KD, A, [LDA], Q, [LDQ],
       VL, VU, IL, IU, ABTOL, NFOUND, W, Z, [LDZ], [WORK], [IWORK2],
       IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER(8) :: N, KD, LDA, LDQ, IL, IU, NFOUND, LDZ, INFO
INTEGER(8), DIMENSION(:) :: IWORK2, IFAIL
REAL(8) :: VL, VU, ABTOL
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: A, Q, Z
```

C INTERFACE

```
#include <sunperf.h>

void dsbevx(char jobz, char range, char uplo, int n, int kd,
          double *a, int lda, double *q, int ldq, double vl,
          double vu, int  il,  int  iu,  double  abtol,  int
          *nfound,  double  *w,  double  *z,  int  ldz,  int
          *ifail, int *info);

void dsbevx_64(char jobz, char range,  char  uplo,  long  n,
          long kd, double *a, long lda, double *q, long ldq,
          double vl, double vu, long  il,  long  iu,  double
          abtol,  long  *nfound,  double *w, double *z, long
          ldz, long *ifail, long *info);
```

PURPOSE

dsbevx computes selected eigenvalues and, optionally, eigenvectors of a real symmetric band matrix A. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

RANGE (input)
          = 'A': all eigenvalues will be found;
          = 'V': all eigenvalues in the  half-open  interval
          (VL,VU]  will  be  found; = 'I': the IL-th through
          IU-th eigenvalues will be found.
```

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  KD >= 0.

A (input/output)
          On entry, the upper or lower triangle of the  sym-
          metric  band  matrix  A,  stored in the first KD+1
          rows of the array.  The j-th column of A is stored
          in  the j-th column of the array A as follows:  if
          UPLO = 'U', A(kd+1+i-j,j) =  A(i,j)  for  max(1,j-
          kd)<=i<=j;  if  UPLO = 'L', A(1+i-j,j)    = A(i,j)
          for j<=i<=min(n,j+kd).

          On exit, A is overwritten by values generated dur-
          ing  the reduction to tridiagonal form.  If UPLO =
          'U', the first superdiagonal and the  diagonal  of
          the  tridiagonal  matrix T are returned in rows KD
          and KD+1 of A, and if UPLO = 'L', the diagonal and
          first  subdiagonal  of T are returned in the first
          two rows of A.

LDA (input)
          The leading dimension of the array A.  LDA >= KD +
          1.

Q (output)
          If JOBZ = 'V', the N-by-N orthogonal  matrix  used
          in  the  reduction to tridiagonal form.  If JOBZ =
          'N', the array Q is not referenced.

LDQ (input)
          The leading dimension of the array Q.  If  JOBZ  =
          'V', then LDQ >= max(1,N).

VL (input)
          If RANGE='V', the lower and upper  bounds  of  the
          interval  to be searched for eigenvalues. VL < VU.
          Not referenced if RANGE = 'A' or 'I'.

VU (input)
          See the description of VL.

IL (input)
```

If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
See the description of IL.

ABTOL (input)
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing A to tri- diagonal form.

Eigenvalues will be computed most accurately when ABTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABTOL to 2*SLAMCH('S').

See "Computing Small Singular Values of Bidiagonal Matrices with Guaranteed High Relative Accuracy," by Demmel and Kahan, LAPACK Working Note #3.

NFOUND (output)
The total number of eigenvalues found. 0 <= NFOUND <= N. If RANGE = 'A', NFOUND = N, and if RANGE = 'I', NFOUND = IU-IL+1.

W (output)
The first NFOUND elements contain the selected eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, the first NFOUND columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). If an eigenvec- tor fails to converge, then that column of Z con- tains the latest approximation to the eigenvector, and the index of the eigenvector is returned in

```
            IFAIL.  If JOBZ = 'N', then Z is  not  referenced.
            Note:   the   user   must  ensure  that  at  least
            max(1,NFOUND) columns are supplied in the array Z;
            if  RANGE  = 'V', the exact value of NFOUND is not
            known in advance and an upper bound must be used.
LDZ (input)
            The leading dimension of the array Z.  LDZ  >=  1,
            and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
            dimension(7*N)

IWORK2 (workspace)

IFAIL (output)
            If JOBZ = 'V', then if INFO = 0, the first  NFOUND
            elements  of  IFAIL  are  zero.  If INFO > 0, then
            IFAIL contains the  indices  of  the  eigenvectors
            that  failed  to  converge.   If  JOBZ = 'N', then
            IFAIL is not referenced.

INFO (output)
            = 0:  successful exit.
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value.
            > 0:  if INFO = i, then i eigenvectors  failed  to
            converge.   Their  indices  are  stored  in  array
            IFAIL.
```

Contents

NAME

dsbgst - reduce a real symmetric-definite banded generalized eigenproblem A*x = lambda*B*x to standard form C*y = lambda*y,

SYNOPSIS

```
SUBROUTINE DSBGST(VECT, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, X, LDX,
      WORK, INFO)

CHARACTER * 1 VECT, UPLO
INTEGER N, KA, KB, LDAB, LDBB, LDX, INFO
DOUBLE PRECISION AB(LDAB,*), BB(LDBB,*), X(LDX,*), WORK(*)

SUBROUTINE DSBGST_64(VECT, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, X,
      LDX, WORK, INFO)

CHARACTER * 1 VECT, UPLO
INTEGER*8 N, KA, KB, LDAB, LDBB, LDX, INFO
DOUBLE PRECISION AB(LDAB,*), BB(LDBB,*), X(LDX,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SBGST(VECT, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB], X,
       [LDX], [WORK], [INFO])

CHARACTER(LEN=1) :: VECT, UPLO
INTEGER :: N, KA, KB, LDAB, LDBB, LDX, INFO
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: AB, BB, X

SUBROUTINE SBGST_64(VECT, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB],
       X, [LDX], [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: VECT, UPLO
INTEGER(8) :: N, KA, KB, LDAB, LDBB, LDX, INFO
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: AB, BB, X
```

C INTERFACE

```
#include <sunperf.h>

void dsbgst(char vect, char uplo, int n,  int  ka,  int  kb,
          double *ab, int ldab, double *bb, int ldbb, double
          *x, int ldx, int *info);

void dsbgst_64(char vect, char uplo, long n, long  ka,  long
          kb,  double *ab, long ldab, double *bb, long ldbb,
          double *x, long ldx, long *info);
```

PURPOSE

dsbgst reduces a real symmetric-definite banded generalized eigenproblem A*x = lambda*B*x to standard form C*y = lambda*y, such that C has the same bandwidth as A.

B must have been previously factorized as S**T*S by SPBSTF, using a split Cholesky factorization. A is overwritten by C = X**T*A*X, where X = S**(-1)*Q and Q is an orthogonal matrix chosen to preserve the bandwidth of A.

ARGUMENTS

VECT (input)
- = 'N': do not form the transformation matrix X;
- = 'V': form X.

UPLO (input)
- = 'U': Upper triangle of A is stored;
- = 'L': Lower triangle of A is stored.

N (input) The order of the matrices A and B. N >= 0.

KA (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KA >= 0.

KB (input)
The number of superdiagonals of the matrix B if UPLO = 'U', or the number of subdiagonals if UPLO

= 'L'. KA >= KB >= 0.

AB (input/output)
On entry, the upper or lower triangle of the sym- metric band matrix A, stored in the first ka+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(ka+1+i-j,j) = A(i,j) for max(1,j- ka)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+ka).

On exit, the transformed matrix X**T*A*X, stored in the same format as A.

LDAB (input)
The leading dimension of the array AB. LDAB >= KA+1.

BB (input)
The banded factor S from the split Cholesky fac- torization of B, as returned by SPBSTF, stored in the first KB+1 rows of the array.

LDBB (input)
The leading dimension of the array BB. LDBB >= KB+1.

X (output)
If VECT = 'V', the n-by-n matrix X. If VECT = 'N', the array X is not referenced.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N) if VECT = 'V'; LDX >= 1 otherwise.

WORK (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille- gal value.

Contents

NAME

dsbgv - compute all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form A*x=(lambda)*B*x

SYNOPSIS

```
SUBROUTINE DSBGV(JOBZ, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, W, Z,
      LDZ, WORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER N, KA, KB, LDAB, LDBB, LDZ, INFO
DOUBLE PRECISION  AB(LDAB,*),  BB(LDBB,*),  W(*),  Z(LDZ,*),
WORK(*)

SUBROUTINE DSBGV_64(JOBZ, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, W, Z,
      LDZ, WORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 N, KA, KB, LDAB, LDBB, LDZ, INFO
DOUBLE PRECISION  AB(LDAB,*),  BB(LDBB,*),  W(*),  Z(LDZ,*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SBGV(JOBZ, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB], W,
       Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: N, KA, KB, LDAB, LDBB, LDZ, INFO
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: AB, BB, Z

SUBROUTINE SBGV_64(JOBZ, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB],
```

```
          W, Z, [LDZ], [WORK], [INFO])

   CHARACTER(LEN=1) :: JOBZ, UPLO
   INTEGER(8) :: N, KA, KB, LDAB, LDBB, LDZ, INFO
   REAL(8), DIMENSION(:) :: W, WORK
   REAL(8), DIMENSION(:,:) :: AB, BB, Z
```

C INTERFACE

```
   #include <sunperf.h>

   void dsbgv(char jobz, char uplo, int n, int ka, int kb, dou-
             ble  *ab,  int  ldab, double *bb, int ldbb, double
             *w, double *z, int ldz, int *info);
   void dsbgv_64(char jobz, char uplo, long n,  long  ka,  long
             kb,  double *ab, long ldab, double *bb, long ldbb,
             double *w, double *z, long ldz, long *info);
```

PURPOSE

dsbgv computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form A*x=(lambda)*B*x. Here A and B are assumed to be symmetric and banded, and B is also positive definite.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

UPLO (input)
= 'U': Upper triangles of A and B are stored;
= 'L': Lower triangles of A and B are stored.

N (input) The order of the matrices A and B. N >= 0.

KA (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KA >= 0.

KB (input)
The number of superdiagonals of the matrix B if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KB >= 0.

AB (input/output)
On entry, the upper or lower triangle of the symmetric band matrix A, stored in the first ka+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(ka+1+i-j,j) = A(i,j) for max(1,j-ka)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+ka).

On exit, the contents of AB are destroyed.

LDAB (input)
The leading dimension of the array AB. LDAB >= KA+1.

BB (input/output)
On entry, the upper or lower triangle of the symmetric band matrix B, stored in the first kb+1 rows of the array. The j-th column of B is stored in the j-th column of the array BB as follows: if UPLO = 'U', BB(kb+1+i-j,j) = B(i,j) for max(1,j-kb)<=i<=j; if UPLO = 'L', BB(1+i-j,j) = B(i,j) for j<=i<=min(n,j+kb).

On exit, the factor S from the split Cholesky factorization B = S**T*S, as returned by SPBSTF.

LDBB (input)
The leading dimension of the array BB. LDBB >= KB+1.

W (output)
If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, Z contains the matrix Z of eigenvectors, with the i-th column of Z holding the eigenvector associated with W(i). The eigenvectors are normalized so that Z**T*B*Z = I. If JOBZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= N.

WORK (workspace)
dimension(3*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, and i is:
<= N: the algorithm failed to converge: i off-diagonal elements of an intermediate tridiagonal form did not converge to zero; > N: if INFO = N + i, for 1 <= i <= N, then SPBSTF
returned INFO = i: B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

Contents

NAME

```
dsbgvd - compute all the eigenvalues,  and  optionally,  the
eigenvectors of a real generalized symmetric-definite banded
eigenproblem, of the form A*x=(lambda)*B*x
```

SYNOPSIS

```
SUBROUTINE DSBGVD(JOBZ, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, W, Z,
      LDZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER N, KA, KB, LDAB, LDBB, LDZ, LWORK, LIWORK, INFO
INTEGER IWORK(*)
DOUBLE PRECISION  AB(LDAB,*),  BB(LDBB,*),  W(*),  Z(LDZ,*),
WORK(*)

SUBROUTINE DSBGVD_64(JOBZ, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, W, Z,
      LDZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 N, KA, KB, LDAB, LDBB, LDZ, LWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION  AB(LDAB,*),  BB(LDBB,*),  W(*),  Z(LDZ,*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SBGVD(JOBZ, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB], W,
       Z, [LDZ], [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: N, KA, KB, LDAB, LDBB, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
```

```
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: AB, BB, Z

SUBROUTINE SBGVD_64(JOBZ, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB],
       W, Z, [LDZ], [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: N, KA, KB, LDAB,  LDBB,  LDZ,  LWORK,  LIWORK,
INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: AB, BB, Z
```

C INTERFACE

```
#include <sunperf.h>
void dsbgvd(char jobz, char uplo, int n,  int  ka,  int  kb,
          double *ab, int ldab, double *bb, int ldbb, double
          *w, double *z, int ldz, int *info);

void dsbgvd_64(char jobz, char uplo, long n, long  ka,  long
          kb,  double *ab, long ldab, double *bb, long ldbb,
          double *w, double *z, long ldz, long *info);
```

PURPOSE

dsbgvd computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form A*x=(lambda)*B*x. Here A and B are assumed to be symmetric and banded, and B is also positive definite. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

JOBZ (input)
- = 'N': Compute eigenvalues only;
- = 'V': Compute eigenvalues and eigenvectors.

UPLO (input)

= 'U': Upper triangles of A and B are stored;
= 'L': Lower triangles of A and B are stored.

N (input) The order of the matrices A and B. N >= 0.

KA (input)
The number of superdiagonals of the matrix A if
UPLO = 'U', or the number of subdiagonals if UPLO
= 'L'. KA >= 0.

KB (input)
The number of superdiagonals of the matrix B if
UPLO = 'U', or the number of subdiagonals if UPLO
= 'L'. KB >= 0.

AB (input/output)
On entry, the upper or lower triangle of the sym-
metric band matrix A, stored in the first ka+1
rows of the array. The j-th column of A is stored
in the j-th column of the array AB as follows: if
UPLO = 'U', AB(ka+1+i-j,j) = A(i,j) for max(1,j-
ka)<=i<=j; if UPLO = 'L', AB(1+i-j,j)   = A(i,j)
for j<=i<=min(n,j+ka).

On exit, the contents of AB are destroyed.

LDAB (input)
The leading dimension of the array AB.   LDAB >=
KA+1.

BB (input/output)
On entry, the upper or lower triangle of the sym-
metric band matrix B, stored in the first kb+1
rows of the array. The j-th column of B is stored
in the j-th column of the array BB as follows: if
UPLO = 'U', BB(ka+1+i-j,j) = B(i,j) for max(1,j-
kb)<=i<=j; if UPLO = 'L', BB(1+i-j,j)   = B(i,j)
for j<=i<=min(n,j+kb).

On exit, the factor S from the split Cholesky fac-
torization B = S**T*S, as returned by SPBSTF.

LDBB (input)
The leading dimension of the array BB.   LDBB >=
KB+1.

W (output)
If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, Z contains the
matrix Z of eigenvectors, with the i-th column of

```
           Z holding the eigenvector  associated  with  W(i).
           The  eigenvectors  are normalized so Z**T*B*Z = I.
           If JOBZ = 'N', then Z is not referenced.

 LDZ (input)
           The leading dimension of the array Z.  LDZ  >=  1,
           and if JOBZ = 'V', LDZ >= max(1,N).

 WORK (workspace)
           On exit, if INFO = 0, WORK(1) returns the  optimal
           LWORK.

 LWORK (input)
           The dimension of the  array  WORK.   If  N  <=  1,
           LWORK  >=  1.   If  JOBZ = 'N' and N > 1, LWORK >=
           3*N.  If JOBZ = 'V' and N > 1, LWORK >= 1 + 5*N  +
           2*N**2.

           If LWORK = -1, then a workspace query is  assumed;
           the  routine  only  calculates the optimal size of
           the WORK array, returns this value  as  the  first
           entry  of  the  WORK  array,  and no error message
           related to LWORK is issued by XERBLA.

 IWORK (workspace/output)
           On exit, if  LIWORK  >  0,  IWORK(1)  returns  the
           optimal LIWORK.

 LIWORK (input)
           The dimension of the array IWORK.  If JOBZ  =  'N'
           or N <= 1, LIWORK >= 1.  If JOBZ  = 'V' and N > 1,
           LIWORK >= 3 + 5*N.

           If LIWORK = -1, then a workspace query is assumed;
           the  routine  only  calculates the optimal size of
           the IWORK array, returns this value as  the  first
           entry  of  the  IWORK  array, and no error message
           related to LIWORK is issued by XERBLA.

 INFO (output)
           = 0:  successful exit
           < 0:  if INFO = -i, the i-th argument had an ille-
           gal value
           > 0:  if INFO = i, and i is:
           <= N:  the algorithm failed to converge:   i  off-
           diagonal  elements  of an intermediate tridiagonal
           form did not converge to zero; > N:   if INFO =  N
           + i, for 1 <= i <= N, then SPBSTF
           returned INFO = i: B  is  not  positive  definite.
           The  factorization of B could not be completed and
```

no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
Mark Fahey, Department of Mathematics, Univ. of Kentucky, USA

Contents

NAME

dsbgvx - compute selected eigenvalues, and optionally, eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form A*x=(lambda)*B*x

SYNOPSIS

```
SUBROUTINE DSBGVX(JOBZ, RANGE, UPLO, N, KA, KB, AB, LDAB, BB, LDBB,
      Q, LDQ, VL, VU, IL, IU, ABSTOL, M, W, Z, LDZ, WORK, IWORK, IFAIL,
      INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER N, KA, KB, LDAB, LDBB, LDQ, IL, IU, M, LDZ, INFO
INTEGER IWORK(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION  AB(LDAB,*),  BB(LDBB,*),  Q(LDQ,*),  W(*),
Z(LDZ,*), WORK(*)

SUBROUTINE DSBGVX_64(JOBZ, RANGE, UPLO, N, KA, KB, AB, LDAB, BB,
      LDBB, Q, LDQ, VL, VU, IL, IU, ABSTOL, M, W, Z, LDZ, WORK, IWORK,
      IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER*8 N, KA, KB, LDAB, LDBB, LDQ, IL, IU, M, LDZ, INFO
INTEGER*8 IWORK(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION  AB(LDAB,*),  BB(LDBB,*),  Q(LDQ,*),  W(*),
Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SBGVX(JOBZ, RANGE, UPLO, [N], KA, KB, AB, [LDAB], BB,
       [LDBB], Q, [LDQ], VL, VU, IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK],
```

```
       [IWORK], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER :: N, KA, KB, LDAB, LDBB, LDQ, IL, IU, M, LDZ, INFO
INTEGER, DIMENSION(:) :: IWORK, IFAIL
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: AB, BB, Q, Z

SUBROUTINE SBGVX_64(JOBZ, RANGE, UPLO, [N], KA, KB, AB, [LDAB], BB,
       [LDBB], Q, [LDQ], VL, VU, IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK],
       [IWORK], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER(8) :: N, KA, KB, LDAB, LDBB, LDQ, IL,  IU,  M,  LDZ,
INFO
INTEGER(8), DIMENSION(:) :: IWORK, IFAIL
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: AB, BB, Q, Z
```

C INTERFACE

```
#include <sunperf.h>

void dsbgvx(char jobz, char range, char uplo, int n, int ka,
          int  kb,  double  *ab,  int  ldab, double *bb, int
          ldbb, double *q, int ldq, double  vl,  double  vu,
          int  il, int iu, double abstol, int *m, double *w,
          double *z, int ldz, int *ifail, int *info);

void dsbgvx_64(char jobz, char range,  char  uplo,  long  n,
          long  ka,  long  kb, double *ab, long ldab, double
          *bb, long ldbb, double *q, long  ldq,  double  vl,
          double  vu,  long il, long iu, double abstol, long
          *m, double *w, double *z, long ldz,  long  *ifail,
          long *info);
```

PURPOSE

dsbgvx computes selected eigenvalues, and optionally, eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form A*x=(lambda)*B*x. Here A and B are assumed to be symmetric and banded, and B is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either all eigenvalues, a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

RANGE (input)
= 'A': all eigenvalues will be found.
= 'V': all eigenvalues in the half-open interval (VL,VU] will be found. = 'I': the IL-th through IU-th eigenvalues will be found.

UPLO (input)
= 'U': Upper triangles of A and B are stored;
= 'L': Lower triangles of A and B are stored.

N (input) The order of the matrices A and B. N >= 0.

KA (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KA >= 0.

KB (input)
The number of superdiagonals of the matrix B if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KB >= 0.

AB (input/output)
On entry, the upper or lower triangle of the symmetric band matrix A, stored in the first ka+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(ka+1+i-j,j) = A(i,j) for max(1,j-ka)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+ka).

On exit, the contents of AB are destroyed.

LDAB (input)
The leading dimension of the array AB. LDAB >= KA+1.

BB (input/output)
On entry, the upper or lower triangle of the symmetric band matrix B, stored in the first kb+1 rows of the array. The j-th column of B is stored in the j-th column of the array BB as follows: if UPLO = 'U', BB(ka+1+i-j,j) = B(i,j) for max(1,j-kb)<=i<=j; if UPLO = 'L', BB(1+i-j,j) = B(i,j) for j<=i<=min(n,j+kb).

On exit, the factor S from the split Cholesky factorization B = S**T*S, as returned by SPBSTF.

LDBB (input)
The leading dimension of the array BB. LDBB >= KB+1.

Q (output)
If JOBZ = 'V', the n-by-n matrix used in the reduction of A*x = (lambda)*B*x to standard form, i.e. C*x = (lambda)*x, and consequently C to tridiagonal form. If JOBZ = 'N', the array Q is not referenced.

LDQ (input)
The leading dimension of the array Q. If JOBZ = 'N', LDQ >= 1. If JOBZ = 'V', LDQ >= max(1,N).

VL (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)
See the description of VL.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
See the description of IL.

ABSTOL (input)
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABSTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABSTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing A to tridiagonal form.

Eigenvalues will be computed most accurately when ABSTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABSTOL to 2*SLAMCH('S').

M (output)
The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)
If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, Z contains the matrix Z of eigenvectors, with the i-th column of Z holding the eigenvector associated with W(i). The eigenvectors are normalized so Z**T*B*Z = I. If JOBZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
dimension(7*N)

IWORK (workspace/output)
dimension(5*N)

IFAIL (input)
If JOBZ = 'V', then if INFO = 0, the first M elements of IFAIL are zero. If INFO > 0, then IFAIL contains the indices of the eigenvalues that failed to converge. If JOBZ = 'N', then IFAIL is not referenced.

INFO (output)
= 0 : successful exit
< 0 : if INFO = -i, the i-th argument had an illegal value
<= N: if INFO = i, then i eigenvectors failed to converge. Their indices are stored in IFAIL. > N : SPBSTF returned an error code; i.e., if INFO = N + i, for 1 <= i <= N, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

```
Based on contributions by
   Mark Fahey, Department of Mathematics, Univ. of Kentucky,
USA
```

Contents

NAME

```
dsbmv - perform the matrix-vector operation   y := alpha*A*x
+ beta*y
```

SYNOPSIS

```
SUBROUTINE DSBMV(UPLO, N, K, ALPHA, A, LDA, X, INCX, BETA, Y,
      INCY)

CHARACTER * 1 UPLO
INTEGER N, K, LDA, INCX, INCY
DOUBLE PRECISION ALPHA, BETA
DOUBLE PRECISION A(LDA,*), X(*), Y(*)

SUBROUTINE DSBMV_64(UPLO, N, K, ALPHA, A, LDA, X, INCX, BETA, Y,
      INCY)

CHARACTER * 1 UPLO
INTEGER*8 N, K, LDA, INCX, INCY
DOUBLE PRECISION ALPHA, BETA
DOUBLE PRECISION A(LDA,*), X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE SBMV(UPLO, [N], K, ALPHA, A, [LDA], X, [INCX], BETA,
       Y, [INCY])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, K, LDA, INCX, INCY
REAL(8) :: ALPHA, BETA
REAL(8), DIMENSION(:) :: X, Y
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE SBMV_64(UPLO, [N], K, ALPHA, A, [LDA], X, [INCX],
```

```
          BETA, Y, [INCY])

   CHARACTER(LEN=1) :: UPLO
   INTEGER(8) :: N, K, LDA, INCX, INCY
   REAL(8) :: ALPHA, BETA
   REAL(8), DIMENSION(:) :: X, Y
   REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dsbmv(char uplo, int n, int k, double alpha, double *a,
          int  lda, double *x, int incx, double beta, double
          *y, int incy);
void dsbmv_64(char uplo, long n, long k, double alpha,  dou-
          ble  *a,  long  lda,  double *x, long incx, double
          beta, double *y, long incy);
```

PURPOSE

dsbmv performs the matrix-vector operation y := alpha*A*x + beta*y, where alpha and beta are scalars, x and y are n element vectors and A is an n by n symmetric band matrix, with k super-diagonals.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the band matrix A is being supplied as follows:

UPLO = 'U' or 'u'   The upper triangular part of A is being supplied.

UPLO = 'L' or 'l'   The lower triangular part of A is being supplied.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

K (input)
On entry, K specifies the number of super-diagonals of the matrix A. K >= 0. Unchanged on

exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
Before entry with UPLO = 'U' or 'u', the leading (k + 1) by n part of the array A must contain the upper triangular band part of the symmetric matrix, supplied column by column, with the leading diagonal of the matrix in row (k + 1) of the array, the first super-diagonal starting at position 2 in row k, and so on. The top left k by k triangle of the array A is not referenced. The following program segment will transfer the upper triangular part of a symmetric band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = K + 1 - J
     DO 10, I = MAX( 1, J - K ), J
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Before entry with UPLO = 'L' or 'l', the leading (k + 1) by n part of the array A must contain the lower triangular band part of the symmetric matrix, supplied column by column, with the leading diagonal of the matrix in row 1 of the array, the first sub-diagonal starting at position 1 in row 2, and so on. The bottom right k by k triangle of the array A is not referenced. The following program segment will transfer the lower triangular part of a symmetric band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = 1 - J
     DO 10, I = J, MIN( N, J + K )
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= (

k + 1). Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
dsbtrd - reduce a real symmetric band matrix A to  symmetric
tridiagonal  form  T by an orthogonal similarity transforma-
tion
```

SYNOPSIS

```
SUBROUTINE DSBTRD(VECT, UPLO, N, KD, AB, LDAB, D, E, Q, LDQ, WORK,
      INFO)

CHARACTER * 1 VECT, UPLO
INTEGER N, KD, LDAB, LDQ, INFO
DOUBLE PRECISION AB(LDAB,*), D(*), E(*), Q(LDQ,*), WORK(*)

SUBROUTINE DSBTRD_64(VECT, UPLO, N, KD, AB, LDAB, D, E, Q, LDQ, WORK,
      INFO)

CHARACTER * 1 VECT, UPLO
INTEGER*8 N, KD, LDAB, LDQ, INFO
DOUBLE PRECISION AB(LDAB,*), D(*), E(*), Q(LDQ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SBTRD(VECT, UPLO, [N], KD, AB, [LDAB], D, E, Q, [LDQ],
       [WORK], [INFO])

CHARACTER(LEN=1) :: VECT, UPLO
INTEGER :: N, KD, LDAB, LDQ, INFO
REAL(8), DIMENSION(:) :: D, E, WORK
REAL(8), DIMENSION(:,:) :: AB, Q

SUBROUTINE SBTRD_64(VECT, UPLO, [N], KD, AB, [LDAB], D, E, Q, [LDQ],
       [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: VECT, UPLO
INTEGER(8) :: N, KD, LDAB, LDQ, INFO
REAL(8), DIMENSION(:) :: D, E, WORK
REAL(8), DIMENSION(:,:) :: AB, Q
```

C INTERFACE

```
#include <sunperf.h>

void dsbtrd(char vect, char uplo, int n, int kd, double *ab,
          int  ldab,  double  *d,  double *e, double *q, int
          ldq, int *info);

void dsbtrd_64(char vect, char uplo, long n, long kd, double
          *ab,  long  ldab, double *d, double *e, double *q,
          long ldq, long *info);
```

PURPOSE

dsbtrd reduces a real symmetric band matrix A to symmetric tridiagonal form T by an orthogonal similarity transformation: Q**T * A * Q = T.

ARGUMENTS

VECT (input)
= 'N': do not form Q;
= 'V': form Q;
= 'U': update a matrix X, by forming X*Q.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

AB (input/output)
On entry, the upper or lower triangle of the symmetric band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(kd+1+i-j,j) = A(i,j) for max(1,j-

kd)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd). On exit, the diagonal elements of AB are overwritten by the diagonal elements of the tridiagonal matrix T; if KD > 0, the elements on the first superdiagonal (if UPLO = 'U') or the first subdiagonal (if UPLO = 'L') are overwritten by the off-diagonal elements of T; the rest of AB is overwritten by values generated during the reduction.

LDAB (input)
The leading dimension of the array AB. LDAB >= KD+1.

D (output)
The diagonal elements of the tridiagonal matrix T.

E (output)
The off-diagonal elements of the tridiagonal matrix T: E(i) = T(i,i+1) if UPLO = 'U'; E(i) = T(i+1,i) if UPLO = 'L'.

Q (input/output)
On entry, if VECT = 'U', then Q must contain an N-by-N matrix X; if VECT = 'N' or 'V', then Q need not be set.

On exit: if VECT = 'V', Q contains the N-by-N orthogonal matrix Q; if VECT = 'U', Q contains the product X*Q; if VECT = 'N', the array Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= 1, and LDQ >= N if VECT = 'V' or 'U'.

WORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

Modified by Linda Kaufman, Bell Labs.

Contents

NAME

```
dscal - Compute y := alpha * y
```

SYNOPSIS

```
SUBROUTINE DSCAL(N, ALPHA, Y, INCY)

INTEGER N, INCY
DOUBLE PRECISION ALPHA
DOUBLE PRECISION Y(*)

SUBROUTINE DSCAL_64(N, ALPHA, Y, INCY)

INTEGER*8 N, INCY
DOUBLE PRECISION ALPHA
DOUBLE PRECISION Y(*)
```

F95 INTERFACE

```
SUBROUTINE SCAL([N], ALPHA, Y, [INCY])

INTEGER :: N, INCY
REAL(8) :: ALPHA
REAL(8), DIMENSION(:) :: Y

SUBROUTINE SCAL_64([N], ALPHA, Y, [INCY])

INTEGER(8) :: N, INCY
REAL(8) :: ALPHA
REAL(8), DIMENSION(:) :: Y
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dscal(int n, double alpha, double *y, int incy);

void dscal_64(long n, double alpha, double *y, long incy);
```

PURPOSE

```
dscal Compute y := alpha * y where alpha is a scalar  and  y
is an n-vector.
```

ARGUMENTS

```
N (input)
          On entry, N specifies the number  of  elements  in
          the  vector.   N  must  be  at  least  one for the
          subroutine to have any visible effect.   Unchanged
          on exit.

ALPHA (input)
          On  entry,  ALPHA  specifies  the  scalar   alpha.
          Unchanged on exit.

Y (input/output)
          ( 1 + ( n - 1 )*abs( INCY  )  ).   On  entry,  the
          incremented  array Y must contain the vector y. On
          exit, Y is overwritten by the updated vector y.

INCY (input)
          On entry, INCY specifies  the  increment  for  the
          elements  of  Y. INCY must not be zero.  Unchanged
          on exit.
```

Contents

NAME

```
dsctr - Scatters elements from x into y.
```

SYNOPSIS

```
 SUBROUTINE DSCTR(NZ, X, INDX, Y)

 DOUBLE PRECISION X(*), Y(*)
 INTEGER NZ
 INTEGER INDX(*)

 SUBROUTINE DSCTR_64(NZ, X, INDX, Y)

 DOUBLE PRECISION X(*), Y(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 SUBROUTINE SCTR([NZ], X, INDX, Y)

 REAL(8), DIMENSION(:) :: X, Y
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 SUBROUTINE SCTR_64([NZ], X, INDX, Y)

 REAL(8), DIMENSION(:) :: X, Y
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
DSCTR - Scatters the components of a sparse vector x stored
in compressed form into specified components of a vector y
```

in full storage form.

```
 do i = 1, n
   y(indx(i)) = x(i)
 enddo
```

ARGUMENTS

NZ (input) - INTEGER
Number of elements in the compressed form.
Unchanged on exit.

X (input)
Vector containing the values to be scattered from compressed form into full storage form. Unchanged on exit.

INDX (input) - INTEGER
Vector containing the indices of the compressed form. It is assumed that the elements in INDX are distinct and greater than zero. Unchanged on exit.

Y (output)
Vector whose elements specified by indx have been set to the corresponding entries of x. Only the elements corresponding to the indices in indx have been modified.

Contents

NAME

```
dsdot - compute the double precision dot product of two sin-
gle precision vectors x and y.
```

SYNOPSIS

```
DOUBLE PRECISION FUNCTION DSDOT(N, X, INCX, Y, INCY)

INTEGER N, INCX, INCY
REAL X(*), Y(*)

DOUBLE PRECISION FUNCTION DSDOT_64(N, X, INCX, Y, INCY)

INTEGER*8 N, INCX, INCY
REAL X(*), Y(*)
```

F95 INTERFACE

```
REAL(8) FUNCTION DSDOT(N, X, INCX, Y, INCY)

INTEGER :: N, INCX, INCY
REAL, DIMENSION(:) :: X, Y

REAL(8) FUNCTION DSDOT_64(N, X, INCX, Y, INCY)

INTEGER(8) :: N, INCX, INCY
REAL, DIMENSION(:) :: X, Y
```

C INTERFACE

```
#include <sunperf.h>

double dsdot(int n, float *x, int incx, float *y, int incy);

double dsdot_64(long n, float *x, long incx, float *y,  long
```

```
          incy);
```

PURPOSE

```
dsdot compute the double precision dot product of  x  and  y
where x and y are single precision n-vectors.
```

ARGUMENTS

```
N (input)
          On entry, N specifies the number  of  elements  in
          the  vector.   If N is not positive then the func-
          tion returns the value 0.0.  Unchanged on exit.
X (input)
          ( 1 + ( n - 1 )*abs( INCX  )  ).   On  entry,  the
          incremented  array  X  must  contain the vector x.
          Unchanged on exit.

INCX (input)
          On entry, INCX specifies  the  increment  for  the
          elements  of  X. INCX must not be zero.  Unchanged
          on exit.

Y (input)
          ( 1 + ( n - 1 )*abs( INCY  )  ).   On  entry,  the
          incremented  array  Y  must  contain the vector y.
          Unchanged on exit.

INCY (input)
          On entry, INCY specifies  the  increment  for  the
          elements  of  Y. INCY must not be zero.  Unchanged
          on exit.
```

Contents

NAME

```
dsecnd - return the user time for a process in seconds
```

SYNOPSIS

```
DOUBLE PRECISION FUNCTION DSECND()

DOUBLE PRECISION FUNCTION DSECND_64()
```

F95 INTERFACE

```
REAL(8) FUNCTION DSECND()

REAL(8) FUNCTION DSECND_64()
```

C INTERFACE

```
#include <sunperf.h>

double dsecnd();

double dsecnd_64();
```

PURPOSE

```
dsecnd returns the user time for a process in seconds.  This
version gets the time from the system function ETIME.
```

Contents

NAME

```
dsinqb - synthesize a Fourier sequence from its  representa-
tion  in  terms of a sine series with odd wave numbers.  The
SINQ operations are unnormalized inverses of themselves,  so
a  call  to  SINQF followed by a call to SINQB will multiply
the input sequence by 4 * N.
```

SYNOPSIS

```
SUBROUTINE DSINQB(N, X, WSAVE)

INTEGER N
DOUBLE PRECISION X(*), WSAVE(*)

SUBROUTINE DSINQB_64(N, X, WSAVE)

INTEGER*8 N
DOUBLE PRECISION X(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE SINQB([N], X, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: X, WSAVE

SUBROUTINE SINQB_64([N], X, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: X, WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void dsinqb(int n, double *x, double *wsave);
```

```
void dsinqb_64(long n, double *x, double *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence to be  transformed.   These
          subroutines are most efficient when N is a product
          of small primes.  N >= 0.

X (input/output)
          On entry, an array  of  length  N  containing  the
          sequence   to   be   transformed.   On  exit,  the
          quarter-wave sine synthesis of the input.
WSAVE (input)
          On entry, an array with dimension of at least (3 *
          N  +  15)  for  scalar subroutines, initialized by
          SINQI.
```

Contents

NAME

```
dsinqf - compute the Fourier coefficients in a  sine  series
representation  with only odd wave numbers.  The SINQ opera-
tions are unnormalized inverses of themselves, so a call  to
SINQF  followed  by  a call to SINQB will multiply the input
sequence by 4 * N.
```

SYNOPSIS

```
SUBROUTINE DSINQF(N, X, WSAVE)

INTEGER N
DOUBLE PRECISION X(*), WSAVE(*)

SUBROUTINE DSINQF_64(N, X, WSAVE)

INTEGER*8 N
DOUBLE PRECISION X(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE SINQF([N], X, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: X, WSAVE

SUBROUTINE SINQF_64([N], X, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: X, WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void dsinqf(int n, double *x, double *wsave);
```

```
void dsinqf_64(long n, double *x, double *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence to be  transformed.   These
          subroutines are most efficient when N is a product
          of small primes.  N >= 0.

X (input/output)
          On entry, an array  of  length  N  containing  the
          sequence   to   be   transformed.   On  exit,  the
          quarter-wave sine transform of the input.
WSAVE (input)
          On entry, an array with dimension of at least   (3
          *  N  + 15) for scalar subroutines, initialized by
          SINQI.
```

Contents

NAME

```
dsinqi - initialize the array xWSAVE, which is used in  both
SINQF and SINQB.
```

SYNOPSIS

```
SUBROUTINE DSINQI(N, WSAVE)

INTEGER N
DOUBLE PRECISION WSAVE(*)

SUBROUTINE DSINQI_64(N, WSAVE)

INTEGER*8 N
DOUBLE PRECISION WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE SINQI(N, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: WSAVE

SUBROUTINE SINQI_64(N, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void dsinqi(int n, double *wsave);

void dsinqi_64(long n, double *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. The method is most efficient when N is a product of small primes.

WSAVE (input)
On entry, an array of dimension (3 * N + 15) or greater. SINQI needs to be called only once to initialize WSAVE before calling SINQF and/or SINQB if N and WSAVE remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
dsint - compute the discrete Fourier sine  transform  of  an
odd sequence.  The SINT transforms are unnormalized inverses
of themselves, so a call of SINT followed by another call of
SINT will multiply the input sequence by 2 * (N+1).
```

SYNOPSIS

```
SUBROUTINE DSINT(N, X, WSAVE)

INTEGER N
DOUBLE PRECISION X(*), WSAVE(*)

SUBROUTINE DSINT_64(N, X, WSAVE)

INTEGER*8 N
DOUBLE PRECISION X(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE SINT([N], X, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: X, WSAVE

SUBROUTINE SINT_64([N], X, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: X, WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void dsint(int n, double *x, double *wsave);
```

```
void dsint_64(long n, double *x, double *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence to be  transformed.   These
          subroutines  are most efficient when N+1 is a pro-
          duct of small primes.  N >= 0.

X (input/output)
          On entry, an array  of  length  N  containing  the
          sequence  to  be  transformed.   On exit, the sine
          transform of the input.
WSAVE (input/output)
          On entry, an array  with  dimension  of  at  least
          int(2.5 * N + 15) initialized by SINTI.
```

Contents

NAME

```
dsinti - initialize the array WSAVE, which is used  in  sub-
routine SINT.
```

SYNOPSIS

```
SUBROUTINE DSINTI(N, WSAVE)

INTEGER N
DOUBLE PRECISION WSAVE(*)

SUBROUTINE DSINTI_64(N, WSAVE)

INTEGER*8 N
DOUBLE PRECISION WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE SINTI(N, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: WSAVE

SUBROUTINE SINTI_64(N, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void dsinti(int n, double *wsave);

void dsinti_64(long n, double *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. N >= 0.

WSAVE (input/output)
On entry, an array of dimension (2N + N/2 + 15) or greater. SINTI is called once to initialize WSAVE before calling SINT and need not be called again between calls to SINT if N and WSAVE remain unchanged. Thus, subsequent transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
dskymm - Skyline format matrix-matrix multiply
```

SYNOPSIS

```
 SUBROUTINE DSKYMM( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, PNTR,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, K, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER    PNTR(*),
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE DSKYMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, PNTR,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, K, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER*8  PNTR(*),
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTR(K+1)-PNTR(1) (upper triangular)
       NNZ = PNTR(M+1)-PNTR(1) (lower triangular)
       PNTR() size = (K+1) (upper triangular)
       PNTR() size = (M+1) (lower triangular)
```

F95 INTERFACE

```
 SUBROUTINE SKYMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL,
*    PNTR,  B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, K
 INTEGER, DIMENSION(:) ::  DESCRA,  PNTR
```

```
      DOUBLE PRECISION    ALPHA, BETA
      DOUBLE PRECISION, DIMENSION(:) ::  VAL
      DOUBLE PRECISION, DIMENSION(:, :) ::  B, C

      SUBROUTINE SKYMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL,
     *    PNTR,  B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
      INTEGER*8    TRANSA, M, K
      INTEGER*8, DIMENSION(:) ::  DESCRA,  PNTR
      DOUBLE PRECISION    ALPHA, BETA
      DOUBLE PRECISION, DIMENSION(:) ::  VAL
      DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
         C <- alpha op(A) B + beta C
where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in skyline format and
op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general (NOT SUPPORTED)
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
```

```
                 DESCRA(2) upper/lower triangular indicator
                   1 : lower
                   2 : upper
                 DESCRA(3) main diagonal type
                   0 : non-unit
                   1 : unit
                 DESCRA(4) Array base  (NOT IMPLEMENTED)
                   0 : C/C++ compatible
                   1 : Fortran compatible
                 DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                   0 : unknown
                   1 : no repeated indices

   VAL()         array contain the nonzeros of A in skyline profile form.
                 Row-oriented if DESCRA(2) = 1 (lower triangular),
                 column oriented if DESCRA(2) = 2 (upper triangular).
   PNTR()        integer array of length M+1 (lower triangular) or
                 K+1 (upper triangular) such that PNTR(I)-PNTR(1)+1
                 points to the location in VAL of the first element of
                 the skyline profile in row (column) I.

   B()           rectangular array with first dimension LDB.

   LDB           leading dimension of B

   BETA          Scalar parameter

   C()           rectangular array with first dimension LDC.

   LDC           leading dimension of C

   WORK()        scratch array of length LWORK. WORK is not
                 referenced in the current version.

   LWORK         length of WORK array. LWORK is not referenced
                 in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

The SKY data structure is not supported for a general matrix structure (*DESCRA*(1)=0).

Also not supported:

1. lower triangular matrix A of size m by n where m > n
2. upper triangular matrix A of size m by n where m < n

Contents

NAME

```
dskysm - Skyline format triangular solve
```

SYNOPSIS

```
 SUBROUTINE DSKYSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, PNTR,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER    PNTR(*),
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE DSKYSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, PNTR,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER*8  PNTR(*),
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTR(M+1)-PNTR(1) (upper triangular)
       NNZ = PNTR(K+1)-PNTR(1) (lower triangular)
       PNTR() size = (M+1) (upper triangular)
       PNTR() size = (K+1) (lower triangular)
```

F95 INTERFACE

```
 SUBROUTINE SKYSM( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
*    PNTR,  B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, UNITD
 INTEGER, DIMENSION(:) ::  DESCRA,  PNTR
```

```
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) ::  VAL, DV
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C

 SUBROUTINE SKYSM_64( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA,
*    VAL, PNTR,  B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, UNITD
 INTEGER*8, DIMENSION(:) ::  DESCRA,  PNTR
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) ::  VAL, DV
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C
where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a diagonal scaling matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in skyline format and
op( A )  is one  of

 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' )).
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row or column scaling (see section
                    NOTES for further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter
```

```
DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))

              Note: For the routine, DESCRA(1)=3 is only supported.
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
                0 : non-unit
                1 : unit
              DESCRA(4) Array base  (NOT IMPLEMENTED)
                0 : C/C++ compatible
                1 : Fortran compatible
              DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                0 : unknown
                1 : no repeated indices

VAL()         array contain the nonzeros of A in skyline profile form.
              Row-oriented if DESCRA(2) = 1 (lower triangular),
              column oriented if DESCRA(2) = 2 (upper triangular).

PNTR()        integer array of length M+1 (lower triangular) or
              K+1 (upper triangular) such that PNTR(I)-PNTR(1)+1
              points to the location in VAL of the first element of
              the skyline profile in row (column) I.

B()           rectangular array with first dimension LDB.

LDB           leading dimension of B

BETA          Scalar parameter

C()           rectangular array with first dimension LDC.

LDC           leading dimension of C

WORK()        scratch array of length LWORK.
              On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.

LWORK         length of WORK array.  LWORK should be at least M.

              For good performance, LWORK should generally be larger.
```

For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. Also not supported:
   a. lower triangular matrix A of size m by n where m > n
   b. upper triangular matrix A of size m by n where m < n

2. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

3. If UNITD =4, the routine scales the rows of A if *DESCRA*(2)=1 and the columns of A if *DESCRA*(2)=2 such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in this particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows (columns) have been scaled. UNITD=2 if *DESCRA*(2)=1 and UNITD=3 if *DESCRA*(2)=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row (column) number which 2-norm is exactly zero.

4. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements

might or might not be referenced in the SKY representation of a sparse matrix. They are not used anyway in these cases. But if UNITD=4, the unit diagonal elements MUST be referenced in the SKY representation.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

dspcon - estimate the reciprocal of the condition number (in the 1-norm) of a real symmetric packed matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by DSPTRF

SYNOPSIS

```
SUBROUTINE DSPCON(UPLO, N, AP, IPIVOT, ANORM, RCOND, WORK, IWORK2,
      INFO)

CHARACTER * 1 UPLO
INTEGER N, INFO
INTEGER IPIVOT(*), IWORK2(*)
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION AP(*), WORK(*)

SUBROUTINE DSPCON_64(UPLO, N, AP, IPIVOT, ANORM, RCOND, WORK, IWORK2,
      INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*), IWORK2(*)
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION AP(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPCON(UPLO, N, AP, IPIVOT, ANORM, RCOND, [WORK], [IWORK2],
       [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT, IWORK2
```

```
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: AP, WORK

SUBROUTINE SPCON_64(UPLO, N, AP, IPIVOT, ANORM, RCOND, [WORK],
       [IWORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, IWORK2
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: AP, WORK
```

C INTERFACE

```
#include <sunperf.h>
void dspcon(char uplo, int n, double *ap, int *ipivot,  dou-
          ble anorm, double *rcond, int *info);

void dspcon_64(char uplo, long n, double *ap, long  *ipivot,
          double anorm, double *rcond, long *info);
```

PURPOSE

dspcon estimates the reciprocal of the condition number (in the 1-norm) of a real symmetric packed matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by DSPTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

AP (input)
Double precision array, dimension (N*(N+1)/2) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by DSPTRF, stored as a packed triangular matrix.

IPIVOT (input)
Integer array, dimension (N) Details of the interchanges and the block structure of D as determined by DSPTRF.

ANORM (input)
The 1-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
Double precision array, dimension(2*N)

IWORK2 (workspace)
Integer array, dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dspev - compute all the eigenvalues and, optionally,  eigen-
vectors of a real symmetric matrix A in packed storage
```

SYNOPSIS

```
SUBROUTINE DSPEV(JOBZ, UPLO, N, AP, W, Z, LDZ, WORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER N, LDZ, INFO
DOUBLE PRECISION AP(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSPEV_64(JOBZ, UPLO, N, AP, W, Z, LDZ, WORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 N, LDZ, INFO
DOUBLE PRECISION AP(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPEV(JOBZ, UPLO, N, AP, W, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: N, LDZ, INFO
REAL(8), DIMENSION(:) :: AP, W, WORK
REAL(8), DIMENSION(:,:) :: Z

SUBROUTINE SPEV_64(JOBZ, UPLO, N, AP, W, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: N, LDZ, INFO
REAL(8), DIMENSION(:) :: AP, W, WORK
REAL(8), DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void dspev(char jobz, char uplo, int n, double  *ap,  double
          *w, double *z, int ldz, int *info);

void dspev_64(char jobz, char uplo, long n, double *ap, dou-
          ble *w, double *z, long ldz, long *info);
```

PURPOSE

dspev computes all the eigenvalues and, optionally, eigenvectors of a real symmetric matrix A in packed storage.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

AP (input/output)
Double precision array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

On exit, AP is overwritten by values generated during the reduction to tridiagonal form. If UPLO = 'U', the diagonal and first superdiagonal of the tridiagonal matrix T overwrite the corresponding elements of A, and if UPLO = 'L', the diagonal and first subdiagonal of T overwrite the corresponding elements of A.

W (output)
Double precision array, dimension (N) If INFO = 0, the eigenvalues in ascending order.

Z (output)

Double precision array, dimension (LDZ, N) If JOBZ = 'V', then if INFO = 0, Z contains the orthonormal eigenvectors of the matrix A, with the i-th column of Z holding the eigenvector associated with W(i). If JOBZ = 'N', then Z is not referenced.

LDZ (input)

The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)

Double precision array, dimension(3*N)

INFO (output)

= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

Contents

NAME

dspevd - compute all the eigenvalues and, optionally, eigen-
vectors of a real symmetric matrix A in packed storage

SYNOPSIS

```
SUBROUTINE DSPEVD(JOBZ, UPLO, N, AP, W, Z, LDZ, WORK, LWORK, IWORK,
      LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER N, LDZ, LWORK, LIWORK, INFO
INTEGER IWORK(*)
DOUBLE PRECISION AP(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSPEVD_64(JOBZ, UPLO, N, AP, W, Z, LDZ, WORK, LWORK,
      IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 N, LDZ, LWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION AP(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPEVD(JOBZ, UPLO, [N], AP, W, Z, [LDZ], [WORK], [LWORK],
       [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: N, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: AP, W, WORK
REAL(8), DIMENSION(:,:) :: Z

SUBROUTINE SPEVD_64(JOBZ, UPLO, [N], AP, W, Z, [LDZ], [WORK], [LWORK],
```

```
       [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: N, LDZ, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: AP, W, WORK
REAL(8), DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void dspevd(char jobz, char uplo, int n, double *ap,  double
          *w, double *z, int ldz, int *info);
void dspevd_64(char jobz, char uplo,  long  n,  double  *ap,
          double *w, double *z, long ldz, long *info);
```

PURPOSE

dspevd computes all the eigenvalues and, optionally, eigenvectors of a real symmetric matrix A in packed storage. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

JOBZ (input)
  = 'N': Compute eigenvalues only;
  = 'V': Compute eigenvalues and eigenvectors.

UPLO (input)
  = 'U': Upper triangle of A is stored;
  = 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

AP (input/output)
  Double precision array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear

array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

On exit, AP is overwritten by values generated during the reduction to tridiagonal form. If UPLO = 'U', the diagonal and first superdiagonal of the tridiagonal matrix T overwrite the corresponding elements of A, and if UPLO = 'L', the diagonal and first subdiagonal of T overwrite the corresponding elements of A.

W (output)

Double precision array, dimension (N) If INFO = 0, the eigenvalues in ascending order.

Z (input) Double precision array, dimension (LDZ, N) If JOBZ = 'V', then if INFO = 0, Z contains the orthonormal eigenvectors of the matrix A, with the i-th column of Z holding the eigenvector associated with W(i). If JOBZ = 'N', then Z is not referenced.

LDZ (input)

The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)

Real array, dimension (LWORK) On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)

The dimension of the array WORK. If N <= 1, LWORK must be at least 1. If JOBZ = 'N' and N > 1, LWORK must be at least 2*N. If JOBZ = 'V' and N > 1, LWORK must be at least 1 + 6*N + N**2.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace/output)

Integer array, dimension (LIWORK) On exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)

The dimension of the array IWORK. If JOBZ = 'N' or N <= 1, LIWORK must be at least 1. If JOBZ =

'V' and N > 1, LIWORK must be at least 3 + 5*N.

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

Contents

NAME

dspevx - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix A in packed storage

SYNOPSIS

```
SUBROUTINE DSPEVX(JOBZ, RANGE, UPLO, N, AP, VL, VU, IL, IU, ABTOL,
      NFOUND, W, Z, LDZ, WORK, IWORK2, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER N, IL, IU, NFOUND, LDZ, INFO
INTEGER IWORK2(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABTOL
DOUBLE PRECISION AP(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSPEVX_64(JOBZ, RANGE, UPLO, N, AP, VL, VU, IL, IU, ABTOL,
      NFOUND, W, Z, LDZ, WORK, IWORK2, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER*8 N, IL, IU, NFOUND, LDZ, INFO
INTEGER*8 IWORK2(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABTOL
DOUBLE PRECISION AP(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPEVX(JOBZ, RANGE, UPLO, N, AP, VL, VU, IL, IU, ABTOL,
       NFOUND, W, Z, [LDZ], [WORK], [IWORK2], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER :: N, IL, IU, NFOUND, LDZ, INFO
INTEGER, DIMENSION(:) :: IWORK2, IFAIL
REAL(8) :: VL, VU, ABTOL
REAL(8), DIMENSION(:) :: AP, W, WORK
```

```
REAL(8), DIMENSION(:,:) :: Z

SUBROUTINE SPEVX_64(JOBZ, RANGE, UPLO, N, AP, VL, VU, IL, IU, ABTOL,
       NFOUND, W, Z, [LDZ], [WORK], [IWORK2], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER(8) :: N, IL, IU, NFOUND, LDZ, INFO
INTEGER(8), DIMENSION(:) :: IWORK2, IFAIL
REAL(8) :: VL, VU, ABTOL
REAL(8), DIMENSION(:) :: AP, W, WORK
REAL(8), DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>
void dspevx(char jobz, char range, char uplo, int n,  double
          *ap,  double vl, double vu, int il, int iu, double
          abtol, int *nfound, double *w, double *z, int ldz,
          int *ifail, int *info);

void dspevx_64(char jobz, char range,  char  uplo,  long  n,
          double  *ap,  double  vl, double vu, long il, long
          iu, double abtol, long *nfound, double *w,  double
          *z, long ldz, long *ifail, long *info);
```

PURPOSE

dspevx computes selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix A in packed storage. Eigenvalues/vectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

JOBZ (input)
- = 'N': Compute eigenvalues only;
- = 'V': Compute eigenvalues and eigenvectors.

RANGE (input)
- = 'A': all eigenvalues will be found;
- = 'V': all eigenvalues in the half-open interval (VL,VU] will be found; = 'I': the IL-th through IU-th eigenvalues will be found.

UPLO (input)
- = 'U': Upper triangle of A is stored;
- = 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

AP (input/output)
Double precision array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

On exit, AP is overwritten by values generated during the reduction to tridiagonal form. If UPLO = 'U', the diagonal and first superdiagonal of the tridiagonal matrix T overwrite the corresponding elements of A, and if UPLO = 'L', the diagonal and first subdiagonal of T overwrite the corresponding elements of A.

VL (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)
See the description of VL.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
See the description of IL.

ABTOL (input)
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABTOL + EPS *   max(|a|,|b|) ,

where EPS is the machine precision. If ABTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing AP to tridiagonal form.

Eigenvalues will be computed most accurately when ABTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABTOL to 2*SLAMCH('S').

See "Computing Small Singular Values of Bidiagonal Matrices with Guaranteed High Relative Accuracy," by Demmel and Kahan, LAPACK Working Note #3.

NFOUND (output)
The total number of eigenvalues found. 0 <= NFOUND <= N. If RANGE = 'A', NFOUND = N, and if RANGE = 'I', NFOUND = IU-IL+1.

W (output)
Double precision array, dimension (N) If INFO = 0, the selected eigenvalues in ascending order.

Z (output)
Double precision array, dimension (LDZ, max(1,M)) If JOBZ = 'V', then if INFO = 0, the first NFOUND columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). If an eigenvector fails to converge, then that column of Z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in IFAIL. If JOBZ = 'N', then Z is not referenced. Note: the user must ensure that at least max(1,NFOUND) columns are supplied in the array Z; if RANGE = 'V', the exact value of NFOUND is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
Double precision array, dimension(8*N)

IWORK2 (workspace)
Integer array, dimension(5*N)

IFAIL (output)
Integer array, dimension(N) If JOBZ = 'V', then if INFO = 0, the first NFOUND elements of IFAIL are zero. If INFO > 0, then IFAIL contains the

indices of the eigenvectors that failed to converge. If JOBZ = 'N', then IFAIL is not referenced.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, then i eigenvectors failed to converge. Their indices are stored in array IFAIL.

Contents

NAME

dspgst - reduce a real symmetric-definite generalized eigen-
problem to standard form, using packed storage

SYNOPSIS

```
SUBROUTINE DSPGST(ITYPE, UPLO, N, AP, BP, INFO)

CHARACTER * 1 UPLO
INTEGER ITYPE, N, INFO
DOUBLE PRECISION AP(*), BP(*)

SUBROUTINE DSPGST_64(ITYPE, UPLO, N, AP, BP, INFO)

CHARACTER * 1 UPLO
INTEGER*8 ITYPE, N, INFO
DOUBLE PRECISION AP(*), BP(*)
```

F95 INTERFACE

```
SUBROUTINE SPGST(ITYPE, UPLO, N, AP, BP, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: ITYPE, N, INFO
REAL(8), DIMENSION(:) :: AP, BP

SUBROUTINE SPGST_64(ITYPE, UPLO, N, AP, BP, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: ITYPE, N, INFO
REAL(8), DIMENSION(:) :: AP, BP
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dspgst(int itype, char uplo, int n, double *ap,  double
          *bp, int *info);

void dspgst_64(long itype, char uplo, long  n,  double  *ap,
          double *bp, long *info);
```

PURPOSE

dspgst reduces a real symmetric-definite generalized eigen-problem to standard form, using packed storage.

If ITYPE = 1, the problem is A*x = lambda*B*x,
and A is overwritten by inv(U**T)*A*inv(U) or inv(L)*A*inv(L**T)
If ITYPE = 2 or 3, the problem is A*B*x = lambda*x or
B*A*x = lambda*x, and A is overwritten by U*A*U**T or L**T*A*L.

B must have been previously factorized as U**T*U or L*L**T by SPPTRF.

ARGUMENTS

ITYPE (input)
= 1: compute inv(U**T)*A*inv(U) or inv(L)*A*inv(L**T);
= 2 or 3: compute U*A*U**T or L**T*A*L.

UPLO (input)
= 'U': Upper triangle of A is stored and B is factored as U**T*U; = 'L': Lower triangle of A is stored and B is factored as L*L**T.

N (input) The order of the matrices A and B. N >= 0.

AP (input/output)
Double precision array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n.

On exit, if INFO = 0, the transformed matrix, stored in the same format as A.

BP (input)
Double precision array, dimension (N*(N+1)/2) The triangular factor from the Cholesky factorization of B, stored in the same format as A, as returned by SPPTRF.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dspgv - compute all the  eigenvalues  and,  optionally,  the
eigenvectors of a real generalized symmetric-definite eigen-
problem, of the form A*x=(lambda)*B*x,  A*Bx=(lambda)*x,  or
B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE DSPGV(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, LDZ, WORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER ITYPE, N, LDZ, INFO
DOUBLE PRECISION AP(*), BP(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSPGV_64(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, LDZ, WORK,
      INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 ITYPE, N, LDZ, INFO
DOUBLE PRECISION AP(*), BP(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPGV(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, [LDZ], [WORK],
       [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: ITYPE, N, LDZ, INFO
REAL(8), DIMENSION(:) :: AP, BP, W, WORK
REAL(8), DIMENSION(:,:) :: Z

SUBROUTINE SPGV_64(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, [LDZ], [WORK],
       [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: ITYPE, N, LDZ, INFO
REAL(8), DIMENSION(:) :: AP, BP, W, WORK
REAL(8), DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void dspgv(int itype, char jobz, char uplo,  int  n,  double
          *ap,  double  *bp,  double *w, double *z, int ldz,
          int *info);

void dspgv_64(long itype, char jobz, char uplo, long n, dou-
          ble  *ap,  double  *bp, double *w, double *z, long
          ldz, long *info);
```

PURPOSE

dspgv computes all the eigenvalues and, optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be symmetric, stored in packed format, and B is also positive definite.

ARGUMENTS

ITYPE (input)
Specifies the problem type to be solved:
= 1: A*x = (lambda)*B*x
= 2: A*B*x = (lambda)*x
= 3: B*A*x = (lambda)*x

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

UPLO (input)
= 'U': Upper triangles of A and B are stored;
= 'L': Lower triangles of A and B are stored.

N (input) The order of the matrices A and B. N >= 0.

AP (input/output)
Double precision array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the

array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

On exit, the contents of A are destroyed.

BP (input/output)

Double precision array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix B, packed columnwise in a linear array. The j-th column of B is stored in the array BP as follows: if UPLO = 'U', BP(i + (j-1)*j/2) = B(i,j) for 1<=i<=j; if UPLO = 'L', BP(i + (j-1)*(2*n-j)/2) = B(i,j) for j<=i<=n.

On exit, the triangular factor U or L from the Cholesky factorization B = U**T*U or B = L*L**T, in the same storage format as B.

W (output)

Double precision array, dimension (N) If INFO = 0, the eigenvalues in ascending order.

Z (output)

Double precision array, dimension (LDZ, N) If JOBZ = 'V', then if INFO = 0, Z contains the matrix Z of eigenvectors. The eigenvectors are normalized as follows: if ITYPE = 1 or 2, Z**T*B*Z = I; if ITYPE = 3, Z**T*inv(B)*Z = I. If JOBZ = 'N', then Z is not referenced.

LDZ (input)

The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)

Double precision array, dimension(3*N)

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: SPPTRF or SSPEV returned an error code:

<= N: if INFO = i, SSPEV failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero. > N: if INFO = n + i, for 1 <= i <= n, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

Contents

NAME

dspgvd - compute all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

SYNOPSIS

```
SUBROUTINE DSPGVD(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, LDZ, WORK,
      LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER ITYPE, N, LDZ, LWORK, LIWORK, INFO
INTEGER IWORK(*)
DOUBLE PRECISION AP(*), BP(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSPGVD_64(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, LDZ, WORK,
      LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 ITYPE, N, LDZ, LWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION AP(*), BP(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPGVD(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, [LDZ], [WORK],
       [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: ITYPE, N, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: AP, BP, W, WORK
```

```
REAL(8), DIMENSION(:,:) :: Z

SUBROUTINE SPGVD_64(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, [LDZ],
       [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: ITYPE, N, LDZ, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: AP, BP, W, WORK
REAL(8), DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void dspgvd(int itype, char jobz, char uplo, int  n,  double
          *ap,  double  *bp,  double *w, double *z, int ldz,
          int *info);

void dspgvd_64(long itype, char jobz,  char  uplo,  long  n,
          double *ap, double *bp, double *w, double *z, long
          ldz, long *info);
```

PURPOSE

dspgvd computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be symmetric, stored in packed format, and B is also positive definite. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

ITYPE (input)

Specifies the problem type to be solved:

```
= 1:  A*x = (lambda)*B*x
= 2:  A*B*x = (lambda)*x
= 3:  B*A*x = (lambda)*x
```

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.

AP (input/output)
          Double precision array, dimension  (N*(N+1)/2)  On
          entry,  the  upper  or  lower triangle of the sym-
          metric matrix A, packed  columnwise  in  a  linear
          array.   The  j-th  column  of  A is stored in the
          array AP as follows:  if UPLO = 'U',  AP(i  +  (j-
          1)*j/2)  = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i
          + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

          On exit, the contents of AP are destroyed.

BP (input/output)
          Double precision array, dimension  (N*(N+1)/2)  On
          entry,  the  upper  or  lower triangle of the sym-
          metric matrix B, packed  columnwise  in  a  linear
          array.   The  j-th  column  of  B is stored in the
          array BP as follows:  if UPLO = 'U',  BP(i  +  (j-
          1)*j/2)  = B(i,j) for 1<=i<=j; if UPLO = 'L', BP(i
          + (j-1)*(2*n-j)/2) = B(i,j) for j<=i<=n.

          On exit, the triangular factor U  or  L  from  the
          Cholesky  factorization  B = U**T*U or B = L*L**T,
          in the same storage format as B.

W (output)
          Double precision array, dimension (N) If INFO = 0,
          the eigenvalues in ascending order.

Z (output)
          Double precision array, dimension (LDZ, N) If JOBZ
          =  'V',  then if INFO = 0, Z contains the matrix Z
          of eigenvectors.  The eigenvectors are  normalized
          as  follows:   if ITYPE = 1 or 2, Z**T*B*Z = I; if
          ITYPE = 3, Z**T*inv(B)*Z = I.  If JOBZ = 'N', then
          Z is not referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).
```

```
WORK (workspace/output)
          Double precision array, dimension (LWORK) On exit,
          if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
          The dimension of the  array  WORK.   If  N  <=  1,
          LWORK  >=  1.   If  JOBZ = 'N' and N > 1, LWORK >=
          2*N.  If JOBZ = 'V' and N > 1, LWORK >= 1 + 6*N  +
          2*N**2.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

IWORK (workspace/output)
          Integer array, dimension (LIWORK) On exit, if INFO
          = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
          The dimension of the array IWORK.  If JOBZ  =  'N'
          or N <= 1, LIWORK >= 1.  If JOBZ  = 'V' and N > 1,
          LIWORK >= 3 + 5*N.

          If LIWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the IWORK array, returns this value as  the  first
          entry  of  the  IWORK  array, and no error message
          related to LIWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  SPPTRF or SSPEVD returned an error code:
          <= N:  if INFO = i, SSPEVD failed to  converge;  i
          off-diagonal elements of an intermediate tridiago-
          nal form did not converge to zero; > N:   if  INFO
          =  N  + i, for 1 <= i <= N, then the leading minor
          of order i of B is  not  positive  definite.   The
          factorization  of  B could not be completed and no
          eigenvalues or eigenvectors were computed.
```

FURTHER DETAILS

```
Based on contributions by
   Mark Fahey, Department of Mathematics, Univ. of Kentucky,
USA
```

Contents

NAME

dspgvx - compute selected eigenvalues, and optionally, eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

SYNOPSIS

```
SUBROUTINE DSPGVX(ITYPE, JOBZ, RANGE, UPLO, N, AP, BP, VL, VU, IL,
      IU, ABSTOL, M, W, Z, LDZ, WORK, IWORK, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER ITYPE, N, IL, IU, M, LDZ, INFO
INTEGER IWORK(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION AP(*), BP(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSPGVX_64(ITYPE, JOBZ, RANGE, UPLO, N, AP, BP, VL, VU, IL,
      IU, ABSTOL, M, W, Z, LDZ, WORK, IWORK, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER*8 ITYPE, N, IL, IU, M, LDZ, INFO
INTEGER*8 IWORK(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION AP(*), BP(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPGVX(ITYPE, JOBZ, RANGE, UPLO, N, AP, BP, VL, VU, IL,
       IU, ABSTOL, M, W, Z, [LDZ], [WORK], [IWORK], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER :: ITYPE, N, IL, IU, M, LDZ, INFO
```

```
INTEGER, DIMENSION(:) :: IWORK, IFAIL
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: AP, BP, W, WORK
REAL(8), DIMENSION(:,:) :: Z

SUBROUTINE SPGVX_64(ITYPE, JOBZ, RANGE, UPLO, N, AP, BP, VL, VU,
       IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK], [IWORK], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER(8) :: ITYPE, N, IL, IU, M, LDZ, INFO
INTEGER(8), DIMENSION(:) :: IWORK, IFAIL
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: AP, BP, W, WORK
REAL(8), DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void dspgvx(int itype, char jobz, char range, char uplo, int
          n,  double  *ap, double *bp, double vl, double vu,
          int il, int iu, double abstol, int *m, double  *w,
          double *z, int ldz, int *ifail, int *info);

void dspgvx_64(long itype, char jobz, char range, char uplo,
          long  n, double *ap, double *bp, double vl, double
          vu, long il, long iu, double abstol, long *m, dou-
          ble  *w,  double  *z,  long ldz, long *ifail, long
          *info);
```

PURPOSE

dspgvx computes selected eigenvalues, and optionally, eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be symmetric, stored in packed storage, and B is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

ITYPE (input)
Specifies the problem type to be solved:
= 1: A*x = (lambda)*B*x
= 2: A*B*x = (lambda)*x
= 3: B*A*x = (lambda)*x

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

RANGE (input)
= 'A': all eigenvalues will be found.
= 'V': all eigenvalues in the half-open interval (VL,VU] will be found. = 'I': the IL-th through IU-th eigenvalues will be found.

UPLO (input)
= 'U': Upper triangle of A and B are stored;
= 'L': Lower triangle of A and B are stored.

N (input) The order of the matrix pencil (A,B). N >= 0.

AP (input/output)
Double precision array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

On exit, the contents of AP are destroyed.

BP (input/output)
Double precision array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix B, packed columnwise in a linear array. The j-th column of B is stored in the array BP as follows: if UPLO = 'U', BP(i + (j-1)*j/2) = B(i,j) for 1<=i<=j; if UPLO = 'L', BP(i + (j-1)*(2*n-j)/2) = B(i,j) for j<=i<=n.

On exit, the triangular factor U or L from the Cholesky factorization B = U**T*U or B = L*L**T, in the same storage format as B.

VL (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)
See the description of VL.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be

returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
See the description of IL.

ABSTOL (input)
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABSTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABSTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing A to tridiagonal form.

Eigenvalues will be computed most accurately when ABSTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABSTOL to 2*SLAMCH('S').

M (output)
The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)
Double precision array, dimension (N) On normal exit, the first M elements contain the selected eigenvalues in ascending order.

Z (output)
Double precision array, dimension (LDZ, max(1,M)) If JOBZ = 'N', then Z is not referenced. If JOBZ = 'V', then if INFO = 0, the first M columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). The eigenvectors are normalized as follows: if ITYPE = 1 or 2, Z**T*B*Z = I; if ITYPE = 3, Z**T*inv(B)*Z = I.

If an eigenvector fails to converge, then that column of Z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in IFAIL. Note: the user must ensure that at least max(1,M) columns are supplied in the array Z; if RANGE = 'V', the exact value of M is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
Double precision array, dimension(8*N)

IWORK (workspace)
Integer array, dimension(5*N)

IFAIL (output)
Integer array, dimension (N) If JOBZ = 'V', then if INFO = 0, the first M elements of IFAIL are zero. If INFO > 0, then IFAIL contains the indices of the eigenvectors that failed to converge. If JOBZ = 'N', then IFAIL is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: SPPTRF or SSPEVX returned an error code:
<= N: if INFO = i, SSPEVX failed to converge; i eigenvectors failed to converge. Their indices are stored in array IFAIL. > N: if INFO = N + i, for 1 <= i <= N, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
Mark Fahey, Department of Mathematics, Univ. of Kentucky, USA

Contents

NAME

```
dspmv - perform the matrix-vector operation   y := alpha*A*x
+ beta*y
```

SYNOPSIS

```
SUBROUTINE DSPMV(UPLO, N, ALPHA, A, X, INCX, BETA, Y, INCY)

CHARACTER * 1 UPLO
INTEGER N, INCX, INCY
DOUBLE PRECISION ALPHA, BETA
DOUBLE PRECISION A(*), X(*), Y(*)

SUBROUTINE DSPMV_64(UPLO, N, ALPHA, A, X, INCX, BETA, Y, INCY)

CHARACTER * 1 UPLO
INTEGER*8 N, INCX, INCY
DOUBLE PRECISION ALPHA, BETA
DOUBLE PRECISION A(*), X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE SPMV(UPLO, N, ALPHA, A, X, [INCX], BETA, Y, [INCY])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INCX, INCY
REAL(8) :: ALPHA, BETA
REAL(8), DIMENSION(:) :: A, X, Y

SUBROUTINE SPMV_64(UPLO, N, ALPHA, A, X, [INCX], BETA, Y, [INCY])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INCX, INCY
REAL(8) :: ALPHA, BETA
```

```
REAL(8), DIMENSION(:) :: A, X, Y
```

C INTERFACE

```
#include <sunperf.h>

void dspmv(char uplo, int n, double alpha, double *a, double
          *x, int incx, double beta, double *y, int incy);

void dspmv_64(char uplo, long n, double  alpha,  double  *a,
          double *x, long incx, double beta, double *y, long
          incy);
```

PURPOSE

dspmv performs the matrix-vector operation y := alpha*A*x + beta*y, where alpha and beta are scalars, x and y are n element vectors and A is an n by n symmetric matrix, supplied in packed form.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the matrix A is supplied in the packed array A as follows:

UPLO = 'U' or 'u'   The upper triangular part of A is supplied in A.

UPLO = 'L' or 'l'   The lower triangular part of A is supplied in A.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
((n*(n + 1))/2). Before entry with UPLO = 'U' or 'u', the array A must contain the upper triangular part of the symmetric matrix packed sequentially, column by column, so that A(1)

contains a(1, 1), A(2) and A(3) contain a(1, 2) and a(2, 2) respectively, and so on. Before entry with UPLO = 'L' or 'l', the array A must contain the lower triangular part of the symmetric matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(2, 1) and a(3, 1) respectively, and so on. Unchanged on exit.

X (input)

(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)

On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

BETA (input)

On entry, BETA specifies the scalar beta. When BETA is supplied as zero then Y need not be set on input. Unchanged on exit.

Y (input/output)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
dspr -  perform  the  symmetric  rank  1  operation    A  :=
alpha*x*x' + A
```

SYNOPSIS

```
SUBROUTINE DSPR(UPLO, N, ALPHA, X, INCX, A)

CHARACTER * 1 UPLO
INTEGER N, INCX
DOUBLE PRECISION ALPHA
DOUBLE PRECISION X(*), A(*)

SUBROUTINE DSPR_64(UPLO, N, ALPHA, X, INCX, A)

CHARACTER * 1 UPLO
INTEGER*8 N, INCX
DOUBLE PRECISION ALPHA
DOUBLE PRECISION X(*), A(*)
```

F95 INTERFACE

```
SUBROUTINE SPR(UPLO, N, ALPHA, X, [INCX], A)

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INCX
REAL(8) :: ALPHA
REAL(8), DIMENSION(:) :: X, A

SUBROUTINE SPR_64(UPLO, N, ALPHA, X, [INCX], A)

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INCX
REAL(8) :: ALPHA
```

```
REAL(8), DIMENSION(:) :: X, A
```

C INTERFACE

```
#include <sunperf.h>

void dspr(char uplo, int n, double  alpha,  double  *x,  int
          incx, double *a);

void dspr_64(char uplo, long n,  double  alpha,  double  *x,
          long incx, double *a);
```

PURPOSE

dspr performs the symmetric rank 1 operation A := alpha*x*x' + A, where alpha is a real scalar, x is an n element vector and A is an n by n symmetric matrix, supplied in packed form.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the matrix A is supplied in the packed array A as follows:

UPLO = 'U' or 'u'   The upper triangular part of A is supplied in A.

UPLO = 'L' or 'l'   The lower triangular part of A is supplied in A.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)

On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

A (input/output)

((n*(n + 1))/2). Before entry with UPLO = 'U' or 'u', the array A must contain the upper triangular part of the symmetric matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(1, 2) and a(2, 2) respectively, and so on. On exit, the array A is overwritten by the upper triangular part of the updated matrix. Before entry with UPLO = 'L' or 'l', the array A must contain the lower triangular part of the symmetric matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(2, 1) and a(3, 1) respectively, and so on. On exit, the array A is overwritten by the lower triangular part of the updated matrix.

Contents

NAME

dspr2 - perform the symmetric rank 2 operation A := alpha*x*y' + alpha*y*x' + A

SYNOPSIS

```
SUBROUTINE DSPR2(UPLO, N, ALPHA, X, INCX, Y, INCY, AP)

CHARACTER * 1 UPLO
INTEGER N, INCX, INCY
DOUBLE PRECISION ALPHA
DOUBLE PRECISION X(*), Y(*), AP(*)

SUBROUTINE DSPR2_64(UPLO, N, ALPHA, X, INCX, Y, INCY, AP)

CHARACTER * 1 UPLO
INTEGER*8 N, INCX, INCY
DOUBLE PRECISION ALPHA
DOUBLE PRECISION X(*), Y(*), AP(*)
```

F95 INTERFACE

```
SUBROUTINE SPR2(UPLO, [N], ALPHA, X, [INCX], Y, [INCY], AP)

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INCX, INCY
REAL(8) :: ALPHA
REAL(8), DIMENSION(:) :: X, Y, AP

SUBROUTINE SPR2_64(UPLO, [N], ALPHA, X, [INCX], Y, [INCY], AP)

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INCX, INCY
REAL(8) :: ALPHA
```

```
REAL(8), DIMENSION(:) :: X, Y, AP
```

C INTERFACE

```
#include <sunperf.h>

void dspr2(char uplo, int n, double alpha,  double  *x,  int
          incx, double *y, int incy, double *ap);

void dspr2_64(char uplo, long n, double  alpha,  double  *x,
          long incx, double *y, long incy, double *ap);
```

PURPOSE

dspr2 performs the symmetric rank 2 operation A := alpha*x*y' + alpha*y*x' + A, where alpha is a scalar, x and y are n element vectors and A is an n by n symmetric matrix, supplied in packed form.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the matrix A is supplied in the packed array AP as follows:

UPLO = 'U' or 'u'   The upper triangular part of A is supplied in AP.

UPLO = 'L' or 'l'   The lower triangular part of A is supplied in AP.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
Double precision array, dimension (1 + (n - 1)*abs(INCX)) Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)

On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

Y (input)

Double precision array, dimension (1 + (n - 1)*abs(INCY)) Before entry, the incremented array Y must contain the n element vector y. Unchanged on exit.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

AP (input/output)

Double precision array, dimension ((n*(n + 1))/2) Before entry with UPLO = 'U' or 'u', the array AP must contain the upper triangular part of the symmetric matrix packed sequentially, column by column, so that AP(1) contains a(1, 1), AP(2) and AP(3) contain a(1, 2) and a(2, 2) respectively, and so on. On exit, the array AP is overwritten by the upper triangular part of the updated matrix. Before entry with UPLO = 'L' or 'l', the array AP must contain the lower triangular part of the symmetric matrix packed sequentially, column by column, so that AP(1) contains a(1, 1), AP(2) and AP(3) contain a(2, 1) and a(3, 1) respectively, and so on. On exit, the array AP is overwritten by the lower triangular part of the updated matrix.

Contents

NAME

```
dsprfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is symmetric indefin-
ite and packed, and provides error bounds and backward error
estimates for the solution
```

SYNOPSIS

```
SUBROUTINE DSPRFS(UPLO, N, NRHS, AP, AF, IPIVOT, B, LDB, X, LDX, FERR,
      BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*), WORK2(*)
DOUBLE PRECISION AP(*), AF(*), B(LDB,*), X(LDX,*),  FERR(*),
BERR(*), WORK(*)

SUBROUTINE DSPRFS_64(UPLO, N, NRHS, AP, AF, IPIVOT, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
DOUBLE PRECISION AP(*), AF(*), B(LDB,*), X(LDX,*),  FERR(*),
BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPRFS(UPLO, N, [NRHS], AP, AF, IPIVOT, B, [LDB], X, [LDX],
       FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
```

```
REAL(8), DIMENSION(:) :: A, AF, FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: B, X

SUBROUTINE SPRFS_64(UPLO, N, [NRHS], AP, AF, IPIVOT, B, [LDB], X, [LDX],
       FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL(8), DIMENSION(:) :: AP, AF, FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: B, X
```

C INTERFACE

```
#include <sunperf.h>
void dsprfs(char uplo, int n, int nrhs, double  *ap,  double
          *af,  int  *ipivot, double *b, int ldb, double *x,
          int ldx, double *ferr, double *berr, int *info);

void dsprfs_64(char uplo, long n,  long  nrhs,  double  *ap,
          double  *af,  long  *ipivot,  double *b, long ldb,
          double *x, long ldx, double *ferr,  double  *berr,
          long *info);
```

PURPOSE

dsprfs improves the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite and packed, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

UPLO (input)
- = 'U': Upper triangle of A is stored;
- = 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

AP (input)
Double precsion array, dimension (N*(N+1)/2) The upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows:

if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

AF (input)
Double precsion array, dimension (N*(N+1)/2) The factored form of the matrix A. AF contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by SSPTRF, stored as a packed triangular matrix.

IPIVOT (input)
Integer array, dimension (N) Details of the interchanges and the block structure of D as determined by SSPTRF.

B (input) Double precision array, dimension (LDB,NRHS) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
Double precision array, dimension (LDX,NRHS) On entry, the solution matrix X, as computed by SSPTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
Double precision array, dimension (NRHS) The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
Double precision array, dimension (NRHS) The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an

exact solution).

WORK (workspace)
Double precision array, dimension(3*N)

WORK2 (workspace)
Integer array, dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-
gal value

Contents

NAME

```
dspsv - compute the solution to  a  real  system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE DSPSV(UPLO, N, NRHS, A, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDB, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION A(*), B(LDB,*)

SUBROUTINE DSPSV_64(UPLO, N, NRHS, A, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION A(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE SPSV(UPLO, N, [NRHS], A, IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: A
REAL(8), DIMENSION(:,:) :: B

SUBROUTINE SPSV_64(UPLO, N, [NRHS], A, IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: A
REAL(8), DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void dspsv(char uplo,  int  n,  int  nrhs,  double  *a,  int
          *ipivot, double *b, int ldb, int *info);

void dspsv_64(char uplo, long n, long nrhs, double *a,  long
          *ipivot, double *b, long ldb, long *info);
```

PURPOSE

dspsv computes the solution to a real system of linear equations
   A * X = B, where A is an N-by-N symmetric matrix stored in packed format and X and B are N-by-NRHS matrices.

The diagonal pivoting method is used to factor A as
   A = U * D * U**T, if UPLO = 'U', or
   A = L * D * L**T, if UPLO = 'L',
where U (or L) is a product of permutation and unit upper (lower) triangular matrices, D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks. The factored form of A is then used to solve the system of equations A * X = B.

ARGUMENTS

UPLO (input)
          = 'U': Upper triangle of A is stored;
          = 'L': Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NRHS (input)
          The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input/output)
          Double precision array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear

array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n. See below for further details.

On exit, the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by SSPTRF, stored as a packed triangular matrix in the same storage format as A.

IPIVOT (output)
Integer array, dimension (N) Details of the interchanges and the block structure of D, as determined by SSPTRF. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged, and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

B (input/output)
Double precision array, dimension (LDB, NRHS) On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, so the solution could not be computed.

FURTHER DETAILS

The packed storage scheme is illustrated by the following example when N = 4, UPLO = 'U':

Two-dimensional storage of the symmetric matrix A:

```
   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = aji)
               a44
```

Packed storage of the upper triangle of A:

A = [a11, a12, a22, a13, a23, a33, a14, a24, a34, a44]

Contents

NAME

```
dspsvx  -  use  the  diagonal  pivoting  factorization  A  =
U*D*U**T  or  A = L*D*L**T to compute the solution to a real
system of linear equations A * X = B, where A is  an  N-by-N
symmetric  matrix stored in packed format and X and B are N-
by-NRHS matrices
```

SYNOPSIS

```
SUBROUTINE DSPSVX(FACT, UPLO, N, NRHS, AP, AF, IPIVOT, B, LDB, X, LDX,
      RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*), WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION AP(*), AF(*), B(LDB,*), X(LDX,*),  FERR(*),
BERR(*), WORK(*)

SUBROUTINE DSPSVX_64(FACT, UPLO, N, NRHS, AP, AF, IPIVOT, B, LDB, X,
      LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION AP(*), AF(*), B(LDB,*), X(LDX,*),  FERR(*),
BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPSVX(FACT, UPLO, N, [NRHS], AP, AF, IPIVOT, B, [LDB], X,
       [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: FACT, UPLO
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: AP, AF, FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: B, X

SUBROUTINE SPSVX_64(FACT, UPLO, N, [NRHS], AP, AF, IPIVOT, B, [LDB], X,
       [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: AP, AF, FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: B, X
```

C INTERFACE

```
#include <sunperf.h>

void dspsvx(char fact, char uplo, int n,  int  nrhs,  double
          *a,  double  *af, int *ipivot, double *b, int ldb,
          double *x, int ldx, double *rcond,  double  *ferr,
          double *berr, int *info);

void dspsvx_64(char fact, char uplo, long n, long nrhs, dou-
          ble  *a, double *af, long *ipivot, double *b, long
          ldb, double *x, long ldx,  double  *rcond,  double
          *ferr, double *berr, long *info);
```

PURPOSE

DSPSVX uses the diagonal pivoting factorization A = U*D*U**T or A = L*D*L**T to compute the solution to a real system of linear equations A * X = B, where A is an N-by-N symmetric matrix stored in packed format and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'N', the diagonal pivoting method is used to factor A as

```
      A = U * D * U**T,  if UPLO = 'U', or
      A = L * D * L**T,  if UPLO = 'L',
```

where U (or L) is a product of permutation and unit upper (lower)

triangular matrices and D is symmetric and block diagonal with
1-by-1 and 2-by-2 diagonal blocks.

2. If some D(i,i)=0, so that D is exactly singular, then the routine
returns with INFO = i. Otherwise, the factored form of A is used
to estimate the condition number of the matrix A. If the
reciprocal of the condition number is less than machine precision,
INFO = N+1 is returned as a warning, but the routine still goes on
to solve for X and compute error bounds as described below.

3. The system of equations is solved for X using the factored form
of A.
4. Iterative refinement is applied to improve the computed solution
matrix and calculate error bounds and backward error estimates
for it.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of A has been supplied on entry. = 'F': On entry, AF and IPIVOT contain the factored form of A. AP, AF and IPIVOT will not be modified. = 'N': The matrix A will be copied to AF and factored.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

AP (input)
Double precision array, dimension (N*(N+1)/2) The upper or lower triangle of the symmetric matrix A,

packed columnwise in a linear array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n. See below for further details.

AF (input/output)
Double precision array, dimension (N*(N+1)/2) If FACT = 'F', then AF is an input argument and on entry contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by SSPTRF, stored as a packed triangular matrix in the same storage format as A.

If FACT = 'N', then AF is an output argument and on exit contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by SSPTRF, stored as a packed triangular matrix in the same storage format as A.

IPIVOT (input or output)
Integer array, dimension (N) If FACT = 'F', then IPIVOT is an input argument and on entry contains details of the interchanges and the block structure of D, as determined by SSPTRF. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

If FACT = 'N', then IPIVOT is an output argument and on exit contains details of the interchanges and the block structure of D, as determined by SSPTRF.

B (input) Double precision array, dimension (LDB, NRHS) The N-by-NRHS right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
Double precision array, dimension (LDX, NRHS) If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A. If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
Double precision array, dimension (NRHS) The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
Double precision array, dimension (NRHS) The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
Double precision array, dimension(3*N)

WORK2 (workspace)
Integer array, dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: D(i,i) is exactly zero. The factorization has been completed but the factor D is exactly singular, so the solution and error bounds could

not be computed. RCOND = 0 is returned. = N+1: D is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

FURTHER DETAILS

The packed storage scheme is illustrated by the following example when N = 4, UPLO = 'U':

Two-dimensional storage of the symmetric matrix A:

```
   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = aji)
               a44
```

Packed storage of the upper triangle of A:

AP = [a11, a12, a22, a13, a23, a33, a14, a24, a34, a44]

Contents

NAME

```
dsptrd - reduce a real symmetric matrix A stored  in  packed
form  to symmetric tridiagonal form T by an orthogonal simi-
larity transformation
```

SYNOPSIS

```
SUBROUTINE DSPTRD(UPLO, N, AP, D, E, TAU, INFO)

CHARACTER * 1 UPLO
INTEGER N, INFO
DOUBLE PRECISION AP(*), D(*), E(*), TAU(*)

SUBROUTINE DSPTRD_64(UPLO, N, AP, D, E, TAU, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, INFO
DOUBLE PRECISION AP(*), D(*), E(*), TAU(*)
```

F95 INTERFACE

```
SUBROUTINE SPTRD(UPLO, N, AP, D, E, TAU, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INFO
REAL(8), DIMENSION(:) :: AP, D, E, TAU

SUBROUTINE SPTRD_64(UPLO, N, AP, D, E, TAU, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INFO
REAL(8), DIMENSION(:) :: AP, D, E, TAU
```

C INTERFACE

```
#include <sunperf.h>

void dsptrd(char uplo, int n, double *ap, double *d,  double
          *e, double *tau, int *info);

void dsptrd_64(char uplo, long n,  double  *ap,  double  *d,
          double *e, double *tau, long *info);
```

PURPOSE

dsptrd reduces a real symmetric matrix A stored in packed form to symmetric tridiagonal form T by an orthogonal similarity transformation: Q**T * A * Q = T.

ARGUMENTS

UPLO (input)
 = 'U': Upper triangle of A is stored;
 = 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

AP (input)
 Double precision array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n. On exit, if UPLO = 'U', the diagonal and first superdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements above the first superdiagonal, with the array TAU, represent the orthogonal matrix Q as a product of elementary reflectors; if UPLO = 'L', the diagonal and first subdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements below the first subdiagonal, with the array TAU, represent the orthogonal matrix Q as a product of elementary reflectors. See Further Details.

D (output)
 Double precision array, dimension (N) The diagonal elements of the tridiagonal matrix T: D(i) =

```
          A(i,i).

E (output)
          Double precision array, dimension (N-1)  The  off-
          diagonal  elements  of  the  tridiagonal matrix T:
          E(i) = A(i,i+1) if UPLO = 'U', E(i) = A(i+1,i)  if
          UPLO = 'L'.

TAU (output)
          Double precision array, dimension (N-1) The scalar
          factors  of the elementary reflectors (see Further
          Details).

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

FURTHER DETAILS

```
If UPLO = 'U', the matrix Q is represented as a  product  of
elementary reflectors

   Q = H(n-1) . . . H(2) H(1).

Each H(i) has the form

   H(i) = I - tau * v * v'

where tau is a real scalar, and v is a real vector with
v(i+1:n) = 0 and v(i) = 1; v(1:i-1) is stored on exit in AP,
overwriting A(1:i-1,i+1), and tau is stored in TAU(i).

If UPLO = 'L', the matrix Q is represented as a  product  of
elementary reflectors

   Q = H(1) H(2) . . . H(n-1).

Each H(i) has the form

   H(i) = I - tau * v * v'

where tau is a real scalar, and v is a real vector with
v(1:i) = 0 and v(i+1) = 1; v(i+2:n) is stored on exit in AP,
overwriting A(i+2:n,i), and tau is stored in TAU(i).
```

Contents

NAME

```
dsptrf - compute  the  factorization  of  a  real  symmetric
matrix  A  stored  in  packed format using the Bunch-Kaufman
diagonal pivoting method
```

SYNOPSIS

```
SUBROUTINE DSPTRF(UPLO, N, AP, IPIVOT, INFO)

CHARACTER * 1 UPLO
INTEGER N, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION AP(*)

SUBROUTINE DSPTRF_64(UPLO, N, AP, IPIVOT, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION AP(*)
```

F95 INTERFACE

```
SUBROUTINE SPTRF(UPLO, N, AP, IPIVOT, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: AP

SUBROUTINE SPTRF_64(UPLO, N, AP, IPIVOT, [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: AP
```

C INTERFACE

```
#include <sunperf.h>

void dsptrf(char uplo, int n, double *ap, int  *ipivot,  int
          *info);

void dsptrf_64(char uplo, long n, double *ap, long  *ipivot,
          long *info);
```

PURPOSE

dsptrf computes the factorization of a real symmetric matrix A stored in packed format using the Bunch-Kaufman diagonal pivoting method:

A = U*D*U**T or A = L*D*L**T

where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

AP (input/output)
Double precision array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n.

On exit, the block diagonal matrix D and the multipliers used to obtain the factor U or L, stored as a packed triangular matrix overwriting A (see below for further details).

IPIVOT (output)

Integer array, dimension (N) Details of the interchanges and the block structure of D. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

5-96 - Based on modifications by J. Lewis, Boeing Computer Services
Company

If UPLO = 'U', then A = U*D*U', where
U = P(n)*U(n)* ... *P(k)U(k)* ...,
i.e., U is a product of terms P(k)*U(k), where k decreases from n to 1 in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIVOT(k), and U(k) is a unit upper triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    v    0   )   k-s
   U(k) =  (   0    I    0   )   s
           (   0    0    I   )   n-k
              k-s   s   n-k
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(1:k-1,k). If s = 2, the upper triangle of D(k) overwrites A(k-1,k-1), A(k-1,k), and A(k,k), and v overwrites A(1:k-2,k-1:k).

If UPLO = 'L', then A = L*D*L', where
L = P(1)*L(1)* ... *P(k)*L(k)* ...,

i.e., L is a product of terms P(k)*L(k), where k increases from 1 to n in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIVOT(k), and L(k) is a unit lower triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    0     0   )  k-1
   L(k) =  (   0    I     0   )  s
           (   0    v     I   )  n-k-s+1
              k-1   s  n-k-s+1
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(k+1:n,k). If s = 2, the lower triangle of D(k) overwrites A(k,k), A(k+1,k), and A(k+1,k+1), and v overwrites A(k+2:n,k:k+1).

Contents

NAME

```
dsptri - compute the inverse of a real symmetric  indefinite
matrix  A  in  packed  storage  using  the factorization A =
U*D*U**T or A = L*D*L**T computed by SSPTRF
```

SYNOPSIS

```
SUBROUTINE DSPTRI(UPLO, N, AP, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
INTEGER N, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION AP(*), WORK(*)

SUBROUTINE DSPTRI_64(UPLO, N, AP, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION AP(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPTRI(UPLO, N, AP, IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: AP, WORK

SUBROUTINE SPTRI_64(UPLO, N, AP, IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INFO
```

```
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: AP, WORK
```

C INTERFACE

```
#include <sunperf.h>

void dsptri(char uplo, int n, double *a,  int  *ipivot,  int
          *info);

void dsptri_64(char uplo, long n, double *a,  long  *ipivot,
          long *info);
```

PURPOSE

dsptri computes the inverse of a real symmetric indefinite matrix A in packed storage using the factorization A = U*D*U**T or A = L*D*L**T computed by SSPTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

AP (input/output)
Double precision array, dimension (N*(N+1)/2) On entry, the block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by SSPTRF, stored as a packed triangular matrix.

On exit, if INFO = 0, the (symmetric) inverse of the original matrix, stored as a packed triangular matrix. The j-th column of inv(A) is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = inv(A)(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2n-j)/2) = inv(A)(i,j) for j<=i<=n.

IPIVOT (input)
Integer array, dimension (N) Details of the interchanges and the block structure of D as determined by DSPTRF.

```
WORK (workspace)
          Double precision array, dimension(N)

INFO (output)
          = 0: successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
          gal value
          > 0: if INFO = i, D(i,i) = 0; the matrix is singu-
          lar and its inverse could not be computed.
```

Contents

NAME

dsptrs - solve a system of linear equations A*X = B with a real symmetric matrix A stored in packed format using the factorization A = U*D*U**T or A = L*D*L**T computed by DSPTRF

SYNOPSIS

```
SUBROUTINE DSPTRS(UPLO, N, NRHS, AP, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDB, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION AP(*), B(LDB,*)

SUBROUTINE DSPTRS_64(UPLO, N, NRHS, AP, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION AP(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE SPTRS(UPLO, N, [NRHS], AP, IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: AP
REAL(8), DIMENSION(:,:) :: B

SUBROUTINE SPTRS_64(UPLO, N, [NRHS], AP, IPIVOT, B, [LDB], [INFO])
```

```
CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: AP
REAL(8), DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void dsptrs(char uplo, int n,  int  nrhs,  double  *ap,  int
          *ipivot, double *b, int ldb, int *info);

void dsptrs_64(char uplo, long n,  long  nrhs,  double  *ap,
          long *ipivot, double *b, long ldb, long *info);
```

PURPOSE

dsptrs solves a system of linear equations A*X = B with a real symmetric matrix A stored in packed format using the factorization A = U*D*U**T or A = L*D*L**T computed by DSPTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

AP (input)
Double precision array, dimension (N*(N+1)/2) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by DSPTRF, stored as a packed triangular matrix.

IPIVOT (input)
Integer array, dimension (N) Details of the interchanges and the block structure of D as determined by DSPTRF.

B (input/output)

Double precision array, dimension (LDB,NRHS) On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)

The leading dimension of the array B. LDB >= max(1,N).

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dstebz - compute the eigenvalues of a symmetric  tridiagonal
matrix T
```

SYNOPSIS

```
SUBROUTINE DSTEBZ(RANGE, ORDER, N, VL, VU, IL, IU, ABSTOL, D, E, M,
      NSPLIT, W, IBLOCK, ISPLIT, WORK, IWORK, INFO)

CHARACTER * 1 RANGE, ORDER
INTEGER N, IL, IU, M, NSPLIT, INFO
INTEGER IBLOCK(*), ISPLIT(*), IWORK(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION D(*), E(*), W(*), WORK(*)

SUBROUTINE DSTEBZ_64(RANGE, ORDER, N, VL, VU, IL, IU, ABSTOL, D, E,
      M, NSPLIT, W, IBLOCK, ISPLIT, WORK, IWORK, INFO)

CHARACTER * 1 RANGE, ORDER
INTEGER*8 N, IL, IU, M, NSPLIT, INFO
INTEGER*8 IBLOCK(*), ISPLIT(*), IWORK(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION D(*), E(*), W(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEBZ(RANGE, ORDER, N, VL, VU, IL, IU, ABSTOL, D, E, M,
       NSPLIT, W, IBLOCK, ISPLIT, [WORK], [IWORK], [INFO])

CHARACTER(LEN=1) :: RANGE, ORDER
INTEGER :: N, IL, IU, M, NSPLIT, INFO
INTEGER, DIMENSION(:) :: IBLOCK, ISPLIT, IWORK
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: D, E, W, WORK
```

```
SUBROUTINE STEBZ_64(RANGE, ORDER, N, VL, VU, IL, IU, ABSTOL, D, E, M,
       NSPLIT, W, IBLOCK, ISPLIT, [WORK], [IWORK], [INFO])

CHARACTER(LEN=1) :: RANGE, ORDER
INTEGER(8) :: N, IL, IU, M, NSPLIT, INFO
INTEGER(8), DIMENSION(:) :: IBLOCK, ISPLIT, IWORK
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: D, E, W, WORK
```

C INTERFACE

```
#include <sunperf.h>

void dstebz(char range, char order, int n, double vl, double
          vu,  int  il,  int  iu,  double abstol, double *d,
          double *e, int *m, int  *nsplit,  double  *w,  int
          *iblock, int *isplit, int *info);

void dstebz_64(char range, char order, long  n,  double  vl,
          double vu, long il, long iu, double abstol, double
          *d, double *e, long *m, long *nsplit,  double  *w,
          long *iblock, long *isplit, long *info);
```

PURPOSE

dstebz computes the eigenvalues of a symmetric tridiagonal matrix T. The user may ask for all eigenvalues, all eigenvalues in the half-open interval (VL, VU], or the IL-th through IU-th eigenvalues.

To avoid overflow, the matrix must be scaled so that its largest element is no greater than overflow**(1/2) * underflow**(1/4) in absolute value, and for greatest accuracy, it should not be much smaller than that.

See W. Kahan "Accurate Eigenvalues of a Symmetric Tridiagonal Matrix", Report CS41, Computer Science Dept., Stanford University, July 21, 1966.

ARGUMENTS

RANGE (input)
= 'A': ("All") all eigenvalues will be found.
= 'V': ("Value") all eigenvalues in the half-open interval (VL, VU] will be found. = 'I': ("Index") the IL-th through IU-th eigenvalues (of the entire matrix) will be found.

```
ORDER (input)
          =  'B':  ("By  Block")  the  eigenvalues  will  be
          grouped  by  split-off  block (see IBLOCK, ISPLIT)
          and ordered from smallest to  largest  within  the
          block.   =  'E': ("Entire matrix") the eigenvalues
          for the entire matrix will be ordered  from  smal-
          lest to largest.

N (input) The order of the tridiagonal matrix T.  N >= 0.

VL (input)
          If RANGE='V', the lower and upper  bounds  of  the
          interval  to  be searched for eigenvalues.  Eigen-
          values less than or equal to VL, or  greater  than
          VU,   will   not  be  returned.   VL  <  VU.   Not
          referenced if RANGE = 'A' or 'I'.

VU (input)
          See the description of VL.

IL (input)
          If RANGE='I', the indices (in ascending order)  of
          the   smallest   and  largest  eigenvalues  to  be
          returned.  1 <= IL <= IU <= N, if N > 0;  IL  =  1
          and  IU  =  0 if N = 0.  Not referenced if RANGE =
          'A' or 'V'.

IU (input)
          See the description of IL.

ABSTOL (input)
          The absolute tolerance for  the  eigenvalues.   An
          eigenvalue   (or  cluster)  is  considered  to  be
          located if it has been determined  to  lie  in  an
          interval whose width is ABSTOL or less.  If ABSTOL
          is less than or equal to zero, then  ULP*|T|  will
          be used, where |T| means the 1-norm of T.

          Eigenvalues will be computed most accurately  when
          ABSTOL  is  set  to  twice the underflow threshold
          2*SLAMCH('S'), not zero.

D (input) The n diagonal elements of the tridiagonal  matrix
          T.

E (input) The (n-1) off-diagonal elements of the tridiagonal
          matrix T.

M (output)
```

The actual number of eigenvalues found. 0 <= M <= N. (See also the description of INFO=2,3.)

NSPLIT (output)
The number of diagonal blocks in the matrix T. 1 <= NSPLIT <= N.

W (output)
On exit, the first M elements of W will contain the eigenvalues. (SSTEBZ may use the remaining N-M elements as workspace.)

IBLOCK (output)
At each row/column j where E(j) is zero or small, the matrix T is considered to split into a block diagonal matrix. On exit, if INFO = 0, IBLOCK(i) specifies to which block (from 1 to the number of blocks) the eigenvalue W(i) belongs. (SSTEBZ may use the remaining N-M elements as workspace.)

ISPLIT (output)
The splitting points, at which T breaks up into submatrices. The first submatrix consists of rows/columns 1 to ISPLIT(1), the second of rows/columns ISPLIT(1)+1 through ISPLIT(2), etc., and the NSPLIT-th consists of rows/columns ISPLIT(NSPLIT-1)+1 through ISPLIT(NSPLIT)=N. (Only the first NSPLIT elements will actually be used, but since the user cannot know a priori what value NSPLIT will have, N words must be reserved for ISPLIT.)

WORK (workspace)
dimension(4*N)

IWORK (workspace)
dimension(3*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: some or all of the eigenvalues failed to converge or
were not computed:
=1 or 3: Bisection failed to converge for some eigenvalues; these eigenvalues are flagged by a negative block number. The effect is that the eigenvalues may not be as accurate as the absolute and relative tolerances. This is generally caused

by unexpectedly inaccurate arithmetic. =2 or 3: RANGE='I' only: Not all of the eigenvalues IL:IU were found.
Effect: M < IU+1-IL
Cause: non-monotonic arithmetic, causing the Sturm sequence to be non-monotonic. Cure: recalculate, using RANGE='A', and pick
out eigenvalues IL:IU. = 4: RANGE='I', and the Gershgorin interval initially used was too small. No eigenvalues were computed.

Contents

NAME

```
dstedc - compute all eigenvalues and, optionally,  eigenvec-
tors  of a symmetric tridiagonal matrix using the divide and
conquer method
```

SYNOPSIS

```
SUBROUTINE DSTEDC(COMPZ, N, D, E, Z, LDZ, WORK, LWORK, IWORK, LIWORK,
      INFO)

CHARACTER * 1 COMPZ
INTEGER N, LDZ, LWORK, LIWORK, INFO
INTEGER IWORK(*)
DOUBLE PRECISION D(*), E(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSTEDC_64(COMPZ, N, D, E, Z, LDZ, WORK, LWORK, IWORK,
      LIWORK, INFO)

CHARACTER * 1 COMPZ
INTEGER*8 N, LDZ, LWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION D(*), E(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEDC(COMPZ, N, D, E, Z, [LDZ], [WORK], [LWORK], [IWORK],
       [LIWORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
INTEGER :: N, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: D, E, WORK
REAL(8), DIMENSION(:,:) :: Z
```

```
SUBROUTINE STEDC_64(COMPZ, N, D, E, Z, [LDZ], [WORK], [LWORK], [IWORK],
       [LIWORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
INTEGER(8) :: N, LDZ, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: D, E, WORK
REAL(8), DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void dstedc(char compz, int n, double *d, double *e,  double
          *z, int ldz, int *info);
void dstedc_64(char compz, long n,  double  *d,  double  *e,
          double *z, long ldz, long *info);
```

PURPOSE

dstedc computes all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the divide and conquer method. The eigenvectors of a full or band real symmetric matrix can also be found if SSYTRD or SSPTRD or SSBTRD has been used to reduce this matrix to tridiagonal form.

This code makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none. See SLAED3 for details.

ARGUMENTS

COMPZ (input)
= 'N': Compute eigenvalues only.
= 'I': Compute eigenvectors of tridiagonal matrix also.
= 'V': Compute eigenvectors of original dense symmetric matrix also. On entry, Z contains the orthogonal matrix used to reduce the original matrix to tridiagonal form.

N (input) The dimension of the symmetric tridiagonal matrix.

```
          N >= 0.

D (input/output)
          On entry, the diagonal elements of the tridiagonal
          matrix.   On exit, if INFO = 0, the eigenvalues in
          ascending order.

E (input/output)
          On entry, the subdiagonal elements of the tridiag-
          onal matrix.  On exit, E has been destroyed.

Z (input) On entry, if COMPZ =  'V',  then  Z  contains  the
          orthogonal  matrix used in the reduction to tridi-
          agonal form.  On exit, if INFO = 0, then if  COMPZ
          =  'V', Z contains the orthonormal eigenvectors of
          the original symmetric matrix, and if COMPZ = 'I',
          Z  contains  the  orthonormal  eigenvectors of the
          symmetric tridiagonal matrix.  If   COMPZ  =  'N',
          then Z is not referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1.
          If eigenvectors are desired, then LDZ >= max(1,N).

WORK (workspace)
          dimension (LWORK) On exit, if INFO  =  0,  WORK(1)
          returns the optimal LWORK.

LWORK (input)
          The dimension of the array WORK.  If COMPZ  =  'N'
          or N <= 1 then LWORK must be at least 1.  If COMPZ
          = 'V' and N > 1 then LWORK must be at least ( 1  +
          3*N  +  2*N*lg N + 3*N**2 ), where lg( N ) = smal-
          lest integer k such that 2**k >= N.   If  COMPZ  =
          'I'  and  N  > 1 then LWORK must be at least ( 1 +
          4*N + N**2 ).

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

IWORK (workspace/output)
          On exit, if INFO = 0, IWORK(1) returns the optimal
          LIWORK.

LIWORK (input)
          The dimension of the array IWORK.  If COMPZ =  'N'
          or  N  <=  1  then  LIWORK must be at least 1.  If
```

COMPZ = 'V' and N > 1 then LIWORK must be at least (6 + 6*N + 5*N*lg N). If COMPZ = 'I' and N > 1 then LIWORK must be at least (3 + 5*N).

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: The algorithm failed to compute an eigenvalue while working on the submatrix lying in rows and columns INFO/(N+1) through mod(INFO,N+1).

FURTHER DETAILS

Based on contributions by
Jeff Rutter, Computer Science Division, University of California
at Berkeley, USA
Modified by Francoise Tisseur, University of Tennessee.

Contents

NAME

```
dstegr - (a) Compute T-sigma_i = L_i D_i  L_i^T,  such  that
L_i D_i L_i^T  is a relatively robust representation
```

SYNOPSIS

```
SUBROUTINE DSTEGR(JOBZ, RANGE, N, D, E, VL, VU, IL, IU, ABSTOL, M, W,
      Z, LDZ, ISUPPZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE
INTEGER N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER ISUPPZ(*), IWORK(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION D(*), E(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSTEGR_64(JOBZ, RANGE, N, D, E, VL, VU, IL, IU, ABSTOL, M,
      W, Z, LDZ, ISUPPZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE
INTEGER*8 N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER*8 ISUPPZ(*), IWORK(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION D(*), E(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEGR(JOBZ, RANGE, [N], D, E, VL, VU, IL, IU, ABSTOL, M,
       W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE
INTEGER :: N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: ISUPPZ, IWORK
REAL(8) :: VL, VU, ABSTOL
```

```
REAL(8), DIMENSION(:) :: D, E, W, WORK
REAL(8), DIMENSION(:,:) :: Z

SUBROUTINE STEGR_64(JOBZ, RANGE, [N], D, E, VL, VU, IL, IU, ABSTOL,
       M, W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE
INTEGER(8) :: N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: ISUPPZ, IWORK
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: D, E, W, WORK
REAL(8), DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>
void dstegr(char jobz, char range, int n, double *d,  double
          *e,  double  vl, double vu, int il, int iu, double
          abstol, int *m, double *w, double *z, int ldz, int
          *isuppz, int *info);

void dstegr_64(char jobz, char range,  long  n,  double  *d,
          double *e, double vl, double vu, long il, long iu,
          double abstol, long *m, double *w, double *z, long
          ldz, long *isuppz, long *info);
```

PURPOSE

```
dstegr b) Compute the  eigenvalues,  lambda_j,  of  L_i  D_i
L_i^T to high
       relative accuracy by the dqds algorithm,
   (c) If there is a cluster of close eigenvalues,  "choose"
sigma_i
       close to the cluster, and go to step (a),
   (d) Given the approximate eigenvalue lambda_j of L_i  D_i
L_i^T,
       compute the corresponding eigenvector by forming a
       rank-revealing twisted factorization.
The desired accuracy of the output can be specified  by  the
input parameter ABSTOL.

For more details, see "A new O(n^2) algorithm for  the  sym-
metric   tridiagonal   eigenvalue/eigenvector  problem",  by
Inderjit Dhillon, Computer Science Division Technical Report
No. UCB/CSD-97-971, UC Berkeley, May 1997.

Note 1 : Currently SSTEGR is only set up to find ALL  the  n
eigenvalues and eigenvectors of T in O(n^2) time
Note 2 : Currently the routine  SSTEIN  is  called  when  an
appropriate  sigma_i  cannot  be  chosen  in step (c) above.
```

SSTEIN invokes modified Gram-Schmidt when eigenvalues are close.
Note 3 : SSTEGR works only on machines which follow ieee-754 floating-point standard in their handling of infinities and NaNs. Normal execution of SSTEGR may create NaNs and infinities and hence may abort due to a floating point exception in environments which do not conform to the ieee standard.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

RANGE (input)
= 'A': all eigenvalues will be found.
= 'V': all eigenvalues in the half-open interval (VL,VU] will be found. = 'I': the IL-th through IU-th eigenvalues will be found.

N (input) The order of the matrix. N >= 0.

D (input/output)
On entry, the n diagonal elements of the tridiagonal matrix T. On exit, D is overwritten.

E (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix T in elements 1 to N-1 of E; E(N) need not be set. On exit, E is overwritten.

VL (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)
See the description of VL.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
See the description of IL.

ABSTOL (input)
The absolute error tolerance for the eigenvalues/eigenvectors. IF JOBZ = 'V', the eigenvalues and eigenvectors output have residual norms bounded by ABSTOL, and the dot products between different eigenvectors are bounded by ABSTOL. If ABSTOL is less than N*EPS*|T|, then N*EPS*|T| will be used in its place, where EPS is the machine precision and |T| is the 1-norm of the tridiagonal matrix. The eigenvalues are computed to an accuracy of EPS*|T| irrespective of ABSTOL. If high relative accuracy is important, set ABSTOL to DLAMCH('Safe minimum'). See Barlow and Demmel "Computing Accurate Eigensystems of Scaled Diagonally Dominant Matrices", LAPACK Working Note #7 for a discussion of which matrices define their eigenvalues to high relative accuracy.

M (output)
The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)
The first M elements contain the selected eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, the first M columns of Z contain the orthonormal eigenvectors of the matrix T corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). If JOBZ = 'N', then Z is not referenced. Note: the user must ensure that at least max(1,M) columns are supplied in the array Z; if RANGE = 'V', the exact value of M is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

ISUPPZ (output)
The support of the eigenvectors in Z, i.e., the indices indicating the nonzero elements in Z. The i-th eigenvector is nonzero only in elements ISUPPZ(2*i-1) through ISUPPZ(2*i).

WORK (workspace)

On exit, if INFO = 0, WORK(1) returns the optimal (and minimal) LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,18*N)
If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of the array IWORK. LIWORK >= max(1,10*N)

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = 1, internal error in SLARRE, if INFO = 2, internal error in SLARRV.

FURTHER DETAILS

Based on contributions by
Inderjit Dhillon, IBM Almaden, USA
Osni Marques, LBNL/NERSC, USA

Contents

NAME

```
dstein - compute the eigenvectors of a real symmetric tridi-
agonal  matrix  T  corresponding  to  specified eigenvalues,
using inverse iteration
```

SYNOPSIS

```
SUBROUTINE DSTEIN(N, D, E, M, W, IBLOCK, ISPLIT, Z, LDZ, WORK, IWORK,
      IFAIL, INFO)

INTEGER N, M, LDZ, INFO
INTEGER IBLOCK(*), ISPLIT(*), IWORK(*), IFAIL(*)
DOUBLE PRECISION D(*), E(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSTEIN_64(N, D, E, M, W, IBLOCK, ISPLIT, Z, LDZ, WORK,
      IWORK, IFAIL, INFO)

INTEGER*8 N, M, LDZ, INFO
INTEGER*8 IBLOCK(*), ISPLIT(*), IWORK(*), IFAIL(*)
DOUBLE PRECISION D(*), E(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEIN(N, D, E, M, W, IBLOCK, ISPLIT, Z, [LDZ], [WORK],
       [IWORK], IFAIL, [INFO])

INTEGER :: N, M, LDZ, INFO
INTEGER, DIMENSION(:) :: IBLOCK, ISPLIT, IWORK, IFAIL
REAL(8), DIMENSION(:) :: D, E, W, WORK
REAL(8), DIMENSION(:,:) :: Z

SUBROUTINE STEIN_64(N, D, E, M, W, IBLOCK, ISPLIT, Z, [LDZ], [WORK],
       [IWORK], IFAIL, [INFO])
```

```
INTEGER(8) :: N, M, LDZ, INFO
INTEGER(8), DIMENSION(:) :: IBLOCK, ISPLIT, IWORK, IFAIL
REAL(8), DIMENSION(:) :: D, E, W, WORK
REAL(8), DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void dstein(int n, double *d, double *e, int m,  double  *w,
          int  *iblock, int *isplit, double *z, int ldz, int
          *ifail, int *info);

void dstein_64(long n, double *d, double *e, long m,  double
          *w,  long  *iblock,  long *isplit, double *z, long
          ldz, long *ifail, long *info);
```

PURPOSE

dstein computes the eigenvectors of a real symmetric tridiagonal matrix T corresponding to specified eigenvalues, using inverse iteration.

The maximum number of iterations allowed for each eigenvector is specified by an internal parameter MAXITS (currently set to 5).

ARGUMENTS

N (input) The order of the matrix. N >= 0.

D (input) The n diagonal elements of the tridiagonal matrix T.

E (input) The (n-1) subdiagonal elements of the tridiagonal matrix T, in elements 1 to N-1. E(N) need not be set.

M (input) The number of eigenvectors to be found. 0 <= M <= N.

W (input) The first M elements of W contain the eigenvalues for which eigenvectors are to be computed. The eigenvalues should be grouped by split-off block and ordered from smallest to largest within the block. (The output array W from SSTEBZ with ORDER = 'B' is expected here.)

IBLOCK (input)

The submatrix indices associated with the corresponding eigenvalues in W; IBLOCK(i)=1 if eigenvalue W(i) belongs to the first submatrix from the top, =2 if W(i) belongs to the second submatrix, etc. (The output array IBLOCK from SSTEBZ is expected here.)

ISPLIT (input)

The splitting points, at which T breaks up into submatrices. The first submatrix consists of rows/columns 1 to ISPLIT(1), the second of rows/columns ISPLIT(1)+1 through ISPLIT(2), etc. (The output array ISPLIT from SSTEBZ is expected here.)

Z (output)

The computed eigenvectors. The eigenvector associated with the eigenvalue W(i) is stored in the i-th column of Z. Any vector which fails to converge is set to its current iterate after MAXITS iterations.

LDZ (input)

The leading dimension of the array Z. LDZ >= max(1,N).

WORK (workspace)

dimension(5*N)

IWORK (workspace)

dimension(N)

IFAIL (output)

On normal exit, all elements of IFAIL are zero. If one or more eigenvectors fail to converge after MAXITS iterations, then their indices are stored in array IFAIL.

INFO (output)

= 0: successful exit.

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, then i eigenvectors failed to converge in MAXITS iterations. Their indices are stored in array IFAIL.

Contents

NAME

```
dsteqr - compute all eigenvalues and, optionally,  eigenvec-
tors of a symmetric tridiagonal matrix using the implicit QL
or QR method
```

SYNOPSIS

```
SUBROUTINE DSTEQR(COMPZ, N, D, E, Z, LDZ, WORK, INFO)

CHARACTER * 1 COMPZ
INTEGER N, LDZ, INFO
DOUBLE PRECISION D(*), E(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSTEQR_64(COMPZ, N, D, E, Z, LDZ, WORK, INFO)

CHARACTER * 1 COMPZ
INTEGER*8 N, LDZ, INFO
DOUBLE PRECISION D(*), E(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEQR(COMPZ, N, D, E, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
INTEGER :: N, LDZ, INFO
REAL(8), DIMENSION(:) :: D, E, WORK
REAL(8), DIMENSION(:,:) :: Z

SUBROUTINE STEQR_64(COMPZ, N, D, E, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
INTEGER(8) :: N, LDZ, INFO
REAL(8), DIMENSION(:) :: D, E, WORK
REAL(8), DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void dsteqr(char compz, int n, double *d, double *e,  double
          *z, int ldz, int *info);

void dsteqr_64(char compz, long n,  double  *d,  double  *e,
          double *z, long ldz, long *info);
```

PURPOSE

dsteqr computes all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the implicit QL or QR method. The eigenvectors of a full or band symmetric matrix can also be found if SSYTRD or SSPTRD or SSBTRD has been used to reduce this matrix to tridiagonal form.

ARGUMENTS

COMPZ (input)
= 'N': Compute eigenvalues only.
= 'V': Compute eigenvalues and eigenvectors of the original symmetric matrix. On entry, Z must contain the orthogonal matrix used to reduce the original matrix to tridiagonal form. = 'I': Compute eigenvalues and eigenvectors of the tridiagonal matrix. Z is initialized to the identity matrix.

N (input) The order of the matrix. N >= 0.

D (input/output)
On entry, the diagonal elements of the tridiagonal matrix. On exit, if INFO = 0, the eigenvalues in ascending order.

E (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix. On exit, E has been destroyed.

Z (input) On entry, if COMPZ = 'V', then Z contains the orthogonal matrix used in the reduction to tridiagonal form. On exit, if INFO = 0, then if COMPZ = 'V', Z contains the orthonormal eigenvectors of the original symmetric matrix, and if COMPZ = 'I', Z contains the orthonormal eigenvectors of the symmetric tridiagonal matrix. If COMPZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if eigenvectors are desired, then LDZ >= max(1,N).

WORK (workspace)
dimension(max(1,2*N-2)) If COMPZ = 'N', then WORK is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: the algorithm has failed to find all the eigenvalues in a total of 30*N iterations; if INFO = i, then i elements of E have not converged to zero; on exit, D and E contain the elements of a symmetric tridiagonal matrix which is orthogonally similar to the original matrix.

Contents

NAME

```
dsterf - compute all eigenvalues of a symmetric  tridiagonal
matrix  using  the  Pal-Walker-Kahan variant of the QL or QR
algorithm
```

SYNOPSIS

```
SUBROUTINE DSTERF(N, D, E, INFO)

INTEGER N, INFO
DOUBLE PRECISION D(*), E(*)

SUBROUTINE DSTERF_64(N, D, E, INFO)

INTEGER*8 N, INFO
DOUBLE PRECISION D(*), E(*)
```

F95 INTERFACE

```
SUBROUTINE STERF([N], D, E, [INFO])

INTEGER :: N, INFO
REAL(8), DIMENSION(:) :: D, E

SUBROUTINE STERF_64([N], D, E, [INFO])

INTEGER(8) :: N, INFO
REAL(8), DIMENSION(:) :: D, E
```

C INTERFACE

```
#include <sunperf.h>

void dsterf(int n, double *d, double *e, int *info);
```

```
void dsterf_64(long n, double *d, double *e, long *info);
```

PURPOSE

dsterf computes all eigenvalues of a symmetric tridiagonal matrix using the Pal-Walker-Kahan variant of the QL or QR algorithm.

ARGUMENTS

N (input) The order of the matrix. N >= 0.

D (input/output)
On entry, the n diagonal elements of the tridiagonal matrix. On exit, if INFO = 0, the eigenvalues in ascending order.

E (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix. On exit, E has been destroyed.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: the algorithm failed to find all of the eigenvalues in a total of 30*N iterations; if INFO = i, then i elements of E have not converged to zero.

Contents

NAME

```
dstev - compute all eigenvalues and,  optionally,  eigenvec-
tors of a real symmetric tridiagonal matrix A
```

SYNOPSIS

```
SUBROUTINE DSTEV(JOBZ, N, DIAG, OFFD, Z, LDZ, WORK, INFO)

CHARACTER * 1 JOBZ
INTEGER N, LDZ, INFO
DOUBLE PRECISION DIAG(*), OFFD(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSTEV_64(JOBZ, N, DIAG, OFFD, Z, LDZ, WORK, INFO)

CHARACTER * 1 JOBZ
INTEGER*8 N, LDZ, INFO
DOUBLE PRECISION DIAG(*), OFFD(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEV(JOBZ, N, DIAG, OFFD, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: JOBZ
INTEGER :: N, LDZ, INFO
REAL(8), DIMENSION(:) :: DIAG, OFFD, WORK
REAL(8), DIMENSION(:,:) :: Z

SUBROUTINE STEV_64(JOBZ, N, DIAG, OFFD, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: JOBZ
INTEGER(8) :: N, LDZ, INFO
REAL(8), DIMENSION(:) :: DIAG, OFFD, WORK
REAL(8), DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void dstev(char jobz, int n,  double  *diag,  double  *offd,
          double *z, int ldz, int *info);

void dstev_64(char jobz, long n, double *diag, double *offd,
          double *z, long ldz, long *info);
```

PURPOSE

```
dstev computes all eigenvalues and, optionally, eigenvectors
of a real symmetric tridiagonal matrix A.
```

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

N (input) The order of the matrix.  N >= 0.

DIAG (input/output)
          On entry, the n diagonal elements of the tridiago-
          nal  matrix  A.   On exit, if INFO = 0, the eigen-
          values in ascending order.

OFFD (input/output)
          On entry, the (n-1) subdiagonal  elements  of  the
          tridiagonal  matrix A, stored in elements 1 to N-1
          of OFFD; OFFD(N) need not be set, but is  used  by
          the  routine.   On  exit, the contents of OFFD are
          destroyed.

Z (input) If JOBZ = 'V', then if INFO = 0,  Z  contains  the
          orthonormal eigenvectors of the matrix A, with the
          i-th column of Z holding the  eigenvector  associ-
          ated  with  DIAG(i).  If JOBZ = 'N', then Z is not
          referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
          If JOBZ = 'N', WORK is not referenced.
```

```
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, the algorithm  failed  to  con-
          verge;  i  off-diagonal  elements  of OFFD did not
          converge to zero.
```

Contents

NAME

```
dstevd - compute all eigenvalues and, optionally,  eigenvec-
tors of a real symmetric tridiagonal matrix
```

SYNOPSIS

```
SUBROUTINE DSTEVD(JOBZ, N, D, E, Z, LDZ, WORK, LWORK, IWORK, LIWORK,
      INFO)

CHARACTER * 1 JOBZ
INTEGER N, LDZ, LWORK, LIWORK, INFO
INTEGER IWORK(*)
DOUBLE PRECISION D(*), E(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSTEVD_64(JOBZ, N, D, E, Z, LDZ, WORK, LWORK, IWORK,
      LIWORK, INFO)

CHARACTER * 1 JOBZ
INTEGER*8 N, LDZ, LWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION D(*), E(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEVD(JOBZ, N, D, E, Z, [LDZ], [WORK], [LWORK], [IWORK],
       [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ
INTEGER :: N, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: D, E, WORK
REAL(8), DIMENSION(:,:) :: Z

SUBROUTINE STEVD_64(JOBZ, N, D, E, Z, [LDZ], [WORK], [LWORK], [IWORK],
```

```
      [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ
INTEGER(8) :: N, LDZ, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: D, E, WORK
REAL(8), DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void dstevd(char jobz, int n, double *d, double  *e,  double
          *z, int ldz, int *info);
void dstevd_64(char jobz, long n, double *d, double *e, dou-
          ble *z, long ldz, long *info);
```

PURPOSE

dstevd computes all eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

JOBZ (input)
  = 'N': Compute eigenvalues only;
  = 'V': Compute eigenvalues and eigenvectors.

N (input) The order of the matrix. N >= 0.

D (input/output)
  On entry, the n diagonal elements of the tridiagonal matrix A. On exit, if INFO = 0, the eigenvalues in ascending order.

E (input/output)
  On entry, the (n-1) subdiagonal elements of the tridiagonal matrix A, stored in elements 1 to N-1 of E; E(N) need not be set, but is used by the

```
          routine.   On  exit,  the  contents  of E are des-
          troyed.

Z (input) If JOBZ = 'V', then if INFO = 0,  Z  contains  the
          orthonormal eigenvectors of the matrix A, with the
          i-th column of Z holding the  eigenvector  associ-
          ated  with  D(i).   If  JOBZ  = 'N', then Z is not
          referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).
WORK (workspace)
          dimension (LWORK) On exit, if INFO  =  0,  WORK(1)
          returns the optimal LWORK.

LWORK (input)
          The dimension of the array WORK.  If JOBZ   =  'N'
          or  N <= 1 then LWORK must be at least 1.  If JOBZ
          = 'V' and N > 1 then LWORK must be at least ( 1  +
          4*N + N**2 ).

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

IWORK (workspace/output)
          On exit, if INFO = 0, IWORK(1) returns the optimal
          LIWORK.

LIWORK (input)
          The dimension of the array IWORK.  If JOBZ  =  'N'
          or N <= 1 then LIWORK must be at least 1.  If JOBZ
          = 'V' and N > 1  then  LIWORK  must  be  at  least
          3+5*N.

          If LIWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the IWORK array, returns this value as  the  first
          entry  of  the  IWORK  array, and no error message
          related to LIWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, the algorithm  failed  to  con-
          verge;  i  off-diagonal elements of E did not con-
```

verge to zero.

Contents

NAME

```
dstevr  -  compute  selected  eigenvalues  and,  optionally,
eigenvectors of a real symmetric tridiagonal matrix T
```

SYNOPSIS

```
SUBROUTINE DSTEVR(JOBZ, RANGE, N, D, E, VL, VU, IL, IU, ABSTOL, M, W,
      Z, LDZ, ISUPPZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE
INTEGER N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER ISUPPZ(*), IWORK(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION D(*), E(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSTEVR_64(JOBZ, RANGE, N, D, E, VL, VU, IL, IU, ABSTOL, M,
      W, Z, LDZ, ISUPPZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE
INTEGER*8 N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER*8 ISUPPZ(*), IWORK(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION D(*), E(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEVR(JOBZ, RANGE, [N], D, E, VL, VU, IL, IU, ABSTOL, M,
       W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE
INTEGER :: N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: ISUPPZ, IWORK
REAL(8) :: VL, VU, ABSTOL
```

```
REAL(8), DIMENSION(:) :: D, E, W, WORK
REAL(8), DIMENSION(:,:) :: Z

SUBROUTINE STEVR_64(JOBZ, RANGE, [N], D, E, VL, VU, IL, IU, ABSTOL,
       M, W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE
INTEGER(8) :: N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: ISUPPZ, IWORK
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: D, E, W, WORK
REAL(8), DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>
void dstevr(char jobz, char range, int n, double *d,  double
          *e,  double  vl, double vu, int il, int iu, double
          abstol, int *m, double *w, double *z, int ldz, int
          *isuppz, int *info);

void dstevr_64(char jobz, char range,  long  n,  double  *d,
          double *e, double vl, double vu, long il, long iu,
          double abstol, long *m, double *w, double *z, long
          ldz, long *isuppz, long *info);
```

PURPOSE

dstevr computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix T. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Whenever possible, SSTEVR calls SSTEGR to compute the eigenspectrum using Relatively Robust Representations. SSTEGR computes eigenvalues by the dqds algorithm, while orthogonal eigenvectors are computed from various "good" L D L^T representations (also known as Relatively Robust Representations). Gram-Schmidt orthogonalization is avoided as far as possible. More specifically, the various steps of the algorithm are as follows. For the i-th unreduced block of T,

```
   (a) Compute T - sigma_i = L_i D_i L_i^T,  such  that  L_i
D_i L_i^T
        is a relatively robust representation,
   (b) Compute the eigenvalues, lambda_j, of L_i  D_i  L_i^T
to high
       relative accuracy by the dqds algorithm,
   (c) If there is a cluster of close eigenvalues,  "choose"
```

```
sigma_i
       close to the cluster, and go to step (a),
   (d) Given the approximate eigenvalue lambda_j of L_i  D_i
L_i^T,
       compute the corresponding eigenvector by forming a
       rank-revealing twisted factorization.
The desired accuracy of the output can be specified  by  the
input parameter ABSTOL.

For more details, see "A new O(n^2) algorithm for  the  sym-
metric   tridiagonal   eigenvalue/eigenvector  problem",  by
Inderjit Dhillon, Computer Science Division Technical Report
No. UCB//CSD-97-971, UC Berkeley, May 1997.

Note 1 : SSTEVR calls  SSTEGR  when  the  full  spectrum  is
requested on machines which conform to the ieee-754 floating
point standard.  SSTEVR calls SSTEBZ and SSTEIN on  non-ieee
machines and
when partial spectrum requests are made.

Normal execution of SSTEGR may create  NaNs  and  infinities
and  hence  may  abort  due to a floating point exception in
environments which do not handle NaNs and infinities in  the
ieee standard default manner.
```

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

RANGE (input)
          = 'A': all eigenvalues will be found.
          = 'V': all eigenvalues in the  half-open  interval
          (VL,VU]  will  be found.  = 'I': the IL-th through
          IU-th eigenvalues will be found.

N (input) The order of the matrix.  N >= 0.

D (input/output)
          On entry, the n diagonal elements of the tridiago-
          nal  matrix  A.  On exit, D may be multiplied by a
          constant factor chosen to avoid over/underflow  in
          computing the eigenvalues.

E (input/output)
          On entry, the (n-1) subdiagonal  elements  of  the
          tridiagonal  matrix  A  in elements 1 to N-1 of E;
```

E(N) need not be set. On exit, E may be multiplied by a constant factor chosen to avoid over/underflow in computing the eigenvalues.

VL (input)

If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)

See the description of VL.

IL (input)

If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)

See the description of IL.

ABSTOL (input)

The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABSTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABSTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing A to tridiagonal form.

See "Computing Small Singular Values of Bidiagonal Matrices with Guaranteed High Relative Accuracy," by Demmel and Kahan, LAPACK Working Note #3.

If high relative accuracy is important, set ABSTOL to SLAMCH('Safe minimum'). Doing so will guarantee that eigenvalues are computed to high relative accuracy when possible in future releases. The current code does not make any guarantees about high relative accuracy, but future releases will. See J. Barlow and J. Demmel, "Computing Accurate Eigensystems of Scaled Diagonally Dominant Matrices", LAPACK Working Note #7, for a discussion of which matrices define their eigenvalues to high relative accuracy.

M (output)
The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)
The first M elements contain the selected eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, the first M columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). Note: the user must ensure that at least max(1,M) columns are supplied in the array Z; if RANGE = 'V', the exact value of M is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

ISUPPZ (output)
The support of the eigenvectors in Z, i.e., the indices indicating the nonzero elements in Z. The i-th eigenvector is nonzero only in elements ISUPPZ(2*i-1) through ISUPPZ(2*i).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal (and minimal) LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 20*N.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if INFO = 0, IWORK(1) returns the optimal (and minimal) LIWORK.

LIWORK (input)
The dimension of the array IWORK. LIWORK >= 10*N.

```
            If LIWORK = -1, then a workspace query is assumed;
            the  routine  only  calculates the optimal size of
            the IWORK array, returns this value as  the  first
            entry  of  the  IWORK  array, and no error message
            related to LIWORK is issued by XERBLA.
INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value
            > 0:  Internal error
```

FURTHER DETAILS

```
Based on contributions by
   Inderjit Dhillon, IBM Almaden, USA
   Osni Marques, LBNL/NERSC, USA
   Ken Stanley, Computer Science Division, University of
     California at Berkeley, USA
```

Contents

NAME

```
dstevx  -  compute  selected  eigenvalues  and,  optionally,
eigenvectors of a real symmetric tridiagonal matrix A
```

SYNOPSIS

```
SUBROUTINE DSTEVX(JOBZ, RANGE, N, DIAG, OFFD, VL, VU, IL, IU, ABTOL,
      NFOUND, W, Z, LDZ, WORK, IWORK2, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE
INTEGER N, IL, IU, NFOUND, LDZ, INFO
INTEGER IWORK2(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABTOL
DOUBLE PRECISION DIAG(*), OFFD(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSTEVX_64(JOBZ, RANGE, N, DIAG, OFFD, VL, VU, IL, IU,
      ABTOL, NFOUND, W, Z, LDZ, WORK, IWORK2, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE
INTEGER*8 N, IL, IU, NFOUND, LDZ, INFO
INTEGER*8 IWORK2(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABTOL
DOUBLE PRECISION DIAG(*), OFFD(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEVX(JOBZ, RANGE, N, DIAG, OFFD, VL, VU, IL, IU, ABTOL,
       NFOUND, W, Z, [LDZ], [WORK], [IWORK2], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE
INTEGER :: N, IL, IU, NFOUND, LDZ, INFO
INTEGER, DIMENSION(:) :: IWORK2, IFAIL
REAL(8) :: VL, VU, ABTOL
REAL(8), DIMENSION(:) :: DIAG, OFFD, W, WORK
```

```
REAL(8), DIMENSION(:,:) :: Z

SUBROUTINE STEVX_64(JOBZ, RANGE, N, DIAG, OFFD, VL, VU, IL, IU,
       ABTOL, NFOUND, W, Z, [LDZ], [WORK], [IWORK2], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE
INTEGER(8) :: N, IL, IU, NFOUND, LDZ, INFO
INTEGER(8), DIMENSION(:) :: IWORK2, IFAIL
REAL(8) :: VL, VU, ABTOL
REAL(8), DIMENSION(:) :: DIAG, OFFD, W, WORK
REAL(8), DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>
void dstevx(char jobz, char range, int n, double *diag, dou-
          ble  *offd,  double vl, double vu, int il, int iu,
          double abtol, int *nfound, double *w,  double  *z,
          int ldz, int *ifail, int *info);

void dstevx_64(char jobz, char range, long n, double  *diag,
          double  *offd, double vl, double vu, long il, long
          iu, double abtol, long *nfound, double *w,  double
          *z, long ldz, long *ifail, long *info);
```

PURPOSE

dstevx computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix A. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

RANGE (input)
= 'A': all eigenvalues will be found.
= 'V': all eigenvalues in the half-open interval (VL,VU] will be found. = 'I': the IL-th through IU-th eigenvalues will be found.

N (input) The order of the matrix. N >= 0.

DIAG (input/output)

On entry, the n diagonal elements of the tridiagonal matrix A. On exit, DIAG may be multiplied by a constant factor chosen to avoid over/underflow in computing the eigenvalues.

OFFD (input/output)

On entry, the (n-1) subdiagonal elements of the tridiagonal matrix A in elements 1 to N-1 of OFFD; OFFD(N) need not be set. On exit, OFFD may be multiplied by a constant factor chosen to avoid over/underflow in computing the eigenvalues.

VL (input)

If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)

See the description of VL.

IL (input)

If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)

See the description of IL.

ABTOL (input)

The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix.

Eigenvalues will be computed most accurately when ABTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABTOL to 2*SLAMCH('S').

See "Computing Small Singular Values of Bidiagonal

Matrices with Guaranteed High Relative Accuracy," by Demmel and Kahan, LAPACK Working Note #3.

NFOUND (output)
The total number of eigenvalues found. 0 <= NFOUND <= N. If RANGE = 'A', NFOUND = N, and if RANGE = 'I', NFOUND = IU-IL+1.

W (output)
The first NFOUND elements contain the selected eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, the first NFOUND columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). If an eigenvector fails to converge (INFO > 0), then that column of Z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in IFAIL. If JOBZ = 'N', then Z is not referenced. Note: the user must ensure that at least max(1,NFOUND) columns are supplied in the array Z; if RANGE = 'V', the exact value of NFOUND is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
dimension(5*N)

IWORK2 (workspace)

IFAIL (output)
If JOBZ = 'V', then if INFO = 0, the first NFOUND elements of IFAIL are zero. If INFO > 0, then IFAIL contains the indices of the eigenvectors that failed to converge. If JOBZ = 'N', then IFAIL is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, then i eigenvectors failed to converge. Their indices are stored in array IFAIL.

Contents

NAME

```
dstsv - compute the solution to a system of linear equations
A * X = B where A is a symmetric tridiagonal matrix
```

SYNOPSIS

```
SUBROUTINE DSTSV(N, NRHS, L, D, SUBL, B, LDB, IPIV, INFO)

INTEGER N, NRHS, LDB, INFO
INTEGER IPIV(*)
DOUBLE PRECISION L(*), D(*), SUBL(*), B(LDB,*)

SUBROUTINE DSTSV_64(N, NRHS, L, D, SUBL, B, LDB, IPIV, INFO)

INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIV(*)
DOUBLE PRECISION L(*), D(*), SUBL(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE STSV(N, NRHS, L, D, SUBL, B, [LDB], IPIV, [INFO])

INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIV
REAL(8), DIMENSION(:) :: L, D, SUBL
REAL(8), DIMENSION(:,:) :: B

SUBROUTINE STSV_64(N, NRHS, L, D, SUBL, B, [LDB], IPIV, [INFO])

INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIV
REAL(8), DIMENSION(:) :: L, D, SUBL
REAL(8), DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void dstsv(int n, int nrhs, double  *l,  double  *d,  double
          *subl, double *b, int ldb, int *ipiv, int *info);

void dstsv_64(long n, long nrhs, double *l, double *d,  dou-
          ble  *subl,  double *b, long ldb, long *ipiv, long
          *info);
```

PURPOSE

dstsv computes the solution to a system of linear equations
A * X = B where A is a symmetric tridiagonal matrix.

ARGUMENTS

N (input) INTEGER
The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides in B.

L (input/output)
REAL array, dimension (N)
On entry, the n-1 subdiagonal elements of the tridiagonal matrix A. On exit, part of the factorization of A.

D (input/output)
REAL array, dimension (N)
On entry, the n diagonal elements of the tridiagonal matrix A. On exit, the n diagonal elements of the diagonal matrix D from the factorization of A.

SUBL (output)
REAL array, dimension (N)
On exit, part of the factorization of A.

B (input/output)
The columns of B contain the right hand sides.

LDB (input)
The leading dimension of B as specified in a type or DIMENSION statement.

IPIV (output)

INTEGER array, dimension (N)
On exit, the pivot indices of the factorization.

INFO (output)
INTEGER
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(k,k) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular and division by zero will occur if it is used to solve a system of equations.

Contents

NAME

```
dsttrf - compute the factorization of a symmetric  tridiago-
nal matrix A
```

SYNOPSIS

```
SUBROUTINE DSTTRF(N, L, D, SUBL, IPIV, INFO)

INTEGER N, INFO
INTEGER IPIV(*)
DOUBLE PRECISION L(*), D(*), SUBL(*)

SUBROUTINE DSTTRF_64(N, L, D, SUBL, IPIV, INFO)

INTEGER*8 N, INFO
INTEGER*8 IPIV(*)
DOUBLE PRECISION L(*), D(*), SUBL(*)
```

F95 INTERFACE

```
SUBROUTINE STTRF([N], L, D, SUBL, IPIV, [INFO])

INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIV
REAL(8), DIMENSION(:) :: L, D, SUBL

SUBROUTINE STTRF_64([N], L, D, SUBL, IPIV, [INFO])

INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIV
REAL(8), DIMENSION(:) :: L, D, SUBL
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dsttrf(int n, double *l, double *d, double  *subl,  int
          *ipiv, int *info);

void dsttrf_64(long n, double *l, double *d,  double  *subl,
          long *ipiv, long *info);
```

PURPOSE

dsttrf computes the factorization of a complex Hermitian tridiagonal matrix A.

ARGUMENTS

N (input) INTEGER
The order of the matrix A. N >= 0.

L (input/output)
REAL array, dimension (N)
On entry, the n-1 subdiagonal elements of the tri- diagonal matrix A. On exit, part of the factori- zation of A.

D (input/output)
REAL array, dimension (N)
On entry, the n diagonal elements of the tridiago- nal matrix A. On exit, the n diagonal elements of the diagonal matrix D from the L*D*L**H factoriza- tion of A.

SUBL (output)
REAL array, dimension (N)
On exit, part of the factorization of A.

IPIV (output)
INTEGER array, dimension (N)
On exit, the pivot indices of the factorization.

INFO (output)
INTEGER
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille- gal value
> 0: if INFO = i, D(k,k) is exactly zero.   The factorization has been completed, but the block diagonal matrix D is exactly singular and division by zero will occur if it is used to solve a system

of equations.

Contents

NAME

```
dsttrs - computes the solution to a real  system  of  linear
equations A * X = B
```

SYNOPSIS

```
SUBROUTINE DSTTRS(N, NRHS, L, D, SUBL, B, LDB, IPIV, INFO)

INTEGER N, NRHS, LDB, INFO
INTEGER IPIV(*)
DOUBLE PRECISION L(*), D(*), SUBL(*), B(LDB,*)

SUBROUTINE DSTTRS_64(N, NRHS, L, D, SUBL, B, LDB, IPIV, INFO)

INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIV(*)
DOUBLE PRECISION L(*), D(*), SUBL(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE STTRS(N, NRHS, L, D, SUBL, B, [LDB], IPIV, [INFO])

INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIV
REAL(8), DIMENSION(:) :: L, D, SUBL
REAL(8), DIMENSION(:,:) :: B

SUBROUTINE STTRS_64(N, NRHS, L, D, SUBL, B, [LDB], IPIV, [INFO])

INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIV
REAL(8), DIMENSION(:) :: L, D, SUBL
REAL(8), DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void dsttrs(int n, int nrhs, double *l,  double  *d,  double
          *subl, double *b, int ldb, int *ipiv, int *info);

void dsttrs_64(long n, long nrhs, double *l, double *d, dou-
          ble  *subl,  double *b, long ldb, long *ipiv, long
          *info);
```

PURPOSE

dsttrs computes the solution to a real system of linear equations A * X = B, where A is an N-by-N symmetric tridiagonal matrix and X and B are N-by-NRHS matrices.

ARGUMENTS

N (input) INTEGER
The order of the matrix A. N >= 0.

NRHS (input)
INTEGER
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

L (input) REAL array, dimension (N-1)
On entry, the subdiagonal elements of LL and DD.

D (input) REAL array, dimension (N)
On entry, the diagonal elements of DD.

SUBL (input)
REAL array, dimension (N-2)
On entry, the second subdiagonal elements of LL.

B (input/output)
REAL array, dimension
(LDB, NRHS) On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
INTEGER
The leading dimension of the array B. LDB >= max(1, N)

```
IPIV (output)
          INTEGER array, dimension (N)
          Details of the interchanges and block  pivot.   If
          IPIV(K)  > 0, 1 by 1 pivot, and if IPIV(K) = K + 1
          an interchange done;  If  IPIV(K)  <  0,  2  by  2
          pivot, no interchange required.

INFO (output)
          INTEGER
          = 0:  successful exit
          < 0:  if INFO = -k, the k-th argument had an ille-
          gal value
```

Contents

NAME

```
dswap - Exchange vectors x and y.
```

SYNOPSIS

```
SUBROUTINE DSWAP(N, X, INCX, Y, INCY)

INTEGER N, INCX, INCY
DOUBLE PRECISION X(*), Y(*)

SUBROUTINE DSWAP_64(N, X, INCX, Y, INCY)

INTEGER*8 N, INCX, INCY
DOUBLE PRECISION X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE SWAP([N], X, [INCX], Y, [INCY])

INTEGER :: N, INCX, INCY
REAL(8), DIMENSION(:) :: X, Y

SUBROUTINE SWAP_64([N], X, [INCX], Y, [INCY])

INTEGER(8) :: N, INCX, INCY
REAL(8), DIMENSION(:) :: X, Y
```

C INTERFACE

```
#include <sunperf.h>

void dswap(int n, double *x, int incx, double *y, int incy);

void dswap_64(long n, double *x, long incx, double *y,  long
          incy);
```

PURPOSE

dswap Exchange x and y where x and y are n-vectors.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input/output)
(1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. On exit, the y vector.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. On exit, the x vector.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
dsycon - estimate the reciprocal of the condition number (in
the  1-norm) of a real symmetric matrix A using the factori-
zation A = U*D*U**T or A = L*D*L**T computed by SSYTRF
```

SYNOPSIS

```
SUBROUTINE DSYCON(UPLO, N, A, LDA, IPIVOT, ANORM, RCOND, WORK,
      IWORK2, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, INFO
INTEGER IPIVOT(*), IWORK2(*)
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION A(LDA,*), WORK(*)

SUBROUTINE DSYCON_64(UPLO, N, A, LDA, IPIVOT, ANORM, RCOND, WORK,
      IWORK2, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, INFO
INTEGER*8 IPIVOT(*), IWORK2(*)
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYCON(UPLO, N, A, [LDA], IPIVOT, ANORM, RCOND, [WORK],
       [IWORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT, IWORK2
REAL(8) :: ANORM, RCOND
```

```
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE SYCON_64(UPLO, N, A, [LDA], IPIVOT, ANORM, RCOND, [WORK],
       [IWORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, IWORK2
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
void dsycon(char uplo,  int  n,  double  *a,  int  lda,  int
          *ipivot, double anorm, double *rcond, int *info);

void dsycon_64(char uplo, long n, double *a, long lda,  long
          *ipivot, double anorm, double *rcond, long *info);
```

PURPOSE

dsycon estimates the reciprocal of the condition number (in the 1-norm) of a real symmetric matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by SSYTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

A (input) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by SSYTRF.

LDA (input)
The leading dimension of the array A. LDA >=

max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by SSYTRF.

ANORM (input)
The 1-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(2*N)

IWORK2 (workspace)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dsyev - compute all eigenvalues and,  optionally,  eigenvec-
tors of a real symmetric matrix A
```

SYNOPSIS

```
SUBROUTINE DSYEV(JOBZ, UPLO, N, A, LDA, W, WORK, LDWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER N, LDA, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), W(*), WORK(*)

SUBROUTINE DSYEV_64(JOBZ, UPLO, N, A, LDA, W, WORK, LDWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 N, LDA, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), W(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYEV(JOBZ, UPLO, N, A, [LDA], W, [WORK], [LDWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: N, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE SYEV_64(JOBZ, UPLO, N, A, [LDA], W, [WORK], [LDWORK],
       [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: N, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dsyev(char jobz, char uplo, int n, double *a, int  lda,
          double *w, int *info);

void dsyev_64(char jobz, char uplo, long n, double *a,  long
          lda, double *w, long *info);
```

PURPOSE

dsyev computes all eigenvalues and, optionally, eigenvectors of a real symmetric matrix A.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the symmetric matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of A contains the lower  triangular  part  of
          the  matrix  A.   On  exit, if JOBZ = 'V', then if
          INFO = 0, A contains the orthonormal  eigenvectors
          of  the matrix A.  If JOBZ = 'N', then on exit the
          lower triangle (if UPLO='L') or the upper triangle
          (if  UPLO='U')  of  A,  including the diagonal, is
          destroyed.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

W (output)
          If INFO = 0, the eigenvalues in ascending order.
```

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of the array WORK. LDWORK >= max(1,3*N-1). For optimal efficiency, LDWORK >= (NB+2)*N, where NB is the blocksize for SSYTRD returned by ILAENV.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

Contents

NAME

```
dsyevd - compute all eigenvalues and, optionally,  eigenvec-
tors of a real symmetric matrix A
```

SYNOPSIS

```
SUBROUTINE DSYEVD(JOBZ, UPLO, N, A, LDA, W, WORK, LWORK, IWORK,
      LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER N, LDA, LWORK, LIWORK, INFO
INTEGER IWORK(*)
DOUBLE PRECISION A(LDA,*), W(*), WORK(*)

SUBROUTINE DSYEVD_64(JOBZ, UPLO, N, A, LDA, W, WORK, LWORK, IWORK,
      LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 N, LDA, LWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION A(LDA,*), W(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYEVD(JOBZ, UPLO, N, A, [LDA], W, [WORK], [LWORK], [IWORK],
       [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: N, LDA, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: A
```

```
SUBROUTINE SYEVD_64(JOBZ, UPLO, N, A, [LDA], W, [WORK], [LWORK],
       [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: N, LDA, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dsyevd(char jobz, char uplo, int n, double *a, int lda,
          double *w, int *info);
void dsyevd_64(char jobz, char uplo, long n, double *a, long
          lda, double *w, long *info);
```

PURPOSE

dsyevd computes all eigenvalues and, optionally, eigenvectors of a real symmetric matrix A. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

Because of large use of BLAS of level 3, SSYEVD needs N**2 more workspace than SSYEVX.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
```

On entry, the symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A. On exit, if JOBZ = 'V', then if INFO = 0, A contains the orthonormal eigenvectors of the matrix A. If JOBZ = 'N', then on exit the lower triangle (if UPLO='L') or the upper triangle (if UPLO='U') of A, including the diagonal, is destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

W (output)
If INFO = 0, the eigenvalues in ascending order.

WORK (workspace)
dimension (LWORK) On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If N <= 1, LWORK must be at least 1. If JOBZ = 'N' and N > 1, LWORK must be at least 2*N+1. If JOBZ = 'V' and N > 1, LWORK must be at least 1 + 6*N + 2*N**2.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of the array IWORK. If N <= 1, LIWORK must be at least 1. If JOBZ = 'N' and N > 1, LIWORK must be at least 1. If JOBZ = 'V' and N > 1, LIWORK must be at least 3 + 5*N.

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

FURTHER DETAILS

Based on contributions by
Jeff Rutter, Computer Science Division, University of California
at Berkeley, USA
Modified by Francoise Tisseur, University of Tennessee.

Contents

NAME

dsyevr - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix T

SYNOPSIS

```
SUBROUTINE DSYEVR(JOBZ, RANGE, UPLO, N, A, LDA, VL, VU, IL, IU,
      ABSTOL, M, W, Z, LDZ, ISUPPZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER N, LDA, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER ISUPPZ(*), IWORK(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION A(LDA,*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSYEVR_64(JOBZ, RANGE, UPLO, N, A, LDA, VL, VU, IL, IU,
      ABSTOL, M, W, Z, LDZ, ISUPPZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER*8 N, LDA, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER*8 ISUPPZ(*), IWORK(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION A(LDA,*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYEVR(JOBZ, RANGE, UPLO, [N], A, [LDA], VL, VU, IL, IU,
       ABSTOL, M, W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [IWORK], [LIWORK],
       [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER :: N, LDA, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: ISUPPZ, IWORK
```

```
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: A, Z

SUBROUTINE SYEVR_64(JOBZ, RANGE, UPLO, [N], A, [LDA], VL, VU, IL, IU,
       ABSTOL, M, W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [IWORK], [LIWORK],
       [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER(8) :: N, LDA, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: ISUPPZ, IWORK
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: A, Z
```

C INTERFACE

```
#include <sunperf.h>

void dsyevr(char jobz, char range, char uplo, int n,  double
          *a, int lda, double vl, double vu, int il, int iu,
          double abstol, int *m, double *w, double  *z,  int
          ldz, int *isuppz, int *info);

void dsyevr_64(char jobz, char range,  char  uplo,  long  n,
          double  *a,  long  lda, double vl, double vu, long
          il, long iu, double abstol, long  *m,  double  *w,
          double *z, long ldz, long *isuppz, long *info);
```

PURPOSE

dsyevr computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix T. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Whenever possible, SSYEVR calls SSTEGR to compute the eigenspectrum using Relatively Robust Representations. SSTEGR computes eigenvalues by the dqds algorithm, while orthogonal eigenvectors are computed from various "good" L D L^T representations (also known as Relatively Robust Representations). Gram-Schmidt orthogonalization is avoided as far as possible. More specifically, the various steps of the algorithm are as follows. For the i-th unreduced block of T,
   (a) Compute T - sigma_i = L_i D_i L_i^T, such that L_i D_i L_i^T
        is a relatively robust representation,
   (b) Compute the eigenvalues, lambda_j, of L_i D_i L_i^T to high

relative accuracy by the dqds algorithm,
(c) If there is a cluster of close eigenvalues, "choose" sigma_i
close to the cluster, and go to step (a),
(d) Given the approximate eigenvalue lambda_j of L_i D_i L_i^T,
compute the corresponding eigenvector by forming a
rank-revealing twisted factorization.
The desired accuracy of the output can be specified by the input parameter ABSTOL.

For more details, see "A new O(n^2) algorithm for the symmetric tridiagonal eigenvalue/eigenvector problem", by Inderjit Dhillon, Computer Science Division Technical Report No. UCB//CSD-97-971, UC Berkeley, May 1997.
Note 1 : SSYEVR calls SSTEGR when the full spectrum is requested on machines which conform to the ieee-754 floating point standard. SSYEVR calls SSTEBZ and SSTEIN on non-ieee machines and
when partial spectrum requests are made.

Normal execution of SSTEGR may create NaNs and infinities and hence may abort due to a floating point exception in environments which do not handle NaNs and infinities in the ieee standard default manner.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

RANGE (input)
= 'A': all eigenvalues will be found.
= 'V': all eigenvalues in the half-open interval (VL,VU] will be found. = 'I': the IL-th through IU-th eigenvalues will be found.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A.

```
            If UPLO = 'L', the leading N-by-N lower triangular
            part of A contains the lower  triangular  part  of
            the  matrix  A.   On  exit, the lower triangle (if
            UPLO='L') or the upper triangle (if  UPLO='U')  of
            A, including the diagonal, is destroyed.

LDA (input)
            The leading dimension of  the  array  A.   LDA  >=
            max(1,N).

VL (input)
            If RANGE='V', the lower and upper  bounds  of  the
            interval  to be searched for eigenvalues. VL < VU.
            Not referenced if RANGE = 'A' or 'I'.

VU (input)
            See the description of VL.

IL (input)
            If RANGE='I', the indices (in ascending order)  of
            the   smallest   and  largest  eigenvalues  to  be
            returned.  1 <= IL <= IU <= N, if N > 0;  IL  =  1
            and  IU  =  0 if N = 0.  Not referenced if RANGE =
            'A' or 'V'.

IU (input)
            See the description of IL.

ABSTOL (input)
            The absolute error tolerance for the  eigenvalues.
            An approximate eigenvalue is accepted as converged
            when it is determined to lie in an interval  [a,b]
            of width less than or equal to

            ABSTOL + EPS *   max( |a|,|b| ) ,

            where EPS is the machine precision.  If ABSTOL  is
            less than or equal to zero, then  EPS*|T|  will be
            used in its place, where |T| is the 1-norm of  the
            tridiagonal  matrix obtained by reducing A to tri-
            diagonal form.

            See "Computing Small Singular Values of Bidiagonal
            Matrices  with Guaranteed High Relative Accuracy,"
            by Demmel and Kahan, LAPACK Working Note #3.

            If high relative accuracy is important, set ABSTOL
            to  SLAMCH(  'Safe  minimum'  ).   Doing  so  will
            guarantee that eigenvalues are  computed  to  high
            relative   accuracy   when   possible   in  future
```

releases. The current code does not make any guarantees about high relative accuracy, but furu- tre releases will. See J. Barlow and J. Demmel, "Computing Accurate Eigensystems of Scaled Diago- nally Dominant Matrices", LAPACK Working Note #7, for a discussion of which matrices define their eigenvalues to high relative accuracy.

M (output)
The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)
The first M elements contain the selected eigen- values in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, the first M columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). If JOBZ = 'N', then Z is not referenced. Note: the user must ensure that at least max(1,M) columns are supplied in the array Z; if RANGE = 'V', the exact value of M is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

ISUPPZ (output)
The support of the eigenvectors in Z, i.e., the indices indicating the nonzero elements in Z. The i-th eigenvector is nonzero only in elements ISUPPZ(2*i-1) through ISUPPZ(2*i).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,26*N). For optimal efficiency, LWORK >= (NB+6)*N, where NB is the max of the blocksize for SSYTRD and SORMTR returned by ILAENV.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of

the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if INFO = 0, IWORK(1) returns the optimal LWORK.

LIWORK (input)
The dimension of the array IWORK. LIWORK >= max(1,10*N).

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille- gal value
> 0: Internal error

FURTHER DETAILS

Based on contributions by
Inderjit Dhillon, IBM Almaden, USA
Osni Marques, LBNL/NERSC, USA
Ken Stanley, Computer Science Division, University of California at Berkeley, USA

Contents

NAME

dsyevx - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix A

SYNOPSIS

```
SUBROUTINE DSYEVX(JOBZ, RANGE, UPLO, N, A, LDA, VL, VU, IL, IU,
      ABTOL, NFOUND, W, Z, LDZ, WORK, LDWORK, IWORK2, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER N, LDA, IL, IU, NFOUND, LDZ, LDWORK, INFO
INTEGER IWORK2(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABTOL
DOUBLE PRECISION A(LDA,*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSYEVX_64(JOBZ, RANGE, UPLO, N, A, LDA, VL, VU, IL, IU,
      ABTOL, NFOUND, W, Z, LDZ, WORK, LDWORK, IWORK2, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER*8 N, LDA, IL, IU, NFOUND, LDZ, LDWORK, INFO
INTEGER*8 IWORK2(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABTOL
DOUBLE PRECISION A(LDA,*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYEVX(JOBZ, RANGE, UPLO, N, A, [LDA], VL, VU, IL, IU,
       ABTOL, NFOUND, W, Z, [LDZ], [WORK], [LDWORK], [IWORK2], IFAIL,
       [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER :: N, LDA, IL, IU, NFOUND, LDZ, LDWORK, INFO
INTEGER, DIMENSION(:) :: IWORK2, IFAIL
REAL(8) :: VL, VU, ABTOL
```

```
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: A, Z

SUBROUTINE SYEVX_64(JOBZ, RANGE, UPLO, N, A, [LDA], VL, VU, IL, IU,
       ABTOL, NFOUND, W, Z, [LDZ], [WORK], [LDWORK], [IWORK2], IFAIL,
       [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER(8) :: N, LDA, IL, IU, NFOUND, LDZ, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK2, IFAIL
REAL(8) :: VL, VU, ABTOL
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: A, Z
```

C INTERFACE

```
#include <sunperf.h>

void dsyevx(char jobz, char range, char uplo, int n,  double
          *a, int lda, double vl, double vu, int il, int iu,
          double abtol, int *nfound, double *w,  double  *z,
          int ldz, int *ifail, int *info);

void dsyevx_64(char jobz, char range,  char  uplo,  long  n,
          double  *a,  long  lda, double vl, double vu, long
          il, long iu, double abtol,  long  *nfound,  double
          *w, double *z, long ldz, long *ifail, long *info);
```

PURPOSE

dsyevx computes selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix A. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

RANGE (input)
          = 'A': all eigenvalues will be found.
          = 'V': all eigenvalues in the  half-open  interval
          (VL,VU]  will  be found.  = 'I': the IL-th through
          IU-th eigenvalues will be found.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
```

```
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the symmetric matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of A contains the lower  triangular  part  of
          the  matrix  A.   On  exit, the lower triangle (if
          UPLO='L') or the upper triangle (if  UPLO='U')  of
          A, including the diagonal, is destroyed.
LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

VL (input)
          If RANGE='V', the lower and upper  bounds  of  the
          interval  to be searched for eigenvalues. VL < VU.
          Not referenced if RANGE = 'A' or 'I'.

VU (input)
          See the description of VL.

IL (input)
          If RANGE='I', the indices (in ascending order)  of
          the   smallest   and  largest  eigenvalues  to  be
          returned.  1 <= IL <= IU <= N, if N > 0;  IL  =  1
          and  IU  =  0 if N = 0.  Not referenced if RANGE =
          'A' or 'V'.

IU (input)
          See the description of IL.

ABTOL (input)
          The absolute error tolerance for the  eigenvalues.
          An approximate eigenvalue is accepted as converged
          when it is determined to lie in an interval  [a,b]
          of width less than or equal to

          ABTOL + EPS *   max( |a|,|b| ) ,

          where EPS is the machine precision.  If  ABTOL  is
          less than or equal to zero, then  EPS*|T|  will be
          used in its place, where |T| is the 1-norm of  the
          tridiagonal  matrix obtained by reducing A to tri-
          diagonal form.

          Eigenvalues will be computed most accurately  when
```

ABTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABTOL to 2*SLAMCH('S').

See "Computing Small Singular Values of Bidiagonal Matrices with Guaranteed High Relative Accuracy," by Demmel and Kahan, LAPACK Working Note #3.

NFOUND (output)
The total number of eigenvalues found. 0 <= NFOUND <= N. If RANGE = 'A', NFOUND = N, and if RANGE = 'I', NFOUND = IU-IL+1.

W (output)
On normal exit, the first NFOUND elements contain the selected eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, the first NFOUND columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). If an eigenvector fails to converge, then that column of Z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in IFAIL. If JOBZ = 'N', then Z is not referenced. Note: the user must ensure that at least max(1,NFOUND) columns are supplied in the array Z; if RANGE = 'V', the exact value of NFOUND is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of the array WORK. LDWORK >= max(1,8*N). For optimal efficiency, LDWORK >= (NB+3)*N, where NB is the max of the blocksize for SSYTRD and SORMTR returned by ILAENV.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

IWORK2 (workspace)
IFAIL (output)
If JOBZ = 'V', then if INFO = 0, the first NFOUND elements of IFAIL are zero. If INFO > 0, then IFAIL contains the indices of the eigenvectors that failed to converge. If JOBZ = 'N', then IFAIL is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, then i eigenvectors failed to converge. Their indices are stored in array IFAIL.

Contents

NAME

```
dsygs2 - reduce a real symmetric-definite generalized eigen-
problem to standard form
```

SYNOPSIS

```
SUBROUTINE DSYGS2(ITYPE, UPLO, N, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER ITYPE, N, LDA, LDB, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*)

SUBROUTINE DSYGS2_64(ITYPE, UPLO, N, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 ITYPE, N, LDA, LDB, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE SYGS2(ITYPE, UPLO, N, A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: ITYPE, N, LDA, LDB, INFO
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE SYGS2_64(ITYPE, UPLO, N, A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: ITYPE, N, LDA, LDB, INFO
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dsygs2(int itype, char uplo, int n, double *a, int lda,
          double *b, int ldb, int *info);

void dsygs2_64(long itype, char uplo,  long  n,  double  *a,
          long lda, double *b, long ldb, long *info);
```

PURPOSE

dsygs2 reduces a real symmetric-definite generalized eigen-problem to standard form.

If ITYPE = 1, the problem is A*x = lambda*B*x,
and A is overwritten by inv(U')*A*inv(U) or inv(L)*A*inv(L')
If ITYPE = 2 or 3, the problem is A*B*x = lambda*x or
B*A*x = lambda*x, and A is overwritten by U*A*U` or L'*A*L.

B must have been previously factorized as U'*U or L*L' by SPOTRF.

ARGUMENTS

ITYPE (input)
= 1: compute inv(U')*A*inv(U) or inv(L)*A*inv(L');
= 2 or 3: compute U*A*U' or L'*A*L.

UPLO (input)
Specifies whether the upper or lower triangular part of the symmetric matrix A is stored, and how B has been factorized. = 'U': Upper triangular
= 'L': Lower triangular

N (input) The order of the matrices A and B. N >= 0.

A (input/output)
On entry, the symmetric matrix A. If UPLO = 'U', the leading n by n upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading n by n lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, if INFO = 0, the transformed matrix, stored in the same format as A.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input) The triangular factor from the Cholesky factorization of B, as returned by SPOTRF.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

```
dsygst - reduce a real symmetric-definite generalized eigen-
problem to standard form
```

SYNOPSIS

```
SUBROUTINE DSYGST(ITYPE, UPLO, N, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER ITYPE, N, LDA, LDB, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*)

SUBROUTINE DSYGST_64(ITYPE, UPLO, N, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 ITYPE, N, LDA, LDB, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE SYGST(ITYPE, UPLO, N, A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: ITYPE, N, LDA, LDB, INFO
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE SYGST_64(ITYPE, UPLO, N, A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: ITYPE, N, LDA, LDB, INFO
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dsygst(int itype, char uplo, int n, double *a, int lda,
          double *b, int ldb, int *info);

void dsygst_64(long itype, char uplo,  long  n,  double  *a,
          long lda, double *b, long ldb, long *info);
```

PURPOSE

dsygst reduces a real symmetric-definite generalized eigen-
problem to standard form.

If ITYPE = 1, the problem is A*x = lambda*B*x,
and   A   is   overwritten   by   inv(U**T)*A*inv(U)   or
inv(L)*A*inv(L**T)
If ITYPE = 2 or 3, the problem is A*B*x = lambda*x or
B*A*x = lambda*x, and A is overwritten by U*A*U**T or
L**T*A*L.

B must have been previously factorized as U**T*U or L*L**T
by SPOTRF.

ARGUMENTS

ITYPE (input)
= 1: compute inv(U**T)*A*inv(U) or inv(L)*A*inv(L**T);
= 2 or 3: compute U*A*U**T or L**T*A*L.

UPLO (input)
= 'U': Upper triangle of A is stored and B is factored as U**T*U; = 'L': Lower triangle of A is stored and B is factored as L*L**T.

N (input) The order of the matrices A and B. N >= 0.

A (input/output)
On entry, the symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, if INFO = 0, the transformed matrix,

stored in the same format as A.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input) The triangular factor from the Cholesky factorization of B, as returned by SPOTRF.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dsygv - compute all the  eigenvalues,  and  optionally,  the
eigenvectors of a real generalized symmetric-definite eigen-
problem, of the form A*x=(lambda)*B*x,  A*Bx=(lambda)*x,  or
B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE DSYGV(ITYPE, JOBZ, UPLO, N, A, LDA, B, LDB, W, WORK,
      LDWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER ITYPE, N, LDA, LDB, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*), W(*), WORK(*)

SUBROUTINE DSYGV_64(ITYPE, JOBZ, UPLO, N, A, LDA, B, LDB, W, WORK,
      LDWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 ITYPE, N, LDA, LDB, LDWORK, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*), W(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYGV(ITYPE, JOBZ, UPLO, N, A, [LDA], B, [LDB], W, [WORK],
       [LDWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: ITYPE, N, LDA, LDB, LDWORK, INFO
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE SYGV_64(ITYPE, JOBZ, UPLO, N, A, [LDA], B, [LDB], W, [WORK],
       [LDWORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: ITYPE, N, LDA, LDB, LDWORK, INFO
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dsygv(int itype, char jobz, char uplo,  int  n,  double
          *a,  int  lda,  double *b, int ldb, double *w, int
          *info);

void dsygv_64(long itype, char jobz, char uplo, long n, dou-
          ble  *a, long lda, double *b, long ldb, double *w,
          long *info);
```

PURPOSE

dsygv computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be symmetric and B is also
positive definite.

ARGUMENTS

```
ITYPE (input)
          Specifies the problem type to be solved:
          = 1:  A*x = (lambda)*B*x
          = 2:  A*B*x = (lambda)*x
          = 3:  B*A*x = (lambda)*x

JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.

A (input/output)
          On entry, the symmetric matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
```

tains the upper triangular part of the matrix A. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A.

On exit, if JOBZ = 'V', then if INFO = 0, A contains the matrix Z of eigenvectors. The eigenvectors are normalized as follows: if ITYPE = 1 or 2, Z**T*B*Z = I; if ITYPE = 3, Z**T*inv(B)*Z = I. If JOBZ = 'N', then on exit the upper triangle (if UPLO='U') or the lower triangle (if UPLO='L') of A, including the diagonal, is destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the symmetric positive definite matrix B. If UPLO = 'U', the leading N-by-N upper triangular part of B contains the upper triangular part of the matrix B. If UPLO = 'L', the leading N-by-N lower triangular part of B contains the lower triangular part of the matrix B.

On exit, if INFO <= N, the part of B containing the matrix is overwritten by the triangular factor U or L from the Cholesky factorization B = U**T*U or B = L*L**T.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

W (output)
If INFO = 0, the eigenvalues in ascending order.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of the array WORK. LDWORK >= max(1,3*N-1). For optimal efficiency, LDWORK >= (NB+2)*N, where NB is the blocksize for SSYTRD returned by ILAENV.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first

entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: SPOTRF or SSYEV returned an error code:
<= N: if INFO = i, SSYEV failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero; > N: if INFO = N + i, for 1 <= i <= N, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

Contents

NAME

```
dsygvd - compute all the eigenvalues,  and  optionally,  the
eigenvectors of a real generalized symmetric-definite eigen-
problem, of the form A*x=(lambda)*B*x,  A*Bx=(lambda)*x,  or
B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE DSYGVD(ITYPE, JOBZ, UPLO, N, A, LDA, B, LDB, W, WORK,
      LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER ITYPE, N, LDA, LDB, LWORK, LIWORK, INFO
INTEGER IWORK(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*), W(*), WORK(*)

SUBROUTINE DSYGVD_64(ITYPE, JOBZ, UPLO, N, A, LDA, B, LDB, W, WORK,
      LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 ITYPE, N, LDA, LDB, LWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*), W(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYGVD(ITYPE, JOBZ, UPLO, [N], A, [LDA], B, [LDB], W, [WORK],
       [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: ITYPE, N, LDA, LDB, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: W, WORK
```

```
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE SYGVD_64(ITYPE, JOBZ, UPLO, [N], A, [LDA], B, [LDB], W,
       [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: ITYPE, N, LDA, LDB, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dsygvd(int itype, char jobz, char uplo, int  n,  double
          *a,  int  lda,  double *b, int ldb, double *w, int
          *info);

void dsygvd_64(long itype, char jobz,  char  uplo,  long  n,
          double  *a,  long lda, double *b, long ldb, double
          *w, long *info);
```

PURPOSE

dsygvd computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be symmetric and B is also positive definite. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

ITYPE (input)

Specifies the problem type to be solved:
= 1: A*x = (lambda)*B*x
= 2: A*B*x = (lambda)*x
= 3: B*A*x = (lambda)*x

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.

A (input/output)
          On entry, the symmetric matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of A contains the lower  triangular  part  of
          the matrix A.

          On exit, if JOBZ = 'V', then if INFO = 0,  A  con-
          tains the matrix Z of eigenvectors.  The eigenvec-
          tors are normalized as follows:  if ITYPE =  1  or
          2,  Z**T*B*Z = I; if ITYPE = 3, Z**T*inv(B)*Z = I.
          If JOBZ = 'N', then on exit the upper triangle (if
          UPLO='U')  or  the lower triangle (if UPLO='L') of
          A, including the diagonal, is destroyed.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

B (input/output)
          On entry, the symmetric matrix B.  If UPLO =  'U',
          the leading N-by-N upper triangular part of B con-
          tains the upper triangular part of the  matrix  B.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of B contains the lower  triangular  part  of
          the matrix B.

          On exit, if INFO <= N, the part  of  B  containing
          the matrix is overwritten by the triangular factor
          U or L from the Cholesky factorization B =  U**T*U
          or B = L*L**T.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

W (output)
          If INFO = 0, the eigenvalues in ascending order.
```

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If N <= 1, LWORK >= 1. If JOBZ = 'N' and N > 1, LWORK >= 2*N+1. If JOBZ = 'V' and N > 1, LWORK >= 1 + 6*N + 2*N**2.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of the array IWORK. If N <= 1, LIWORK >= 1. If JOBZ = 'N' and N > 1, LIWORK >= 1. If JOBZ = 'V' and N > 1, LIWORK >= 3 + 5*N.

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: SPOTRF or SSYEVD returned an error code:
<= N: if INFO = i, SSYEVD failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero; > N: if INFO = N + i, for 1 <= i <= N, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
Mark Fahey, Department of Mathematics, Univ. of Kentucky, USA

Contents

NAME

dsygvx - compute selected eigenvalues, and optionally, eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

SYNOPSIS

```
SUBROUTINE DSYGVX(ITYPE, JOBZ, RANGE, UPLO, N, A, LDA, B, LDB, VL,
      VU, IL, IU, ABSTOL, M, W, Z, LDZ, WORK, LWORK, IWORK, IFAIL,
      INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER ITYPE, N, LDA, LDB, IL, IU, M, LDZ, LWORK, INFO
INTEGER IWORK(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION A(LDA,*), B(LDB,*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE DSYGVX_64(ITYPE, JOBZ, RANGE, UPLO, N, A, LDA, B, LDB, VL,
      VU, IL, IU, ABSTOL, M, W, Z, LDZ, WORK, LWORK, IWORK, IFAIL,
      INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER*8 ITYPE, N, LDA, LDB, IL, IU, M, LDZ, LWORK, INFO
INTEGER*8 IWORK(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION A(LDA,*), B(LDB,*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYGVX(ITYPE, JOBZ, RANGE, UPLO, [N], A, [LDA], B, [LDB],
       VL, VU, IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK], [LWORK], [IWORK],
       IFAIL, [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER :: ITYPE, N, LDA, LDB, IL, IU, M, LDZ, LWORK, INFO
INTEGER, DIMENSION(:) :: IWORK, IFAIL
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: A, B, Z

SUBROUTINE SYGVX_64(ITYPE, JOBZ, RANGE, UPLO, [N], A, [LDA], B, [LDB],
       VL, VU, IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK], [LWORK], [IWORK],
       IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER(8) :: ITYPE, N, LDA, LDB, IL,  IU,  M,  LDZ,  LWORK,
INFO
INTEGER(8), DIMENSION(:) :: IWORK, IFAIL
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: W, WORK
REAL(8), DIMENSION(:,:) :: A, B, Z
```

C INTERFACE

```
#include <sunperf.h>

void dsygvx(int itype, char jobz, char range, char uplo, int
          n,  double *a, int lda, double *b, int ldb, double
          vl, double vu, int il, int iu, double abstol,  int
          *m, double *w, double *z, int ldz, int *ifail, int
          *info);

void dsygvx_64(long itype, char jobz, char range, char uplo,
          long  n, double *a, long lda, double *b, long ldb,
          double vl, double vu, long  il,  long  iu,  double
          abstol,  long  *m, double *w, double *z, long ldz,
          long *ifail, long *info);
```

PURPOSE

dsygvx computes selected eigenvalues, and optionally, eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be symmetric and B is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

```
ITYPE (input)
          Specifies the problem type to be solved:
          = 1:  A*x = (lambda)*B*x
          = 2:  A*B*x = (lambda)*x
          = 3:  B*A*x = (lambda)*x

JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

RANGE (input)
          = 'A': all eigenvalues will be found.
          = 'V': all eigenvalues in the  half-open  interval
          (VL,VU]  will  be found.  = 'I': the IL-th through
          IU-th eigenvalues will be found.

UPLO (input)
          = 'U':  Upper triangle of A and B are stored;
          = 'L':  Lower triangle of A and B are stored.

N (input) The order of the matrix pencil (A,B).  N >= 0.

A (input/output)
          On entry, the symmetric matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of A contains the lower  triangular  part  of
          the matrix A.

          On exit, the lower triangle (if UPLO='L')  or  the
          upper  triangle  (if UPLO='U') of A, including the
          diagonal, is destroyed.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

B (input/output)
          On entry, the symmetric matrix B.  If UPLO =  'U',
          the leading N-by-N upper triangular part of B con-
          tains the upper triangular part of the  matrix  B.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of B contains the lower  triangular  part  of
          the matrix B.

          On exit, if INFO <= N, the part  of  B  containing
          the matrix is overwritten by the triangular factor
          U or L from the Cholesky factorization B =  U**T*U
          or B = L*L**T.
```

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

VL (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)
See the description of VL.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
See the description of IL.

ABSTOL (input)
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABSTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABSTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing A to tri- diagonal form.

Eigenvalues will be computed most accurately when ABSTOL is set to twice the underflow threshold 2*DLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABSTOL to 2*SLAMCH('S').

M (output)
The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)
On normal exit, the first M elements contain the selected eigenvalues in ascending order.

```
Z (input) If JOBZ = 'N', then Z is not referenced.  If  JOBZ
          =  'V', then if INFO = 0, the first M columns of Z
          contain the orthonormal eigenvectors of the matrix
          A  corresponding to the selected eigenvalues, with
          the i-th column of Z holding the eigenvector asso-
          ciated with W(i).  The eigenvectors are normalized
          as follows:  if ITYPE = 1 or 2, Z**T*B*Z =  I;  if
          ITYPE = 3, Z**T*inv(B)*Z = I.
          If an eigenvector fails  to  converge,  then  that
          column  of  Z contains the latest approximation to
          the eigenvector, and the index of the  eigenvector
          is  returned in IFAIL.  Note: the user must ensure
          that at least max(1,M) columns are supplied in the
          array  Z;  if RANGE = 'V', the exact value of M is
          not known in advance and an upper  bound  must  be
          used.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The  length  of  the   array   WORK.    LWORK   >=
          max(1,8*N).   For  optimal  efficiency,  LWORK  >=
          (NB+3)*N, where NB is  the  blocksize  for  SSYTRD
          returned by ILAENV.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

IWORK (workspace)
          dimension(5*N)

IFAIL (output)
          If JOBZ = 'V', then if INFO = 0, the first M  ele-
          ments  of IFAIL are zero.  If INFO > 0, then IFAIL
          contains the  indices  of  the  eigenvectors  that
          failed  to converge.  If JOBZ = 'N', then IFAIL is
          not referenced.

INFO (output)
          = 0:  successful exit
```

< 0: if INFO = -i, the i-th argument had an illegal value
> 0: SPOTRF or SSYEVX returned an error code:
<= N: if INFO = i, SSYEVX failed to converge; i eigenvectors failed to converge. Their indices are stored in array IFAIL. > N: if INFO = N + i, for 1 <= i <= N, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
Mark Fahey, Department of Mathematics, Univ. of Kentucky, USA

Contents

NAME

dsymm - perform one of the matrix-matrix operations C := alpha*A*B + beta*C or C := alpha*B*A + beta*C

SYNOPSIS

```
SUBROUTINE DSYMM(SIDE, UPLO, M, N, ALPHA, A, LDA, B, LDB, BETA, C,
      LDC)

CHARACTER * 1 SIDE, UPLO
INTEGER M, N, LDA, LDB, LDC
DOUBLE PRECISION ALPHA, BETA
DOUBLE PRECISION A(LDA,*), B(LDB,*), C(LDC,*)

SUBROUTINE DSYMM_64(SIDE, UPLO, M, N, ALPHA, A, LDA, B, LDB, BETA, C,
      LDC)

CHARACTER * 1 SIDE, UPLO
INTEGER*8 M, N, LDA, LDB, LDC
DOUBLE PRECISION ALPHA, BETA
DOUBLE PRECISION A(LDA,*), B(LDB,*), C(LDC,*)
```

F95 INTERFACE

```
SUBROUTINE SYMM(SIDE, UPLO, [M], [N], ALPHA, A, [LDA], B, [LDB],
       BETA, C, [LDC])

CHARACTER(LEN=1) :: SIDE, UPLO
INTEGER :: M, N, LDA, LDB, LDC
REAL(8) :: ALPHA, BETA
REAL(8), DIMENSION(:,:) :: A, B, C

SUBROUTINE SYMM_64(SIDE, UPLO, [M], [N], ALPHA, A, [LDA], B, [LDB],
       BETA, C, [LDC])
```

```
CHARACTER(LEN=1) :: SIDE, UPLO
INTEGER(8) :: M, N, LDA, LDB, LDC
REAL(8) :: ALPHA, BETA
REAL(8), DIMENSION(:,:) :: A, B, C
```

C INTERFACE

```
#include <sunperf.h>

void dsymm(char side, char uplo, int m, int n, double alpha,
          double  *a,  int  lda,  double *b, int ldb, double
          beta, double *c, int ldc);

void dsymm_64(char side, char uplo, long m, long  n,  double
          alpha,  double  *a, long lda, double *b, long ldb,
          double beta, double *c, long ldc);
```

PURPOSE

dsymm performs one of the matrix-matrix operations C := alpha*A*B + beta*C or C := alpha*B*A + beta*C where alpha and beta are scalars, A is a symmetric matrix and B and C are m by n matrices.

ARGUMENTS

SIDE (input)
On entry, SIDE specifies whether the symmetric matrix A appears on the left or right in the operation as follows:

SIDE = 'L' or 'l'   C := alpha*A*B + beta*C,

SIDE = 'R' or 'r'   C := alpha*B*A + beta*C,

Unchanged on exit.

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the symmetric matrix A is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of the symmetric matrix is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of the symmetric matrix is to be referenced.

Unchanged on exit.

M (input)
On entry, M specifies the number of rows of the matrix C. M >= 0. Unchanged on exit.

N (input)
On entry, N specifies the number of columns of the matrix C. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
DOUBLE PRECISION array of DIMENSION (LDA, ka), where ka is m when SIDE = 'L' or 'l' and is n otherwise.

Before entry with SIDE = 'L' or 'l', the m by m part of the array A must contain the symmetric matrix, such that when UPLO = 'U' or 'u', the leading m by m upper triangular part of the array A must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of A is not referenced, and when UPLO = 'L' or 'l', the leading m by m lower triangular part of the array A must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of A is not referenced.

Before entry with SIDE = 'R' or 'r', the n by n part of the array A must contain the symmetric matrix, such that when UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of A is not referenced, and when UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of A is not referenced.

Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When

SIDE = 'L' or 'l' then LDA >= max(1, m), otherwise LDA >= max(1, n). Unchanged on exit.

B (input)
DOUBLE PRECISION array of DIMENSION (LDB, n). Before entry, the leading m by n part of the array B must contain the matrix B. Unchanged on exit.

LDB (input)
On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. LDB >= max(1, m). Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. When BETA is supplied as zero then C need not be set on input. Unchanged on exit.

C (input/output)
DOUBLE PRECISION array of DIMENSION (LDC, n). Before entry, the leading m by n part of the array C must contain the matrix C, except when beta is zero, in which case C need not be set on entry. On exit, the array C is overwritten by the m by n updated matrix.

LDC (input)
On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC >= max(1, m). Unchanged on exit.

Contents

NAME

```
dsymv - perform the matrix-vector operation   y := alpha*A*x
+ beta*y
```

SYNOPSIS

```
SUBROUTINE DSYMV(UPLO, N, ALPHA, A, LDA, X, INCX, BETA, Y, INCY)

CHARACTER * 1 UPLO
INTEGER N, LDA, INCX, INCY
DOUBLE PRECISION ALPHA, BETA
DOUBLE PRECISION A(LDA,*), X(*), Y(*)

SUBROUTINE DSYMV_64(UPLO, N, ALPHA, A, LDA, X, INCX, BETA, Y, INCY)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, INCX, INCY
DOUBLE PRECISION ALPHA, BETA
DOUBLE PRECISION A(LDA,*), X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE SYMV(UPLO, [N], ALPHA, A, [LDA], X, [INCX], BETA, Y, [INCY])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, INCX, INCY
REAL(8) :: ALPHA, BETA
REAL(8), DIMENSION(:) :: X, Y
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE SYMV_64(UPLO, [N], ALPHA, A, [LDA], X, [INCX], BETA, Y,
       [INCY])

CHARACTER(LEN=1) :: UPLO
```

```
INTEGER(8) :: N, LDA, INCX, INCY
REAL(8) :: ALPHA, BETA
REAL(8), DIMENSION(:) :: X, Y
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dsymv(char uplo, int n, double alpha,  double  *a,  int
          lda,  double *x, int incx, double beta, double *y,
          int incy);

void dsymv_64(char uplo, long n, double  alpha,  double  *a,
          long  lda, double *x, long incx, double beta, dou-
          ble *y, long incy);
```

PURPOSE

dsymv performs the matrix-vector operation y := alpha*A*x + beta*y, where alpha and beta are scalars, x and y are n element vectors and A is an n by n symmetric matrix.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array A is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of A is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of A is to be referenced.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
Before entry with UPLO = 'U' or 'u', the leading

n by n upper triangular part of the array A must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of A is not referenced. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of A is not referenced. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

X (input)

(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)

On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

BETA (input)

On entry, BETA specifies the scalar beta. When BETA is supplied as zero then Y need not be set on input. Unchanged on exit.

Y (input/output)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

dsyr - perform the symmetric rank 1 operation A := alpha*x*x' + A

SYNOPSIS

```
SUBROUTINE DSYR(UPLO, N, ALPHA, X, INCX, A, LDA)

CHARACTER * 1 UPLO
INTEGER N, INCX, LDA
DOUBLE PRECISION ALPHA
DOUBLE PRECISION X(*), A(LDA,*)

SUBROUTINE DSYR_64(UPLO, N, ALPHA, X, INCX, A, LDA)

CHARACTER * 1 UPLO
INTEGER*8 N, INCX, LDA
DOUBLE PRECISION ALPHA
DOUBLE PRECISION X(*), A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE SYR(UPLO, [N], ALPHA, X, [INCX], A, [LDA])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INCX, LDA
REAL(8) :: ALPHA
REAL(8), DIMENSION(:) :: X
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE SYR_64(UPLO, [N], ALPHA, X, [INCX], A, [LDA])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INCX, LDA
```

```
REAL(8) :: ALPHA
REAL(8), DIMENSION(:) :: X
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dsyr(char uplo, int n, double  alpha,  double  *x,  int
          incx, double *a, int lda);

void dsyr_64(char uplo, long n,  double  alpha,  double  *x,
          long incx, double *a, long lda);
```

PURPOSE

dsyr performs the symmetric rank 1 operation A := alpha*x*x' + A, where alpha is a real scalar, x is an n element vector and A is an n by n symmetric matrix.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array A is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of A is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of A is to be referenced.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)

On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

A (input/output)

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of A is not referenced. On exit, the upper triangular part of the array A is overwritten by the upper triangular part of the updated matrix. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of A is not referenced. On exit, the lower triangular part of the array A is overwritten by the lower triangular part of the updated matrix.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

Contents

NAME

dsyr2 - perform the symmetric rank 2 operation A := alpha*x*y' + alpha*y*x' + A

SYNOPSIS

```
SUBROUTINE DSYR2(UPLO, N, ALPHA, X, INCX, Y, INCY, A, LDA)

CHARACTER * 1 UPLO
INTEGER N, INCX, INCY, LDA
DOUBLE PRECISION ALPHA
DOUBLE PRECISION X(*), Y(*), A(LDA,*)

SUBROUTINE DSYR2_64(UPLO, N, ALPHA, X, INCX, Y, INCY, A, LDA)

CHARACTER * 1 UPLO
INTEGER*8 N, INCX, INCY, LDA
DOUBLE PRECISION ALPHA
DOUBLE PRECISION X(*), Y(*), A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE SYR2(UPLO, [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INCX, INCY, LDA
REAL(8) :: ALPHA
REAL(8), DIMENSION(:) :: X, Y
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE SYR2_64(UPLO, [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INCX, INCY, LDA
```

```
REAL(8) :: ALPHA
REAL(8), DIMENSION(:) :: X, Y
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dsyr2(char uplo, int n, double alpha,  double  *x,  int
          incx, double *y, int incy, double *a, int lda);

void dsyr2_64(char uplo, long n, double  alpha,  double  *x,
          long  incx,  double *y, long incy, double *a, long
          lda);
```

PURPOSE

dsyr2 performs the symmetric rank 2 operation A := alpha*x*y' + alpha*y*x' + A, where alpha is a scalar, x and y are n element vectors and A is an n by n symmetric matrix.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array A is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of A is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of A is to be referenced.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

Y (input)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. Unchanged on exit.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

A (input/output)
Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of A is not referenced. On exit, the upper triangular part of the array A is overwritten by the upper triangular part of the updated matrix. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of A is not referenced. On exit, the lower triangular part of the array A is overwritten by the lower triangular part of the updated matrix.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

Contents

NAME

```
dsyr2k - perform one of the symmetric rank 2k operations   C
:=  alpha*A*B'  +  alpha*B*A'  + beta*C or C := alpha*A'*B +
alpha*B'*A + beta*C
```

SYNOPSIS

```
SUBROUTINE DSYR2K(UPLO, TRANSA, N, K, ALPHA, A, LDA, B, LDB, BETA, C,
      LDC)

CHARACTER * 1 UPLO, TRANSA
INTEGER N, K, LDA, LDB, LDC
DOUBLE PRECISION ALPHA, BETA
DOUBLE PRECISION A(LDA,*), B(LDB,*), C(LDC,*)

SUBROUTINE DSYR2K_64(UPLO, TRANSA, N, K, ALPHA, A, LDA, B, LDB, BETA,
      C, LDC)

CHARACTER * 1 UPLO, TRANSA
INTEGER*8 N, K, LDA, LDB, LDC
DOUBLE PRECISION ALPHA, BETA
DOUBLE PRECISION A(LDA,*), B(LDB,*), C(LDC,*)
```

F95 INTERFACE

```
SUBROUTINE SYR2K(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], B, [LDB],
       BETA, C, [LDC])

CHARACTER(LEN=1) :: UPLO, TRANSA
INTEGER :: N, K, LDA, LDB, LDC
REAL(8) :: ALPHA, BETA
REAL(8), DIMENSION(:,:) :: A, B, C

SUBROUTINE SYR2K_64(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], B,
```

```
          [LDB], BETA, C, [LDC])

   CHARACTER(LEN=1) :: UPLO, TRANSA
   INTEGER(8) :: N, K, LDA, LDB, LDC
   REAL(8) :: ALPHA, BETA
   REAL(8), DIMENSION(:,:) :: A, B, C
```

C INTERFACE

```
#include <sunperf.h>

void dsyr2k(char uplo, char transa, int  n,  int  k,  double
          alpha,  double  *a,  int  lda, double *b, int ldb,
          double beta, double *c, int ldc);
void dsyr2k_64(char uplo, char transa, long n, long k,  dou-
          ble  alpha,  double  *a, long lda, double *b, long
          ldb, double beta, double *c, long ldc);
```

PURPOSE

dsyr2k performs one of the symmetric rank 2k operations C := alpha*A*B' + alpha*B*A' + beta*C or C := alpha*A'*B + alpha*B'*A + beta*C where alpha and beta are scalars, C is an n by n symmetric matrix and A and B are n by k matrices in the first case and k by n matrices in the second case.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array C is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of C is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of C is to be referenced.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   C := alpha*A*B' + alpha*B*A' + beta*C.

TRANSA = 'T' or 't'   C := alpha*A'*B + alpha*B'*A + beta*C.

TRANSA = 'C' or 'c'   C := alpha*A'*B + alpha*B'*A + beta*C.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input)
On entry, N specifies the order of the matrix C. N must be at least zero. Unchanged on exit.

K (input)
On entry with TRANSA = 'N' or 'n', K specifies the number of columns of the matrices A and B, and on entry with TRANSA = 'T' or 't' or 'C' or 'c', K specifies the number of rows of the matrices A and B. K must be at least zero. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
DOUBLE PRECISION array of DIMENSION (LDA, ka), where ka is k when TRANSA = 'N' or 'n', and is n otherwise. Before entry with TRANSA = 'N' or 'n', the leading n by k part of the array A must contain the matrix A, otherwise the leading k by n part of the array A must contain the matrix A. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDA must be at least max(1, n), otherwise LDA must be at least max(1, k). Unchanged on exit.

B (input)
DOUBLE PRECISION array of DIMENSION (LDB, kb), where kb is k when TRANSA = 'N' or 'n', and is n otherwise. Before entry with TRANSA = 'N' or 'n', the leading n by k part of the array B must contain the matrix B, otherwise the leading k by n part of the array B must contain the matrix B. Unchanged on exit.

LDB (input)

On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDB must be at least max(1, n), otherwise LDB must be at least max(1, k). Unchanged on exit.

BETA (input)

On entry, BETA specifies the scalar beta. Unchanged on exit.

C (input/output)

DOUBLE PRECISION array of DIMENSION (LDC, n).

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array C must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of C is not referenced. On exit, the upper triangular part of the array C is overwritten by the upper triangular part of the updated matrix.

Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array C must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of C is not referenced. On exit, the lower triangular part of the array C is overwritten by the lower triangular part of the updated matrix.

LDC (input)

On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC must be at least max(1, n). Unchanged on exit.

Contents

NAME

```
dsyrfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is symmetric indefin-
ite, and provides error bounds and backward error  estimates
for the solution
```

SYNOPSIS

```
SUBROUTINE DSYRFS(UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*), WORK2(*)
DOUBLE PRECISION A(LDA,*), AF(LDAF,*),  B(LDB,*),  X(LDX,*),
FERR(*), BERR(*), WORK(*)

SUBROUTINE DSYRFS_64(UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
DOUBLE PRECISION A(LDA,*), AF(LDAF,*),  B(LDB,*),  X(LDX,*),
FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYRFS(UPLO, N, NRHS, A, [LDA], AF, [LDAF], IPIVOT, B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
```

```
REAL(8), DIMENSION(:) :: FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: A, AF, B, X

SUBROUTINE SYRFS_64(UPLO, N, NRHS, A, [LDA], AF, [LDAF], IPIVOT, B,
       [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL(8), DIMENSION(:) :: FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
#include <sunperf.h>
void dsyrfs(char uplo, int n, int nrhs, double *a, int  lda,
          double  *af, int ldaf, int *ipivot, double *b, int
          ldb, double *x,  int  ldx,  double  *ferr,  double
          *berr, int *info);

void dsyrfs_64(char uplo, long n, long nrhs, double *a, long
          lda,  double  *af, long ldaf, long *ipivot, double
          *b, long ldb, double *x, long ldx,  double  *ferr,
          double *berr, long *info);
```

PURPOSE

dsyrfs improves the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not refer-

enced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input)
The factored form of the matrix A. AF contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by SSYTRF.

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by SSYTRF.

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by SSYTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)

```
            The componentwise relative backward error of  each
            solution  vector X(j) (i.e., the smallest relative
            change in any element of A or B that makes X(j) an
            exact solution).

WORK (workspace)
            dimension(3*N)

WORK2 (workspace)
            dimension(N)

INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value
```

Contents

NAME

```
dsyrk - perform one of the symmetric rank k  operations    C
:= alpha*A*A' + beta*C or C := alpha*A'*A + beta*C
```

SYNOPSIS

```
SUBROUTINE DSYRK(UPLO, TRANSA, N, K, ALPHA, A, LDA, BETA, C, LDC)

CHARACTER * 1 UPLO, TRANSA
INTEGER N, K, LDA, LDC
DOUBLE PRECISION ALPHA, BETA
DOUBLE PRECISION A(LDA,*), C(LDC,*)

SUBROUTINE DSYRK_64(UPLO, TRANSA, N, K, ALPHA, A, LDA, BETA, C, LDC)

CHARACTER * 1 UPLO, TRANSA
INTEGER*8 N, K, LDA, LDC
DOUBLE PRECISION ALPHA, BETA
DOUBLE PRECISION A(LDA,*), C(LDC,*)
```

F95 INTERFACE

```
SUBROUTINE SYRK(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], BETA, C,
       [LDC])

CHARACTER(LEN=1) :: UPLO, TRANSA
INTEGER :: N, K, LDA, LDC
REAL(8) :: ALPHA, BETA
REAL(8), DIMENSION(:,:) :: A, C

SUBROUTINE SYRK_64(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], BETA,
       C, [LDC])

CHARACTER(LEN=1) :: UPLO, TRANSA
```

```
INTEGER(8) :: N, K, LDA, LDC
REAL(8) :: ALPHA, BETA
REAL(8), DIMENSION(:,:) :: A, C
```

C INTERFACE

```
#include <sunperf.h>

void dsyrk(char uplo, char transa,  int  n,  int  k,  double
          alpha, double *a, int lda, double beta, double *c,
          int ldc);

void dsyrk_64(char uplo, char transa, long n, long k, double
          alpha,  double  *a,  long lda, double beta, double
          *c, long ldc);
```

PURPOSE

dsyrk performs one of the symmetric rank k operations C := alpha*A*A' + beta*C or C := alpha*A'*A + beta*C where alpha and beta are scalars, C is an n by n symmetric matrix and A is an n by k matrix in the first case and a k by n matrix in the second case.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array C is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of C is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of C is to be referenced.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   C := alpha*A*A' + beta*C.

TRANSA = 'T' or 't'   C := alpha*A'*A + beta*C.

TRANSA = 'C' or 'c'   C := alpha*A'*A + beta*C.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input)

On entry, N specifies the order of the matrix C. N must be at least zero. Unchanged on exit.

K (input)

On entry with TRANSA = 'N' or 'n', K specifies the number of columns of the matrix A, and on entry with TRANSA = 'T' or 't' or 'C' or 'c', K specifies the number of rows of the matrix A. K must be at least zero. Unchanged on exit.

ALPHA (input)

On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)

DOUBLE PRECISION array of DIMENSION (LDA, ka), where ka is k when TRANSA = 'N' or 'n', and is n otherwise. Before entry with TRANSA = 'N' or 'n', the leading n by k part of the array A must contain the matrix A, otherwise the leading k by n part of the array A must contain the matrix A. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDA must be at least max(1, n), otherwise LDA must be at least max(1, k). Unchanged on exit.

BETA (input)

On entry, BETA specifies the scalar beta. Unchanged on exit.

C (input/output)

DOUBLE PRECISION array of DIMENSION (LDC, n).

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array C must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of C is not referenced. On exit, the upper triangular part of the array C is overwritten by the upper triangular part of the updated

matrix.

Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array C must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of C is not referenced. On exit, the lower triangular part of the array C is overwritten by the lower triangular part of the updated matrix.

LDC (input)

On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC must be at least max(1, n). Unchanged on exit.

Contents

NAME

```
dsysv - compute the solution to  a  real  system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE DSYSV(UPLO, N, NRHS, A, LDA, IPIV, B, LDB, WORK, LWORK,
      INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDA, LDB, LWORK, INFO
INTEGER IPIV(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*), WORK(*)

SUBROUTINE DSYSV_64(UPLO, N, NRHS, A, LDA, IPIV, B, LDB, WORK, LWORK,
      INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDA, LDB, LWORK, INFO
INTEGER*8 IPIV(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYSV(UPLO, [N], [NRHS], A, [LDA], IPIV, B, [LDB], [WORK],
       [LWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDA, LDB, LWORK, INFO
INTEGER, DIMENSION(:) :: IPIV
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE SYSV_64(UPLO, [N], [NRHS], A, [LDA], IPIV, B, [LDB], [WORK],
```

```
          [LWORK], [INFO])

    CHARACTER(LEN=1) :: UPLO
    INTEGER(8) :: N, NRHS, LDA, LDB, LWORK, INFO
    INTEGER(8), DIMENSION(:) :: IPIV
    REAL(8), DIMENSION(:) :: WORK
    REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
    #include <sunperf.h>

    void dsysv(char uplo, int n, int nrhs, double *a,  int  lda,
              int *ipiv, double *b, int ldb, int *info);
    void dsysv_64(char uplo, long n, long nrhs, double *a,  long
              lda, long *ipiv, double *b, long ldb, long *info);
```

PURPOSE

```
    dsysv computes the solution to a real system of linear equa-
    tions
       A * X = B, where A is an N-by-N symmetric  matrix  and  X
    and B are N-by-NRHS matrices.

    The diagonal pivoting method is used to factor A as
       A = U * D * U**T,  if UPLO = 'U', or
       A = L * D * L**T,  if UPLO = 'L',
    where U (or L) is a product of permutation  and  unit  upper
    (lower)  triangular  matrices,  and D is symmetric and block
    diagonal with 1-by-1 and 2-by-2 diagonal blocks.   The  fac-
    tored  form  of  A is then used to solve the system of equa-
    tions A * X = B.
```

ARGUMENTS

```
    UPLO (input)
              = 'U':  Upper triangle of A is stored;
              = 'L':  Lower triangle of A is stored.

    N (input) The number of linear equations, i.e., the order of
              the matrix A.  N >= 0.

    NRHS (input)
              The number of right hand sides, i.e.,  the  number
              of columns of the matrix B.  NRHS >= 0.

    A (input/output)
              On entry, the symmetric matrix A.  If UPLO =  'U',
```

the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, if INFO = 0, the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by SSYTRF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIV (output)
Details of the interchanges and the block structure of D, as determined by SSYTRF. If IPIV(k) > 0, then rows and columns k and IPIV(k) were interchanged, and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIV(k) = IPIV(k-1) < 0, then rows and columns k-1 and -IPIV(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIV(k) = IPIV(k+1) < 0, then rows and columns k+1 and -IPIV(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The length of WORK. LWORK >= 1, and for best performance LWORK >= N*NB, where NB is the optimal blocksize for SSYTRF.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, so the solution could not be computed.

Contents

NAME

```
dsysvx - use the diagonal pivoting factorization to  compute
the solution to a real system of linear equations A * X = B,
```

SYNOPSIS

```
SUBROUTINE DSYSVX(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B,
      LDB, X, LDX, RCOND, FERR, BERR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER IPIVOT(*), WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(LDA,*), AF(LDAF,*),  B(LDB,*),  X(LDX,*),
FERR(*), BERR(*), WORK(*)

SUBROUTINE DSYSVX_64(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT,
      B, LDB, X, LDX, RCOND, FERR, BERR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(LDA,*), AF(LDAF,*),  B(LDB,*),  X(LDX,*),
FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYSVX(FACT, UPLO, N, NRHS, A, [LDA], AF, [LDAF], IPIVOT,
       B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [LDWORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: FACT, UPLO
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
```

```
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: A, AF, B, X

SUBROUTINE SYSVX_64(FACT, UPLO, N, NRHS, A, [LDA], AF, [LDAF],
       IPIVOT, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [LDWORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
#include <sunperf.h>

void dsysvx(char fact, char uplo, int n,  int  nrhs,  double
          *a,  int  lda,  double *af, int ldaf, int *ipivot,
          double *b, int ldb, double  *x,  int  ldx,  double
          *rcond, double *ferr, double *berr, int *info);

void dsysvx_64(char fact, char uplo, long n, long nrhs, dou-
          ble  *a,  long  lda,  double  *af, long ldaf, long
          *ipivot, double *b, long ldb, double *x, long ldx,
          double  *rcond,  double  *ferr, double *berr, long
          *info);
```

PURPOSE

```
dsysvx uses the diagonal pivoting factorization  to  compute
the solution to a real system of linear equations A * X = B,
where A is an N-by-N symmetric matrix and X and B are  N-by-
NRHS matrices.

Error bounds on the solution and a  condition  estimate  are
also provided.

The following steps are performed:

1. If FACT = 'N', the diagonal pivoting method  is  used  to
factor A.
   The form of the factorization is
      A = U * D * U**T,  if UPLO = 'U', or
      A = L * D * L**T,  if UPLO = 'L',
   where U (or L) is a product of permutation and unit upper
(lower)
   triangular matrices, and D is symmetric and block  diago-
```

nal with
1-by-1 and 2-by-2 diagonal blocks.

2. If some D(i,i)=0, so that D is exactly singular, then the routine returns with INFO = i. Otherwise, the factored form of A is used to estimate the condition number of the matrix A. If the reciprocal of the condition number is less than machine precision, INFO = N+1 is returned as a warning, but the routine still goes on to solve for X and compute error bounds as described below.

3. The system of equations is solved for X using the factored form of A.

4. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of A has been supplied on entry. = 'F': On entry, AF and IPIVOT contain the factored form of A. AF and IPIVOT will not be modified. = 'N': The matrix A will be copied to AF and factored.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not refer-

enced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input/output)
If FACT = 'F', then AF is an input argument and on entry contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by SSYTRF.
If FACT = 'N', then AF is an output argument and on exit returns the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T.

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

IPIVOT (input or output)
If FACT = 'F', then IPIVOT is an input argument and on entry contains details of the interchanges and the block structure of D, as determined by SSYTRF. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

If FACT = 'N', then IPIVOT is an output argument and on exit contains details of the interchanges and the block structure of D, as determined by SSYTRF.

B (input) The N-by-NRHS right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A. If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of WORK. LDWORK >= 3*N, and for best performance LDWORK >= N*NB, where NB is the optimal blocksize for SSYTRF.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: D(i,i) is exactly zero. The factorization has been completed but the factor D is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: D is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
dsytd2 - reduce a real symmetric matrix A to symmetric  tri-
diagonal form T by an orthogonal similarity transformation
```

SYNOPSIS

```
SUBROUTINE DSYTD2(UPLO, N, A, LDA, D, E, TAU, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, INFO
DOUBLE PRECISION A(LDA,*), D(*), E(*), TAU(*)

SUBROUTINE DSYTD2_64(UPLO, N, A, LDA, D, E, TAU, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, INFO
DOUBLE PRECISION A(LDA,*), D(*), E(*), TAU(*)
```

F95 INTERFACE

```
SUBROUTINE SYTD2(UPLO, N, A, [LDA], D, E, TAU, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, INFO
REAL(8), DIMENSION(:) :: D, E, TAU
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE SYTD2_64(UPLO, N, A, [LDA], D, E, TAU, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, INFO
REAL(8), DIMENSION(:) :: D, E, TAU
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dsytd2(char uplo, int n, double *a, int lda, double *d,
          double *e, double *tau, int *info);

void dsytd2_64(char uplo, long n, double *a, long lda,  dou-
          ble *d, double *e, double *tau, long *info);
```

PURPOSE

dsytd2 reduces a real symmetric matrix A to symmetric tridiagonal form T by an orthogonal similarity transformation: Q' * A * Q = T.

ARGUMENTS

UPLO (input)
Specifies whether the upper or lower triangular part of the symmetric matrix A is stored:
= 'U': Upper triangular
= 'L': Lower triangular

N (input) The order of the matrix A. N >= 0.

A (input) On entry, the symmetric matrix A. If UPLO = 'U', the leading n-by-n upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced. On exit, if UPLO = 'U', the diagonal and first superdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements above the first superdiagonal, with the array TAU, represent the orthogonal matrix Q as a product of elementary reflectors; if UPLO = 'L', the diagonal and first subdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements below the first subdiagonal, with the array TAU, represent the orthogonal matrix Q as a product of elementary reflectors. See Further Details.

```
LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

D (output)
          The diagonal elements of the tridiagonal matrix T:
          D(i) = A(i,i).

E (output)
          The  off-diagonal  elements  of  the   tridiagonal
          matrix  T:   E(i) = A(i,i+1) if UPLO = 'U', E(i) =
          A(i+1,i) if UPLO = 'L'.

TAU (output)
          The scalar factors of  the  elementary  reflectors
          (see Further Details).
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value.
```

FURTHER DETAILS

```
If UPLO = 'U', the matrix Q is represented as a  product  of
elementary reflectors

   Q = H(n-1) . . . H(2) H(1).

Each H(i) has the form

   H(i) = I - tau * v * v'

where tau is a real scalar, and v is a real vector with
v(i+1:n) = 0 and v(i) = 1; v(1:i-1) is stored on exit in
A(1:i-1,i+1), and tau in TAU(i).

If UPLO = 'L', the matrix Q is represented as a  product  of
elementary reflectors

   Q = H(1) H(2) . . . H(n-1).

Each H(i) has the form

   H(i) = I - tau * v * v'

where tau is a real scalar, and v is a real vector with
v(1:i) = 0 and v(i+1) = 1; v(i+2:n) is  stored  on  exit  in
A(i+2:n,i), and tau in TAU(i).

The contents of A on exit are illustrated by  the  following
```

```
examples with n = 5:

if UPLO = 'U':                       if UPLO = 'L':

  (  d   e   v2  v3  v4  )                (  d
)
  (     d   e   v3  v4  )               (  e   d
)
  (         d   e   v4  )              (  v1  e   d
)
  (            d   e  )             (  v1  v2  e   d
)
  (                d  )            (  v1  v2  v3  e   d
)

where d and e denote diagonal and off-diagonal  elements  of
T, and vi denotes an element of the vector defining H(i).
```

Contents

NAME

dsytf2 - compute the factorization of a real symmetric matrix A using the Bunch-Kaufman diagonal pivoting method

SYNOPSIS

```
SUBROUTINE DSYTF2(UPLO, N, A, LDA, IPIV, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, INFO
INTEGER IPIV(*)
DOUBLE PRECISION A(LDA,*)

SUBROUTINE DSYTF2_64(UPLO, N, A, LDA, IPIV, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, INFO
INTEGER*8 IPIV(*)
DOUBLE PRECISION A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE SYTF2(UPLO, [N], A, [LDA], IPIV, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIV
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE SYTF2_64(UPLO, [N], A, [LDA], IPIV, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, INFO
```

```
INTEGER(8), DIMENSION(:) :: IPIV
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dsytf2(char uplo, int n, double *a, int lda, int *ipiv,
          int *info);

void dsytf2_64(char uplo, long n, double *a, long lda,  long
          *ipiv, long *info);
```

PURPOSE

dsytf2 computes the factorization of a real symmetric matrix A using the Bunch-Kaufman diagonal pivoting method:

A = U*D*U' or A = L*D*L'

where U (or L) is a product of permutation and unit upper (lower) triangular matrices, U' is the transpose of U, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

This is the unblocked version of the algorithm, calling Level 2 BLAS.

ARGUMENTS

UPLO (input)
Specifies whether the upper or lower triangular part of the symmetric matrix A is stored:
= 'U': Upper triangular
= 'L': Lower triangular

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the symmetric matrix A. If UPLO = 'U', the leading n-by-n upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, the block diagonal matrix D and the mul-

tipliers used to obtain the factor U or L (see below for further details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIV (output)
Details of the interchanges and the block structure of D. If IPIV(k) > 0, then rows and columns k and IPIV(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIV(k) = IPIV(k-1) < 0, then rows and columns k-1 and -IPIV(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIV(k) = IPIV(k+1) < 0, then rows and columns k+1 and -IPIV(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, D(k,k) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

1-96 - Based on modifications by J. Lewis, Boeing Computer Services
Company

If UPLO = 'U', then A = U*D*U', where
U = P(n)*U(n)* ... *P(k)U(k)* ...,
i.e., U is a product of terms P(k)*U(k), where k decreases from n to 1 in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIV(k), and U(k) is a unit upper triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    v    0   )   k-s
   U(k) =  (   0    I    0   )   s
           (   0    0    I   )   n-k
              k-s   s   n-k
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(1:k-

1,k).   If s = 2, the upper triangle of D(k) overwrites A(k-1,k-1), A(k-1,k), and A(k,k), and v overwrites A(1:k-2,k-1:k).

If UPLO = 'L', then A = L*D*L', where
   L = P(1)*L(1)* ... *P(k)*L(k)* ...,
i.e., L is a product of terms P(k)*L(k), where k increases from 1 to n in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIV(k), and L(k) is a unit lower triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    0     0   )  k-1
   L(k) =  (   0    I     0   )  s
           (   0    v     I   )  n-k-s+1
              k-1   s  n-k-s+1
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(k+1:n,k). If s = 2, the lower triangle of D(k) overwrites A(k,k), A(k+1,k), and A(k+1,k+1), and v overwrites A(k+2:n,k:k+1).

Contents

NAME

```
dsytrd - reduce a real symmetric matrix A to real  symmetric
tridiagonal  form  T by an orthogonal similarity transforma-
tion
```

SYNOPSIS

```
SUBROUTINE DSYTRD(UPLO, N, A, LDA, D, E, TAU, WORK, LWORK, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, LWORK, INFO
DOUBLE PRECISION A(LDA,*), D(*), E(*), TAU(*), WORK(*)

SUBROUTINE DSYTRD_64(UPLO, N, A, LDA, D, E, TAU, WORK, LWORK, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, LWORK, INFO
DOUBLE PRECISION A(LDA,*), D(*), E(*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYTRD(UPLO, N, A, [LDA], D, E, TAU, [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, LWORK, INFO
REAL(8), DIMENSION(:) :: D, E, TAU, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE SYTRD_64(UPLO, N, A, [LDA], D, E, TAU, [WORK], [LWORK],
       [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, LWORK, INFO
```

```
REAL(8), DIMENSION(:) :: D, E, TAU, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dsytrd(char uplo, int n, double *a, int lda, double *d,
          double *e, double *tau, int *info);

void dsytrd_64(char uplo, long n, double *a, long lda,  dou-
          ble *d, double *e, double *tau, long *info);
```

PURPOSE

dsytrd reduces a real symmetric matrix A to real symmetric tridiagonal form T by an orthogonal similarity transformation: Q**T * A * Q = T.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input) On entry, the symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced. On exit, if UPLO = 'U', the diagonal and first superdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements above the first superdiagonal, with the array TAU, represent the orthogonal matrix Q as a product of elementary reflectors; if UPLO = 'L', the diagonal and first subdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements below the first subdiagonal, with the array TAU, represent the orthogonal matrix Q as a product of elementary reflectors. See Further Details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

D (output)
The diagonal elements of the tridiagonal matrix T: D(i) = A(i,i).

E (output)
The off-diagonal elements of the tridiagonal matrix T: E(i) = A(i,i+1) if UPLO = 'U', E(i) = A(i+1,i) if UPLO = 'L'.

TAU (output)
The scalar factors of the elementary reflectors (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 1. For optimum performance LWORK >= N*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

If UPLO = 'U', the matrix Q is represented as a product of elementary reflectors

```
Q = H(n-1) . . . H(2) H(1).
```

Each H(i) has the form

```
H(i) = I - tau * v * v'
```

where tau is a real scalar, and v is a real vector with v(i+1:n) = 0 and v(i) = 1; v(1:i-1) is stored on exit in

```
A(1:i-1,i+1), and tau in TAU(i).

If UPLO = 'L', the matrix Q is represented as a  product  of
elementary reflectors

   Q = H(1) H(2) . . . H(n-1).

Each H(i) has the form

   H(i) = I - tau * v * v'

where tau is a real scalar, and v is a real vector with
v(1:i) = 0 and v(i+1) = 1; v(i+2:n) is  stored  on  exit  in
A(i+2:n,i), and tau in TAU(i).
The contents of A on exit are illustrated by  the  following
examples with n = 5:

if UPLO = 'U':                       if UPLO = 'L':

  (    d     e     v2    v3    v4   )                (     d
)
  (       d    e    v3    v4   )                (    e     d
)
  (           d    e    v4  )              (   v1   e     d
)
  (              d   e  )              (   v1   v2   e    d
)
  (                  d  )              (  v1  v2  v3  e    d
)

where d and e denote diagonal and off-diagonal  elements  of
T, and vi denotes an element of the vector defining H(i).
```

Contents

NAME

dsytrf - compute the factorization of a real symmetric matrix A using the Bunch-Kaufman diagonal pivoting method

SYNOPSIS

```
SUBROUTINE DSYTRF(UPLO, N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, LDWORK, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION A(LDA,*), WORK(*)

SUBROUTINE DSYTRF_64(UPLO, N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, LDWORK, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYTRF(UPLO, N, A, [LDA], IPIVOT, [WORK], [LDWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, LDWORK, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE SYTRF_64(UPLO, N, A, [LDA], IPIVOT, [WORK], [LDWORK],
       [INFO])
```

```
CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dsytrf(char uplo,  int  n,  double  *a,  int  lda,  int
          *ipivot, int *info);

void dsytrf_64(char uplo, long n, double *a, long lda,  long
          *ipivot, long *info);
```

PURPOSE

dsytrf computes the factorization of a real symmetric matrix A using the Bunch-Kaufman diagonal pivoting method. The form of the factorization is

```
A = U*D*U**T  or  A = L*D*L**T
```

where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

This is the blocked version of the algorithm, calling Level 3 BLAS.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, the block diagonal matrix D and the multipliers used to obtain the factor U or L (see below for further details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (output)
Details of the interchanges and the block structure of D. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of WORK. LDWORK >=1. For best performance LDWORK >= N*NB, where NB is the block size returned by ILAENV.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

If UPLO = 'U', then A = U*D*U', where

```
   U = P(n)*U(n)* ... *P(k)U(k)* ...,
```
i.e., U is a product of terms P(k)*U(k), where k decreases from n to 1 in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIVOT(k), and U(k) is a unit upper triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    v    0   )   k-s
   U(k) =  (   0    I    0   )   s
           (   0    0    I   )   n-k
              k-s   s   n-k
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(1:k-1,k). If s = 2, the upper triangle of D(k) overwrites A(k-1,k-1), A(k-1,k), and A(k,k), and v overwrites A(1:k-2,k-1:k).

If UPLO = 'L', then A = L*D*L', where
```
   L = P(1)*L(1)* ... *P(k)*L(k)* ...,
```
i.e., L is a product of terms P(k)*L(k), where k increases from 1 to n in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIVOT(k), and L(k) is a unit lower triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    0     0   )  k-1
   L(k) =  (   0    I     0   )  s
           (   0    v     I   )  n-k-s+1
              k-1   s  n-k-s+1
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(k+1:n,k). If s = 2, the lower triangle of D(k) overwrites A(k,k), A(k+1,k), and A(k+1,k+1), and v overwrites A(k+2:n,k:k+1).

Contents

NAME

```
dsytri - compute the inverse of a real symmetric  indefinite
matrix  A  using  the  factorization  A  =  U*D*U**T  or A =
L*D*L**T computed by SSYTRF
```

SYNOPSIS

```
SUBROUTINE DSYTRI(UPLO, N, A, LDA, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION A(LDA,*), WORK(*)

SUBROUTINE DSYTRI_64(UPLO, N, A, LDA, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYTRI(UPLO, N, A, [LDA], IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE SYTRI_64(UPLO, N, A, [LDA], IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dsytri(char uplo,  int  n,  double  *a,  int  lda,  int
          *ipivot, int *info);

void dsytri_64(char uplo, long n, double *a, long lda,  long
          *ipivot, long *info);
```

PURPOSE

dsytri computes the inverse of a real symmetric indefinite matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by SSYTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by SSYTRF.

On exit, if INFO = 0, the (symmetric) inverse of the original matrix. If UPLO = 'U', the upper triangular part of the inverse is formed and the part of A below the diagonal is not referenced; if UPLO = 'L' the lower triangular part of the inverse is formed and the part of A above the diagonal is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by SSYTRF.

WORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) = 0; the matrix is singular and its inverse could not be computed.

Contents

NAME

```
dsytrs - solve a system of linear equations A*X = B  with  a
real symmetric matrix A using the factorization A = U*D*U**T
or A = L*D*L**T computed by SSYTRF
```

SYNOPSIS

```
SUBROUTINE DSYTRS(UPLO, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*)

SUBROUTINE DSYTRS_64(UPLO, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE SYTRS(UPLO, N, NRHS, A, [LDA], IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE SYTRS_64(UPLO, N, NRHS, A, [LDA], IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
```

```
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dsytrs(char uplo, int n, int nrhs, double *a, int  lda,
          int *ipivot, double *b, int ldb, int *info);

void dsytrs_64(char uplo, long n, long nrhs, double *a, long
          lda,  long  *ipivot,  double  *b,  long  ldb, long
          *info);
```

PURPOSE

dsytrs solves a system of linear equations A*X = B with a real symmetric matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by SSYTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by SSYTRF.

LDA (input)
The leading dimension of the array A.   LDA >= max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by SSYTRF.

B (input/output)
On entry, the right hand side matrix B. On exit,

the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dtbcon - estimate the reciprocal of the condition number  of
a  triangular  band  matrix  A,  in either the 1-norm or the
infinity-norm
```

SYNOPSIS

```
SUBROUTINE DTBCON(NORM, UPLO, DIAG, N, KD, A, LDA, RCOND, WORK,
      WORK2, INFO)

CHARACTER * 1 NORM, UPLO, DIAG
INTEGER N, KD, LDA, INFO
INTEGER WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(LDA,*), WORK(*)

SUBROUTINE DTBCON_64(NORM, UPLO, DIAG, N, KD, A, LDA, RCOND, WORK,
      WORK2, INFO)

CHARACTER * 1 NORM, UPLO, DIAG
INTEGER*8 N, KD, LDA, INFO
INTEGER*8 WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TBCON(NORM, UPLO, DIAG, N, KD, A, [LDA], RCOND, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
INTEGER :: N, KD, LDA, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL(8) :: RCOND
```

```
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE TBCON_64(NORM, UPLO, DIAG, N, KD, A, [LDA], RCOND,
       [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
INTEGER(8) :: N, KD, LDA, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
void dtbcon(char norm, char uplo, char diag, int n, int  kd,
          double *a, int lda, double *rcond, int *info);

void dtbcon_64(char norm, char uplo, char diag, long n, long
          kd,  double  *a,  long  lda,  double  *rcond, long
          *info);
```

PURPOSE

```
dtbcon estimates the reciprocal of the condition number of a
triangular  band  matrix  A,  in  either  the  1-norm or the
infinity-norm.

The norm of A is computed and an estimate  is  obtained  for
norm(inv(A)), then the reciprocal of the condition number is
computed as
   RCOND = 1 / ( norm(A) * norm(inv(A)) ).
```

ARGUMENTS

```
NORM (input)
          Specifies whether the 1-norm condition  number  or
          the infinity-norm condition number is required:
          = '1' or 'O':  1-norm;
          = 'I':         Infinity-norm.

UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

DIAG (input)
          = 'N':  A is non-unit triangular;
```

```
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals  or  subdiagonals  of
          the triangular band matrix A.  KD >= 0.

A (input) The upper  or  lower  triangular  band  matrix  A,
          stored  in  the  first kd+1 rows of the array. The
          j-th column of A is stored in the j-th  column  of
          the  array A as follows:  if UPLO = 'U', A(kd+1+i-
          j,j) = A(i,j) for  max(1,j-kd)<=i<=j;  if  UPLO  =
          'L', A(1+i-j,j)    = A(i,j) for j<=i<=min(n,j+kd).
          If DIAG = 'U', the diagonal elements of A are  not
          referenced and are assumed to be 1.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          KD+1.

RCOND (output)
          The reciprocal of  the  condition  number  of  the
          matrix   A,  computed  as  RCOND  =  1/(norm(A)  *
          norm(inv(A))).

WORK (workspace)
          dimension(3*N)

WORK2 (workspace)
          dimension(N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
dtbmv - perform one of the matrix-vector operations    x  :=
A*x, or x := A'*x
```

SYNOPSIS

```
SUBROUTINE DTBMV(UPLO, TRANSA, DIAG, N, K, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, K, LDA, INCY
DOUBLE PRECISION A(LDA,*), Y(*)

SUBROUTINE DTBMV_64(UPLO, TRANSA, DIAG, N, K, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, K, LDA, INCY
DOUBLE PRECISION A(LDA,*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE TBMV(UPLO, [TRANSA], DIAG, [N], K, A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, K, LDA, INCY
REAL(8), DIMENSION(:) :: Y
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE TBMV_64(UPLO, [TRANSA], DIAG, [N], K, A, [LDA], Y,
       [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, K, LDA, INCY
REAL(8), DIMENSION(:) :: Y
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dtbmv(char uplo, char transa, char diag, int n, int  k,
          double *a, int lda, double *y, int incy);

void dtbmv_64(char uplo, char transa,  char  diag,  long  n,
          long  k,  double  *a,  long  lda,  double *y, long
          incy);
```

PURPOSE

dtbmv performs one of the matrix-vector operations x := A*x, or x := A'*x, where x is an n element vector and A is an n by n unit, or non-unit, upper or lower triangular band matrix, with (k + 1) diagonals.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   x := A*x.

TRANSA = 'T' or 't'   x := A'*x.

TRANSA = 'C' or 'c'   x := A'*x.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)

On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit triangular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

K (input)
On entry with UPLO = 'U' or 'u', K specifies the number of super-diagonals of the matrix A. On entry with UPLO = 'L' or 'l', K specifies the number of sub-diagonals of the matrix A. K >= 0. Unchanged on exit.

A (input)
Before entry with UPLO = 'U' or 'u', the leading (k + 1) by n part of the array A must contain the upper triangular band part of the matrix of coefficients, supplied column by column, with the leading diagonal of the matrix in row (k + 1) of the array, the first super-diagonal starting at position 2 in row k, and so on. The top left k by k triangle of the array A is not referenced. The following program segment will transfer an upper triangular band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = K + 1 - J
     DO 10, I = MAX( 1, J - K ), J
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Before entry with UPLO = 'L' or 'l', the leading (k + 1) by n part of the array A must contain the lower triangular band part of the matrix of coefficients, supplied column by column, with the leading diagonal of the matrix in row 1 of the array, the first sub-diagonal starting at position 1 in row 2, and so on. The bottom right k by k triangle of the array A is not referenced. The following program segment will transfer a lower

triangular band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = 1 - J
     DO 10, I = J, MIN( N, J + K )
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Note that when DIAG = 'U' or 'u' the elements of the array A corresponding to the diagonal elements of the matrix are not referenced, but are assumed to be unity. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= (k + 1). Unchanged on exit.

Y (input/output)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector x. On exit, Y is overwritten with the tran- formed vector x.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
dtbrfs - provide error bounds and backward  error  estimates
for the solution to a system of linear equations with a tri-
angular band coefficient matrix
```

SYNOPSIS

```
SUBROUTINE DTBRFS(UPLO, TRANSA, DIAG, N, KD, NRHS, A, LDA, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, KD, NRHS, LDA, LDB, LDX, INFO
INTEGER WORK2(*)
DOUBLE  PRECISION  A(LDA,*),  B(LDB,*),  X(LDX,*),  FERR(*),
BERR(*), WORK(*)

SUBROUTINE DTBRFS_64(UPLO, TRANSA, DIAG, N, KD, NRHS, A, LDA, B,
      LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, KD, NRHS, LDA, LDB, LDX, INFO
INTEGER*8 WORK2(*)
DOUBLE  PRECISION  A(LDA,*),  B(LDB,*),  X(LDX,*),  FERR(*),
BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TBRFS(UPLO, [TRANSA], DIAG, N, KD, NRHS, A, [LDA], B,
       [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, KD, NRHS, LDA, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL(8), DIMENSION(:) :: FERR, BERR, WORK
```

```
REAL(8), DIMENSION(:,:) :: A, B, X

SUBROUTINE TBRFS_64(UPLO, [TRANSA], DIAG, N, KD, NRHS, A, [LDA],
       B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, KD, NRHS, LDA, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL(8), DIMENSION(:) :: FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: A, B, X
```

C INTERFACE

```
#include <sunperf.h>
void dtbrfs(char uplo, char transa, char diag,  int  n,  int
          kd,  int  nrhs, double *a, int lda, double *b, int
          ldb, double *x,  int  ldx,  double  *ferr,  double
          *berr, int *info);

void dtbrfs_64(char uplo, char transa, char  diag,  long  n,
          long  kd,  long  nrhs, double *a, long lda, double
          *b, long ldb, double *x, long ldx,  double  *ferr,
          double *berr, long *info);
```

PURPOSE

dtbrfs provides error bounds and backward error estimates for the solution to a system of linear equations with a triangular band coefficient matrix.

The solution matrix X must be computed by STBTRS or some other means before entering this routine. STBRFS does not do iterative refinement because doing so cannot improve the backward error.

ARGUMENTS

UPLO (input)
= 'U': A is upper triangular;
= 'L': A is lower triangular.

TRANSA (input)
Specifies the form of the system of equations:
= 'N': A * X = B (No transpose)
= 'T': A**T * X = B (Transpose)
= 'C': A**H * X = B (Conjugate transpose = Transpose)

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
= 'N': A is non-unit triangular;
= 'U': A is unit triangular.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals or subdiagonals of the triangular band matrix A. KD >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The upper or lower triangular band matrix A, stored in the first kd+1 rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd). If DIAG = 'U', the diagonal elements of A are not referenced and are assumed to be 1.

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input) The solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dtbsv - solve one of the systems of equations A*x  =  b,  or
A'*x = b
```

SYNOPSIS

```
SUBROUTINE DTBSV(UPLO, TRANSA, DIAG, N, K, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, K, LDA, INCY
DOUBLE PRECISION A(LDA,*), Y(*)

SUBROUTINE DTBSV_64(UPLO, TRANSA, DIAG, N, K, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, K, LDA, INCY
DOUBLE PRECISION A(LDA,*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE TBSV(UPLO, [TRANSA], DIAG, [N], K, A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, K, LDA, INCY
REAL(8), DIMENSION(:) :: Y
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE TBSV_64(UPLO, [TRANSA], DIAG, [N], K, A, [LDA], Y,
       [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, K, LDA, INCY
REAL(8), DIMENSION(:) :: Y
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dtbsv(char uplo, char transa, char diag, int n, int  k,
          double *a, int lda, double *y, int incy);

void dtbsv_64(char uplo, char transa,  char  diag,  long  n,
          long  k,  double  *a,  long  lda,  double *y, long
          incy);
```

PURPOSE

dtbsv solves one of the systems of equations A*x = b, or A'*x = b, where b and x are n element vectors and A is an n by n unit, or non-unit, upper or lower triangular band matrix, with (k + 1) diagonals.

No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u' A is an upper triangular matrix.

UPLO = 'L' or 'l' A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the equations to be solved as follows:

TRANSA = 'N' or 'n' A*x = b.

TRANSA = 'T' or 't' A'*x = b.

TRANSA = 'C' or 'c' A'*x = b.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit triangular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

K (input)
On entry with UPLO = 'U' or 'u', K specifies the number of super-diagonals of the matrix A. On entry with UPLO = 'L' or 'l', K specifies the number of sub-diagonals of the matrix A. K >= 0. Unchanged on exit.

A (input)
Before entry with UPLO = 'U' or 'u', the leading (k + 1) by n part of the array A must contain the upper triangular band part of the matrix of coefficients, supplied column by column, with the leading diagonal of the matrix in row (k + 1) of the array, the first super-diagonal starting at position 2 in row k, and so on. The top left k by k triangle of the array A is not referenced. The following program segment will transfer an upper triangular band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = K + 1 - J
     DO 10, I = MAX( 1, J - K ), J
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Before entry with UPLO = 'L' or 'l', the leading (k + 1) by n part of the array A must contain the lower triangular band part of the matrix of coefficients, supplied column by column, with the

leading diagonal of the matrix in row 1 of the array, the first sub-diagonal starting at position 1 in row 2, and so on. The bottom right k by k triangle of the array A is not referenced. The following program segment will transfer a lower triangular band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = 1 - J
     DO 10, I = J, MIN( N, J + K )
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Note that when DIAG = 'U' or 'u' the elements of the array A corresponding to the diagonal elements of the matrix are not referenced, but are assumed to be unity. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= (k + 1). Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element right-hand side vector b. On exit, Y is overwritten with the solution vector x.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
dtbtrs - solve a triangular system of the form   A * X  =  B
or A**T * X = B,
```

SYNOPSIS

```
SUBROUTINE DTBTRS(UPLO, TRANSA, DIAG, N, KD, NRHS, A, LDA, B, LDB,
      INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, KD, NRHS, LDA, LDB, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*)

SUBROUTINE DTBTRS_64(UPLO, TRANSA, DIAG, N, KD, NRHS, A, LDA, B,
      LDB, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, KD, NRHS, LDA, LDB, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE TBTRS(UPLO, TRANSA, DIAG, N, KD, NRHS, A, [LDA], B,
       [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, KD, NRHS, LDA, LDB, INFO
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE TBTRS_64(UPLO, TRANSA, DIAG, N, KD, NRHS, A, [LDA], B,
       [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, KD, NRHS, LDA, LDB, INFO
```

```
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dtbtrs(char uplo, char transa, char diag,  int  n,  int
          kd,  int  nrhs, double *a, int lda, double *b, int
          ldb, int *info);

void dtbtrs_64(char uplo, char transa, char  diag,  long  n,
          long  kd,  long  nrhs, double *a, long lda, double
          *b, long ldb, long *info);
```

PURPOSE

dtbtrs solves a triangular system of the form

where A is a triangular band matrix of order N, and B is an N-by NRHS matrix. A check is made to verify that A is non-singular.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

TRANSA (input)
          Specifies the form the system of equations:
          = 'N':  A * X = B  (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose = Tran-
          spose)

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals  or  subdiagonals  of
          the triangular band matrix A.  KD >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.
```

A (input) The upper or lower triangular band matrix A, stored in the first kd+1 rows of A. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd). If DIAG = 'U', the diagonal elements of A are not referenced and are assumed to be 1.

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

B (input/output)
On entry, the right hand side matrix B. On exit, if INFO = 0, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the i-th diagonal element of A is zero, indicating that the matrix is singular and the solutions X have not been computed.

Contents

NAME

```
dtgevc - compute some or all of the right and/or  left  gen-
eralized  eigenvectors  of  a  pair of real upper triangular
matrices (A,B)
```

SYNOPSIS

```
SUBROUTINE DTGEVC(SIDE, HOWMNY, SELECT, N, A, LDA, B, LDB, VL, LDVL,
      VR, LDVR, MM, M, WORK, INFO)

CHARACTER * 1 SIDE, HOWMNY
INTEGER N, LDA, LDB, LDVL, LDVR, MM, M, INFO
LOGICAL SELECT(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*), VL(LDVL,*), VR(LDVR,*),
WORK(*)

SUBROUTINE DTGEVC_64(SIDE, HOWMNY, SELECT, N, A, LDA, B, LDB, VL,
      LDVL, VR, LDVR, MM, M, WORK, INFO)

CHARACTER * 1 SIDE, HOWMNY
INTEGER*8 N, LDA, LDB, LDVL, LDVR, MM, M, INFO
LOGICAL*8 SELECT(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*), VL(LDVL,*), VR(LDVR,*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TGEVC(SIDE, HOWMNY, SELECT, N, A, [LDA], B, [LDB], VL,
       [LDVL], VR, [LDVR], MM, M, [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, HOWMNY
INTEGER :: N, LDA, LDB, LDVL, LDVR, MM, M, INFO
LOGICAL, DIMENSION(:) :: SELECT
```

```
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A, B, VL, VR

SUBROUTINE TGEVC_64(SIDE, HOWMNY, SELECT, N, A, [LDA], B, [LDB], VL,
       [LDVL], VR, [LDVR], MM, M, [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, HOWMNY
INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, MM, M, INFO
LOGICAL(8), DIMENSION(:) :: SELECT
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A, B, VL, VR
```

C INTERFACE

```
#include <sunperf.h>
void dtgevc(char side, char howmny, int *select, int n, dou-
          ble  *a,  int lda, double *b, int ldb, double *vl,
          int ldvl, double *vr, int ldvr, int  mm,  int  *m,
          int *info);

void dtgevc_64(char side, char howmny, long *select, long n,
          double  *a,  long lda, double *b, long ldb, double
          *vl, long ldvl, double *vr, long  ldvr,  long  mm,
          long *m, long *info);
```

PURPOSE

dtgevc computes some or all of the right and/or left generalized eigenvectors of a pair of real upper triangular matrices (A,B).

The right generalized eigenvector x and the left generalized eigenvector y of (A,B) corresponding to a generalized eigenvalue w are defined by:

(A - wB) * x = 0 and y**H * (A - wB) = 0

where y**H denotes the conjugate tranpose of y.

If an eigenvalue w is determined by zero diagonal elements of both A and B, a unit vector is returned as the corresponding eigenvector.

If all eigenvectors are requested, the routine may either return the matrices X and/or Y of right or left eigenvectors of (A,B), or the products Z*X and/or Q*Y, where Z and Q are input orthogonal matrices. If (A,B) was obtained from the generalized real-Schur factorization of an original pair of matrices
   (A0,B0) = (Q*A*Z**H,Q*B*Z**H),

then Z*X and Q*Y are the matrices of right or left eigenvectors of A.

A must be block upper triangular, with 1-by-1 and 2-by-2 diagonal blocks. Corresponding to each 2-by-2 diagonal block is a complex conjugate pair of eigenvalues and eigenvectors; only one
eigenvector of the pair is computed, namely the one corresponding to the eigenvalue with positive imaginary part.

ARGUMENTS

SIDE (input)
= 'R': compute right eigenvectors only;
= 'L': compute left eigenvectors only;
= 'B': compute both right and left eigenvectors.

HOWMNY (input)
= 'A': compute all right and/or left eigenvectors;
= 'B': compute all right and/or left eigenvectors, and backtransform them using the input matrices supplied in VR and/or VL; = 'S': compute selected right and/or left eigenvectors, specified by the logical array SELECT.

SELECT (input)
If HOWMNY='S', SELECT specifies the eigenvectors to be computed. If HOWMNY='A' or 'B', SELECT is not referenced. To select the real eigenvector corresponding to the real eigenvalue w(j), SELECT(j) must be set to .TRUE. To select the complex eigenvector corresponding to a complex conjugate pair w(j) and w(j+1), either SELECT(j) or SELECT(j+1) must be set to .TRUE..

N (input) The order of the matrices A and B. N >= 0.

A (input) The upper quasi-triangular matrix A.

LDA (input)
The leading dimension of array A. LDA >= max(1, N).

B (input) The upper triangular matrix B. If A has a 2-by-2 diagonal block, then the corresponding 2-by-2 block of B must be diagonal with positive elements.

LDB (input)
The leading dimension of array B. LDB >= max(1,N).

VL (input/output)
On entry, if SIDE = 'L' or 'B' and HOWMNY = 'B', VL must contain an N-by-N matrix Q (usually the orthogonal matrix Q of left Schur vectors returned by SHGEQZ). On exit, if SIDE = 'L' or 'B', VL contains: if HOWMNY = 'A', the matrix Y of left eigenvectors of (A,B); if HOWMNY = 'B', the matrix Q*Y; if HOWMNY = 'S', the left eigenvectors of (A,B) specified by SELECT, stored consecutively in the columns of VL, in the same order as their eigenvalues. If SIDE = 'R', VL is not referenced.

A complex eigenvector corresponding to a complex eigenvalue is stored in two consecutive columns, the first holding the real part, and the second the imaginary part.

LDVL (input)
The leading dimension of array VL. LDVL >= max(1,N) if SIDE = 'L' or 'B'; LDVL >= 1 otherwise.

VR (input/output)
On entry, if SIDE = 'R' or 'B' and HOWMNY = 'B', VR must contain an N-by-N matrix Q (usually the orthogonal matrix Z of right Schur vectors returned by SHGEQZ). On exit, if SIDE = 'R' or 'B', VR contains: if HOWMNY = 'A', the matrix X of right eigenvectors of (A,B); if HOWMNY = 'B', the matrix Z*X; if HOWMNY = 'S', the right eigenvectors of (A,B) specified by SELECT, stored consecutively in the columns of VR, in the same order as their eigenvalues. If SIDE = 'L', VR is not referenced.

A complex eigenvector corresponding to a complex eigenvalue is stored in two consecutive columns, the first holding the real part and the second the imaginary part.

LDVR (input)
The leading dimension of the array VR. LDVR >= max(1,N) if SIDE = 'R' or 'B'; LDVR >= 1 otherwise.

MM (input)
The number of columns in the arrays VL and/or VR. MM >= M.

M (output)
The number of columns in the arrays VL and/or VR actually used to store the eigenvectors. If HOWMNY = 'A' or 'B', M is set to N. Each selected real eigenvector occupies one column and each selected complex eigenvector occupies two columns.

WORK (workspace)
dimension(6*N)

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: the 2-by-2 block (INFO:INFO+1) does not have a complex eigenvalue.

FURTHER DETAILS

```
Allocation of workspace:
---------- -- ---------

   WORK( j ) = 1-norm of j-th column of A, above the  diago-
nal
   WORK( N+j ) = 1-norm of j-th column of B, above the diag-
onal
   WORK( 2*N+1:3*N ) = real part of eigenvector
   WORK( 3*N+1:4*N ) = imaginary part of eigenvector
   WORK( 4*N+1:5*N ) = real part of back-transformed  eigen-
vector
   WORK( 5*N+1:6*N ) = imaginary  part  of  back-transformed
eigenvector

Rowwise vs. columnwise solution methods:
------- --  ---------- -------- -------

Finding a  generalized  eigenvector  consists  basically  of
solving the singular triangular system

 (A - w B) x = 0     (for right) or:   (A - w B)**H  y  =  0
(for left)

Consider finding the  i-th  right  eigenvector  (assume  all
eigenvalues are real). The equation to be solved is:
0 = sum  C(j,k) v(k)  = sum  C(j,k) v(k)     for j =  i,.  .
.,1
```

```
    k=j                 k=j

where  C = (A - w B)  (The components v(i+1:n) are 0.)

The "rowwise" method is:

(1)  v(i) := 1
for j = i-1,. . .,1:
                        i
    (2) compute  s = - sum C(j,k) v(k)   and
                      k=j+1
    (3) v(j) := s / C(j,j)
```

Step 2 is sometimes called the "dot product" step, since it is an inner product between the j-th row and the portion of the eigenvector that has been computed so far.

The "columnwise" method consists basically in doing the sums for all the rows in parallel. As each v(j) is computed, the contribution of v(j) times the j-th column of C is added to the partial sums. Since FORTRAN arrays are stored column-wise, this has the advantage that at each step, the elements of C that are accessed are adjacent to one another, whereas with the rowwise method, the elements accessed at a step are spaced LDA (and LDB) words apart.

When finding left eigenvectors, the matrix in question is the transpose of the one in storage, so the rowwise method then actually accesses columns of A and B at each step, and so is the preferred method.

Contents

NAME

```
dtgexc - reorder the generalized real Schur decomposition of
a  real  matrix  pair  (A,B) using an orthogonal equivalence
transformation   (A, B) = Q * (A, B) * Z',
```

SYNOPSIS

```
SUBROUTINE DTGEXC(WANTQ, WANTZ, N, A, LDA, B, LDB, Q, LDQ, Z, LDZ,
      IFST, ILST, WORK, LWORK, INFO)

INTEGER N, LDA, LDB, LDQ, LDZ, IFST, ILST, LWORK, INFO
LOGICAL WANTQ, WANTZ
DOUBLE PRECISION  A(LDA,*),  B(LDB,*),  Q(LDQ,*),  Z(LDZ,*),
WORK(*)

SUBROUTINE DTGEXC_64(WANTQ, WANTZ, N, A, LDA, B, LDB, Q, LDQ, Z, LDZ,
      IFST, ILST, WORK, LWORK, INFO)

INTEGER*8 N, LDA, LDB, LDQ, LDZ, IFST, ILST, LWORK, INFO
LOGICAL*8 WANTQ, WANTZ
DOUBLE PRECISION  A(LDA,*),  B(LDB,*),  Q(LDQ,*),  Z(LDZ,*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TGEXC(WANTQ, WANTZ, N, A, [LDA], B, [LDB], Q, [LDQ], Z,
       [LDZ], IFST, ILST, [WORK], [LWORK], [INFO])

INTEGER :: N, LDA, LDB, LDQ, LDZ, IFST, ILST, LWORK, INFO
LOGICAL :: WANTQ, WANTZ
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A, B, Q, Z
```

```
SUBROUTINE TGEXC_64(WANTQ, WANTZ, N, A, [LDA], B, [LDB], Q, [LDQ], Z,
       [LDZ], IFST, ILST, [WORK], [LWORK], [INFO])

INTEGER(8) :: N, LDA, LDB, LDQ, LDZ, IFST, ILST, LWORK, INFO
LOGICAL(8) :: WANTQ, WANTZ
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A, B, Q, Z
```

C INTERFACE

```
#include <sunperf.h>

void dtgexc(int wantq, int wantz, int n, double *a, int lda,
          double *b, int ldb, double *q, int ldq, double *z,
          int ldz, int *ifst, int *ilst, int *info);
void dtgexc_64(long wantq, long wantz, long  n,  double  *a,
          long  lda,  double  *b,  long ldb, double *q, long
          ldq, double *z, long ldz, long *ifst, long  *ilst,
          long *info);
```

PURPOSE

dtgexc reorders the generalized real Schur decomposition of a real matrix pair (A,B) using an orthogonal equivalence transformation

so that the diagonal block of (A, B) with row index IFST is moved to row ILST.

(A, B) must be in generalized real Schur canonical form (as returned by SGGES), i.e. A is block upper triangular with 1-by-1 and 2-by-2 diagonal blocks. B is upper triangular.

Optionally, the matrices Q and Z of generalized Schur vectors are updated.

```
       Q(in) * A(in) * Z(in)' = Q(out) * A(out) * Z(out)'
       Q(in) * B(in) * Z(in)' = Q(out) * B(out) * Z(out)'
```

ARGUMENTS

WANTQ (input)

WANTZ (input)

N (input) The order of the matrices A and B. N >= 0.

A (input/output)

On entry, the matrix A in generalized real Schur canonical form. On exit, the updated matrix A, again in generalized real Schur canonical form.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the matrix B in generalized real Schur canonical form (A,B). On exit, the updated matrix B, again in generalized real Schur canonical form (A,B).

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

Q (input/output)
On entry, if WANTQ = .TRUE., the orthogonal matrix Q. On exit, the updated matrix Q. If WANTQ = .FALSE., Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= 1. If WANTQ = .TRUE., LDQ >= N.

Z (input/output)
On entry, if WANTZ = .TRUE., the orthogonal matrix Z. On exit, the updated matrix Z. If WANTZ = .FALSE., Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1. If WANTZ = .TRUE., LDZ >= N.

IFST (input/output)
Specify the reordering of the diagonal blocks of (A, B). The block with row index IFST is moved to row ILST, by a sequence of swapping between adjacent blocks. On exit, if IFST pointed on entry to the second row of a 2-by-2 block, it is changed to point to the first row; ILST always points to the first row of the block in its final position (which may differ from its input value by +1 or -1). 1 <= IFST, ILST <= N.

ILST (input/output)
See the description of IFST.

```
WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK. LWORK  >=  4*N  +
          16.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          =0:  successful exit.
          <0:  if INFO = -i, the i-th argument had an  ille-
          gal value.
          =1:  The transformed matrix pair (A, B)  would  be
          too  far  from generalized Schur form; the problem
          is ill- conditioned. (A, B)  may  have  been  par-
          tially reordered, and ILST points to the first row
          of the current position of the block being moved.
```

FURTHER DETAILS

```
Based on contributions by
   Bo Kagstrom and Peter Poromaa,  Department  of  Computing
Science,
   Umea University, S-901 87 Umea, Sweden.

[1] B. Kagstrom; A Direct Method for Reordering  Eigenvalues
in the
    Generalized Real Schur Form of a Regular Matrix Pair (A,
B), in
    M.S. Moonen et al (eds), Linear Algebra for Large  Scale
and
    Real-Time Applications, Kluwer Academic Publ.  1993,  pp
195-218.
```

Contents

NAME

```
dtgsen - reorder the generalized real Schur decomposition of
a  real  matrix  pair  (A,  B)  (in  terms of an orthonormal
equivalence trans- formation Q' * (A, B) *  Z),  so  that  a
selected cluster of eigenvalues appears in the leading diag-
onal blocks of the upper quasi-triangular matrix A  and  the
upper triangular B
```

SYNOPSIS

```
SUBROUTINE DTGSEN(IJOB, WANTQ, WANTZ, SELECT, N, A, LDA, B, LDB,
      ALPHAR, ALPHAI, BETA, Q, LDQ, Z, LDZ, M, PL, PR, DIF, WORK,
      LWORK, IWORK, LIWORK, INFO)

INTEGER IJOB, N, LDA, LDB, LDQ, LDZ, M, LWORK, LIWORK, INFO
INTEGER IWORK(*)
LOGICAL WANTQ, WANTZ
LOGICAL SELECT(*)
DOUBLE PRECISION PL, PR
DOUBLE PRECISION A(LDA,*), B(LDB,*),  ALPHAR(*),  ALPHAI(*),
BETA(*), Q(LDQ,*), Z(LDZ,*), DIF(*), WORK(*)

SUBROUTINE DTGSEN_64(IJOB, WANTQ, WANTZ, SELECT, N, A, LDA, B, LDB,
      ALPHAR, ALPHAI, BETA, Q, LDQ, Z, LDZ, M, PL, PR, DIF, WORK,
      LWORK, IWORK, LIWORK, INFO)

INTEGER*8 IJOB, N, LDA, LDB, LDQ,  LDZ,  M,  LWORK,  LIWORK,
INFO
INTEGER*8 IWORK(*)
LOGICAL*8 WANTQ, WANTZ
LOGICAL*8 SELECT(*)
DOUBLE PRECISION PL, PR
```

```
DOUBLE PRECISION A(LDA,*), B(LDB,*),  ALPHAR(*),  ALPHAI(*),
BETA(*), Q(LDQ,*), Z(LDZ,*), DIF(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TGSEN(IJOB, WANTQ, WANTZ, SELECT, N, A, [LDA], B, [LDB],
       ALPHAR, ALPHAI, BETA, Q, [LDQ], Z, [LDZ], M, PL, PR, DIF, [WORK],
       [LWORK], [IWORK], [LIWORK], [INFO])

INTEGER :: IJOB, N, LDA, LDB, LDQ, LDZ,  M,  LWORK,  LIWORK,
INFO
INTEGER, DIMENSION(:) :: IWORK
LOGICAL :: WANTQ, WANTZ
LOGICAL, DIMENSION(:) :: SELECT
REAL(8) :: PL, PR
REAL(8), DIMENSION(:) :: ALPHAR, ALPHAI, BETA, DIF, WORK
REAL(8), DIMENSION(:,:) :: A, B, Q, Z
SUBROUTINE TGSEN_64(IJOB, WANTQ, WANTZ, SELECT, N, A, [LDA], B, [LDB],
       ALPHAR, ALPHAI, BETA, Q, [LDQ], Z, [LDZ], M, PL, PR, DIF, [WORK],
       [LWORK], [IWORK], [LIWORK], [INFO])

INTEGER(8) :: IJOB, N, LDA, LDB, LDQ, LDZ, M, LWORK, LIWORK,
INFO
INTEGER(8), DIMENSION(:) :: IWORK
LOGICAL(8) :: WANTQ, WANTZ
LOGICAL(8), DIMENSION(:) :: SELECT
REAL(8) :: PL, PR
REAL(8), DIMENSION(:) :: ALPHAR, ALPHAI, BETA, DIF, WORK
REAL(8), DIMENSION(:,:) :: A, B, Q, Z
```

C INTERFACE

```
#include <sunperf.h>

void dtgsen(int ijob, int wantq, int wantz, int *select, int
          n,  double *a, int lda, double *b, int ldb, double
          *alphar, double *alphai, double *beta, double  *q,
          int  ldq,  double *z, int ldz, int *m, double *pl,
          double *pr, double *dif, int *info);

void dtgsen_64(long  ijob,  long  wantq,  long  wantz,  long
          *select,  long  n, double *a, long lda, double *b,
          long ldb, double *alphar, double  *alphai,  double
          *beta,  double  *q, long ldq, double *z, long ldz,
          long *m, double *pl, double *pr, double *dif, long
          *info);
```

PURPOSE

dtgsen reorders the generalized real Schur decomposition of a real matrix pair (A, B) (in terms of an orthonormal equivalence trans- formation Q' * (A, B) * Z), so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper quasi-triangular matrix A and the upper triangular B. The leading columns of Q and Z form orthonormal bases of the corresponding left and right eigenspaces (deflating subspaces). (A, B) must be in generalized real Schur canonical form (as returned by SGGES), i.e. A is block upper triangular with 1-by-1 and 2-by-2 diagonal blocks. B is upper triangular.

DTGSEN also computes the generalized eigenvalues

w(j) = (ALPHAR(j) + i*ALPHAI(j))/BETA(j)

of the reordered matrix pair (A, B).

Optionally, DTGSEN computes the estimates of reciprocal condition numbers for eigenvalues and eigenspaces. These are Difu[(A11,B11), (A22,B22)] and Difl[(A11,B11), (A22,B22)], i.e. the separation(s) between the matrix pairs (A11, B11) and (A22,B22) that correspond to the selected cluster and the eigenvalues outside the cluster, resp., and norms of "projections" onto left and right eigenspaces w.r.t. the selected cluster in the (1,1)-block.

ARGUMENTS

IJOB (input)
Specifies whether condition numbers are required for the cluster of eigenvalues (PL and PR) or the deflating subspaces (Difu and Difl):
=0: Only reorder w.r.t. SELECT. No extras.
=1: Reciprocal of norms of "projections" onto left and right eigenspaces w.r.t. the selected cluster (PL and PR). =2: Upper bounds on Difu and Difl. F-norm-based estimate
(DIF(1:2)).
=3: Estimate of Difu and Difl. 1-norm-based estimate
(DIF(1:2)). About 5 times as expensive as IJOB = 2. =4: Compute PL, PR and DIF (i.e. 0, 1 and 2 above): Economic version to get it all. =5: Compute PL, PR and DIF (i.e. 0, 1 and 3 above)

WANTQ (input)

WANTZ (input)

SELECT (input)
SELECT specifies the eigenvalues in the selected cluster. To select a real eigenvalue w(j), SELECT(j) must be set to w(j) and w(j+1), corresponding to a 2-by-2 diagonal block, either SELECT(j) or SELECT(j+1) or both must be set to either both included in the cluster or both excluded.

N (input) The order of the matrices A and B. N >= 0.

A (input/output)
On entry, the upper quasi-triangular matrix A, with (A, B) in generalized real Schur canonical form. On exit, A is overwritten by the reordered matrix A.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the upper triangular matrix B, with (A, B) in generalized real Schur canonical form. On exit, B is overwritten by the reordered matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

ALPHAR (output)
On exit, (ALPHAR(j) + ALPHAI(j)*i)/BETA(j), j=1,...,N, will be the generalized eigenvalues. ALPHAR(j) + ALPHAI(j)*i and BETA(j),j=1,...,N are the diagonals of the complex Schur form (S,T) that would result if the 2-by-2 diagonal blocks of the real generalized Schur form of (A,B) were further reduced to triangular form using complex unitary transformations. If ALPHAI(j) is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-st eigenvalues are a complex conjugate pair, with ALPHAI(j+1) negative.

ALPHAI (output)
See the description of ALPHAR.

BETA (output)
See the description of ALPHAR.

Q (input/output)

On entry, if WANTQ = .TRUE., Q is an N-by-N matrix. On exit, Q has been postmultiplied by the left orthogonal transformation matrix which reorder (A, B); The leading M columns of Q form orthonormal bases for the specified pair of left eigenspaces (deflating subspaces). If WANTQ = .FALSE., Q is not referenced.

LDQ (input)

The leading dimension of the array Q. LDQ >= 1; and if WANTQ = .TRUE., LDQ >= N.

Z (input/output)

On entry, if WANTZ = .TRUE., Z is an N-by-N matrix. On exit, Z has been postmultiplied by the left orthogonal transformation matrix which reorder (A, B); The leading M columns of Z form orthonormal bases for the specified pair of left eigenspaces (deflating subspaces). If WANTZ = .FALSE., Z is not referenced.

LDZ (input)

The leading dimension of the array Z. LDZ >= 1; If WANTZ = .TRUE., LDZ >= N.

M (output)

The dimension of the specified pair of left and right eigen- spaces (deflating subspaces). 0 <= M <= N.

PL (output)

If IJOB = 1, 4 or 5, PL, PR are lower bounds on the reciprocal of the norm of "projections" onto left and right eigenspaces with respect to the selected cluster. 0 < PL, PR <= 1. If M = 0 or M = N, PL = PR = 1. If IJOB = 0, 2 or 3, PL and PR are not referenced.

PR (output)

See the description of PL.

DIF (output)

If IJOB >= 2, DIF(1:2) store the estimates of Difu and Difl.
If IJOB = 2 or 4, DIF(1:2) are F-norm-based upper bounds on
Difu and Difl. If IJOB = 3 or 5, DIF(1:2) are 1-norm-based estimates of Difu and Difl. If M = 0 or N, DIF(1:2) = F-norm([A, B]). If IJOB = 0 or 1, DIF is not referenced.

WORK (workspace)
If IJOB = 0, WORK is not referenced. Otherwise, on exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 4*N+16. If IJOB = 1, 2 or 4, LWORK >= MAX(4*N+16, 2*M*(N-M)). If IJOB = 3 or 5, LWORK >= MAX(4*N+16, 4*M*(N-M)).

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace/output)
If IJOB = 0, IWORK is not referenced. Otherwise, on exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of the array IWORK. LIWORK >= 1. If IJOB = 1, 2 or 4, LIWORK >= N+6. If IJOB = 3 or 5, LIWORK >= MAX(2*M*(N-M), N+6).

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
=0: Successful exit.
<0: If INFO = -i, the i-th argument had an illegal value.
=1: Reordering of (A, B) failed because the transformed matrix pair (A, B) would be too far from generalized Schur form; the problem is very ill-conditioned. (A, B) may have been partially reordered. If requested, 0 is returned in DIF(*), PL and PR.

FURTHER DETAILS

DTGSEN first collects the selected eigenvalues by computing orthogonal U and W that move them to the top left corner of (A, B). In other words, the selected eigenvalues are the eigenvalues of (A11, B11) in:

```
              U'*(A, B)*W = (A11 A12) (B11 B12) n1
                            ( 0  A22),( 0  B22) n2
                              n1  n2    n1  n2
```

where N = n1+n2 and U' means the transpose of U. The first n1 columns of U and W span the specified pair of left and right eigenspaces (deflating subspaces) of (A, B).

If (A, B) has been obtained from the generalized real Schur decomposition of a matrix pair (C, D) = Q*(A, B)*Z', then the reordered generalized real Schur form of (C, D) is given by

```
         (C, D) = (Q*U)*(U'*(A, B)*W)*(Z*W)',
```

and the first n1 columns of Q*U and Z*W span the corresponding deflating subspaces of (C, D) (Q and Z store Q*U and Z*W, resp.).

Note that if the selected eigenvalue is sufficiently ill-conditioned, then its value may differ significantly from its value before reordering.

The reciprocal condition numbers of the left and right eigenspaces spanned by the first n1 columns of U and W (or Q*U and Z*W) may be returned in DIF(1:2), corresponding to Difu and Difl, resp.

The Difu and Difl are defined as:
ifu[(A11, B11), (A22, B22)] = sigma-min(Zu)
and

where sigma-min(Zu) is the smallest singular value of the (2*n1*n2)-by-(2*n1*n2) matrix

```
u = [ kron(In2, A11)  -kron(A22', In1) ]
          [ kron(In2, B11)  -kron(B22', In1) ].
```

Here, Inx is the identity matrix of size nx and A22' is the transpose of A22. kron(X, Y) is the Kronecker product between the matrices X and Y.

When DIF(2) is small, small changes in (A, B) can cause large changes in the deflating subspace. An approximate (asymptotic) bound on the maximum angular error in the computed deflating subspaces is PS * norm((A, B)) / DIF(2),

where EPS is the machine precision.

```
The reciprocal norm of the projectors on the left and  right
eigenspaces associated with (A11, B11) may be returned in PL
and PR.  They are computed as follows. First  we  compute  L
and R so that P*(A, B)*Q is block diagonal, where
 = ( I -L ) n1           Q = ( I R ) n1
        ( 0  I ) n2    and        ( 0 I ) n2
          n1 n2                    n1 n2

and (L, R) is the  solution  to  the  generalized  Sylvester
equation 11*R - L*A22 = -A12

Then  PL   =   (F-norm(L)**2+1)**(-1/2)   and   PR   =   (F-
norm(R)**2+1)**(-1/2).  An approximate (asymptotic) bound on
the average absolute error of the selected eigenvalues is
PS * norm((A, B)) / PL.

There are also global error bounds which valid for perturba-
tions up to a certain restriction:  A lower bound (x) on the
smallest F-norm(E,F) for which an eigenvalue of  (A11,  B11)
may move and coalesce with an eigenvalue of (A22, B22) under
perturbation (E,F), (i.e. (A + E, B + F), is

 x                                                         =
min(Difu,Difl)/((1/(PL*PL)+1/(PR*PR))**(1/2)+2*max(1/PL,1/PR)).

An approximate bound on x can be computed from DIF(1:2),  PL
and PR.

If y = ( F-norm(E,F) / x) <= 1, the angles between the  per-
turbed  (L',  R')  and  unperturbed  (L,  R)  left and right
deflating subspaces associated with the selected cluster  in
the (1,1)-blocks can be bounded as

 max-angle(L, L') <= arctan( y * PL / (1 - y *  (1  -  PL  *
PL)**(1/2))
 max-angle(R, R') <= arctan( y * PR / (1 - y *  (1  -  PR  *
PR)**(1/2))

See LAPACK User's Guide section 4.11 or the following refer-
ences for more information.

Note  that  if  the  default  method   for   computing   the
Frobenius-norm-  based  estimate  DIF  is  not  wanted  (see
SLATDF), then the parameter IDIFJB  (see  below)  should  be
changed  from  3  to  4  (routine  SLATDF  (IJOB = 2 will be
used)). See STGSYL for more details.

Based on contributions by
   Bo Kagstrom and Peter Poromaa,  Department  of  Computing
Science,
```

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References
==========

[1] B. Kagstrom; A Direct Method for Reordering Eigenvalues in the Generalized Real Schur Form of a Regular Matrix Pair (A, B), in M.S. Moonen et al (eds), Linear Algebra for Large Scale and Real-Time Applications, Kluwer Academic Publ. 1993, pp 195-218.

[2] B. Kagstrom and P. Poromaa; Computing Eigenspaces with Specified Eigenvalues of a Regular Matrix Pair (A, B) and Condition Estimation: Theory, Algorithms and Software, Report UMINF - 94.04, Department of Computing Science, Umea University, S-901 87 Umea, Sweden, 1994. Also as LAPACK Working Note 87. To appear in Numerical Algorithms, 1996.

[3] B. Kagstrom and P. Poromaa, LAPACK-Style Algorithms and Software for Solving the Generalized Sylvester Equation and Estimating the Separation between Regular Matrix Pairs, Report UMINF - 93.23, Department of Computing Science, Umea University, S-901 87 Umea, Sweden, December 1993, Revised April 1994, Also as LAPACK Working Note 75. To appear in ACM Trans. on Math. Software, Vol 22, No 1, 1996.

Contents

NAME

```
dtgsja - compute the generalized singular  value  decomposi-
tion  (GSVD)  of  two real upper triangular (or trapezoidal)
matrices A and B
```

SYNOPSIS

```
SUBROUTINE DTGSJA(JOBU, JOBV, JOBQ, M, P, N, K, L, A, LDA, B, LDB,
      TOLA, TOLB, ALPHA, BETA, U, LDU, V, LDV, Q, LDQ, WORK, NCYCLE,
      INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
INTEGER M, P, N, K, L, LDA, LDB, LDU, LDV, LDQ, NCYCLE, INFO
DOUBLE PRECISION TOLA, TOLB
DOUBLE  PRECISION  A(LDA,*),  B(LDB,*),  ALPHA(*),  BETA(*),
U(LDU,*), V(LDV,*), Q(LDQ,*), WORK(*)

SUBROUTINE DTGSJA_64(JOBU, JOBV, JOBQ, M, P, N, K, L, A, LDA, B, LDB,
      TOLA, TOLB, ALPHA, BETA, U, LDU, V, LDV, Q, LDQ, WORK, NCYCLE,
      INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
INTEGER*8 M, P, N, K, L, LDA, LDB, LDU,  LDV,  LDQ,  NCYCLE,
INFO
DOUBLE PRECISION TOLA, TOLB
DOUBLE  PRECISION  A(LDA,*),  B(LDB,*),  ALPHA(*),  BETA(*),
U(LDU,*), V(LDV,*), Q(LDQ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TGSJA(JOBU, JOBV, JOBQ, M, P, N, K, L, A, [LDA], B, [LDB],
       TOLA, TOLB, ALPHA, BETA, U, [LDU], V, [LDV], Q, [LDQ], [WORK],
       NCYCLE, [INFO])
```

```
CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
INTEGER :: M, P, N, K, L, LDA, LDB, LDU, LDV,  LDQ,  NCYCLE,
INFO
REAL(8) :: TOLA, TOLB
REAL(8), DIMENSION(:) :: ALPHA, BETA, WORK
REAL(8), DIMENSION(:,:) :: A, B, U, V, Q

SUBROUTINE TGSJA_64(JOBU, JOBV, JOBQ, M, P, N, K, L, A, [LDA], B,
       [LDB], TOLA, TOLB, ALPHA, BETA, U, [LDU], V, [LDV], Q, [LDQ],
       [WORK], NCYCLE, [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
INTEGER(8) :: M, P, N, K, L, LDA, LDB, LDU, LDV,  LDQ,  NCY-
CLE, INFO
REAL(8) :: TOLA, TOLB
REAL(8), DIMENSION(:) :: ALPHA, BETA, WORK
REAL(8), DIMENSION(:,:) :: A, B, U, V, Q
```

C INTERFACE

```
#include <sunperf.h>

void dtgsja(char jobu, char jobv, char jobq, int m,  int  p,
          int  n,  int  k, int l, double *a, int lda, double
          *b, int ldb,  double  tola,  double  tolb,  double
          *alpha,  double  *beta, double *u, int ldu, double
          *v, int ldv, double *q, int ldq, int *ncycle,  int
          *info);

void dtgsja_64(char jobu, char jobv, char jobq, long m, long
          p,  long  n,  long k, long l, double *a, long lda,
          double *b, long ldb,  double  tola,  double  tolb,
          double  *alpha, double *beta, double *u, long ldu,
          double *v, long ldv, double  *q,  long  ldq,  long
          *ncycle, long *info);
```

PURPOSE

dtgsja computes the generalized singular value decomposition (GSVD) of two real upper triangular (or trapezoidal) matrices A and B.

On entry, it is assumed that matrices A and B have the following forms, which may be obtained by the preprocessing subroutine SGGSVP from a general M-by-N matrix A and P-by-N matrix B:

```
             N-K-L  K    L
   A =    K ( 0    A12  A13 ) if M-K-L >= 0;
          L ( 0     0   A23 )
      M-K-L ( 0     0    0  )
```

```
           N-K-L  K    L
   A =  K ( 0    A12  A13 ) if M-K-L < 0;
      M-K ( 0     0   A23 )

           N-K-L  K    L
   B =  L ( 0     0   B13 )
      P-L ( 0     0    0  )
```

where the K-by-K matrix A12 and L-by-L matrix B13 are non-singular upper triangular; A23 is L-by-L upper triangular if M-K-L >= 0, otherwise A23 is (M-K)-by-L upper trapezoidal.

On exit,

```
            U'*A*Q = D1*( 0 R ),    V'*B*Q = D2*( 0 R ),
```

where U, V and Q are orthogonal matrices, Z' denotes the transpose of Z, R is a nonsingular upper triangular matrix, and D1 and D2 are ``diagonal'' matrices, which are of the following structures:

If M-K-L >= 0,

```
                    K  L
       D1 =     K ( I  0 )
                L ( 0  C )
            M-K-L ( 0  0 )

                  K  L
       D2 = L   ( 0  S )
            P-L ( 0  0 )

              N-K-L  K    L
  ( 0 R ) = K (  0   R11  R12 ) K
            L (  0    0   R22 ) L
```

where

```
  C = diag( ALPHA(K+1), ... , ALPHA(K+L) ),
  S = diag( BETA(K+1),  ... , BETA(K+L) ),
  C**2 + S**2 = I.

  R is stored in A(1:K+L,N-K-L+1:N) on exit.
```

If M-K-L < 0,

```
              K M-K K+L-M
    D1 =   K ( I  0    0   )
         M-K ( 0  C    0   )
```

```
                 K M-K K+L-M
    D2 =   M-K ( 0  S    0   )
         K+L-M ( 0  0    I   )
           P-L ( 0  0    0   )

               N-K-L  K   M-K  K+L-M

          M-K ( 0     0   R22  R23  )
        K+L-M ( 0     0    0   R33  )
```

where
C = diag(ALPHA(K+1), ... , ALPHA(M)),
S = diag(BETA(K+1), ... , BETA(M)),
C**2 + S**2 = I.

R = (R11 R12 R13) is stored in A(1:M, N-K-L+1:N) and R33 is stored
    (0 R22 R23)
in B(M-K+1:L,N+M-K-L+1:N) on exit.

The computation of the orthogonal transformation matrices U, V or Q is optional. These matrices may either be formed explicitly, or they may be postmultiplied into input matrices U1, V1, or Q1.
STGSJA essentially uses a variant of Kogbetliantz algorithm to reduce min(L,M-K)-by-L triangular (or trapezoidal) matrix A23 and L-by-L matrix B13 to the form:
   U1'*A13*Q1 = C1*R1; V1'*B13*Q1 = S1*R1,
where U1, V1 and Q1 are orthogonal matrix, and Z' is the transpose of Z. C1 and S1 are diagonal matrices satisfying
   C1**2 + S1**2 = I,
and R1 is an L-by-L nonsingular upper triangular matrix.

ARGUMENTS

JOBU (input)
= 'U': U must contain an orthogonal matrix U1 on entry, and the product U1*U is returned; = 'I': U is initialized to the unit matrix, and the orthogonal matrix U is returned; = 'N': U is not computed.

JOBV (input)
= 'V': V must contain an orthogonal matrix V1 on entry, and the product V1*V is returned; = 'I': V is initialized to the unit matrix, and the orthogonal matrix V is returned; = 'N': V is not computed.

JOBQ (input)
= 'Q': Q must contain an orthogonal matrix Q1 on entry, and the product Q1*Q is returned; = 'I': Q is initialized to the unit matrix, and the orthogonal matrix Q is returned; = 'N': Q is not computed.

M (input) The number of rows of the matrix A. M >= 0.

P (input) The number of rows of the matrix B. P >= 0.

N (input) The number of columns of the matrices A and B. N >= 0.

K (input) K and L specify the subblocks in the input matrices A and B:
A23 = A(K+1:MIN(K+L,M),N-L+1:N) and B13 = B(1:L,N-L+1:N) of A and B, whose GSVD is going to be computed by STGSJA. See Further details.

L (input) See the description of K.

A (input/output)
On entry, the M-by-N matrix A. On exit, A(N-K+1:N,1:MIN(K+L,M)) contains the triangular matrix R or part of R. See Purpose for details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the P-by-N matrix B. On exit, if necessary, B(M-K+1:L,N+M-K-L+1:N) contains a part of R. See Purpose for details.

LDB (input)
The leading dimension of the array B. LDB >= max(1,P).

TOLA (input)
TOLA and TOLB are the convergence criteria for the Jacobi- Kogbetliantz iteration procedure. Generally, they are the same as used in the preprocessing step, say TOLA = max(M,N)*norm(A)*MACHEPS, TOLB = max(P,N)*norm(B)*MACHEPS.

TOLB (input)
See the description of TOLA.

```
ALPHA (output)
          On exit, ALPHA and BETA  contain  the  generalized
          singular value pairs of A and B; ALPHA(1:K) = 1,
          BETA(1:K)  = 0, and if M-K-L >= 0,  ALPHA(K+1:K+L)
          = diag(C),
          BETA(K+1:K+L)   =  diag(S),  or  if  M-K-L  <   0,
          ALPHA(K+1:M)= C, ALPHA(M+1:K+L)= 0
          BETA(K+1:M) = S, BETA(M+1:K+L) = 1.   Furthermore,
          if K+L < N, ALPHA(K+L+1:N) = 0 and
          BETA(K+L+1:N)  = 0.
BETA (output)
          See the description of ALPHA.

U (input) On entry, if JOBU = 'U', U must contain  a  matrix
          U1  (usually  the  orthogonal  matrix  returned by
          SGGSVP).  On exit, if JOBU = 'I', U  contains  the
          orthogonal matrix U; if JOBU = 'U', U contains the
          product U1*U.  If JOBU = 'N', U is not referenced.

LDU (input)
          The leading dimension  of  the  array  U.  LDU  >=
          max(1,M) if JOBU = 'U'; LDU >= 1 otherwise.

V (input) On entry, if JOBV = 'V', V must contain  a  matrix
          V1  (usually  the  orthogonal  matrix  returned by
          SGGSVP).  On exit, if JOBV = 'I', V  contains  the
          orthogonal matrix V; if JOBV = 'V', V contains the
          product V1*V.  If JOBV = 'N', V is not referenced.

LDV (input)
          The leading dimension  of  the  array  V.  LDV  >=
          max(1,P) if JOBV = 'V'; LDV >= 1 otherwise.

Q (input) On entry, if JOBQ = 'Q', Q must contain  a  matrix
          Q1  (usually  the  orthogonal  matrix  returned by
          SGGSVP).  On exit, if JOBQ = 'I', Q  contains  the
          orthogonal matrix Q; if JOBQ = 'Q', Q contains the
          product Q1*Q.  If JOBQ = 'N', Q is not referenced.

LDQ (input)
          The leading dimension  of  the  array  Q.  LDQ  >=
          max(1,N) if JOBQ = 'Q'; LDQ >= 1 otherwise.

WORK (workspace)
          dimension(2*N)

NCYCLE (output)
          The number of cycles required for convergence.
```

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
= 1: the procedure does not converge after MAXIT cycles.

Contents

NAME

```
dtgsna - estimate reciprocal condition numbers for specified
eigenvalues  and/or  eigenvectors of a matrix pair (A, B) in
generalized real Schur canonical form (or of any matrix pair
(Q*A*Z',  Q*B*Z') with orthogonal matrices Q and Z, where Z'
denotes the transpose of Z
```

SYNOPSIS

```
SUBROUTINE DTGSNA(JOB, HOWMNT, SELECT, N, A, LDA, B, LDB, VL, LDVL,
      VR, LDVR, S, DIF, MM, M, WORK, LWORK, IWORK, INFO)

CHARACTER * 1 JOB, HOWMNT
INTEGER N, LDA, LDB, LDVL, LDVR, MM, M, LWORK, INFO
INTEGER IWORK(*)
LOGICAL SELECT(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*), VL(LDVL,*), VR(LDVR,*),
S(*), DIF(*), WORK(*)

SUBROUTINE DTGSNA_64(JOB, HOWMNT, SELECT, N, A, LDA, B, LDB, VL,
      LDVL, VR, LDVR, S, DIF, MM, M, WORK, LWORK, IWORK, INFO)

CHARACTER * 1 JOB, HOWMNT
INTEGER*8 N, LDA, LDB, LDVL, LDVR, MM, M, LWORK, INFO
INTEGER*8 IWORK(*)
LOGICAL*8 SELECT(*)
DOUBLE PRECISION A(LDA,*), B(LDB,*), VL(LDVL,*), VR(LDVR,*),
S(*), DIF(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TGSNA(JOB, HOWMNT, SELECT, [N], A, [LDA], B, [LDB], VL,
       [LDVL], VR, [LDVR], S, DIF, MM, M, [WORK], [LWORK], [IWORK],
```

```
       [INFO])

CHARACTER(LEN=1) :: JOB, HOWMNT
INTEGER :: N, LDA, LDB, LDVL, LDVR, MM, M, LWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
LOGICAL, DIMENSION(:) :: SELECT
REAL(8), DIMENSION(:) :: S, DIF, WORK
REAL(8), DIMENSION(:,:) :: A, B, VL, VR

SUBROUTINE TGSNA_64(JOB, HOWMNT, SELECT, [N], A, [LDA], B, [LDB], VL,
       [LDVL], VR, [LDVR], S, DIF, MM, M, [WORK], [LWORK], [IWORK],
       [INFO])

CHARACTER(LEN=1) :: JOB, HOWMNT
INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, MM, M, LWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
LOGICAL(8), DIMENSION(:) :: SELECT
REAL(8), DIMENSION(:) :: S, DIF, WORK
REAL(8), DIMENSION(:,:) :: A, B, VL, VR
```

C INTERFACE

```
#include <sunperf.h>

void dtgsna(char job, char howmnt, int *select, int n,  dou-
          ble  *a,  int lda, double *b, int ldb, double *vl,
          int ldvl, double *vr, int ldvr, double *s,  double
          *dif, int mm, int *m, int *info);

void dtgsna_64(char job, char howmnt, long *select, long  n,
          double  *a,  long lda, double *b, long ldb, double
          *vl, long ldvl, double *vr, long ldvr, double  *s,
          double *dif, long mm, long *m, long *info);
```

PURPOSE

dtgsna estimates reciprocal condition numbers for specified eigenvalues and/or eigenvectors of a matrix pair (A, B) in generalized real Schur canonical form (or of any matrix pair (Q*A*Z', Q*B*Z') with orthogonal matrices Q and Z, where Z' denotes the transpose of Z.

(A, B) must be in generalized real Schur form (as returned by SGGES), i.e. A is block upper triangular with 1-by-1 and 2-by-2 diagonal blocks. B is upper triangular.

ARGUMENTS

JOB (input)
Specifies whether condition numbers are required for eigenvalues (S) or eigenvectors (DIF):
= 'E': for eigenvalues only (S);
= 'V': for eigenvectors only (DIF);
= 'B': for both eigenvalues and eigenvectors (S and DIF).

HOWMNT (input)
= 'A': compute condition numbers for all eigenpairs;
= 'S': compute condition numbers for selected eigenpairs specified by the array SELECT.

SELECT (input)
If HOWMNT = 'S', SELECT specifies the eigenpairs for which condition numbers are required. To select condition numbers for the eigenpair corresponding to a real eigenvalue w(j), SELECT(j) must be set to .TRUE.. To select condition numbers corresponding to a complex conjugate pair of eigenvalues w(j) and w(j+1), either SELECT(j) or SELECT(j+1) or both, must be set to .TRUE.. If HOWMNT = 'A', SELECT is not referenced.

N (input) The order of the square matrix pair (A, B). N >= 0.

A (input) The upper quasi-triangular matrix A in the pair (A,B).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input) The upper triangular matrix B in the pair (A,B).

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

VL (input)
If JOB = 'E' or 'B', VL must contain left eigenvectors of (A, B), corresponding to the eigenpairs specified by HOWMNT and SELECT. The eigenvectors must be stored in consecutive columns of VL, as returned by STGEVC. If JOB = 'V', VL is not referenced.

LDVL (input)

The leading dimension of the array VL. LDVL >= 1. If JOB = 'E' or 'B', LDVL >= N.

VR (input)
If JOB = 'E' or 'B', VR must contain right eigenvectors of (A, B), corresponding to the eigenpairs specified by HOWMNT and SELECT. The eigenvectors must be stored in consecutive columns ov VR, as returned by STGEVC. If JOB = 'V', VR is not referenced.

LDVR (input)
The leading dimension of the array VR. LDVR >= 1. If JOB = 'E' or 'B', LDVR >= N.

S (output)
If JOB = 'E' or 'B', the reciprocal condition numbers of the selected eigenvalues, stored in consecutive elements of the array. For a complex conjugate pair of eigenvalues two consecutive elements of S are set to the same value. Thus S(j), DIF(j), and the j-th columns of VL and VR all correspond to the same eigenpair (but not in general the j-th eigenpair, unless all eigenpairs are selected). If JOB = 'V', S is not referenced.

DIF (output)
If JOB = 'V' or 'B', the estimated reciprocal condition numbers of the selected eigenvectors, stored in consecutive elements of the array. For a complex eigenvector two consecutive elements of DIF are set to the same value. If the eigenvalues cannot be reordered to compute DIF(j), DIF(j) is set to 0; this can only occur when the true value would be very small anyway. If JOB = 'E', DIF is not referenced.

MM (input)
The number of elements in the arrays S and DIF. MM >= M.

M (output)
The number of elements of the arrays S and DIF used to store the specified condition numbers; for each selected real eigenvalue one element is used, and for each selected complex conjugate pair of eigenvalues, two elements are used. If HOWMNT = 'A', M is set to N.

WORK (workspace)
If JOB = 'E', WORK is not referenced. Otherwise,

```
          on  exit, if INFO = 0, WORK(1) returns the optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK. LWORK >=  N.   If
          JOB = 'V' or 'B' LWORK >= 2*N*(N+2)+16.
          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

IWORK (workspace)
          dimension(N+6) If JOB = 'E', IWORK is  not  refer-
          enced.

INFO (output)
          =0: Successful exit
          <0: If INFO = -i, the i-th argument had an illegal
          value
```

FURTHER DETAILS

```
The reciprocal of the  condition  number  of  a  generalized
eigenvalue w = (a, b) is defined as
(w) = (|u'Av|**2 + |u'Bv|**2)**(1/2) / (norm(u)*norm(v))

where u and v are the left and right eigenvectors of (A,  B)
corresponding  to  w;  |z| denotes the absolute value of the
complex number, and norm(u) denotes the 2-norm of the vector
u.
The pair (a, b) corresponds to an  eigenvalue  w  =  a/b  (=
u'Av/u'Bv)  of the matrix pair (A, B). If both a and b equal
zero, then (A B) is singular and S(I) = -1 is returned.

An approximate error bound on the chordal  distance  between
the   i-th   computed   generalized  eigenvalue  w  and  the
corresponding exact eigenvalue lambda is
hord(w, lambda) <= EPS * norm(A, B) / S(I)

where EPS is the machine precision.

The reciprocal of  the  condition  number  DIF(i)  of  right
eigenvector  u  and  left eigenvector v corresponding to the
generalized eigenvalue w is defined as follows:

a) If the i-th eigenvalue w = (a,b) is real

   Suppose U and V are orthogonal transformations such that
```

```
              U'*(A, B)*V  = (S, T) = ( a   *  ) ( b   *   )
1
                                        ( 0  S22 ),( 0  T22  )
n-1
                                          1  n-1     1 n-1

   Then the reciprocal condition number DIF(i) is
              Difl((a, b), (S22, T22)) = sigma-min( Zl ),

   where sigma-min(Zl) denotes the smallest  singular  value
of the
   2(n-1)-by-2(n-1) matrix

       Zl = [ kron(a, In-1)  -kron(1, S22) ]
            [ kron(b, In-1)  -kron(1, T22) ] .

   Here In-1 is the identity matrix of size n-1. kron(X,  Y)
is the
   Kronecker product between the matrices X and Y.

   Note that if the default method for computing  DIF(i)  is
wanted
   (see SLATDF),  then  the  parameter  DIFDRI  (see  below)
should be
   changed from 3 to 4 (routine  SLATDF(IJOB  =  2  will  be
used)).
   See STGSYL for more details.

b) If the i-th and (i+1)-th eigenvalues are  complex  conju-
gate pair,

   Suppose U and V are orthogonal transformations such that

              U'*(A, B)*V = (S, T) = ( S11  *   ) (  T11   *
)  2
                                       (  0      S22   ),(   0
T22) n-2
                                         2    n-2     2    n-2

   and (S11,  T11)  corresponds  to  the  complex  conjugate
eigenvalue
   pair (w, conjg(w)). There exist unitary matrices  U1  and
V1 such
   that

       U1'*S11*V1 = ( s11 s12 )   and U1'*T11*V1 = ( t11 t12
)
                    (  0  s22 )                    (  0  t22
)
```

```
   where the generalized eigenvalues w = s11/t11 and
   conjg(w) = s22/t22.

   Then the reciprocal condition number DIF(i) is bounded by

       min( d1, max( 1, |real(s11)/real(s22)| )*d2 )

   where, d1 = Difl((s11, t11), (s22, t22)) = sigma-min(Z1),
where
   Z1 is the complex 2-by-2 matrix

            Z1 =  [ s11  -s22 ]
                  [ t11  -t22 ],

   This is done by computing (using real arithmetic) the
   roots of the characteristical polynomial det(Z1' *  Z1  -
lambda I),
   where Z1' denotes  the  conjugate  transpose  of  Z1  and
det(X) denotes
   the determinant of X.

   and d2 is an upper bound on Difl((S11, T11), (S22, T22)),
i.e. an
   upper bound on sigma-min(Z2), where Z2 is  (2n-2)-by-(2n-
2)

            Z2 = [ kron(S11', In-2)  -kron(I2, S22) ]
                 [ kron(T11', In-2)  -kron(I2, T22) ]

   Note that if the default  method  for  computing  DIF  is
wanted (see
   SLATDF), then the parameter DIFDRI (see below) should  be
changed
   from 3 to 4 (routine SLATDF(IJOB = 2 will be used)).  See
STGSYL
   for more details.

For each eigenvalue/vector specified by SELECT, DIF stores a
Frobenius norm-based estimate of Difl.

An approximate error bound for the i-th computed eigenvector
VL(i) or VR(i) is given by

           EPS * norm(A, B) / DIF(i).

See ref. [2-3] for more details and further references.

Based on contributions by
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Science,
```

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References

[1] B. Kagstrom; A Direct Method for Reordering Eigenvalues in the Generalized Real Schur Form of a Regular Matrix Pair (A, B), in M.S. Moonen et al (eds), Linear Algebra for Large Scale and Real-Time Applications, Kluwer Academic Publ. 1993, pp 195-218.

[2] B. Kagstrom and P. Poromaa; Computing Eigenspaces with Specified Eigenvalues of a Regular Matrix Pair (A, B) and Condi- tion Estimation: Theory, Algorithms and Software, Report UMINF - 94.04, Department of Computing Science, Umea University, S-901 87 Umea, Sweden, 1994. Also as LAPACK Working Note 87. To appear in Numerical Algorithms, 1996.

[3] B. Kagstrom and P. Poromaa, LAPACK-Style Algorithms and Software for Solving the Generalized Sylvester Equation and Estimating the Separation between Regular Matrix Pairs, Report UMINF - 93.23, Department of Computing Science, Umea University, S-901 87 Umea, Sweden, December 1993, Revised April 1994, Also as LAPACK Working Note 75. To appear in ACM Trans. on Math. Software, Vol 22, No 1, 1996.

Contents

NAME

```
dtgsyl - solve the generalized Sylvester equation
```

SYNOPSIS

```
SUBROUTINE DTGSYL(TRANS, IJOB, M, N, A, LDA, B, LDB, C, LDC, D, LDD,
      E, LDE, F, LDF, SCALE, DIF, WORK, LWORK, IWORK, INFO)

CHARACTER * 1 TRANS
INTEGER IJOB, M, N, LDA, LDB, LDC,  LDD,  LDE,  LDF,  LWORK,
INFO
INTEGER IWORK(*)
DOUBLE PRECISION SCALE, DIF
DOUBLE PRECISION  A(LDA,*),  B(LDB,*),  C(LDC,*),  D(LDD,*),
E(LDE,*), F(LDF,*), WORK(*)

SUBROUTINE DTGSYL_64(TRANS, IJOB, M, N, A, LDA, B, LDB, C, LDC, D,
      LDD, E, LDE, F, LDF, SCALE, DIF, WORK, LWORK, IWORK, INFO)

CHARACTER * 1 TRANS
INTEGER*8 IJOB, M, N, LDA, LDB, LDC, LDD, LDE,  LDF,  LWORK,
INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION SCALE, DIF
DOUBLE PRECISION  A(LDA,*),  B(LDB,*),  C(LDC,*),  D(LDD,*),
E(LDE,*), F(LDF,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TGSYL(TRANS, IJOB, [M], [N], A, [LDA], B, [LDB], C, [LDC],
       D, [LDD], E, [LDE], F, [LDF], SCALE, DIF, [WORK], [LWORK], [IWORK],
       [INFO])
```

```
CHARACTER(LEN=1) :: TRANS
INTEGER :: IJOB, M, N, LDA, LDB, LDC, LDD, LDE, LDF,  LWORK,
INFO
INTEGER, DIMENSION(:) :: IWORK
REAL(8) :: SCALE, DIF
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A, B, C, D, E, F

SUBROUTINE TGSYL_64(TRANS, IJOB, [M], [N], A, [LDA], B, [LDB], C,
       [LDC], D, [LDD], E, [LDE], F, [LDF], SCALE, DIF, [WORK], [LWORK],
       [IWORK], [INFO])

CHARACTER(LEN=1) :: TRANS
INTEGER(8) :: IJOB, M, N, LDA,  LDB,  LDC,  LDD,  LDE,  LDF,
LWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8) :: SCALE, DIF
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A, B, C, D, E, F
```

C INTERFACE

```
#include <sunperf.h>

void dtgsyl(char trans, int ijob, int m, int n,  double  *a,
          int  lda,  double *b, int ldb, double *c, int ldc,
          double *d, int ldd, double *e, int lde, double *f,
          int ldf, double *scale, double *dif, int *info);

void dtgsyl_64(char trans, long ijob, long m, long n, double
          *a, long lda, double *b, long ldb, double *c, long
          ldc, double *d, long ldd,  double  *e,  long  lde,
          double  *f,  long ldf, double *scale, double *dif,
          long *info);
```

PURPOSE

dtgsyl solves the generalized Sylvester equation:

```
            A * R - L * B = scale * C                 (1)
            D * R - L * E = scale * F
```

where R and L are unknown m-by-n matrices, (A, D), (B, E) and (C, F) are given matrix pairs of size m-by-m, n-by-n and m-by-n, respectively, with real entries. (A, D) and (B, E) must be in generalized (real) Schur canonical form, i.e. A, B are upper quasi triangular and D, E are upper triangular.

The solution (R, L) overwrites (C, F). 0 <= SCALE <= 1 is an output scaling factor chosen to avoid overflow.

In matrix notation (1) is equivalent to solve Zx = scale b, where Z is defined as

```
           Z = [ kron(In, A)  -kron(B', Im) ]         (2)
               [ kron(In, D)  -kron(E', Im) ].
```

Here Ik is the identity matrix of size k and X' is the transpose of X. kron(X, Y) is the Kronecker product between the matrices X and Y.

If TRANS = 'T', STGSYL solves the transposed system Z'*y = scale*b, which is equivalent to solve for R and L in

```
            A' * R  + D' * L   = scale *  C           (3)
            R  * B' + L  * E'  = scale * (-F)
```

This case (TRANS = 'T') is used to compute an one-norm-based estimate of Dif[(A,D), (B,E)], the separation between the matrix pairs (A,D) and (B,E), using SLACON.

If IJOB >= 1, STGSYL computes a Frobenius norm-based estimate of Dif[(A,D),(B,E)]. That is, the reciprocal of a lower bound on the reciprocal of the smallest singular value of Z. See [1-2] for more information.

This is a level 3 BLAS algorithm.

ARGUMENTS

TRANS (input)
: = 'N', solve the generalized Sylvester equation (1). = 'T', solve the 'transposed' system (3).

IJOB (input)
: Specifies what kind of functionality to be performed. =0: solve (1) only.
=1: The functionality of 0 and 3.
=2: The functionality of 0 and 4.
=3: Only an estimate of Dif[(A,D), (B,E)] is computed. (look ahead strategy IJOB = 1 is used).
=4: Only an estimate of Dif[(A,D), (B,E)] is computed. (SGECON on sub-systems is used). Not referenced if TRANS = 'T'.

M (input)
: The order of the matrices A and D, and the row dimension of the matrices C, F, R and L.

```
N (input) The order of the matrices B and E, and the  column
          dimension of the matrices C, F, R and L.

A (input) The upper quasi triangular matrix A.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1, M).

B (input) The upper quasi triangular matrix B.

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          max(1, N).
C (input/output)
          On entry, C contains the  right-hand-side  of  the
          first  matrix equation in (1) or (3).  On exit, if
          IJOB = 0, 1 or 2, C has been  overwritten  by  the
          solution  R.  If  IJOB = 3 or 4 and TRANS = 'N', C
          holds R, the solution achieved during the computa-
          tion of the Dif-estimate.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1, M).

D (input) The upper triangular matrix D.

LDD (input)
          The leading dimension  of  the  array  D.  LDD  >=
          max(1, M).

E (input) The upper triangular matrix E.

LDE (input)
          The leading dimension  of  the  array  E.  LDE  >=
          max(1, N).

F (input/output)
          On entry, F contains the  right-hand-side  of  the
          second matrix equation in (1) or (3).  On exit, if
          IJOB = 0, 1 or 2, F has been  overwritten  by  the
          solution  L.  If  IJOB = 3 or 4 and TRANS = 'N', F
          holds L, the solution achieved during the computa-
          tion of the Dif-estimate.

LDF (input)
          The leading dimension  of  the  array  F.  LDF  >=
          max(1, M).
```

SCALE (output)
On exit SCALE is the reciprocal of a lower bound of the reciprocal of the Dif-function, i.e. SCALE is an upper bound of Dif[(A,D), (B,E)] = sigma_min(Z), where Z as in (2). If IJOB = 0 or TRANS = 'T', SCALE is not touched.

DIF (output)
On exit SCALE is the reciprocal of a lower bound of the reciprocal of the Dif-function, i.e. SCALE is an upper bound of Dif[(A,D), (B,E)] = sigma_min(Z), where Z as in (2). If IJOB = 0 or TRANS = 'T', SCALE is not touched.

WORK (workspace)
If IJOB = 0, WORK is not referenced. Otherwise, on exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK > = 1. If IJOB = 1 or 2 and TRANS = 'N', LWORK >= 2*M*N.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace)
dimension(M+N+2)

INFO (output)
=0: successful exit
<0: If INFO = -i, the i-th argument had an illegal value.
>0: (A, D) and (B, E) have common or close eigen- values.

FURTHER DETAILS

Based on contributions by
Bo Kagstrom and Peter Poromaa, Department of Computing Science,
Umea University, S-901 87 Umea, Sweden.

[1] B. Kagstrom and P. Poromaa, LAPACK-Style Algorithms and Software
for Solving the Generalized Sylvester Equation and Estimating the
Separation between Regular Matrix Pairs, Report UMINF -

93.23,
    Department of Computing Science, Umea University, S-901
87 Umea,
    Sweden, December 1993, Revised April 1994, Also as
LAPACK Working
    Note 75. To appear in ACM Trans. on Math. Software, Vol
22,
    No 1, 1996.

[2] B. Kagstrom, A Perturbation Analysis of the Generalized
Sylvester
    Equation (AR - LB, DR - LE) = (C, F), SIAM J. Matrix
Anal.
    Appl., 15(4):1045-1060, 1994

[3] B. Kagstrom and L. Westin, Generalized Schur Methods
with
    Condition Estimators for Solving the Generalized Sylves-
ter
    Equation, IEEE Transactions on Automatic Control, Vol.
34, No. 7,
    July 1989, pp 745-751.

Contents

NAME

```
dtpcon - estimate the reciprocal of the condition number  of
a  packed  triangular  matrix A, in either the 1-norm or the
infinity-norm
```

SYNOPSIS

```
SUBROUTINE DTPCON(NORM, UPLO, DIAG, N, A, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 NORM, UPLO, DIAG
INTEGER N, INFO
INTEGER WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(*), WORK(*)

SUBROUTINE DTPCON_64(NORM, UPLO, DIAG, N, A, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 NORM, UPLO, DIAG
INTEGER*8 N, INFO
INTEGER*8 WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TPCON(NORM, UPLO, DIAG, N, A, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: A, WORK
```

```
SUBROUTINE TPCON_64(NORM, UPLO, DIAG, N, A, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: A, WORK
```

C INTERFACE

```
#include <sunperf.h>

void dtpcon(char norm, char uplo, char diag, int  n,  double
          *a, double *rcond, int *info);
void dtpcon_64(char norm, char uplo, char diag, long n, dou-
          ble *a, double *rcond, long *info);
```

PURPOSE

dtpcon estimates the reciprocal of the condition number of a packed triangular matrix A, in either the 1-norm or the infinity-norm.

The norm of A is computed and an estimate is obtained for norm(inv(A)), then the reciprocal of the condition number is computed as

RCOND = 1 / (norm(A) * norm(inv(A))).

ARGUMENTS

```
NORM (input)
          Specifies whether the 1-norm condition  number  or
          the infinity-norm condition number is required:
          = '1' or 'O':  1-norm;
          = 'I':         Infinity-norm.

UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.
```

A (input) The upper or lower triangular matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n. If DIAG = 'U', the diagonal elements of A are not referenced and are assumed to be 1.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(norm(A) * norm(inv(A))).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dtpmv - perform one of the  matrix-vector  operations  x  :=
A*x, or x := A'*x
```

SYNOPSIS

```
SUBROUTINE DTPMV(UPLO, TRANSA, DIAG, N, A, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, INCY
DOUBLE PRECISION A(*), Y(*)

SUBROUTINE DTPMV_64(UPLO, TRANSA, DIAG, N, A, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, INCY
DOUBLE PRECISION A(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE TPMV(UPLO, [TRANSA], DIAG, [N], A, Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, INCY
REAL(8), DIMENSION(:) :: A, Y

SUBROUTINE TPMV_64(UPLO, [TRANSA], DIAG, [N], A, Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, INCY
REAL(8), DIMENSION(:) :: A, Y
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dtpmv(char uplo, char transa, char diag, int n,  double
          *a, double *y, int incy);

void dtpmv_64(char uplo, char transa,  char  diag,  long  n,
          double *a, double *y, long incy);
```

PURPOSE

dtpmv performs one of the matrix-vector operations x := A*x, or x := A'*x, where x is an n element vector and A is an n by n unit, or non-unit, upper or lower triangular matrix, supplied in packed form.

ARGUMENTS

```
UPLO (input)
          On entry, UPLO specifies whether the matrix is  an
          upper or lower triangular matrix as follows:

          UPLO = 'U' or  'u'    A  is  an  upper  triangular
          matrix.

          UPLO = 'L'  or  'l'    A  is  a  lower  triangular
          matrix.

          Unchanged on exit.

TRANSA (input)
          On entry, TRANSA specifies  the  operation  to  be
          performed as follows:

          TRANSA = 'N' or 'n'   x := A*x.

          TRANSA = 'T' or 't'   x := A'*x.

          TRANSA = 'C' or 'c'   x := A'*x.

          Unchanged on exit.

          TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
          On entry, DIAG specifies whether or not A is  unit
          triangular as follows:

          DIAG = 'U' or 'u'   A is assumed to be  unit  tri-
```

angular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

N (input)

On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

A (input)

((n*(n + 1))/2). Before entry with UPLO = 'U' or 'u', the array A must contain the upper triangular matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(1, 2) and a(2, 2) respectively, and so on. Before entry with UPLO = 'L' or 'l', the array A must contain the lower triangular matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(2, 1) and a(3, 1) respectively, and so on. Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced, but are assumed to be unity. Unchanged on exit.

Y (input/output)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector x. On exit, Y is overwritten with the tranformed vector x.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
dtprfs - provide error bounds and backward  error  estimates
for the solution to a system of linear equations with a tri-
angular packed coefficient matrix
```

SYNOPSIS

```
SUBROUTINE DTPRFS(UPLO, TRANSA, DIAG, N, NRHS, A, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER WORK2(*)
DOUBLE PRECISION A(*), B(LDB,*), X(LDX,*), FERR(*), BERR(*),
WORK(*)

SUBROUTINE DTPRFS_64(UPLO, TRANSA, DIAG, N, NRHS, A, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 WORK2(*)
DOUBLE PRECISION A(*), B(LDB,*), X(LDX,*), FERR(*), BERR(*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TPRFS(UPLO, [TRANSA], DIAG, N, NRHS, A, B, [LDB], X, [LDX],
       FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL(8), DIMENSION(:) :: A, FERR, BERR, WORK
```

```
REAL(8), DIMENSION(:,:) :: B, X

SUBROUTINE TPRFS_64(UPLO, [TRANSA], DIAG, N, NRHS, A, B, [LDB], X,
       [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL(8), DIMENSION(:) :: A, FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: B, X
```

C INTERFACE

```
#include <sunperf.h>
void dtprfs(char uplo, char transa, char diag,  int  n,  int
          nrhs,  double  *a,  double *b, int ldb, double *x,
          int ldx, double *ferr, double *berr, int *info);

void dtprfs_64(char uplo, char transa, char  diag,  long  n,
          long  nrhs, double *a, double *b, long ldb, double
          *x, long ldx, double  *ferr,  double  *berr,  long
          *info);
```

PURPOSE

dtprfs provides error bounds and backward error estimates for the solution to a system of linear equations with a triangular packed coefficient matrix.

The solution matrix X must be computed by STPTRS or some other means before entering this routine. STPRFS does not do iterative refinement because doing so cannot improve the backward error.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B  (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose = Tran-
          spose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.
```

DIAG (input)
= 'N': A is non-unit triangular;
= 'U': A is unit triangular.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The upper or lower triangular matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n. If DIAG = 'U', the diagonal elements of A are not referenced and are assumed to be 1.

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input) The solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dtpsv - solve one of the systems of equations A*x  =  b,  or
A'*x = b
```

SYNOPSIS

```
SUBROUTINE DTPSV(UPLO, TRANSA, DIAG, N, A, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, INCY
DOUBLE PRECISION A(*), Y(*)

SUBROUTINE DTPSV_64(UPLO, TRANSA, DIAG, N, A, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, INCY
DOUBLE PRECISION A(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE TPSV(UPLO, [TRANSA], DIAG, [N], A, Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, INCY
REAL(8), DIMENSION(:) :: A, Y

SUBROUTINE TPSV_64(UPLO, [TRANSA], DIAG, [N], A, Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, INCY
REAL(8), DIMENSION(:) :: A, Y
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dtpsv(char uplo, char transa, char diag, int n,  double
          *a, double *y, int incy);

void dtpsv_64(char uplo, char transa,  char  diag,  long  n,
          double *a, double *y, long incy);
```

PURPOSE

dtpsv solves one of the systems of equations A*x = b, or A'*x = b, where b and x are n element vectors and A is an n by n unit, or non-unit, upper or lower triangular matrix, supplied in packed form.

No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the equations to be solved as follows:

TRANSA = 'N' or 'n'   A*x = b.

TRANSA = 'T' or 't'   A'*x = b.

TRANSA = 'C' or 'c'   A'*x = b.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)

On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u' A is assumed to be unit triangular.

DIAG = 'N' or 'n' A is not assumed to be unit triangular.

Unchanged on exit.

N (input)

On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

A (input)

((n*(n + 1))/2). Before entry with UPLO = 'U' or 'u', the array A must contain the upper triangular matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(1, 2) and a(2, 2) respectively, and so on. Before entry with UPLO = 'L' or 'l', the array A must contain the lower triangular matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(2, 1) and a(3, 1) respectively, and so on. Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced, but are assumed to be unity. Unchanged on exit.

Y (input/output)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element right-hand side vector b. On exit, Y is overwritten with the solution vector x.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
dtptri - compute the inverse of a real upper or  lower  tri-
angular matrix A stored in packed format
```

SYNOPSIS

```
SUBROUTINE DTPTRI(UPLO, DIAG, N, A, INFO)

CHARACTER * 1 UPLO, DIAG
INTEGER N, INFO
DOUBLE PRECISION A(*)

SUBROUTINE DTPTRI_64(UPLO, DIAG, N, A, INFO)

CHARACTER * 1 UPLO, DIAG
INTEGER*8 N, INFO
DOUBLE PRECISION A(*)
```

F95 INTERFACE

```
SUBROUTINE TPTRI(UPLO, DIAG, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
INTEGER :: N, INFO
REAL(8), DIMENSION(:) :: A

SUBROUTINE TPTRI_64(UPLO, DIAG, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
INTEGER(8) :: N, INFO
REAL(8), DIMENSION(:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dtptri(char uplo, char diag,  int  n,  double  *a,  int
          *info);

void dtptri_64(char uplo, char diag, long n, double *a, long
          *info);
```

PURPOSE

dtptri computes the inverse of a real upper or lower triangular matrix A stored in packed format.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the upper or lower triangular matrix  A,
          stored  columnwise  in  a  linear array.  The j-th
          column of A is stored in the array A  as  follows:
          if  UPLO  =  'U',  A(i  +  (j-1)*j/2) = A(i,j) for
          1<=i<=j; if UPLO = 'L', A(i + (j-1)*((2*n-j)/2)  =
          A(i,j)   for   j<=i<=n.   See  below  for  further
          details.  On exit, the (triangular) inverse of the
          original  matrix,  in the same packed storage for-
          mat.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, A(i,i) is  exactly  zero.   The
          triangular  matrix is singular and its inverse can
          not be computed.
```

FURTHER DETAILS

A triangular matrix A can be transferred to packed storage using one of the following program segments:

UPLO = 'U':

```
      JC = 1
      DO 2 J = 1, N
         DO 1 I = 1, J
            A(JC+I-1) = A(I,J)
    1    CONTINUE
         JC = JC + J
    2 CONTINUE
```

UPLO = 'L':

```
      JC = 1
      DO 2 J = 1, N
         DO 1 I = J, N
            A(JC+I-J) = A(I,J)
    1    CONTINUE
         JC = JC + N - J + 1
    2 CONTINUE
```

Contents

NAME

```
dtptrs - solve a triangular system of the form   A * X  =  B
or A**T * X = B,
```

SYNOPSIS

```
SUBROUTINE DTPTRS(UPLO, TRANSA, DIAG, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, NRHS, LDB, INFO
DOUBLE PRECISION A(*), B(LDB,*)

SUBROUTINE DTPTRS_64(UPLO, TRANSA, DIAG, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, NRHS, LDB, INFO
DOUBLE PRECISION A(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE TPTRS(UPLO, TRANSA, DIAG, N, NRHS, A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, NRHS, LDB, INFO
REAL(8), DIMENSION(:) :: A
REAL(8), DIMENSION(:,:) :: B

SUBROUTINE TPTRS_64(UPLO, TRANSA, DIAG, N, NRHS, A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, NRHS, LDB, INFO
REAL(8), DIMENSION(:) :: A
REAL(8), DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void dtptrs(char uplo, char transa, char diag,  int  n,  int
          nrhs, double *a, double *b, int ldb, int *info);

void dtptrs_64(char uplo, char transa, char  diag,  long  n,
          long  nrhs,  double  *a, double *b, long ldb, long
          *info);
```

PURPOSE

dtptrs solves a triangular system of the form

where A is a triangular matrix of order N stored in packed format, and B is an N-by-NRHS matrix. A check is made to verify that A is nonsingular.

ARGUMENTS

UPLO (input)
= 'U': A is upper triangular;
= 'L': A is lower triangular.

TRANSA (input)
Specifies the form of the system of equations:
= 'N': A * X = B (No transpose)
= 'T': A**T * X = B (Transpose)
= 'C': A**H * X = B (Conjugate transpose = Transpose)

DIAG (input)
= 'N': A is non-unit triangular;
= 'U': A is unit triangular.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The upper or lower triangular matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

B (input/output)
On entry, the right hand side matrix B. On exit, if INFO = 0, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the i-th diagonal element of A is zero, indicating that the matrix is singular and the solutions X have not been computed.

Contents

NAME

```
dtrans - transpose and scale source matrix
```

SYNOPSIS

```
SUBROUTINE DTRANS(PLACE, SCALE, SOURCE, M, N, DEST)

CHARACTER * 1 PLACE
INTEGER M, N
DOUBLE PRECISION SCALE
DOUBLE PRECISION SOURCE(*), DEST(*)

SUBROUTINE DTRANS_64(PLACE, SCALE, SOURCE, M, N, DEST)

CHARACTER * 1 PLACE
INTEGER*8 M, N
DOUBLE PRECISION SCALE
DOUBLE PRECISION SOURCE(*), DEST(*)
```

F95 INTERFACE

```
SUBROUTINE TRANS([PLACE], SCALE, SOURCE, M, N, [DEST])

CHARACTER(LEN=1) :: PLACE
INTEGER :: M, N
REAL(8) :: SCALE
REAL(8), DIMENSION(:) :: SOURCE, DEST

SUBROUTINE TRANS_64([PLACE], SCALE, SOURCE, M, N, [DEST])

CHARACTER(LEN=1) :: PLACE
INTEGER(8) :: M, N
REAL(8) :: SCALE
REAL(8), DIMENSION(:) :: SOURCE, DEST
```

C INTERFACE

```
#include <sunperf.h>

void dtrans(char place, double scale, double *source, int m,
          int n, double *dest);

void dtrans_64(char place,  double  scale,  double  *source,
          long m, long n, double *dest);
```

PURPOSE

dtrans scales and transposes the source matrix. The N2 x N1 result is written into SOURCE when PLACE = 'I' or 'i', and DEST when PLACE = 'O' or 'o'.

PLACE = 'I' or 'i': SOURCE = SCALE * SOURCE'

PLACE = 'O' or 'o': DEST = SCALE * SOURCE'

ARGUMENTS

PLACE (input)
: Type of transpose. 'I' or 'i' for in-place, 'O' or 'o' for out-of-place. 'I' is default.

SCALE (input)
: Scale factor on the SOURCE matrix.

SOURCE (input/output)
: (M, N) on input. Array of (N, M) on output if in-place transpose.

M (input)
: Number of rows in the SOURCE matrix on input.

N (input)
: Number of columns in the SOURCE matrix on input.

DEST (output)
: Scaled and transposed SOURCE matrix if out-of-place transpose. Not referenced if in-place transpose.

Contents

NAME

dtrcon - estimate the reciprocal of the condition number of a triangular matrix A, in either the 1-norm or the infinity-norm

SYNOPSIS

```
SUBROUTINE DTRCON(NORM, UPLO, DIAG, N, A, LDA, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 NORM, UPLO, DIAG
INTEGER N, LDA, INFO
INTEGER WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(LDA,*), WORK(*)

SUBROUTINE DTRCON_64(NORM, UPLO, DIAG, N, A, LDA, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 NORM, UPLO, DIAG
INTEGER*8 N, LDA, INFO
INTEGER*8 WORK2(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TRCON(NORM, UPLO, DIAG, N, A, [LDA], RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL(8) :: RCOND
```

```
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE TRCON_64(NORM, UPLO, DIAG, N, A, [LDA], RCOND, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
void dtrcon(char norm, char uplo, char diag, int  n,  double
          *a, int lda, double *rcond, int *info);

void dtrcon_64(char norm, char uplo, char diag, long n, dou-
          ble *a, long lda, double *rcond, long *info);
```

PURPOSE

dtrcon estimates the reciprocal of the condition number of a triangular matrix A, in either the 1-norm or the infinity-norm.

The norm of A is computed and an estimate is obtained for norm(inv(A)), then the reciprocal of the condition number is computed as

```
RCOND = 1 / ( norm(A) * norm(inv(A)) ).
```

ARGUMENTS

```
NORM (input)
          Specifies whether the 1-norm condition  number  or
          the infinity-norm condition number is required:
          = '1' or 'O':  1-norm;
          = 'I':         Infinity-norm.

UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.
```

N (input) The order of the matrix A. N >= 0.

A (input) The triangular matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of the array A contains the upper triangular matrix, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of the array A contains the lower triangular matrix, and the strictly upper triangular part of A is not referenced. If DIAG = 'U', the diagonal elements of A are also not referenced and are assumed to be 1.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(norm(A) * norm(inv(A))).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
dtrevc - compute some or all of the right and/or left eigen-
vectors of a real upper quasi-triangular matrix T
```

SYNOPSIS

```
SUBROUTINE DTREVC(SIDE, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR,
      LDVR, MM, M, WORK, INFO)

CHARACTER * 1 SIDE, HOWMNY
INTEGER N, LDT, LDVL, LDVR, MM, M, INFO
LOGICAL SELECT(*)
DOUBLE PRECISION T(LDT,*), VL(LDVL,*), VR(LDVR,*), WORK(*)

SUBROUTINE DTREVC_64(SIDE, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR,
      LDVR, MM, M, WORK, INFO)

CHARACTER * 1 SIDE, HOWMNY
INTEGER*8 N, LDT, LDVL, LDVR, MM, M, INFO
LOGICAL*8 SELECT(*)
DOUBLE PRECISION T(LDT,*), VL(LDVL,*), VR(LDVR,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TREVC(SIDE, HOWMNY, SELECT, N, T, [LDT], VL, [LDVL], VR,
       [LDVR], MM, M, [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, HOWMNY
INTEGER :: N, LDT, LDVL, LDVR, MM, M, INFO
LOGICAL, DIMENSION(:) :: SELECT
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: T, VL, VR
```

```
SUBROUTINE TREVC_64(SIDE, HOWMNY, SELECT, N, T, [LDT], VL, [LDVL],
       VR, [LDVR], MM, M, [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, HOWMNY
INTEGER(8) :: N, LDT, LDVL, LDVR, MM, M, INFO
LOGICAL(8), DIMENSION(:) :: SELECT
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: T, VL, VR
```

C INTERFACE

```
#include <sunperf.h>

void dtrevc(char side, char howmny, int *select, int n, dou-
          ble *t, int ldt, double *vl, int ldvl, double *vr,
          int ldvr, int mm, int *m, int *info);
void dtrevc_64(char side, char howmny, long *select, long n,
          double *t, long ldt, double *vl, long ldvl, double
          *vr, long ldvr, long mm, long *m, long *info);
```

PURPOSE

dtrevc computes some or all of the right and/or left eigenvectors of a real upper quasi-triangular matrix T.

The right eigenvector x and the left eigenvector y of T corresponding to an eigenvalue w are defined by:

T*x = w*x,   y'*T = w*y'

where y' denotes the conjugate transpose of the vector y.

If all eigenvectors are requested, the routine may either return the matrices X and/or Y of right or left eigenvectors of T, or the products Q*X and/or Q*Y, where Q is an input orthogonal
matrix. If T was obtained from the real-Schur factorization of an original matrix A = Q*T*Q', then Q*X and Q*Y are the matrices of right or left eigenvectors of A.

T must be in Schur canonical form (as returned by SHSEQR), that is, block upper triangular with 1-by-1 and 2-by-2 diagonal blocks; each 2-by-2 diagonal block has its diagonal elements equal and its off-diagonal elements of opposite sign. Corresponding to each 2-by-2 diagonal block is a complex conjugate pair of eigenvalues and eigenvectors; only one eigenvector of the pair is computed, namely the one corresponding to the eigenvalue with positive imaginary part.

ARGUMENTS

```
SIDE (input)
          = 'R':  compute right eigenvectors only;
          = 'L':  compute left eigenvectors only;
          = 'B':  compute both right and left eigenvectors.

HOWMNY (input)
          = 'A':  compute all right  and/or  left  eigenvec-
          tors;
          = 'B':  compute all right  and/or  left  eigenvec-
          tors,  and  backtransform  them  using  the  input
          matrices supplied in VR and/or VL; = 'S':  compute
          selected right and/or left eigenvectors, specified
          by the logical array SELECT.
SELECT (input/output)
          If HOWMNY = 'S', SELECT specifies the eigenvectors
          to be computed.  If HOWMNY = 'A' or 'B', SELECT is
          not referenced.  To select  the  real  eigenvector
          corresponding to a real eigenvalue w(j), SELECT(j)
          must be set to  .TRUE..   To  select  the  complex
          eigenvector  corresponding  to a complex conjugate
          pair  w(j)  and  w(j+1),   either   SELECT(j)   or
          SELECT(j+1)  must  be  set to .TRUE.; then on exit
          SELECT(j) is .TRUE. and SELECT(j+1) is .FALSE..

N (input) The order of the matrix T. N >= 0.

T (input/output)
          The  upper  quasi-triangular  matrix  T  in  Schur
          canonical form.

LDT (input)
          The leading dimension  of  the  array  T.  LDT  >=
          max(1,N).

VL (input/output)
          On entry, if SIDE = 'L' or 'B' and HOWMNY  =  'B',
          VL  must  contain  an N-by-N matrix Q (usually the
          orthogonal matrix Q of Schur vectors  returned  by
          SHSEQR).   On  exit, if SIDE = 'L' or 'B', VL con-
          tains:  if HOWMNY = 'A',  the  matrix  Y  of  left
          eigenvectors  of  T;  VL  has the same quasi-lower
          triangular form as T'. If T(i,i) is a real  eigen-
          value,  then  the  i-th column VL(i) of VL  is its
          corresponding eigenvector. If T(i:i+1,i:i+1) is  a
          2-by-2   block   whose  eigenvalues  are  complex-
          conjugate  eigenvalues  of  T,  then  VL(i)+sqrt(-
```

1)*VL(i+1) is the complex eigenvector corresponding to the eigenvalue with positive real part. if HOWMNY = 'B', the matrix Q*Y; if HOWMNY = 'S', the left eigenvectors of T specified by SELECT, stored consecutively in the columns of VL, in the same order as their eigenvalues. A complex eigenvector corresponding to a complex eigenvalue is stored in two consecutive columns, the first holding the real part, and the second the imaginary part. If SIDE = 'R', VL is not referenced.

LDVL (input)
The leading dimension of the array VL. LDVL >= max(1,N) if SIDE = 'L' or 'B'; LDVL >= 1 otherwise.

VR (input/output)
On entry, if SIDE = 'R' or 'B' and HOWMNY = 'B', VR must contain an N-by-N matrix Q (usually the orthogonal matrix Q of Schur vectors returned by SHSEQR). On exit, if SIDE = 'R' or 'B', VR contains: if HOWMNY = 'A', the matrix X of right eigenvectors of T; VR has the same quasi-upper triangular form as T. If T(i,i) is a real eigenvalue, then the i-th column VR(i) of VR is its corresponding eigenvector. If T(i:i+1,i:i+1) is a 2-by-2 block whose eigenvalues are complex-conjugate eigenvalues of T, then VR(i)+sqrt(-1)*VR(i+1) is the complex eigenvector corresponding to the eigenvalue with positive real part. if HOWMNY = 'B', the matrix Q*X; if HOWMNY = 'S', the right eigenvectors of T specified by SELECT, stored consecutively in the columns of VR, in the same order as their eigenvalues. A complex eigenvector corresponding to a complex eigenvalue is stored in two consecutive columns, the first holding the real part and the second the imaginary part. If SIDE = 'L', VR is not referenced.

LDVR (input)
The leading dimension of the array VR. LDVR >= max(1,N) if SIDE = 'R' or 'B'; LDVR >= 1 otherwise.

MM (input)
The number of columns in the arrays VL and/or VR. MM >= M.

M (output)
The number of columns in the arrays VL and/or VR

actually used to store the eigenvectors. If HOWMNY = 'A' or 'B', M is set to N. Each selected real eigenvector occupies one column and each selected complex eigenvector occupies two columns.

WORK (workspace)
dimension(3*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The algorithm used in this program is basically backward (forward) substitution, with scaling to make the the code robust against possible overflow.

Each eigenvector is normalized so that the element of largest magnitude has magnitude 1; here the magnitude of a complex number (x,y) is taken to be |x| + |y|.

Contents

NAME

```
dtrexc - reorder the real  Schur  factorization  of  a  real
matrix  A  =  Q*T*Q**T, so that the diagonal block of T with
row index IFST is moved to row ILST
```

SYNOPSIS

```
SUBROUTINE DTREXC(COMPQ, N, T, LDT, Q, LDQ, IFST, ILST, WORK, INFO)

CHARACTER * 1 COMPQ
INTEGER N, LDT, LDQ, IFST, ILST, INFO
DOUBLE PRECISION T(LDT,*), Q(LDQ,*), WORK(*)

SUBROUTINE DTREXC_64(COMPQ, N, T, LDT, Q, LDQ, IFST, ILST, WORK,
      INFO)

CHARACTER * 1 COMPQ
INTEGER*8 N, LDT, LDQ, IFST, ILST, INFO
DOUBLE PRECISION T(LDT,*), Q(LDQ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TREXC(COMPQ, N, T, [LDT], Q, [LDQ], IFST, ILST, [WORK],
       [INFO])

CHARACTER(LEN=1) :: COMPQ
INTEGER :: N, LDT, LDQ, IFST, ILST, INFO
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: T, Q

SUBROUTINE TREXC_64(COMPQ, N, T, [LDT], Q, [LDQ], IFST, ILST, [WORK],
       [INFO])

CHARACTER(LEN=1) :: COMPQ
```

```
INTEGER(8) :: N, LDT, LDQ, IFST, ILST, INFO
REAL(8), DIMENSION(:) :: WORK
REAL(8), DIMENSION(:,:) :: T, Q
```

C INTERFACE

```
#include <sunperf.h>

void dtrexc(char compq, int n, double *t,  int  ldt,  double
          *q, int ldq, int *ifst, int *ilst, int *info);

void dtrexc_64(char compq, long n, double *t, long ldt, dou-
          ble  *q,  long  ldq,  long *ifst, long *ilst, long
          *info);
```

PURPOSE

dtrexc reorders the real Schur factorization of a real matrix A = Q*T*Q**T, so that the diagonal block of T with row index IFST is moved to row ILST.

The real Schur form T is reordered by an orthogonal similarity transformation Z**T*T*Z, and optionally the matrix Q of Schur vectors is updated by postmultiplying it with Z.

T must be in Schur canonical form (as returned by SHSEQR), that is, block upper triangular with 1-by-1 and 2-by-2 diagonal blocks; each 2-by-2 diagonal block has its diagonal elements equal and its off-diagonal elements of opposite sign.

ARGUMENTS

COMPQ (input)
= 'V': update the matrix Q of Schur vectors;
= 'N': do not update Q.

N (input) The order of the matrix T. N >= 0.

T (input/output)
On entry, the upper quasi-triangular matrix T, in Schur Schur canonical form. On exit, the reordered upper quasi-triangular matrix, again in Schur canonical form.

LDT (input)
The leading dimension of the array T. LDT >= max(1,N).

Q (input) On entry, if COMPQ = 'V', the matrix Q of Schur vectors. On exit, if COMPQ = 'V', Q has been postmultiplied by the orthogonal transformation matrix Z which reorders T. If COMPQ = 'N', Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= max(1,N).

IFST (input/output)
Specify the reordering of the diagonal blocks of T. The block with row index IFST is moved to row ILST, by a sequence of transpositions between adjacent blocks. On exit, if IFST pointed on entry to the second row of a 2-by-2 block, it is changed to point to the first row; ILST always points to the first row of the block in its final position (which may differ from its input value by +1 or -1). 1 <= IFST <= N; 1 <= ILST <= N.

ILST (input/output)
See the description of IFST.

WORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
= 1: two adjacent blocks were too close to swap (the problem is very ill-conditioned); T may have been partially reordered, and ILST points to the first row of the current position of the block being moved.

Contents

NAME

dtrmm - perform one of the matrix-matrix operations B := alpha*op(A)*B, or B := alpha*B*op(A)

SYNOPSIS

```
SUBROUTINE DTRMM(SIDE, UPLO, TRANSA, DIAG, M, N, ALPHA, A, LDA, B,
      LDB)

CHARACTER * 1 SIDE, UPLO, TRANSA, DIAG
INTEGER M, N, LDA, LDB
DOUBLE PRECISION ALPHA
DOUBLE PRECISION A(LDA,*), B(LDB,*)

SUBROUTINE DTRMM_64(SIDE, UPLO, TRANSA, DIAG, M, N, ALPHA, A, LDA, B,
      LDB)

CHARACTER * 1 SIDE, UPLO, TRANSA, DIAG
INTEGER*8 M, N, LDA, LDB
DOUBLE PRECISION ALPHA
DOUBLE PRECISION A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE TRMM(SIDE, UPLO, [TRANSA], DIAG, [M], [N], ALPHA, A, [LDA],
       B, [LDB])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANSA, DIAG
INTEGER :: M, N, LDA, LDB
REAL(8) :: ALPHA
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE TRMM_64(SIDE, UPLO, [TRANSA], DIAG, [M], [N], ALPHA, A,
       [LDA], B, [LDB])
```

```
CHARACTER(LEN=1) :: SIDE, UPLO, TRANSA, DIAG
INTEGER(8) :: M, N, LDA, LDB
REAL(8) :: ALPHA
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dtrmm(char side, char uplo, char transa, char diag, int
          m, int n, double alpha, double *a, int lda, double
          *b, int ldb);

void dtrmm_64(char side, char uplo, char transa, char  diag,
          long m, long n, double alpha, double *a, long lda,
          double *b, long ldb);
```

PURPOSE

dtrmm performs one of the matrix-matrix operations B := alpha*op(A)*B, or B := alpha*B*op(A) where alpha is a scalar, B is an m by n matrix, A is a unit, or non-unit, upper or lower triangular matrix and op(A) is one of

op(A) = A   or   op(A) = A'.

ARGUMENTS

SIDE (input)
On entry, SIDE specifies whether op(A) multiplies B from the left or right as follows:

SIDE = 'L' or 'l'   B := alpha*op(A)*B.

SIDE = 'R' or 'r'   B := alpha*B*op(A).

Unchanged on exit.

UPLO (input)
On entry, UPLO specifies whether the matrix A is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the form of op(A) to be used in the matrix multiplication as follows:

TRANSA = 'N' or 'n' op(A) = A.

TRANSA = 'T' or 't' op(A) = A'.

TRANSA = 'C' or 'c' op(A) = A'.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u' A is assumed to be unit triangular.

DIAG = 'N' or 'n' A is not assumed to be unit triangular.

Unchanged on exit.

M (input)
On entry, M specifies the number of rows of B. M >= 0. Unchanged on exit.

N (input)
On entry, N specifies the number of columns of B. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. When alpha is zero then A is not referenced and B need not be set before entry. Unchanged on exit.

A (input)
DOUBLE PRECISION array of DIMENSION (LDA, k), where k is m when SIDE = 'L' or 'l' and is n when SIDE = 'R' or 'r'.

Before entry with UPLO = 'U' or 'u', the leading k by k upper triangular part of the array A must contain the upper triangular matrix and the strictly lower triangular part of A is not refer-

enced.

Before entry with UPLO = 'L' or 'l', the leading k by k lower triangular part of the array A must contain the lower triangular matrix and the strictly upper triangular part of A is not referenced.

Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced either, but are assumed to be one. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When SIDE = 'L' or 'l' then LDA >= max(1, m), when SIDE = 'R' or 'r' then LDA >= max(1, n). Unchanged on exit.

B (input/output)
DOUBLE PRECISION array of DIMENSION (LDB, n). Before entry, the leading m by n part of the array B must contain the matrix B, and on exit is overwritten by the transformed matrix.

LDB (input)
On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. LDB >= max(1, m). Unchanged on exit.

Contents

NAME

dtrmv - perform one of the matrix-vector operations x := A*x, or x := A'*x

SYNOPSIS

```
SUBROUTINE DTRMV(UPLO, TRANSA, DIAG, N, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, LDA, INCY
DOUBLE PRECISION A(LDA,*), Y(*)

SUBROUTINE DTRMV_64(UPLO, TRANSA, DIAG, N, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, LDA, INCY
DOUBLE PRECISION A(LDA,*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE TRMV(UPLO, [TRANSA], DIAG, [N], A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, LDA, INCY
REAL(8), DIMENSION(:) :: Y
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE TRMV_64(UPLO, [TRANSA], DIAG, [N], A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, LDA, INCY
REAL(8), DIMENSION(:) :: Y
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dtrmv(char uplo, char transa, char diag, int n,  double
          *a, int lda, double *y, int incy);

void dtrmv_64(char uplo, char transa,  char  diag,  long  n,
          double *a, long lda, double *y, long incy);
```

PURPOSE

dtrmv performs one of the matrix-vector operations x := A*x, or x := A'*x, where x is an n element vector and A is an n by n unit, or non-unit, upper or lower triangular matrix.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   x := A*x.

TRANSA = 'T' or 't'   x := A'*x.

TRANSA = 'C' or 'c'   x := A'*x.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit triangular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

A (input)
Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular matrix and the strictly lower triangular part of A is not referenced. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular matrix and the strictly upper triangular part of A is not referenced. Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced either, but are assumed to be unity. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector x. On exit, Y is overwritten with the tranformed vector x.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

dtrrfs - provide error bounds and backward error estimates for the solution to a system of linear equations with a triangular coefficient matrix

SYNOPSIS

```
SUBROUTINE DTRRFS(UPLO, TRANSA, DIAG, N, NRHS, A, LDA, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, NRHS, LDA, LDB, LDX, INFO
INTEGER WORK2(*)
DOUBLE  PRECISION  A(LDA,*),  B(LDB,*),  X(LDX,*),  FERR(*),
BERR(*), WORK(*)

SUBROUTINE DTRRFS_64(UPLO, TRANSA, DIAG, N, NRHS, A, LDA, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, NRHS, LDA, LDB, LDX, INFO
INTEGER*8 WORK2(*)
DOUBLE  PRECISION  A(LDA,*),  B(LDB,*),  X(LDX,*),  FERR(*),
BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TRRFS(UPLO, [TRANSA], DIAG, N, NRHS, A, [LDA], B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, NRHS, LDA, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL(8), DIMENSION(:) :: FERR, BERR, WORK
```

```
REAL(8), DIMENSION(:,:) :: A, B, X

SUBROUTINE TRRFS_64(UPLO, [TRANSA], DIAG, N, NRHS, A, [LDA], B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, NRHS, LDA, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL(8), DIMENSION(:) :: FERR, BERR, WORK
REAL(8), DIMENSION(:,:) :: A, B, X
```

C INTERFACE

```
#include <sunperf.h>
void dtrrfs(char uplo, char transa, char diag,  int  n,  int
          nrhs, double *a, int lda, double *b, int ldb, dou-
          ble *x, int ldx, double *ferr, double  *berr,  int
          *info);

void dtrrfs_64(char uplo, char transa, char  diag,  long  n,
          long  nrhs,  double  *a, long lda, double *b, long
          ldb, double *x, long  ldx,  double  *ferr,  double
          *berr, long *info);
```

PURPOSE

dtrrfs provides error bounds and backward error estimates for the solution to a system of linear equations with a triangular coefficient matrix.

The solution matrix X must be computed by STRTRS or some other means before entering this routine. STRRFS does not do iterative refinement because doing so cannot improve the backward error.

ARGUMENTS

UPLO (input)
- = 'U': A is upper triangular;
- = 'L': A is lower triangular.

TRANSA (input)
Specifies the form of the system of equations:
- = 'N': A * X = B (No transpose)
- = 'T': A**T * X = B (Transpose)
- = 'C': A**H * X = B (Conjugate transpose = Transpose)

```
          TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.
A (input) The triangular matrix A.  If UPLO = 'U', the lead-
          ing  N-by-N  upper  triangular part of the array A
          contains the  upper  triangular  matrix,  and  the
          strictly  lower triangular part of A is not refer-
          enced.  If UPLO = 'L', the  leading  N-by-N  lower
          triangular  part of the array A contains the lower
          triangular matrix, and the strictly upper triangu-
          lar  part  of A is not referenced.  If DIAG = 'U',
          the diagonal elements of A are also not referenced
          and are assumed to be 1.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

B (input) The right hand side matrix B.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

X (input) The solution matrix X.

LDX (input)
          The leading dimension of  the  array  X.   LDX  >=
          max(1,N).

FERR (output)
          The estimated forward error bound for  each  solu-
          tion  vector X(j) (the j-th column of the solution
          matrix  X).   If  XTRUE  is  the   true   solution
          corresponding  to  X(j),  FERR(j)  is an estimated
          upper bound for the magnitude of the largest  ele-
          ment in (X(j) - XTRUE) divided by the magnitude of
          the largest element in X(j).  The estimate  is  as
          reliable  as the estimate for RCOND, and is almost
          always a slight overestimate of the true error.

BERR (output)
```

```
            The componentwise relative backward error of  each
            solution  vector X(j) (i.e., the smallest relative
            change in any element of A or B that makes X(j) an
            exact solution).
WORK (workspace)
            dimension(3*N)

WORK2 (workspace)
            dimension(N)

INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value
```

Contents

NAME

dtrsen - reorder the real Schur factorization of a real matrix A = Q*T*Q**T, so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper quasi-triangular matrix T,

SYNOPSIS

```
SUBROUTINE DTRSEN(JOB, COMPQ, SELECT, N, T, LDT, Q, LDQ, WR, WI, M,
      S, SEP, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOB, COMPQ
INTEGER N, LDT, LDQ, M, LWORK, LIWORK, INFO
INTEGER IWORK(*)
LOGICAL SELECT(*)
DOUBLE PRECISION S, SEP
DOUBLE PRECISION T(LDT,*), Q(LDQ,*), WR(*), WI(*), WORK(*)

SUBROUTINE DTRSEN_64(JOB, COMPQ, SELECT, N, T, LDT, Q, LDQ, WR, WI,
      M, S, SEP, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOB, COMPQ
INTEGER*8 N, LDT, LDQ, M, LWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
LOGICAL*8 SELECT(*)
DOUBLE PRECISION S, SEP
DOUBLE PRECISION T(LDT,*), Q(LDQ,*), WR(*), WI(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TRSEN(JOB, COMPQ, SELECT, N, T, [LDT], Q, [LDQ], WR, WI,
       M, S, SEP, [WORK], [LWORK], [IWORK], [LIWORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOB, COMPQ
INTEGER :: N, LDT, LDQ, M, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
LOGICAL, DIMENSION(:) :: SELECT
REAL(8) :: S, SEP
REAL(8), DIMENSION(:) :: WR, WI, WORK
REAL(8), DIMENSION(:,:) :: T, Q

SUBROUTINE TRSEN_64(JOB, COMPQ, SELECT, N, T, [LDT], Q, [LDQ], WR,
       WI, M, S, SEP, [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOB, COMPQ
INTEGER(8) :: N, LDT, LDQ, M, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
LOGICAL(8), DIMENSION(:) :: SELECT
REAL(8) :: S, SEP
REAL(8), DIMENSION(:) :: WR, WI, WORK
REAL(8), DIMENSION(:,:) :: T, Q
```

C INTERFACE

```
#include <sunperf.h>

void dtrsen(char job, char compq, int *select, int n, double
          *t,  int ldt, double *q, int ldq, double *wr, dou-
          ble *wi, int  *m,  double  *s,  double  *sep,  int
          *info);

void dtrsen_64(char job, char compq, long *select,  long  n,
          double  *t,  long ldt, double *q, long ldq, double
          *wr, double *wi, long *m, double *s, double  *sep,
          long *info);
```

PURPOSE

dtrsen reorders the real Schur factorization of a real matrix A = Q*T*Q**T, so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper quasi-triangular matrix T, and the leading columns of Q form an orthonormal basis of the corresponding right invariant subspace.

Optionally the routine computes the reciprocal condition numbers of the cluster of eigenvalues and/or the invariant subspace.

T must be in Schur canonical form (as returned by SHSEQR), that is, block upper triangular with 1-by-1 and 2-by-2 diagonal blocks; each 2-by-2 diagonal block has its diagonal elemnts equal and its off-diagonal elements of opposite sign.

ARGUMENTS

```
JOB (input)
          Specifies whether condition numbers  are  required
          for  the cluster of eigenvalues (S) or the invari-
          ant subspace (SEP):
          = 'N': none;
          = 'E': for eigenvalues only (S);
          = 'V': for invariant subspace only (SEP);
          = 'B': for both eigenvalues and invariant subspace
          (S and SEP).

COMPQ (input)
          = 'V': update the matrix Q of Schur vectors;
          = 'N': do not update Q.

SELECT (input)
          SELECT specifies the eigenvalues in  the  selected
          cluster.   To   select  a  real  eigenvalue  w(j),
          SELECT(j)  must  be  set  to  w(j)   and   w(j+1),
          corresponding  to  a 2-by-2 diagonal block, either
          SELECT(j) or SELECT(j+1) or both must  be  set  to
          either  both  included  in  the  cluster  or  both
          excluded.

N (input) The order of the matrix T. N >= 0.

T (input/output)
          On entry, the upper quasi-triangular matrix T,  in
          Schur  canonical  form.  On exit, T is overwritten
          by the reordered matrix T, again in Schur  canoni-
          cal  form,  with  the  selected eigenvalues in the
          leading diagonal blocks.

LDT (input)
          The leading dimension  of  the  array  T.  LDT  >=
          max(1,N).

Q (input) On entry, if COMPQ = 'V', the matrix  Q  of  Schur
          vectors.   On  exit,  if  COMPQ  = 'V', Q has been
          postmultiplied by  the  orthogonal  transformation
          matrix  which reorders T; the leading M columns of
          Q form an  orthonormal  basis  for  the  specified
          invariant  subspace.   If  COMPQ  =  'N', Q is not
          referenced.

LDQ (input)
```

The leading dimension of the array Q. LDQ >= 1; and if COMPQ = 'V', LDQ >= N.

WR (output)
The real and imaginary parts, respectively, of the reordered eigenvalues of T. The eigenvalues are stored in the same order as on the diagonal of T, with WR(i) = T(i,i) and, if T(i:i+1,i:i+1) is a 2-by-2 diagonal block, WI(i) > 0 and WI(i+1) = -WI(i). Note that if a complex eigenvalue is sufficiently ill-conditioned, then its value may differ significantly from its value before reordering.

WI (output)
See the description of WR.

M (output)
The dimension of the specified invariant subspace. 0 < = M <= N.

S (output)
If JOB = 'E' or 'B', S is a lower bound on the reciprocal condition number for the selected cluster of eigenvalues. S cannot underestimate the true reciprocal condition number by more than a factor of sqrt(N). If M = 0 or N, S = 1. If JOB = 'N' or 'V', S is not referenced.

SEP (output)
If JOB = 'V' or 'B', SEP is the estimated reciprocal condition number of the specified invariant subspace. If M = 0 or N, SEP = norm(T). If JOB = 'N' or 'E', SEP is not referenced.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If JOB = 'N', LWORK >= max(1,N); if JOB = 'E', LWORK >= M*(N-M); if JOB = 'V' or 'B', LWORK >= 2*M*(N-M).

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace/output)
If JOB = 'N' or 'E', IWORK is not referenced.

LIWORK (input)
The dimension of the array IWORK. If JOB = 'N' or 'E', LIWORK >= 1; if JOB = 'V' or 'B', LIWORK >= M*(N-M).

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
= 1: reordering of T failed because some eigenvalues are too close to separate (the problem is very ill-conditioned); T may have been partially reordered, and WR and WI contain the eigenvalues in the same order as in T; S and SEP (if requested) are set to zero.

FURTHER DETAILS

STRSEN first collects the selected eigenvalues by computing an orthogonal transformation Z to move them to the top left corner of T. In other words, the selected eigenvalues are the eigenvalues of T11 in:

```
Z'*T*Z = ( T11 T12 ) n1
         (  0  T22 ) n2
            n1  n2
```

where N = n1+n2 and Z' means the transpose of Z. The first n1 columns of Z span the specified invariant subspace of T.

If T has been obtained from the real Schur factorization of a matrix A = Q*T*Q', then the reordered real Schur factorization of A is given by A = (Q*Z)*(Z'*T*Z)*(Q*Z)', and the first n1 columns of Q*Z span the corresponding invariant subspace of A.

The reciprocal condition number of the average of the eigenvalues of T11 may be returned in S. S lies between 0 (very badly conditioned) and 1 (very well conditioned). It is computed as follows. First we compute R so that

```
                     P = ( I  R ) n1
                         ( 0  0 ) n2
                           n1 n2
```

is the projector on the invariant subspace associated with T11. R is the solution of the Sylvester equation:

```
                    T11*R - R*T22 = T12.
```

Let F-norm(M) denote the Frobenius-norm of M and 2-norm(M) denote the two-norm of M. Then S is computed as the lower bound

```
                  (1 + F-norm(R)**2)**(-1/2)
```

on the reciprocal of 2-norm(P), the true reciprocal condition number. S cannot underestimate 1 / 2-norm(P) by more than a factor of sqrt(N).

An approximate error bound for the computed average of the eigenvalues of T11 is

```
                     EPS * norm(T) / S
```

where EPS is the machine precision.

The reciprocal condition number of the right invariant subspace spanned by the first n1 columns of Z (or of Q*Z) is returned in SEP. SEP is defined as the separation of T11 and T22:

```
                 sep( T11, T22 ) = sigma-min( C )
```

where sigma-min(C) is the smallest singular value of the n1*n2-by-n1*n2 matrix

```
   C  = kprod( I(n2), T11 ) - kprod( transpose(T22), I(n1) )
```

I(m) is an m by m identity matrix, and kprod denotes the Kronecker product. We estimate sigma-min(C) by the reciprocal of an estimate of the 1-norm of inverse(C). The true reciprocal 1-norm of inverse(C) cannot differ from sigma-min(C) by more than a factor of sqrt(n1*n2).

When SEP is small, small changes in T can cause large changes in the invariant subspace. An approximate bound on the maximum angular error in the computed right invariant subspace is

```
                  EPS * norm(T) / SEP
```

Contents

NAME

```
dtrsm - solve one of the matrix  equations    op(  A  )*X  =
alpha*B, or X*op( A ) = alpha*B
```

SYNOPSIS

```
SUBROUTINE DTRSM(SIDE, UPLO, TRANSA, DIAG, M, N, ALPHA, A, LDA, B,
      LDB)

CHARACTER * 1 SIDE, UPLO, TRANSA, DIAG
INTEGER M, N, LDA, LDB
DOUBLE PRECISION ALPHA
DOUBLE PRECISION A(LDA,*), B(LDB,*)

SUBROUTINE DTRSM_64(SIDE, UPLO, TRANSA, DIAG, M, N, ALPHA, A, LDA, B,
      LDB)

CHARACTER * 1 SIDE, UPLO, TRANSA, DIAG
INTEGER*8 M, N, LDA, LDB
DOUBLE PRECISION ALPHA
DOUBLE PRECISION A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE TRSM(SIDE, UPLO, [TRANSA], DIAG, [M], [N], ALPHA, A, [LDA],
       B, [LDB])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANSA, DIAG
INTEGER :: M, N, LDA, LDB
REAL(8) :: ALPHA
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE TRSM_64(SIDE, UPLO, [TRANSA], DIAG, [M], [N], ALPHA, A,
       [LDA], B, [LDB])
```

```
CHARACTER(LEN=1) :: SIDE, UPLO, TRANSA, DIAG
INTEGER(8) :: M, N, LDA, LDB
REAL(8) :: ALPHA
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dtrsm(char side, char uplo, char transa, char diag, int
          m, int n, double alpha, double *a, int lda, double
          *b, int ldb);

void dtrsm_64(char side, char uplo, char transa, char  diag,
          long m, long n, double alpha, double *a, long lda,
          double *b, long ldb);
```

PURPOSE

dtrsm solves one of the matrix equations op(A)*X = alpha*B, or X*op(A) = alpha*B where alpha is a scalar, X and B are m by n matrices, A is a unit, or non-unit, upper or lower triangular matrix and op(A) is one of

op(A) = A   or   op(A) = A'.

The matrix X is overwritten on B.

ARGUMENTS

SIDE (input)
On entry, SIDE specifies whether op(A) appears on the left or right of X as follows:

SIDE = 'L' or 'l'   op(A)*X = alpha*B.

SIDE = 'R' or 'r'   X*op(A) = alpha*B.

Unchanged on exit.

UPLO (input)
On entry, UPLO specifies whether the matrix A is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)

On entry, TRANSA specifies the form of op(A) to be used in the matrix multiplication as follows:

TRANSA = 'N' or 'n'   op(A) = A.

TRANSA = 'T' or 't'   op(A) = A'.

TRANSA = 'C' or 'c'   op(A) = A'.

Unchanged on exit.
TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)

On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit triangular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

M (input)

On entry, M specifies the number of rows of B. M >= 0. Unchanged on exit.

N (input)

On entry, N specifies the number of columns of B. N >= 0. Unchanged on exit.

ALPHA (input)

On entry, ALPHA specifies the scalar alpha. When alpha is zero then A is not referenced and B need not be set before entry. Unchanged on exit.

A (input)

DOUBLE PRECISION array of DIMENSION (LDA, k), where k is m when SIDE = 'L' or 'l' and is n when SIDE = 'R' or 'r'.
Before entry with UPLO = 'U' or 'u', the leading k by k upper triangular part of the array A must contain the upper triangular matrix and the

strictly lower triangular part of A is not referenced.
Before entry with UPLO = 'L' or 'l', the leading k by k lower triangular part of the array A must contain the lower triangular matrix and the strictly upper triangular part of A is not referenced.
Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced either, but are assumed to be one. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When SIDE = 'L' or 'l' then LDA >= max(1, m), when SIDE = 'R' or 'r' then LDA >= max(1, n). Unchanged on exit.

B (input/output)
DOUBLE PRECISION array of DIMENSION (LDB, n). Before entry, the leading m by n part of the array B must contain the right-hand side matrix B, and on exit is overwritten by the solution matrix X.

LDB (input)
On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. LDB >= max(1, m). Unchanged on exit.

Contents

NAME

```
dtrsna - estimate reciprocal condition numbers for specified
eigenvalues  and/or  right  eigenvectors  of  a  real  upper
quasi-triangular matrix T (or of any matrix Q*T*Q**T with  Q
orthogonal)
```

SYNOPSIS

```
SUBROUTINE DTRSNA(JOB, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR, LDVR,
      S, SEP, MM, M, WORK, LDWORK, WORK1, INFO)

CHARACTER * 1 JOB, HOWMNY
INTEGER N, LDT, LDVL, LDVR, MM, M, LDWORK, INFO
INTEGER WORK1(*)
LOGICAL SELECT(*)
DOUBLE PRECISION  T(LDT,*),  VL(LDVL,*),  VR(LDVR,*),  S(*),
SEP(*), WORK(LDWORK,*)

SUBROUTINE DTRSNA_64(JOB, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR,
      LDVR, S, SEP, MM, M, WORK, LDWORK, WORK1, INFO)

CHARACTER * 1 JOB, HOWMNY
INTEGER*8 N, LDT, LDVL, LDVR, MM, M, LDWORK, INFO
INTEGER*8 WORK1(*)
LOGICAL*8 SELECT(*)
DOUBLE PRECISION  T(LDT,*),  VL(LDVL,*),  VR(LDVR,*),  S(*),
SEP(*), WORK(LDWORK,*)
```

F95 INTERFACE

```
SUBROUTINE TRSNA(JOB, HOWMNY, SELECT, N, T, [LDT], VL, [LDVL], VR,
       [LDVR], S, SEP, MM, M, [WORK], [LDWORK], [WORK1], [INFO])
```

```
CHARACTER(LEN=1) :: JOB, HOWMNY
INTEGER :: N, LDT, LDVL, LDVR, MM, M, LDWORK, INFO
INTEGER, DIMENSION(:) :: WORK1
LOGICAL, DIMENSION(:) :: SELECT
REAL(8), DIMENSION(:) :: S, SEP
REAL(8), DIMENSION(:,:) :: T, VL, VR, WORK

SUBROUTINE TRSNA_64(JOB, HOWMNY, SELECT, N, T, [LDT], VL, [LDVL], VR,
       [LDVR], S, SEP, MM, M, [WORK], [LDWORK], [WORK1], [INFO])

CHARACTER(LEN=1) :: JOB, HOWMNY
INTEGER(8) :: N, LDT, LDVL, LDVR, MM, M, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: WORK1
LOGICAL(8), DIMENSION(:) :: SELECT
REAL(8), DIMENSION(:) :: S, SEP
REAL(8), DIMENSION(:,:) :: T, VL, VR, WORK
```

C INTERFACE

```
#include <sunperf.h>

void dtrsna(char job, char howmny, int *select, int n,  dou-
          ble *t, int ldt, double *vl, int ldvl, double *vr,
          int ldvr, double *s, double *sep, int mm, int  *m,
          int ldwork, int *info);

void dtrsna_64(char job, char howmny, long *select, long  n,
          double *t, long ldt, double *vl, long ldvl, double
          *vr, long ldvr, double *s, double *sep,  long  mm,
          long *m, long ldwork, long *info);
```

PURPOSE

dtrsna estimates reciprocal condition numbers for specified eigenvalues and/or right eigenvectors of a real upper quasi-triangular matrix T (or of any matrix Q*T*Q**T with Q orthogonal).

T must be in Schur canonical form (as returned by SHSEQR), that is, block upper triangular with 1-by-1 and 2-by-2 diagonal blocks; each 2-by-2 diagonal block has its diagonal elements equal and its off-diagonal elements of opposite sign.

ARGUMENTS

JOB (input)
Specifies whether condition numbers are required for eigenvalues (S) or eigenvectors (SEP):

= 'E': for eigenvalues only (S);
= 'V': for eigenvectors only (SEP);
= 'B': for both eigenvalues and eigenvectors (S and SEP).

HOWMNY (input)
= 'A': compute condition numbers for all eigenpairs;
= 'S': compute condition numbers for selected eigenpairs specified by the array SELECT.

SELECT (input)
If HOWMNY = 'S', SELECT specifies the eigenpairs for which condition numbers are required. To select condition numbers for the eigenpair corresponding to a real eigenvalue w(j), SELECT(j) must be set to .TRUE.. To select condition numbers corresponding to a complex conjugate pair of eigenvalues w(j) and w(j+1), either SELECT(j) or SELECT(j+1) or both, must be set to .TRUE.. If HOWMNY = 'A', SELECT is not referenced.

N (input) The order of the matrix T. N >= 0.

T (input) The upper quasi-triangular matrix T, in Schur canonical form.

LDT (input)
The leading dimension of the array T. LDT >= max(1,N).

VL (input)
If JOB = 'E' or 'B', VL must contain left eigenvectors of T (or of any Q*T*Q**T with Q orthogonal), corresponding to the eigenpairs specified by HOWMNY and SELECT. The eigenvectors must be stored in consecutive columns of VL, as returned by SHSEIN or STREVC. If JOB = 'V', VL is not referenced.

LDVL (input)
The leading dimension of the array VL. LDVL >= 1; and if JOB = 'E' or 'B', LDVL >= N.

VR (input)
If JOB = 'E' or 'B', VR must contain right eigenvectors of T (or of any Q*T*Q**T with Q orthogonal), corresponding to the eigenpairs specified by HOWMNY and SELECT. The eigenvectors must be stored in consecutive columns of VR, as returned by

SHSEIN or STREVC. If JOB = 'V', VR is not referenced.

LDVR (input)
The leading dimension of the array VR. LDVR >= 1; and if JOB = 'E' or 'B', LDVR >= N.

S (output)
If JOB = 'E' or 'B', the reciprocal condition numbers of the selected eigenvalues, stored in consecutive elements of the array. For a complex conjugate pair of eigenvalues two consecutive elements of S are set to the same value. Thus S(j), SEP(j), and the j-th columns of VL and VR all correspond to the same eigenpair (but not in general the j-th eigenpair, unless all eigenpairs are selected). If JOB = 'V', S is not referenced.

SEP (output)
If JOB = 'V' or 'B', the estimated reciprocal condition numbers of the selected eigenvectors, stored in consecutive elements of the array. For a complex eigenvector two consecutive elements of SEP are set to the same value. If the eigenvalues cannot be reordered to compute SEP(j), SEP(j) is set to 0; this can only occur when the true value would be very small anyway. If JOB = 'E', SEP is not referenced.

MM (input)
The number of elements in the arrays S (if JOB = 'E' or 'B') and/or SEP (if JOB = 'V' or 'B'). MM >= M.

M (output)
The number of elements of the arrays S and/or SEP actually used to store the estimated condition numbers. If HOWMNY = 'A', M is set to N.

WORK (workspace)
dimension(LDWORK,N+1) If JOB = 'E', WORK is not referenced.

LDWORK (input)
The leading dimension of the array WORK. LDWORK >= 1; and if JOB = 'V' or 'B', LDWORK >= N.

WORK1 (workspace)
dimension(N) If JOB = 'E', WORK1 is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The reciprocal of the condition number of an eigenvalue lambda is defined as

```
        S(lambda) = |v'*u| / (norm(u)*norm(v))
```

where u and v are the right and left eigenvectors of T corresponding to lambda; v' denotes the conjugate-transpose of v, and norm(u) denotes the Euclidean norm. These reciprocal condition numbers always lie between zero (very badly conditioned) and one (very well conditioned). If n = 1, S(lambda) is defined to be 1.

An approximate error bound for a computed eigenvalue W(i) is given by

```
                    EPS * norm(T) / S(i)
```

where EPS is the machine precision.

The reciprocal of the condition number of the right eigenvector u corresponding to lambda is defined as follows. Suppose

```
            T = ( lambda  c  )
                (   0    T22 )
```

Then the reciprocal condition number is

```
        SEP( lambda, T22 ) = sigma-min( T22 - lambda*I )
```

where sigma-min denotes the smallest singular value. We approximate the smallest singular value by the reciprocal of an estimate of the one-norm of the inverse of T22 - lambda*I. If n = 1, SEP(1) is defined to be abs(T(1,1)).

An approximate error bound for a computed right eigenvector VR(i) is given by

```
                    EPS * norm(T) / SEP(i)
```

Contents

NAME

```
dtrsv - solve one of the systems of equations A*x  =  b,  or
A'*x = b
```

SYNOPSIS

```
SUBROUTINE DTRSV(UPLO, TRANSA, DIAG, N, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, LDA, INCY
DOUBLE PRECISION A(LDA,*), Y(*)

SUBROUTINE DTRSV_64(UPLO, TRANSA, DIAG, N, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, LDA, INCY
DOUBLE PRECISION A(LDA,*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE TRSV(UPLO, [TRANSA], DIAG, [N], A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, LDA, INCY
REAL(8), DIMENSION(:) :: Y
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE TRSV_64(UPLO, [TRANSA], DIAG, [N], A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, LDA, INCY
REAL(8), DIMENSION(:) :: Y
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void dtrsv(char uplo, char transa, char diag, int n,  double
          *a, int lda, double *y, int incy);

void dtrsv_64(char uplo, char transa,  char  diag,  long  n,
          double *a, long lda, double *y, long incy);
```

PURPOSE

dtrsv solves one of the systems of equations A*x = b, or A'*x = b, where b and x are n element vectors and A is an n by n unit, or non-unit, upper or lower triangular matrix. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u' A is an upper triangular matrix.

UPLO = 'L' or 'l' A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the equations to be solved as follows:

TRANSA = 'N' or 'n' A*x = b.

TRANSA = 'T' or 't' A'*x = b.

TRANSA = 'C' or 'c' A'*x = b.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)

On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit triangular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

A (input)
Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular matrix and the strictly lower triangular part of A is not referenced. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular matrix and the strictly upper triangular part of A is not referenced. Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced either, but are assumed to be unity. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element right-hand side vector b. On exit, Y is overwritten with the solution vector x.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
dtrsyl - solve the real Sylvester matrix equation
```

SYNOPSIS

```
SUBROUTINE DTRSYL(TRANA, TRANB, ISGN, M, N, A, LDA, B, LDB, C, LDC,
      SCALE, INFO)

CHARACTER * 1 TRANA, TRANB
INTEGER ISGN, M, N, LDA, LDB, LDC, INFO
DOUBLE PRECISION SCALE
DOUBLE PRECISION A(LDA,*), B(LDB,*), C(LDC,*)

SUBROUTINE DTRSYL_64(TRANA, TRANB, ISGN, M, N, A, LDA, B, LDB, C,
      LDC, SCALE, INFO)

CHARACTER * 1 TRANA, TRANB
INTEGER*8 ISGN, M, N, LDA, LDB, LDC, INFO
DOUBLE PRECISION SCALE
DOUBLE PRECISION A(LDA,*), B(LDB,*), C(LDC,*)
```

F95 INTERFACE

```
SUBROUTINE TRSYL(TRANA, TRANB, ISGN, M, N, A, [LDA], B, [LDB], C,
       [LDC], SCALE, [INFO])

CHARACTER(LEN=1) :: TRANA, TRANB
INTEGER :: ISGN, M, N, LDA, LDB, LDC, INFO
REAL(8) :: SCALE
REAL(8), DIMENSION(:,:) :: A, B, C

SUBROUTINE TRSYL_64(TRANA, TRANB, ISGN, M, N, A, [LDA], B, [LDB], C,
       [LDC], SCALE, [INFO])
```

```
CHARACTER(LEN=1) :: TRANA, TRANB
INTEGER(8) :: ISGN, M, N, LDA, LDB, LDC, INFO
REAL(8) :: SCALE
REAL(8), DIMENSION(:,:) :: A, B, C
```

C INTERFACE

```
#include <sunperf.h>

void dtrsyl(char trana, char tranb, int isgn, int m, int  n,
          double *a, int lda, double *b, int ldb, double *c,
          int ldc, double *scale, int *info);

void dtrsyl_64(char trana, char tranb, long  isgn,  long  m,
          long  n, double *a, long lda, double *b, long ldb,
          double *c, long ldc, double *scale, long *info);
```

PURPOSE

dtrsyl solves the real Sylvester matrix equation:

```
op(A)*X + X*op(B) = scale*C or
op(A)*X - X*op(B) = scale*C,
```

where op(A) = A or A**T, and A and B are both upper quasi-triangular. A is M-by-M and B is N-by-N; the right hand side C and the solution X are M-by-N; and scale is an output scale factor, set <= 1 to avoid overflow in X.

A and B must be in Schur canonical form (as returned by SHSEQR), that is, block upper triangular with 1-by-1 and 2-by-2 diagonal blocks; each 2-by-2 diagonal block has its diagonal elements equal and its off-diagonal elements of opposite sign.

ARGUMENTS

TRANA (input)
Specifies the option op(A):
= 'N': op(A) = A   (No transpose)
= 'T': op(A) = A**T (Transpose)
= 'C': op(A) = A**H (Conjugate transpose = Transpose)

TRANB (input)
Specifies the option op(B):
= 'N': op(B) = B   (No transpose)
= 'T': op(B) = B**T (Transpose)

```
          = 'C': op(B) = B**H (Conjugate transpose  =  Tran-
          spose)

ISGN (input)
          Specifies the sign in the equation:
          = +1: solve op(A)*X + X*op(B) = scale*C
          = -1: solve op(A)*X - X*op(B) = scale*C

M (input) The order of the matrix A, and the number of  rows
          in the matrices X and C. M >= 0.

N (input) The order of the  matrix  B,  and  the  number  of
          columns in the matrices X and C. N >= 0.

A (input) The upper  quasi-triangular  matrix  A,  in  Schur
          canonical form.
LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,M).

B (input) The upper  quasi-triangular  matrix  B,  in  Schur
          canonical form.

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          max(1,N).

C (input/output)
          On entry, the M-by-N right hand side matrix C.  On
          exit, C is overwritten by the solution matrix X.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M)

SCALE (output)
          The scale factor, scale, set <= 1 to  avoid  over-
          flow in X.

INFO (output)
          = 0: successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
          gal value
          = 1: A and B have  common  or  very  close  eigen-
          values;  perturbed  values  were used to solve the
          equation (but the matrices A and B are unchanged).
```

Contents

NAME

```
dtrti2 - compute the inverse of a real upper or  lower  tri-
angular matrix
```

SYNOPSIS

```
SUBROUTINE DTRTI2(UPLO, DIAG, N, A, LDA, INFO)

CHARACTER * 1 UPLO, DIAG
INTEGER N, LDA, INFO
DOUBLE PRECISION A(LDA,*)

SUBROUTINE DTRTI2_64(UPLO, DIAG, N, A, LDA, INFO)

CHARACTER * 1 UPLO, DIAG
INTEGER*8 N, LDA, INFO
DOUBLE PRECISION A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE TRTI2(UPLO, DIAG, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
INTEGER :: N, LDA, INFO
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE TRTI2_64(UPLO, DIAG, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
INTEGER(8) :: N, LDA, INFO
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dtrti2(char uplo, char diag, int n, double *a, int lda,
          int *info);

void dtrti2_64(char uplo, char diag, long n, double *a, long
          lda, long *info);
```

PURPOSE

```
dtrti2 computes the inverse of a real upper  or  lower  tri-
angular matrix.

This is the Level 2 BLAS version of the algorithm.
```

ARGUMENTS

```
UPLO (input)
          Specifies whether the matrix A is upper  or  lower
          triangular.  = 'U':  Upper triangular
          = 'L':  Lower triangular

DIAG (input)
          Specifies whether or not the matrix A is unit tri-
          angular.  = 'N':  Non-unit triangular
          = 'U':  Unit triangular

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the triangular matrix A.  If UPLO = 'U',
          the  leading  n  by n upper triangular part of the
          array A contains the upper triangular matrix,  and
          the  strictly  lower  triangular  part of A is not
          referenced.  If UPLO = 'L', the  leading  n  by  n
          lower  triangular part of the array A contains the
          lower triangular matrix, and  the  strictly  upper
          triangular part of A is not referenced.  If DIAG =
          'U', the diagonal  elements  of  A  are  also  not
          referenced and are assumed to be 1.

          On exit, the (triangular) inverse of the  original
          matrix, in the same storage format.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).
```

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value

Contents

NAME

```
dtrtri - compute the inverse of a real upper or  lower  tri-
angular matrix A
```

SYNOPSIS

```
SUBROUTINE DTRTRI(UPLO, DIAG, N, A, LDA, INFO)

CHARACTER * 1 UPLO, DIAG
INTEGER N, LDA, INFO
DOUBLE PRECISION A(LDA,*)

SUBROUTINE DTRTRI_64(UPLO, DIAG, N, A, LDA, INFO)

CHARACTER * 1 UPLO, DIAG
INTEGER*8 N, LDA, INFO
DOUBLE PRECISION A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE TRTRI(UPLO, DIAG, N, A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
INTEGER :: N, LDA, INFO
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE TRTRI_64(UPLO, DIAG, N, A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
INTEGER(8) :: N, LDA, INFO
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dtrtri(char uplo, char diag, int n, double *a, int lda,
          int *info);

void dtrtri_64(char uplo, char diag, long n, double *a, long
          lda, long *info);
```

PURPOSE

dtrtri computes the inverse of a real upper or lower triangular matrix A.

This is the Level 3 BLAS version of the algorithm.

ARGUMENTS

UPLO (input)
= 'U': A is upper triangular;
= 'L': A is lower triangular.

DIAG (input)
= 'N': A is non-unit triangular;
= 'U': A is unit triangular.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the triangular matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of the array A contains the upper triangular matrix, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of the array A contains the lower triangular matrix, and the strictly upper triangular part of A is not referenced. If DIAG = 'U', the diagonal elements of A are also not referenced and are assumed to be 1. On exit, the (triangular) inverse of the original matrix, in the same storage format.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-

gal value
> 0: if INFO = i, A(i,i) is exactly zero. The triangular matrix is singular and its inverse can not be computed.

Contents

NAME

```
dtrtrs - solve a triangular system of the form   A * X  =  B
or A**T * X = B,
```

SYNOPSIS

```
SUBROUTINE DTRTRS(UPLO, TRANSA, DIAG, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, NRHS, LDA, LDB, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*)

SUBROUTINE DTRTRS_64(UPLO, TRANSA, DIAG, N, NRHS, A, LDA, B, LDB,
      INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, NRHS, LDA, LDB, INFO
DOUBLE PRECISION A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE TRTRS(UPLO, [TRANSA], DIAG, N, NRHS, A, [LDA], B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, NRHS, LDA, LDB, INFO
REAL(8), DIMENSION(:,:) :: A, B

SUBROUTINE TRTRS_64(UPLO, [TRANSA], DIAG, N, NRHS, A, [LDA], B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
REAL(8), DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void dtrtrs(char uplo, char transa, char diag,  int  n,  int
          nrhs,  double *a, int lda, double *b, int ldb, int
          *info);

void dtrtrs_64(char uplo, char transa, char  diag,  long  n,
          long  nrhs,  double  *a, long lda, double *b, long
          ldb, long *info);
```

PURPOSE

dtrtrs solves a triangular system of the form
where A is a triangular matrix of order N, and B is an N-by-NRHS matrix. A check is made to verify that A is non-singular.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B  (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose = Tran-
          spose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

A (input) The triangular matrix A.  If UPLO = 'U', the lead-
          ing  N-by-N  upper  triangular part of the array A
          contains the  upper  triangular  matrix,  and  the
```

strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of the array A contains the lower triangular matrix, and the strictly upper triangular part of A is not referenced. If DIAG = 'U', the diagonal elements of A are also not referenced and are assumed to be 1.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the right hand side matrix B. On exit, if INFO = 0, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the i-th diagonal element of A is zero, indicating that the matrix is singular and the solutions X have not been computed.

Contents

NAME

dtzrqf - routine is deprecated and has been replaced by routine STZRZF

SYNOPSIS

```
SUBROUTINE DTZRQF(M, N, A, LDA, TAU, INFO)

INTEGER M, N, LDA, INFO
DOUBLE PRECISION A(LDA,*), TAU(*)

SUBROUTINE DTZRQF_64(M, N, A, LDA, TAU, INFO)

INTEGER*8 M, N, LDA, INFO
DOUBLE PRECISION A(LDA,*), TAU(*)
```

F95 INTERFACE

```
SUBROUTINE TZRQF(M, N, A, [LDA], TAU, [INFO])

INTEGER :: M, N, LDA, INFO
REAL(8), DIMENSION(:) :: TAU
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE TZRQF_64(M, N, A, [LDA], TAU, [INFO])

INTEGER(8) :: M, N, LDA, INFO
REAL(8), DIMENSION(:) :: TAU
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dtzrqf(int m, int n, double *a, int lda,  double  *tau,
          int *info);

void dtzrqf_64(long m, long n, double *a, long  lda,  double
          *tau, long *info);
```

PURPOSE

dtzrqf routine is deprecated and has been replaced by routine STZRZF.

STZRQF reduces the M-by-N (M<=N) real upper trapezoidal matrix A to upper triangular form by means of orthogonal transformations.

The upper trapezoidal matrix A is factored as

```
   A = ( R  0 ) * Z,
```

where Z is an N-by-N orthogonal matrix and R is an M-by-M upper triangular matrix.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= M.

A (input/output)
On entry, the leading M-by-N upper trapezoidal part of the array A must contain the matrix to be factorized. On exit, the leading M-by-M upper triangular part of A contains the upper triangular matrix R, and elements M+1 to N of the first M rows of A, with the array TAU, represent the orthogonal matrix Z as a product of M elementary reflectors.

LDA (input)
The leading dimension of the array A.   LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-

gal value

FURTHER DETAILS

The factorization is obtained by Householder's method. The kth transformation matrix, Z(k), which is used to introduce zeros into the (m - k + 1)th row of A, is given in the form

```
   Z( k ) = ( I     0   ),
            ( 0  T( k ) )
```

where

```
   T( k ) = I - tau*u( k )*u( k )',   u( k ) = (   1    ),
                                               (   0    )
                                               ( z( k ) )
```

tau is a scalar and z(k) is an (n - m) element vector. tau and z(k) are chosen to annihilate the elements of the kth row of X.

The scalar tau is returned in the kth element of TAU and the vector u(k) in the kth row of A, such that the elements of z(k) are in a(k, m + 1), ..., a(k, n). The elements of R are returned in the upper triangular part of A.

Z is given by

```
   Z =  Z( 1 ) * Z( 2 ) * ... * Z( m ).
```

Contents

NAME

```
dtzrzf - reduce the M-by-N ( M<=N ) real  upper  trapezoidal
matrix  A  to  upper  triangular form by means of orthogonal
transformations
```

SYNOPSIS

```
SUBROUTINE DTZRZF(M, N, A, LDA, TAU, WORK, LWORK, INFO)

INTEGER M, N, LDA, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)

SUBROUTINE DTZRZF_64(M, N, A, LDA, TAU, WORK, LWORK, INFO)

INTEGER*8 M, N, LDA, LWORK, INFO
DOUBLE PRECISION A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TZRZF([M], [N], A, [LDA], TAU, [WORK], [LWORK], [INFO])

INTEGER :: M, N, LDA, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A

SUBROUTINE TZRZF_64([M], [N], A, [LDA], TAU, [WORK], [LWORK], [INFO])

INTEGER(8) :: M, N, LDA, LWORK, INFO
REAL(8), DIMENSION(:) :: TAU, WORK
REAL(8), DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void dtzrzf(int m, int n, double *a, int lda,  double  *tau,
          int *info);

void dtzrzf_64(long m, long n, double *a, long  lda,  double
          *tau, long *info);
```

PURPOSE

dtzrzf reduces the M-by-N (M<=N) real upper trapezoidal matrix A to upper triangular form by means of orthogonal transformations.

The upper trapezoidal matrix A is factored as

```
   A = ( R  0 ) * Z,
```

where Z is an N-by-N orthogonal matrix and R is an M-by-M upper triangular matrix.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the leading M-by-N upper trapezoidal part of the array A must contain the matrix to be factorized. On exit, the leading M-by-M upper triangular part of A contains the upper triangular matrix R, and elements M+1 to N of the first M rows of A, with the array TAU, represent the orthogonal matrix Z as a product of M elementary reflectors.

LDA (input)
The leading dimension of the array A.   LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)

The dimension of the array WORK. LWORK >= max(1,M). For optimum performance LWORK >= M*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

Based on contributions by
A. Petitet, Computer Science Dept., Univ. of Tenn., Knoxville, USA

The factorization is obtained by Householder's method. The kth transformation matrix, Z(k), which is used to introduce zeros into the (m - k + 1)th row of A, is given in the form

```
   Z( k ) = ( I     0   ),
            ( 0  T( k ) )
```

where

```
   T( k ) = I - tau*u( k )*u( k )',   u( k ) = (   1    ),
                                               (   0    )
                                               ( z( k ) )
```

tau is a scalar and z(k) is an (n - m) element vector. tau and z(k) are chosen to annihilate the elements of the kth row of X.

The scalar tau is returned in the kth element of TAU and the vector u(k) in the kth row of A, such that the elements of z(k) are in a(k, m + 1), ..., a(k, n). The elements of R are returned in the upper triangular part of A.

Z is given by

```
   Z =  Z( 1 ) * Z( 2 ) * ... * Z( m ).
```

Contents

NAME

```
dvbrmm - variable block sparse row format matrix-matrix
multiply
```

SYNOPSIS

```
 SUBROUTINE DVBRMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, MB, N, KB, DESCRA(5), LDB, LDC, LWORK
 INTEGER    INDX(*), BINDX(*), RPNTR(MB+1), CPNTR(KB+1),
*           BPNTRB(MB), BPNTRE(MB)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION VAL(*), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE DVBRMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), LDB, LDC, LWORK
 INTEGER*8  INDX(*), BINDX(*), RPNTR(MB+1), CPNTR(KB+1),
*           BPNTRB(MB), BPNTRE(MB)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION VAL(*), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE VBRMM(TRANSA, MB, [N], KB, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, KB
 INTEGER, DIMENSION(:) ::  DESCRA, INDX, BINDX
 INTEGER, DIMENSION(:) ::  RPNTR, CPNTR, BPNTRB, BPNTRE
 REAL*8    ALPHA, BETA
 REAL*8, DIMENSION(:) :: VAL
```

```
 REAL*8, DIMENSION(:, :) ::  B, C

 SUBROUTINE VBRMM_64(TRANSA, MB, [N], KB, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, KB
 INTEGER*8, DIMENSION(:) ::  DESCRA, INDX, BINDX
 INTEGER*8, DIMENSION(:) ::  RPNTR, CPNTR, BPNTRB, BPNTRE
 REAL*8    ALPHA, BETA
 REAL*8, DIMENSION(:) :: VAL
 REAL*8, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
         C <- alpha op(A) B + beta C
where ALPHA and BETA are scalar, C and B are  matrices,
A is a matrix represented in variable block sparse row  format
and op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if the matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
```

```
                  DESCRA(2) upper/lower triangular indicator
                    1 : lower
                    2 : upper
                  DESCRA(3) main diagonal type
                    0 : non-unit
                    1 : unit
                  DESCRA(4) Array base  (NOT IMPLEMENTED)
                    0 : C/C++ compatible
                    1 : Fortran compatible
                  DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                    0 : unknown
                    1 : no repeated indices

VAL()             scalar array of length NNZ consisting of the block entries
                  of A where each block entry is a dense rectangular matrix
                  stored column by column.
                  NNZ is the total number of point entries in all nonzero
                  block  entries of a matrix A.
INDX()            integer array of length BNNZ+1 where BNNZ is the number of
                  block entries of a matrix A such that the I-th element of
                  INDX[] points to the location in VAL of the (1,1) element
                  of the I-th block entry.

BINDX()           integer array of length BNNZ consisting of the block
                  column indices of the block entries of A where BNNZ is
                  the number block entries of a matrix A.

RPNTR()           integer array of length MB+1 such that RPNTR(I)-RPNTR(1)+1
                  is the row index of the first point row in the I-th block
                  row.
                  RPNTR(MB+1) is set to M+RPNTR(1) where M is the number of
                  rows in matrix A.
                  Thus, the number of point rows in the I-th block row is
                  RPNTR(I+1)-RPNTR(I).

CPNTR()           integer array of length KB+1 such that CPNTR(J)-CPNTR(1)+1
                  is the column index of the first point column in the J-th
                  block column. CPNTR(KB+1) is set to K+CPNTR(1) where K is
                  the number of columns in matrix A.
                  Thus, the number of point columns in the J-th block column
                  is CPNTR(J+1)-CPNTR(J).

BPNTRB()          integer array of length MB such that BPNTRB(I)-BPNTRB(1)+1
                  points to location in BINDX of the first block entry of
                  the I-th block row of A.

BPNTRE()          integer array of length MB such that BPNTRE(I)-BPNTRB(1)
                  points to location in BINDX of the last block entry of
                  the I-th block row of A.
```

B()   rectangular array with first dimension LDB.

LDB   leading dimension of B

BETA   Scalar parameter

C()   rectangular array with first dimension LDC.

LDC   leading dimension of C

WORK()   scratch array of length LWORK. WORK is not referenced in the current version.

LWORK   length of WORK array. LWORK is not referenced in the current version.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. For a general matrix (*DESCRA*(1)=0), array CPNTR can be different from RPNTR. For all other matrix types, RPNTR must equal CPNTR and a single array can be passed for both arguments.

2. It is known that there exists another representation of the variable block sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of six array instead of the seven used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block row in the array BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of variable block sparse row format the following calling sequence should be used

```
  SUBROUTINE SVBRMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *           VAL, INDX, BINDX, RPNTR, CPNTR, IA, IA(2),
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

dvbrsm - variable block sparse row format triangular solve

SYNOPSIS

```
 SUBROUTINE DVBRSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*            B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), LDB, LDC, LWORK
 INTEGER    INDX(*), BINDX(*), RPNTR(MB+1), CPNTR(MB+1),
*            BPNTRB(MB), BPNTRE(MB)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION DV(*), VAL(*), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE DVBRSM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*            B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), LDB, LDC, LWORK
 INTEGER*8  INDX(*), BINDX(*), RPNTR(MB+1), CPNTR(MB+1),
*            BPNTRB(MB), BPNTRE(MB)
 DOUBLE PRECISION ALPHA, BETA
 DOUBLE PRECISION DV(*), VAL(*), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE VBRSM(TRANSA, MB, [N], UNITD, DV, ALPHA, DESCRA,
*            VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*            B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, UNITD
 INTEGER, DIMENSION(:) ::  DESCRA, INDX, BINDX
 INTEGER, DIMENSION(:) ::  RPNTR, CPNTR, BPNTRB, BPNTRE
 DOUBLE PRECISION    ALPHA, BETA
 DOUBLE PRECISION, DIMENSION(:) :: VAL, DV
 DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

```
      SUBROUTINE VBRSM_64(TRANSA, MB, [N], UNITD, DV, ALPHA, DESCRA,
     *           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
     *           B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
      INTEGER*8    TRANSA, MB, UNITD
      INTEGER*8, DIMENSION(:) ::  DESCRA, INDX, BINDX
      INTEGER*8, DIMENSION(:) ::  RPNTR, CPNTR, BPNTRB, BPNTRE
      DOUBLE PRECISION    ALPHA, BETA
      DOUBLE PRECISION, DIMENSION(:) :: VAL, DV
      DOUBLE PRECISION, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
       C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
       C <- ALPHA op(A) D B + BETA C
     where ALPHA and BETA are scalar, C and B are m by n dense matrices,
     D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
     lower triangular matrix represented in variable block sparse row
     format and  op( A )  is one  of

      op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
      (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
     TRANSA        Indicates how to operate with the sparse matrix
                     0 : operate with matrix
                     1 : operate with transpose matrix
                     2 : operate with the conjugate transpose of matrix.
                         2 is equivalent to 1 if matrix is real.

     MB            Number of block rows in matrix A

     N             Number of columns in matrix C

     UNITD         Type of scaling:
                     1 : Identity matrix (argument DV[] is ignored)
                     2 : Scale on left (row block scaling)
                     3 : Scale on right (column block scaling)

     DV()          Array containing the block entries of the block
                   diagonal matrix D.  The size of the J-th block is
                   RPNTR(J+1)-RPNTR(J) and each block contains matrix
                   entries stored column-major.  The total length of
                   array DV is given by the formula:

                   sum over J from 1 to MB:
```

```
                 ((RPNTR(J+1)-RPNTR(J))*(RPNTR(J+1)-RPNTR(J)))

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              Note: For the routine, DESCRA(1)=3 is only supported.
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
                0 : non-identity blocks on the main diagonal
                1 : identity diagonal block
                2 : diagonal blocks are dense matrices
              DESCRA(4) Array base  (NOT IMPLEMENTED)
                0 : C/C++ compatible
                1 : Fortran compatible
              DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                0 : unknown
                1 : no repeated indices

VAL()         scalar array of length NNZ consisting of the block entries
              of A where each block entry is a dense rectangular matrix
              stored column by column.
              NNZ is the total number of point entries in all nonzero
              block  entries of a matrix A.

INDX()        integer array of length BNNZ+1 where BNNZ is the number
              block entries of a matrix A such that the I-th element of
              INDX[] points to the location in VAL of the (1,1) element
              of the I-th block entry.

BINDX()       integer array of length BNNZ consisting of the block
              column indices of the block entries of A where BNNZ is
              the number block entries of a matrix A.   Block column
              indices MUST be sorted in increasing order for each block
              row.

RPNTR()       integer array of length MB+1 such that RPNTR(I)-RPNTR(1)+1
              is the row index of the first point row in the I-th block
              row.
              RPNTR(MB+1) is set to M+RPNTR(1) where M is the number
              of rows  in square triangular matrix A.
```

Thus, the number of point rows in the I-th block row is RPNTR(I+1)-RPNTR(I).

NOTE: For the current version CPNTR must equal RPNTR and a single array can be passed for both arguments

CPNTR() integer array of length MB+1 such that CPNTR(J)-CPNTR(1)+1 is the column index of the first point column in the J-th block column. CPNTR(MB+1) is set to M+CPNTR(1). Thus, the number of point columns in the J-th block column is CPNTR(J+1)-CPNTR(J).

NOTE: For the current version CPNTR must equal RPNTR and a single array can be passed for both arguments

BPNTRB() integer array of length MB such that BPNTRB(I)-BPNTRB(1)+1 points to location in BINDX of the first block entry of the I-th block row of A.

BPNTRE() integer array of length MB such that BPNTRE(I)-BPNTRB(1) points to location in BINDX of the last block entry of the I-th block row of A.

B() rectangular array with first dimension LDB.

LDB leading dimension of B

BETA Scalar parameter

C() rectangular array with first dimension LDC.

LDC leading dimension of C

WORK() scratch array of length LWORK. On exit, if LWORK= -1, WORK(1) returns the optimum size of LWORK.

LWORK length of WORK array. LWORK should be at least M = RPNTR(MB+1)-RPNTR(1).

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued

by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If *DESCRA*(3)=0,the lower or upper triangular part of each diagonal block is used by the routine depending on *DESCRA*(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might not be referenced in the VBR representation of a sparse matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense matrices and the LU factorization with partial pivoting is used by the routine. *WORK*(1)=0 on return if the factorization for all diagonal blocks has been completed successfully, otherwise *WORK*(1) = -i where i is the block number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3 in this case.

6. It is known that there exists another representation of the variable block sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of six array instead of the seven used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block row in the array BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of variable block sparse row

format the following calling sequence should be used

```
  SUBROUTINE DVBRSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, INDX, BINDX, RPNTR, CPNTR, IA, IA(2),
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

```
dwiener - perform Wiener deconvolution of two signals
```

SYNOPSIS

```
SUBROUTINE DWIENER(N_POINTS, ACOR, XCOR, FLTR, EROP, ISW, IERR)

INTEGER N_POINTS, ISW, IERR
DOUBLE PRECISION ACOR(*), XCOR(*), FLTR(*), EROP(*)

SUBROUTINE DWIENER_64(N_POINTS, ACOR, XCOR, FLTR, EROP, ISW, IERR)

INTEGER*8 N_POINTS, ISW, IERR
DOUBLE PRECISION ACOR(*), XCOR(*), FLTR(*), EROP(*)
```

F95 INTERFACE

```
SUBROUTINE WIENER(N_POINTS, ACOR, XCOR, FLTR, EROP, ISW, IERR)

INTEGER :: N_POINTS, ISW, IERR
REAL(8), DIMENSION(:) :: ACOR, XCOR, FLTR, EROP

SUBROUTINE WIENER_64(N_POINTS, ACOR, XCOR, FLTR, EROP, ISW, IERR)

INTEGER(8) :: N_POINTS, ISW, IERR
REAL(8), DIMENSION(:) :: ACOR, XCOR, FLTR, EROP
```

C INTERFACE

```
#include <sunperf.h>

void dwiener(int n_points, double *acor, double *xcor,  dou-
          ble *fltr, double *erop, int *isw, int *ierr);

void dwiener_64(long n_points, double *acor,  double  *xcor,
```

```
double   *fltr,  double  *erop,  long  *isw,  long
*ierr);
```

PURPOSE

dwiener performs Wiener deconvolution of two signals.

ARGUMENTS

N_POINTS (input)
On entry, the number of points in the input correlations. Unchanged on exit.

ACOR (input)
On entry, autocorrelation coefficients. Unchanged on exit.

XCOR (input)
On entry, cross-correlation coefficients. Unchanged on exit.

FLTR (output)
On exit, filter coefficients.

EROP (output)
On exit, the prediction error.

ISW (input)
On entry, if ISW .EQ. 0 then perform spiking deconvolution, otherwise perform general deconvolution. Unchanged on exit.

IERR (output)
On exit, the deconvolution was successful iff IERR .EQ. 0, otherwise there was an error.

Contents

NAME

```
dzasum - Return the sum of the absolute values of  a  vector
x.
```

SYNOPSIS

```
DOUBLE PRECISION FUNCTION DZASUM(N, X, INCX)

DOUBLE COMPLEX X(*)
INTEGER N, INCX

DOUBLE PRECISION FUNCTION DZASUM_64(N, X, INCX)

DOUBLE COMPLEX X(*)
INTEGER*8 N, INCX
```

F95 INTERFACE

```
REAL(8) FUNCTION ASUM([N], X, [INCX])

COMPLEX(8), DIMENSION(:) :: X
INTEGER :: N, INCX

REAL(8) FUNCTION ASUM_64([N], X, [INCX])

COMPLEX(8), DIMENSION(:) :: X
INTEGER(8) :: N, INCX
```

C INTERFACE

```
#include <sunperf.h>

double dzasum(int n, doublecomplex *x, int incx);

double dzasum_64(long n, doublecomplex *x, long incx);
```

PURPOSE

dzasum Return the sum of the absolute values of the elements of x where x is an n-vector. This is the sum of the absolute values of the real and complex elements and not the sum of the squares of the real and complex elements.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Contents

NAME

```
dznrm2 - Return the Euclidian norm of a vector.
```

SYNOPSIS

```
DOUBLE PRECISION FUNCTION DZNRM2(N, X, INCX)

DOUBLE COMPLEX X(*)
INTEGER N, INCX

DOUBLE PRECISION FUNCTION DZNRM2_64(N, X, INCX)

DOUBLE COMPLEX X(*)
INTEGER*8 N, INCX
```

F95 INTERFACE

```
REAL(8) FUNCTION NRM2([N], X, [INCX])

COMPLEX(8), DIMENSION(:) :: X
INTEGER :: N, INCX

REAL(8) FUNCTION NRM2_64([N], X, [INCX])

COMPLEX(8), DIMENSION(:) :: X
INTEGER(8) :: N, INCX
```

C INTERFACE

```
#include <sunperf.h>

double dznrm2(int n, doublecomplex *x, int incx);

double dznrm2_64(long n, doublecomplex *x, long incx);
```

PURPOSE

dznrm2 Return the Euclidian norm of a vector x where x is an n-vector.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must be positive. Unchanged on exit.

Contents

NAME

```
ezfftb - computes a periodic sequence from its Fourier coef-
ficients.   EZFFTB  is  a  simplified  but slower version of
RFFTB.
```

SYNOPSIS

```
SUBROUTINE EZFFTB(N, R, AZERO, A, B, WSAVE)

INTEGER N
REAL AZERO
REAL R(*), A(*), B(*), WSAVE(*)

SUBROUTINE EZFFTB_64(N, R, AZERO, A, B, WSAVE)

INTEGER*8 N
REAL AZERO
REAL R(*), A(*), B(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE EZFFTB(N, R, AZERO, A, B, WSAVE)

INTEGER :: N
REAL :: AZERO
REAL, DIMENSION(:) :: R, A, B, WSAVE

SUBROUTINE EZFFTB_64(N, R, AZERO, A, B, WSAVE)

INTEGER(8) :: N
REAL :: AZERO
REAL, DIMENSION(:) :: R, A, B, WSAVE
```

C INTERFACE

```
#include <sunperf.h>
```

```
void ezfftb(int n, float *r, float azero,  float  *a,  float
          *b, float *wsave);

void ezfftb_64(long n, float  *r,  float  azero,  float  *a,
          float *b, float *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence  to  be  synthesized.   The
          method  is most efficient when N is the product of
          small primes.  N >= 0.

R (output)
          On exit, the Fourier synthesis of the inputs.
AZERO (input)
          On entry, the  constant  Fourier  coefficient  A0.
          Unchanged on exit.

A (input/output)
          On  entry,  arrays  that  contain  the   remaining
          Fourier  coefficients.   On exit, these arrays are
          unchanged.

B (input/output)
          On  entry,  arrays  that  contain  the   remaining
          Fourier  coefficients.   On exit, these arrays are
          unchanged.

WSAVE (input)
          On entry, an array with dimension of at least (3 *
          N + 15), initialized by EZFFTI.
```

Contents

NAME

```
ezfftf - computes the Fourier  coefficients  of  a  periodic
sequence.   EZFFTF  is  a  simplified  but slower version of
RFFTF.
```

SYNOPSIS

```
SUBROUTINE EZFFTF(N, R, AZERO, A, B, WSAVE)

INTEGER N
REAL AZERO
REAL R(*), A(*), B(*), WSAVE(*)

SUBROUTINE EZFFTF_64(N, R, AZERO, A, B, WSAVE)

INTEGER*8 N
REAL AZERO
REAL R(*), A(*), B(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE EZFFTF(N, R, AZERO, A, B, WSAVE)

INTEGER :: N
REAL :: AZERO
REAL, DIMENSION(:) :: R, A, B, WSAVE

SUBROUTINE EZFFTF_64(N, R, AZERO, A, B, WSAVE)

INTEGER(8) :: N
REAL :: AZERO
REAL, DIMENSION(:) :: R, A, B, WSAVE
```

C INTERFACE

```
#include <sunperf.h>
```

```
void ezfftf(int n, float *r, float azero,  float  *a,  float
          *b, float *wsave);

void ezfftf_64(long n, float  *r,  float  azero,  float  *a,
          float *b, float *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. The method is most efficient when N is the product of small primes. N >= 0.

R (output)
A real array of length N containing the sequence to be transformed. On exit, R is unchanged.

AZERO (output)
On exit, the sum from i=1 to i=n of r(i)/n.

A (input/output)
On entry, arrays that contain the remaining Fourier coefficients. On exit, these arrays are unchanged.

B (input/output)
On entry, arrays that contain the remaining Fourier coefficients. On exit, these arrays are unchanged.

WSAVE (input)
On entry, an array with dimension of at least (3 * N + 15), initialized by EZFFTI.

Contents

NAME

```
ezffti - initializes the array WSAVE, which is used in  both
EZFFTF and EZFFTB.
```

SYNOPSIS

```
SUBROUTINE EZFFTI(N, WSAVE)

INTEGER N
REAL WSAVE(*)

SUBROUTINE EZFFTI_64(N, WSAVE)

INTEGER*8 N
REAL WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE EZFFTI(N, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: WSAVE

SUBROUTINE EZFFTI_64(N, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void ezffti(int n, float *wsave);

void ezffti_64(long n, float *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. N >= 0.

WSAVE (input)
On entry, an array with a dimension of at least (3 * N + 15). The same work array can be used for both EZFFTF and EZFFTB as long as N remains unchanged. Different WSAVE arrays are required for different values of N. This initialization does not have to be repeated between calls to EZFFTF or EZFFTB as long as N and WSAVE remain unchanged, thus subsequent transforms can be obtained faster than the first.

Contents

NAME

fft - Fast Fourier transform subroutines

OVERVIEW

The signal processing software in Sun Performance Library includes a set of routines based on public domain packages FFTPACK and VFFPACK that computes the Fast Fourier Transform. These routines are now being replaced by a new interface (Perflib interface).

MAPPING

Below is a mapping of routines from the FFTPACK interface and the new Perflib interface. See individual man pages for more detail.

FFTPACK interface	Perflib interface
RFFTF (DFFTF)	SFFTC (DFFTZ)
RFFTB (DFFTB)	CFFTS (ZFFTD)
CFFTF (ZFFTF)	CFFTC (ZFFTZ)
EZFFTF (DEZFFTF)	SFFTC (DFFTZ)
EZFFTB (DEZFFTB)	CFFTS (ZFFTD)
CFFTB (ZFFTB)	CFFTC (ZFFTZ)
RFFT2F (DFFT2F)	SFFT2C (DFFT2Z)
RFFT2B (DFFT2B)	CFFT2S (ZFFT2D)
CFFT2F (ZFFT2F)	CFFT2C (ZFFT2Z)
CFFT2B (ZFFT2B)	CFFT2C (ZFFT2Z)
RFFT3F (DFFT3F)	SFFT3C (DFFT3Z)
RFFT3B (DFFT3B)	CFFT3S (ZFFT3D)
CFFT3B (ZFFT3B)	CFFT3C (ZFFT3Z)
CFFT3F (ZFFT3F)	CFFT3C (ZFFT3Z)

```
VCFFTF (VZFFTF)         CFFTCM (ZFFTZM)
VCFFTB (VZFFTB)         CFFTCM (ZFFTZM)
VRFFTF (VDFFTF)         SFFTCM (DFFTZM)
VRFFTB (VDFFTB)         CFFTSM (ZFFTDM)
RFFTI (DFFTI)           SFFTC (DFFTZ), CFFTS (ZFFTD)
CFFTI (ZFFTI)           CFFTC (ZFFTZ)
EZFFTI (DEZFFTI)        SFFTC (DFFTZ), CFFTS (ZFFTD)
RFFT2I (DFFT2I)         SFFTC2 (DFFTZ2), CFFTS2 (ZFFTD2)
RFFT3I (DFFT3I)         SFFTC3 (DFFTZ3), CFFTS3 (ZFFTD3)
CFFT2I (ZFFT2I)         CFFTC2 (ZFFTZ2)
CFFT3I (ZFFT3I)         CFFTC3 (ZFFTZ3)
VCFFTI (VZFFTI)         CFFTCM (ZFFTZM)
VRFFTI (VDFFTI)         SFFTCM (DFFTZM), CFFTSM (ZFFTDM)
```

NOTES

Unlike the FFTPACK interface, the Perflib interface does not provide separate routines for initialization. Computation and initialization can be selected by an argument in the calling sequence of each routine. Similar to the FFTPACK routines, the weight and factor tables need to be initialized once for a particular transform length. Once these tables are initialized, they can be used repeatedly to compute the forward and inverse tranforms for different data sets until, of course, the transform length is changed. The appropriate transform routine is then called to initialize the tables for the new length.

The Perflib interface gives the user the option of computing the FFT in-place (input overwritten by transform results) or out-of-place (input unchanged) in every routine. When an out-of-place transform is requested, the input and output arrays must not overlap in memory. In-place transforms require that there be perfect overlay between the input and output arrays. That is, the arrays must begin at the same memory location. The routines assume (and therefore do not check) that these conditions are satisfied. In some cases, the dimension(s) of the input and output arrays are related to each other. Below is a summary of requirements of the array dimensions. LDX1 and LDX are leading dimensions of the input arrays and LDY1 and LDY are leading dimensions of the output arrays. LDX2 and LDY2 are the second dimensions of the input and output arrays, respectively. N1 and N2 are the first and second actual dimensions of the problem.

```
Routine name      in-place       out-of-place

SFFTCM, DFFTZM    LDX = 2*LDY    LDX >= N1
                  LDY >= N1/2+1  LDY >= N1/2+1
```

CFFTSM, ZFFTDM	LDX >= N1/2+1	LDX >= N1/2+1
	LDY = 2*LDX	LDY >= N1
CFFTCM, ZFFTZM	LDX >= N1	LDX >= N1
	LDY = LDX	LDY >= N1
SFFTC2, DFFTZ2	LDX = 2*LDY	LDX >= N1
	LDY >= N1/2+1	LDY >= N1/2+1
CFFTS2, ZFFTD2	LDX >= N1/2+1	LDX >= N1/2+1
	LDY = 2*LDX	LDY >= 2*LDX; LDY is even
CFFTC2, ZFFTZ2	LDX >= N1	LDX >= N1
	LDY = LDX	LDY >= N1
CFFTS3, ZFFTD3	LDX1 >= N1/2+1	LDX1 >= N1/2+1
	LDX2 >= N2	LDX2 >= N2
	LDY1 = 2*LDX1	LDY1 >= 2*LDX1; LDY1 is even
	LDY2 = LDX2	LDY2 >= N2
CFFTC3, ZFFTZ3	LDX1 >= N1	LDX1 >= N1
	LDX2 >= N2	LDX2 >= N2
	LDY1 = LDX1	LDY1 >= N1
	LDY2 = LDX2	LDY2 >= N2
SFFTC3, DFFTZ3	LDX1 = 2*LDY1	LDX1 >= N1
	LDX2 >= N2	LDX2 >= N2
	LDY1 >= N1/2+1	LDY1 >= N1/2+1
	LDY2 = LDX2	LDY2 >= N2

In routines that compute transforms between complex and real data type such as SFFTC2 or CFFTS3 even though the transform length is N1, only (N1/2+1) complex data points are referenced or computed. These data points make up the positive-frequency half of the spectrum of the Discrete Fourier Transform. The remaining N1-(N1/2+1) data points can be easily derived since they are complex conjugates and therefore are not stored or referenced.

Contents

NAME

```
icamax - return the index of the element with largest  abso-
lute value.
```

SYNOPSIS

```
INTEGER FUNCTION ICAMAX(N, X, INCX)

COMPLEX X(*)
INTEGER N, INCX

INTEGER*8 FUNCTION ICAMAX_64(N, X, INCX)

COMPLEX X(*)
INTEGER*8 N, INCX
```

F95 INTERFACE

```
INTEGER FUNCTION IAMAX([N], X, [INCX])

COMPLEX, DIMENSION(:) :: X
INTEGER :: N, INCX

INTEGER(8) FUNCTION IAMAX_64([N], X, [INCX])

COMPLEX, DIMENSION(:) :: X
INTEGER(8) :: N, INCX
```

C INTERFACE

```
#include <sunperf.h>

int icamax(int n, complex *x, int incx);
```

```
long icamax_64(long n, complex *x, long incx);
```

PURPOSE

icamax return the index of the element in x with largest absolute value where x is an n-vector and absolute value is defined as the sum of the absolute value of the real part and the absolute value of the imaginary part.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must be positive. Unchanged on exit.

NOTES

If the vector contains all NaNs, the function returns 1. If the vector contains valid complex numbers and one or more NaNs, the routine returns the index of the element containing the largest absolute value.

Contents

NAME

```
idamax - return the index of the element with largest  abso-
lute value.
```

SYNOPSIS

```
INTEGER FUNCTION IDAMAX(N, X, INCX)

INTEGER N, INCX
DOUBLE PRECISION X(*)

INTEGER*8 FUNCTION IDAMAX_64(N, X, INCX)

INTEGER*8 N, INCX
DOUBLE PRECISION X(*)
```

F95 INTERFACE

```
INTEGER FUNCTION IAMAX([N], X, [INCX])

INTEGER :: N, INCX
REAL(8), DIMENSION(:) :: X

INTEGER(8) FUNCTION IAMAX_64([N], X, [INCX])

INTEGER(8) :: N, INCX
REAL(8), DIMENSION(:) :: X
```

C INTERFACE

```
#include <sunperf.h>

int idamax(int n, double *x, int incx);
```

```
long idamax_64(long n, double *x, long incx);
```

PURPOSE

idamax return the index of the element in x with largest absolute value where x is an n-vector.

ARGUMENTS

N (input)
    On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input)
    (1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
    On entry, INCX specifies the increment for the elements of X. INCX must be positive. Unchanged on exit.

NOTES

If the vector contains all NaNs, the function returns 1. If the vector contains valid floating point numbers and one or more NaNs, the routine returns the index of the lement containing the largest absolute value.

Contents

NAME

```
ilaenv - The name of the calling subroutine, in either upper
case or lower case.
```

SYNOPSIS

```
INTEGER FUNCTION ILAENV(ISPEC, NAME, OPTS, N1, N2, N3, N4)

CHARACTER * 6 NAME CHARACTER * 4 OPTS
INTEGER ISPEC, N1, N2, N3, N4

INTEGER*8 FUNCTION ILAENV_64(ISPEC, NAME, OPTS, N1, N2, N3, N4)

CHARACTER * 6 NAME CHARACTER * 4 OPTS
INTEGER*8 ISPEC, N1, N2, N3, N4
```

F95 INTERFACE

```
INTEGER FUNCTION ILAENV(ISPEC, NAME, OPTS, N1, N2, N3, N4)

CHARACTER(LEN=6) :: NAME CHARACTER(LEN=4) :: OPTS
INTEGER :: ISPEC, N1, N2, N3, N4

INTEGER(8) FUNCTION ILAENV_64(ISPEC, NAME, OPTS, N1, N2, N3, N4)

CHARACTER(LEN=6) :: NAME CHARACTER(LEN=4) :: OPTS
INTEGER(8) :: ISPEC, N1, N2, N3, N4
```

C INTERFACE

```
#include <sunperf.h>

int ilaenv(int ispec, char *name, char *opts,  int  n1,  int
          n2, int n3, int n4);
```

```
long ilaenv_64(long ispec, char *name, char *opts, long  n1,
          long n2, long n3, long n4);
```

PURPOSE

ilaenv is called from the LAPACK routines to choose problem-dependent parameters for the local environment. See ISPEC for a description of the parameters.

This version provides a set of parameters which should give good, but not optimal, performance on many of the currently available computers. Users are encouraged to modify this subroutine to set the tuning parameters for their particular machine using the option and problem size information in the arguments.
This routine will not function correctly if it is converted to all lower case. Converting it to all upper case is allowed.

ARGUMENTS

ISPEC (input)
Specifies the parameter to be returned as the value of ILAENV. = 1: the optimal blocksize; if this value is 1, an unblocked algorithm will give the best performance. = 2: the minimum block size for which the block routine should be used; if the usable block size is less than this value, an unblocked routine should be used. = 3: the cross-over point (in a block routine, for N less than this value, an unblocked routine should be used) = 4: the number of shifts, used in the nonsymmetric eigenvalue routines = 5: the minimum column dimension for blocking to be used; rectangular blocks must have dimension at least k by m, where k is given by ILAENV(2,...) and m by ILAENV(5,...) = 6: the crossover point for the SVD (when reducing an m by n matrix to bidiagonal form, if max(m,n)/min(m,n) exceeds this value, a QR factorization is used first to reduce the matrix to a triangular form.) = 7: the number of processors = 8: the crossover point for the multishift QR and QZ methods for nonsymmetric eigenvalue problems. = 9: maximum size of the subproblems at the bottom of the computation tree in the divide-and-conquer algorithm (used by xGELSD and xGESDD) =10: ieee NaN arithmetic can be trusted not to trap

=11: infinity arithmetic can be trusted not to trap

NAME (input)

The name of the calling subroutine, in either upper case or lower case.

OPTS (input)

The character options to the subroutine NAME, concatenated into a single character string. For example, UPLO = 'U', TRANS = 'T', and DIAG = 'N' for a triangular routine would be specified as OPTS = 'UTN'.

N1 (input)

INTEGER

N2 (input)

INTEGER

N3 (input)

INTEGER

N4 (input)

INTEGER

N1, N2, N3, N4 are problem dimensions for the subroutine NAME; these may not all be required.

>= 0: the value of the parameter specified by ISPEC

< 0: if ILAENV = -k, the k-th argument had an illegal value.

< 0: if ILAENV = -k, the k-th argument had an illegal value.

FURTHER DETAILS

The following conventions have been used when calling ILAENV from the LAPACK routines:

1) OPTS is a concatenation of all of the character options to
subroutine NAME, in the same order that they appear in the
argument list for NAME, even if they are not used in determining
the value of the parameter specified by ISPEC.

2) The problem dimensions N1, N2, N3, N4 are specified in the order
that they appear in the argument list for NAME. N1 is used
first, N2 second, and so on, and unused problem dimensions are

passed a value of -1.
3) The parameter value returned by ILAENV is checked for validity in
the calling subroutine. For example, ILAENV is used to retrieve
the optimal blocksize for STRTRI as follows:

```
NB = ILAENV( 1, 'STRTRI', UPLO // DIAG, N, -1, -1, -1 )
IF( NB.LE.1 ) NB = MAX( 1, N )
```

Contents

NAME

```
isamax - return the index of the element with largest  abso-
lute value.
```

SYNOPSIS

```
INTEGER FUNCTION ISAMAX(N, X, INCX)

INTEGER N, INCX
REAL X(*)

INTEGER*8 FUNCTION ISAMAX_64(N, X, INCX)

INTEGER*8 N, INCX
REAL X(*)
```

F95 INTERFACE

```
INTEGER FUNCTION IAMAX([N], X, [INCX])

INTEGER :: N, INCX
REAL, DIMENSION(:) :: X

INTEGER(8) FUNCTION IAMAX_64([N], X, [INCX])

INTEGER(8) :: N, INCX
REAL, DIMENSION(:) :: X
```

C INTERFACE

```
#include <sunperf.h>

int isamax(int n, float *x, int incx);
```

```
long isamax_64(long n, float *x, long incx);
```

PURPOSE

isamax return the index of the element in x with largest absolute value where x is an n-vector.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must be positive. Unchanged on exit.

NOTES

If the vector contains all NaNs, the function returns 1. If the vector contains valid floating point numbers and one or more NaNs, the routine returns the index of the element containing the largest absolute value.

Contents

NAME

```
izamax - return the index of the element with largest  abso-
lute value.
```

SYNOPSIS

```
INTEGER FUNCTION IZAMAX(N, X, INCX)

DOUBLE COMPLEX X(*)
INTEGER N, INCX

INTEGER*8 FUNCTION IZAMAX_64(N, X, INCX)

DOUBLE COMPLEX X(*)
INTEGER*8 N, INCX
```

F95 INTERFACE

```
INTEGER FUNCTION IAMAX([N], X, [INCX])

COMPLEX(8), DIMENSION(:) :: X
INTEGER :: N, INCX

INTEGER(8) FUNCTION IAMAX_64([N], X, [INCX])

COMPLEX(8), DIMENSION(:) :: X
INTEGER(8) :: N, INCX
```

C INTERFACE

```
#include <sunperf.h>

int izamax(int n, doublecomplex *x, int incx);
```

```
long izamax_64(long n, doublecomplex *x, long incx);
```

PURPOSE

izamax return the index of the element in x with largest absolute value where x is an n-vector and absolute value is defined as the sum of the absolute value of the real part and the absolute value of the imaginary part.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the sub- routine to have any visible effect. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must be positive. Unchanged on exit.

NOTES

If the vector contains all NaNs, the function returns 1. If the vector contains valid double complex numbers and one or more NaNs, the routine returns the index of the element con- taining the largest absolute value.

Contents

NAME

```
lsame - returns .TRUE. if  CA  is  the  same  letter  as  CB
regardless of case
```

SYNOPSIS

```
LOGICAL FUNCTION LSAME(CA, CB)

CHARACTER * 1 CA, CB

LOGICAL*8 FUNCTION LSAME_64(CA, CB)

CHARACTER * 1 CA, CB
```

F95 INTERFACE

```
LOGICAL FUNCTION LSAME(CA, CB)

CHARACTER(LEN=1) :: CA, CB

LOGICAL(8) FUNCTION LSAME_64(CA, CB)

CHARACTER(LEN=1) :: CA, CB
```

C INTERFACE

```
#include <sunperf.h>

int lsame(char ca, char cb);

long lsame_64(char ca, char cb);
```

PURPOSE

lsame returns .TRUE. if CA is the same letter as CB regardless of case.

ARGUMENTS

CA (input)
On entry, CA is a single character to compare with CB. Unchanged on exit.

CB (input)
On entry, CB is a single character to compare with CA. Unchanged on exit.

Contents

NAME

```
rfft2b - compute a periodic sequence from its Fourier  coef-
ficients.   The  RFFT operations are unnormalized, so a call
of RFFT2F followed by a call of  RFFT2B  will  multiply  the
input sequence by M*N.
```

SYNOPSIS

```
SUBROUTINE RFFT2B(PLACE, M, N, A, LDA, B, LDB, WORK, LWORK)

CHARACTER * 1 PLACE
INTEGER M, N, LDA, LDB, LWORK
REAL A(LDA,*), B(LDB,*), WORK(*)

SUBROUTINE RFFT2B_64(PLACE, M, N, A, LDA, B, LDB, WORK, LWORK)

CHARACTER * 1 PLACE
INTEGER*8 M, N, LDA, LDB, LWORK
REAL A(LDA,*), B(LDB,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT2B(PLACE, [M], [N], A, [LDA], B, [LDB], WORK, LWORK)

CHARACTER(LEN=1) :: PLACE
INTEGER :: M, N, LDA, LDB, LWORK
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE FFT2B_64(PLACE, [M], [N], A, [LDA], B, [LDB], WORK, LWORK)

CHARACTER(LEN=1) :: PLACE
INTEGER(8) :: M, N, LDA, LDB, LWORK
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void rfft2b(char place, int m, int n,  float  *a,  int  lda,
          float *b, int ldb, float *work, int lwork);

void rfft2b_64(char place, long m, long n,  float  *a,  long
          lda, float *b, long ldb, float *work, long lwork);
```

ARGUMENTS

PLACE (input)
Character. If PLACE = 'I' or 'i' (for in-place) , the input and output data are stored in array A. If PLACE = 'O' or 'o' (for out-of-place), the input data is stored in array B while the output is stored in A.

M (input) Integer specifying the number of rows to be transformed. It is most efficient when M is a product of small primes. M >= 0; when M = 0, the subroutine returns immediately without changing any data.

N (input) Integer specifying the number of columns to be transformed. It is most most efficient when N is a product of small primes. N >= 0; when N = 0, the subroutine returns immediately without changing any data.

A (input/output)
Real array of dimension (LDA,N). On entry, the two-dimensional array A(LDA,N) contains the input data to be transformed if an in-place transform is requested. Otherwise, it is not referenced. Upon exit, results are stored in A(1:M,1:N).

LDA (input)
Integer specifying the leading dimension of A. If an out-of-place transform is desired LDA >= M. Else if an in-place transform is desired LDA >= 2*(M/2+1).

B (input/output)
Real array of dimension (2*LDB, N). On entry, if an out-of-place transform is requested B contains the input data. Otherwise, B is not referenced. B is unchanged upon exit.

LDB (input)

Integer. If an out-of-place transform is desired, 2*LDB is the leading dimension of the array B which contains the data to be transformed and 2*LDB >= 2*(M/2+1). Otherwise it is not referenced.

WORK (input/output)

One-dimensional real array of length at least LWORK. On input, WORK must have been initialized by RFFT2I.

LWORK (input)

Integer. LWORK >= (M + 2*N + MAX(M, 2*N) + 30)

Contents

NAME

```
rfft2f - compute the  Fourier  coefficients  of  a  periodic
sequence.   The  RFFT operations are unnormalized, so a call
of RFFT2F followed by a call of  RFFT2B  will  multiply  the
input sequence by M*N.
```

SYNOPSIS

```
SUBROUTINE RFFT2F(PLACE, FULL, M, N, A, LDA, B, LDB, WORK, LWORK)

CHARACTER * 1 PLACE, FULL
INTEGER M, N, LDA, LDB, LWORK
REAL A(LDA,*), B(LDB,*), WORK(*)

SUBROUTINE RFFT2F_64(PLACE, FULL, M, N, A, LDA, B, LDB, WORK, LWORK)

CHARACTER * 1 PLACE, FULL
INTEGER*8 M, N, LDA, LDB, LWORK
REAL A(LDA,*), B(LDB,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT2F(PLACE, FULL, [M], [N], A, [LDA], B, [LDB], WORK,
       LWORK)

CHARACTER(LEN=1) :: PLACE, FULL
INTEGER :: M, N, LDA, LDB, LWORK
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE FFT2F_64(PLACE, FULL, [M], [N], A, [LDA], B, [LDB], WORK,
       LWORK)

CHARACTER(LEN=1) :: PLACE, FULL
INTEGER(8) :: M, N, LDA, LDB, LWORK
```

```
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void rfft2f(char place, char full, int m, int n,  float  *a,
          int  lda,  float  *b,  int  ldb,  float *work, int
          lwork);

void rfft2f_64(char place, char full, long m, long n,  float
          *a,  long  lda,  float  *b, long ldb, float *work,
          long lwork);
```

ARGUMENTS

PLACE (input)
Character. If PLACE = 'I' or 'i' (for in-place) , the input and output data are stored in array A. If PLACE = 'O' or 'o' (for out-of-place), the input data is stored in array B while the output is stored in A.

FULL (input)
Indicates whether or not to generate the full result matrix. 'F' or 'f' will cause RFFT2F to generate the full result matrix. Otherwise only a partial matrix that takes advantage of symmetry will be generated.

M (input) Integer specifying the number of rows to be transformed. It is most efficient when M is a product of small primes. M >= 0; when M = 0, the subroutine returns immediately without changing any data.

N (input) Integer specifying the number of columns to be transformed. It is most most efficient when N is a product of small primes. N >= 0; when N = 0, the subroutine returns immediately without changing any data.

A (input/output)
On entry, a two-dimensional array A(LDA,N) that contains the data to be transformed. Upon exit, A is unchanged if an out-of-place transform is done. If an in-place transform with partial result is requested, A(1:(M/2+1)*2,1:N) will contain the transformed results. If an in-place transform

with full result is requested, A(1:2*M,1:N) will contain complete transformed results.

LDA (input)
Leading dimension of the array containing the data to be transformed. LDA must be even if the transformed sequences are to be stored in A.
If PLACE = ('O' or 'o') LDA >= M
If PLACE = ('I' or 'i') LDA must be even. If
FULL = ('F' or 'f'), LDA >= 2*M
FULL is not ('F' or 'f'), LDA >= (M/2+1)*2

B (input/output)
Upon exit, a two-dimensional array B(2*LDB,N) that contains the transformed results if an out-of-place transform is done. Otherwise, B is not used.
If an out-of-place transform is done and FULL is not 'F' or 'f', B(1:(M/2+1)*2,1:N) will contain the partial transformed results. If FULL = 'F' or 'f', B(1:2*M,1:N) will contain the complete transformed results.

LDB (input)
2*LDB is the leading dimension of the array B. If an in-place transform is desired LDB is ignored.
If PLACE is ('O' or 'o') and
FULL is ('F' or 'f'), LDB >= M
FULL is not ('F' or 'f'), LDB >= M/2+1
Note that even though LDB is used in the argument list, 2*LDB is the actual leading dimension of B.

WORK (input/output)
One-dimensional real array of length at least LWORK. On input, WORK must have been initialized by RFFT2I.

LWORK (input)
Integer. LWORK >= (M + 2*N + MAX(M, 2*N) + 30)

Contents

NAME

```
rfft2i - initialize the array WSAVE, which is used  in  both
the forward and backward transforms.
```

SYNOPSIS

```
SUBROUTINE RFFT2I(M, N, WORK)

INTEGER M, N
REAL WORK(*)

SUBROUTINE RFFT2I_64(M, N, WORK)

INTEGER*8 M, N
REAL WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT2I(M, N, WORK)

INTEGER :: M, N
REAL, DIMENSION(:) :: WORK

SUBROUTINE FFT2I_64(M, N, WORK)

INTEGER(8) :: M, N
REAL, DIMENSION(:) :: WORK
```

C INTERFACE

```
#include <sunperf.h>

void rfft2i(int m, int n, float *work);

void rfft2i_64(long m, long n, float *work);
```

ARGUMENTS

M (input) Number of rows to be transformed. M >= 0.

N (input) Number of columns to be transformed. N >= 0.

WORK (input/output)
On entry, an array of dimension (M + 2*N + MAX(M, 2*N) + 30) or greater. RFFT2I needs to be called only once to initialize array WORK before calling RFFT2F and/or RFFT2B if M, N and WORK remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
rfft3b - compute a periodic sequence from its Fourier  coef-
ficients.   The  RFFT operations are unnormalized, so a call
of RFFT3F followed by a call of  RFFT3B  will  multiply  the
input sequence by M*N*K.
```

SYNOPSIS

```
SUBROUTINE RFFT3B(PLACE, M, N, K, A, LDA, B, LDB, WORK, LWORK)

CHARACTER * 1 PLACE
INTEGER M, N, K, LDA, LDB, LWORK
REAL A(LDA,N,*), B(LDB,N,*), WORK(*)

SUBROUTINE RFFT3B_64(PLACE, M, N, K, A, LDA, B, LDB, WORK, LWORK)

CHARACTER * 1 PLACE
INTEGER*8 M, N, K, LDA, LDB, LWORK
REAL A(LDA,N,*), B(LDB,N,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT3B(PLACE, [M], [N], [K], A, [LDA], B, [LDB], WORK,
       LWORK)

CHARACTER(LEN=1) :: PLACE
INTEGER :: M, N, K, LDA, LDB, LWORK
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:,:) :: A, B

SUBROUTINE FFT3B_64(PLACE, [M], [N], [K], A, [LDA], B, [LDB], WORK,
       LWORK)

CHARACTER(LEN=1) :: PLACE
INTEGER(8) :: M, N, K, LDA, LDB, LWORK
```

```
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void rfft3b(char place, int m, int n, int k, float  *a,  int
          lda, float *b, int ldb, float *work, int lwork);

void rfft3b_64(char place, long m, long n, long k, float *a,
          long  lda,  float  *b, long ldb, float *work, long
          lwork);
```

ARGUMENTS

PLACE (input)
Select an in-place ('I' or 'i') or out-of-place ('O' or 'o') transform.

M (input) Integer specifying the number of rows to be transformed. It is most efficient when M is a product of small primes. M >= 0; when M = 0, the subroutine returns immediately without changing any data.

N (input) Integer specifying the number of columns to be transformed. It is most efficient when N is a product of small primes. N >= 0; when N = 0, the subroutine returns immediately without changing any data.

K (input) Integer specifying the number of planes to be transformed. It is most efficient when K is a product of small primes. K >= 0; when K = 0, the subroutine returns immediately without changing any data.

A (input/output)
On entry, the three-dimensional array A(LDA,N,K) contains the data to be transformed if an in-place transform is requested. Otherwise, it is not referenced. Upon exit, results are stored in A(1:M,1:N,1:K).

LDA (input)
Integer specifying the leading dimension of A. If an out-of-place transform is desired LDA >= M. Else if an in-place transform is desired LDA >= 2*(M/2+1).

B (input/output)
Real array of dimension B(2*LDB,N,K). On entry, if an out-of-place transform is requested B(1:2*(M/2+1),1:N,1:K) contains the input data. Otherwise, B is not referenced. B is unchanged upon exit.

LDB (input)
If an out-of-place transform is desired, 2*LDB is the leading dimension of the array B which contains the data to be transformed and 2*LDB >= 2*(M/2+1). Otherwise it is not referenced.

WORK (input/output)
One-dimensional real array of length at least LWORK. On input, WORK must have been initialized by RFFT3I.

LWORK (input)
Integer. LWORK >= (M + 2*(N + K) + 4*K + 45).

Contents

NAME

```
rfft3f - compute the Fourier coefficients of a real periodic
sequence.   The  RFFT operations are unnormalized, so a call
of RFFT3F followed by a call of  RFFT3B  will  multiply  the
input sequence by M*N*K.
```

SYNOPSIS

```
SUBROUTINE RFFT3F(PLACE, FULL, M, N, K, A, LDA, B, LDB, WORK, LWORK)

CHARACTER * 1 PLACE, FULL
INTEGER M, N, K, LDA, LDB, LWORK
REAL A(LDA,N,*), B(LDB,N,*), WORK(*)

SUBROUTINE RFFT3F_64(PLACE, FULL, M, N, K, A, LDA, B, LDB, WORK,
      LWORK)

CHARACTER * 1 PLACE, FULL
INTEGER*8 M, N, K, LDA, LDB, LWORK
REAL A(LDA,N,*), B(LDB,N,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT3F(PLACE, FULL, [M], [N], [K], A, [LDA], B, [LDB],
       WORK, LWORK)

CHARACTER(LEN=1) :: PLACE, FULL
INTEGER :: M, N, K, LDA, LDB, LWORK
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:,:) :: A, B

SUBROUTINE FFT3F_64(PLACE, FULL, [M], [N], [K], A, [LDA], B, [LDB],
       WORK, LWORK)

CHARACTER(LEN=1) :: PLACE, FULL
```

```
INTEGER(8) :: M, N, K, LDA, LDB, LWORK
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void rfft3f(char place, char full, int  m,  int  n,  int  k,
          float *a, int lda, float *b, int ldb, float *work,
          int lwork);

void rfft3f_64(char place, char full, long m, long  n,  long
          k,  float  *a, long lda, float *b, long ldb, float
          *work, long lwork);
```

ARGUMENTS

PLACE (input)
Select an in-place ('I' or 'i') or out-of-place ('O' or 'o') transform.

FULL (input)
Select a full ('F' or 'f') or partial (' ') representation of the results. If the caller selects full representation then an MxNxK real array will transform to produce an MxNxK complex array. If the caller does not select full representation then an MxNxK real array will transform to a (M/2+1)xNxK complex array that takes advantage of the symmetry properties of a transformed real sequence.

M (input) Integer specifying the number of rows to be transformed. It is most efficient when M is a product of small primes. M >= 0; when M = 0, the subroutine returns immediately without changing any data.

N (input) Integer specifying the number of columns to be transformed. It is most efficient when N is a product of small primes. N >= 0; when N = 0, the subroutine returns immediately without changing any data.

K (input) Integer specifying the number of planes to be transformed. It is most efficient when K is a product of small primes. K >= 0; when K = 0, the subroutine returns immediately without changing any data.

```
A (input/output)
          On entry,  a  three-dimensional  array  A(LDA,N,K)
          that  contains  input  data to be transformed.  On
          exit, if an in-place transform is done and FULL is
          not  'F'  or 'f', A(1:2*(M/2+1),1:N,1:K) will con-
          tain the partial transformed results.  If  FULL  =
          'F' or 'f', A(1:2*M,1:N,1:K) will contain the com-
          plete transformed results.

LDA (input)
          Leading dimension of the array containing the data
          to  be  transformed.   LDA  must  be  even  if the
          transformed sequences are to be stored in A.
          If PLACE = ('O' or 'o') LDA >= M
          If PLACE = ('I' or 'i') LDA must be even.  If
          FULL = ('F' or 'f'), LDA >= 2*M
          FULL is not ('F' or 'f'), LDA >= 2*(M/2+1)

B (input/output)
          Upon exit, a three-dimensional array  B(2*LDB,N,K)
          that  contains  the transformed results if an out-
          of-place transform is done.  Otherwise, B  is  not
          used.
          If an out-of-place transform is done and  FULL  is
          not  'F'  or 'f', B(1:2*(M/2+1),1:N,1:K) will con-
          tain the partial transformed results.  If  FULL  =
          'F' or 'f', B(1:2*M,1:N,1:K) will contain the com-
          plete transformed results.

LDB (input)
          2*LDB is the leading dimension of the array B.  If
          an in-place transform is desired LDB is ignored.
          If PLACE is ('O' or 'o') and
          FULL is ('F' or 'f'), then LDB >= M
          FULL is not ('F' or 'f'), then LDB >= M/2 + 1
          Note that even though LDB is used in the  argument
          list, 2*LDB is the actual leading dimension of B.

WORK (input/output)
          One-dimensional real  array  of  length  at  least
          LWORK.  WORK must have been initialized by RFFT3I.

LWORK (input)
          Integer.  LWORK >= (M + 2*(N + K) + 4*K + 45).
```

Contents

NAME

```
rfft3i - initialize the array WSAVE, which is used  in  both
RFFT3F and RFFT3B.
```

SYNOPSIS

```
SUBROUTINE RFFT3I(M, N, K, WORK)

INTEGER M, N, K
REAL WORK(*)

SUBROUTINE RFFT3I_64(M, N, K, WORK)

INTEGER*8 M, N, K
REAL WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT3I(M, N, K, WORK)

INTEGER :: M, N, K
REAL, DIMENSION(:) :: WORK

SUBROUTINE FFT3I_64(M, N, K, WORK)

INTEGER(8) :: M, N, K
REAL, DIMENSION(:) :: WORK
```

C INTERFACE

```
#include <sunperf.h>

void rfft3i(int m, int n, int k, float *work);

void rfft3i_64(long m, long n, long k, float *work);
```

ARGUMENTS

M (input) Number of rows to be transformed. M >= 0.

N (input) Number of columns to be transformed. N >= 0.

K (input) Number of planes to be transformed. K >= 0.

WORK (input/output)
On entry, an array of dimension (M + 2*(N + K) + 4*K + 45) or greater. RFFT3I needs to be called only once to initialize array WORK before calling RFFT3F and/or RFFT3B if M, N, K and WORK remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

rfftb - compute a periodic sequence from its Fourier coefficients. The RFFT operations are unnormalized, so a call of RFFTF followed by a call of RFFTB will multiply the input sequence by N.

SYNOPSIS

```
SUBROUTINE RFFTB(N, X, WSAVE)

INTEGER N
REAL X(*), WSAVE(*)

SUBROUTINE RFFTB_64(N, X, WSAVE)

INTEGER*8 N
REAL X(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE FFTB([N], X, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: X, WSAVE

SUBROUTINE FFTB_64([N], X, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: X, WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void rfftb(int n, float *x, float *wsave);
```

```
void rfftb_64(long n, float *x, float *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence to be  transformed.   These
          subroutines are most efficient when N is a product
          of small primes.  N >= 0.

X (input) On entry, an array  of  length  N  containing  the
          sequence to be transformed.

WSAVE (input)
          On entry, WSAVE must be an array of dimension (2 *
          N  + 15) or greater and must have been initialized
          by RFFTI.
```

Contents

NAME

```
rfftf - compute  the  Fourier  coefficients  of  a  periodic
sequence.  The FFT operations are unnormalized, so a call of
RFFTF followed by a call of RFFTB will  multiply  the  input
sequence by N.
```

SYNOPSIS

```
SUBROUTINE RFFTF(N, X, WSAVE)

INTEGER N
REAL X(*), WSAVE(*)

SUBROUTINE RFFTF_64(N, X, WSAVE)

INTEGER*8 N
REAL X(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE FFTF([N], X, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: X, WSAVE

SUBROUTINE FFTF_64([N], X, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: X, WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void rfftf(int n, float *x, float *wsave);
```

```
void rfftf_64(long n, float *x, float *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence to be  transformed.   These
          subroutines are most efficient when N is a product
          of small primes.  N >= 0.

X (input) On entry, an array  of  length  N  containing  the
          sequence to be transformed.

WSAVE (input)
          On entry, WSAVE must be an array of dimension (2 *
          N  + 15) or greater and must have been initialized
          by RFFTI.
```

Contents

NAME

```
rffti - initialize the array WSAVE, which is  used  in  both
RFFTF and RFFTB.
```

SYNOPSIS

```
SUBROUTINE RFFTI(N, WSAVE)

INTEGER N
REAL WSAVE(*)

SUBROUTINE RFFTI_64(N, WSAVE)

INTEGER*8 N
REAL WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE FFTI(N, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: WSAVE

SUBROUTINE FFTI_64(N, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void rffti(int n, float *wsave);

void rffti_64(long n, float *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. N >= 0.

WSAVE (input)
On entry, an array of dimension (2 * N + 15) or greater. RFFTI needs to be called only once to initialize array WORK before calling RFFTF and/or RFFTB if N and WSAVE remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
rfftopt - compute the length of the closest fast FFT
```

SYNOPSIS

```
INTEGER FUNCTION RFFTOPT(LEN)

INTEGER LEN

INTEGER*8 FUNCTION RFFTOPT_64(LEN)

INTEGER*8 LEN
```

F95 INTERFACE

```
INTEGER FUNCTION RFFTOPT(LEN)

INTEGER :: LEN

INTEGER(8) FUNCTION RFFTOPT_64(LEN)

INTEGER(8) :: LEN
```

C INTERFACE

```
#include <sunperf.h>

int rfftopt(int len);

long rfftopt_64(long len);
```

PURPOSE

```
rfftopt computes the length of the closest fast  FFT.   Fast
```

Fourier transform algorithms, including those used in Performance Library, work best with vector lengths that are products of small primes. For example, an FFT of length 32=2**5 will run faster than an FFT of prime length 31 because 32 is a product of small primes and 31 is not. If your application is such that you can taper or zero pad your vector to a larger length then this function may help you select a better length and run your FFT faster.

RFFTOPT will return an integer no smaller than the input argument N that is the closest number that is the product of small primes. RFFTOPT will return 16 for an input of N=16 and return 18=2*3*3 for an input of N=17.

Note that the length computed here is not guaranteed to be optimal, only to be a
product of small primes. Also, the value returned may change as the underlying
FFTs become capable of handling larger primes. For example, passing in N=51 to day will return 52=2*2*13 rather than 51=3*17 because the FFTs in Performance Li brary do not have fast radix 17 code. In the future, radix 17 code may be added
and then N=51 will return 51.

Contents

NAME

```
sasum - Return the sum of the absolute values of a vector x.
```

SYNOPSIS

```
REAL FUNCTION SASUM(N, X, INCX)

INTEGER N, INCX
REAL X(*)

REAL FUNCTION SASUM_64(N, X, INCX)

INTEGER*8 N, INCX
REAL X(*)
```

F95 INTERFACE

```
REAL FUNCTION ASUM([N], X, [INCX])

INTEGER :: N, INCX
REAL, DIMENSION(:) :: X

REAL FUNCTION ASUM_64([N], X, [INCX])

INTEGER(8) :: N, INCX
REAL, DIMENSION(:) :: X
```

C INTERFACE

```
#include <sunperf.h>

float sasum(int n, float *x, int incx);

float sasum_64(long n, float *x, long incx);
```

PURPOSE

sasum Return the sum of the absolute values of x where x is an n-vector.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Contents

NAME

```
saxpy - compute y := alpha * x + y
```

SYNOPSIS

```
SUBROUTINE SAXPY(N, ALPHA, X, INCX, Y, INCY)

INTEGER N, INCX, INCY
REAL ALPHA
REAL X(*), Y(*)

SUBROUTINE SAXPY_64(N, ALPHA, X, INCX, Y, INCY)

INTEGER*8 N, INCX, INCY
REAL ALPHA
REAL X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE AXPY([N], ALPHA, X, [INCX], Y, [INCY])

INTEGER :: N, INCX, INCY
REAL :: ALPHA
REAL, DIMENSION(:) :: X, Y

SUBROUTINE AXPY_64([N], ALPHA, X, [INCX], Y, [INCY])

INTEGER(8) :: N, INCX, INCY
REAL :: ALPHA
REAL, DIMENSION(:) :: X, Y
```

C INTERFACE

```
#include <sunperf.h>
```

```
void saxpy(int n, float alpha, float *x, int incx, float *y,
          int incy);

void saxpy_64(long n, float  alpha,  float  *x,  long  incx,
          float *y, long incy);
```

PURPOSE

saxpy compute y := alpha * x + y where alpha is a scalar and x and y are n-vectors.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
saxpyi - Compute y := alpha * x + y
```

SYNOPSIS

```
 SUBROUTINE SAXPYI(NZ, A, X, INDX, Y)

 REAL A
 REAL X(*), Y(*)
 INTEGER NZ
 INTEGER INDX(*)

 SUBROUTINE SAXPYI_64(NZ, A, X, INDX, Y)

 REAL A
 REAL X(*), Y(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 SUBROUTINE AXPYI([NZ], [A], X, INDX, Y)

 REAL :: A
 REAL, DIMENSION(:) :: X, Y
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 SUBROUTINE AXPYI_64([NZ], [A], X, INDX, Y)

 REAL :: A
 REAL, DIMENSION(:) :: X, Y
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
SAXPYI Compute y := alpha * x + y where alpha is a scalar, x
is a sparse vector, and y is a vector in full storage form

 do i = 1, n
   y(indx(i)) = alpha * x(i) + y(indx(i))
 enddo
```

ARGUMENTS

```
NZ (input) - INTEGER
        Number of elements in the compressed form.
        Unchanged on exit.

A (input)
        On entry, A(LPHA) specifies the scaling value.
        Unchanged on exit.  A is defaulted to 1.0E0 for F95
        INTERFACE.
X (input)
        Vector containing the values of the compressed form.
        Unchanged on exit.

INDX (input) - INTEGER
        Vector containing the indices of the compressed
        form.  It is assumed that the elements in INDX are
        distinct and greater than zero.  Unchanged on exit.

Y (output)
        Vector on input which contains the vector Y in full
        storage form.  On exit, only the elements
        corresponding to the indices in INDX have been
        modified.
```

Contents

NAME

sbcomm - block coordinate matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE SBCOMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BJNDX, BNNZ, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, KB, DESCRA(5), BNNZ, LB,
*           LDB, LDC, LWORK
 INTEGER    BINDX(BNNZ), BJNDX(BNNZ)
 REAL       ALPHA, BETA
 REAL       VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE SBCOMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BJNDX, BNNZ, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), BNNZ, LB,
*           LDB, LDC, LWORK
 INTEGER*8  BINDX(BNNZ), BJNDX(BNNZ)
 REAL       ALPHA, BETA
 REAL       VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
  SUBROUTINE BCOMM(TRANSA,MB,N,KB,ALPHA,DESCRA,VAL,BINDX, BJNDX,
*   BNNZ, LB, B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, N, KB, BNNZ, LB
 INTEGER, DIMENSION(:) ::   DESCRA, BINDX, BJNDX
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL
 REAL, DIMENSION(:, :) ::  B, C

  SUBROUTINE BCOMM_64(TRANSA,MB,N,KB,ALPHA,DESCRA,VAL,BINDX, BJNDX,
```

```
*   BNNZ, LB, B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, N, KB, BNNZ, LB
 INTEGER*8, DIMENSION(:) ::  DESCRA,  BINDX, BJNDX
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL
 REAL, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
         C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block coordinate format and
op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                      ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if the matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
```

```
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

 VAL()          scalar array of length LB*LB*BNNZ consisting of
                the non-zero block entries of A, in any order.
                Each block is stored in standard column-major form.

 BINDX()        integer array of length BNNZ consisting of the
                block row indices of the block entries of A.

 BJNDX()        integer array of length BNNZ consisting of the
                block column indices of the block entries of A.

 BNNZ           number of block entries

 LB             dimension of dense blocks composing A.

 B()            rectangular array with first dimension LDB.
 LDB            leading dimension of B

 BETA           Scalar parameter

 C()            rectangular array with first dimension LDC.

 LDC            leading dimension of C

 WORK()         scratch array of length LWORK. WORK is not
                referenced in the current version.

 LWORK          length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

```
sbdimm - block diagonal format matrix-matrix multiply
```

SYNOPSIS

```
  SUBROUTINE SBDIMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *           VAL, BLDA, IBDIAG, NBDIAG, LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
  INTEGER    TRANSA, MB, N, KB, DESCRA(5), BLDA, NBDIAG, LB,
 *           LDB, LDC, LWORK
  INTEGER    IBDIAG(NBDIAG)
  REAL       ALPHA, BETA
  REAL       VAL(LB*LB*BLDA*NBDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)

  SUBROUTINE SBDIMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *           VAL, BLDA, IBDIAG, NBDIAG, LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
  INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), BLDA, NBDIAG, LB,
 *           LDB, LDC, LWORK
  INTEGER*8  IBDIAG(NBDIAG)
  REAL       ALPHA, BETA
  REAL       VAL(LB*LB*BLDA*NBDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
   SUBROUTINE BDIMM(TRANSA,MB, [N], KB, ALPHA, DESCRA, VAL, BLDA,
 *    IBDIAG, NBDIAG, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
  INTEGER    TRANSA, MB, KB, BLDA, NBDIAG, LB
  INTEGER, DIMENSION(:) ::    DESCRA, IBDIAG
  REAL    ALPHA, BETA
  REAL, DIMENSION(:) :: VAL
  REAL, DIMENSION(:, :) ::  B, C

   SUBROUTINE BDIMM_64(TRANSA,MB, [N], KB, ALPHA, DESCRA, VAL, BLDA,
```

```
*    IBDIAG, NBDIAG, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, KB, BLDA, NBDIAG, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, IBDIAG
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL
 REAL, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
         C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block diagonal format and
op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix  is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
```

```
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

  VAL()         two-dimensional LB*LB*BLDA-by-NBDIAG scalar array
                consisting of the NBDIAG nonzero block diagonal in
                any order. Each dense block is stored in standard
                column-major form.

  BLDA          leading block dimension of VAL().

  IBDIAG()      integer array of length NBDIAG consisting of the
                corresponding diagonal offsets of the non-zero
                block diagonals of A in VAL.  Lower triangular
                block diagonals have negative offsets, the main
                block diagonal has offset 0, and upper triangular
                block diagonals have positive offset.

  NBDIAG        the number of non-zero block diagonals in A.
  LB            dimension of dense blocks composing A.

  B()           rectangular array with first dimension LDB.

  LDB           leading dimension of B

  BETA          Scalar parameter

  C()           rectangular array with first dimension LDC.

  LDC           leading dimension of C

  WORK()        scratch array of length LWORK. WORK is not
                referenced in the current version.

  LWORK         length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

```
sbdism -  block diagonal format triangular solve
```

SYNOPSIS

```
 SUBROUTINE SBDISM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, BLDA, IBDIAG, NBDIAG, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), BLDA, NBDIAG, LB,
*           LDB, LDC, LWORK
 INTEGER    IBDIAG(NBDIAG)
 REAL       ALPHA, BETA
 REAL       DV(MB*LB*LB), VAL(LB*LB*BLDA, NBDIAG), B(LDB,*), C(LDC,*),
*           WORK(LWORK)

 SUBROUTINE SBDISM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, BLDA, IBDIAG, NBDIAG, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), BLDA, NBDIAG, LB,
*           LDB, LDC, LWORK
 INTEGER*8  IBDIAG(NBDIAG)
 REAL       ALPHA, BETA
 REAL       DV(MB*LB*LB), VAL(LB*LB*BLDA, NBDIAG), B(LDB,*), C(LDC,*),
*           WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE BDISM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BLDA,
*   IBDIAG, NBDIAG, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, N, UNITD, BLDA, NBDIAG, LB
 INTEGER, DIMENSION(:) ::    DESCRA, IBDIAG
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL, DV
 REAL, DIMENSION(:, :) ::   B, C
```

```
 SUBROUTINE BDISM_64(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BLDA,
*   IBDIAG, NBDIAG, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, N, UNITD, BLDA, NBDIAG, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, IBDIAG
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL, DV
 REAL, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in block diagonal format
and  op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)

DV()          Array of length MB*LB*LB containing the elements of
              the diagonal blocks of the matrix D.  The size of each
              square block is LB-by-LB and each block
              is stored in standard column-major form.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                 0 : general
                 1 : symmetric (A=A')
                 2 : Hermitian (A= CONJG(A'))
                 3 : Triangular
                 4 : Skew(Anti)-Symmetric (A=-A')
                 5 : Diagonal
                 6 : Skew-Hermitian (A= -CONJG(A'))
               Note: For the routine, DESCRA(1)=3 is only supported.

               DESCRA(2) upper/lower triangular indicator
                 1 : lower
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-identity blocks on the main diagonal
                 1 : identity diagonal blocks
                 2 : diagonal blocks are dense matrices
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices
VAL()          Two-dimensional LB*LB*BLDA-by-NBDIAG scalar array
               consisting of the NBDIAG non-zero block diagonal.
               Each dense block is stored in standard column-major form.

BLDA           Leading block dimension of VAL().  Should be greater
               than or equal to MB.

IBDIAG()       integer array of length NBDIAG consisting of the
               corresponding diagonal offsets of the non-zero block
               diagonals of A in VAL.  Lower triangular block diagonals
               have negative offsets, the main block diagonal has offset
               0, and upper triangular block diagonals have positive offset.
               Elements of IBDIAG  MUST be sorted in  increasing order.

NBDIAG         The number of non-zero block diagonals in A.

LB             Dimension of dense blocks composing A.

B()            Rectangular array with first dimension LDB.

LDB            Leading dimension of B.

BETA           Scalar parameter.

C()            Rectangular array with first dimension LDC.

LDC            Leading dimension of C.
```

```
WORK()        scratch array of length LWORK.
              On exit, if LWORK= -1, WORK(1) returns the optimum  size
              of LWORK.

LWORK        length of WORK array. LWORK should be at least
             MB*LB.

             For good performance, LWORK should generally be larger.
             For optimum performance on multiple processors, LWORK
             >=MB*LB*N_CPUS where N_CPUS is the maximum number of
             processors available to the program.

             If LWORK=0, the routine is to allocate workspace needed.

             If LWORK = -1, then a workspace query is assumed; the
             routine only calculates the optimum size of the WORK array,
             returns this value as the first entry of the WORK array,
             and no error message related to LWORK is issued by XERBLA.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If *DESCRA*(3)=0, the lower or upper triangular part of each diagonal block is used by the routine depending on *DESCRA*(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might not be referenced in the BDI representation of a sparse matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense matrices and the LU factorization with partial pivoting is used by the routine. *WORK*(1)=0 on return if the factorization for all diagonal blocks has been completed successfully, otherwise *WORK*(1) = -i where i is the block

number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

```
sbdsdc - compute the singular value decomposition (SVD) of a
real N-by-N (upper or lower) bidiagonal matrix B
```

SYNOPSIS

```
SUBROUTINE SBDSDC(UPLO, COMPQ, N, D, E, U, LDU, VT, LDVT, Q, IQ,
      WORK, IWORK, INFO)

CHARACTER * 1 UPLO, COMPQ
INTEGER N, LDU, LDVT, INFO
INTEGER IQ(*), IWORK(*)
REAL D(*), E(*), U(LDU,*), VT(LDVT,*), Q(*), WORK(*)

SUBROUTINE SBDSDC_64(UPLO, COMPQ, N, D, E, U, LDU, VT, LDVT, Q, IQ,
      WORK, IWORK, INFO)

CHARACTER * 1 UPLO, COMPQ
INTEGER*8 N, LDU, LDVT, INFO
INTEGER*8 IQ(*), IWORK(*)
REAL D(*), E(*), U(LDU,*), VT(LDVT,*), Q(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE BDSDC(UPLO, COMPQ, [N], D, E, U, [LDU], VT, [LDVT], Q, IQ,
       [WORK], [IWORK], [INFO])

CHARACTER(LEN=1) :: UPLO, COMPQ
INTEGER :: N, LDU, LDVT, INFO
INTEGER, DIMENSION(:) :: IQ, IWORK
REAL, DIMENSION(:) :: D, E, Q, WORK
REAL, DIMENSION(:,:) :: U, VT
```

```
SUBROUTINE BDSDC_64(UPLO, COMPQ, [N], D, E, U, [LDU], VT, [LDVT], Q,
       IQ, [WORK], [IWORK], [INFO])

CHARACTER(LEN=1) :: UPLO, COMPQ
INTEGER(8) :: N, LDU, LDVT, INFO
INTEGER(8), DIMENSION(:) :: IQ, IWORK
REAL, DIMENSION(:) :: D, E, Q, WORK
REAL, DIMENSION(:,:) :: U, VT
```

C INTERFACE

```
#include <sunperf.h>

void sbdsdc(char uplo, char compq, int n,  float  *d,  float
          *e,  float *u, int ldu, float *vt, int ldvt, float
          *q, int *iq, int *info);
void sbdsdc_64(char uplo, char  compq,  long  n,  float  *d,
          float  *e,  float  *u,  long  ldu, float *vt, long
          ldvt, float *q, long *iq, long *info);
```

PURPOSE

sbdsdc computes the singular value decomposition (SVD) of a real N-by-N (upper or lower) bidiagonal matrix B: B = U * S * VT, using a divide and conquer method, where S is a diagonal matrix with non-negative diagonal elements (the singular values of B), and U and VT are orthogonal matrices of left and right singular vectors, respectively. SBDSDC can be used to compute all singular values, and optionally, singular vectors or singular vectors in compact form.

This code makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none. See SLASD3 for details.

The code currently call SLASDQ if singular values only are desired. However, it can be slightly modified to compute singular values using the divide and conquer method.

ARGUMENTS

UPLO (input)
          = 'U': B is upper bidiagonal.
          = 'L': B is lower bidiagonal.

COMPQ (input)
Specifies whether singular vectors are to be computed as follows:
= 'N': Compute singular values only;
= 'P': Compute singular values and compute singular vectors in compact form; = 'I': Compute singular values and singular vectors.

N (input) The order of the matrix B. N >= 0.

D (input/output)
On entry, the n diagonal elements of the bidiagonal matrix B. On exit, if INFO=0, the singular values of B.

E (input/output)
On entry, the elements of E contain the offdiagonal elements of the bidiagonal matrix whose SVD is desired. On exit, E has been destroyed.

U (output)
If COMPQ = 'I', then: On exit, if INFO = 0, U contains the left singular vectors of the bidiagonal matrix. For other values of COMPQ, U is not referenced.

LDU (input)
The leading dimension of the array U. LDU >= 1. If singular vectors are desired, then LDU >= max(1, N).

VT (output)
If COMPQ = 'I', then: On exit, if INFO = 0, VT' contains the right singular vectors of the bidiagonal matrix. For other values of COMPQ, VT is not referenced.

LDVT (input)
The leading dimension of the array VT. LDVT >= 1. If singular vectors are desired, then LDVT >= max(1, N).

Q (input) If COMPQ = 'P', then: On exit, if INFO = 0, Q and IQ contain the left and right singular vectors in a compact form, requiring O(N log N) space instead of 2*N**2. In particular, Q contains all the REAL data in LDQ >= N*(11 + 2*SMLSIZ + 8*INT(LOG_2(N/(SMLSIZ+1)))) words of memory, where SMLSIZ is returned by ILAENV and is equal to the maximum size of the subproblems at the bottom of

```
            the computation  tree  (usually  about  25).   For
            other values of COMPQ, Q is not referenced.

  IQ (output)
            If  COMPQ = 'P', then:  On exit, if INFO  =  0,  Q
            and IQ contain the left and right singular vectors
            in a compact form,  requiring  O(N  log  N)  space
            instead of 2*N**2.  In particular, IQ contains all
            INTEGER    data    in    LDIQ    >=     N*(3     +
            3*INT(LOG_2(N/(SMLSIZ+1)))) words of memory, where
            SMLSIZ is returned by ILAENV and is equal  to  the
            maximum  size  of the subproblems at the bottom of
            the computation  tree  (usually  about  25).   For
            other values of COMPQ, IQ is not referenced.

  WORK (workspace)
            If COMPQ = 'N' then LWORK >= (2 * N).  If COMPQ  =
            'P'  then  LWORK  >= (6 * N).  If COMPQ = 'I' then
            LWORK >= (3 * N**2 + 4 * N).

  IWORK (workspace)
            dimension(8*N)

  INFO (output)
            = 0:  successful exit.
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value.
            > 0:  The algorithm failed to compute an  singular
            value.   The  update process of divide and conquer
            failed.
```

FURTHER DETAILS

```
  Based on contributions by
     Ming Gu and Huan Ren, Computer Science Division,  Univer-
  sity of
     California at Berkeley, USA
```

Contents

NAME

```
sbdsqr - compute the singular value decomposition (SVD) of a
real N-by-N (upper or lower) bidiagonal matrix B.
```

SYNOPSIS

```
SUBROUTINE SBDSQR(UPLO, N, NCVT, NRU, NCC, D, E, VT, LDVT, U, LDU, C,
      LDC, WORK, INFO)

CHARACTER * 1 UPLO
INTEGER N, NCVT, NRU, NCC, LDVT, LDU, LDC, INFO
REAL D(*), E(*), VT(LDVT,*), U(LDU,*), C(LDC,*), WORK(*)

SUBROUTINE SBDSQR_64(UPLO, N, NCVT, NRU, NCC, D, E, VT, LDVT, U, LDU,
      C, LDC, WORK, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NCVT, NRU, NCC, LDVT, LDU, LDC, INFO
REAL D(*), E(*), VT(LDVT,*), U(LDU,*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE BDSQR(UPLO, [N], [NCVT], [NRU], [NCC], D, E, VT, [LDVT],
       U, [LDU], C, [LDC], [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NCVT, NRU, NCC, LDVT, LDU, LDC, INFO
REAL, DIMENSION(:) :: D, E, WORK
REAL, DIMENSION(:,:) :: VT, U, C

SUBROUTINE BDSQR_64(UPLO, [N], [NCVT], [NRU], [NCC], D, E, VT, [LDVT],
       U, [LDU], C, [LDC], [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
INTEGER(8) :: N, NCVT, NRU, NCC, LDVT, LDU, LDC, INFO
REAL, DIMENSION(:) :: D, E, WORK
REAL, DIMENSION(:,:) :: VT, U, C
```

C INTERFACE

```
#include <sunperf.h>

void sbdsqr(char uplo, int n, int ncvt, int  nru,  int  ncc,
          float *d, float *e, float *vt, int ldvt, float *u,
          int ldu, float *c, int ldc, int *info);

void sbdsqr_64(char uplo, long n, long ncvt, long nru,  long
          ncc,  float  *d,  float  *e, float *vt, long ldvt,
          float *u, long  ldu,  float  *c,  long  ldc,  long
          *info);
```

PURPOSE

sbdsqr computes the singular value decomposition (SVD) of a real N-by-N (upper or lower) bidiagonal matrix B: B = Q * S * P' (P' denotes the transpose of P), where S is a diagonal matrix with non-negative diagonal elements (the singular values of B), and Q and P are orthogonal matrices.

The routine computes S, and optionally computes U * Q, P' * VT, or Q' * C, for given real input matrices U, VT, and C.

See "Computing Small Singular Values of Bidiagonal Matrices With Guaranteed High Relative Accuracy," by J. Demmel and W. Kahan, LAPACK Working Note #3 (or SIAM J. Sci. Statist. Comput. vol. 11, no. 5, pp. 873-912, Sept 1990) and "Accurate singular values and differential qd algorithms," by B. Parlett and V. Fernando, Technical Report CPAM-554, Mathematics Department, University of California at Berkeley, July 1992 for a detailed description of the algorithm.

ARGUMENTS

UPLO (input)
  = 'U': B is upper bidiagonal;
  = 'L': B is lower bidiagonal.

N (input) The order of the matrix B. N >= 0.

NCVT (input)
  The number of columns of the matrix VT. NCVT >= 0.

NRU (input)
The number of rows of the matrix U. NRU >= 0.

NCC (input)
The number of columns of the matrix C. NCC >= 0.

D (input/output)
On entry, the n diagonal elements of the bidiagonal matrix B. On exit, if INFO=0, the singular values of B in decreasing order.

E (input/output)
On entry, the elements of E contain the offdiagonal elements of the bidiagonal matrix whose SVD is desired. On normal exit (INFO = 0), E is destroyed. If the algorithm does not converge (INFO > 0), D and E will contain the diagonal and superdiagonal elements of a bidiagonal matrix orthogonally equivalent to the one given as input. E(N) is used for workspace.

VT (input/output)
On entry, an N-by-NCVT matrix VT. On exit, VT is overwritten by P' * VT. VT is not referenced if NCVT = 0.

LDVT (input)
The leading dimension of the array VT. LDVT >= max(1,N) if NCVT > 0; LDVT >= 1 if NCVT = 0.

U (input/output)
On entry, an NRU-by-N matrix U. On exit, U is overwritten by U * Q. U is not referenced if NRU = 0.

LDU (input)
The leading dimension of the array U. LDU >= max(1,NRU).

C (input/output)
On entry, an N-by-NCC matrix C. On exit, C is overwritten by Q' * C. C is not referenced if NCC = 0.

LDC (input)
The leading dimension of the array C. LDC >= max(1,N) if NCC > 0; LDC >=1 if NCC = 0.

WORK (workspace)
dimension(4*N)

INFO (output)
= 0: successful exit
< 0: If INFO = -i, the i-th argument had an illegal value
> 0: the algorithm did not converge; D and E contain the elements of a bidiagonal matrix which is orthogonally similar to the input matrix B; if INFO = i, i elements of E have not converged to zero.

Contents

NAME

sbelmm - block Ellpack format matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE SBELMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BLDA, MAXBNZ, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, KB, DESCRA(5), BLDA, MAXBNZ, LB,
*           LDB, LDC, LWORK
 INTEGER    BINDX(BLDA,MAXBNZ)
 REAL       ALPHA, BETA
 REAL       VAL(LB*LB*BLDA*MAXBNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE SBELMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BLDA, MAXBNZ, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), BLDA, MAXBNZ, LB,
*           LDB, LDC, LWORK
 INTEGER*8  BINDX(BLDA,MAXBNZ)
 REAL       ALPHA, BETA
 REAL       VAL(LB*LB*BLDA*MAXBNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE BELMM( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
*           BLDA, MAXBNZ, LB, B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, KB, BLDA, MAXBNZ, LB
 INTEGER, DIMENSION(:) ::    DESCRA, BINDX
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL
 REAL, DIMENSION(:, :) ::  B, C

 SUBROUTINE BELMM_64( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
```

```
*            BLDA, MAXBNZ, LB, B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, KB, BLDA, MAXBNZ, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL
 REAL, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
         C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block Ellpack format and
op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
```

```
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

 VAL()          scalar array of length LB*LB*BLDA*MAXBNZ containing
                matrix entries, stored column-major within each dense
                block.

 BINDX()        two-dimensional integer BLDA-by-MAXBNZ array such
                BINDX(i,:) consists of the block column indices of the
                nonzero blocks in block row i, padded by the integer
                value i if the number of nonzero blocks is less than
                MAXBNZ.

 BLDA           leading dimension of BINDX(:,:).

 MAXBNZ         max number of nonzeros blocks per row.
 LB             row and column dimension of the dense blocks composing
                VAL.

 B()            rectangular array with first dimension LDB.

 LDB            leading dimension of B

 BETA           Scalar parameter

 C()            rectangular array with first dimension LDC.

 LDC            leading dimension of C

 WORK()         scratch array of length LWORK. WORK is not
                referenced in the current version.

 LWORK          length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS)

Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

sbelsm - block Ellpack format triangular solve

SYNOPSIS

```
 SUBROUTINE SBELSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, BINDX, BLDA, MAXBNZ, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), BLDA, MAXBNZ, LB,
*           LDB, LDC, LWORK
 INTEGER    BINDX(BLDA,MAXBNZ)
 REAL       ALPHA, BETA
 REAL       DV(MB*LB*LB), VAL(LB*LB*BLDA*MAXBNZ), B(LDB,*), C(LDC,*),
*           WORK(LWORK)

 SUBROUTINE SBELSM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, BINDX, BLDA, MAXBNZ, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), BLDA, MAXBNZ, LB,
*           LDB, LDC, LWORK
 INTEGER*8  BINDX(BLDA,MAXBNZ)
 REAL       ALPHA, BETA
 REAL       DV(MB*LB*LB), VAL(LB*LB*BLDA*MAXBNZ), B(LDB,*), C(LDC,*),
*           WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE BELSM( TRANSA, MB, [N], UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*   BLDA, MAXBNZ, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, UNITD,  BLDA, MAXBNZ, LB
 INTEGER, DIMENSION(:) ::    DESCRA,  BINDX
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL, DV
 REAL, DIMENSION(:, :) ::   B, C
```

```
 SUBROUTINE BELSM_64( TRANSA, MB, [N], UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*   BLDA, MAXBNZ, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8  TRANSA, MB, UNITD,  BLDA, MAXBNZ, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA,  BINDX
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL, DV
 REAL, DIMENSION(:, :) ::   B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in block Ellpack format and
op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)

DV()          Array of the length MB*LB*LB consisting of the block
              entries of block diagonal matrix D where each
              block is stored in standard column-major form.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
```

```
                    1 : symmetric (A=A')
                    2 : Hermitian (A= CONJG(A'))
                    3 : Triangular
                    4 : Skew(Anti)-Symmetric (A=-A')
                    5 : Diagonal
                    6 : Skew-Hermitian (A= -CONJG(A'))
                  Note: For the routine, DESCRA(1)=3 is only supported.

                  DESCRA(2) upper/lower triangular indicator
                    1 : lower
                    2 : upper
                  DESCRA(3) main diagonal type
                    0 : non-identity blocks on the main diagonal
                    1 : identity diagonal blocks
                    2 : diagonal blocks are dense matrices
                  DESCRA(4) Array base  (NOT IMPLEMENTED)
                    0 : C/C++ compatible
                    1 : Fortran compatible
                  DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                    0 : unknown
                    1 : no repeated indices
    VAL()         scalar array of length LB*LB*BLDA*MAXBNZ containing
                  matrix entries, stored column-major within each dense
                  block.

    BINDX()       two-dimensional integer BLDA-by-MAXBNZ array such
                  BINDX(i,:) consists of the block column indices of the
                  nonzero blocks in block row i, padded by the integer
                  value i if the number of nonzero blocks is less than
                  MAXBNZ.  The block column indices MUST be sorted
                  in increasing order for each block row.

    BLDA          leading dimension of BINDX(:,:).

    MAXBNZ        max number of nonzeros blocks per row.

    LB            row and column dimension of the dense blocks composing A.

    B()           rectangular array with first dimension LDB.

    LDB           leading dimension of B

    BETA          Scalar parameter

    C()           rectangular array with first dimension LDC.

    LDC           leading dimension of C

    WORK()        scratch array of length LWORK.
                  On exit, if LWORK= -1, WORK(1) returns the minimum
```

size of LWORK.

LWORK length of WORK array. LWORK should be at least MB*LB.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=MB*LB*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If *DESCRA*(3)=0,the lower or upper triangular part of each diagonal block is used by the routine depending on *DESCRA*(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might not be referenced in the BEL representation of a sparse matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense matrices and the LU factorization with partial pivoting is used by the routine. *WORK*(1)=0 on return if the factorization for all diagonal blocks has been completed successfully, otherwise *WORK*(1) = -i where i is the block number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

```
sbscmm - block sparse column matrix-matrix multiply
```

SYNOPSIS

```
 SUBROUTINE SBSCMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BPNTRB, BPNTRE, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, KB, DESCRA(5), LB,
*           LDB, LDC, LWORK
 INTEGER    BINDX(BNNZ), BPNTRB(KB), BPNTRE(KB)
 REAL       ALPHA, BETA
 REAL       VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE SBSCMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BPNTRB, BPNTRE, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), LB,
*           LDB, LDC, LWORK
 INTEGER*8  BINDX(BNNZ), BPNTRB(KB), BPNTRE(KB)
 REAL       ALPHA, BETA
 REAL       VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where: BNNZ = BPNTRE(KB)-BPNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE BSCMM( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB,  KB, LB
 INTEGER, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL
 REAL, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE BSCMM_64( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, KB, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL
 REAL, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block sparse column format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
```

```
                 DESCRA(3) main diagonal type
                   0 : non-unit
                   1 : unit
                 DESCRA(4) Array base  (NOT IMPLEMENTED)
                   0 : C/C++ compatible
                   1 : Fortran compatible
                 DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                   0 : unknown
                   1 : no repeated indices

   VAL()         scalar array of length LB*LB*BNNZ consisting of
                 the block entries stored column-major within each
                 dense block.

   BINDX()       integer array of length BNNZ consisting of the
                 block row indices of the block entries of A.

   BPNTRB()      integer array of length KB such that
                 BPNTRB(J)-BPNTRB(1)+1 points to location in BINDX
                 of the first block entry of the J-th block column
                 of A.
   BPNTRE()      integer array of length KB such that
                 BPNTRE(J)-BPNTRB(1) points to location in BINDX
                 of the last block entry of the J-th block column
                 of A.

   LB            dimension of dense blocks composing A.

   B()           rectangular array with first dimension LDB.

   LDB           leading dimension of B

   BETA          Scalar parameter

   C()           rectangular array with first dimension LDC.

   LDC           leading dimension of C

   WORK()        scratch array of length LWORK. WORK is not
                 referenced in the current version.

   LWORK         length of WORK array. LWORK is not referenced
                 in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

It is known that there exists another representation of the block sparse column format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block column in the arrays VAL and BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of block sparse column format the following calling sequence should be used

```
  CALL SBSCMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *           VAL, BINDX, IA, IA(2), LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

sbscsm - block sparse column format triangular solve

SYNOPSIS

```
 SUBROUTINE SBSCSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, BINDX, BPNTRB, BPNTRE, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), LB,
*           LDB, LDC, LWORK
 INTEGER    BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
 REAL       ALPHA, BETA
 REAL       DV(MB*LB*LB), VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE SBSCSM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, BINDX, BPNTRB, BPNTRE, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), LB,
*           LDB, LDC, LWORK
 INTEGER*8  BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
 REAL       ALPHA, BETA
 REAL       DV(MB*LB*LB), VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where: BNNZ = BPNTRE(MB)- BPNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE BSCSM(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, N, UNITD, LB
 INTEGER, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL, DV
 REAL, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE BSCSM_64(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, N, UNITD, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL, DV
 REAL, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in block sparse column format
and  op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row block scaling)
                3 : Scale on right (column block scaling)

DV()          Array of the length MB*LB*LB consisting of the block
              entries of block diagonal matrix D where each
              block is stored in standard column-major form.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
```

```
                 1 : symmetric (A=A')
                 2 : Hermitian (A= CONJG(A'))
                 3 : Triangular
                 4 : Skew(Anti)-Symmetric (A=-A')
                 5 : Diagonal
                 6 : Skew-Hermitian (A= -CONJG(A'))

               Note: For the routine, DESCRA(1)=3 is only supported.

               DESCRA(2) upper/lower triangular indicator
                 1 : lower
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-identity blocks on the main diagonal
                 1 : identity diagonal blocks
                 2 : diagonal blocks are dense matrices
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices
VAL()          scalar array of length LB*LB*BNNZ consisting of the block
               entries stored column-major within each dense block.

BINDX()        integer array of length BNNZ consisting of the
               block row indices of the block entries of A.
               The block row indices MUST be sorted
               in increasing order for each block column.

BPNTRB()       integer array of length MB such that
               BPNTRB(J)-BPNTRB(1)+1 points to location in BINDX
               of the first block entry of the J-th block column of A.

BPNTRE()       integer array of length MB such that
               BPNTRE(J)-BPNTRB(1) points to location in BINDX
               of the last block entry of the J-th block column of A.

LB             dimension of dense blocks composing A.

B()            rectangular array with first dimension LDB.

LDB            leading dimension of B

BETA           Scalar parameter

C()            rectangular array with first dimension LDC.

LDC            leading dimension of C
```

```
 WORK()        scratch array of length LWORK.
               On exit, if LWORK= -1, WORK(1) returns the optimum
               size of LWORK.

 LWORK         length of WORK array. LWORK should be at least
               MB*LB.

               For good performance, LWORK should generally be larger.
               For optimum performance on multiple processors, LWORK
               >=MB*LB*N_CPUS where N_CPUS is the maximum number of
               processors available to the program.

               If LWORK=0, the routine is to allocate workspace needed.

               If LWORK = -1, then a workspace query is assumed; the
               routine only calculates the optimum size of the WORK
               array, returns this value as the first entry of the WORK
               array, and no error message related to LWORK is issued
               by XERBLA.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If *DESCRA*(3)=0,the lower or upper triangular part of each diagonal block is used by the routine depending on *DESCRA*(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might not be referenced in the BSC representation of a sparse matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense matrices and the LU factorization with partial pivoting is used by the routine. *WORK*(1)=0 on return if the factorization for all diagonal blocks has been completed

successfully, otherwise *WORK*(1) = -i where i is the block number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3 in this case.

6. It is known that there exists another representation of the block sparse column format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block column in the arrays VAL and BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of block sparse column format the following calling sequence should be used

```
  CALL SBSCSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, BINDX, IA, IA(2), LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

sbsrmm - block sparse row format matrix-matrix multiply

SYNOPSIS

```
  SUBROUTINE SBSRMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *           VAL, BINDX, BPNTRB, BPNTRE, LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
  INTEGER    TRANSA, MB, N, KB, DESCRA(5), LB,
 *           LDB, LDC, LWORK
  INTEGER    BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
  REAL       ALPHA, BETA
  REAL       VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

  SUBROUTINE SBSRMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *           VAL, BINDX, BPNTRB, BPNTRE, LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
  INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), LB,
 *           LDB, LDC, LWORK
  INTEGER*8  BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
  REAL       ALPHA, BETA
  REAL       VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

  where: BNNZ = BPNTRE(MB)-BPNTRB(1)
```

F95 INTERFACE

```
  SUBROUTINE BSRMM( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
 *     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
  INTEGER    TRANSA, MB,  KB, LB
  INTEGER, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
  REAL    ALPHA, BETA
  REAL, DIMENSION(:) :: VAL
  REAL, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE BSRMM_64( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, KB, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL
 REAL, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block sparse row format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix A is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
```

```
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

 VAL()          scalar array of length LB*LB*BNNZ consisting
                of the block  entries stored column-major within
                each dense block.

 BINDX()        integer array of length BNNZ consisting of the
                block column indices of the block entries of A.

 BPNTRB()       integer array of length MB such that
                BPNTRB(J)-BPNTRB(1)+1 points to location in BINDX
                of the first block entry of the J-th block row of A.
 BPNTRE()       integer array of length MB such that
                BPNTRE(J)-BPNTRB(1) points to location in BINDX
                of the last block entry of the J-th block row of A.

 LB             dimension of dense blocks composing A.

 B()            rectangular array with first dimension LDB.

 LDB            leading dimension of B

 BETA           Scalar parameter

 C()            rectangular array with first dimension LDC.

 LDC            leading dimension of C

 WORK()         scratch array of length LWORK. WORK is not
                referenced in the current version.

 LWORK          length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

It is known that there exists another representation of the block sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block row in the arrays VAL and BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of block sparse row format the following calling sequence should be used

```
  CALL SBSRMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *           VAL, BINDX, IA, IA(2), LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

sbsrsm - block sparse row format triangular solve

SYNOPSIS

```
 SUBROUTINE SBSRSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, BINDX, BPNTRB, BPNTRE, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), LB,
*           LDB, LDC, LWORK
 INTEGER    BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
 REAL       ALPHA, BETA
 REAL       DV(MB*LB*LB), VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE SBSRSM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, BINDX, BPNTRB, BPNTRE, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), LB,
*           LDB, LDC, LWORK
 INTEGER*8  BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
 REAL       ALPHA, BETA
 REAL       DV(MB*LB*LB), VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where: BNNZ = BPNTRE(MB)-BPNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE BSRSM(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, N, UNITD, LB
 INTEGER, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL, DV
 REAL, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE BSRSM_64(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, N, UNITD, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL, DV
 REAL, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in block sparse row format
format and  op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row block scaling)
                3 : Scale on right (column block scaling)

DV()          Array of the length MB*LB*LB consisting of the block
              entries of block diagonal matrix D where each
              block is stored in standard column-major form.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
```

```
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))

                Note: For the routine, DESCRA(1)=3 is only supported.

                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-identity blocks on the main diagonal
                  1 : identity diagonal blocks
                  2 : diagonal blocks are dense matrices
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices
  VAL()         scalar array of length LB*LB*BNNZ consisting of the block
                entries stored column-major within each dense block.

  BINDX()       integer array of length BNNZ consisting of the
                block column indices of the block entries of A.
                The block column indices MUST be sorted
                in increasing order for each block row.

  BPNTRB()      integer array of length MB such that
                BPNTRB(J)-BPNTRB(1)+1 points to location in BINDX
                of the first block entry of the J-th block row of A.

  BPNTRE()      integer array of length MB such that
                BPNTRE(J)-BPNTRB(1) points to location in BINDX
                of the last block entry of the J-th block row of A.

  LB            dimension of dense blocks composing A.

  B()           rectangular array with first dimension LDB.

  LDB           leading dimension of B

  BETA          Scalar parameter

  C()           rectangular array with first dimension LDC.

  LDC           leading dimension of C
```

WORK()	scratch array of length LWORK. On exit, if LWORK= -1, WORK(1) returns the optimum size of LWORK.
LWORK	length of WORK array. LWORK should be at least MB*LB. For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=MB*LB*N_CPUS where N_CPUS is the maximum number of processors available to the program. If LWORK=0, the routine is to allocate workspace needed. If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If *DESCRA*(3)=0,the lower or upper triangular part of each diagonal block is used by the routine depending on *DESCRA*(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might not be referenced in the BSR representation of a sparse matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense matrices and the LU factorization with partial pivoting is used by the routine. *WORK*(1)=0 on return if the factorization for all diagonal blocks has been completed

successfully, otherwise *WORK*(1) = -i where i is the block number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3 in this case.

2. It is known that there exists another representation of the block sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block row in the arrays VAL and BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of block sparse row format the following calling sequence should be used

```
  CALL SBSRSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, BINDX, IA, IA(2), LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

```
scasum - Return the sum of the absolute values of  a  vector
x.
```

SYNOPSIS

```
REAL FUNCTION SCASUM(N, X, INCX)

COMPLEX X(*)
INTEGER N, INCX

REAL FUNCTION SCASUM_64(N, X, INCX)

COMPLEX X(*)
INTEGER*8 N, INCX
```

F95 INTERFACE

```
REAL FUNCTION ASUM([N], X, [INCX])

COMPLEX, DIMENSION(:) :: X
INTEGER :: N, INCX

REAL FUNCTION ASUM_64([N], X, [INCX])

COMPLEX, DIMENSION(:) :: X
INTEGER(8) :: N, INCX
```

C INTERFACE

```
#include <sunperf.h>

float scasum(int n, complex *x, int incx);

float scasum_64(long n, complex *x, long incx);
```

PURPOSE

scasum Return the sum of the absolute values of the elements of x where x is an n-vector. This is the sum of the absolute values of the real and complex elements and not the sum of the squares of the real and complex elements.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Contents

NAME

```
scnrm2 - Return the Euclidian norm of a vector.
```

SYNOPSIS

```
REAL FUNCTION SCNRM2(N, X, INCX)

COMPLEX X(*)
INTEGER N, INCX

REAL FUNCTION SCNRM2_64(N, X, INCX)

COMPLEX X(*)
INTEGER*8 N, INCX
```

F95 INTERFACE

```
REAL FUNCTION NRM2([N], X, [INCX])

COMPLEX, DIMENSION(:) :: X
INTEGER :: N, INCX

REAL FUNCTION NRM2_64([N], X, [INCX])

COMPLEX, DIMENSION(:) :: X
INTEGER(8) :: N, INCX
```

C INTERFACE

```
#include <sunperf.h>

float scnrm2(int n, complex *x, int incx);

float scnrm2_64(long n, complex *x, long incx);
```

PURPOSE

scnrm2 Return the Euclidian norm of a vector x where x is an n-vector.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must be positive. Unchanged on exit.

Contents

NAME

scnvcor - compute the convolution or correlation of real vectors

SYNOPSIS

```
SUBROUTINE SCNVCOR(CNVCOR, FOUR, NX, X, IFX, INCX, NY, NPRE, M, Y,
      IFY, INC1Y, INC2Y, NZ, K, Z, IFZ, INC1Z, INC2Z, WORK, LWORK)

CHARACTER * 1 CNVCOR, FOUR
INTEGER NX, IFX, INCX, NY, NPRE, M, IFY, INC1Y,  INC2Y,  NZ,
K, IFZ, INC1Z, INC2Z, LWORK
REAL X(*), Y(*), Z(*), WORK(*)

SUBROUTINE SCNVCOR_64(CNVCOR, FOUR, NX, X, IFX, INCX, NY, NPRE, M, Y,
      IFY, INC1Y, INC2Y, NZ, K, Z, IFZ, INC1Z, INC2Z, WORK, LWORK)

CHARACTER * 1 CNVCOR, FOUR
INTEGER*8 NX, IFX, INCX, NY, NPRE, M, IFY, INC1Y, INC2Y, NZ,
K, IFZ, INC1Z, INC2Z, LWORK
REAL X(*), Y(*), Z(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE CNVCOR(CNVCOR, FOUR, [NX], X, IFX, [INCX], NY, NPRE, M, Y,
       IFY, INC1Y, INC2Y, NZ, K, Z, IFZ, INC1Z, INC2Z, WORK, [LWORK])

CHARACTER(LEN=1) :: CNVCOR, FOUR
INTEGER :: NX, IFX, INCX, NY, NPRE, M,  IFY,  INC1Y,  INC2Y,
NZ, K, IFZ, INC1Z, INC2Z, LWORK
REAL, DIMENSION(:) :: X, Y, Z, WORK

SUBROUTINE CNVCOR_64(CNVCOR, FOUR, [NX], X, IFX, [INCX], NY, NPRE, M,
```

```
          Y, IFY, INC1Y, INC2Y, NZ, K, Z, IFZ, INC1Z, INC2Z, WORK, [LWORK])

    CHARACTER(LEN=1) :: CNVCOR, FOUR
    INTEGER(8) :: NX, IFX, INCX, NY, NPRE, M, IFY, INC1Y, INC2Y,
    NZ, K, IFZ, INC1Z, INC2Z, LWORK
    REAL, DIMENSION(:) :: X, Y, Z, WORK
```

C INTERFACE

```
    #include <sunperf.h>

    void scnvcor(char cnvcor, char four, int nx, float  *x,  int
              ifx,  int incx, int ny, int npre, int m, float *y,
              int ify, int inc1y, int  inc2y,  int  nz,  int  k,
              float  *z,  int  ifz,  int inc1z, int inc2z, float
              *work, int lwork);
    void scnvcor_64(char cnvcor, char four, long nx,  float  *x,
              long  ifx,  long incx, long ny, long npre, long m,
              float *y, long ify, long inc1y, long  inc2y,  long
              nz,  long  k, float *z, long ifz, long inc1z, long
              inc2z, float *work, long lwork);
```

PURPOSE

scnvcor computes the convolution or correlation of real vectors.

ARGUMENTS

CNVCOR (input)
'V' or 'v' if convolution is desired, 'R' or 'r' if correlation is desired.

FOUR (input)
'T' or 't' if the Fourier transform method is to be used, 'D' or 'd' if the computation should be done directly from the definition. The Fourier transform method is generally faster, but it may introduce noticeable errors into certain results, notably when both the filter and data vectors consist entirely of integers or vectors where elements of either the filter vector or a given data vector differ significantly in magnitude from the 1-norm of the vector.

NX (input)
Length of the filter vector. NX >= 0. SCNVCOR will return immediately if NX = 0.

X (input)
Filter vector.

IFX (input)
Index of the first element of X. NX >= IFX >= 1.

INCX (input)
Stride between elements of the filter vector in X. INCX > 0.

NY (input)
Length of the input vectors. NY >= 0. SCNVCOR will return immediately if NY = 0.

NPRE (input)
The number of implicit zeros prepended to the Y vectors. NPRE >= 0.

M (input)
Number of input vectors. M >= 0. SCNVCOR will return immediately if M = 0.

Y (input)
Input vectors.

IFY (input)
Index of the first element of Y. NY >= IFY >= 1.

INC1Y (input)
Stride between elements of the input vectors in Y. INC1Y > 0.

INC2Y (input)
Stride between the input vectors in Y. INC2Y > 0.

NZ (input)
Length of the output vectors. NZ >= 0. SCNVCOR will return immediately if NZ = 0. See the Notes section below for information about how this argument interacts with NX and NY to control circular versus end-off shifting.

K (input)
Number of Z vectors. K >= 0. If K = 0 then SCNVCOR will return immediately. If K < M then only the first K input vectors will be processed. If K > M then M input vectors will be processed.

Z (output)
Result vectors.

IFZ (input)
Index of the first element of Z. NZ >= IFZ >= 1.

INC1Z (input)
Stride between elements of the output vectors in Z. INC1Z > 0.

INC2Z (input)
Stride between the output vectors in Z. INC2Z > 0.

WORK (input/output)
Scratch space. Before the first call to SCNVCOR with particular values of the integer arguments the first element of WORK must be set to zero. If WORK is written between calls to SCNVCOR or if SCNVCOR is called with different values of the integer arguments then the first element of WORK must again be set to zero before each call. If WORK has not been written and the same values of the integer arguments are used then the first element of WORK to zero. This can avoid certain initializations that store their results into WORK, and avoiding the initialization can make SCNVCOR run faster.

LWORK (input)
Length of WORK. LWORK >= 4*max(NX,NPRE+NY,NZ)+15.

NOTES

If any vector overlaps a writable vector, either because of argument aliasing or ill-chosen values of the various INC arguments, the results are undefined and may vary from one run to the next.

The most common form of the computation, and the case that executes fastest, is applying a filter vector X to a series of vectors stored in the columns of Y with the result placed into the columns of Z. In that case, INCX = 1, INC1Y = 1, INC2Y >= NY, INC1Z = 1, INC2Z >= NZ. Another common form is applying a filter vector X to a series of vectors stored in the rows of Y and store the result in the row of Z, in which case INCX = 1, INC1Y >= NY, INC2Y = 1, INC1Z >= NZ, and INC2Z = 1.

Contents

NAME

```
scnvcor2 - compute the convolution or  correlation  of  real
matrices
```

SYNOPSIS

```
SUBROUTINE SCNVCOR2(CNVCOR, METHOD, TRANSX, SCRATCHX, TRANSY,
      SCRATCHY, MX, NX, X, LDX, MY, NY, MPRE, NPRE, Y, LDY, MZ, NZ, Z,
      LDZ, WORKIN, LWORK)

CHARACTER * 1  CNVCOR,  METHOD,  TRANSX,  SCRATCHX,  TRANSY,
SCRATCHY
COMPLEX WORKIN(*)
INTEGER MX, NX, LDX, MY, NY, MPRE, NPRE, LDY, MZ,  NZ,  LDZ,
LWORK
REAL X(LDX,*), Y(LDY,*), Z(LDZ,*)

SUBROUTINE SCNVCOR2_64(CNVCOR, METHOD, TRANSX, SCRATCHX, TRANSY,
      SCRATCHY, MX, NX, X, LDX, MY, NY, MPRE, NPRE, Y, LDY, MZ, NZ, Z,
      LDZ, WORKIN, LWORK)

CHARACTER * 1  CNVCOR,  METHOD,  TRANSX,  SCRATCHX,  TRANSY,
SCRATCHY
COMPLEX WORKIN(*)
INTEGER*8 MX, NX, LDX, MY, NY, MPRE, NPRE, LDY, MZ, NZ, LDZ,
LWORK
REAL X(LDX,*), Y(LDY,*), Z(LDZ,*)
```

F95 INTERFACE

```
SUBROUTINE CNVCOR2(CNVCOR, METHOD, TRANSX, SCRATCHX, TRANSY,
       SCRATCHY, [MX], [NX], X, [LDX], [MY], [NY], MPRE, NPRE, Y, [LDY],
       [MZ], [NZ], Z, [LDZ], WORKIN, [LWORK])
```

```
CHARACTER(LEN=1)  ::  CNVCOR,  METHOD,   TRANSX,   SCRATCHX,
TRANSY, SCRATCHY
COMPLEX, DIMENSION(:) :: WORKIN
INTEGER :: MX, NX, LDX, MY, NY, MPRE,  NPRE,  LDY,  MZ,  NZ,
LDZ, LWORK
REAL, DIMENSION(:,:) :: X, Y, Z

SUBROUTINE CNVCOR2_64(CNVCOR, METHOD, TRANSX, SCRATCHX, TRANSY,
       SCRATCHY, [MX], [NX], X, [LDX], [MY], [NY], MPRE, NPRE, Y, [LDY],
       [MZ], [NZ], Z, [LDZ], WORKIN, [LWORK])

CHARACTER(LEN=1)  ::  CNVCOR,  METHOD,   TRANSX,   SCRATCHX,
TRANSY, SCRATCHY
COMPLEX, DIMENSION(:) :: WORKIN
INTEGER(8) :: MX, NX, LDX, MY, NY, MPRE, NPRE, LDY, MZ,  NZ,
LDZ, LWORK
REAL, DIMENSION(:,:) :: X, Y, Z
```

C INTERFACE

```
#include <sunperf.h>

void scnvcor2(char cnvcor, char method,  char  transx,  char
          scratchx,  char transy, char scratchy, int mx, int
          nx, float *x, int ldx, int my, int ny,  int  mpre,
          int npre, float *y, int ldy, int mz, int nz, float
          *z, int ldz, complex *workin, int lwork);

void scnvcor2_64(char cnvcor, char method, char transx, char
          scratchx,  char  transy,  char  scratchy, long mx,
          long nx, float *x, long ldx,  long  my,  long  ny,
          long mpre, long npre, float *y, long ldy, long mz,
          long nz, float *z, long ldz, complex *workin, long
          lwork);
```

PURPOSE

scnvcor2 computes the convolution or correlation of real matrices.

ARGUMENTS

CNVCOR (input)
'V' or 'v' to compute convolution, 'R' or 'r' to compute correlation.

METHOD (input)
'T' or 't' if the Fourier transform method is to be used, 'D' or 'd' to compute directly from the

definition.

TRANSX (input)
'N' or 'n' if X is the filter matrix, 'T' or 't' if transpose(X) is the filter matrix.

SCRATCHX (input)
'N' or 'n' if X must be preserved, 'S' or 's' if X can be used as scratch space. The contents of X are undefined after returning from a call in which X is allowed to be used for scratch.

TRANSY (input)
'N' or 'n' if Y is the input matrix, 'T' or 't' if transpose(Y) is the input matrix.

SCRATCHY (input)
'N' or 'n' if Y must be preserved, 'S' or 's' if Y can be used as scratch space. The contents of Y are undefined after returning from a call in which Y is allowed to be used for scratch.

MX (input)
Number of rows in the filter matrix. MX >= 0.

NX (input)
Number of columns in the filter matrix. NX >= 0.

X (input) dimension(LDX,NX)
On entry, the filter matrix. Unchanged on exit if SCRATCHX is 'N' or 'n', undefined on exit if SCRATCHX is 'S' or 's'.

LDX (input)
Leading dimension of the array that contains the filter matrix.

MY (input)
Number of rows in the input matrix. MY >= 0.

NY (input)
Number of columns in the input matrix. NY >= 0.

MPRE (input)
Number of implicit zeros to prepend to each row of the input matrix. MPRE >= 0.

NPRE (input)
Number of implicit zeros to prepend to each column of the input matrix. NPRE >= 0.

Y (input) dimension(LDY,*)
Input matrix. Unchanged on exit if SCRATCHY is 'N' or 'n', undefined on exit if SCRATCHY is 'S' or 's'.

LDY (input)
Leading dimension of the array that contains the input matrix.

MZ (input)
Number of rows in the output matrix. MZ >= 0. SCNVCOR2 will return immediately if MZ = 0.

NZ (input)
Number of columns in the output matrix. NZ >= 0. SCNVCOR2 will return immediately if NZ = 0.

Z (output)
dimension(LDZ,*)
Result matrix.

LDZ (input)
Leading dimension of the array that contains the result matrix. LDZ >= MAX(1,MZ).

WORKIN (input/output)
(input/scratch) dimension(LWORK)
On entry for the first call to SCNVCOR2, REAL(WORKIN(1)) must contain 0.0. After the first call, REAL(WORKIN(1)) must be set to 0.0 iff WORKIN has been altered since the last call to this subroutine or if the sizes of the arrays have changed.

LWORK (input)
Length of the work vector. The upper bound of the workspace length requirement is 2 * (MYC + NYC) + 15, where MYC = MAX(MAX(MX,NX), MAX(MY,NY)+NPRE) and NYC = MAX(MAX(MX,NX), MAX(MY,NY)+MPRE). If LWORK indicates a workspace that is too small, the routine will allocate its own workspace. If the FFT is not used, the value of LWORK is unimportant.

Contents

NAME

```
scoomm - coordinate matrix-matrix multiply
```

SYNOPSIS

```
 SUBROUTINE SCOOMM( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, JNDX, NNZ,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, K, DESCRA(5), NNZ
*           LDB, LDC, LWORK
 INTEGER    INDX(NNZ), JNDX(NNZ)
 REAL       ALPHA, BETA
 REAL       VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE SCOOMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, JNDX, NNZ,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, K, DESCRA(5), NNZ
*           LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), JNDX(NNZ)
 REAL       ALPHA, BETA
 REAL       VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE COOMM( TRANSA, M, [N], K, ALPHA, DESCRA,
*           VAL, INDX, JNDX, NNZ, B, [LDB], BETA, C, [LDC],
*           [WORK], [LWORK] )
 INTEGER TRANSA, M, K,  NNZ
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, JNDX
 REAL   ALPHA, BETA
 REAL, DIMENSION(:) :: VAL
 REAL, DIMENSION(:, :) ::  B, C
```

```
       SUBROUTINE COOMM_64( TRANSA, M, [N], K, ALPHA, DESCRA,
      *           VAL, INDX, JNDX, NNZ, B, [LDB], BETA, C, [LDC],
      *           [WORK], [LWORK] )
       INTEGER*8 TRANSA, M, K,  NNZ
       INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, JNDX
       REAL   ALPHA, BETA
       REAL, DIMENSION(:) :: VAL
       REAL, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
              C <- alpha op(A) B + beta C

      where ALPHA and BETA are scalar, C and B are dense matrices,
      A is a matrix represented in coordinate format and
      op( A )  is one  of
      op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                          ( ' indicates matrix transpose)
```

ARGUMENTS

```
      TRANSA        Indicates how to operate with the sparse matrix
                      0 : operate with matrix
                      1 : operate with transpose matrix
                      2 : operate with the conjugate transpose of matrix.
                          2 is equivalent to 1 if matrix is real.

      M             Number of rows in matrix A

      N             Number of columns in matrix C

      K             Number of columns in matrix A

      ALPHA         Scalar parameter

      DESCRA()      Descriptor argument.  Five element integer array
                    DESCRA(1) matrix structure
                      0 : general
                      1 : symmetric (A=A')
                      2 : Hermitian (A= CONJG(A'))
                      3 : Triangular
                      4 : Skew(Anti)-Symmetric (A=-A')
                      5 : Diagonal
                      6 : Skew-Hermitian (A= -CONJG(A'))
                    DESCRA(2) upper/lower triangular indicator
                      1 : lower
                      2 : upper
```

```
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

 VAL()          scalar array of length NNZ consisting of the
                non-zero entries of A, in any order.

 INDX()         integer array of length NNZ consisting of the
                corresponding row indices of the entries of A.

 JNDX()         integer array of length NNZ consisting of the
                corresponding column indices of the entries of A.

 NNZ            number of non-zero elements in A.
 B()            rectangular array with first dimension LDB.

 LDB            leading dimension of B

 BETA           Scalar parameter

 C()            rectangular array with first dimension LDC.

 LDC            leading dimension of C

 WORK()         scratch array of length LWORK. WORK is not
                referenced in the current version.

 LWORK          length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS)
Standard", University of Tennessee, Knoxville, Tennessee,
1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

```
scopy - Copy x to y
```

SYNOPSIS

```
SUBROUTINE SCOPY(N, X, INCX, Y, INCY)

INTEGER N, INCX, INCY
REAL X(*), Y(*)

SUBROUTINE SCOPY_64(N, X, INCX, Y, INCY)

INTEGER*8 N, INCX, INCY
REAL X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE COPY([N], X, [INCX], Y, [INCY])

INTEGER :: N, INCX, INCY
REAL, DIMENSION(:) :: X, Y

SUBROUTINE COPY_64([N], X, [INCX], Y, [INCY])

INTEGER(8) :: N, INCX, INCY
REAL, DIMENSION(:) :: X, Y
```

C INTERFACE

```
#include <sunperf.h>

void scopy(int n, float *x, int incx, float *y, int incy);

void scopy_64(long n, float *x, long incx,  float  *y,  long
          incy);
```

PURPOSE

scopy Copy x to y where x and y are n-vectors.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (output)
(1 + (m - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the vector x.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
scscmm - compressed sparse column format matrix-matrix
multiply
```

SYNOPSIS

```
 SUBROUTINE SCSCMM( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, K, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTRB(K), PNTRE(K)
 REAL       ALPHA, BETA
 REAL       VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE SCSCMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, K, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTRB(K), PNTRE(K)
 REAL       ALPHA, BETA
 REAL       VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTRE(K)-PNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE CSCMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER TRANSA, M, K
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL
```

```
 REAL, DIMENSION(:, :) ::  B, C

 SUBROUTINE CSCMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER*8 TRANSA, M, K
 INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL
 REAL, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in compressed sparse column format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                  ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
```

```
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices

 VAL()         scalar array of length NNZ consisting of nonzero
               entries of A.

 INDX()        integer array of length NNZ consisting of the row
               indices of nonzero entries of A.

 PNTRB()       integer array of length K such that PNTRB(J)-PNTRB(1)+1
               points to location in VAL of the first nonzero element
               in column J.
 PNTRE()       integer array of length K such that PNTRE(J)-PNTRB(1)
               points to location in VAL of the last nonzero element
               in column J.

 B()           rectangular array with first dimension LDB.

 LDB           leading dimension of B

 BETA          Scalar parameter

 C()           rectangular array with first dimension LDC.

 LDC           leading dimension of C

 WORK()        scratch array of length LWORK. WORK is not
               referenced in the current version.

 LWORK         length of WORK array. LWORK is not referenced
               in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS)
Standard", University of Tennessee, Knoxville, Tennessee,

1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

It is known that there exists another representation of the compressed sparse column format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each column in the arrays VAL and INDX is used instead of two arrays PNTRB and PNTRE. To use the routine with this kind of sparse column format the following calling sequence should be used

```
  SUBROUTINE SCSCMM( TRANSA, M, N, K, ALPHA, DESCRA,
 *           VAL, INDX, IA, IA(2), B, LDB, BETA,
 *           C, LDC, WORK, LWORK )
```

Contents

NAME

```
scscsm - compressed sparse column format triangular solve
```

SYNOPSIS

```
 SUBROUTINE SCSCSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTRB(M), PNTRE(M)
 REAL       ALPHA, BETA
 REAL       DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE SCSCSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTRB(M), PNTRE(M)
 REAL       ALPHA, BETA
 REAL       DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTRE(M)-PNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE CSCSM( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER TRANSA, M, UNITD
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL, DV
 REAL, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE CSCSM_64(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER*8 TRANSA, M, UNITD
 INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL, DV
 REAL, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a diagonal scaling matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in compressed sparse column
format and op( A )  is one  of
op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic column scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                  0 : general
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))

                Note: For the routine, DESCRA(1)=3 is only supported.

                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices
VAL()           scalar array of length NNZ consisting of nonzero entries
                of A.

INDX()          integer array of length NNZ consisting of the row indices
                of nonzero entries of A. (Row indices MUST be sorted in
                increasing order for each column).

PNTRB()         integer array of length M such that PNTRB(J)-PNTRB(1)+1
                points to location in VAL of the first nonzero element
                in column J.

PNTRE()         integer array of length M such that PNTRE(J)-PNTRB(1)
                points to location in VAL of the last nonzero element
                in column J.

B()             rectangular array with first dimension LDB.

LDB             leading dimension of B

BETA            Scalar parameter

C()             rectangular array with first dimension LDC.

LDC             leading dimension of C

WORK()          scratch array of length LWORK.

                On exit,  if LWORK = -1, WORK(1) returns the optimum  LWORK.
```

LWORK length of WORK array. LWORK should be at least M.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the columns of A such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the columns have been scaled. UNITD=3 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the column number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the CSC representation

of a sparse matrix. They are not used anyway in these cases. But if UNITD=4, the unit diagonal elements MUST be referenced in the CSC representation.

4. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

5. It is known that there exists another representation of the compressed sparse column format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each column in the arrays VAL and INDX is used instead of two arrays PNTRB and PNTRE. To use the routine with this kind of sparse column format the following calling sequence should be used

```
  SUBROUTINE SCSCSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, INDX, IA, IA(2), B, LDB, BETA,
 *           C, LDC, WORK, LWORK )
```

Contents

NAME

```
scsrmm - compressed sparse row format matrix-matrix multiply
```

SYNOPSIS

```
 SUBROUTINE SCSRMM( TRANSA, M, N, K, ALPHA, DESCRA,
*            VAL, INDX, PNTRB, PNTRE,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, K, DESCRA(5),
*            LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTRB(M), PNTRE(M)
 REAL       ALPHA, BETA
 REAL       VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE SCSRMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*            VAL, INDX, PNTRB, PNTRE,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, K, DESCRA(5),
*            LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTRB(M), PNTRE(M)
 REAL       ALPHA, BETA
 REAL       VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTRE(M)-PNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE CSRMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER TRANSA, M, K
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL
 REAL, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE CSRMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER*8 TRANSA, M, K
 INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL
 REAL, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in compressed sparse row format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
```

```
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

 VAL()          scalar array of length NNZ consisting of nonzero
                entries of A.

 INDX()         integer array of length NNZ consisting of the
                column indices of nonzero entries of A.

 PNTRB()        integer array of length M such that PNTRB(J)-PNTRB(1)+1
                points to location in VAL of the first nonzero element
                in row J.
 PNTRE()        integer array of length M such that PNTRE(J)-PNTRB(1)
                points to location in VAL of the last nonzero element
                in row J.

 B()            rectangular array with first dimension LDB.

 LDB            leading dimension of B

 BETA           Scalar parameter

 C()            rectangular array with first dimension LDC.

 LDC            leading dimension of C

 WORK()         scratch array of length LWORK. WORK is not
                referenced in the current version.

 LWORK          length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

It is known that there exists another representation of the compressed sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each row in the arrays VAL and INDX is used instead of two arrays PNTRB and PNTRE. To use the routine with this kind of compressed sparse row format the following calling sequence should be used

```
  SUBROUTINE SCSRMM( TRANSA, M, N, K, ALPHA, DESCRA,
 *           VAL, INDX, IA, IA(2), B, LDB, BETA,
 *           C, LDC, WORK, LWORK )
```

Contents

NAME

scsrsm - compressed sparse row format triangular solve

SYNOPSIS

```
 SUBROUTINE SCSRSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTRB(M), PNTRE(M)
 REAL       ALPHA, BETA
 REAL       DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE SCSRSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTRB(M), PNTRE(M)
 REAL       ALPHA, BETA
 REAL       DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTRE(M)-PNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE CSRSM( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER TRANSA, M, UNITD
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL, DV
 REAL, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE CSRSM_64(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER*8 TRANSA, M, UNITD
 INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL, DV
 REAL, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C
```

where ALPHA and BETA are scalar, C and B are m by n dense matrices, D is a diagonal scaling matrix, A is a unit, or non-unit, upper or lower triangular matrix represented in compressed sparse row format and op(A) is one of

op(A) = inv(A) or op(A) = inv(A') or op(A) =inv(conjg(A'))
(inv denotes matrix inverse, ' indicates matrix transpose)

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of
              the scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                 0 : general
                 1 : symmetric (A=A')
                 2 : Hermitian (A= CONJG(A'))
                 3 : Triangular
                 4 : Skew(Anti)-Symmetric (A=-A')
                 5 : Diagonal
                 6 : Skew-Hermitian (A= -CONJG(A'))

               Note: For the routine, only DESCRA(1)=3 is supported.

               DESCRA(2) upper/lower triangular indicator
                 1 : lower
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices
VAL()          scalar array of length NNZ consisting of nonzero entries
               of A.

INDX()         integer array of length NNZ consisting of the column
               indices of nonzero entries of A (column indices MUST be
               sorted in increasing order for each row)

PNTRB()        integer array of length M such that PNTRB(J)-PNTRB(1)+1
               points to location in VAL of the first nonzero element
               in row J.

PNTRE()        integer array of length M such that PNTRE(J)-PNTRB(1)
               points to location in VAL of the last nonzero element
               in row J.

B()            rectangular array with first dimension LDB.

LDB            leading dimension of B

BETA           Scalar parameter

C()            rectangular array with first dimension LDC.

LDC            leading dimension of C

WORK()         scratch array of length LWORK.

               On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.
```

LWORK length of WORK array. LWORK should be at least M.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the rows of A such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows have been scaled. UNITD=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the CSR representation of a sparse matrix. They are not used anyway in these cases.

But if UNITD=4, the unit diagonal elements MUST be referenced in the CSR representation.

4. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

5. It is known that there exists another representation of the compressed sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each row in the arrays VAL and INDX is used instead of two arrays PNTRB and PNTRE. To use the routine with this kind of compressed sparse row format the following calling sequence should be used

```
  SUBROUTINE SCSRSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, INDX, IA, IA(2), B, LDB, BETA, C,
 *            LDC, WORK, LWORK )
```

Contents

NAME

sdiamm - diagonal format matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE SDIAMM( TRANSA, M, N, K, ALPHA, DESCRA,
*            VAL, LDA, IDIAG, NDIAG,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, K, DESCRA(5), LDA, NDIAG,
*            LDB, LDC, LWORK
 INTEGER    IDIAG(NDIAG)
 REAL       ALPHA, BETA
 REAL       VAL(LDA,NDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE SDIAMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*            VAL, LDA, IDIAG, NDIAG,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, K, DESCRA(5), LDA, NDIAG,
*            LDB, LDC, LWORK
 INTEGER*8  IDIAG(NDIAG)
 REAL       ALPHA, BETA
```

F95 INTERFACE

```
  SUBROUTINE DIAMM(TRANSA, M, [N], K, ALPHA, DESCRA, VAL, [LDA],
*    IDIAG, NDIAG, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, K,  NDIAG
 INTEGER, DIMENSION(:) ::    DESCRA, IDIAG
 REAL    ALPHA, BETA
 REAL, DIMENSION(:, :) ::  VAL, B, C

  SUBROUTINE DIAMM_64(TRANSA, M, [N], K, ALPHA, DESCRA, VAL, [LDA],
*    IDIAG, NDIAG, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, K,  NDIAG
```

```
INTEGER*8, DIMENSION(:) ::    DESCRA, IDIAG
REAL    ALPHA, BETA
REAL, DIMENSION(:, :) ::  VAL, B, C
```

DESCRIPTION

```
         C <- alpha op(A) B + beta C

 where ALPHA and BETA are scalar, C and B are dense matrices,
 A is a matrix represented in diagonal format and op( A ) is one  of

 op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)

 TRANSA        Indicates how to operate with the sparse matrix
                 0 : operate with matrix
                 1 : operate with transpose matrix
                 2 : operate with the conjugate transpose of matrix.
                     2 is equivalent to 1 if matrix is real.

 M             Number of rows in matrix A

 N             Number of columns in matrix C

 K             Number of columns in matrix A

 ALPHA         Scalar parameter

 DESCRA()      Descriptor argument.  Five element integer array
                 0 : general
                 1 : symmetric (A=A')
                 2 : Hermitian (A= CONJG(A'))
                 3 : Triangular
                 4 : Skew(Anti)-Symmetric (A=-A')
                 5 : Diagonal
                 6 : Skew-Hermitian (A= -CONJG(A'))
               DESCRA(2) upper/lower triangular indicator
                 1 : lower
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices
```

```
VAL()          two-dimensional LDA-by-NDIAG array such that VAL(:,I)
               consists of non-zero elements on diagonal IDIAG(I)
               of A.  Diagonals in the lower triangular part of A
               are padded from the top, and those in the upper
               triangular part are padded from the bottom.

LDA            leading dimension of VAL, must be .GE. MIN(M,K)

IDIAG()        integer array of length NDIAG consisting of the
               corresponding diagonal offsets of the non-zero
               diagonals of A in VAL.  Lower triangular diagonals
               have negative offsets, the main diagonal has offset
               0, and upper triangular diagonals have positive offset.

NDIAG          number of non-zero diagonals in A.

B()            rectangular array with first dimension LDB.
LDB            leading dimension of B

BETA           Scalar parameter

C()            rectangular array with first dimension LDC.

LDC            leading dimension of C

WORK()         scratch array of length LWORK. WORK is not
               referenced in the current version.

LWORK          length of WORK array. LWORK is not referenced
               in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS)
Standard", University of Tennessee, Knoxville, Tennessee,
1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

sdiasm - diagonal format triangular solve

SYNOPSIS

```
 SUBROUTINE SDIASM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, LDA, IDIAG, NDIAG,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5), LDA, NDIAG,
*            LDB, LDC, LWORK
 INTEGER    IDIAG(NDIAG)
 REAL       ALPHA, BETA
 REAL       DV(M), VAL(LDA,NDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE SDIASM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, LDA, IDIAG, NDIAG,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5), LDA, NDIAG,
*            LDB, LDC, LWORK
 INTEGER*8  IDIAG(NDIAG)
 REAL       ALPHA, BETA
 REAL       DV(M), VAL(LDA,NDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
  SUBROUTINE DIASM(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
*   [LDA], IDIAG, NDIAG, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, NDIAG
 INTEGER, DIMENSION(:) ::    DESCRA, IDIAG
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) ::  DV
 REAL, DIMENSION(:, :) ::  VAL, B, C

  SUBROUTINE DIASM_64(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
```

```
*   [LDA], IDIAG, NDIAG, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, NDIAG
 INTEGER*8, DIMENSION(:) ::    DESCRA, IDIAG
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) ::  DV
 REAL, DIMENSION(:, :) ::  VAL, B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C
```

where ALPHA and BETA are scalar, C and B are m by n dense matrices, D is a diagonal scaling matrix, A is a unit, or non-unit, upper or lower triangular matrix represented in diagonal format and op(A) is one of

```
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
```

```
                 1 : symmetric (A=A')
                 2 : Hermitian (A= CONJG(A'))
                 3 : Triangular
                 4 : Skew(Anti)-Symmetric (A=-A')
                 5 : Diagonal
                 6 : Skew-Hermitian (A= -CONJG(A'))
               Note: For the routine, only DESCRA(1)=3 is supported.

               DESCRA(2) upper/lower triangular indicator
                 1 : lower
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices
VAL()          two-dimensional LDA-by-NDIAG array such that VAL(:,I)
               consists of non-zero elements on diagonal IDIAG(I)
               of A.  Diagonals in the lower triangular part of A
               are padded from the top, and those in the upper
               triangular part are padded from the bottom.

LDA            leading dimension of VAL, must be .GE. MIN(M,K)

IDIAG()        integer array of length NDIAG consisting of the
               corresponding diagonal offsets of the non-zero
               diagonals of A in VAL.  Lower triangular diagonals
               have negative offsets, the main diagonal has offset
               0, and upper triangular diagonals have positive offset.
               Elements of IDIAG of MUST be sorted in  increasing order.

NDIAG          number of non-zero diagonals in A.

B()            rectangular array with first dimension LDB.

LDB            leading dimension of B

BETA           Scalar parameter

C()            rectangular array with first dimension LDC.

LDC            leading dimension of C

WORK()         scratch array of length LWORK.
               On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.
```

LWORK length of WORK array. LWORK should be at least M.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/
"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the rows of A such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows have been scaled. UNITD=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the DIA representation of a sparse matrix. They are not used anyway in these cases. But if UNITD=4, the unit diagonal elements MUST be referenced in the DIA representation.

4. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

sdisna - compute the reciprocal condition numbers for the eigenvectors of a real symmetric or complex Hermitian matrix or for the left or right singular vectors of a general m-by-n matrix

SYNOPSIS

```
SUBROUTINE SDISNA(JOB, M, N, D, SEP, INFO)

CHARACTER * 1 JOB
INTEGER M, N, INFO
REAL D(*), SEP(*)

SUBROUTINE SDISNA_64(JOB, M, N, D, SEP, INFO)

CHARACTER * 1 JOB
INTEGER*8 M, N, INFO
REAL D(*), SEP(*)
```

F95 INTERFACE

```
SUBROUTINE DISNA(JOB, M, N, D, SEP, [INFO])

CHARACTER(LEN=1) :: JOB
INTEGER :: M, N, INFO
REAL, DIMENSION(:) :: D, SEP

SUBROUTINE DISNA_64(JOB, M, N, D, SEP, [INFO])

CHARACTER(LEN=1) :: JOB
INTEGER(8) :: M, N, INFO
REAL, DIMENSION(:) :: D, SEP
```

C INTERFACE

```
#include <sunperf.h>

void sdisna(char job, int m, int n, float  *d,  float  *sep,
          int *info);

void sdisna_64(char job, long m, long  n,  float  *d,  float
          *sep, long *info);
```

PURPOSE

sdisna computes the reciprocal condition numbers for the eigenvectors of a real symmetric or complex Hermitian matrix or for the left or right singular vectors of a general m-by-n matrix. The reciprocal condition number is the 'gap' between the corresponding eigenvalue or singular value and the nearest other one.

The bound on the error, measured by angle in radians, in the I-th computed vector is given by

```
SLAMCH( 'E' ) * ( ANORM / SEP( I ) )
```

where ANORM = 2-norm(A) = max(abs(D(j))). SEP(I) is not allowed to be smaller than SLAMCH('E')*ANORM in order to limit the size of the error bound.

SDISNA may also be used to compute error bounds for eigenvectors of the generalized symmetric definite eigenproblem.

ARGUMENTS

JOB (input)
Specifies for which problem the reciprocal condition numbers should be computed:
= 'E': the eigenvectors of a symmetric/Hermitian matrix;
= 'L': the left singular vectors of a general matrix;
= 'R': the right singular vectors of a general matrix.

M (input) The number of rows of the matrix. M >= 0.

N (input) If JOB = 'L' or 'R', the number of columns of the matrix, in which case N >= 0. Ignored if JOB = 'E'.

D (input) dimension (min(M,N)) if JOB = 'L' or 'R' The eigenvalues (if JOB = 'E') or singular values (if JOB = 'L' or 'R') of the matrix, in either increasing or decreasing order. If singular values, they must be non-negative.

SEP (output)
dimension (min(M,N)) if JOB = 'L' or 'R' The reciprocal condition numbers of the vectors.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

```
sdot - compute the dot product of two vectors x and y.
```

SYNOPSIS

```
REAL FUNCTION SDOT(N, X, INCX, Y, INCY)

INTEGER N, INCX, INCY
REAL X(*), Y(*)

REAL FUNCTION SDOT_64(N, X, INCX, Y, INCY)

INTEGER*8 N, INCX, INCY
REAL X(*), Y(*)
```

F95 INTERFACE

```
REAL FUNCTION DOT([N], X, [INCX], Y, [INCY])

INTEGER :: N, INCX, INCY
REAL, DIMENSION(:) :: X, Y

REAL FUNCTION DOT_64([N], X, [INCX], Y, [INCY])

INTEGER(8) :: N, INCX, INCY
REAL, DIMENSION(:) :: X, Y
```

C INTERFACE

```
#include <sunperf.h>

float sdot(int n, float *x, int incx, float *y, int incy);

float sdot_64(long n, float *x, long incx,  float  *y,  long
          incy);
```

PURPOSE

sdot compute the dot product of x and y where x and y are n-vectors.

ARGUMENTS

N (input)
: On entry, N specifies the number of elements in the vector. If N is not positive then the function returns the value 0.0. Unchanged on exit.

X (input)
: (1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
: On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input)
: (1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. Unchanged on exit.

INCY (input)
: On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
sdoti - Compute the indexed dot product.
```

SYNOPSIS

```
 REAL FUNCTION SDOTI(NZ, X, INDX, Y)

 REAL X(*), Y(*)
 INTEGER NZ
 INTEGER INDX(*)

 REAL FUNCTION SDOTI_64(NZ, X, INDX, Y)

 REAL X(*), Y(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 REAL FUNCTION DOTI([NZ], X, INDX, Y)

 REAL, DIMENSION(:) :: X, Y
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 REAL FUNCTION DOTI_64([NZ], X, INDX, Y)

 REAL, DIMENSION(:) :: X, Y
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
SDOTI Compute the indexed dot product of a real sparse
vector x stored in compressed form with a real vector y in
```

full storage form.

```
dot = 0
do i = 1, n
  dot = dot + x(i) * y(indx(i))
enddo
```

ARGUMENTS

NZ (input)
Number of elements in the compressed form. Unchanged on exit.

X (input)
Vector in compressed form. Unchanged on exit.

INDX (input)
Vector containing the indices of the compressed form. It is assumed that the elements in INDX are distinct and greater than zero. Unchanged on exit.

Y (input)
Vector in full storage form. Only the elements corresponding to the indices in INDX will be accessed.

Contents

NAME

```
sdsdot - compute a constant plus the  double  precision  dot
product of two single precision vectors x and y
```

SYNOPSIS

```
REAL FUNCTION SDSDOT(N, SB, SX, INCX, SY, INCY)

INTEGER N, INCX, INCY
REAL SB
REAL SX(*), SY(*)

REAL FUNCTION SDSDOT_64(N, SB, SX, INCX, SY, INCY)

INTEGER*8 N, INCX, INCY
REAL SB
REAL SX(*), SY(*)
```

F95 INTERFACE

```
REAL FUNCTION SDSDOT(N, SB, SX, INCX, SY, INCY)

INTEGER :: N, INCX, INCY
REAL :: SB
REAL, DIMENSION(:) :: SX, SY

REAL FUNCTION SDSDOT_64(N, SB, SX, INCX, SY, INCY)

INTEGER(8) :: N, INCX, INCY
REAL :: SB
REAL, DIMENSION(:) :: SX, SY
```

C INTERFACE

```
#include <sunperf.h>
```

```
float sdsdot(int n, float sb, float  *sx,  int  incx,  float
          *sy, int incy);

float sdsdot_64(long n, float  sb,  float  *sx,  long  incx,
          float *sy, long incy);
```

PURPOSE

sdsdot Computes a constant plus the double precision dot product of x and y where x and y are single precision n-vectors.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. If N is not positive then the function returns the value 0.0. Unchanged on exit.

SB (input)
On entry, the constant that is added to the dot product before the result is returned. Unchanged on exit.

SX (input)
(1 + (n - 1)*abs(INCX)). On entry, the incremented array SX must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of SX. INCX must not be zero. Unchanged on exit.

SY (input)
(1 + (n - 1)*abs(INCY)). On entry, the incremented array SY must contain the vector y. Unchanged on exit.

INCY (input)
On entry, INCY specifies the increment for the elements of SY. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
second - return the user time for a process in seconds
```

SYNOPSIS

```
REAL FUNCTION SECOND()

REAL FUNCTION SECOND_64()
```

F95 INTERFACE

```
REAL FUNCTION SECOND()

REAL FUNCTION SECOND_64()
```

C INTERFACE

```
#include <sunperf.h>

float second();

float second_64();
```

PURPOSE

```
second returns the user time for a process in seconds.  This
version gets the time from the system function ETIME.
```

Contents

NAME

```
sellmm - Ellpack format matrix-matrix multiply
```

SYNOPSIS

```
  SUBROUTINE SELLMM( TRANSA, M, N, K, ALPHA, DESCRA,
 *           VAL, INDX, LDA, MAXNZ,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
  INTEGER    TRANSA, M, N, K, DESCRA(5), LDA, MAXNZ,
 *           LDB, LDC, LWORK
  INTEGER    INDX(LDA,MAXNZ)
  REAL       ALPHA, BETA
  REAL       VAL(LDA,MAXNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

  SUBROUTINE SELLMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
 *           VAL, INDX, LDA, MAXNZ,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
  INTEGER*8  TRANSA, M, N, K, DESCRA(5), LDA, MAXNZ,
 *           LDB, LDC, LWORK
  INTEGER*8  INDX(LDA,MAXNZ)
  REAL       ALPHA, BETA
  REAL       VAL(LDA,MAXNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
  SUBROUTINE ELLMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
 *        [LDA], MAXNZ, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
  INTEGER    TRANSA, M, K, MAXNZ
  INTEGER, DIMENSION(:) ::  DESCRA
  INTEGER, DIMENSION(:, :) ::    INDX
  REAL    ALPHA, BETA
  REAL, DIMENSION(:, :) ::  VAL, B, C

  SUBROUTINE ELLMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
```

```
*        [LDA], MAXNZ, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, K, MAXNZ
 INTEGER*8, DIMENSION(:) ::  DESCRA
 INTEGER*8, DIMENSION(:, :) ::    INDX
 REAL    ALPHA, BETA
 REAL, DIMENSION(:, :) ::  VAL, B, C
```

DESCRIPTION

```
where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in  Ellpack format format and
op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
                0 : non-unit
                1 : unit
```

```
                  DESCRA(4) Array base  (NOT IMPLEMENTED)
                    0 : C/C++ compatible
                    1 : Fortran compatible
                  DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                    0 : unknown
                    1 : no repeated indices

    VAL()         two-dimensional LDA-by-MAXNZ array such that VAL(I,:)
                  consists of non-zero elements in row I of A, padded by
                  zero values if the row contains less than MAXNZ.

    INDX()        two-dimensional integer LDA-by-MAXNZ array such
                  INDX(I,:) consists of the column indices of the
                  nonzero elements in row I, padded by the integer
                  value I if the number of nonzeros is less than MAXNZ.

    LDA           leading dimension of VAL and INDX.

    MAXNZ         max number of nonzeros elements per row.

    B()           rectangular array with first dimension LDB.
    LDB           leading dimension of B

    BETA          Scalar parameter

    C()           rectangular array with first dimension LDC.

    LDC           leading dimension of C

    WORK()        scratch array of length LWORK. WORK is not
                  referenced in the current version.

    LWORK         length of WORK array. LWORK is not referenced
                  in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

```
sellsm - Ellpack format triangular solve
```

SYNOPSIS

```
  SUBROUTINE SELLSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, INDX, LDA, MAXNZ,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
  INTEGER    TRANSA, M, N, UNITD, DESCRA(5), LDA, MAXNZ,
 *           LDB, LDC, LWORK
  INTEGER    INDX(LDA,MAXNZ)
  REAL       ALPHA, BETA
  REAL       DV(M), VAL(LDA,MAXNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

  SUBROUTINE SELLSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, INDX, LDA, MAXNZ,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
  INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5), LDA, MAXNZ,
 *           LDB, LDC, LWORK
  INTEGER*8  INDX(LDA,MAXNZ)
  REAL       ALPHA, BETA
  REAL       DV(M), VAL(LDA,MAXNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
  SUBROUTINE ELLSM( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
 *    INDX, [LDA], MAXNZ, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
  INTEGER    TRANSA, M,  MAXNZ
  INTEGER, DIMENSION(:) ::  DESCRA
  INTEGER, DIMENSION(:, :) ::    INDX
  REAL    ALPHA, BETA
  REAL, DIMENSION(:) ::  DV
  REAL, DIMENSION(:, :) ::  VAL, B, C
```

```
 SUBROUTINE ELLSM_64( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
*    INDX, [LDA], MAXNZ, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M,  MAXNZ
 INTEGER*8, DIMENSION(:) ::  DESCRA
 INTEGER*8, DIMENSION(:, :) ::    INDX
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) ::  DV
 REAL, DIMENSION(:, :) ::  VAL, B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a diagonal scaling matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in Ellpack format and
op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                 0 : general
                 1 : symmetric (A=A')
                 2 : Hermitian (A= CONJG(A'))
                 3 : Triangular
                 4 : Skew(Anti)-Symmetric (A=-A')
                 5 : Diagonal
                 6 : Skew-Hermitian (A= -CONJG(A'))

               Note: For the routine, only DESCRA(1)=3 is supported.

               DESCRA(2) upper/lower triangular indicator
                 1 : lower
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices
VAL()          two-dimensional LDA-by-MAXNZ array such that VAL(I,:)
               consists of non-zero elements in row I of A, padded by
               zero values if the row contains less than MAXNZ.

INDX()         two-dimensional integer LDA-by-MAXNZ array such
               INDX(I,:) consists of the column indices of the
               nonzero elements in row I, padded by the integer
               value I if the number of nonzeros is less than MAXNZ.
               The column indices MUST be sorted in increasing order
               for each row.

LDA            leading dimension of VAL and INDX.

MAXNZ          max number of nonzeros elements per row.

B()            rectangular array with first dimension LDB.

LDB            leading dimension of B

BETA           Scalar parameter

C()            rectangular array with first dimension LDC.

LDC            leading dimension of C

WORK()         scratch array of length LWORK.

               On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.
```

LWORK length of WORK array. LWORK should be at least M.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the rows of A such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows have been scaled. UNITD=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the ELL representation of a sparse matrix. They are not used anyway in these cases.

But if UNITD=4, the unit diagonal elements MUST be referenced in the ELL representation.

4. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

sfftc - initialize the trigonometric weight and factor tables or compute the forward Fast Fourier Transform of a real sequence.

SYNOPSIS

```
SUBROUTINE SFFTC(IOPT, N, SCALE, X, Y, TRIGS, IFAC, WORK, LWORK, IERR)

INTEGER IOPT, N, IFAC(*), LWORK, IERR
COMPLEX Y(*)
REAL X(*), SCALE, TRIGS(*), WORK(*)

SUBROUTINE SFFTC_64(IOPT, N, SCALE, X, Y, TRIGS, IFAC, WORK, LWORK, IERR)

INTEGER*8 IOPT, N, IFAC(*), LWORK, IERR
REAL X(*), SCALE, TRIGS(*), WORK(*)
COMPLEX Y(*)
```

F95 INTERFACE

```
SUBROUTINE FFT(IOPT, N, SCALE, X, Y, TRIGS, IFAC, WORK, [LWORK], IERR)

INTEGER*4, INTENT(IN) :: IOPT
INTEGER*4, INTENT(IN), OPTIONAL :: N, LWORK
REAL, INTENT(IN), OPTIONAL :: SCALE
REAL, INTENT(IN), DIMENSION(:) :: X
COMPLEX, INTENT(OUT), DIMENSION(:) :: Y
REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
INTEGER*4, INTENT(INOUT), DIMENSION(:) :: IFAC
REAL, INTENT(OUT), DIMENSION(:) :: WORK
INTEGER*4, INTENT(OUT) :: IERR

SUBROUTINE FFT_64(IOPT, [N], [SCALE], X, Y, TRIGS, IFAC, WORK, [LWORK], IERR)
```

```
INTEGER(8), INTENT(IN) :: IOPT
INTEGER(8), INTENT(IN), OPTIONAL :: N, LWORK
REAL, INTENT(IN), OPTIONAL :: SCALE
REAL, INTENT(IN), DIMENSION(:) :: X
COMPLEX, INTENT(OUT), DIMENSION(:) :: Y
REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
REAL, INTENT(OUT), DIMENSION(:) :: WORK
INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
#include <sunperf.h>

void sfftc_ (int *iopt, int *n, float *scale, float *x, com-
          plex *y, float *trigs, int *ifac, float *work, int
          *lwork, int *ierr);

void sfftc_64_ (long *iopt, long *n, float *scale, float *x,
          complex *y, float *trigs, long *ifac, float *work,
          long *lwork, long *ierr);
```

PURPOSE

sfftc initializes the trigonometric weight and factor tables or computes the forward Fast Fourier Transform of a real sequence as follows:

```
               N-1
Y(k) = scale * SUM  W*X(j)
               j=0
```

where
k ranges from 0 to N-1
i = sqrt(-1)
isign = -1 for forward transform
W = exp(isign*i*j*k*2*pi/N)
In real-to-complex transform of length N, the (N/2+1) complex output data points stored are the positive-frequency half of the spectrum of the Discrete Fourier Transform. The other half can be obtained through complex conjugation and therefore is not stored.

ARGUMENTS

IOPT (input)
Integer specifying the operation to be performed:

```
          IOPT = 0 computes the trigonometric  weight  table
          and factor table
          IOPT = -1 computes forward FFT

N (input)
          Integer specifying length of the input sequence X.
          N  is most efficient when it is a product of small
          primes.  N >= 0.  Unchanged on exit.

SCALE (input)
          Real scalar by which transform results are scaled.
          Unchanged  on exit.  SCALE is defaulted to 1.0 for
          F95 INTERFACE.

X (input) On entry, X is a real array whose first N elements
          contain the sequence to be transformed.

Y (output)
          Complex array whose first (N/2+1) elements contain
          the  transform  results.   X and Y may be the same
          array starting at the  same  memory  location,  in
          which  case  the  dimension  of X must be at least
          2*(N/2+1).  Otherwise, it is assumed that there is
          no overlap between X and Y in memory.

TRIGS (input/output)
          Real array of length 2*N that  contains  the  tri-
          gonometric weights.  The weights are computed when
          the routine is called with IOPT = 0 and  they  are
          used   in   subsequent   calls  when  IOPT  =  -1.
          Unchanged on exit.

IFAC (input/output)
          Integer array of dimension at least 128 that  con-
          tains  the factors of N.  The factors are computed
          when the routine is called with IOPT = 0 and  they
          are  used  in  subsequent  calls  where IOPT = -1.
          Unchanged on exit.

WORK (workspace)
          Real array of dimension at least N.  The user  can
          also  choose  to have the routine allocate its own
          workspace (see LWORK).

LWORK (input)
          Integer specifying workspace size.  If LWORK =  0,
          the routine will allocate its own workspace.

IERR (output)
          On exit, integer IERR has  one  of  the  following
```

```
values:
0 = normal return
-1 = IOPT is not 0 or -1
-2 = N < 0
-3 = (LWORK is not 0) and (LWORK is less than N)
-4 = memory allocation for workspace failed
```

SEE ALSO

fft

Contents

NAME

```
     sfftc2 - initialize  the  trigonometric  weight  and  factor
     tables  or  compute the two-dimensional forward Fast Fourier
     Transform of a two-dimensional real array.
```

SYNOPSIS

```
     SUBROUTINE SFFTC2(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     COMPLEX Y(LDY, *)
     REAL X(LDX, *), SCALE, TRIGS(*), WORK(*)

     SUBROUTINE SFFTC2_64(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER*8 IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     REAL X(LDX, *), SCALE, TRIGS(*), WORK(*)
     COMPLEX Y(LDY, *)
```

F95 INTERFACE

```
     SUBROUTINE FFT2(IOPT, [N1], [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
     &          IFAC, WORK, [LWORK], IERR)

     INTEGER*4, INTENT(IN) :: IOPT
     INTEGER*4, INTENT(IN), OPTIONAL :: N1, N2, LDX, LDY, LWORK
     REAL, INTENT(IN), OPTIONAL :: SCALE
     REAL, INTENT(IN), DIMENSION(:,:) :: X
     COMPLEX, INTENT(OUT), DIMENSION(:,:) :: Y
     REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER*4, INTENT(INOUT), DIMENSION(:) :: IFAC
```

```
     REAL, INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER*4, INTENT(OUT) :: IERR

     SUBROUTINE FFT2_64(IOPT, [N1], [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
IFAC, WORK, [LWORK], IERR)

     INTEGER(8), INTENT(IN) :: IOPT
     INTEGER(8), INTENT(IN), OPTIONAL :: N1, N2, LDX, LDY, LWORK
     REAL, INTENT(IN), OPTIONAL :: SCALE
     REAL, INTENT(IN), DIMENSION(:,:) :: X
     COMPLEX, INTENT(OUT), DIMENSION(:,:) :: Y
     REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL, INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
     #include <sunperf.h>
     void sfftc2_ (int *iopt, int *n1,  int  *n2,  float  *scale,
               float  *x,  int  *ldx, complex *y, int *ldy, float
               *trigs, int *ifac, float *work,  int  *lwork,  int
               *ierr);

     void sfftc2_64_ (long  *iopt,  long  *n1,  long  *n2,  float
               *scale,  float  *x,  long  *ldx,  complex *y, long
               *ldy, float *trigs, long *ifac, float *work,  long
               *lwork, long *ierr);
```

PURPOSE

sfftc2 initializes the trigonometric weight and factor tables or computes the two-dimensional forward Fast Fourier Transform of a two-dimensional real array. In computing the two-dimensional FFT, one-dimensional FFTs are computed along the columns of the input array. One-dimensional FFTs are then computed along the rows of the intermediate results.

```
                   N2-1  N1-1
Y(k1,k2) = scale * SUM   SUM   W2*W1*X(j1,j2)
                   j2=0  j1=0

where
k1 ranges from 0 to N1-1 and k2 ranges from 0 to N2-1
i = sqrt(-1)
isign = -1 for forward transform
W1 = exp(isign*i*j1*k1*2*pi/N1)
W2 = exp(isign*i*j2*k2*2*pi/N2)
```

In real-to-complex transform of length N1, the (N1/2+1) complex output data points stored are the positive-frequency half of the spectrum of the Discrete Fourier Transform. The other half can be obtained through complex conjugation and therefore is not stored.

ARGUMENTS

IOPT (input)
Integer specifying the operation to be performed:
IOPT = 0 computes the trigonometric weight table and factor table
IOPT = -1 computes forward FFT

N1 (input)
Integer specifying length of the transform in the first dimension. N1 is most efficient when it is a product of small primes. N1 >= 0. Unchanged on exit.

N2 (input)
Integer specifying length of the transform in the second dimension. N2 is most efficient when it is a product of small primes N2 >= 0. Unchanged on exit.

SCALE (input)
Real scalar by which transform results are scaled. Unchanged on exit. SCALE is defaulted to 1.0 for F95 INTERFACE.

X (input) X is a complex array of dimensions (LDX, N2) that contains input data to be transformed. X and Y can be the same array.

LDX (input)
Leading dimension of X. LDX >= N1 if X is not the same array as Y. Else, LDX = 2*LDY. Unchanged on exit.

Y (output)
Y is a complex array of dimensions (LDY, N2) that contains the transform results. X and Y can be the same array starting at the same memory location, in which case the input data are overwritten by their transform results. Otherwise, it is assumed that there is no overlap between X and Y in memory.

LDY (input)

Leading dimension of Y. LDY >= N1/2+1 Unchanged on exit.

TRIGS (input/output)

Real array of length 2*(N1+N2) that contains the trigonometric weights. The weights are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = -1. Unchanged on exit.

IFAC (input/output)

Integer array of dimension at least 2*128 that contains the factors of N1 and N2. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = -1. Unchanged on exit.

WORK (workspace)

Real array of dimension at least MAX(N1, 2*N2)*NCPUS, where NCPUS is the number of threads used to execute the routine. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)

Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)

On exit, integer IERR has one of the following values:

0 = normal return
-1 = IOPT is not 0 or -1
-2 = N1 < 0
-3 = N2 < 0
-4 = (LDX < N1) or (LDX not equal 2*LDY when X and Y are same array)
-5 = (LDY < N1/2+1)
-6 = (LWORK not equal 0) and (LWORK < MAX(N1,2*N2)*NCPUS)
-7 = memory allocation failed

SEE ALSO

fft

CAUTIONS

Y(N1/2+1:LDY,:) is used as scratch space. Upon returning, the original contents of Y(N1/2+1:LDY,:) will be lost, whereas Y(1:N1/2+1,1:N2) contains the transform results.

Contents

NAME

```
     sfftc3 - initialize  the  trigonometric  weight  and  factor
     tables or compute the three-dimensional forward Fast Fourier
     Transform of a three-dimensional complex array.
```

SYNOPSIS

```
     SUBROUTINE SFFTC3(IOPT, N1, N2, N3, SCALE, X, LDX1, LDX2, Y, LDY1, LDY2,
TRIGS, IFAC, WORK, LWORK, IERR)

     INTEGER IOPT, N1, N2, N3, LDX1, LDX2, LDY1,  LDY2,  IFAC(*),
     LWORK, IERR
     COMPLEX Y(LDY1, LDY2, *)
     REAL X(LDX1, LDX2, *), SCALE, TRIGS(*), WORK(*)

     SUBROUTINE SFFTC3_64(IOPT, N1, N2, N3, SCALE, X, LDX1, LDX2, Y, LDY1, LDY2,
TRIGS, IFAC, WORK, LWORK, IERR)

     INTEGER*8 IOPT, N1, N2, N3, LDX1, LDX2, LDY1, LDY2, IFAC(*),
     LWORK, IERR
     COMPLEX Y(LDY1, LDY2, *)
     REAL X(LDX1, LDX2, *), SCALE, TRIGS(*), WORK(*)
```

F95 INTERFACE

```
     SUBROUTINE FFT3(IOPT, [N1], [N2], [N3], [SCALE], X, [LDX1], LDX2, Y, [LDY1],
LDY2, TRIGS, IFAC, WORK, [LWORK], IERR)

     INTEGER*4, INTENT(IN) :: IOPT, LDX2, LDY2
     INTEGER*4, INTENT(IN), OPTIONAL :: N1, N2, N3,  LDX1,  LDY1,
     LWORK
     REAL, INTENT(IN), OPTIONAL :: SCALE
     REAL, INTENT(IN), DIMENSION(:,:) :: X
```

```
     COMPLEX, INTENT(OUT), DIMENSION(:,:) :: Y
     REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER*4, INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL, INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER*4, INTENT(OUT) :: IERR

     SUBROUTINE FFT3_64(IOPT, [N1], [N2], [N3], [SCALE], X, [LDX1], LDX2, Y,
[LDY1], LDY2, TRIGS, IFAC, WORK, [LWORK], IERR)

     INTEGER(8), INTENT(IN) :: IOPT, LDX2, LDY2
     INTEGER(8), INTENT(IN), OPTIONAL :: N1, N2, N3, LDX1,  LDY1,
     LWORK
     REAL, INTENT(IN), OPTIONAL :: SCALE
     REAL, INTENT(IN), DIMENSION(:,:) :: X
     COMPLEX, INTENT(OUT), DIMENSION(:,:) :: Y
     REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL, INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
     #include <sunperf.h>

     void sfftc3_ (int *iopt, int *n1, int *n2,  int  *n3,  float
               *scale,  float  *x,  int *ldx1, int *ldx2, complex
               *y, int *ldy1, int *ldy2, float *trigs, int *ifac,
               float *work, int *lwork, int *ierr);

     void sfftc3_64_ (long *iopt, long *n1, long *n2,  long  *n3,
               float  *scale,  float  *x, long *ldx1, long *ldx2,
               complex *y, long *ldy1, long *ldy2, float  *trigs,
               long *ifac, float *work, long *lwork, long *ierr);
```

PURPOSE

```
     sfftc3  initializes  the  trigonometric  weight  and  factor
     tables   or  computes  the  three-dimensional  forward  Fast
     Fourier Transform of a three-dimensional complex array.

                           N3-1  N2-1  N1-1
     Y(k1,k2,k3) = scale * SUM   SUM   SUM   W3*W2*W1*X(j1,j2,j3)
                           j3=0  j2=0  j1=0

     where
     k1 ranges from 0 to N1-1; k2 ranges from 0 to  N2-1  and  k3
     ranges from 0 to N3-1
     i = sqrt(-1)
     isign = -1 for forward transform
     W1 = exp(isign*i*j1*k1*2*pi/N1)
     W2 = exp(isign*i*j2*k2*2*pi/N2)
```

```
W3 = exp(isign*i*j3*k3*2*pi/N3)
```

ARGUMENTS

```
IOPT (input)
          Integer specifying the operation to be performed:
          IOPT = 0 computes the trigonometric  weight  table
          and factor table
          IOPT = -1 computes forward FFT

N1 (input)
          Integer specifying length of the transform in  the
          first  dimension.  N1 is most efficient when it is
          a product of small primes.  N1 >= 0.  Unchanged on
          exit.

N2 (input)
          Integer specifying length of the transform in  the
          second dimension.  N2 is most efficient when it is
          a product of small primes.  N2 >= 0.  Unchanged on
          exit.
N3 (input)
          Integer specifying length of the transform in  the
          third  dimension.  N3 is most efficient when it is
          a product of small primes.  N3 >= 0.  Unchanged on
          exit.

SCALE (input)
          Real scalar by which transform results are scaled.
          Unchanged  on exit.  SCALE is defaulted to 1.0 for
          F95 INTERFACE.

X (input) X is a real array of dimensions (LDX1,  LDX2,  N3)
          that contains input data to be transformed.  X can
          be same array as Y.

LDX1 (input)
          first dimension of X.  If X is not same  array  as
          Y,  LDX1  >=  N1  Else, LDX1 = 2*LDY1 Unchanged on
          exit.

LDX2 (input)
          second dimension of X.  LDX2 >=  N2  Unchanged  on
          exit.

Y (output)
          Y is a complex array of  dimensions  (LDY1,  LDY2,
          N3)  that contains the transform results.  X and Y
```

can be the same array starting at the same memory location, in which case the input data are overwritten by their transform results. Otherwise, it is assumed that there is no overlap between X and Y in memory.

LDY1 (input)

first dimension of Y. LDY1 >= N1/2+1 Unchanged on exit.

LDY2 (input)

second dimension of Y. If X and Y are the same array, LDY2 = LDX2 Else LDY2 >= N2 Unchanged on exit.

TRIGS (input/output)

Real array of length 2*(N1+N2+N3) that contains the trigonometric weights. The weights are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = -1. Unchanged on exit.

IFAC (input/output)

Integer array of dimension at least 3*128 that contains the factors of N1, N2 and N3. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = -1. Unchanged on exit.

WORK (workspace)

Real array of dimension at least (MAX(N,2*N2,2*N3) + 16*N3) * NCPUS where NCPUS is the number of threads used to execute the routine. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)

Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)

On exit, integer IERR has one of the following values:

0 = normal return

-1 = IOPT is not 0 or -1

-2 = N1 < 0

-3 = N2 < 0

-4 = N3 < 0

-5 = (LDX1 < N1) or (LDX not equal 2*LDY when X and Y are same array)

-6 = (LDX2 < N2)
-7 = (LDY1 < N1/2+1)
-8 = (LDY2 < N2) or (LDY2 not equal LDX2 when X and Y are same array)
-9 = (LWORK not equal 0) and (LWORK < (MAX(N,2*N2,2*N3) + 16*N3)*NCPUS)
-10 = memory allocation failed

SEE ALSO

fft

CAUTIONS

This routine uses Y((N1/2+1)+1:LDY1,:,:) as scratch space. Therefore, the original contents of this subarray will be lost upon returning from routine while subarray Y(1:N1/2+1,1:N2,1:N3) contains the transform results.

Contents

NAME

sfftcm - initialize the trigonometric weight and factor tables or compute the one-dimensional forward Fast Fourier Transform of a set of real data sequences stored in a two-dimensional array.

SYNOPSIS

```
     SUBROUTINE SFFTCM(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     REAL X(LDX, *), SCALE, TRIGS(*), WORK(*)
     COMPLEX Y(LDY, *)

     SUBROUTINE SFFTCM_64(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER*8 IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     REAL X(LDX, *), SCALE, TRIGS(*), WORK(*)
     COMPLEX Y(LDY, *)
```

F95 INTERFACE

```
     SUBROUTINE FFTM(IOPT, [N1], [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
               IFAC, WORK, [LWORK], IERR)

     INTEGER*4, INTENT(IN) :: IOPT
     INTEGER*4, INTENT(IN), OPTIONAL :: N1, N2, LDX, LDY, LWORK
     REAL, INTENT(IN), OPTIONAL :: SCALE
     REAL, INTENT(IN), DIMENSION(:,:) :: X
     COMPLEX, INTENT(OUT), DIMENSION(:,:) :: Y
     REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER*4, INTENT(INOUT), DIMENSION(:) :: IFAC
```

```
     REAL, INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER*4, INTENT(OUT) :: IERR

     SUBROUTINE FFTM_64(IOPT, [N1], [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
IFAC, WORK, [LWORK], IERR)

     INTEGER(8), INTENT(IN) :: IOPT
     INTEGER(8), INTENT(IN), OPTIONAL :: N1, N2, LDX, LDY, LWORK
     REAL, INTENT(IN), OPTIONAL :: SCALE
     REAL, INTENT(IN), DIMENSION(:,:) :: X
     COMPLEX, INTENT(OUT), DIMENSION(:,:) :: Y
     REAL, INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL, INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
     #include <sunperf.h>
     void sfftcm_ (int *iopt, int *n1,  int  *n2,  float  *scale,
               float  *x,  int  *ldx, complex *y, int *ldy, float
               *trigs, int *ifac, float *work,  int  *lwork,  int
               *ierr);

     void sfftcm_64_ (long  *iopt,  long  *n1,  long  *n2,  float
               *scale,  float  *x,  long  *ldx,  complex *y, long
               *ldy, float *trigs, long *ifac, float *work,  long
               *lwork, long *ierr);
```

PURPOSE

sfftcm initializes the trigonometric weight and factor tables or computes the one-dimensional forward Fast Fourier Transform of a set of real data sequences stored in a two-dimensional array:

```
                 N1-1
Y(k,l) = scale * SUM  W*X(j,l)
                 j=0
```

where
k ranges from 0 to N1-1 and l ranges from 0 to N2-1
i = sqrt(-1)
isign = -1 for forward transform
W = exp(isign*i*j*k*2*pi/N1)
In real-to-complex transform of length N1, the (N1/2+1) complex output data points stored are the positive-frequency half of the spectrum of the discrete Fourier transform. The other half can be obtained through complex conjugation and therefore is not stored.

ARGUMENTS

IOPT (input)
Integer specifying the operation to be performed:
IOPT = 0 computes the trigonometric weight table and factor table
IOPT = -1 computes forward FFT

N1 (input)
Integer specifying length of the input sequences. N1 is most efficient when it is a product of small primes. N1 >= 0. Unchanged on exit.

N2 (input)
Integer specifying number of input sequences. N2 >= 0. Unchanged on exit.

SCALE (input)
Real scalar by which transform results are scaled. Unchanged on exit. SCALE is defaulted to 1.0 for F95 INTERFACE.

X (input) X is a real array of dimensions (LDX, N2) that contains the sequences to be transformed stored in its columns.

LDX (input)
Leading dimension of X. If X and Y are the same array, LDX = 2*LDY Else LDX >= N1 Unchanged on exit.

Y (output)
Y is a complex array of dimensions (LDY, N2) that contains the transform results of the input sequences. X and Y can be the same array starting at the same memory location, in which case the input sequences are overwritten by their transform results. Otherwise, it is assumed that there is no overlap between X and Y in memory.

LDY (input)
Leading dimension of Y. LDY >= N1/2 + 1 Unchanged on exit.

TRIGS (input/output)
Real array of length 2*N1 that contains the trigonometric weights. The weights are computed when

the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = -1. Unchanged on exit.

IFAC (input/output)
Integer array of dimension at least 128 that contains the factors of N1. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = -1. Unchanged on exit.

WORK (workspace)
Real array of dimension at least N1. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)
Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)
On exit, integer IERR has one of the following values:
0 = normal return
-1 = IOPT is not 0 or -1
-2 = N1 < 0
-3 = N2 < 0
-4 = (LDX < N1) or (LDX not equal 2*LDY when X and Y are same array)
-4 = (LDY < N1/2 + 1)
-6 = (LWORK not equal 0) and (LWORK < N1)
-7 = memory allocation failed

SEE ALSO

fft

Contents

NAME

```
sgbbrd - reduce a real general m-by-n band matrix A to upper
bidiagonal form B by an orthogonal transformation
```

SYNOPSIS

```
SUBROUTINE SGBBRD(VECT, M, N, NCC, KL, KU, AB, LDAB, D, E, Q, LDQ,
      PT, LDPT, C, LDC, WORK, INFO)

CHARACTER * 1 VECT
INTEGER M, N, NCC, KL, KU, LDAB, LDQ, LDPT, LDC, INFO
REAL AB(LDAB,*), D(*), E(*), Q(LDQ,*), PT(LDPT,*), C(LDC,*),
WORK(*)

SUBROUTINE SGBBRD_64(VECT, M, N, NCC, KL, KU, AB, LDAB, D, E, Q, LDQ,
      PT, LDPT, C, LDC, WORK, INFO)

CHARACTER * 1 VECT
INTEGER*8 M, N, NCC, KL, KU, LDAB, LDQ, LDPT, LDC, INFO
REAL AB(LDAB,*), D(*), E(*), Q(LDQ,*), PT(LDPT,*), C(LDC,*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GBBRD(VECT, M, [N], [NCC], KL, KU, AB, [LDAB], D, E, Q,
       [LDQ], PT, [LDPT], C, [LDC], [WORK], [INFO])

CHARACTER(LEN=1) :: VECT
INTEGER :: M, N, NCC, KL, KU, LDAB, LDQ, LDPT, LDC, INFO
REAL, DIMENSION(:) :: D, E, WORK
REAL, DIMENSION(:,:) :: AB, Q, PT, C

SUBROUTINE GBBRD_64(VECT, M, [N], [NCC], KL, KU, AB, [LDAB], D, E,
       Q, [LDQ], PT, [LDPT], C, [LDC], [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: VECT
INTEGER(8) :: M, N, NCC, KL, KU, LDAB, LDQ, LDPT, LDC, INFO
REAL, DIMENSION(:) :: D, E, WORK
REAL, DIMENSION(:,:) :: AB, Q, PT, C
```

C INTERFACE

```
#include <sunperf.h>

void sgbbrd(char vect, int m, int n, int ncc,  int  kl,  int
          ku, float *ab, int ldab, float *d, float *e, float
          *q, int ldq, float *pt, int ldpt,  float  *c,  int
          ldc, int *info);
void sgbbrd_64(char vect, long m, long n, long ncc, long kl,
          long ku, float *ab, long ldab, float *d, float *e,
          float *q, long ldq, float *pt,  long  ldpt,  float
          *c, long ldc, long *info);
```

PURPOSE

sgbbrd reduces a real general m-by-n band matrix A to upper bidiagonal form B by an orthogonal transformation: Q' * A * P = B.

The routine computes B, and optionally forms Q or P', or computes Q'*C for a given matrix C.

ARGUMENTS

VECT (input)
Specifies whether or not the matrices Q and P' are to be formed. = 'N': do not form Q or P';
= 'Q': form Q only;
= 'P': form P' only;
= 'B': form both.

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

NCC (input)
The number of columns of the matrix C. NCC >= 0.

KL (input)
The number of subdiagonals of the matrix A. KL >= 0.

KU (input)
The number of superdiagonals of the matrix A. KU >= 0.

AB (input/output)
REAL array, dimension(LDAB,N) On entry, the m-by-n band matrix A, stored in rows 1 to KL+KU+1. The j-th column of A is stored in the j-th column of the array AB as follows: AB(ku+1+i-j,j) = A(i,j) for max(1,j-ku)<=i<=min(m,j+kl). On exit, A is overwritten by values generated during the reduction.

LDAB (input)
The leading dimension of the array A. LDAB >= KL+KU+1.

D (output)
REAL array, dimension(min(M,N)) The diagonal elements of the bidiagonal matrix B.

E (output)
REAL array, dimension(min(M,N)-1) The superdiagonal elements of the bidiagonal matrix B.

Q (output)
REAL array, dimension(LDQ,M) If VECT = 'Q' or 'B', the m-by-m orthogonal matrix Q. If VECT = 'N' or 'P', the array Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= max(1,M) if VECT = 'Q' or 'B'; LDQ >= 1 otherwise.

PT (output)
REAL array, dimension(LDPT,N) If VECT = 'P' or 'B', the n-by-n orthogonal matrix P'. If VECT = 'N' or 'Q', the array PT is not referenced.

LDPT (input)
The leading dimension of the array PT. LDPT >= max(1,N) if VECT = 'P' or 'B'; LDPT >= 1 otherwise.

C (input/output)
REAL array, dimension(LDC.NCC) On entry, an m-by-ncc matrix C. On exit, C is overwritten by Q'*C. C is not referenced if NCC = 0.

LDC (input)

The leading dimension of the array C. LDC >= max(1,M) if NCC > 0; LDC >= 1 if NCC = 0.

WORK (workspace)
REAL array, dimension(2*MAX(M,N))

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

sgbcon - estimate the reciprocal of the condition number of a real general band matrix A, in either the 1-norm or the infinity-norm,

SYNOPSIS

```
SUBROUTINE SGBCON(NORM, N, NSUB, NSUPER, A, LDA, IPIVOT, ANORM,
      RCOND, WORK, WORK2, INFO)

CHARACTER * 1 NORM
INTEGER N, NSUB, NSUPER, LDA, INFO
INTEGER IPIVOT(*), WORK2(*)
REAL ANORM, RCOND
REAL A(LDA,*), WORK(*)

SUBROUTINE SGBCON_64(NORM, N, NSUB, NSUPER, A, LDA, IPIVOT, ANORM,
      RCOND, WORK, WORK2, INFO)

CHARACTER * 1 NORM
INTEGER*8 N, NSUB, NSUPER, LDA, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
REAL ANORM, RCOND
REAL A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GBCON(NORM, [N], NSUB, NSUPER, A, [LDA], IPIVOT, ANORM,
       RCOND, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM
INTEGER :: N, NSUB, NSUPER, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
REAL :: ANORM, RCOND
```

```
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE GBCON_64(NORM, [N], NSUB, NSUPER, A, [LDA], IPIVOT, ANORM,
       RCOND, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM
INTEGER(8) :: N, NSUB, NSUPER, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
void sgbcon(char norm, int n, int nsub,  int  nsuper,  float
          *a,  int  lda,  int  *ipivot,  float  anorm, float
          *rcond, int *info);

void sgbcon_64(char norm, long n, long  nsub,  long  nsuper,
          float  *a,  long  lda,  long *ipivot, float anorm,
          float *rcond, long *info);
```

PURPOSE

sgbcon estimates the reciprocal of the condition number of a real general band matrix A, in either the 1-norm or the infinity-norm, using the LU factorization computed by SGBTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as

RCOND = 1 / (norm(A) * norm(inv(A))).

ARGUMENTS

NORM (input)
Specifies whether the 1-norm condition number or the infinity-norm condition number is required:
= '1' or 'O': 1-norm;
= 'I': Infinity-norm.

N (input) The order of the matrix A. N >= 0.

NSUB (input)
The number of subdiagonals within the band of A. NSUB >= 0.

```
NSUPER (input)
          The number of superdiagonals within the band of A.
          NSUPER >= 0.

A (input) Details of the LU factorization of the band matrix
          A, as computed by SGBTRF.  U is stored as an upper
          triangular band matrix with NSUB+NSUPER superdiag-
          onals  in  rows 1 to NSUB+NSUPER+1, and the multi-
          pliers used during the factorization are stored in
          rows NSUB+NSUPER+2 to 2*NSUB+NSUPER+1.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          2*NSUB+NSUPER+1.

IPIVOT (input)
          The pivot indices; for 1 <= i <= N, row i  of  the
          matrix was interchanged with row IPIVOT(i).

ANORM (input)
          If NORM = '1' or 'O', the 1-norm of  the  original
          matrix A.  If NORM = 'I', the infinity-norm of the
          original matrix A.

RCOND (output)
          The reciprocal of  the  condition  number  of  the
          matrix   A,  computed  as  RCOND  =  1/(norm(A)  *
          norm(inv(A))).

WORK (workspace)
          dimension(3*N)

WORK2 (workspace)
          dimension (N)

INFO (output)
          = 0:  successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
          gal value
```

Contents

NAME

```
sgbequ - compute row and column scalings intended to equili-
brate  an  M-by-N  band  matrix  A  and reduce its condition
number
```

SYNOPSIS

```
SUBROUTINE SGBEQU(M, N, KL, KU, A, LDA, R, C, ROWCN,
      COLCN, AMAX, INFO)

INTEGER M, N, KL, KU, LDA, INFO
REAL ROWCN, COLCN, AMAX
REAL A(LDA,*), R(*), C(*)

SUBROUTINE SGBEQU_64(M, N, KL, KU, A, LDA, R, C, ROWCN,
      COLCN, AMAX, INFO)

INTEGER*8 M, N, KL, KU, LDA, INFO
REAL ROWCN, COLCN, AMAX
REAL A(LDA,*), R(*), C(*)
```

F95 INTERFACE

```
SUBROUTINE GBEQU([M], [N], KL, KU, A, [LDA], R, C,
       ROWCN, COLCN, AMAX, [INFO])

INTEGER :: M, N, KL, KU, LDA, INFO
REAL :: ROWCN, COLCN, AMAX
REAL, DIMENSION(:) :: R, C
REAL, DIMENSION(:,:) :: A

SUBROUTINE GBEQU_64([M], [N], KL, KU, A, [LDA], R, C,
       ROWCN, COLCN, AMAX, [INFO])
```

```
INTEGER(8) :: M, N, KL, KU, LDA, INFO
REAL :: ROWCN, COLCN, AMAX
REAL, DIMENSION(:) :: R, C
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void sgbequ(int m, int n, int kl, int ku, float *a, int lda,
          float  *r,  float  *c, float *rowcn, float *colcn,
          float *amax, int *info);

void sgbequ_64(long m, long n, long kl, long ku,  float  *a,
          long  lda, float *r, float *c, float *rowcn, float
          *colcn, float *amax, long *info);
```

PURPOSE

sgbequ computes row and column scalings intended to equilibrate an M-by-N band matrix A and reduce its condition number. R returns the row scale factors and C the column scale factors, chosen to try to make the largest element in each row and column of the matrix B with elements B(i,j)=R(i)*A(i,j)*C(j) have absolute value 1.

R(i) and C(j) are restricted to be between SMLNUM = smallest safe number and BIGNUM = largest safe number. Use of these scaling factors is not guaranteed to reduce the condition number of A but works well in practice.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

KL (input)
The number of subdiagonals within the band of A. KL >= 0.

KU (input)
The number of superdiagonals within the band of A. KU >= 0.

A (input) The band matrix A, stored in rows 1 to KL+KU+1. The j-th column of A is stored in the j-th column of the array A as follows: A(ku+1+i-j,j) = A(i,j) for max(1,j-ku)<=i<=min(m,j+kl).

LDA (input)
The leading dimension of the array A. LDA >= KL+KU+1.

R (output)
If INFO = 0, or INFO > M, R contains the row scale factors for A.

C (output)
If INFO = 0, C contains the column scale factors for A.

ROWCN (output)
If INFO = 0 or INFO > M, ROWCN contains the ratio of the smallest R(i) to the largest R(i). If ROWCN >= 0.1 and AMAX is neither too large nor too small, it is not worth scaling by R.

COLCN (output)
If INFO = 0, COLCN contains the ratio of the smallest C(i) to the largest C(i). If COLCN >= 0.1, it is not worth scaling by C.

AMAX (output)
Absolute value of largest matrix element. If AMAX is very close to overflow or very close to underflow, the matrix should be scaled.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= M: the i-th row of A is exactly zero
> M: the (i-M)-th column of A is exactly zero

Contents

NAME

sgbmv - perform one of the matrix-vector operations y := alpha*A*x + beta*y or y := alpha*A'*x + beta*y

SYNOPSIS

```
SUBROUTINE SGBMV(TRANSA, M, N, NSUB, NSUPER, ALPHA, A, LDA, X, INCX,
      BETA, Y, INCY)

CHARACTER * 1 TRANSA
INTEGER M, N, NSUB, NSUPER, LDA, INCX, INCY
REAL ALPHA, BETA
REAL A(LDA,*), X(*), Y(*)

SUBROUTINE SGBMV_64(TRANSA, M, N, NSUB, NSUPER, ALPHA, A, LDA, X,
      INCX, BETA, Y, INCY)

CHARACTER * 1 TRANSA
INTEGER*8 M, N, NSUB, NSUPER, LDA, INCX, INCY
REAL ALPHA, BETA
REAL A(LDA,*), X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE GBMV([TRANSA], [M], [N], NSUB, NSUPER, ALPHA, A, [LDA], X,
       [INCX], BETA, Y, [INCY])

CHARACTER(LEN=1) :: TRANSA
INTEGER :: M, N, NSUB, NSUPER, LDA, INCX, INCY
REAL :: ALPHA, BETA
REAL, DIMENSION(:) :: X, Y
REAL, DIMENSION(:,:) :: A

SUBROUTINE GBMV_64([TRANSA], [M], [N], NSUB, NSUPER, ALPHA, A, [LDA],
```

```
       X, [INCX], BETA, Y, [INCY])

CHARACTER(LEN=1) :: TRANSA
INTEGER(8) :: M, N, NSUB, NSUPER, LDA, INCX, INCY
REAL :: ALPHA, BETA
REAL, DIMENSION(:) :: X, Y
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void sgbmv(char transa, int m, int n, int nsub, int  nsuper,
          float  alpha,  float  *a,  int  lda, float *x, int
          incx, float beta, float *y, int incy);
void sgbmv_64(char transa, long m, long n, long  nsub,  long
          nsuper, float alpha, float *a, long lda, float *x,
          long incx, float beta, float *y, long incy);
```

PURPOSE

sgbmv performs one of the matrix-vector operations y := alpha*A*x + beta*y or y := alpha*A'*x + beta*y, where alpha and beta are scalars, x and y are vectors and A is an m by n band matrix, with nsub sub-diagonals and nsuper super-diagonals.

ARGUMENTS

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   y := alpha*A*x + beta*y.

TRANSA = 'T' or 't'   y := alpha*A'*x + beta*y.

TRANSA = 'C' or 'c'   y := alpha*A'*x + beta*y.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

M (input)
On entry, M specifies the number of rows of the matrix A. M >= 0. Unchanged on exit.

N (input)

On entry, N specifies the number of columns of the matrix A. N >= 0. Unchanged on exit.

NSUB (input)
On entry, NSUB specifies the number of sub-diagonals of the matrix A. NSUB >= 0. Unchanged on exit.

NSUPER (input)
On entry, NSUPER specifies the number of super-diagonals of the matrix A. NSUPER >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
Before entry, the leading (nsub + nsuper + 1) by n part of the array A must contain the matrix of coefficients, supplied column by column, with the leading diagonal of the matrix in row (nsuper + 1) of the array, the first super-diagonal starting at position 2 in row nsuper, the first sub-diagonal starting at position 1 in row (nsuper + 2), and so on. Elements in the array A that do not correspond to elements in the band matrix (such as the top left nsuper by nsuper triangle) are not referenced. The following program segment will transfer a band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     K = NSUPER + 1 - J
     DO 10, I = MAX( 1, J - NSUPER ), MIN( M, J  +
NSUB )
       A( K + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= (nsub + nsuper + 1). Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)) when TRANSA = 'N' or 'n' and at least (1 + (m - 1)*abs(INCX)) otherwise. Before entry, the incremented array X

must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. When BETA is supplied as zero then Y need not be set on input. Unchanged on exit.

Y (input/output)
(1 + (m - 1)*abs(INCY)) when TRANSA = 'N' or 'n' and at least (1 + (n - 1)*abs(INCY)) otherwise. Before entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
sgbrfs - improve the computed solution to a system of linear
equations  when  the  coefficient matrix is banded, and pro-
vides error bounds and  backward  error  estimates  for  the
solution
```

SYNOPSIS

```
SUBROUTINE SGBRFS(TRANSA, N, KL, KU, NRHS, A, LDA, AF, LDAF,
      IPIVOT, B, LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
INTEGER N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*), WORK2(*)
REAL  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),  FERR(*),
BERR(*), WORK(*)

SUBROUTINE SGBRFS_64(TRANSA, N, KL, KU, NRHS, A, LDA, AF, LDAF,
      IPIVOT, B, LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
INTEGER*8 N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
REAL  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),  FERR(*),
BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GBRFS([TRANSA], [N], KL, KU, [NRHS], A, [LDA], AF,
       [LDAF], IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: TRANSA
INTEGER :: N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
```

```
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
REAL, DIMENSION(:) :: FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, AF, B, X

SUBROUTINE GBRFS_64([TRANSA], [N], KL, KU, [NRHS], A, [LDA],
       AF, [LDAF], IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: TRANSA
INTEGER(8) :: N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL, DIMENSION(:) :: FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
#include <sunperf.h>

void sgbrfs(char transa, int n, int kl, int  ku,  int  nrhs,
          float  *a,  int  lda,  float  *af,  int  ldaf, int
          *ipivot, float *b, int ldb,  float  *x,  int  ldx,
          float *ferr, float *berr, int *info);

void sgbrfs_64(char transa, long n, long kl, long  ku,  long
          nrhs,  float  *a,  long lda, float *af, long ldaf,
          long *ipivot, float *b, long ldb, float  *x,  long
          ldx, float *ferr, float *berr, long *info);
```

PURPOSE

sgbrfs improves the computed solution to a system of linear equations when the coefficient matrix is banded, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

```
TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose = Tran-
          spose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A.  N >= 0.

KL (input)
```

The number of subdiagonals within the band of A. KL >= 0.

KU (input)
The number of superdiagonals within the band of A. KU >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The original band matrix A, stored in rows 1 to KL+KU+1. The j-th column of A is stored in the j-th column of the array A as follows: A(ku+1+i-j,j) = A(i,j) for max(1,j-ku)<=i<=min(n,j+kl).

LDA (input)
The leading dimension of the array A. LDA >= KL+KU+1.

AF (input)
Details of the LU factorization of the band matrix A, as computed by SGBTRF. U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1.

LDAF (input)
The leading dimension of the array AF. LDAF >= 2*KL*KU+1.

IPIVOT (input)
The pivot indices from SGBTRF; for 1<=i<=N, row i of the matrix was interchanged with row IPIVOT(i).

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by SGBTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)

The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)

The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)

dimension(3*N)

WORK2 (workspace)

dimension(N)

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
sgbsv - compute the solution to  a  real  system  of  linear
equations  A  *  X  = B, where A is a band matrix of order N
with KL subdiagonals and KU superdiagonals, and X and B  are
N-by-NRHS matrices
```

SYNOPSIS

```
SUBROUTINE SGBSV(N, KL, KU, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

INTEGER N, KL, KU, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)
REAL A(LDA,*), B(LDB,*)

SUBROUTINE SGBSV_64(N, KL, KU, NRHS, A, LDA, IPIVOT, B, LDB,
      INFO)

INTEGER*8 N, KL, KU, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
REAL A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE GBSV([N], KL, KU, [NRHS], A, [LDA], IPIVOT, B, [LDB],
       [INFO])

INTEGER :: N, KL, KU, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE GBSV_64([N], KL, KU, [NRHS], A, [LDA], IPIVOT, B,
       [LDB], [INFO])
```

```
INTEGER(8) :: N, KL, KU, NRHS, LDA, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void sgbsv(int n, int kl, int ku, int nrhs,  float  *a,  int
          lda, int *ipivot, float *b, int ldb, int *info);

void sgbsv_64(long n, long kl, long ku, long nrhs, float *a,
          long  lda,  long *ipivot, float *b, long ldb, long
          *info);
```

PURPOSE

sgbsv computes the solution to a real system of linear equations A * X = B, where A is a band matrix of order N with KL subdiagonals and KU superdiagonals, and X and B are N-by-NRHS matrices.

The LU decomposition with partial pivoting and row interchanges is used to factor A as A = L * U, where L is a product of permutation and unit lower triangular matrices with KL subdiagonals, and U is upper triangular with KL+KU superdiagonals. The factored form of A is then used to solve the system of equations A * X = B.

ARGUMENTS

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

KL (input)
The number of subdiagonals within the band of A. KL >= 0.

KU (input)
The number of superdiagonals within the band of A. KU >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input/output)
On entry, the matrix A in band storage, in rows

KL+1 to 2*KL+KU+1; rows 1 to KL of the array need not be set. The j-th column of A is stored in the j-th column of the array A as follows: A(KL+KU+1+i-j,j) = A(i,j) for max(1,j-KU)<=i<=min(N,j+KL) On exit, details of the factorization: U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1. See below for further details.

LDA (input)
The leading dimension of the array A. LDA >= 2*KL+KU+1.

IPIVOT (output)
The pivot indices that define the permutation matrix P; row i of the matrix was interchanged with row IPIVOT(i).

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly singular, and the solution has not been computed.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when M = N = 6, KL = 2, KU = 1:

On entry: On exit:

```
    *    *    *    +    +    +       *    *    *   u14   u25
u36
    *    *    +    +    +    +       *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
```

```
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
   a21  a32  a43  a54  a65   *      m21  m32  m43  m54   m65
*
   a31  a42  a53  a64   *    *      m31  m42  m53   m64    *
*
```

Array elements marked * are not used by the routine; elements marked + need not be set on entry, but are required by the routine to store elements of U because of fill-in resulting from the row interchanges.

Contents

NAME

```
sgbsvx - use the LU factorization to compute the solution to
a  real  system of linear equations A * X = B, A**T * X = B,
or A**H * X = B,
```

SYNOPSIS

```
SUBROUTINE SGBSVX(FACT, TRANSA, N, KL, KU, NRHS, A, LDA, AF,
      LDAF, IPIVOT, EQUED, R, C, B, LDB, X, LDX, RCOND, FERR,
      BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, TRANSA, EQUED
INTEGER N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*), WORK2(*)
REAL RCOND
REAL A(LDA,*), AF(LDAF,*), R(*), C(*),  B(LDB,*),  X(LDX,*),
FERR(*), BERR(*), WORK(*)

SUBROUTINE SGBSVX_64(FACT, TRANSA, N, KL, KU, NRHS, A, LDA, AF,
      LDAF, IPIVOT, EQUED, R, C, B, LDB, X, LDX, RCOND, FERR,
      BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, TRANSA, EQUED
INTEGER*8 N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
REAL RCOND
REAL A(LDA,*), AF(LDAF,*), R(*), C(*),  B(LDB,*),  X(LDX,*),
FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GBSVX(FACT, [TRANSA], [N], KL, KU, [NRHS], A, [LDA],
       AF, [LDAF], IPIVOT, EQUED, R, C, B, [LDB], X, [LDX],
       RCOND, FERR, BERR, [WORK], [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: FACT, TRANSA, EQUED
INTEGER :: N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
REAL :: RCOND
REAL, DIMENSION(:) :: R, C, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, AF, B, X

SUBROUTINE GBSVX_64(FACT, [TRANSA], [N], KL, KU, [NRHS], A,
       [LDA], AF, [LDAF], IPIVOT, EQUED, R, C, B, [LDB], X, [LDX],
       RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, TRANSA, EQUED
INTEGER(8) :: N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL :: RCOND
REAL, DIMENSION(:) :: R, C, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
#include <sunperf.h>

void sgbsvx(char fact, char transa, int n, int kl,  int  ku,
          int  nrhs, float *a, int lda, float *af, int ldaf,
          int *ipivot, char equed, float *r, float *c, float
          *b,  int  ldb,  float  *x,  int ldx, float *rcond,
          float *ferr, float *berr, int *info);

void sgbsvx_64(char fact, char transa, long n, long kl, long
          ku, long nrhs, float *a, long lda, float *af, long
          ldaf, long *ipivot, char equed,  float  *r,  float
          *c,  float *b, long ldb, float *x, long ldx, float
          *rcond, float *ferr, float *berr, long *info);
```

PURPOSE

sgbsvx uses the LU factorization to compute the solution to a real system of linear equations A * X = B, A**T * X = B, or A**H * X = B, where A is a band matrix of order N with KL subdiagonals and KU superdiagonals, and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed by this subroutine:

1. If FACT = 'E', real scaling factors are computed to equilibrate

```
   the system:
      TRANS = 'N':  diag(R)*A*diag(C)     *inv(diag(C))*X  =
diag(R)*B
      TRANS = 'T': (diag(R)*A*diag(C))**T *inv(diag(R))*X  =
diag(C)*B
      TRANS = 'C': (diag(R)*A*diag(C))**H *inv(diag(R))*X  =
diag(C)*B
   Whether or not the system will be equilibrated depends on
the
   scaling of the matrix A, but if equilibration is used,  A
is
   overwritten by diag(R)*A*diag(C) and B by  diag(R)*B  (if
TRANS='N')
   or diag(C)*B (if TRANS = 'T' or 'C').

2. If FACT = 'N' or 'E', the LU  decomposition  is  used  to
factor the
   matrix A (after equilibration if FACT = 'E') as
      A = L * U,
   where L is a product of permutation and unit  lower  tri-
angular
   matrices with KL subdiagonals, and U is upper  triangular
with
   KL+KU superdiagonals.

3. If some U(i,i)=0, so that U is exactly singular, then the
routine
   returns with INFO = i. Otherwise, the factored form of  A
is used
   to estimate the condition number of the matrix A.  If the
   reciprocal of the condition number is less  than  machine
precision,
   INFO = N+1 is returned as  a  warning,  but  the  routine
still goes on
   to solve for X and  compute  error  bounds  as  described
below.

4. The system of equations is solved for X  using  the  fac-
tored form
   of A.

5. Iterative refinement is applied to improve  the  computed
solution
   matrix and calculate  error  bounds  and  backward  error
estimates
   for it.

6. If equilibration was used, the matrix X is  premultiplied
by
   diag(C) (if TRANS = 'N') or diag(R) (if TRANS  =  'T'  or
```

```
'C') so
   that it solves the original system before equilibration.
```

ARGUMENTS

```
FACT (input)
          Specifies whether or not the factored form of  the
          matrix A is supplied on entry, and if not, whether
          the matrix A should be equilibrated before  it  is
          factored.  = 'F':  On entry, AF and IPIVOT contain
          the factored form of A.  If EQUED is not 'N',  the
          matrix  A  has been equilibrated with scaling fac-
          tors given by R and C.  A, AF, and IPIVOT are  not
          modified.   = 'N':  The matrix A will be copied to
          AF and factored.
          = 'E':  The  matrix  A  will  be  equilibrated  if
          necessary, then copied to AF and factored.
TRANSA (input)
          Specifies the form of the system of equations.   =
          'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Transpose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

KL (input)
          The number of subdiagonals within the band  of  A.
          KL >= 0.

KU (input)
          The number of superdiagonals within the band of A.
          KU >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input/output)
          On entry, the matrix A in band storage, in rows  1
          to KL+KU+1.  The j-th column of A is stored in the
          j-th column of the array A as follows:   A(KU+1+i-
          j,j) = A(i,j) for max(1,j-KU)<=i<=min(N,j+kl)

          If FACT = 'F' and EQUED is not 'N',  then  A  must
          have been equilibrated by the scaling factors in R
          and/or C.  A is not modified if FACT = 'F' or 'N',
```

or if FACT = 'E' and EQUED = 'N' on exit.

On exit, if EQUED .ne. 'N', A is scaled as follows: EQUED = 'R': A := diag(R) * A
EQUED = 'C': A := A * diag(C)
EQUED = 'B': A := diag(R) * A * diag(C).

LDA (input)
The leading dimension of the array A. LDA >= KL+KU+1.

AF (input/output)
If FACT = 'F', then AF is an input argument and on entry contains details of the LU factorization of the band matrix A, as computed by SGBTRF. U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1. If EQUED .ne. 'N', then AF is the factored form of the equilibrated matrix A.

If FACT = 'N', then AF is an output argument and on exit returns details of the LU factorization of A.

If FACT = 'E', then AF is an output argument and on exit returns details of the LU factorization of the equilibrated matrix A (see the description of A for the form of the equilibrated matrix).

LDAF (input)
The leading dimension of the array AF. LDAF >= 2*KL+KU+1.

IPIVOT (input)
If FACT = 'F', then IPIVOT is an input argument and on entry contains the pivot indices from the factorization A = L*U as computed by SGBTRF; row i of the matrix was interchanged with row IPIVOT(i).

If FACT = 'N', then IPIVOT is an output argument and on exit contains the pivot indices from the factorization A = L*U of the original matrix A.

If FACT = 'E', then IPIVOT is an output argument and on exit contains the pivot indices from the factorization A = L*U of the equilibrated matrix A.

EQUED (input)
Specifies the form of equilibration that was done.
= 'N': No equilibration (always true if FACT = 'N').
= 'R': Row equilibration, i.e., A has been premultiplied by diag(R). = 'C': Column equilibration, i.e., A has been postmultiplied by diag(C). = 'B': Both row and column equilibration, i.e., A has been replaced by diag(R) * A * diag(C). EQUED is an input argument if FACT = 'F'; otherwise, it is an output argument.

R (input/output)
The row scale factors for A. If EQUED = 'R' or 'B', A is multiplied on the left by diag(R); if EQUED = 'N' or 'C', R is not accessed. R is an input argument if FACT = 'F'; otherwise, R is an output argument. If FACT = 'F' and EQUED = 'R' or 'B', each element of R must be positive.

C (input/output)
The column scale factors for A. If EQUED = 'C' or 'B', A is multiplied on the right by diag(C); if EQUED = 'N' or 'R', C is not accessed. C is an input argument if FACT = 'F'; otherwise, C is an output argument. If FACT = 'F' and EQUED = 'C' or 'B', each element of C must be positive.

B (input/output)
On entry, the right hand side matrix B. On exit, if EQUED = 'N', B is not modified; if TRANSA = 'N' and EQUED = 'R' or 'B', B is overwritten by diag(R)*B; if TRANSA = 'T' or 'C' and EQUED = 'C' or 'B', B is overwritten by diag(C)*B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X to the original system of equations. Note that A and B are modified on exit if EQUED .ne. 'N', and the solution to the equilibrated system is inv(diag(C))*X if TRANSA = 'N' and EQUED = 'C' or 'B', or inv(diag(R))*X if TRANSA = 'T' or 'C' and EQUED = 'R' or 'B'.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)

The estimate of the reciprocal condition number of the matrix A after equilibration (if done). If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)

The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)

The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)

dimension(3*N) On exit, WORK(1) contains the reciprocal pivot growth factor norm(A)/norm(U). The "max absolute element" norm is used. If WORK(1) is much less than 1, then the stability of the LU factorization of the (equilibrated) matrix A could be poor. This also means that the solution X, condition estimator RCOND, and forward error bound FERR could be unreliable. If factorization fails with 0<INFO<=N, then WORK(1) contains the reciprocal pivot growth factor for the leading INFO columns of A.

WORK2 (workspace)

dimension(N)

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, and i is

<= N: U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution and error bounds could

not be computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
sgbtf2 - compute an LU factorization of a real  m-by-n  band
matrix A using partial pivoting with row interchanges
```

SYNOPSIS

```
SUBROUTINE SGBTF2(M, N, KL, KU, AB, LDAB, IPIV, INFO)

INTEGER M, N, KL, KU, LDAB, INFO
INTEGER IPIV(*)
REAL AB(LDAB,*)

SUBROUTINE SGBTF2_64(M, N, KL, KU, AB, LDAB, IPIV, INFO)

INTEGER*8 M, N, KL, KU, LDAB, INFO
INTEGER*8 IPIV(*)
REAL AB(LDAB,*)
```

F95 INTERFACE

```
SUBROUTINE GBTF2([M], [N], KL, KU, AB, [LDAB], IPIV, [INFO])

INTEGER :: M, N, KL, KU, LDAB, INFO
INTEGER, DIMENSION(:) :: IPIV
REAL, DIMENSION(:,:) :: AB

SUBROUTINE GBTF2_64([M], [N], KL, KU, AB, [LDAB], IPIV, [INFO])

INTEGER(8) :: M, N, KL, KU, LDAB, INFO
INTEGER(8), DIMENSION(:) :: IPIV
REAL, DIMENSION(:,:) :: AB
```

C INTERFACE

```
#include <sunperf.h>

void sgbtf2(int m, int n, int kl, int  ku,  float  *ab,  int
          ldab, int *ipiv, int *info);

void sgbtf2_64(long m, long n, long kl, long ku, float  *ab,
          long ldab, long *ipiv, long *info);
```

PURPOSE

sgbtf2 computes an LU factorization of a real m-by-n band matrix A using partial pivoting with row interchanges.

This is the unblocked version of the algorithm, calling Level 2 BLAS.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

KL (input)
The number of subdiagonals within the band of A. KL >= 0.

KU (input)
The number of superdiagonals within the band of A. KU >= 0.

AB (input/output)
On entry, the matrix A in band storage, in rows KL+1 to 2*KL+KU+1; rows 1 to KL of the array need not be set. The j-th column of A is stored in the j-th column of the array AB as follows: AB(kl+ku+1+i-j,j) = A(i,j) for max(1,j-ku)<=i<=min(m,j+kl)

On exit, details of the factorization: U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1. See below for further details.

LDAB (input)
The leading dimension of the array AB. LDAB >=

```
          2*KL+KU+1.

IPIV (output)
          The pivot indices; for 1 <= i <= min(M,N),  row  i
          of the matrix was interchanged with row IPIV(i).

INFO (output)
          = 0: successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
          gal value
          > 0: if INFO = +i, U(i,i)  is  exactly  zero.  The
          factorization has been completed, but the factor U
          is exactly singular, and  division  by  zero  will
          occur  if  it  is  used to solve a system of equa-
          tions.
```

FURTHER DETAILS

```
The band storage scheme  is  illustrated  by  the  following
example, when M = N = 6, KL = 2, KU = 1:

On entry:                       On exit:

    *    *    *    +    +    +       *    *    *   u14   u25
u36
    *    *    +    +    +    +       *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
   a21  a32  a43  a54  a65   *      m21  m32  m43  m54   m65
*
   a31  a42  a53  a64   *    *      m31  m42  m53   m64    *
*

Array elements marked * are not used by  the  routine;  ele-
ments marked + need not be set on entry, but are required by
the routine to store  elements  of  U,  because  of  fill-in
resulting from the row
interchanges.
```

Contents

NAME

```
sgbtrf - compute an LU factorization of a real  m-by-n  band
matrix A using partial pivoting with row interchanges
```

SYNOPSIS

```
SUBROUTINE SGBTRF(M, N, KL, KU, AB, LDAB, IPIVOT, INFO)

INTEGER M, N, KL, KU, LDAB, INFO
INTEGER IPIVOT(MIN(M,N))
REAL AB(LDAB, N)

SUBROUTINE SGBTRF_64(M, N, KL, KU, AB, LDAB, IPIVOT, INFO)

INTEGER*8 M, N, KL, KU, LDAB, INFO
INTEGER*8 IPIVOT(MIN(M,N))
REAL AB(LDAB,N)
```

F95 INTERFACE

```
SUBROUTINE GBTRF(M, [N], KL, KU, AB, [LDAB], IPIVOT, [INFO])

INTEGER :: M, N, KL, KU, LDAB, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:,:) :: AB

SUBROUTINE GBTRF_64(M, [N], KL, KU, AB, [LDAB], IPIVOT, [INFO])

INTEGER(8) :: M, N, KL, KU, LDAB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:,:) :: AB
```

C INTERFACE

```
#include <sunperf.h>

void sgbtrf(int m, int n, int kl, int  ku,  float  *ab,  int
          ldab, int *ipivot, int *info);

void sgbtrf_64(long m, long n, long kl, long ku, float  *ab,
          long ldab, long *ipivot, long *info);
```

PURPOSE

sgbtrf computes an LU factorization of a real m-by-n band matrix A using partial pivoting with row interchanges.

This is the blocked version of the algorithm, calling Level 3 BLAS.

ARGUMENTS

M (input) Integer
The number of rows of the matrix A. M >= 0.

N (input) Integer
The number of columns of the matrix A. N >= 0.

KL (input) Integer
The number of subdiagonals within the band of A. KL >= 0.

KU (input) Integer
The number of superdiagonals within the band of A. KU >= 0.

AB (input/output) Real array of dimension (LDAB, N).
On entry, the matrix A in band storage, in rows KL+1 to 2*KL+KU+1; rows 1 to KL of the array need not be set. The j-th column of A is stored in the j-th column of the array AB as follows: AB(KL+KU+1+I-J,J) = A(I,J) for MAX(1,J-KU)<=I<=MIN(M,J+KL)

On exit, details of the factorization: U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1. See below for further details.

LDAB (input) Integer
The leading dimension of the array AB. LDAB >= 2*KL+KU+1.

IPIVOT (output) Integer array of dimension MIN(M,N)
The pivot indices; for 1 <= I <= MIN(M,N), row I of the matrix was interchanged with row IPIVOT(I).

INFO (output) Integer
= 0: successful exit
< 0: if INFO = -I, the I-th argument had an illegal value
> 0: if INFO = +I, U(I,I) is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when M = N = 6, KL = 2, KU = 1:

```
On entry:                       On exit:

    *    *    *    +    +    +       *    *    *   u14   u25
u36
    *    *    +    +    +    +       *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
   a21  a32  a43  a54  a65   *      m21  m32  m43  m54   m65
*
   a31  a42  a53  a64   *    *      m31  m42  m53   m64    *
*
```

Array elements marked * are not used by the routine; elements marked + need not be set on entry, but are required by the routine to store elements of U because of fill-in resulting from the row interchanges.

Contents

NAME

sgbtrs - solve a system of linear equations A * X = B or A' * X = B with a general band matrix A using the LU factorization computed by SGBTRF

SYNOPSIS

```
SUBROUTINE SGBTRS(TRANSA, N, NSUB, NSUPER, NRHS, A, LDA, IPIVOT, B,
      LDB, INFO)

CHARACTER * 1 TRANSA
INTEGER N, NSUB, NSUPER, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)
REAL A(LDA,*), B(LDB,*)

SUBROUTINE SGBTRS_64(TRANSA, N, NSUB, NSUPER, NRHS, A, LDA, IPIVOT,
      B, LDB, INFO)

CHARACTER * 1 TRANSA
INTEGER*8 N, NSUB, NSUPER, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
REAL A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE GBTRS([TRANSA], [N], NSUB, NSUPER, [NRHS], A, [LDA],
       IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: TRANSA
INTEGER :: N, NSUB, NSUPER, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE GBTRS_64([TRANSA], [N], NSUB, NSUPER, [NRHS], A, [LDA],
```

```
          IPIVOT, B, [LDB], [INFO])

    CHARACTER(LEN=1) :: TRANSA
    INTEGER(8) :: N, NSUB, NSUPER, NRHS, LDA, LDB, INFO
    INTEGER(8), DIMENSION(:) :: IPIVOT
    REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
    #include <sunperf.h>

    void sgbtrs(char transa, int n, int nsub,  int  nsuper,  int
              nrhs,  float  *a,  int lda, int *ipivot, float *b,
              int ldb, int *info);
    void sgbtrs_64(char transa, long n, long nsub, long  nsuper,
              long nrhs, float *a, long lda, long *ipivot, float
              *b, long ldb, long *info);
```

PURPOSE

```
    sgbtrs solves a system of linear equations
       A * X = B  or  A' * X = B with a general  band  matrix  A
    using the LU factorization computed by SGBTRF.
```

ARGUMENTS

```
    TRANSA (input)
              Specifies the form of the system of equations.   =
              'N':  A * X = B  (No transpose)
              = 'T':  A'* X = B  (Transpose)
              = 'C':  A'* X = B  (Conjugate  transpose  =  Tran-
              spose)

              TRANSA is defaulted to 'N' for F95 INTERFACE.

    N (input) The order of the matrix A.  N >= 0.

    NSUB (input)
              The number of subdiagonals within the band  of  A.
              NSUB >= 0.

    NSUPER (input)
              The number of superdiagonals within the band of A.
              NSUPER >= 0.

    NRHS (input)
              The number of right hand sides, i.e.,  the  number
              of columns of the matrix B.  NRHS >= 0.
```

A (input) Details of the LU factorization of the band matrix A, as computed by SGBTRF. U is stored as an upper triangular band matrix with NSUB+NSUPER superdiag- onals in rows 1 to NSUB+NSUPER+1, and the multi- pliers used during the factorization are stored in rows NSUB+NSUPER+2 to 2*NSUB+NSUPER+1.

LDA (input)
The leading dimension of the array A. LDA >= 2*NSUB+NSUPER+1.

IPIVOT (input)
The pivot indices; for 1 <= i <= N, row i of the matrix was interchanged with row IPIVOT(i).

B (input/output)
On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille- gal value

Contents

NAME

```
sgebak - form the right or left eigenvectors of a real  gen-
eral  matrix  by  backward  transformation  on  the computed
eigenvectors of the balanced matrix output by SGEBAL
```

SYNOPSIS

```
SUBROUTINE SGEBAK(JOB, SIDE, N, ILO, IHI, SCALE, M, V, LDV, INFO)

CHARACTER * 1 JOB, SIDE
INTEGER N, ILO, IHI, M, LDV, INFO
REAL SCALE(*), V(LDV,*)

SUBROUTINE SGEBAK_64(JOB, SIDE, N, ILO, IHI, SCALE, M, V, LDV, INFO)

CHARACTER * 1 JOB, SIDE
INTEGER*8 N, ILO, IHI, M, LDV, INFO
REAL SCALE(*), V(LDV,*)
```

F95 INTERFACE

```
SUBROUTINE GEBAK(JOB, SIDE, [N], ILO, IHI, SCALE, [M], V, [LDV],
       [INFO])

CHARACTER(LEN=1) :: JOB, SIDE
INTEGER :: N, ILO, IHI, M, LDV, INFO
REAL, DIMENSION(:) :: SCALE
REAL, DIMENSION(:,:) :: V

SUBROUTINE GEBAK_64(JOB, SIDE, [N], ILO, IHI, SCALE, [M], V, [LDV],
       [INFO])

CHARACTER(LEN=1) :: JOB, SIDE
INTEGER(8) :: N, ILO, IHI, M, LDV, INFO
```

```
REAL, DIMENSION(:) :: SCALE
REAL, DIMENSION(:,:) :: V
```

C INTERFACE

```
#include <sunperf.h>

void sgebak(char job, char side, int n, int  ilo,  int  ihi,
          float  *scale,  int  m,  float  *v,  int  ldv, int
          *info);

void sgebak_64(char job, char side, long n, long  ilo,  long
          ihi,  float  *scale,  long  m, float *v, long ldv,
          long *info);
```

PURPOSE

sgebak forms the right or left eigenvectors of a real general matrix by backward transformation on the computed eigenvectors of the balanced matrix output by SGEBAL.

ARGUMENTS

JOB (input)
Specifies the type of backward transformation required: = 'N', do nothing, return immediately; = 'P', do backward transformation for permutation only; = 'S', do backward transformation for scaling only; = 'B', do backward transformations for both permutation and scaling. JOB must be the same as the argument JOB supplied to SGEBAL.

SIDE (input)
= 'R': V contains right eigenvectors;
= 'L': V contains left eigenvectors.

N (input) The number of rows of the matrix V. N >= 0.

ILO (input)
The integers ILO and IHI determined by SGEBAL. 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

IHI (input)
See the description for ILO.

SCALE (input)
Details of the permutation and scaling factors, as

returned by SGEBAL.

M (input) The number of columns of the matrix V. M >= 0.

V (input/output)
On entry, the matrix of right or left eigenvectors to be transformed, as returned by SHSEIN or STREVC. On exit, V is overwritten by the transformed eigenvectors.

LDV (input)
The leading dimension of the array V. LDV >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

```
sgebal - balance a general real matrix A
```

SYNOPSIS

```
SUBROUTINE SGEBAL(JOB, N, A, LDA, ILO, IHI, SCALE, INFO)

CHARACTER * 1 JOB
INTEGER N, LDA, ILO, IHI, INFO
REAL A(LDA,*), SCALE(*)

SUBROUTINE SGEBAL_64(JOB, N, A, LDA, ILO, IHI, SCALE, INFO)

CHARACTER * 1 JOB
INTEGER*8 N, LDA, ILO, IHI, INFO
REAL A(LDA,*), SCALE(*)
```

F95 INTERFACE

```
SUBROUTINE GEBAL(JOB, [N], A, [LDA], ILO, IHI, SCALE, [INFO])

CHARACTER(LEN=1) :: JOB
INTEGER :: N, LDA, ILO, IHI, INFO
REAL, DIMENSION(:) :: SCALE
REAL, DIMENSION(:,:) :: A

SUBROUTINE GEBAL_64(JOB, [N], A, [LDA], ILO, IHI, SCALE, [INFO])

CHARACTER(LEN=1) :: JOB
INTEGER(8) :: N, LDA, ILO, IHI, INFO
REAL, DIMENSION(:) :: SCALE
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void sgebal(char job, int n, float *a, int  lda,  int  *ilo,
          int *ihi, float *scale, int *info);

void sgebal_64(char job, long n, float *a,  long  lda,  long
          *ilo, long *ihi, float *scale, long *info);
```

PURPOSE

sgebal balances a general real matrix A. This involves, first, permuting A by a similarity transformation to isolate eigenvalues in the first 1 to ILO-1 and last IHI+1 to N elements on the diagonal; and second, applying a diagonal similarity transformation to rows and columns ILO to IHI to make the rows and columns as close in norm as possible. Both steps are optional.

Balancing may reduce the 1-norm of the matrix, and improve the accuracy of the computed eigenvalues and/or eigenvectors.

ARGUMENTS

JOB (input)
Specifies the operations to be performed on A:
= 'N': none: simply set ILO = 1, IHI = N, SCALE(I) = 1.0 for i = 1,...,N; = 'P': permute only;
= 'S': scale only;
= 'B': both permute and scale.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the input matrix A. On exit, A is overwritten by the balanced matrix. If JOB = 'N', A is not referenced. See Further Details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

ILO (output)
ILO and IHI are set to integers such that on exit A(i,j) = 0 if i > j and j = 1,...,ILO-1 or I =

IHI+1,...,N. If JOB = 'N' or 'S', ILO = 1 and IHI = N.

IHI (output)
See the description for ILO.

SCALE (output)
Details of the permutations and scaling factors applied to A. If P(j) is the index of the row and column interchanged with row and column j and D(j) is the scaling factor applied to row and column j, then SCALE(j) = P(j) for j = 1,...,ILO-1 = D(j) for j = ILO,...,IHI = P(j) for j = IHI+1,...,N. The order in which the interchanges are made is N to IHI+1, then 1 to ILO-1.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

The permutations consist of row and column interchanges which put the matrix in the form

```
          ( T1   X   Y  )
  P A P = (  0   B   Z  )
          (  0   0   T2 )
```

where T1 and T2 are upper triangular matrices whose eigenvalues lie along the diagonal. The column indices ILO and IHI mark the starting and ending columns of the submatrix B. Balancing consists of applying a diagonal similarity transformation inv(D) * B * D to make the 1-norms of each row of B and its corresponding column nearly equal. The output matrix is

```
  ( T1     X*D          Y    )
  (  0  inv(D)*B*D  inv(D)*Z ).
  (  0      0           T2   )
```

Information about the permutations P and the diagonal matrix D is returned in the vector SCALE.

This subroutine is based on the EISPACK routine BALANC.

Modified by Tzu-Yi Chen, Computer Science Division, University of
California at Berkeley, USA

Contents

NAME

```
sgebrd - reduce a general real M-by-N matrix A to  upper  or
lower bidiagonal form B by an orthogonal transformation
```

SYNOPSIS

```
SUBROUTINE SGEBRD(M, N, A, LDA, D, E, TAUQ, TAUP, WORK, LWORK, INFO)

INTEGER M, N, LDA, LWORK, INFO
REAL A(LDA,*), D(*), E(*), TAUQ(*), TAUP(*), WORK(*)

SUBROUTINE SGEBRD_64(M, N, A, LDA, D, E, TAUQ, TAUP, WORK, LWORK,
      INFO)

INTEGER*8 M, N, LDA, LWORK, INFO
REAL A(LDA,*), D(*), E(*), TAUQ(*), TAUP(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEBRD([M], [N], A, [LDA], D, E, TAUQ, TAUP, [WORK], [LWORK],
       [INFO])

INTEGER :: M, N, LDA, LWORK, INFO
REAL, DIMENSION(:) :: D, E, TAUQ, TAUP, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE GEBRD_64([M], [N], A, [LDA], D, E, TAUQ, TAUP, [WORK],
       [LWORK], [INFO])

INTEGER(8) :: M, N, LDA, LWORK, INFO
REAL, DIMENSION(:) :: D, E, TAUQ, TAUP, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void sgebrd(int m, int n, float *a, int lda, float *d, float
          *e, float *tauq, float *taup, int *info);

void sgebrd_64(long m, long n, float *a, long lda, float *d,
          float *e, float *tauq, float *taup, long *info);
```

PURPOSE

sgebrd reduces a general real M-by-N matrix A to upper or lower bidiagonal form B by an orthogonal transformation: Q**T * A * P = B.

If m >= n, B is upper bidiagonal; if m < n, B is lower bidiagonal.

ARGUMENTS

M (input) The number of rows in the matrix A. M >= 0.

N (input) The number of columns in the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N general matrix to be reduced. On exit, if m >= n, the diagonal and the first superdiagonal are overwritten with the upper bidiagonal matrix B; the elements below the diagonal, with the array TAUQ, represent the orthogonal matrix Q as a product of elementary reflectors, and the elements above the first superdiagonal, with the array TAUP, represent the orthogonal matrix P as a product of elementary reflectors; if m < n, the diagonal and the first subdiagonal are overwritten with the lower bidiagonal matrix B; the elements below the first subdiagonal, with the array TAUQ, represent the orthogonal matrix Q as a product of elementary reflectors, and the elements above the diagonal, with the array TAUP, represent the orthogonal matrix P as a product of elementary reflectors. See Further Details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

D (output)

The diagonal elements of the bidiagonal matrix B: D(i) = A(i,i).

E (output)

The off-diagonal elements of the bidiagonal matrix B: if m >= n, E(i) = A(i,i+1) for i = 1,2,...,n-1; if m < n, E(i) = A(i+1,i) for i = 1,2,...,m-1.

TAUQ (output)

The scalar factors of the elementary reflectors which represent the orthogonal matrix Q. See Further Details.

TAUP (output)

The scalar factors of the elementary reflectors which represent the orthogonal matrix P. See Further Details.

WORK (workspace)

On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)

The length of the array WORK. LWORK >= max(1,M,N). For optimum performance LWORK >= (M+N)*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

The matrices Q and P are represented as products of elementary reflectors:

If m >= n,

```
   Q = H(1) H(2) . . . H(n)  and  P = G(1) G(2) . . . G(n-1)
```

Each H(i) and G(i) has the form:

```
   H(i) = I - tauq * v * v'  and G(i) = I - taup * u * u'
```

```
where tauq and taup are real scalars, and v and u  are  real
vectors;  v(1:i-1)  = 0, v(i) = 1, and v(i+1:m) is stored on
exit in A(i+1:m,i); u(1:i) = 0, u(i+1) = 1, and u(i+2:n)  is
stored  on exit in A(i,i+2:n); tauq is stored in TAUQ(i) and
taup in TAUP(i).

If m < n,

   Q = H(1) H(2) . . . H(m-1)  and  P = G(1) G(2) . . . G(m)

Each H(i) and G(i) has the form:
   H(i) = I - tauq * v * v'  and G(i) = I - taup * u * u'

where tauq and taup are real scalars, and v and u  are  real
vectors;  v(1:i)  = 0, v(i+1) = 1, and v(i+2:m) is stored on
exit in A(i+2:m,i); u(1:i-1) = 0, u(i) = 1, and u(i+1:n)  is
stored  on exit in A(i,i+1:n); tauq is stored in TAUQ(i) and
taup in TAUP(i).

The contents of A on exit are illustrated by  the  following
examples:

m = 6 and n = 5 (m > n):          m = 5 and n = 6 (m < n):

  (  d   e   u1  u1  u1 )           (  d   u1   u1   u1   u1
u1 )
  (  v1  d   e   u2  u2 )           (  e   d    u2   u2   u2
u2 )
  (  v1  v2  d   e   u3 )           (  v1  e    d    u3   u3
u3 )
  (  v1  v2  v3  d   e  )           (  v1  v2   e    d    u4
u4 )
  (  v1  v2  v3  v4  d  )           (  v1   v2   v3   e    d
u5 )
  (  v1  v2  v3  v4  v5 )

where d and e denote diagonal and off-diagonal  elements  of
B, vi denotes an element of the vector defining H(i), and ui
an element of the vector defining G(i).
```

Contents

NAME

```
sgecon - estimate the reciprocal of the condition number  of
a  general  real  matrix  A,  in  either  the  1-norm or the
infinity-norm, using the LU factorization computed by SGETRF
```

SYNOPSIS

```
SUBROUTINE SGECON(NORM, N, A, LDA, ANORM, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 NORM
INTEGER N, LDA, INFO
INTEGER WORK2(*)
REAL ANORM, RCOND
REAL A(LDA,*), WORK(*)

SUBROUTINE SGECON_64(NORM, N, A, LDA, ANORM, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 NORM
INTEGER*8 N, LDA, INFO
INTEGER*8 WORK2(*)
REAL ANORM, RCOND
REAL A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GECON(NORM, [N], A, [LDA], ANORM, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: NORM
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: WORK
```

```
REAL, DIMENSION(:,:) :: A

SUBROUTINE GECON_64(NORM, [N], A, [LDA], ANORM, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: NORM
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
void sgecon(char norm, int  n,  float  *a,  int  lda,  float
          anorm, float *rcond, int *info);

void sgecon_64(char norm, long n, float *a, long lda,  float
          anorm, float *rcond, long *info);
```

PURPOSE

sgecon estimates the reciprocal of the condition number of a general real matrix A, in either the 1-norm or the infinity-norm, using the LU factorization computed by SGETRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as
   RCOND = 1 / (norm(A) * norm(inv(A))).

ARGUMENTS

NORM (input)
Specifies whether the 1-norm condition number or the infinity-norm condition number is required:
= '1' or 'O': 1-norm;
= 'I':   Infinity-norm.

N (input) The order of the matrix A. N >= 0.

A (input) The factors L and U from the factorization A = P*L*U as computed by SGETRF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

ANORM (input)
If NORM = '1' or 'O', the 1-norm of the original matrix A. If NORM = 'I', the infinity-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(norm(A) * norm(inv(A))).

WORK (workspace)
dimension(4*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

sgeequ - compute row and column scalings intended to equilibrate an M-by-N matrix A and reduce its condition number

SYNOPSIS

```
SUBROUTINE SGEEQU(M, N, A, LDA, R, C, ROWCN, COLCN, AMAX,
      INFO)

INTEGER M, N, LDA, INFO
REAL ROWCN, COLCN, AMAX
REAL A(LDA,*), R(*), C(*)

SUBROUTINE SGEEQU_64(M, N, A, LDA, R, C, ROWCN, COLCN, AMAX,
      INFO)

INTEGER*8 M, N, LDA, INFO
REAL ROWCN, COLCN, AMAX
REAL A(LDA,*), R(*), C(*)
```

F95 INTERFACE

```
SUBROUTINE GEEQU([M], [N], A, [LDA], R, C, ROWCN, COLCN,
       AMAX, [INFO])

INTEGER :: M, N, LDA, INFO
REAL :: ROWCN, COLCN, AMAX
REAL, DIMENSION(:) :: R, C
REAL, DIMENSION(:,:) :: A

SUBROUTINE GEEQU_64([M], [N], A, [LDA], R, C, ROWCN, COLCN,
       AMAX, [INFO])

INTEGER(8) :: M, N, LDA, INFO
```

```
REAL :: ROWCN, COLCN, AMAX
REAL, DIMENSION(:) :: R, C
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void sgeequ(int m, int n, float *a, int lda, float *r, float
          *c,  float  *rowcn, float *colcn, float *amax, int
          *info);

void sgeequ_64(long m, long n, float *a, long lda, float *r,
          float *c, float *rowcn, float *colcn, float *amax,
          long *info);
```

PURPOSE

sgeequ computes row and column scalings intended to equilibrate an M-by-N matrix A and reduce its condition number. R returns the row scale factors and C the column scale factors, chosen to try to make the largest element in each row and column of the matrix B with elements B(i,j)=R(i)*A(i,j)*C(j) have absolute value 1.

R(i) and C(j) are restricted to be between SMLNUM = smallest safe number and BIGNUM = largest safe number. Use of these scaling factors is not guaranteed to reduce the condition number of A but works well in practice.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input) The M-by-N matrix whose equilibration factors are to be computed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

R (output)
If INFO = 0 or INFO > M, R contains the row scale factors for A.

C (output)

If INFO = 0, C contains the column scale factors for A.

ROWCN (output)
If INFO = 0 or INFO > M, ROWCN contains the ratio of the smallest R(i) to the largest R(i). If ROWCN >= 0.1 and AMAX is neither too large nor too small, it is not worth scaling by R.

COLCN (output)
If INFO = 0, COLCN contains the ratio of the smallest C(i) to the largest C(i). If COLCN >= 0.1, it is not worth scaling by C.

AMAX (output)
Absolute value of largest matrix element. If AMAX is very close to overflow or very close to underflow, the matrix should be scaled.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= M: the i-th row of A is exactly zero
> M: the (i-M)-th column of A is exactly zero

Contents

NAME

```
sgees - compute for an N-by-N real  nonsymmetric  matrix  A,
the eigenvalues, the real Schur form T, and, optionally, the
matrix of Schur vectors Z
```

SYNOPSIS

```
SUBROUTINE SGEES(JOBZ, SORTEV, SELECT, N, A, LDA, NOUT, WR, WI, Z,
      LDZ, WORK, LDWORK, WORK3, INFO)

CHARACTER * 1 JOBZ, SORTEV
INTEGER N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL SELECT
LOGICAL WORK3(*)
REAL A(LDA,*), WR(*), WI(*), Z(LDZ,*), WORK(*)

SUBROUTINE SGEES_64(JOBZ, SORTEV, SELECT, N, A, LDA, NOUT, WR, WI, Z,
      LDZ, WORK, LDWORK, WORK3, INFO)

CHARACTER * 1 JOBZ, SORTEV
INTEGER*8 N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL*8 SELECT
LOGICAL*8 WORK3(*)
REAL A(LDA,*), WR(*), WI(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEES(JOBZ, SORTEV, SELECT, [N], A, [LDA], NOUT, WR, WI, Z,
       [LDZ], [WORK], [LDWORK], [WORK3], [INFO])

CHARACTER(LEN=1) :: JOBZ, SORTEV
INTEGER :: N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL :: SELECT
LOGICAL, DIMENSION(:) :: WORK3
```

```
REAL, DIMENSION(:) :: WR, WI, WORK
REAL, DIMENSION(:,:) :: A, Z

SUBROUTINE GEES_64(JOBZ, SORTEV, SELECT, [N], A, [LDA], NOUT, WR, WI,
       Z, [LDZ], [WORK], [LDWORK], [WORK3], [INFO])

CHARACTER(LEN=1) :: JOBZ, SORTEV
INTEGER(8) :: N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL(8) :: SELECT
LOGICAL(8), DIMENSION(:) :: WORK3
REAL, DIMENSION(:) :: WR, WI, WORK
REAL, DIMENSION(:,:) :: A, Z
```

C INTERFACE

```
#include <sunperf.h>
void       sgees(char       jobz,        char        sortev,
          int(*select)(float,float),  int  n,  float *a, int
          lda, int *nout, float *wr, float  *wi,  float  *z,
          int ldz, int *info);

void      sgees_64(char       jobz,       char       sortev,
          long(*select)(float,float), long n, float *a, long
          lda, long *nout, float *wr, float *wi,  float  *z,
          long ldz, long *info);
```

PURPOSE

sgees computes for an N-by-N real nonsymmetric matrix A, the eigenvalues, the real Schur form T, and, optionally, the matrix of Schur vectors Z. This gives the Schur factorization A = Z*T*(Z**T).

Optionally, it also orders the eigenvalues on the diagonal of the real Schur form so that selected eigenvalues are at the top left. The leading columns of Z then form an orthonormal basis for the invariant subspace corresponding to the selected eigenvalues.

A matrix is in real Schur form if it is upper quasi-triangular with 1-by-1 and 2-by-2 blocks. 2-by-2 blocks will be standardized in the form

```
        [  a  b  ]
        [  c  a  ]
```

where b*c < 0. The eigenvalues of such a block are a +- sqrt(bc).

ARGUMENTS

JOBZ (input)
= 'N': Schur vectors are not computed;
= 'V': Schur vectors are computed.

SORTEV (input)
Specifies whether or not to order the eigenvalues on the diagonal of the Schur form. = 'N': Eigenvalues are not ordered;
= 'S': Eigenvalues are ordered (see SELECT).

SELECT (input)
SELECT must be declared EXTERNAL in the calling subroutine. If SORTEV = 'S', SELECT is used to select eigenvalues to sort to the top left of the Schur form. If SORTEV = 'N', SELECT is not referenced. An eigenvalue WR(j)+sqrt(-1)*WI(j) is selected if SELECT(WR(j),WI(j)) is true; i.e., if either one of a complex conjugate pair of eigenvalues is selected, then both complex eigenvalues are selected. Note that a selected complex eigenvalue may no longer satisfy SELECT(WR(j),WI(j)) = .TRUE. after ordering, since ordering may change the value of complex eigenvalues (especially if the eigenvalue is ill-conditioned); in this case INFO is set to N+2 (see INFO below).

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the N-by-N matrix A. On exit, A has been overwritten by its real Schur form T.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

NOUT (output)
If SORTEV = 'N', NOUT = 0. If SORTEV = 'S', NOUT = number of eigenvalues (after sorting) for which SELECT is true. (Complex conjugate pairs for which SELECT is true for either eigenvalue count as 2.)

WR (output)
WR and WI contain the real and imaginary parts, respectively, of the computed eigenvalues in the same order that they appear on the diagonal of the output Schur form T. Complex conjugate pairs of eigenvalues will appear consecutively with the

eigenvalue having the positive imaginary part first.

WI (output)

See the description for WR.

Z (output)

If JOBZ = 'V', Z contains the orthogonal matrix Z of Schur vectors. If JOBZ = 'N', Z is not referenced.

LDZ (input)

The leading dimension of the array Z. LDZ >= 1; if JOBZ = 'V', LDZ >= N.

WORK (workspace)

On exit, if INFO = 0, WORK(1) contains the optimal LDWORK.

LDWORK (input)

The dimension of the array WORK. LDWORK >= max(1,3*N). For good performance, LDWORK must generally be larger.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK3 (workspace)

dimension(N) Not referenced if SORTEV = 'N'.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value.

> 0: if INFO = i, and i is

<= N: the QR algorithm failed to compute all the eigenvalues; elements 1:ILO-1 and i+1:N of WR and WI contain those eigenvalues which have converged; if JOBZ = 'V', Z contains the matrix which reduces A to its partially converged Schur form. = N+1: the eigenvalues could not be reordered because some eigenvalues were too close to separate (the problem is very ill-conditioned); = N+2: after reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the Schur form no longer satisfy SELECT=.TRUE. This could also be caused by underflow due to scaling.

Contents

NAME

```
sgeesx - compute for an N-by-N real nonsymmetric  matrix  A,
the eigenvalues, the real Schur form T, and, optionally, the
matrix of Schur vectors Z
```

SYNOPSIS

```
SUBROUTINE SGEESX(JOBZ, SORTEV, SELECT, SENSE, N, A, LDA, NOUT, WR,
      WI, Z, LDZ, SRCONE, RCONV, WORK, LDWORK, IWORK2, LDWRK2, BWORK3,
      INFO)

CHARACTER * 1 JOBZ, SORTEV, SENSE
INTEGER N, LDA, NOUT, LDZ, LDWORK, LDWRK2, INFO
INTEGER IWORK2(*)
LOGICAL SELECT
LOGICAL BWORK3(*)
REAL SRCONE, RCONV
REAL A(LDA,*), WR(*), WI(*), Z(LDZ,*), WORK(*)

SUBROUTINE SGEESX_64(JOBZ, SORTEV, SELECT, SENSE, N, A, LDA, NOUT,
      WR, WI, Z, LDZ, SRCONE, RCONV, WORK, LDWORK, IWORK2, LDWRK2,
      BWORK3, INFO)

CHARACTER * 1 JOBZ, SORTEV, SENSE
INTEGER*8 N, LDA, NOUT, LDZ, LDWORK, LDWRK2, INFO
INTEGER*8 IWORK2(*)
LOGICAL*8 SELECT
LOGICAL*8 BWORK3(*)
REAL SRCONE, RCONV
REAL A(LDA,*), WR(*), WI(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEESX(JOBZ, SORTEV, SELECT, SENSE, [N], A, [LDA], NOUT,
```

```
       WR, WI, Z, [LDZ], SRCONE, RCONV, [WORK], [LDWORK], [IWORK2],
       [LDWRK2], [BWORK3], [INFO])

CHARACTER(LEN=1) :: JOBZ, SORTEV, SENSE
INTEGER :: N, LDA, NOUT, LDZ, LDWORK, LDWRK2, INFO
INTEGER, DIMENSION(:) :: IWORK2
LOGICAL :: SELECT
LOGICAL, DIMENSION(:) :: BWORK3
REAL :: SRCONE, RCONV
REAL, DIMENSION(:) :: WR, WI, WORK
REAL, DIMENSION(:,:) :: A, Z

SUBROUTINE GEESX_64(JOBZ, SORTEV, SELECT, SENSE, [N], A, [LDA], NOUT,
       WR, WI, Z, [LDZ], SRCONE, RCONV, [WORK], [LDWORK], [IWORK2],
       [LDWRK2], [BWORK3], [INFO])

CHARACTER(LEN=1) :: JOBZ, SORTEV, SENSE
INTEGER(8) :: N, LDA, NOUT, LDZ, LDWORK, LDWRK2, INFO
INTEGER(8), DIMENSION(:) :: IWORK2
LOGICAL(8) :: SELECT
LOGICAL(8), DIMENSION(:) :: BWORK3
REAL :: SRCONE, RCONV
REAL, DIMENSION(:) :: WR, WI, WORK
REAL, DIMENSION(:,:) :: A, Z
```

C INTERFACE

```
#include <sunperf.h>

void      sgeesx(char       jobz,       char        sortev,
          int(*select)(float,float),   char  sense,  int  n,
          float *a, int lda, int  *nout,  float  *wr,  float
          *wi,  float  *z,  int  ldz,  float  *srcone, float
          *rconv, int *info);

void      sgeesx_64(char      jobz,       char       sortev,
          long(*select)(float,float),  char  sense,  long n,
          float *a, long lda, long *nout, float  *wr,  float
          *wi,  float  *z,  long  ldz,  float *srcone, float
          *rconv, long *info);
```

PURPOSE

sgeesx computes for an N-by-N real nonsymmetric matrix A, the eigenvalues, the real Schur form T, and, optionally, the matrix of Schur vectors Z. This gives the Schur factorization A = Z*T*(Z**T).

Optionally, it also orders the eigenvalues on the diagonal of the real Schur form so that selected eigenvalues are at the top left; computes a reciprocal condition number for the average of the selected eigenvalues (RCONDE); and computes a reciprocal condition number for the right invariant subspace corresponding to the selected eigenvalues (RCONDV). The leading columns of Z form an orthonormal basis for this invariant subspace.

For further explanation of the reciprocal condition numbers RCONDE and RCONDV, see Section 4.10 of the LAPACK Users' Guide (where these quantities are called s and sep respectively).

A real matrix is in real Schur form if it is upper quasi-triangular with 1-by-1 and 2-by-2 blocks. 2-by-2 blocks will be standardized in the form

```
          [  a  b  ]
          [  c  a  ]
```

where b*c < 0. The eigenvalues of such a block are a +- sqrt(bc).

ARGUMENTS

JOBZ (input)
  = 'N': Schur vectors are not computed;
  = 'V': Schur vectors are computed.

SORTEV (input)
  Specifies whether or not to order the eigenvalues on the diagonal of the Schur form. = 'N': Eigenvalues are not ordered;
  = 'S': Eigenvalues are ordered (see SELECT).

SELECT (input)
  SELECT must be declared EXTERNAL in the calling subroutine. If SORTEV = 'S', SELECT is used to select eigenvalues to sort to the top left of the Schur form. If SORTEV = 'N', SELECT is not referenced. An eigenvalue WR(j)+sqrt(-1)*WI(j) is selected if SELECT(WR(j),WI(j)) is true; i.e., if either one of a complex conjugate pair of eigenvalues is selected, then both are. Note that a selected complex eigenvalue may no longer satisfy SELECT(WR(j),WI(j)) = .TRUE. after ordering, since ordering may change the value of complex eigenvalues (especially if the eigenvalue is ill-conditioned); in this case INFO may be set to N+3 (see INFO below).

```
SENSE (input)
          Determines which reciprocal condition numbers  are
          computed.  = 'N': None are computed;
          = 'E': Computed for  average  of  selected  eigen-
          values only;
          = 'V': Computed for selected right invariant  sub-
          space only;
          = 'B': Computed for both.  If SENSE = 'E', 'V'  or
          'B', SORTEV must equal 'S'.

N (input) The order of the matrix A. N >= 0.

A (input/output)
          On entry, the N-by-N matrix  A.   On  exit,  A  is
          overwritten by its real Schur form T.
LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

NOUT (output)
          If SORTEV = 'N', NOUT = 0.  If SORTEV = 'S',  NOUT
          =  number of eigenvalues (after sorting) for which
          SELECT is true. (Complex conjugate pairs for which
          SELECT is true for either eigenvalue count as 2.)

WR (output)
          WR and WI contain the real  and  imaginary  parts,
          respectively,  of the computed eigenvalues, in the
          same order that they appear on the diagonal of the
          output  Schur  form T.  Complex conjugate pairs of
          eigenvalues appear consecutively with  the  eigen-
          value having the positive imaginary part first.

WI (output)
          See the description for WR.

Z (output)
          If JOBZ = 'V', Z contains the orthogonal matrix  Z
          of  Schur vectors.  If JOBZ = 'N', Z is not refer-
          enced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= N.

SRCONE (output)
          If  SENSE  =  'E'  or  'B',  SRCONE  contains  the
          reciprocal condition number for the average of the
          selected eigenvalues.  Not referenced if  SENSE  =
          'N' or 'V'.
```

```
RCONV (output)
         If SENSE = 'V' or 'B', RCONV contains the recipro-
         cal   condition  number  for  the  selected  right
         invariant subspace.  Not referenced if SENSE = 'N'
         or 'E'.

WORK (workspace)
         On exit, if INFO = 0, WORK(1) returns the  optimal
         LDWORK.

LDWORK (input)
         The  dimension  of  the  array  WORK.   LDWORK  >=
         max(1,3*N).   Also,  if SENSE = 'E' or 'V' or 'B',
         LDWORK >= N+2*NOUT*(N-NOUT),  where  NOUT  is  the
         number  of  selected  eigenvalues computed by this
         routine.  Note that N+2*NOUT*(N-NOUT) <=  N+N*N/2.
         For  good  performance,  LDWORK  must generally be
         larger.

IWORK2 (workspace/output)
         Not referenced if SENSE = 'N' or 'E'.  On exit, if
         INFO = 0, IWORK2(1) returns the optimal LDWRK2.

LDWRK2 (input)
         The dimension of the array IWORK2.  LDWRK2  >=  1;
         if SENSE = 'V' or 'B', LDWRK2 >= NOUT*(N-NOUT).

BWORK3 (workspace)
         dimension(N) Not referenced if SORTEV = 'N'.

INFO (output)
         = 0: successful exit
         < 0: if INFO = -i, the i-th argument had an  ille-
         gal value.
         > 0: if INFO = i, and i is
         <= N: the QR algorithm failed to compute all the
         eigenvalues; elements 1:ILO-1 and i+1:N of WR  and
         WI contain those eigenvalues which have converged;
         if JOBZ = 'V', Z contains the transformation which
         reduces  A  to its partially converged Schur form.
         = N+1: the  eigenvalues  could  not  be  reordered
         because   some   eigenvalues  were  too  close  to
         separate (the problem is very ill-conditioned);  =
         N+2:  after reordering, roundoff changed values of
         some complex eigenvalues so  that  leading  eigen-
         values   in  the  Schur  form  no  longer  satisfy
         SELECT=.TRUE.  This could also be caused by under-
         flow due to scaling.
```

Contents

NAME

```
sgeev - compute for an N-by-N real  nonsymmetric  matrix  A,
the  eigenvalues  and,  optionally,  the  left  and/or right
eigenvectors
```

SYNOPSIS

```
SUBROUTINE SGEEV(JOBVL, JOBVR, N, A, LDA, WR, WI, VL, LDVL, VR, LDVR,
      WORK, LDWORK, INFO)

CHARACTER * 1 JOBVL, JOBVR
INTEGER N, LDA, LDVL, LDVR, LDWORK, INFO
REAL A(LDA,*), WR(*), WI(*), VL(LDVL,*), VR(LDVR,*), WORK(*)

SUBROUTINE SGEEV_64(JOBVL, JOBVR, N, A, LDA, WR, WI, VL, LDVL, VR,
      LDVR, WORK, LDWORK, INFO)

CHARACTER * 1 JOBVL, JOBVR
INTEGER*8 N, LDA, LDVL, LDVR, LDWORK, INFO
REAL A(LDA,*), WR(*), WI(*), VL(LDVL,*), VR(LDVR,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEEV(JOBVL, JOBVR, [N], A, [LDA], WR, WI, VL, [LDVL], VR,
       [LDVR], [WORK], [LDWORK], [INFO])

CHARACTER(LEN=1) :: JOBVL, JOBVR
INTEGER :: N, LDA, LDVL, LDVR, LDWORK, INFO
REAL, DIMENSION(:) :: WR, WI, WORK
REAL, DIMENSION(:,:) :: A, VL, VR

SUBROUTINE GEEV_64(JOBVL, JOBVR, [N], A, [LDA], WR, WI, VL, [LDVL],
       VR, [LDVR], [WORK], [LDWORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOBVL, JOBVR
INTEGER(8) :: N, LDA, LDVL, LDVR, LDWORK, INFO
REAL, DIMENSION(:) :: WR, WI, WORK
REAL, DIMENSION(:,:) :: A, VL, VR
```

C INTERFACE

```
#include <sunperf.h>

void sgeev(char jobvl, char jobvr, int n, float *a, int lda,
          float  *wr,  float *wi, float *vl, int ldvl, float
          *vr, int ldvr, int *info);

void sgeev_64(char jobvl, char jobvr, long n, float *a, long
          lda,  float  *wr, float *wi, float *vl, long ldvl,
          float *vr, long ldvr, long *info);
```

PURPOSE

sgeev computes for an N-by-N real nonsymmetric matrix A, the eigenvalues and, optionally, the left and/or right eigenvectors.

```
The right eigenvector v(j) of A satisfies
                 A * v(j) = lambda(j) * v(j)
where lambda(j) is its eigenvalue.
The left eigenvector u(j) of A satisfies
              u(j)**H * A = lambda(j) * u(j)**H
where u(j)**H denotes the conjugate transpose of u(j).
```

The computed eigenvectors are normalized to have Euclidean norm equal to 1 and largest component real.

ARGUMENTS

JOBVL (input)
= 'N': left eigenvectors of A are not computed;
= 'V': left eigenvectors of A are computed.

JOBVR (input)
= 'N': right eigenvectors of A are not computed;
= 'V': right eigenvectors of A are computed.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the N-by-N matrix A. On exit, A has been overwritten.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

WR (output)
WR and WI contain the real and imaginary parts, respectively, of the computed eigenvalues. Complex conjugate pairs of eigenvalues appear consecutively with the eigenvalue having the positive imaginary part first.

WI (output)
See the description for WR.

VL (output)
If JOBVL = 'V', the left eigenvectors u(j) are stored one after another in the columns of VL, in the same order as their eigenvalues. If JOBVL = 'N', VL is not referenced. If the j-th eigenvalue is real, then u(j) = VL(:,j), the j-th column of VL. If the j-th and (j+1)-st eigenvalues form a complex conjugate pair, then u(j) = VL(:,j) + i*VL(:,j+1) and
u(j+1) = VL(:,j) - i*VL(:,j+1).

LDVL (input)
The leading dimension of the array VL. LDVL >= 1; if JOBVL = 'V', LDVL >= N.

VR (input)
If JOBVR = 'V', the right eigenvectors v(j) are stored one after another in the columns of VR, in the same order as their eigenvalues. If JOBVR = 'N', VR is not referenced. If the j-th eigenvalue is real, then v(j) = VR(:,j), the j-th column of VR. If the j-th and (j+1)-st eigenvalues form a complex conjugate pair, then v(j) = VR(:,j) + i*VR(:,j+1) and
v(j+1) = VR(:,j) - i*VR(:,j+1).

LDVR (input)
The leading dimension of the array VR. LDVR >= 1; if JOBVR = 'V', LDVR >= N.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >=

max(1,3*N), and if JOBVL = 'V' or JOBVR = 'V', LDWORK >= 4*N. For good performance, LDWORK must generally be larger.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value.

> 0: if INFO = i, the QR algorithm failed to compute all the eigenvalues, and no eigenvectors have been computed; elements i+1:N of WR and WI contain eigenvalues which have converged.

Contents

NAME

```
sgeevx - compute for an N-by-N real nonsymmetric  matrix  A,
the  eigenvalues  and,  optionally,  the  left  and/or right
eigenvectors
```

SYNOPSIS

```
SUBROUTINE SGEEVX(BALANC, JOBVL, JOBVR, SENSE, N, A, LDA, WR, WI, VL,
      LDVL, VR, LDVR, ILO, IHI, SCALE, ABNRM, RCONE, RCONV, WORK,
      LDWORK, IWORK2, INFO)

CHARACTER * 1 BALANC, JOBVL, JOBVR, SENSE
INTEGER N, LDA, LDVL, LDVR, ILO, IHI, LDWORK, INFO
INTEGER IWORK2(*)
REAL ABNRM
REAL  A(LDA,*),  WR(*),   WI(*),   VL(LDVL,*),   VR(LDVR,*),
SCALE(*), RCONE(*), RCONV(*), WORK(*)

SUBROUTINE SGEEVX_64(BALANC, JOBVL, JOBVR, SENSE, N, A, LDA, WR, WI,
      VL, LDVL, VR, LDVR, ILO, IHI, SCALE, ABNRM, RCONE, RCONV, WORK,
      LDWORK, IWORK2, INFO)

CHARACTER * 1 BALANC, JOBVL, JOBVR, SENSE
INTEGER*8 N, LDA, LDVL, LDVR, ILO, IHI, LDWORK, INFO
INTEGER*8 IWORK2(*)
REAL ABNRM
REAL  A(LDA,*),  WR(*),   WI(*),   VL(LDVL,*),   VR(LDVR,*),
SCALE(*), RCONE(*), RCONV(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEEVX(BALANC, JOBVL, JOBVR, SENSE, [N], A, [LDA], WR, WI,
       VL, [LDVL], VR, [LDVR], ILO, IHI, SCALE, ABNRM, RCONE, RCONV,
       [WORK], [LDWORK], [IWORK2], [INFO])
```

```
CHARACTER(LEN=1) :: BALANC, JOBVL, JOBVR, SENSE
INTEGER :: N, LDA, LDVL, LDVR, ILO, IHI, LDWORK, INFO
INTEGER, DIMENSION(:) :: IWORK2
REAL :: ABNRM
REAL, DIMENSION(:) :: WR, WI, SCALE, RCONE, RCONV, WORK
REAL, DIMENSION(:,:) :: A, VL, VR

SUBROUTINE GEEVX_64(BALANC, JOBVL, JOBVR, SENSE, [N], A, [LDA], WR,
       WI, VL, [LDVL], VR, [LDVR], ILO, IHI, SCALE, ABNRM, RCONE, RCONV,
       [WORK], [LDWORK], [IWORK2], [INFO])

CHARACTER(LEN=1) :: BALANC, JOBVL, JOBVR, SENSE
INTEGER(8) :: N, LDA, LDVL, LDVR, ILO, IHI, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK2
REAL :: ABNRM
REAL, DIMENSION(:) :: WR, WI, SCALE, RCONE, RCONV, WORK
REAL, DIMENSION(:,:) :: A, VL, VR
```

C INTERFACE

```
#include <sunperf.h>

void sgeevx(char balanc, char jobvl, char jobvr, char sense,
          int  n,  float  *a, int lda, float *wr, float *wi,
          float *vl, int ldvl,  float  *vr,  int  ldvr,  int
          *ilo,  int *ihi, float *scale, float *abnrm, float
          *rcone, float *rconv, int *info);

void sgeevx_64(char balanc, char  jobvl,  char  jobvr,  char
          sense,  long  n,  float  *a,  long lda, float *wr,
          float *wi, float *vl, long ldvl, float  *vr,  long
          ldvr,  long  *ilo,  long *ihi, float *scale, float
          *abnrm, float *rcone, float *rconv, long *info);
```

PURPOSE

sgeevx computes for an N-by-N real nonsymmetric matrix A, the eigenvalues and, optionally, the left and/or right eigenvectors.

Optionally also, it computes a balancing transformation to improve the conditioning of the eigenvalues and eigenvectors (ILO, IHI, SCALE, and ABNRM), reciprocal condition numbers for the eigenvalues (RCONDE), and reciprocal condition numbers for the right
eigenvectors (RCONDV).

The right eigenvector v(j) of A satisfies

```
                A * v(j) = lambda(j) * v(j)
```

```
where lambda(j) is its eigenvalue.
The left eigenvector u(j) of A satisfies
              u(j)**H * A = lambda(j) * u(j)**H
where u(j)**H denotes the conjugate transpose of u(j).

The computed eigenvectors are normalized to  have  Euclidean
norm equal to 1 and largest component real.

Balancing a matrix means permuting the rows and  columns  to
make  it more nearly upper triangular, and applying a diago-
nal similarity transformation D * A * D**(-1), where D is  a
diagonal matrix, to make its rows and columns closer in norm
and the condition numbers of its eigenvalues  and  eigenvec-
tors  smaller.   The  computed  reciprocal condition numbers
correspond to  the  balanced  matrix.   Permuting  rows  and
columns  will  not  change  the  condition numbers (in exact
arithmetic) but diagonal scaling will.  For further explana-
tion  of  balancing, see section 4.10.2 of the LAPACK Users'
Guide.
```

ARGUMENTS

```
BALANC (input)
          Indicates how the input matrix  should  be  diago-
          nally scaled and/or permuted to improve the condi-
          tioning of its eigenvalues.  = 'N': Do not  diago-
          nally scale or permute;
          = 'P': Perform permutations  to  make  the  matrix
          more  nearly  upper  triangular. Do not diagonally
          scale; = 'S': Diagonally scale  the  matrix,  i.e.
          replace  A  by  D*A*D**(-1), where D is a diagonal
          matrix chosen to make the rows and  columns  of  A
          more  equal  in  norm. Do not permute; = 'B': Both
          diagonally scale and permute A.

          Computed reciprocal condition numbers will be  for
          the  matrix after balancing and/or permuting. Per-
          muting does not change condition numbers (in exact
          arithmetic), but balancing does.

JOBVL (input)
          = 'N': left eigenvectors of A are not computed;
          = 'V': left eigenvectors of A  are  computed.   If
          SENSE = 'E' or 'B', JOBVL must = 'V'.

JOBVR (input)
          = 'N': right eigenvectors of A are not computed;
          = 'V': right eigenvectors of A are  computed.   If
```

```
          SENSE = 'E' or 'B', JOBVR must = 'V'.

SENSE (input)
          Determines which reciprocal condition numbers  are
          computed.  = 'N': None are computed;
          = 'E': Computed for eigenvalues only;
          = 'V': Computed for right eigenvectors only;
          = 'B': Computed for eigenvalues and  right  eigen-
          vectors.

          If SENSE = 'E' or 'B', both left and right  eigen-
          vectors  must  also  be  computed (JOBVL = 'V' and
          JOBVR = 'V').

N (input) The order of the matrix A. N >= 0.
A (input/output)
          On entry, the N-by-N matrix A.   On  exit,  A  has
          been  overwritten.  If JOBVL = 'V' or JOBVR = 'V',
          A contains the real Schur  form  of  the  balanced
          version of the input matrix A.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

WR (output)
          WR and WI contain the real  and  imaginary  parts,
          respectively,  of  the computed eigenvalues.  Com-
          plex conjugate pairs of  eigenvalues  will  appear
          consecutively with the eigenvalue having the posi-
          tive imaginary part first.

WI (output)
          See the description for WR.

VL (output)
          If JOBVL = 'V', the  left  eigenvectors  u(j)  are
          stored  one after another in the columns of VL, in
          the same order as their eigenvalues.  If  JOBVL  =
          'N', VL is not referenced.  If the j-th eigenvalue
          is real, then u(j) = VL(:,j), the j-th  column  of
          VL.   If  the j-th and (j+1)-st eigenvalues form a
          complex conjugate pair,  then  u(j)  =  VL(:,j)  +
          i*VL(:,j+1) and
          u(j+1) = VL(:,j) - i*VL(:,j+1).

LDVL (input)
          The leading dimension of the array VL.  LDVL >= 1;
          if JOBVL = 'V', LDVL >= N.
```

VR (output)
If JOBVR = 'V', the right eigenvectors v(j) are stored one after another in the columns of VR, in the same order as their eigenvalues. If JOBVR = 'N', VR is not referenced. If the j-th eigenvalue is real, then v(j) = VR(:,j), the j-th column of VR. If the j-th and (j+1)-st eigenvalues form a complex conjugate pair, then v(j) = VR(:,j) + i*VR(:,j+1) and
v(j+1) = VR(:,j) - i*VR(:,j+1).

LDVR (input)
The leading dimension of the array VR. LDVR >= 1, and if JOBVR = 'V', LDVR >= N.

ILO (output)
ILO and IHI are integer values determined when A was balanced. The balanced A(i,j) = 0 if I > J and J = 1,...,ILO-1 or I = IHI+1,...,N.

IHI (output)
See the description of ILO.

SCALE (output)
Details of the permutations and scaling factors applied when balancing A. If P(j) is the index of the row and column interchanged with row and column j, and D(j) is the scaling factor applied to row and column j, then SCALE(J) = P(J), for J = 1,...,ILO-1 = D(J), for J = ILO,...,IHI = P(J) for J = IHI+1,...,N. The order in which the interchanges are made is N to IHI+1, then 1 to ILO-1.

ABNRM (output)
The one-norm of the balanced matrix (the maximum of the sum of absolute values of elements of any column).

RCONE (output)
RCONE(j) is the reciprocal condition number of the j-th eigenvalue.

RCONV (output)
RCONV(j) is the reciprocal condition number of the j-th right eigenvector.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)

The dimension of the array WORK. If SENSE = 'N' or 'E', LDWORK >= max(1,2*N), and if JOBVL = 'V' or JOBVR = 'V', LDWORK >= 3*N. If SENSE = 'V' or 'B', LDWORK >= N*(N+6). For good performance, LDWORK must generally be larger.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

IWORK2 (workspace)

dimension(2*N-2) If SENSE = 'N' or 'E', not referenced.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value.

> 0: if INFO = i, the QR algorithm failed to compute all the eigenvalues, and no eigenvectors or condition numbers have been computed; elements 1:ILO-1 and i+1:N of WR and WI contain eigenvalues which have converged.

Contents

NAME

```
sgegs - routine is deprecated and has been replaced by  rou-
tine SGGES
```

SYNOPSIS

```
SUBROUTINE SGEGS(JOBVSL, JOBVSR, N, A, LDA, B, LDB, ALPHAR, ALPHAI,
      BETA, VSL, LDVSL, VSR, LDVSR, WORK, LDWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR
INTEGER N, LDA, LDB, LDVSL, LDVSR, LDWORK, INFO
REAL  A(LDA,*),  B(LDB,*),  ALPHAR(*),  ALPHAI(*),  BETA(*),
VSL(LDVSL,*), VSR(LDVSR,*), WORK(*)

SUBROUTINE SGEGS_64(JOBVSL, JOBVSR, N, A, LDA, B, LDB, ALPHAR,
      ALPHAI, BETA, VSL, LDVSL, VSR, LDVSR, WORK, LDWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR
INTEGER*8 N, LDA, LDB, LDVSL, LDVSR, LDWORK, INFO
REAL  A(LDA,*),  B(LDB,*),  ALPHAR(*),  ALPHAI(*),  BETA(*),
VSL(LDVSL,*), VSR(LDVSR,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEGS(JOBVSL, JOBVSR, [N], A, [LDA], B, [LDB], ALPHAR,
       ALPHAI, BETA, VSL, [LDVSL], VSR, [LDVSR], [WORK], [LDWORK], [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR
INTEGER :: N, LDA, LDB, LDVSL, LDVSR, LDWORK, INFO
REAL, DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
REAL, DIMENSION(:,:) :: A, B, VSL, VSR

SUBROUTINE GEGS_64(JOBVSL, JOBVSR, [N], A, [LDA], B, [LDB], ALPHAR,
       ALPHAI, BETA, VSL, [LDVSL], VSR, [LDVSR], [WORK], [LDWORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOBVSL, JOBVSR
INTEGER(8) :: N, LDA, LDB, LDVSL, LDVSR, LDWORK, INFO
REAL, DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
REAL, DIMENSION(:,:) :: A, B, VSL, VSR
```

C INTERFACE

```
#include <sunperf.h>

void sgegs(char jobvsl, char jobvsr, int n,  float  *a,  int
          lda,  float  *b,  int  ldb,  float  *alphar, float
          *alphai, float *beta, float *vsl, int ldvsl, float
          *vsr, int ldvsr, int *info);
void sgegs_64(char jobvsl, char jobvsr, long  n,  float  *a,
          long lda, float *b, long ldb, float *alphar, float
          *alphai, float  *beta,  float  *vsl,  long  ldvsl,
          float *vsr, long ldvsr, long *info);
```

PURPOSE

sgegs routine is deprecated and has been replaced by routine SGGES.

SGEGS computes for a pair of N-by-N real nonsymmetric matrices A, B: the generalized eigenvalues (alphar +/- alphai*i, beta), the real Schur form (A, B), and optionally left and/or right Schur vectors (VSL and VSR).

(If only the generalized eigenvalues are needed, use the driver SGEGV instead.)

A generalized eigenvalue for a pair of matrices (A,B) is, roughly speaking, a scalar w or a ratio alpha/beta = w, such that A - w*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0, and even for both being zero. A good beginning reference is the book, "Matrix Computations", by G. Golub & C. van Loan (Johns Hopkins U. Press)

The (generalized) Schur form of a pair of matrices is the result of multiplying both matrices on the left by one orthogonal matrix and both on the right by another orthogonal matrix, these two orthogonal matrices being chosen so as to bring the pair of matrices into (real) Schur form.

A pair of matrices A, B is in generalized real Schur form if B is upper triangular with non-negative diagonal and A is block upper triangular with 1-by-1 and 2-by-2 blocks. 1-by-1 blocks correspond to real generalized eigenvalues,

```
while 2-by-2 blocks of A will be  "standardized"  by  making
the corresponding elements of B have the form:
        [  a  0  ]
        [  0  b  ]

and the pair of corresponding 2-by-2 blocks in A and B  will
have a complex conjugate pair of generalized eigenvalues.

The left and right Schur vectors are the columns of VSL  and
VSR,  respectively,  where  VSL  and  VSR are the orthogonal
matrices which reduce A and B to Schur form:

Schur form of (A,B) = ( (VSL)**T A (VSR), (VSL)**T B (VSR) )
```

ARGUMENTS

```
JOBVSL (input)
          = 'N':  do not compute the left Schur vectors;
          = 'V':  compute the left Schur vectors.

JOBVSR (input)
          = 'N':  do not compute the right Schur vectors;
          = 'V':  compute the right Schur vectors.

N (input) The order of the matrices A, B, VSL, and  VSR.   N
          >= 0.

A (input/output)
          On entry, the first of the pair of matrices  whose
          generalized  eigenvalues  and  (optionally)  Schur
          vectors are to be computed.  On exit, the general-
          ized  Schur  form  of A.  Note: to avoid overflow,
          the Frobenius norm of the matrix A should be  less
          than the overflow threshold.

LDA (input)
          The leading dimension of A.  LDA >= max(1,N).

B (input/output)
          On entry, the second of the pair of matrices whose
          generalized  eigenvalues  and  (optionally)  Schur
          vectors are to be computed.  On exit, the general-
          ized  Schur  form  of B.  Note: to avoid overflow,
          the Frobenius norm of the matrix B should be  less
          than the overflow threshold.

LDB (input)
          The leading dimension of B.  LDB >= max(1,N).
```

ALPHAR (output)

On exit, (ALPHAR(j) + ALPHAI(j)*i)/BETA(j), j=1,...,N, will be the generalized eigenvalues. ALPHAR(j) + ALPHAI(j)*i, j=1,...,N and BETA(j),j=1,...,N are the diagonals of the complex Schur form (A,B) that would result if the 2-by-2 diagonal blocks of the real Schur form of (A,B) were further reduced to triangular form using 2-by-2 complex unitary transformations. If ALPHAI(j) is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-st eigenvalues are a complex conjugate pair, with ALPHAI(j+1) negative.

Note: the quotients ALPHAR(j)/BETA(j) and ALPHAI(j)/BETA(j) may easily over- or underflow, and BETA(j) may even be zero. Thus, the user should avoid naively computing the ratio alpha/beta. However, ALPHAR and ALPHAI will be always less than and usually comparable with norm(A) in magnitude, and BETA always less than and usually comparable with norm(B).

ALPHAI (output)

See the description for ALPHAR.

BETA (output)

See the description for ALPHAR.

VSL (input)

If JOBVSL = 'V', VSL will contain the left Schur vectors. (See "Purpose", above.) Not referenced if JOBVSL = 'N'.

LDVSL (input)

The leading dimension of the matrix VSL. LDVSL >=1, and if JOBVSL = 'V', LDVSL >= N.

VSR (input)

If JOBVSR = 'V', VSR will contain the right Schur vectors. (See "Purpose", above.) Not referenced if JOBVSR = 'N'.

LDVSR (input)

The leading dimension of the matrix VSR. LDVSR >= 1, and if JOBVSR = 'V', LDVSR >= N.

WORK (workspace)

On exit, if INFO = 0, WORK(1) returns the optimal

LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,4*N). For good performance, LDWORK must generally be larger. To compute the optimal value of LDWORK, call ILAENV to get blocksizes (for SGEQRF, SORMQR, and SORGQR.) Then compute: NB -- MAX of the blocksizes for SGEQRF, SORMQR, and SORGQR The optimal LDWORK is 2*N + N*(NB+1).

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille- gal value.
= 1,...,N: The QZ iteration failed. (A,B) are not in Schur form, but ALPHAR(j), ALPHAI(j), and BETA(j) should be correct for j=INFO+1,...,N. > N: errors that usually indicate LAPACK problems:
=N+1: error return from SGGBAL
=N+2: error return from SGEQRF
=N+3: error return from SORMQR
=N+4: error return from SORGQR
=N+5: error return from SGGHRD
=N+6: error return from SHGEQZ (other than failed iteration) =N+7: error return from SGGBAK (comput- ing VSL)
=N+8: error return from SGGBAK (computing VSR)
=N+9: error return from SLASCL (various places)

Contents

NAME

```
sgegv - routine is deprecated and has been replaced by  rou-
tine SGGEV
```

SYNOPSIS

```
SUBROUTINE SGEGV(JOBVL, JOBVR, N, A, LDA, B, LDB, ALPHAR, ALPHAI,
      BETA, VL, LDVL, VR, LDVR, WORK, LDWORK, INFO)

CHARACTER * 1 JOBVL, JOBVR
INTEGER N, LDA, LDB, LDVL, LDVR, LDWORK, INFO
REAL  A(LDA,*),  B(LDB,*),  ALPHAR(*),  ALPHAI(*),  BETA(*),
VL(LDVL,*), VR(LDVR,*), WORK(*)

SUBROUTINE SGEGV_64(JOBVL, JOBVR, N, A, LDA, B, LDB, ALPHAR, ALPHAI,
      BETA, VL, LDVL, VR, LDVR, WORK, LDWORK, INFO)

CHARACTER * 1 JOBVL, JOBVR
INTEGER*8 N, LDA, LDB, LDVL, LDVR, LDWORK, INFO
REAL  A(LDA,*),  B(LDB,*),  ALPHAR(*),  ALPHAI(*),  BETA(*),
VL(LDVL,*), VR(LDVR,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEGV(JOBVL, JOBVR, [N], A, [LDA], B, [LDB], ALPHAR,
       ALPHAI, BETA, VL, [LDVL], VR, [LDVR], [WORK], [LDWORK], [INFO])

CHARACTER(LEN=1) :: JOBVL, JOBVR
INTEGER :: N, LDA, LDB, LDVL, LDVR, LDWORK, INFO
REAL, DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
REAL, DIMENSION(:,:) :: A, B, VL, VR

SUBROUTINE GEGV_64(JOBVL, JOBVR, [N], A, [LDA], B, [LDB], ALPHAR,
```

```
          ALPHAI, BETA, VL, [LDVL], VR, [LDVR], [WORK], [LDWORK], [INFO])

   CHARACTER(LEN=1) :: JOBVL, JOBVR
   INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, LDWORK, INFO
   REAL, DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
   REAL, DIMENSION(:,:) :: A, B, VL, VR
```

C INTERFACE

```
#include <sunperf.h>

void sgegv(char jobvl, char jobvr, int n, float *a, int lda,
          float  *b,  int ldb, float *alphar, float *alphai,
          float *beta, float *vl, int ldvl, float  *vr,  int
          ldvr, int *info);
void sgegv_64(char jobvl, char jobvr, long n, float *a, long
          lda,  float  *b,  long  ldb,  float *alphar, float
          *alphai, float *beta, float *vl, long ldvl,  float
          *vr, long ldvr, long *info);
```

PURPOSE

sgegv routine is deprecated and has been replaced by routine SGGEV.

SGEGV computes for a pair of n-by-n real nonsymmetric matrices A and B, the generalized eigenvalues (alphar +/- alphai*i, beta), and optionally, the left and/or right generalized eigenvectors (VL and VR).

A generalized eigenvalue for a pair of matrices (A,B) is, roughly speaking, a scalar w or a ratio alpha/beta = w, such that A - w*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0, and even for both being zero. A good beginning reference is the book, "Matrix Computations", by G. Golub & C. van Loan (Johns Hopkins U. Press)

A right generalized eigenvector corresponding to a generalized eigenvalue w for a pair of matrices (A,B) is a vector r such that (A - w B) r = 0 . A left generalized eigenvector is a vector l such that l**H * (A - w B) = 0, where l**H is the
conjugate-transpose of l.

Note: this routine performs "full balancing" on A and B -- see "Further Details", below.

ARGUMENTS

JOBVL (input)
= 'N': do not compute the left generalized eigenvectors;
= 'V': compute the left generalized eigenvectors.

JOBVR (input)
= 'N': do not compute the right generalized eigenvectors;
= 'V': compute the right generalized eigenvectors.

N (input) The order of the matrices A, B, VL, and VR. N >= 0.

A (input/output)
On entry, the first of the pair of matrices whose generalized eigenvalues and (optionally) generalized eigenvectors are to be computed. On exit, the contents will have been destroyed. (For a description of the contents of A on exit, see "Further Details", below.)

LDA (input)
The leading dimension of A. LDA >= max(1,N).

B (input/output)
On entry, the second of the pair of matrices whose generalized eigenvalues and (optionally) generalized eigenvectors are to be computed. On exit, the contents will have been destroyed. (For a description of the contents of B on exit, see "Further Details", below.)

LDB (input)
The leading dimension of B. LDB >= max(1,N).

ALPHAR (output)
On exit, (ALPHAR(j) + ALPHAI(j)*i)/BETA(j), j=1,...,N, will be the generalized eigenvalues. If ALPHAI(j) is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-st eigenvalues are a complex conjugate pair, with ALPHAI(j+1) negative.

Note: the quotients ALPHAR(j)/BETA(j) and ALPHAI(j)/BETA(j) may easily over- or underflow, and BETA(j) may even be zero. Thus, the user should avoid naively computing the ratio alpha/beta. However, ALPHAR and ALPHAI will be always less than and usually comparable with norm(A) in magnitude, and BETA always less than and usually comparable with norm(B).

ALPHAI (output)

See the description of ALPHAR.

BETA (output)

See the description of ALPHAR.

VL (output)

If JOBVL = 'V', the left generalized eigenvectors. (See "Purpose", above.) Real eigenvectors take one column, complex take two columns, the first for the real part and the second for the imaginary part. Complex eigenvectors correspond to an eigenvalue with positive imaginary part. Each eigenvector will be scaled so the largest component will have abs(real part) + abs(imag. part) = 1, *except* that for eigenvalues with alpha=beta=0, a zero vector will be returned as the corresponding eigenvector. Not referenced if JOBVL = 'N'.

LDVL (input)

The leading dimension of the matrix VL. LDVL >= 1, and if JOBVL = 'V', LDVL >= N.

VR (output)

If JOBVR = 'V', the right generalized eigenvectors. (See "Purpose", above.) Real eigenvectors take one column, complex take two columns, the first for the real part and the second for the imaginary part. Complex eigenvectors correspond to an eigenvalue with positive imaginary part. Each eigenvector will be scaled so the largest component will have abs(real part) + abs(imag. part) = 1, *except* that for eigenvalues with alpha=beta=0, a zero vector will be returned as the corresponding eigenvector. Not referenced if JOBVR = 'N'.

LDVR (input)

The leading dimension of the matrix VR. LDVR >= 1, and if JOBVR = 'V', LDVR >= N.

WORK (workspace)

On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)

The dimension of the array WORK. LDWORK >= max(1,8*N). For good performance, LDWORK must generally be larger. To compute the optimal value of LDWORK, call ILAENV to get blocksizes (for SGEQRF, SORMQR, and SORGQR.) Then compute: NB -- MAX of the blocksizes for SGEQRF, SORMQR, and SORGQR; The optimal LDWORK is: 2*N + MAX(6*N, N*(NB+1)).

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
= 1,...,N: The QZ iteration failed. No eigenvectors have been calculated, but ALPHAR(j), ALPHAI(j), and BETA(j) should be correct for j=INFO+1,...,N. > N: errors that usually indicate LAPACK problems:
=N+1: error return from SGGBAL
=N+2: error return from SGEQRF
=N+3: error return from SORMQR
=N+4: error return from SORGQR
=N+5: error return from SGGHRD
=N+6: error return from SHGEQZ (other than failed iteration) =N+7: error return from STGEVC
=N+8: error return from SGGBAK (computing VL)
=N+9: error return from SGGBAK (computing VR)
=N+10: error return from SLASCL (various calls)

FURTHER DETAILS

Balancing

This driver calls SGGBAL to both permute and scale rows and columns of A and B. The permutations PL and PR are chosen so that PL*A*PR and PL*B*R will be upper triangular except for the diagonal blocks A(i:j,i:j) and B(i:j,i:j), with i and j as close together as possible. The diagonal scaling matrices DL and DR are chosen so that the pair DL*PL*A*PR*DR, DL*PL*B*PR*DR have elements close to one (except for the elements that start out zero.)

After the eigenvalues and eigenvectors of the balanced matrices have been computed, SGGBAK transforms the eigenvectors back to what they would have been (in perfect arithmetic) if they had not been balanced.

Contents of A and B on Exit
-------- -- - --- - -- ----

If any eigenvectors are computed (either JOBVL='V' or JOBVR='V' or both), then on exit the arrays A and B will contain the real Schur form[*] of the "balanced" versions of A and B. If no eigenvectors are computed, then only the diagonal blocks will be correct.

[*] See SHGEQZ, SGEGS, or read the book "Matrix Computations",
    by Golub & van Loan, pub. by Johns Hopkins U. Press.

Contents

NAME

```
sgehrd - reduce a real general matrix A to upper  Hessenberg
form H by an orthogonal similarity transformation
```

SYNOPSIS

```
SUBROUTINE SGEHRD(N, ILO, IHI, A, LDA, TAU, WORKIN, LWORKIN, INFO)

INTEGER N, ILO, IHI, LDA, LWORKIN, INFO
REAL A(LDA,*), TAU(*), WORKIN(*)

SUBROUTINE SGEHRD_64(N, ILO, IHI, A, LDA, TAU, WORKIN, LWORKIN, INFO)

INTEGER*8 N, ILO, IHI, LDA, LWORKIN, INFO
REAL A(LDA,*), TAU(*), WORKIN(*)
```

F95 INTERFACE

```
SUBROUTINE GEHRD([N], ILO, IHI, A, [LDA], TAU, [WORKIN], [LWORKIN],
       [INFO])

INTEGER :: N, ILO, IHI, LDA, LWORKIN, INFO
REAL, DIMENSION(:) :: TAU, WORKIN
REAL, DIMENSION(:,:) :: A

SUBROUTINE GEHRD_64([N], ILO, IHI, A, [LDA], TAU, [WORKIN], [LWORKIN],
       [INFO])

INTEGER(8) :: N, ILO, IHI, LDA, LWORKIN, INFO
REAL, DIMENSION(:) :: TAU, WORKIN
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void sgehrd(int n, int ilo, int  ihi,  float  *a,  int  lda,
          float *tau, int *info);

void sgehrd_64(long n, long ilo, long ihi,  float  *a,  long
          lda, float *tau, long *info);
```

PURPOSE

sgehrd reduces a real general matrix A to upper Hessenberg form H by an orthogonal similarity transformation: Q' * A * Q = H .

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

ILO (input)
It is assumed that A is already upper triangular in rows and columns 1:ILO-1 and IHI+1:N. ILO and IHI are normally set by a previous call to SGEBAL; otherwise they should be set to 1 and N respectively. See Further Details.

IHI (input)
See the description of ILO.

A (input/output)
On entry, the N-by-N general matrix to be reduced. On exit, the upper triangle and the first subdiagonal of A are overwritten with the upper Hessenberg matrix H, and the elements below the first subdiagonal, with the array TAU, represent the orthogonal matrix Q as a product of elementary reflectors. See Further Details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

TAU (output)
The scalar factors of the elementary reflectors (see Further Details). Elements 1:ILO-1 and IHI:N-1 of TAU are set to zero.

WORKIN (workspace)

On exit, if INFO = 0, WORKIN(1) returns the optimal LWORKIN.

LWORKIN (input)
The length of the array WORKIN. LWORKIN >= max(1,N). For optimum performance LWORKIN >= N*NB, where NB is the optimal blocksize.

If LWORKIN = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORKIN array, returns this value as the first entry of the WORKIN array, and no error message related to LWORKIN is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

The matrix Q is represented as a product of (ihi-ilo) elementary reflectors

```
   Q = H(ilo) H(ilo+1) . . . H(ihi-1).
```

Each H(i) has the form

```
   H(i) = I - tau * v * v'
```

where tau is a real scalar, and v is a real vector with v(1:i) = 0, v(i+1) = 1 and v(ihi+1:n) = 0; v(i+2:ihi) is stored on exit in A(i+2:ihi,i), and tau in TAU(i).

The contents of A are illustrated by the following example, with n = 7, ilo = 2 and ihi = 6:

```
on entry,                        on exit,

( a   a   a   a   a   a   a )    (  a   a   h    h    h    h
a  )  (      a   a   a   a   a   a )    (      a   h   h   h
h   a ) (     a   a   a   a   a   a )    (       h    h    h
h    h    h  ) (     a   a   a   a   a   a )    (      v2  h
h   h   h   h ) (     a   a   a   a   a   a )     (       v2
v3   h    h    h    h  )  (     a   a   a   a   a   a )    (
v2  v3  v4  h   h   h ) (                         a  )     (
a )
```

where a denotes an element of the original matrix A, h denotes a modified element of the upper Hessenberg matrix H, and vi denotes an element of the vector defining H(i).

Contents

NAME

```
sgelqf - compute an LQ factorization of a real M-by-N matrix
A
```

SYNOPSIS

```
SUBROUTINE SGELQF(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER M, N, LDA, LDWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)

SUBROUTINE SGELQF_64(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER*8 M, N, LDA, LDWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GELQF([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER :: M, N, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE GELQF_64([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER(8) :: M, N, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void sgelqf(int m, int n, float *a, int lda, float *tau, int
          *info);

void sgelqf_64(long m, long n, float  *a,  long  lda,  float
          *tau, long *info);
```

PURPOSE

sgelqf computes an LQ factorization of a real M-by-N matrix A: A = L * Q.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, the elements on and below the diagonal of the array contain the m-by-min(m,n) lower trapezoidal matrix L (L is lower triangular if m <= n); the elements above the diagonal, with the array TAU, represent the orthogonal matrix Q as a product of elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,M). For optimum performance LDWORK >= M*NB, where NB is the optimal blocksize.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

```
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

FURTHER DETAILS

```
The matrix Q is  represented  as  a  product  of  elementary
reflectors

   Q = H(k) . . . H(2) H(1), where k = min(m,n).

Each H(i) has the form
   H(i) = I - tau * v * v'

where tau is a real scalar, and v is a real vector with
v(1:i-1) = 0 and v(i) = 1; v(i+1:n) is  stored  on  exit  in
A(i,i+1:n), and tau in TAU(i).
```

Contents

NAME

```
sgels - solve overdetermined or underdetermined real  linear
systems  involving  an  M-by-N  matrix  A, or its transpose,
using a QR or LQ factorization of A
```

SYNOPSIS

```
SUBROUTINE SGELS(TRANSA, M, N, NRHS, A, LDA, B, LDB, WORK, LDWORK,
      INFO)

CHARACTER * 1 TRANSA
INTEGER M, N, NRHS, LDA, LDB, LDWORK, INFO
REAL A(LDA,*), B(LDB,*), WORK(*)

SUBROUTINE SGELS_64(TRANSA, M, N, NRHS, A, LDA, B, LDB, WORK, LDWORK,
      INFO)

CHARACTER * 1 TRANSA
INTEGER*8 M, N, NRHS, LDA, LDB, LDWORK, INFO
REAL A(LDA,*), B(LDB,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GELS([TRANSA], [M], [N], [NRHS], A, [LDA], B, [LDB], [WORK],
       LDWORK, [INFO])

CHARACTER(LEN=1) :: TRANSA
INTEGER :: M, N, NRHS, LDA, LDB, LDWORK, INFO
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE GELS_64([TRANSA], [M], [N], [NRHS], A, [LDA], B, [LDB],
       [WORK], LDWORK, [INFO])
```

```
CHARACTER(LEN=1) :: TRANSA
INTEGER(8) :: M, N, NRHS, LDA, LDB, LDWORK, INFO
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void sgels (char, int, int, int, float*, int,  float*,  int,
          int*);

void sgels_64 (char, long, long, long, float*, long, float*,
          long, long*);
```

PURPOSE

sgels solves overdetermined or underdetermined real linear systems involving an M-by-N matrix A, or its transpose, using a QR or LQ factorization of A. It is assumed that A has full rank.

The following options are provided:

1. If TRANS = 'N' and m >= n: find the least squares solution of
   an overdetermined system, i.e., solve the least squares problem
                minimize || B - A*X ||.

2. If TRANS = 'N' and m < n: find the minimum norm solution of
   an underdetermined system A * X = B.

3. If TRANS = 'T' and m >= n: find the minimum norm solution of
   an undetermined system A**T * X = B.

4. If TRANS = 'T' and m < n: find the least squares solution of
   an overdetermined system, i.e., solve the least squares problem
                minimize || B - A**T * X ||.

Several right hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the M-by-NRHS right hand side matrix B and the N-by-NRHS solution matrix X.

ARGUMENTS

```
TRANSA (input)
          = 'N': the linear system involves A;
          = 'T': the linear system involves A**T.

          TRANSA is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix A.  M >= 0.

N (input) The number of columns of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X. NRHS >=0.
A (input/output)
          On entry, the M-by-N matrix A.  On exit, if  M  >=
          N,  A is overwritten by details of its QR factori-
          zation as returned by SGEQRF; if  M  <   N,  A  is
          overwritten  by details of its LQ factorization as
          returned by SGELQF.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,M).

B (input/output)
          On entry, the matrix B of right hand side vectors,
          stored columnwise; B is M-by-NRHS if TRANSA = 'N',
          or N-by-NRHS if TRANSA  =  'T'.   On  exit,  B  is
          overwritten   by   the  solution  vectors,  stored
          columnwise:  if TRANSA = 'N' and m >= n, rows 1 to
          n of B contain the least squares solution vectors;
          the residual sum of squares for  the  solution  in
          each column is given by the sum of squares of ele-
          ments N+1 to M in that column; if TRANSA = 'N' and
          m  <  n, rows 1 to N of B contain the minimum norm
          solution vectors; if TRANSA = 'T' and m >= n, rows
          1 to M of B contain the minimum norm solution vec-
          tors; if TRANSA = 'T' and m < n, rows 1 to M of  B
          contain  the  least  squares solution vectors; the
          residual sum of squares for the solution  in  each
          column  is given by the sum of squares of elements
          M+1 to N in that column.

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          MAX(1,M,N).
```

```
WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LDWORK.

LDWORK (output)
          The dimension of the array WORK.  LDWORK  >=  max(
          1,  MN  +  max( MN, NRHS ) ).  For optimal perfor-
          mance, LDWORK >= max( 1, MN + max( MN,  NRHS  )*NB
          ).   where  MN  =  min(M,N)  and NB is the optimum
          block size.

          If LDWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LDWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
sgelsd - compute the minimum-norm solution to a real  linear
least squares problem
```

SYNOPSIS

```
SUBROUTINE SGELSD(M, N, NRHS, A, LDA, B, LDB, S, RCOND, RANK, WORK,
      LWORK, IWORK, INFO)

INTEGER M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER IWORK(*)
REAL RCOND
REAL A(LDA,*), B(LDB,*), S(*), WORK(*)

SUBROUTINE SGELSD_64(M, N, NRHS, A, LDA, B, LDB, S, RCOND, RANK,
      WORK, LWORK, IWORK, INFO)

INTEGER*8 M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER*8 IWORK(*)
REAL RCOND
REAL A(LDA,*), B(LDB,*), S(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GELSD([M], [N], [NRHS], A, [LDA], B, [LDB], S, RCOND,
       RANK, [WORK], [LWORK], [IWORK], [INFO])

INTEGER :: M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL :: RCOND
REAL, DIMENSION(:) :: S, WORK
REAL, DIMENSION(:,:) :: A, B
```

```
SUBROUTINE GELSD_64([M], [N], [NRHS], A, [LDA], B, [LDB], S, RCOND,
       RANK, [WORK], [LWORK], [IWORK], [INFO])

INTEGER(8) :: M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL :: RCOND
REAL, DIMENSION(:) :: S, WORK
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void sgelsd(int m, int n, int nrhs, float *a, int lda, float
          *b, int ldb, float *s, float rcond, int *rank, int
          *info);
void sgelsd_64(long m, long n, long  nrhs,  float  *a,  long
          lda,  float  *b,  long ldb, float *s, float rcond,
          long *rank, long *info);
```

PURPOSE

sgelsd computes the minimum-norm solution to a real linear least squares problem:

    minimize 2-norm(| b - A*x |)

using the singular value decomposition (SVD) of A. A is an M-by-N matrix which may be rank-deficient.

Several right hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the M-by-NRHS right hand side matrix B and the N-by-NRHS solution matrix X.

The problem is solved in three steps:
(1) Reduce the coefficient matrix A to bidiagonal form with
    Householder transformations, reducing the original prob-
lem
    into a "bidiagonal least squares problem" (BLS)
(2) Solve the BLS using a divide and conquer approach.
(3) Apply back all the Householder tranformations to solve
    the original least squares problem.

The effective rank of A is determined by treating as zero those singular values which are less than RCOND times the largest singular value.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray

X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

M (input) The number of rows of A. M >= 0.

N (input) The number of columns of A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, A has been destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the M-by-NRHS right hand side matrix B. On exit, B is overwritten by the N-by-NRHS solution matrix X. If m >= n and RANK = n, the residual sum-of-squares for the solution in the i-th column is given by the sum of squares of elements n+1:m in that column.

LDB (input)
The leading dimension of the array B. LDB >= max(1,max(M,N)).

S (output)
The singular values of A in decreasing order. The condition number of A in the 2-norm = S(1)/S(min(m,n)).

RCOND (input)
RCOND is used to determine the effective rank of A. Singular values S(i) <= RCOND*S(1) are treated as zero. If RCOND < 0, machine precision is used instead.

RANK (output)
The effective rank of A, i.e., the number of singular values which are greater than RCOND*S(1).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 1. The exact minimum amount of workspace needed depends on M, N and NRHS. As long as LWORK is at least 12*N + 2*N*SMLSIZ + 8*N*NLVL + N*NRHS * (SMLSIZ+1)**2, if M is greater than or equal to N or 12*M + 2*M*SMLSIZ + 8*M*NLVL + M*NRHS + (SMLSIZ+1)**2, if M is less than N, the code will execute correctly. SMLSIZ is returned by ILAENV and is equal to the maximum size of the subproblems at the bottom of the computation tree (usually about 25), and NLVL = INT(LOG_2(MIN(M,N)/(SMLSIZ+1))) + 1 For good performance, LWORK should generally be larger.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace)
LIWORK >= 3 * MINMN * NLVL + 11 * MINMN, where MINMN = MIN(M,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: the algorithm for computing the SVD failed to converge; if INFO = i, i off-diagonal elements of an intermediate bidiagonal form did not converge to zero.

FURTHER DETAILS

Based on contributions by
Ming Gu and Ren-Cang Li, Computer Science Division, University of California at Berkeley, USA
Osni Marques, LBNL/NERSC, USA

Contents

NAME

```
sgelss - compute the minimum norm solution to a real  linear
least squares problem
```

SYNOPSIS

```
SUBROUTINE SGELSS(M, N, NRHS, A, LDA, B, LDB, SING, RCOND, IRANK,
      WORK, LDWORK, INFO)

INTEGER M, N, NRHS, LDA, LDB, IRANK, LDWORK, INFO
REAL RCOND
REAL A(LDA,*), B(LDB,*), SING(*), WORK(*)

SUBROUTINE SGELSS_64(M, N, NRHS, A, LDA, B, LDB, SING, RCOND, IRANK,
      WORK, LDWORK, INFO)

INTEGER*8 M, N, NRHS, LDA, LDB, IRANK, LDWORK, INFO
REAL RCOND
REAL A(LDA,*), B(LDB,*), SING(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GELSS([M], [N], [NRHS], A, [LDA], B, [LDB], SING, RCOND,
       IRANK, [WORK], [LDWORK], [INFO])

INTEGER :: M, N, NRHS, LDA, LDB, IRANK, LDWORK, INFO
REAL :: RCOND
REAL, DIMENSION(:) :: SING, WORK
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE GELSS_64([M], [N], [NRHS], A, [LDA], B, [LDB], SING,
       RCOND, IRANK, [WORK], [LDWORK], [INFO])

INTEGER(8) :: M, N, NRHS, LDA, LDB, IRANK, LDWORK, INFO
```

```
REAL :: RCOND
REAL, DIMENSION(:) :: SING, WORK
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void sgelss(int m, int n, int nrhs, float *a, int lda, float
          *b, int ldb, float *sing, float rcond, int *irank,
          int *info);

void sgelss_64(long m, long n, long  nrhs,  float  *a,  long
          lda, float *b, long ldb, float *sing, float rcond,
          long *irank, long *info);
```

PURPOSE

sgelss computes the minimum norm solution to a real linear least squares problem:

Minimize 2-norm(| b - A*x |).

using the singular value decomposition (SVD) of A. A is an M-by-N matrix which may be rank-deficient.

Several right hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the M-by-NRHS right hand side matrix B and the N-by-NRHS solution matrix X.

The effective rank of A is determined by treating as zero those singular values which are less than RCOND times the largest singular value.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, the first min(m,n) rows of A are overwritten with its right

singular vectors, stored rowwise.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the M-by-NRHS right hand side matrix B. On exit, B is overwritten by the N-by-NRHS solution matrix X. If m >= n and IRANK = n, the residual sum-of-squares for the solution in the i-th column is given by the sum of squares of elements n+1:m in that column.

LDB (input)
The leading dimension of the array B. LDB >= max(1,max(M,N)).

SING (output)
The singular values of A in decreasing order. The condition number of A in the 2-norm = SING(1)/SING(min(m,n)).

RCOND (input)
RCOND is used to determine the effective rank of A. Singular values SING(i) <= RCOND*SING(1) are treated as zero. If RCOND < 0, machine precision is used instead.

IRANK (output)
The effective rank of A, i.e., the number of singular values which are greater than RCOND*SING(1).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= 1, and also: LDWORK >= 3*min(M,N) + max(2*min(M,N), max(M,N), NRHS) For good performance, LDWORK should generally be larger.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)

= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: the algorithm for computing the SVD failed to converge; if INFO = i, i off-diagonal elements of an intermediate bidiagonal form did not converge to zero.

Contents

NAME

```
sgelsx - routine is deprecated and has been replaced by rou-
tine SGELSY
```

SYNOPSIS

```
SUBROUTINE SGELSX(M, N, NRHS, A, LDA, B, LDB, JPIVOT, RCOND, IRANK,
      WORK, INFO)

INTEGER M, N, NRHS, LDA, LDB, IRANK, INFO
INTEGER JPIVOT(*)
REAL RCOND
REAL A(LDA,*), B(LDB,*), WORK(*)

SUBROUTINE SGELSX_64(M, N, NRHS, A, LDA, B, LDB, JPIVOT, RCOND,
      IRANK, WORK, INFO)

INTEGER*8 M, N, NRHS, LDA, LDB, IRANK, INFO
INTEGER*8 JPIVOT(*)
REAL RCOND
REAL A(LDA,*), B(LDB,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GELSX([M], [N], [NRHS], A, [LDA], B, [LDB], JPIVOT, RCOND,
       IRANK, [WORK], [INFO])

INTEGER :: M, N, NRHS, LDA, LDB, IRANK, INFO
INTEGER, DIMENSION(:) :: JPIVOT
REAL :: RCOND
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE GELSX_64([M], [N], [NRHS], A, [LDA], B, [LDB], JPIVOT,
```

```
          RCOND, IRANK, [WORK], [INFO])

INTEGER(8) :: M, N, NRHS, LDA, LDB, IRANK, INFO
INTEGER(8), DIMENSION(:) :: JPIVOT
REAL :: RCOND
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void sgelsx(int m, int n, int nrhs, float *a, int lda, float
          *b, int ldb, int *jpivot, float rcond, int *irank,
          int *info);
void sgelsx_64(long m, long n, long  nrhs,  float  *a,  long
          lda,  float  *b,  long  ldb,  long  *jpivot, float
          rcond, long *irank, long *info);
```

PURPOSE

sgelsx routine is deprecated and has been replaced by routine SGELSY.

SGELSX computes the minimum-norm solution to a real linear least squares problem:

```
    minimize || A * X - B ||
```

using a complete orthogonal factorization of A. A is an M-by-N matrix which may be rank-deficient.

Several right hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the M-by-NRHS right hand side matrix B and the N-by-NRHS solution matrix X.

The routine first computes a QR factorization with column pivoting:

```
    A * P = Q * [ R11 R12 ]
                [  0  R22 ]
```

with R11 defined as the largest leading submatrix whose estimated condition number is less than 1/RCOND. The order of R11, RANK, is the effective rank of A.

Then, R22 is considered to be negligible, and R12 is annihilated by orthogonal transformations from the right, arriving at the complete orthogonal factorization:

```
   A * P = Q * [ T11 0 ] * Z
               [  0  0 ]
```

The minimum-norm solution is then

```
   X = P * Z' [ inv(T11)*Q1'*B ]
```

```
                [        0       ]
where Q1 consists of the first RANK columns of Q.
```

ARGUMENTS

```
M (input) The number of rows of the matrix A.  M >= 0.

N (input) The number of columns of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of matrices B and X. NRHS >= 0.
A (input/output)
          On entry, the M-by-N matrix A.   On  exit,  A  has
          been   overwritten  by  details  of  its  complete
          orthogonal factorization.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,M).

B (input/output)
          On entry, the M-by-NRHS right hand side matrix  B.
          On exit, the N-by-NRHS solution matrix X.  If m >=
          n and IRANK = n, the residual  sum-of-squares  for
          the  solution  in  the i-th column is given by the
          sum of squares of elements N+1:M in that column.

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          max(1,M,N).

JPIVOT (input/output)
          On entry, if JPIVOT(i) .ne. 0, the i-th column  of
          A  is  an  initial  column, otherwise it is a free
          column.  Before the QR  factorization  of  A,  all
          initial  columns are permuted to the leading posi-
          tions; only the remaining free columns  are  moved
          as a result of column pivoting during the factori-
          zation.  On exit, if JPIVOT(i) = k, then the  i-th
          column of A*P was the k-th column of A.

RCOND (input)
          RCOND is used to determine the effective  rank  of
          A,  which  is  defined as the order of the largest
          leading triangular submatrix R11 in the QR factor-
          ization with pivoting of A, whose estimated condi-
          tion number < 1/RCOND.
```

```
IRANK (output)
          The effective rank of A, i.e., the  order  of  the
          submatrix  R11.   This is the same as the order of
          the submatrix T11 in the complete orthogonal  fac-
          torization of A.

WORK (workspace)
          (max( min(M,N)+3*N, 2*min(M,N)+NRHS )),
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

sgelsy - compute the minimum-norm solution to a real linear least squares problem

SYNOPSIS

```
SUBROUTINE SGELSY(M, N, NRHS, A, LDA, B, LDB, JPVT, RCOND, RANK,
      WORK, LWORK, INFO)

INTEGER M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER JPVT(*)
REAL RCOND
REAL A(LDA,*), B(LDB,*), WORK(*)

SUBROUTINE SGELSY_64(M, N, NRHS, A, LDA, B, LDB, JPVT, RCOND, RANK,
      WORK, LWORK, INFO)

INTEGER*8 M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER*8 JPVT(*)
REAL RCOND
REAL A(LDA,*), B(LDB,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GELSY([M], [N], [NRHS], A, [LDA], B, [LDB], JPVT, RCOND,
       RANK, [WORK], [LWORK], [INFO])

INTEGER :: M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER, DIMENSION(:) :: JPVT
REAL :: RCOND
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A, B
```

```
SUBROUTINE GELSY_64([M], [N], [NRHS], A, [LDA], B, [LDB], JPVT,
       RCOND, RANK, [WORK], [LWORK], [INFO])

INTEGER(8) :: M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER(8), DIMENSION(:) :: JPVT
REAL :: RCOND
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void sgelsy(int m, int n, int nrhs, float *a, int lda, float
          *b,  int  ldb,  int *jpvt, float rcond, int *rank,
          int *info);
void sgelsy_64(long m, long n, long  nrhs,  float  *a,  long
          lda,  float *b, long ldb, long *jpvt, float rcond,
          long *rank, long *info);
```

PURPOSE

```
sgelsy computes the minimum-norm solution to a  real  linear
least squares problem:
    minimize || A * X - B ||
using a complete orthogonal factorization of A.  A is an  M-
by-N matrix which may be rank-deficient.

Several right hand side vectors b and solution vectors x can
be  handled in a single call; they are stored as the columns
of the M-by-NRHS right hand side matrix B and the  N-by-NRHS
solution matrix X.

The routine first computes a QR  factorization  with  column
pivoting:
    A * P = Q * [ R11 R12 ]
                [  0  R22 ]
with R11 defined as  the  largest  leading  submatrix  whose
estimated  condition number is less than 1/RCOND.  The order
of R11, RANK, is the effective rank of A.

Then, R22 is considered to be negligible, and R12 is annihi-
lated by orthogonal transformations from the right, arriving
at the complete orthogonal factorization:
   A * P = Q * [ T11 0 ] * Z
               [  0  0 ]
The minimum-norm solution is then
   X = P * Z' [ inv(T11)*Q1'*B ]
              [        0       ]
where Q1 consists of the first RANK columns of Q.
```

```
This routine is basically identical to the  original  xGELSX
except three differences:
  o The call to the subroutine xGEQPF has  been  substituted
by the
    the call to the subroutine xGEQP3. This subroutine is  a
Blas-3
    version of the QR factorization with column pivoting.
  o Matrix B (the right hand side) is updated with Blas-3.
  o The permutation of matrix B (the  right  hand  side)  is
faster and
    more simple.
```

ARGUMENTS

```
M (input) The number of rows of the matrix A.  M >= 0.
N (input) The number of columns of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of matrices B and X. NRHS >= 0.

A (input/output)
          On entry, the M-by-N matrix A.   On  exit,  A  has
          been   overwritten  by  details  of  its  complete
          orthogonal factorization.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,M).

B (input/output)
          On entry, the M-by-NRHS right hand side matrix  B.
          On exit, the N-by-NRHS solution matrix X.

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          max(1,M,N).

JPVT (input/output)
          On entry, if JPVT(i) .ne. 0, the i-th column of  A
          is permuted to the front of AP, otherwise column i
          is a free column.  On exit, if JPVT(i) =  k,  then
          the i-th column of AP was the k-th column of A.

RCOND (input)
          RCOND is used to determine the effective  rank  of
          A,  which  is  defined as the order of the largest
```

leading triangular submatrix R11 in the QR factorization with pivoting of A, whose estimated condition number < 1/RCOND.

RANK (output)

The effective rank of A, i.e., the order of the submatrix R11. This is the same as the order of the submatrix T11 in the complete orthogonal factorization of A.

WORK (workspace)

On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)

The dimension of the array WORK. The unblocked strategy requires that: LWORK >= MAX(MN+3*N+1, 2*MN+NRHS), where MN = min(M, N). The block algorithm requires that: LWORK >= MAX(MN+2*N+NB*(N+1), 2*MN+NB*NRHS), where NB is an upper bound on the blocksize returned by ILAENV for the routines SGEQP3, STZRZF, STZRQF, SORMQR, and SORMRZ.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: If INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

Based on contributions by

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Contents

NAME

```
sgemm - perform one of the matrix-matrix operations    C  :=
alpha*op( A )*op( B ) + beta*C
```

SYNOPSIS

```
SUBROUTINE SGEMM(TRANSA, TRANSB, M, N, K, ALPHA, A, LDA, B, LDB,
      BETA, C, LDC)

CHARACTER * 1 TRANSA, TRANSB
INTEGER M, N, K, LDA, LDB, LDC
REAL ALPHA, BETA
REAL A(LDA,*), B(LDB,*), C(LDC,*)

SUBROUTINE SGEMM_64(TRANSA, TRANSB, M, N, K, ALPHA, A, LDA, B, LDB,
      BETA, C, LDC)

CHARACTER * 1 TRANSA, TRANSB
INTEGER*8 M, N, K, LDA, LDB, LDC
REAL ALPHA, BETA
REAL A(LDA,*), B(LDB,*), C(LDC,*)
```

F95 INTERFACE

```
SUBROUTINE GEMM([TRANSA], [TRANSB], [M], [N], [K], ALPHA, A, [LDA],
       B, [LDB], BETA, C, [LDC])

CHARACTER(LEN=1) :: TRANSA, TRANSB
INTEGER :: M, N, K, LDA, LDB, LDC
REAL :: ALPHA, BETA
REAL, DIMENSION(:,:) :: A, B, C

SUBROUTINE GEMM_64([TRANSA], [TRANSB], [M], [N], [K], ALPHA, A, [LDA],
       B, [LDB], BETA, C, [LDC])
```

```
CHARACTER(LEN=1) :: TRANSA, TRANSB
INTEGER(8) :: M, N, K, LDA, LDB, LDC
REAL :: ALPHA, BETA
REAL, DIMENSION(:,:) :: A, B, C
```

C INTERFACE

```
#include <sunperf.h>

void sgemm(char transa, char transb, int m, int  n,  int  k,
          float alpha, float *a, int lda, float *b, int ldb,
          float beta, float *c, int ldc);

void sgemm_64(char transa, char transb, long m, long n, long
          k, float alpha, float *a, long lda, float *b, long
          ldb, float beta, float *c, long ldc);
```

PURPOSE

sgemm performs one of the matrix-matrix operations C := alpha*op(A)*op(B) + beta*C where op(X) is one of

op(X) = X   or   op(X) = X',

alpha and beta are scalars, and A, B and C are matrices, with op(A) an m by k matrix, op(B) a k by n matrix and C an m by n matrix.

ARGUMENTS

TRANSA (input)
On entry, TRANSA specifies the form of op(A) to be used in the matrix multiplication as follows:

TRANSA = 'N' or 'n', op(A) = A.

TRANSA = 'T' or 't', op(A) = A'.

TRANSA = 'C' or 'c', op(A) = A'.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

TRANSB (input)
On entry, TRANSB specifies the form of op(B) to be used in the matrix multiplication as follows:

TRANSB = 'N' or 'n', op(B) = B.

TRANSB = 'T' or 't', op(B) = B'.

TRANSB = 'C' or 'c', op(B) = B'.

Unchanged on exit.

TRANSB is defaulted to 'N' for F95 INTERFACE.

M (input)
On entry, M specifies the number of rows of the matrix op(A) and of the matrix C. M must be at least zero. Unchanged on exit.

N (input)
On entry, N specifies the number of columns of the matrix op(B) and the number of columns of the matrix C. N must be at least zero. Unchanged on exit.

K (input)
On entry, K specifies the number of columns of the matrix op(A) and the number of rows of the matrix op(B). K must be at least zero. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
REAL array of DIMENSION (LDA, ka), where ka is k when TRANSA = 'N' or 'n', and is m otherwise. Before entry with TRANSA = 'N' or 'n', the leading m by k part of the array A must contain the matrix A, otherwise the leading k by m part of the array A must contain the matrix A. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDA >= max(1, m), otherwise LDA >= max(1, k). Unchanged on exit.

B (input)
REAL array of DIMENSION (LDB, kb), where kb is n when TRANSB = 'N' or 'n', and is k otherwise. Before entry with TRANSB = 'N' or 'n', the lead-

ing k by n part of the array B must contain the matrix B, otherwise the leading n by k part of the array B must contain the matrix B. Unchanged on exit.

LDB (input)
On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. When TRANSB = 'N' or 'n' then LDB >= max(1, k), otherwise LDB >= max(1, n). Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. When BETA is supplied as zero then C need not be set on input. Unchanged on exit.

C (input/output)
REAL array of DIMENSION (LDC, n). Before entry, the leading m by n part of the array C must contain the matrix C, except when beta is zero, in which case C need not be set on entry. On exit, the array C is overwritten by the m by n matrix (alpha*op(A)*op(B) + beta*C).

LDC (input)
On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC >= max(1, m). Unchanged on exit.

Contents

NAME

```
sgemv - perform one of the matrix-vector operations    y  :=
alpha*A*x + beta*y or y := alpha*A'*x + beta*y
```

SYNOPSIS

```
SUBROUTINE SGEMV(TRANSA, M, N, ALPHA, A, LDA, X, INCX, BETA, Y, INCY)

CHARACTER * 1 TRANSA
INTEGER M, N, LDA, INCX, INCY
REAL ALPHA, BETA
REAL A(LDA,*), X(*), Y(*)

SUBROUTINE SGEMV_64(TRANSA, M, N, ALPHA, A, LDA, X, INCX, BETA, Y,
      INCY)

CHARACTER * 1 TRANSA
INTEGER*8 M, N, LDA, INCX, INCY
REAL ALPHA, BETA
REAL A(LDA,*), X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE GEMV([TRANSA], [M], [N], ALPHA, A, [LDA], X, [INCX], BETA,
       Y, [INCY])

CHARACTER(LEN=1) :: TRANSA
INTEGER :: M, N, LDA, INCX, INCY
REAL :: ALPHA, BETA
REAL, DIMENSION(:) :: X, Y
REAL, DIMENSION(:,:) :: A

SUBROUTINE GEMV_64([TRANSA], [M], [N], ALPHA, A, [LDA], X, [INCX],
       BETA, Y, [INCY])
```

```
CHARACTER(LEN=1) :: TRANSA
INTEGER(8) :: M, N, LDA, INCX, INCY
REAL :: ALPHA, BETA
REAL, DIMENSION(:) :: X, Y
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void sgemv(char transa, int m, int n, float alpha, float *a,
          int lda, float *x, int incx, float beta, float *y,
          int incy);
void sgemv_64(char transa, long  m,  long  n,  float  alpha,
          float  *a,  long  lda,  float *x, long incx, float
          beta, float *y, long incy);
```

PURPOSE

sgemv performs one of the matrix-vector operations y := alpha*A*x + beta*y, or y := alpha*A'*x + beta*y, where alpha and beta are scalars, x and y are vectors and A is an m by n matrix.

ARGUMENTS

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   y := alpha*A*x + beta*y.

TRANSA = 'T' or 't'   y := alpha*A'*x + beta*y.

TRANSA = 'C' or 'c'   y := alpha*A'*x + beta*y.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

M (input)
On entry, M specifies the number of rows of the matrix A. M >= 0. Unchanged on exit.

N (input)
On entry, N specifies the number of columns of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
Before entry, the leading m by n part of the array A must contain the matrix of coefficients. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, m). Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)) when TRANSA = 'N' or 'n' and at least (1 + (m - 1)*abs(INCX)) otherwise. Before entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. When BETA is supplied as zero then Y need not be set on input. Unchanged on exit.

Y (input/output)
(1 + (m - 1)*abs(INCY)) when TRANSA = 'N' or 'n' and at least (1 + (n - 1)*abs(INCY)) otherwise. Before entry with BETA non-zero, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
sgeqlf - compute a QL factorization of a real M-by-N  matrix
A
```

SYNOPSIS

```
SUBROUTINE SGEQLF(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER M, N, LDA, LDWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)

SUBROUTINE SGEQLF_64(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER*8 M, N, LDA, LDWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEQLF([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER :: M, N, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE GEQLF_64([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER(8) :: M, N, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void sgeqlf(int m, int n, float *a, int lda, float *tau, int
          *info);

void sgeqlf_64(long m, long n, float  *a,  long  lda,  float
          *tau, long *info);
```

PURPOSE

sgeqlf computes a QL factorization of a real M-by-N matrix A: A = Q * L.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, if m >= n, the lower triangle of the subarray A(m-n+1:m,1:n) contains the N-by-N lower triangular matrix L; if m <= n, the elements on and below the (n-m)-th superdiagonal contain the M-by-N lower trapezoidal matrix L; the remaining elements, with the array TAU, represent the orthogonal matrix Q as a product of elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A.   LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK.   LDWORK >= max(1,N). For optimum performance LDWORK >= N*NB, where NB is the optimal blocksize.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first

entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The matrix Q is represented as a product of elementary reflectors

```
   Q = H(k) . . . H(2) H(1), where k = min(m,n).
```

Each H(i) has the form

```
   H(i) = I - tau * v * v'
```

where tau is a real scalar, and v is a real vector with v(m-k+i+1:m) = 0 and v(m-k+i) = 1; v(1:m-k+i-1) is stored on exit in A(1:m-k+i-1,n-k+i), and tau in TAU(i).

Contents

NAME

```
sgeqp3 - compute a QR factorization with column pivoting  of
a matrix A
```

SYNOPSIS

```
SUBROUTINE SGEQP3(M, N, A, LDA, JPVT, TAU, WORK, LWORK, INFO)

INTEGER M, N, LDA, LWORK, INFO
INTEGER JPVT(*)
REAL A(LDA,*), TAU(*), WORK(*)

SUBROUTINE SGEQP3_64(M, N, A, LDA, JPVT, TAU, WORK, LWORK, INFO)

INTEGER*8 M, N, LDA, LWORK, INFO
INTEGER*8 JPVT(*)
REAL A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEQP3([M], [N], A, [LDA], JPVT, TAU, [WORK], [LWORK],
       [INFO])

INTEGER :: M, N, LDA, LWORK, INFO
INTEGER, DIMENSION(:) :: JPVT
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE GEQP3_64([M], [N], A, [LDA], JPVT, TAU, [WORK], [LWORK],
       [INFO])

INTEGER(8) :: M, N, LDA, LWORK, INFO
INTEGER(8), DIMENSION(:) :: JPVT
```

```
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void sgeqp3(int m, int n, float  *a,  int  lda,  int  *jpvt,
          float *tau, int *info);

void sgeqp3_64(long m, long n,  float  *a,  long  lda,  long
          *jpvt, float *tau, long *info);
```

PURPOSE

sgeqp3 computes a QR factorization with column pivoting of a matrix A: A*P = Q*R using Level 3 BLAS.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, the upper triangle of the array contains the min(M,N)-by-N upper trapezoidal matrix R; the elements below the diagonal, together with the array TAU, represent the orthogonal matrix Q as a product of min(M,N) elementary reflectors.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

JPVT (input/output)
On entry, if JPVT(J).ne.0, the J-th column of A is permuted to the front of A*P (a leading column); if JPVT(J)=0, the J-th column of A is a free column.   On exit, if JPVT(J)=K, then the J-th column of A*P was the the K-th column of A.

TAU (output)
The scalar factors of the elementary reflectors.

WORK (workspace)
On exit, if INFO=0, WORK(1) returns the optimal

```
          LWORK.

LWORK (input)
          The dimension of the array WORK. LWORK  >=  3*N+1.
          For  optimal  performance LWORK >= 2*N+( N+1 )*NB,
          where NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0: successful exit.
          < 0: if INFO = -i, the i-th argument had an  ille-
          gal value.
```

FURTHER DETAILS

```
The matrix Q is  represented  as  a  product  of  elementary
reflectors

   Q = H(1) H(2) . . . H(k), where k = min(m,n).

Each H(i) has the form

   H(i) = I - tau * v * v'

where tau is a real/complex scalar, and v is a  real/complex
vector with v(1:i-1) = 0 and v(i) = 1; v(i+1:m) is stored on
exit in A(i+1:m,i), and tau in TAU(i).

Based on contributions by
  G. Quintana-Orti, Depto. de Informatica, Universidad Jaime
I, Spain
  X. Sun, Computer Science Dept., Duke University, USA
```

Contents

NAME

```
sgeqpf - routine is deprecated and has been replaced by rou-
tine SGEQP3
```

SYNOPSIS

```
SUBROUTINE SGEQPF(M, N, A, LDA, JPIVOT, TAU, WORK, INFO)

INTEGER M, N, LDA, INFO
INTEGER JPIVOT(*)
REAL A(LDA,*), TAU(*), WORK(*)

SUBROUTINE SGEQPF_64(M, N, A, LDA, JPIVOT, TAU, WORK, INFO)

INTEGER*8 M, N, LDA, INFO
INTEGER*8 JPIVOT(*)
REAL A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEQPF([M], [N], A, [LDA], JPIVOT, TAU, [WORK], [INFO])

INTEGER :: M, N, LDA, INFO
INTEGER, DIMENSION(:) :: JPIVOT
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE GEQPF_64([M], [N], A, [LDA], JPIVOT, TAU, [WORK], [INFO])

INTEGER(8) :: M, N, LDA, INFO
INTEGER(8), DIMENSION(:) :: JPIVOT
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void sgeqpf(int m, int n, float *a, int  lda,  int  *jpivot,
          float *tau, int *info);

void sgeqpf_64(long m, long n,  float  *a,  long  lda,  long
          *jpivot, float *tau, long *info);
```

PURPOSE

sgeqpf routine is deprecated and has been replaced by routine SGEQP3.

SGEQPF computes a QR factorization with column pivoting of a real M-by-N matrix A: A*P = Q*R.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0

A (input/output)
On entry, the M-by-N matrix A. On exit, the upper triangle of the array contains the min(M,N)-by-N upper triangular matrix R; the elements below the diagonal, together with the array TAU, represent the orthogonal matrix Q as a product of min(m,n) elementary reflectors.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

JPIVOT (input/output)
On entry, if JPIVOT(i) .ne. 0, the i-th column of A is permuted to the front of A*P (a leading column); if JPIVOT(i) = 0, the i-th column of A is a free column. On exit, if JPIVOT(i) = k, then the i-th column of A*P was the k-th column of A.

TAU (output)
The scalar factors of the elementary reflectors.

WORK (workspace)

```
          dimension(N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

FURTHER DETAILS

```
The matrix Q is  represented  as  a  product  of  elementary
reflectors

   Q = H(1) H(2) . . . H(n)

Each H(i) has the form
   H = I - tau * v * v'

where tau is a real scalar, and v is a real vector with
v(1:i-1) = 0 and v(i) = 1; v(i+1:m) is  stored  on  exit  in
A(i+1:m,i).

The matrix P is represented in jpvt as follows: If
   jpvt(j) = i
then the jth column of P is the ith canonical unit vector.
```

Contents

NAME

```
sgeqrf - compute a QR factorization of a real M-by-N  matrix
A
```

SYNOPSIS

```
SUBROUTINE SGEQRF(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER M, N, LDA, LDWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)

SUBROUTINE SGEQRF_64(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER*8 M, N, LDA, LDWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEQRF([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER :: M, N, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE GEQRF_64([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER(8) :: M, N, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void sgeqrf(int m, int n, float *a, int lda, float *tau, int
          *info);

void sgeqrf_64(long m, long n, float  *a,  long  lda,  float
          *tau, long *info);
```

PURPOSE

```
sgeqrf computes a QR factorization of a real  M-by-N  matrix
A:  A = Q * R.
```

ARGUMENTS

```
M (input) The number of rows of the matrix A.  M >= 0.
N (input) The number of columns of the matrix A.  N >= 0.

A (input/output)
          On entry, the M-by-N matrix A.  On exit, the  ele-
          ments  on and above the diagonal of the array con-
          tain the min(M,N)-by-N upper trapezoidal matrix  R
          (R  is  upper  triangular if m >= n); the elements
          below the diagonal, with the array TAU,  represent
          the  orthogonal  matrix Q as a product of min(m,n)
          elementary reflectors (see Further Details).

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,M).

TAU (output)
          The scalar factors of  the  elementary  reflectors
          (see Further Details).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LDWORK.

LDWORK (input)
          The  dimension  of  the  array  WORK.   LDWORK  >=
          max(1,N).  For optimum performance LDWORK >= N*NB,
          where NB is the optimal blocksize.

          If LDWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LDWORK is issued by XERBLA.
```

```
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

FURTHER DETAILS

```
The matrix Q is  represented  as  a  product  of  elementary
reflectors

   Q = H(1) H(2) . . . H(k), where k = min(m,n).

Each H(i) has the form
   H(i) = I - tau * v * v'

where tau is a real scalar, and v is a real vector with
v(1:i-1) = 0 and v(i) = 1; v(i+1:m) is  stored  on  exit  in
A(i+1:m,i), and tau in TAU(i).
```

Contents

NAME

```
sger - perform the rank 1 operation   A := alpha*x*y' + A
```

SYNOPSIS

```
SUBROUTINE SGER(M, N, ALPHA, X, INCX, Y, INCY, A, LDA)

INTEGER M, N, INCX, INCY, LDA
REAL ALPHA
REAL X(*), Y(*), A(LDA,*)

SUBROUTINE SGER_64(M, N, ALPHA, X, INCX, Y, INCY, A, LDA)

INTEGER*8 M, N, INCX, INCY, LDA
REAL ALPHA
REAL X(*), Y(*), A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE GER([M], [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

INTEGER :: M, N, INCX, INCY, LDA
REAL :: ALPHA
REAL, DIMENSION(:) :: X, Y
REAL, DIMENSION(:,:) :: A

SUBROUTINE GER_64([M], [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

INTEGER(8) :: M, N, INCX, INCY, LDA
REAL :: ALPHA
REAL, DIMENSION(:) :: X, Y
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void sger(int m, int n, float alpha,  float  *x,  int  incx,
          float *y, int incy, float *a, int lda);

void sger_64(long m, long n, float  alpha,  float  *x,  long
          incx, float *y, long incy, float *a, long lda);
```

PURPOSE

sger performs the rank 1 operation A := alpha*x*y' + A, where alpha is a scalar, x is an m element vector, y is an n element vector and A is an m by n matrix.

ARGUMENTS

M (input)
On entry, M specifies the number of rows of the matrix A. M >= 0. Unchanged on exit.

N (input)
On entry, N specifies the number of columns of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (m - 1)*abs(INCX)). Before entry, the incremented array X must contain the m element vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

Y (input)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. Unchanged on exit.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

A (input/output)

Before entry, the leading m by n part of the array A must contain the matrix of coefficients. On exit, A is overwritten by the updated matrix.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, m). Unchanged on exit.

Contents

NAME

sgerfs - improve the computed solution to a system of linear equations and provides error bounds and backward error estimates for the solution

SYNOPSIS

```
SUBROUTINE SGERFS(TRANSA, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*), WORK2(*)
REAL  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),  FERR(*),
BERR(*), WORK(*)

SUBROUTINE SGERFS_64(TRANSA, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B,
      LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
REAL  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),  FERR(*),
BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GERFS([TRANSA], [N], [NRHS], A, [LDA], AF, [LDAF], IPIVOT,
       B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: TRANSA
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
REAL, DIMENSION(:) :: FERR, BERR, WORK
```

```
REAL, DIMENSION(:,:) :: A, AF, B, X

SUBROUTINE GERFS_64([TRANSA], [N], [NRHS], A, [LDA], AF, [LDAF],
       IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: TRANSA
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL, DIMENSION(:) :: FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
#include <sunperf.h>
void sgerfs(char transa, int n, int nrhs, float *a, int lda,
          float  *af,  int  ldaf, int *ipivot, float *b, int
          ldb, float *x, int ldx, float *ferr, float  *berr,
          int *info);

void sgerfs_64(char transa, long n,  long  nrhs,  float  *a,
          long  lda,  float  *af,  long  ldaf, long *ipivot,
          float *b, long ldb,  float  *x,  long  ldx,  float
          *ferr, float *berr, long *info);
```

PURPOSE

sgerfs improves the computed solution to a system of linear equations and provides error bounds and backward error estimates for the solution.

ARGUMENTS

TRANSA (input)
Specifies the form of the system of equations:
```
= 'N':  A * X = B     (No transpose)
= 'T':  A**T * X = B  (Transpose)
= 'C':  A**H * X = B  (Conjugate transpose = Tran-
spose)
```

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The original N-by-N matrix A.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input)
The factors L and U from the factorization A = P*L*U as computed by SGETRF.

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

IPIVOT (input)
The pivot indices from SGETRF; for 1<=i<=N, row i of the matrix was interchanged with row IPIVOT(i).

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by SGETRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-
gal value

Contents

NAME

```
sgerqf - compute an RQ factorization of a real M-by-N matrix
A
```

SYNOPSIS

```
SUBROUTINE SGERQF(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER M, N, LDA, LDWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)

SUBROUTINE SGERQF_64(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER*8 M, N, LDA, LDWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GERQF([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER :: M, N, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE GERQF_64([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER(8) :: M, N, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void sgerqf(int m, int n, float *a, int lda, float *tau, int
          *info);

void sgerqf_64(long m, long n, float  *a,  long  lda,  float
          *tau, long *info);
```

PURPOSE

```
sgerqf computes an RQ factorization of a real M-by-N  matrix
A:  A = R * Q.
```

ARGUMENTS

```
M (input) The number of rows of the matrix A.  M >= 0.
N (input) The number of columns of the matrix A.  N >= 0.

A (input/output)
          On entry, the M-by-N matrix A.  On exit, if  m  <=
          n,  the  upper  triangle  of the subarray A(1:m,n-
          m+1:n) contains the M-by-M upper triangular matrix
          R;  if  m  >= n, the elements on and above the (m-
          n)-th subdiagonal contain the  M-by-N  upper  tra-
          pezoidal  matrix  R;  the remaining elements, with
          the array TAU, represent the orthogonal  matrix  Q
          as  a  product  of  min(m,n) elementary reflectors
          (see Further Details).

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,M).

TAU (output)
          The scalar factors of  the  elementary  reflectors
          (see Further Details).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LDWORK.

LDWORK (input)
          The  dimension  of  the  array  WORK.   LDWORK  >=
          max(1,M).  For optimum performance LDWORK >= M*NB,
          where NB is the optimal blocksize.

          If LDWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
```

entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The matrix Q is represented as a product of elementary reflectors

```
   Q = H(1) H(2) . . . H(k), where k = min(m,n).
```

Each H(i) has the form

```
   H(i) = I - tau * v * v'
```

where tau is a real scalar, and v is a real vector with v(n-k+i+1:n) = 0 and v(n-k+i) = 1; v(1:n-k+i-1) is stored on exit in A(m-k+i,1:n-k+i-1), and tau in TAU(i).

Contents

NAME

```
sgesdd - compute the singular value decomposition (SVD) of a
real  M-by-N  matrix  A,  optionally  computing the left and
right singular vectors
```

SYNOPSIS

```
SUBROUTINE SGESDD(JOBZ, M, N, A, LDA, S, U, LDU, VT, LDVT, WORK,
      LWORK, IWORK, INFO)

CHARACTER * 1 JOBZ
INTEGER M, N, LDA, LDU, LDVT, LWORK, INFO
INTEGER IWORK(*)
REAL A(LDA,*), S(*), U(LDU,*), VT(LDVT,*), WORK(*)

SUBROUTINE SGESDD_64(JOBZ, M, N, A, LDA, S, U, LDU, VT, LDVT, WORK,
      LWORK, IWORK, INFO)

CHARACTER * 1 JOBZ
INTEGER*8 M, N, LDA, LDU, LDVT, LWORK, INFO
INTEGER*8 IWORK(*)
REAL A(LDA,*), S(*), U(LDU,*), VT(LDVT,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GESDD(JOBZ, [M], [N], A, [LDA], S, U, [LDU], VT, [LDVT],
       [WORK], [LWORK], [IWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ
INTEGER :: M, N, LDA, LDU, LDVT, LWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: S, WORK
REAL, DIMENSION(:,:) :: A, U, VT
```

```
SUBROUTINE GESDD_64(JOBZ, [M], [N], A, [LDA], S, U, [LDU], VT, [LDVT],
       [WORK], [LWORK], [IWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ
INTEGER(8) :: M, N, LDA, LDU, LDVT, LWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: S, WORK
REAL, DIMENSION(:,:) :: A, U, VT
```

C INTERFACE

```
#include <sunperf.h>

void sgesdd(char jobz, int m, int  n,  float  *a,  int  lda,
          float  *s, float *u, int ldu, float *vt, int ldvt,
          int *info);
void sgesdd_64(char jobz, long m, long  n,  float  *a,  long
          lda, float *s, float *u, long ldu, float *vt, long
          ldvt, long *info);
```

PURPOSE

sgesdd computes the singular value decomposition (SVD) of a real M-by-N matrix A, optionally computing the left and right singular vectors. If singular vectors are desired, it uses a divide-and-conquer algorithm.

The SVD is written
 = U * SIGMA * transpose(V)

where SIGMA is an M-by-N matrix which is zero except for its min(m,n) diagonal elements, U is an M-by-M orthogonal matrix, and V is an N-by-N orthogonal matrix. The diagonal elements of SIGMA are the singular values of A; they are real and non-negative, and are returned in descending order. The first min(m,n) columns of U and V are the left and right singular vectors of A.

Note that the routine returns VT = V**T, not V.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

```
JOBZ (input)
          Specifies options for computing all or part of the
          matrix U:
          = 'A':  all M columns of U and all N rows of  V**T
          are  returned  in the arrays U and VT; = 'S':  the
          first min(M,N) columns of U and the first min(M,N)
          rows  of V**T are returned in the arrays U and VT;
          = 'O':  If M >= N, the first N columns  of  U  are
          overwritten  on  the  array A and all rows of V**T
          are returned  in  the  array  VT;  otherwise,  all
          columns  of  U are returned in the array U and the
          first M rows of V**T are overwritten in the  array
          VT;  =  'N':   no columns of U or rows of V**T are
          computed.

M (input) The number of rows of the input matrix A.  M >= 0.
N (input) The number of columns of the input matrix A.  N >=
          0.

A (input/output)
          On entry, the M-by-N matrix A.  On exit, if JOBZ =
          'O',  A is overwritten with the first N columns of
          U (the left singular vectors,  stored  columnwise)
          if  M >= N; A is overwritten with the first M rows
          of V**T (the right singular vectors,  stored  row-
          wise)  otherwise.   if JOBZ .ne. 'O', the contents
          of A are destroyed.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,M).

S (output)
          The singular values of A, sorted so that  S(i)  >=
          S(i+1).

U (output)
          UCOL = M if JOBZ = 'A' or JOBZ = 'O' and  M  <  N;
          UCOL  =  min(M,N) if JOBZ = 'S'.  If JOBZ = 'A' or
          JOBZ = 'O' and  M  <  N,  U  contains  the  M-by-M
          orthogonal matrix U; if JOBZ = 'S', U contains the
          first min(M,N) columns of  U  (the  left  singular
          vectors,  stored  columnwise); if JOBZ = 'O' and M
          >= N, or JOBZ = 'N', U is not referenced.

LDU (input)
          The leading dimension of the array U.  LDU  >=  1;
```

if JOBZ = 'S' or 'A' or JOBZ = 'O' and M < N, LDU >= M.

VT (output)

If JOBZ = 'A' or JOBZ = 'O' and M >= N, VT contains the N-by-N orthogonal matrix V**T; if JOBZ = 'S', VT contains the first min(M,N) rows of V**T (the right singular vectors, stored rowwise); if JOBZ = 'O' and M < N, or JOBZ = 'N', VT is not referenced.

LDVT (input)

The leading dimension of the array VT. LDVT >= 1; if JOBZ = 'A' or JOBZ = 'O' and M >= N, LDVT >= N; if JOBZ = 'S', LDVT >= min(M,N).

WORK (workspace)

On exit, if INFO = 0, WORK(1) returns the optimal LWORK;

LWORK (input)

The dimension of the array WORK. LWORK >= 1. If JOBZ = 'N', LWORK >= 3*min(M,N) + max(max(M,N),6*min(M,N)). If JOBZ = 'O', LWORK >= 3*min(M,N)*min(M,N) + max(max(M,N),5*min(M,N)* min(M,N)+4*min(M,N)). If JOBZ = 'S' or 'A' LWORK >= 3*min(M,N)*min(M,N) + max(max(M,N),4*min(M,N)* min(M,N)+4*min(M,N)). For good performance, LWORK should generally be larger. If LWORK < 0 but other input arguments are legal, WORK(1) returns optimal LWORK.

IWORK (workspace)

dimension(8*MIN(M,N))

INFO (output)

= 0: successful exit.

< 0: if INFO = -i, the i-th argument had an illegal value.

> 0: SBDSDC did not converge, updating process failed.

FURTHER DETAILS

Based on contributions by
Ming Gu and Huan Ren, Computer Science Division, University of
California at Berkeley, USA

Contents

NAME

```
sgesv - compute the solution to  a  real  system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE SGESV(N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

INTEGER N, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)
REAL A(LDA,*), B(LDB,*)

SUBROUTINE SGESV_64(N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

INTEGER*8 N, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
REAL A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE GESV([N], [NRHS], A, [LDA], IPIVOT, B, [LDB], [INFO])

INTEGER :: N, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE GESV_64([N], [NRHS], A, [LDA], IPIVOT, B, [LDB], [INFO])

INTEGER(8) :: N, NRHS, LDA, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>
```

```
void sgesv(int n, int nrhs, float *a, int lda, int  *ipivot,
          float *b, int ldb, int *info);

void sgesv_64(long n, long nrhs, float *a,  long  lda,  long
          *ipivot, float *b, long ldb, long *info);
```

PURPOSE

sgesv computes the solution to a real system of linear equations

A * X = B, where A is an N-by-N matrix and X and B are N-by-NRHS matrices.

The LU decomposition with partial pivoting and row interchanges is used to factor A as

A = P * L * U,

where P is a permutation matrix, L is unit lower triangular, and U is upper triangular. The factored form of A is then used to solve the system of equations A * X = B.

ARGUMENTS

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input/output)
On entry, the N-by-N coefficient matrix A. On exit, the factors L and U from the factorization A = P*L*U; the unit diagonal elements of L are not stored.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (output)
The pivot indices that define the permutation matrix P; row i of the matrix was interchanged with row IPIVOT(i).

B (input/output)
On entry, the N-by-NRHS matrix of right hand side

matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution could not be computed.

Contents

NAME

```
sgesvd - compute the singular value decomposition (SVD) of a
real  M-by-N  matrix A, optionally computing the left and/or
right singular vectors
```

SYNOPSIS

```
SUBROUTINE SGESVD(JOBU, JOBVT, M, N, A, LDA, SING, U, LDU, VT, LDVT,
      WORK, LDWORK, INFO)

CHARACTER * 1 JOBU, JOBVT
INTEGER M, N, LDA, LDU, LDVT, LDWORK, INFO
REAL A(LDA,*), SING(*), U(LDU,*), VT(LDVT,*), WORK(*)

SUBROUTINE SGESVD_64(JOBU, JOBVT, M, N, A, LDA, SING, U, LDU, VT,
      LDVT, WORK, LDWORK, INFO)

CHARACTER * 1 JOBU, JOBVT
INTEGER*8 M, N, LDA, LDU, LDVT, LDWORK, INFO
REAL A(LDA,*), SING(*), U(LDU,*), VT(LDVT,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GESVD(JOBU, JOBVT, [M], [N], A, [LDA], SING, U, [LDU], VT,
       [LDVT], [WORK], [LDWORK], [INFO])

CHARACTER(LEN=1) :: JOBU, JOBVT
INTEGER :: M, N, LDA, LDU, LDVT, LDWORK, INFO
REAL, DIMENSION(:) :: SING, WORK
REAL, DIMENSION(:,:) :: A, U, VT

SUBROUTINE GESVD_64(JOBU, JOBVT, [M], [N], A, [LDA], SING, U, [LDU],
       VT, [LDVT], [WORK], [LDWORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOBU, JOBVT
INTEGER(8) :: M, N, LDA, LDU, LDVT, LDWORK, INFO
REAL, DIMENSION(:) :: SING, WORK
REAL, DIMENSION(:,:) :: A, U, VT
```

C INTERFACE

```
#include <sunperf.h>

void sgesvd(char jobu, char jobvt, int m, int n,  float  *a,
          int  lda,  float  *sing,  float *u, int ldu, float
          *vt, int ldvt, int *info);

void sgesvd_64(char jobu, char jobvt, long m, long n,  float
          *a,  long  lda,  float  *sing, float *u, long ldu,
          float *vt, long ldvt, long *info);
```

PURPOSE

sgesvd computes the singular value decomposition (SVD) of a real M-by-N matrix A, optionally computing the left and/or right singular vectors. The SVD is written
 = U * SIGMA * transpose(V)

where SIGMA is an M-by-N matrix which is zero except for its min(m,n) diagonal elements, U is an M-by-M orthogonal matrix, and V is an N-by-N orthogonal matrix. The diagonal elements of SIGMA are the singular values of A; they are real and non-negative, and are returned in descending order. The first min(m,n) columns of U and V are the left and right singular vectors of A.

Note that the routine returns V**T, not V.

ARGUMENTS

JOBU (input)
Specifies options for computing all or part of the matrix U:
= 'A': all M columns of U are returned in array U:
= 'S': the first min(m,n) columns of U (the left singular vectors) are returned in the array U; = 'O': the first min(m,n) columns of U (the left singular vectors) are overwritten on the array A;
= 'N': no columns of U (no left singular vectors) are computed.

```
JOBVT (input)
          Specifies options for computing all or part of the
          matrix V**T:
          = 'A':  all N rows of V**T  are  returned  in  the
          array VT;
          = 'S':  the first min(m,n) rows of V**T (the right
          singular  vectors) are returned in the array VT; =
          'O':  the first min(m,n) rows of V**T  (the  right
          singular  vectors) are overwritten on the array A;
          = 'N':  no rows of V**T (no  right  singular  vec-
          tors) are computed.

          JOBVT and JOBU cannot both be 'O'.

M (input) The number of rows of the input matrix A.  M >= 0.

N (input) The number of columns of the input matrix A.  N >=
          0.
A (input/output)
          On entry, the M-by-N matrix A.  On exit, if JOBU =
          'O',   A  is  overwritten  with the first min(m,n)
          columns of U (the left  singular  vectors,  stored
          columnwise); if JOBVT = 'O', A is overwritten with
          the first min(m,n) rows of V**T (the right  singu-
          lar vectors, stored rowwise); if JOBU .ne. 'O' and
          JOBVT .ne. 'O', the contents of A are destroyed.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,M).

SING (output)
          The singular values of A, sorted so  that  SING(i)
          >= SING(i+1).

U (input) (LDU,M) if JOBU = 'A' or (LDU,min(M,N)) if JOBU  =
          'S'.  If JOBU = 'A', U contains the M-by-M orthog-
          onal matrix U; if JOBU = 'S', U contains the first
          min(m,n)  columns of U (the left singular vectors,
          stored columnwise); if JOBU = 'N' or 'O', U is not
          referenced.

LDU (input)
          The leading dimension of the array U.  LDU  >=  1;
          if JOBU = 'S' or 'A', LDU >= M.

VT (input)
          If JOBVT = 'A', VT contains the N-by-N  orthogonal
          matrix V**T; if JOBVT = 'S', VT contains the first
          min(m,n) rows of V**T (the right singular vectors,
```

stored rowwise); if JOBVT = 'N' or 'O', VT is not referenced.

LDVT (input)
The leading dimension of the array VT. LDVT >= 1; if JOBVT = 'A', LDVT >= N; if JOBVT = 'S', LDVT >= min(M,N).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK; if INFO > 0, WORK(2:MIN(M,N)) contains the unconverged superdiagonal elements of an upper bidiagonal matrix B whose diagonal is in SING (not necessarily sorted). B satisfies A = U * B * VT, so it has the same singular values as A, and singular vectors related by U and VT.

LDWORK (input)
The dimension of the array WORK. LDWORK >= 1. LDWORK >= MAX(3*MIN(M,N)+MAX(M,N),5*MIN(M,N)). For good performance, LDWORK should generally be larger.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if SBDSQR did not converge, INFO specifies how many superdiagonals of an intermediate bidiagonal form B did not converge to zero. See the description of WORK above for details.

Contents

NAME

```
sgesvx - use the LU factorization to compute the solution to
a real system of linear equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE SGESVX(FACT, TRANSA, N, NRHS, A, LDA, AF, LDAF, IPIVOT,
      EQUED, R, C, B, LDB, X, LDX, RCOND, FERR, BERR, WORK,
      WORK2, INFO)

CHARACTER * 1 FACT, TRANSA, EQUED
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*), WORK2(*)
REAL RCOND
REAL A(LDA,*), AF(LDAF,*), R(*), C(*),  B(LDB,*),  X(LDX,*),
FERR(*), BERR(*), WORK(*)

SUBROUTINE SGESVX_64(FACT, TRANSA, N, NRHS, A, LDA, AF, LDAF, IPIVOT,
      EQUED, R, C, B, LDB, X, LDX, RCOND, FERR, BERR, WORK,
      WORK2, INFO)

CHARACTER * 1 FACT, TRANSA, EQUED
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
REAL RCOND
REAL A(LDA,*), AF(LDAF,*), R(*), C(*),  B(LDB,*),  X(LDX,*),
FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GESVX(FACT, [TRANSA], [N], [NRHS], A, [LDA], AF, [LDAF],
       IPIVOT, EQUED, R, C, B, [LDB], X, [LDX], RCOND, FERR,
       BERR, [WORK], [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: FACT, TRANSA, EQUED
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
REAL :: RCOND
REAL, DIMENSION(:) :: R, C, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, AF, B, X

SUBROUTINE GESVX_64(FACT, [TRANSA], [N], [NRHS], A, [LDA], AF, [LDAF],
       IPIVOT, EQUED, R, C, B, [LDB], X, [LDX], RCOND, FERR,
       BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, TRANSA, EQUED
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL :: RCOND
REAL, DIMENSION(:) :: R, C, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
#include <sunperf.h>

void sgesvx(char fact, char transa, int n, int  nrhs,  float
          *a,  int  lda,  float  *af, int ldaf, int *ipivot,
          char equed, float *r, float *c, float *b, int ldb,
          float  *x,  int  ldx,  float  *rcond, float *ferr,
          float *berr, int *info);

void sgesvx_64(char fact, char transa, long  n,  long  nrhs,
          float  *a,  long  lda,  float *af, long ldaf, long
          *ipivot, char equed, float *r, float *c, float *b,
          long  ldb, float *x, long ldx, float *rcond, float
          *ferr, float *berr, long *info);
```

PURPOSE

```
sgesvx uses the LU factorization to compute the solution  to
a real system of linear equations
   A * X = B, where A is an N-by-N matrix and X  and  B  are
N-by-NRHS matrices.

Error bounds on the solution and a  condition  estimate  are
also provided.

The following steps are performed:

1. If FACT = 'E',  real  scaling  factors  are  computed  to
equilibrate
   the system:
      TRANS = 'N':  diag(R)*A*diag(C)     *inv(diag(C))*X  =
diag(R)*B
```

```
      TRANS = 'T': (diag(R)*A*diag(C))**T *inv(diag(R))*X  =
diag(C)*B
      TRANS = 'C': (diag(R)*A*diag(C))**H *inv(diag(R))*X  =
diag(C)*B
   Whether or not the system will be equilibrated depends on
the
   scaling of the matrix A, but if equilibration is used,  A
is
   overwritten by diag(R)*A*diag(C) and B by  diag(R)*B  (if
TRANS='N')
   or diag(C)*B (if TRANS = 'T' or 'C').

2. If FACT = 'N' or 'E', the LU  decomposition  is  used  to
factor the
   matrix A (after equilibration if FACT = 'E') as
      A = P * L * U,
   where P is a permutation matrix, L is a unit  lower  tri-
angular
   matrix, and U is upper triangular.
3. If some U(i,i)=0, so that U is exactly singular, then the
routine
   returns with INFO = i. Otherwise, the factored form of  A
is used
   to estimate the condition number of the matrix A.  If the
   reciprocal of the condition number is less  than  machine
precision,
   INFO = N+1 is returned as  a  warning,  but  the  routine
still goes on
   to solve for X and  compute  error  bounds  as  described
below.

4. The system of equations is solved for X  using  the  fac-
tored form
   of A.

5. Iterative refinement is applied to improve  the  computed
solution
   matrix and calculate  error  bounds  and  backward  error
estimates
   for it.

6. If equilibration was used, the matrix X is  premultiplied
by
   diag(C) (if TRANS = 'N') or diag(R) (if TRANS  =  'T'  or
'C') so
   that it solves the original system before equilibration.
```

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored. = 'F': On entry, AF and IPIVOT contain the factored form of A. If EQUED is not 'N', the matrix A has been equilibrated with scaling factors given by R and C. A, AF, and IPIVOT are not modified. = 'N': The matrix A will be copied to AF and factored.
= 'E': The matrix A will be equilibrated if necessary, then copied to AF and factored.

TRANSA (input)
Specifies the form of the system of equations:
= 'N': A * X = B (No transpose)
= 'T': A**T * X = B (Transpose)
= 'C': A**H * X = B (Transpose)

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input/output)
On entry, the N-by-N matrix A. If FACT = 'F' and EQUED is not 'N', then A must have been equilibrated by the scaling factors in R and/or C. A is not modified if FACT = 'F' or 'N', or if FACT = 'E' and EQUED = 'N' on exit.

On exit, if EQUED .ne. 'N', A is scaled as follows: EQUED = 'R': A := diag(R) * A
EQUED = 'C': A := A * diag(C)
EQUED = 'B': A := diag(R) * A * diag(C).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input/output)
If FACT = 'F', then AF is an input argument and on entry contains the factors L and U from the factorization A = P*L*U as computed by SGETRF. If EQUED .ne. 'N', then AF is the factored form of the equilibrated matrix A.

If FACT = 'N', then AF is an output argument and on exit returns the factors L and U from the factorization A = P*L*U of the original matrix A.

If FACT = 'E', then AF is an output argument and on exit returns the factors L and U from the factorization A = P*L*U of the equilibrated matrix A (see the description of A for the form of the equilibrated matrix).

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

IPIVOT (input or output)
If FACT = 'F', then IPIVOT is an input argument and on entry contains the pivot indices from the factorization A = P*L*U as computed by SGETRF; row i of the matrix was interchanged with row IPIVOT(i).

If FACT = 'N', then IPIVOT is an output argument and on exit contains the pivot indices from the factorization A = P*L*U of the original matrix A.

If FACT = 'E', then IPIVOT is an output argument and on exit contains the pivot indices from the factorization A = P*L*U of the equilibrated matrix A.

EQUED (input/output)
Specifies the form of equilibration that was done. = 'N': No equilibration (always true if FACT = 'N').
= 'R': Row equilibration, i.e., A has been premultiplied by diag(R). = 'C': Column equilibration, i.e., A has been postmultiplied by diag(C). = 'B': Both row and column equilibration, i.e., A has been replaced by diag(R) * A * diag(C). EQUED is an input argument if FACT = 'F'; otherwise, it is an output argument.

R (input/output)
The row scale factors for A. If EQUED = 'R' or 'B', A is multiplied on the left by diag(R); if EQUED = 'N' or 'C', R is not accessed. R is an input argument if FACT = 'F'; otherwise, R is an output argument. If FACT = 'F' and EQUED = 'R' or 'B', each element of R must be positive.

C (input/output)

The column scale factors for A. If EQUED = 'C' or 'B', A is multiplied on the right by diag(C); if EQUED = 'N' or 'R', C is not accessed. C is an input argument if FACT = 'F'; otherwise, C is an output argument. If FACT = 'F' and EQUED = 'C' or 'B', each element of C must be positive.

B (input/output)

On entry, the N-by-NRHS right hand side matrix B. On exit, if EQUED = 'N', B is not modified; if TRANSA = 'N' and EQUED = 'R' or 'B', B is overwritten by diag(R)*B; if TRANSA = 'T' or 'C' and EQUED = 'C' or 'B', B is overwritten by diag(C)*B.

LDB (input)

The leading dimension of the array B. LDB >= max(1,N).

X (output)

If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X to the original system of equations. Note that A and B are modified on exit if EQUED .ne. 'N', and the solution to the equilibrated system is inv(diag(C))*X if TRANSA = 'N' and EQUED = 'C' or 'B', or inv(diag(R))*X if TRANSA = 'T' or 'C' and EQUED = 'R' or 'B'.

LDX (input)

The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)

The estimate of the reciprocal condition number of the matrix A after equilibration (if done). If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)

The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(4*N) On exit, WORK(1) contains the reciprocal pivot growth factor norm(A)/norm(U). The "max absolute element" norm is used. If WORK(1) is much less than 1, then the stability of the LU factorization of the (equilibrated) matrix A could be poor. This also means that the solution X, condition estimator RCOND, and forward error bound FERR could be unreliable. If factorization fails with 0<INFO<=N, then WORK(1) contains the reciprocal pivot growth factor for the leading INFO columns of A.

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille- gal value
> 0: if INFO = i, and i is
<= N: U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
sgetf2 - compute an LU factorization  of  a  general  m-by-n
matrix A using partial pivoting with row interchanges
```

SYNOPSIS

```
SUBROUTINE SGETF2(M, N, A, LDA, IPIV, INFO)

INTEGER M, N, LDA, INFO
INTEGER IPIV(*)
REAL A(LDA,*)

SUBROUTINE SGETF2_64(M, N, A, LDA, IPIV, INFO)

INTEGER*8 M, N, LDA, INFO
INTEGER*8 IPIV(*)
REAL A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE GETF2([M], [N], A, [LDA], IPIV, [INFO])

INTEGER :: M, N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIV
REAL, DIMENSION(:,:) :: A

SUBROUTINE GETF2_64([M], [N], A, [LDA], IPIV, [INFO])

INTEGER(8) :: M, N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIV
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void sgetf2(int m, int n, float *a, int lda, int *ipiv,  int
          *info);

void sgetf2_64(long m, long n,  float  *a,  long  lda,  long
          *ipiv, long *info);
```

PURPOSE

sgetf2 computes an LU factorization of a general m-by-n matrix A using partial pivoting with row interchanges.

The factorization has the form
   A = P * L * U
where P is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if m > n), and U is upper triangular (upper trapezoidal if m < n).

This is the right-looking Level 2 BLAS version of the algorithm.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the m by n matrix to be factored. On exit, the factors L and U from the factorization A = P*L*U; the unit diagonal elements of L are not stored.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

IPIV (output)
The pivot indices; for 1 <= i <= min(M,N), row i of the matrix was interchanged with row IPIV(i).

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, U(k,k) is exactly zero. The factorization has been completed, but the factor U is

exactly singular, and division by zero will occur if it is used to solve a system of equations.

Contents

NAME

```
sgetrf - compute an LU factorization  of  a  general  M-by-N
matrix A using partial pivoting with row interchanges
```

SYNOPSIS

```
SUBROUTINE SGETRF(M, N, A, LDA, IPIVOT, INFO)

INTEGER M, N, LDA, INFO
INTEGER IPIVOT(*)
REAL A(LDA,*)

SUBROUTINE SGETRF_64(M, N, A, LDA, IPIVOT, INFO)

INTEGER*8 M, N, LDA, INFO
INTEGER*8 IPIVOT(*)
REAL A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE GETRF([M], [N], A, [LDA], IPIVOT, [INFO])

INTEGER :: M, N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:,:) :: A

SUBROUTINE GETRF_64([M], [N], A, [LDA], IPIVOT, [INFO])

INTEGER(8) :: M, N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void sgetrf(int m, int n, float *a, int  lda,  int  *ipivot,
          int *info);

void sgetrf_64(long m, long n,  float  *a,  long  lda,  long
          *ipivot, long *info);
```

PURPOSE

```
sgetrf computes an LU  factorization  of  a  general  M-by-N
matrix A using partial pivoting with row interchanges.

The factorization has the form
   A = P * L * U
where P is a permutation matrix, L is lower triangular  with
unit  diagonal  elements (lower trapezoidal if m > n), and U
is upper triangular (upper trapezoidal if m < n).

This is the right-looking Level 3 BLAS version of the  algo-
rithm.
```

ARGUMENTS

```
M (input) The number of rows of the matrix A.  M >= 0.

N (input) The number of columns of the matrix A.  N >= 0.

A (input/output)
          On entry, the M-by-N matrix to  be  factored.   On
          exit, the factors L and U from the factorization A
          = P*L*U; the unit diagonal elements of L  are  not
          stored.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,M).

IPIVOT (output)
          The pivot indices; for 1 <= i <= min(M,N),  row  i
          of the matrix was interchanged with row IPIVOT(i).

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, U(i,i)  is  exactly  zero.  The
          factorization has been completed, but the factor U
```

is exactly singular, and division by zero will occur if it is used to solve a system of equations.

Contents

NAME

```
sgetri - compute the inverse of a matrix using the  LU  fac-
torization computed by SGETRF
```

SYNOPSIS

```
SUBROUTINE SGETRI(N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

INTEGER N, LDA, LDWORK, INFO
INTEGER IPIVOT(*)
REAL A(LDA,*), WORK(*)

SUBROUTINE SGETRI_64(N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

INTEGER*8 N, LDA, LDWORK, INFO
INTEGER*8 IPIVOT(*)
REAL A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GETRI([N], A, [LDA], IPIVOT, [WORK], [LDWORK], [INFO])

INTEGER :: N, LDA, LDWORK, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE GETRI_64([N], A, [LDA], IPIVOT, [WORK], [LDWORK], [INFO])

INTEGER(8) :: N, LDA, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void sgetri(int n, float  *a,  int  lda,  int  *ipivot,  int
          *info);

void sgetri_64(long n, float *a,  long  lda,  long  *ipivot,
          long *info);
```

PURPOSE

sgetri computes the inverse of a matrix using the LU factor-
ization computed by SGETRF.

This method inverts U and then computes inv(A) by solving
the system inv(A)*L = inv(U) for inv(A).

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the factors L and U from the factoriza-
tion A = P*L*U as computed by SGETRF. On exit, if
INFO = 0, the inverse of the original matrix A.

LDA (input)
The leading dimension of the array A.   LDA >=
max(1,N).

IPIVOT (input)
The pivot indices from SGETRF; for 1<=i<=N, row i
of the matrix was interchanged with row IPIVOT(i).

WORK (workspace)
On exit, if INFO=0, then WORK(1) returns the
optimal LDWORK.

LDWORK (input)
The dimension of the array WORK.   LDWORK >=
max(1,N). For optimal performance LDWORK >= N*NB,
where NB is the optimal blocksize returned by
ILAENV.

If LDWORK = -1, then a workspace query is assumed;
the routine only calculates the optimal size of
the WORK array, returns this value as the first

entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, U(i,i) is exactly zero; the matrix is singular and its inverse could not be computed.

Contents

NAME

```
sgetrs - solve a system of linear equations  A * X = B or A'
* X = B with a general N-by-N matrix A using the LU factori-
zation computed by SGETRF
```

SYNOPSIS

```
SUBROUTINE SGETRS(TRANSA, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 TRANSA
INTEGER N, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)
REAL A(LDA,*), B(LDB,*)

SUBROUTINE SGETRS_64(TRANSA, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 TRANSA
INTEGER*8 N, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
REAL A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE GETRS([TRANSA], [N], [NRHS], A, [LDA], IPIVOT, B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: TRANSA
INTEGER :: N, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE GETRS_64([TRANSA], [N], [NRHS], A, [LDA], IPIVOT, B, [LDB],
       [INFO])
```

```
CHARACTER(LEN=1) :: TRANSA
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void sgetrs(char transa, int n, int nrhs, float *a, int lda,
          int *ipivot, float *b, int ldb, int *info);

void sgetrs_64(char transa, long n,  long  nrhs,  float  *a,
          long  lda,  long *ipivot, float *b, long ldb, long
          *info);
```

PURPOSE

```
sgetrs solves a system of linear equations
   A * X = B  or  A' * X = B with a general N-by-N matrix  A
using the LU factorization computed by SGETRF.
```

ARGUMENTS

```
TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B  (No transpose)
          = 'T':  A'* X = B  (Transpose)
          = 'C':  A'* X = B  (Conjugate  transpose  =  Tran-
          spose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

A (input) The factors L and U from  the  factorization  A  =
          P*L*U as computed by SGETRF.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

IPIVOT (input)
          The pivot indices from SGETRF; for 1<=i<=N, row  i
```

of the matrix was interchanged with row IPIVOT(i).

B (input/output)
On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
sggbak - form the right or left eigenvectors of a real  gen-
eralized  eigenvalue  problem  A*x = lambda*B*x, by backward
transformation on the computed eigenvectors of the  balanced
pair of matrices output by SGGBAL
```

SYNOPSIS

```
SUBROUTINE SGGBAK(JOB, SIDE, N, ILO, IHI, LSCALE, RSCALE, M, V, LDV,
      INFO)

CHARACTER * 1 JOB, SIDE
INTEGER N, ILO, IHI, M, LDV, INFO
REAL LSCALE(*), RSCALE(*), V(LDV,*)

SUBROUTINE SGGBAK_64(JOB, SIDE, N, ILO, IHI, LSCALE, RSCALE, M, V,
      LDV, INFO)

CHARACTER * 1 JOB, SIDE
INTEGER*8 N, ILO, IHI, M, LDV, INFO
REAL LSCALE(*), RSCALE(*), V(LDV,*)
```

F95 INTERFACE

```
SUBROUTINE GGBAK(JOB, SIDE, [N], ILO, IHI, LSCALE, RSCALE, [M], V,
       [LDV], [INFO])

CHARACTER(LEN=1) :: JOB, SIDE
INTEGER :: N, ILO, IHI, M, LDV, INFO
REAL, DIMENSION(:) :: LSCALE, RSCALE
REAL, DIMENSION(:,:) :: V

SUBROUTINE GGBAK_64(JOB, SIDE, [N], ILO, IHI, LSCALE, RSCALE, [M], V,
```

```
       [LDV], [INFO])

CHARACTER(LEN=1) :: JOB, SIDE
INTEGER(8) :: N, ILO, IHI, M, LDV, INFO
REAL, DIMENSION(:) :: LSCALE, RSCALE
REAL, DIMENSION(:,:) :: V
```

C INTERFACE

```
#include <sunperf.h>

void sggbak(char job, char side, int n, int  ilo,  int  ihi,
          float *lscale, float *rscale, int m, float *v, int
          ldv, int *info);

void sggbak_64(char job, char side, long n, long  ilo,  long
          ihi,  float  *lscale, float *rscale, long m, float
          *v, long ldv, long *info);
```

PURPOSE

sggbak forms the right or left eigenvectors of a real generalized eigenvalue problem A*x = lambda*B*x, by backward transformation on the computed eigenvectors of the balanced pair of matrices output by SGGBAL.

ARGUMENTS

```
JOB (input)
          Specifies  the  type  of  backward  transformation
          required:
          = 'N':  do nothing, return immediately;
          = 'P':  do backward transformation for permutation
          only;
          = 'S':  do  backward  transformation  for  scaling
          only;
          = 'B':  do backward transformations for both  per-
          mutation and scaling.  JOB must be the same as the
          argument JOB supplied to SGGBAL.

SIDE (input)
          = 'R':  V contains right eigenvectors;
          = 'L':  V contains left eigenvectors.

N (input) The number of rows of the matrix V.  N >= 0.

ILO (input)
          The integers ILO and IHI determined by SGGBAL.   1
```

```
          <=  ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if
          N=0.

IHI (input)
          See the description for ILO.

LSCALE (input)
          Details of the permutations and/or scaling factors
          applied  to  the left side of A and B, as returned
          by SGGBAL.

RSCALE (input)
          Details of the permutations and/or scaling factors
          applied  to the right side of A and B, as returned
          by SGGBAL.

M (input) The number of columns of the matrix V.  M >= 0.

V (input/output)
          On entry, the matrix of right or left eigenvectors
          to  be  transformed,  as  returned  by STGEVC.  On
          exit, V is overwritten by the  transformed  eigen-
          vectors.

LDV (input)
          The leading dimension of  the  matrix  V.  LDV  >=
          max(1,N).

INFO (output)
          = 0:  successful exit.
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value.
```

FURTHER DETAILS

```
See R.C. Ward, Balancing the generalized eigenvalue problem,
               SIAM J. Sci. Stat. Comp. 2 (1981), 141-152.
```

Contents

NAME

```
sggbal - balance a pair of general real matrices (A,B)
```

SYNOPSIS

```
SUBROUTINE SGGBAL(JOB, N, A, LDA, B, LDB, ILO, IHI, LSCALE, RSCALE,
      WORK, INFO)

CHARACTER * 1 JOB
INTEGER N, LDA, LDB, ILO, IHI, INFO
REAL A(LDA,*), B(LDB,*), LSCALE(*), RSCALE(*), WORK(*)

SUBROUTINE SGGBAL_64(JOB, N, A, LDA, B, LDB, ILO, IHI, LSCALE,
      RSCALE, WORK, INFO)

CHARACTER * 1 JOB
INTEGER*8 N, LDA, LDB, ILO, IHI, INFO
REAL A(LDA,*), B(LDB,*), LSCALE(*), RSCALE(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGBAL(JOB, [N], A, [LDA], B, [LDB], ILO, IHI, LSCALE,
       RSCALE, [WORK], [INFO])

CHARACTER(LEN=1) :: JOB
INTEGER :: N, LDA, LDB, ILO, IHI, INFO
REAL, DIMENSION(:) :: LSCALE, RSCALE, WORK
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE GGBAL_64(JOB, [N], A, [LDA], B, [LDB], ILO, IHI, LSCALE,
       RSCALE, [WORK], [INFO])

CHARACTER(LEN=1) :: JOB
```

```
INTEGER(8) :: N, LDA, LDB, ILO, IHI, INFO
REAL, DIMENSION(:) :: LSCALE, RSCALE, WORK
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void sggbal(char job, int n, float *a, int  lda,  float  *b,
          int  ldb, int *ilo, int *ihi, float *lscale, float
          *rscale, int *info);

void sggbal_64(char job, long n, float *a, long  lda,  float
          *b, long ldb, long *ilo, long *ihi, float *lscale,
          float *rscale, long *info);
```

PURPOSE

sggbal balances a pair of general real matrices (A,B). This involves, first, permuting A and B by similarity transformations to isolate eigenvalues in the first 1 to ILO$-$1 and last IHI+1 to N elements on the diagonal; and second, applying a diagonal similarity transformation to rows and columns ILO to IHI to make the rows and columns as close in norm as possible. Both steps are optional.

Balancing may reduce the 1-norm of the matrices, and improve the accuracy of the computed eigenvalues and/or eigenvectors in the generalized eigenvalue problem A*x = lambda*B*x.

ARGUMENTS

JOB (input)
Specifies the operations to be performed on A and B:
= 'N': none: simply set ILO = 1, IHI = N, LSCALE(I) = 1.0 and RSCALE(I) = 1.0 for i = 1,...,N. = 'P': permute only;
= 'S': scale only;
= 'B': both permute and scale.

N (input) The order of the matrices A and B. N >= 0.

A (input/output)
On entry, the input matrix A. On exit, A is overwritten by the balanced matrix. If JOB = 'N', A is not referenced.

```
LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,N).

B (input) On entry, the input matrix  B.   On  exit,   B  is
          overwritten by the balanced matrix.  If JOB = 'N',
          B is not referenced.

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          max(1,N).

ILO (output)
          ILO and IHI are set to integers such that on  exit
          A(i,j)  =  0  and  B(i,j)  =  0  if  i > j and j =
          1,...,ILO-1 or i = IHI+1,...,N.  If JOB =  'N'  or
          'S', ILO = 1 and IHI = N.

IHI (output)
          See the description for ILO.

LSCALE (input)
          Details of the permutations  and  scaling  factors
          applied  to  the left side of A and B.  If P(j) is
          the index of the row interchanged with row j,  and
          D(j)  is the scaling factor applied to row j, then
          LSCALE(j) = P(j)    for J  =  1,...,ILO-1  =  D(j)
          for J = ILO,...,IHI = P(j)    for J = IHI+1,...,N.
          The order in which the interchanges are made is  N
          to IHI+1, then 1 to ILO-1.

RSCALE (input)
          Details of the permutations  and  scaling  factors
          applied  to the right side of A and B.  If P(j) is
          the index of the column interchanged  with  column
          j,  and  D(j)  is  the  scaling  factor applied to
          column  j,  then  LSCALE(j)  =  P(j)     for  J  =
          1,...,ILO-1  =  D(j)    for J = ILO,...,IHI = P(j)
          for J =  IHI+1,...,N.   The  order  in  which  the
          interchanges  are  made  is  N to IHI+1, then 1 to
          ILO-1.

WORK (workspace)
          dimension(6*N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value.
```

FURTHER DETAILS

```
See R.C. WARD, Balancing the generalized eigenvalue problem,
               SIAM J. Sci. Stat. Comp. 2 (1981), 141-152.
```

Contents

NAME

sgges - compute for a pair of N-by-N real nonsymmetric matrices (A,B),

SYNOPSIS

```
SUBROUTINE SGGES(JOBVSL, JOBVSR, SORT, SELCTG, N, A, LDA, B, LDB,
      SDIM, ALPHAR, ALPHAI, BETA, VSL, LDVSL, VSR, LDVSR, WORK, LWORK,
      BWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR, SORT
INTEGER N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, INFO
LOGICAL SELCTG
LOGICAL BWORK(*)
REAL  A(LDA,*),  B(LDB,*),  ALPHAR(*),  ALPHAI(*),  BETA(*),
VSL(LDVSL,*), VSR(LDVSR,*), WORK(*)

SUBROUTINE SGGES_64(JOBVSL, JOBVSR, SORT, SELCTG, N, A, LDA, B, LDB,
      SDIM, ALPHAR, ALPHAI, BETA, VSL, LDVSL, VSR, LDVSR, WORK, LWORK,
      BWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR, SORT
INTEGER*8 N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, INFO
LOGICAL*8 SELCTG
LOGICAL*8 BWORK(*)
REAL  A(LDA,*),  B(LDB,*),  ALPHAR(*),  ALPHAI(*),  BETA(*),
VSL(LDVSL,*), VSR(LDVSR,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGES(JOBVSL, JOBVSR, SORT, [SELCTG], [N], A, [LDA], B, [LDB],
       SDIM, ALPHAR, ALPHAI, BETA, VSL, [LDVSL], VSR, [LDVSR], [WORK],
       [LWORK], [BWORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOBVSL, JOBVSR, SORT
INTEGER :: N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, INFO
LOGICAL :: SELCTG
LOGICAL, DIMENSION(:) :: BWORK
REAL, DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
REAL, DIMENSION(:,:) :: A, B, VSL, VSR

SUBROUTINE GGES_64(JOBVSL, JOBVSR, SORT, [SELCTG], [N], A, [LDA], B,
       [LDB], SDIM, ALPHAR, ALPHAI, BETA, VSL, [LDVSL], VSR, [LDVSR],
       [WORK], [LWORK], [BWORK], [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR, SORT
INTEGER(8) :: N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, INFO
LOGICAL(8) :: SELCTG
LOGICAL(8), DIMENSION(:) :: BWORK
REAL, DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
REAL, DIMENSION(:,:) :: A, B, VSL, VSR
```

C INTERFACE

```
#include <sunperf.h>

void   sgges(char   jobvsl,   char   jobvsr,   char    sort,
          int(*selctg)(float,float,float),  int n, float *a,
          int lda, float  *b,  int  ldb,  int  *sdim,  float
          *alphar,  float  *alphai, float *beta, float *vsl,
          int ldvsl, float *vsr, int ldvsr, int *info);

void  sgges_64(char  jobvsl,   char   jobvsr,   char   sort,
          long(*selctg)(float,float,float),  long  n,  float
          *a, long lda, float  *b,  long  ldb,  long  *sdim,
          float  *alphar,  float *alphai, float *beta, float
          *vsl, long ldvsl, float  *vsr,  long  ldvsr,  long
          *info);
```

PURPOSE

sgges computes for a pair of N-by-N real nonsymmetric matrices (A,B), the generalized eigenvalues, the generalized real Schur form (S,T), optionally, the left and/or right matrices of Schur vectors (VSL and VSR). This gives the generalized Schur factorization

(A,B) = ((VSL)*S*(VSR)**T, (VSL)*T*(VSR)**T)

Optionally, it also orders the eigenvalues so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper quasi-triangular matrix S and the upper triangular matrix T.The leading columns of VSL and VSR then form an orthonormal basis for the corresponding left and right eigenspaces (deflating subspaces).

(If only the generalized eigenvalues are needed, use the driver SGGEV instead, which is faster.)

A generalized eigenvalue for a pair of matrices (A,B) is a scalar w or a ratio alpha/beta = w, such that A - w*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0 or both being zero.

A pair of matrices (S,T) is in generalized real Schur form if T is upper triangular with non-negative diagonal and S is block upper triangular with 1-by-1 and 2-by-2 blocks. 1-by-1 blocks correspond to real generalized eigenvalues, while 2-by-2 blocks of S will be "standardized" by making the corresponding elements of T have the form:

```
        [  a  0  ]
        [  0  b  ]
```

and the pair of corresponding 2-by-2 blocks in S and T will have a complex conjugate pair of generalized eigenvalues.

ARGUMENTS

JOBVSL (input)
= 'N': do not compute the left Schur vectors;
= 'V': compute the left Schur vectors.

JOBVSR (input)
= 'N': do not compute the right Schur vectors;
= 'V': compute the right Schur vectors.

SORT (input)
Specifies whether or not to order the eigenvalues on the diagonal of the generalized Schur form. = 'N': Eigenvalues are not ordered;
= 'S': Eigenvalues are ordered (see SELCTG);

SELCTG (input)
SELCTG must be declared EXTERNAL in the calling subroutine. If SORT = 'N', SELCTG is not referenced. If SORT = 'S', SELCTG is used to select eigenvalues to sort to the top left of the Schur form. An eigenvalue (ALPHAR(j)+ALPHAI(j))/BETA(j) is selected if SELCTG(ALPHAR(j),ALPHAI(j),BETA(j)) is true; i.e. if either one of a complex conjugate pair of eigenvalues is selected, then both complex eigenvalues are selected.

Note that in the ill-conditioned case, a selected complex eigenvalue may no longer satisfy SELCTG(ALPHAR(j),ALPHAI(j), BETA(j)) = .TRUE. after ordering. INFO is to be set to N+2 in this case.

N (input) The order of the matrices A, B, VSL, and VSR. N >= 0.

A (input/output)
On entry, the first of the pair of matrices. On exit, A has been overwritten by its generalized Schur form S.

LDA (input)
The leading dimension of A. LDA >= max(1,N).

B (input/output)
On entry, the second of the pair of matrices. On exit, B has been overwritten by its generalized Schur form T.

LDB (input)
The leading dimension of B. LDB >= max(1,N).

SDIM (output)
If SORT = 'N', SDIM = 0. If SORT = 'S', SDIM = number of eigenvalues (after sorting) for which SELCTG is true. (Complex conjugate pairs for which SELCTG is true for either eigenvalue count as 2.)

ALPHAR (output)
On exit, (ALPHAR(j) + ALPHAI(j)*i)/BETA(j), j=1,...,N, will be the generalized eigenvalues. ALPHAR(j) + ALPHAI(j)*i, and BETA(j),j=1,...,N are the diagonals of the complex Schur form (S,T) that would result if the 2-by-2 diagonal blocks of the real Schur form of (A,B) were further reduced to triangular form using 2-by-2 complex unitary transformations. If ALPHAI(j) is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-st eigenvalues are a complex conjugate pair, with ALPHAI(j+1) negative.

Note: the quotients ALPHAR(j)/BETA(j) and ALPHAI(j)/BETA(j) may easily over- or underflow, and BETA(j) may even be zero. Thus, the user should avoid naively computing the ratio. However, ALPHAR and ALPHAI will be always less than and usually comparable with norm(A) in magnitude, and BETA always less than and usually comparable with norm(B).

ALPHAI (output)
See the description for ALPHAR.

BETA (output)
See the description for ALPHAR.

VSL (input)
If JOBVSL = 'V', VSL will contain the left Schur vectors. Not referenced if JOBVSL = 'N'.

LDVSL (input)
The leading dimension of the matrix VSL. LDVSL >=1, and if JOBVSL = 'V', LDVSL >= N.

VSR (input)
If JOBVSR = 'V', VSR will contain the right Schur vectors. Not referenced if JOBVSR = 'N'.

LDVSR (input)
The leading dimension of the matrix VSR. LDVSR >= 1, and if JOBVSR = 'V', LDVSR >= N.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 8*N+16.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

BWORK (workspace)
dimension(N) Not referenced if SORT = 'N'.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
= 1,...,N: The QZ iteration failed. (A,B) are not in Schur form, but ALPHAR(j), ALPHAI(j), and BETA(j) should be correct for j=INFO+1,...,N. > N: =N+1: other than QZ iteration failed in

SHGEQZ.
=N+2: after reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the Generalized Schur form no longer satisfy SELCTG=.TRUE. This could also be caused due to scaling. =N+3: reordering failed in STGSEN.

Contents

NAME

```
sggesx - compute for a  pair  of  N-by-N  real  nonsymmetric
matrices  (A,B), the generalized eigenvalues, the real Schur
form (S,T), and,
```

SYNOPSIS

```
SUBROUTINE SGGESX(JOBVSL, JOBVSR, SORT, SELCTG, SENSE, N, A, LDA, B,
      LDB, SDIM, ALPHAR, ALPHAI, BETA, VSL, LDVSL, VSR, LDVSR, RCONDE,
      RCONDV, WORK, LWORK, IWORK, LIWORK, BWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR, SORT, SENSE
INTEGER N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, LIWORK, INFO
INTEGER IWORK(*)
LOGICAL SELCTG
LOGICAL BWORK(*)
REAL  A(LDA,*),  B(LDB,*),  ALPHAR(*),  ALPHAI(*),  BETA(*),
VSL(LDVSL,*), VSR(LDVSR,*), RCONDE(*), RCONDV(*), WORK(*)

SUBROUTINE SGGESX_64(JOBVSL, JOBVSR, SORT, SELCTG, SENSE, N, A, LDA,
      B, LDB, SDIM, ALPHAR, ALPHAI, BETA, VSL, LDVSL, VSR, LDVSR,
      RCONDE, RCONDV, WORK, LWORK, IWORK, LIWORK, BWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR, SORT, SENSE
INTEGER*8 N, LDA, LDB, SDIM, LDVSL,  LDVSR,  LWORK,  LIWORK,
INFO
INTEGER*8 IWORK(*)
LOGICAL*8 SELCTG
LOGICAL*8 BWORK(*)
REAL  A(LDA,*),  B(LDB,*),  ALPHAR(*),  ALPHAI(*),  BETA(*),
VSL(LDVSL,*), VSR(LDVSR,*), RCONDE(*), RCONDV(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGESX(JOBVSL, JOBVSR, SORT, [SELCTG], SENSE, [N], A, [LDA],
       B, [LDB], SDIM, ALPHAR, ALPHAI, BETA, VSL, [LDVSL], VSR, [LDVSR],
       RCONDE, RCONDV, [WORK], [LWORK], [IWORK], [LIWORK], [BWORK],
       [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR, SORT, SENSE
INTEGER :: N, LDA, LDB, SDIM, LDVSL, LDVSR,  LWORK,  LIWORK,
INFO
INTEGER, DIMENSION(:) :: IWORK
LOGICAL :: SELCTG
LOGICAL, DIMENSION(:) :: BWORK
REAL, DIMENSION(:) :: ALPHAR, ALPHAI, BETA, RCONDE,  RCONDV,
WORK
REAL, DIMENSION(:,:) :: A, B, VSL, VSR
SUBROUTINE GGESX_64(JOBVSL, JOBVSR, SORT, [SELCTG], SENSE, [N], A, [LDA],
       B, [LDB], SDIM, ALPHAR, ALPHAI, BETA, VSL, [LDVSL], VSR, [LDVSR],
       RCONDE, RCONDV, [WORK], [LWORK], [IWORK], [LIWORK], [BWORK],
       [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR, SORT, SENSE
INTEGER(8) ::  N,  LDA,  LDB,  SDIM,  LDVSL,  LDVSR,  LWORK,
LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
LOGICAL(8) :: SELCTG
LOGICAL(8), DIMENSION(:) :: BWORK
REAL, DIMENSION(:) :: ALPHAR, ALPHAI, BETA, RCONDE,  RCONDV,
WORK
REAL, DIMENSION(:,:) :: A, B, VSL, VSR
```

C INTERFACE

```
#include <sunperf.h>

void   sggesx(char   jobvsl,   char   jobvsr,   char   sort,
          int(*selctg)(float,float,float),  char  sense, int
          n, float *a, int  lda,  float  *b,  int  ldb,  int
          *sdim,  float *alphar, float *alphai, float *beta,
          float *vsl, int  ldvsl,  float  *vsr,  int  ldvsr,
          float *rconde, float *rcondv, int *info);

void  sggesx_64(char  jobvsl,  char   jobvsr,   char   sort,
          long(*selctg)(float,float,float), char sense, long
          n, float *a, long lda, float *b,  long  ldb,  long
          *sdim,  float *alphar, float *alphai, float *beta,
          float *vsl, long ldvsl, float  *vsr,  long  ldvsr,
          float *rconde, float *rcondv, long *info);
```

PURPOSE

sggesx computes for a pair of N-by-N real nonsymmetric

matrices (A,B), the generalized eigenvalues, the real Schur form (S,T), and, optionally, the left and/or right matrices of Schur vectors (VSL and VSR). This gives the generalized Schur factorization
A,B) = ((VSL) S (VSR)**T, (VSL) T (VSR)**T)

Optionally, it also orders the eigenvalues so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper quasi-triangular matrix S and the upper triangular matrix T; computes a reciprocal condition number for the average of the selected eigenvalues (RCONDE); and computes a reciprocal condition number for the right and left deflating subspaces corresponding to the selected eigenvalues (RCONDV). The leading columns of VSL and VSR then form an orthonormal basis for the corresponding left and right eigenspaces (deflating subspaces).

A generalized eigenvalue for a pair of matrices (A,B) is a scalar w or a ratio alpha/beta = w, such that A - w*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0 or for both being zero.

A pair of matrices (S,T) is in generalized real Schur form if T is upper triangular with non-negative diagonal and S is block upper triangular with 1-by-1 and 2-by-2 blocks. 1-by-1 blocks correspond to real generalized eigenvalues, while 2-by-2 blocks of S will be "standardized" by making the corresponding elements of T have the form:

```
[  a  0  ]
[  0  b  ]
```

and the pair of corresponding 2-by-2 blocks in S and T will have a complex conjugate pair of generalized eigenvalues.

ARGUMENTS

JOBVSL (input)
= 'N': do not compute the left Schur vectors;
= 'V': compute the left Schur vectors.

JOBVSR (input)
= 'N': do not compute the right Schur vectors;
= 'V': compute the right Schur vectors.

SORT (input)
Specifies whether or not to order the eigenvalues on the diagonal of the generalized Schur form. =

'N': Eigenvalues are not ordered;
= 'S': Eigenvalues are ordered (see SELCTG).

SELCTG (input)
SELCTG must be declared EXTERNAL in the calling subroutine. If SORT = 'N', SELCTG is not referenced. If SORT = 'S', SELCTG is used to select eigenvalues to sort to the top left of the Schur form. An eigenvalue (ALPHAR(j)+ALPHAI(j))/BETA(j) is selected if SELCTG(ALPHAR(j),ALPHAI(j),BETA(j)) is true; i.e. if either one of a complex conjugate pair of eigenvalues is selected, then both complex eigenvalues are selected. Note that a selected complex eigenvalue may no longer satisfy SELCTG(ALPHAR(j),ALPHAI(j),BETA(j)) = .TRUE. after ordering, since ordering may change the value of complex eigenvalues (especially if the eigenvalue is ill-conditioned), in this case INFO is set to N+3.

SENSE (input)
Determines which reciprocal condition numbers are computed. = 'N' : None are computed;
= 'E' : Computed for average of selected eigenvalues only;
= 'V' : Computed for selected deflating subspaces only;
= 'B' : Computed for both. If SENSE = 'E', 'V', or 'B', SORT must equal 'S'.

N (input) The order of the matrices A, B, VSL, and VSR. N >= 0.

A (input/output)
On entry, the first of the pair of matrices. On exit, A has been overwritten by its generalized Schur form S.

LDA (input)
The leading dimension of A. LDA >= max(1,N).

B (input/output)
On entry, the second of the pair of matrices. On exit, B has been overwritten by its generalized Schur form T.

LDB (input)
The leading dimension of B. LDB >= max(1,N).

SDIM (output)

If SORT = 'N', SDIM = 0. If SORT = 'S', SDIM = number of eigenvalues (after sorting) for which SELCTG is true. (Complex conjugate pairs for which SELCTG is true for either eigenvalue count as 2.)

ALPHAR (output)

On exit, (ALPHAR(j) + ALPHAI(j)*i)/BETA(j), j=1,...,N, will be the generalized eigenvalues. ALPHAR(j) + ALPHAI(j)*i and BETA(j),j=1,...,N are the diagonals of the complex Schur form (S,T) that would result if the 2-by-2 diagonal blocks of the real Schur form of (A,B) were further reduced to triangular form using 2-by-2 complex unitary transformations. If ALPHAI(j) is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-st eigenvalues are a complex conjugate pair, with ALPHAI(j+1) negative.

Note: the quotients ALPHAR(j)/BETA(j) and ALPHAI(j)/BETA(j) may easily over- or underflow, and BETA(j) may even be zero. Thus, the user should avoid naively computing the ratio. However, ALPHAR and ALPHAI will be always less than and usually comparable with norm(A) in magnitude, and BETA always less than and usually comparable with norm(B).

ALPHAI (output)

See the description for ALPHAR.

BETA (output)

See the description for ALPHAR.

VSL (input)

If JOBVSL = 'V', VSL will contain the left Schur vectors. Not referenced if JOBVSL = 'N'.

LDVSL (input)

The leading dimension of the matrix VSL. LDVSL >=1, and if JOBVSL = 'V', LDVSL >= N.

VSR (input)

If JOBVSR = 'V', VSR will contain the right Schur vectors. Not referenced if JOBVSR = 'N'.

LDVSR (input)

The leading dimension of the matrix VSR. LDVSR >= 1, and if JOBVSR = 'V', LDVSR >= N.

RCONDE (output)
If SENSE = 'E' or 'B', RCONDE(1) and RCONDE(2) contain the reciprocal condition numbers for the average of the selected eigenvalues. Not referenced if SENSE = 'N' or 'V'.

RCONDV (output)
If SENSE = 'V' or 'B', RCONDV(1) and RCONDV(2) contain the reciprocal condition numbers for the selected deflating subspaces. Not referenced if SENSE = 'N' or 'E'.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 8*(N+1)+16. If SENSE = 'E', 'V', or 'B', LWORK >= MAX(8*(N+1)+16, 2*SDIM*(N-SDIM)).

IWORK (workspace)
Not referenced if SENSE = 'N'.

LIWORK (input)
The dimension of the array WORK. LIWORK >= N+6.

BWORK (workspace)
dimension(N) Not referenced if SORT = 'N'.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
= 1,...,N: The QZ iteration failed. (A,B) are not in Schur form, but ALPHAR(j), ALPHAI(j), and BETA(j) should be correct for j=INFO+1,...,N. > N: =N+1: other than QZ iteration failed in SHGEQZ =N+2: after reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the Generalized Schur form no longer satisfy SELCTG=.TRUE. This could also be caused due to scaling. =N+3: reordering failed in STGSEN.

Further details ===============

An approximate (asymptotic) bound on the average absolute error of the selected eigenvalues is

EPS * norm((A, B)) / RCONDE(1).

An approximate (asymptotic) bound on the maximum angular error in the computed deflating subspaces is

EPS * norm((A, B)) / RCONDV(2).

See LAPACK User's Guide, section 4.11 for more information.

Contents

NAME

```
sggev - compute for  a  pair  of  N-by-N  real  nonsymmetric
matrices (A,B)
```

SYNOPSIS

```
SUBROUTINE SGGEV(JOBVL, JOBVR, N, A, LDA, B, LDB, ALPHAR, ALPHAI,
      BETA, VL, LDVL, VR, LDVR, WORK, LWORK, INFO)

CHARACTER * 1 JOBVL, JOBVR
INTEGER N, LDA, LDB, LDVL, LDVR, LWORK, INFO
REAL  A(LDA,*),  B(LDB,*),  ALPHAR(*),  ALPHAI(*),  BETA(*),
VL(LDVL,*), VR(LDVR,*), WORK(*)

SUBROUTINE SGGEV_64(JOBVL, JOBVR, N, A, LDA, B, LDB, ALPHAR, ALPHAI,
      BETA, VL, LDVL, VR, LDVR, WORK, LWORK, INFO)

CHARACTER * 1 JOBVL, JOBVR
INTEGER*8 N, LDA, LDB, LDVL, LDVR, LWORK, INFO
REAL  A(LDA,*),  B(LDB,*),  ALPHAR(*),  ALPHAI(*),  BETA(*),
VL(LDVL,*), VR(LDVR,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGEV(JOBVL, JOBVR, [N], A, [LDA], B, [LDB], ALPHAR,
       ALPHAI, BETA, VL, [LDVL], VR, [LDVR], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: JOBVL, JOBVR
INTEGER :: N, LDA, LDB, LDVL, LDVR, LWORK, INFO
REAL, DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
REAL, DIMENSION(:,:) :: A, B, VL, VR

SUBROUTINE GGEV_64(JOBVL, JOBVR, [N], A, [LDA], B, [LDB], ALPHAR,
       ALPHAI, BETA, VL, [LDVL], VR, [LDVR], [WORK], [LWORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOBVL, JOBVR
INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, LWORK, INFO
REAL, DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
REAL, DIMENSION(:,:) :: A, B, VL, VR
```

C INTERFACE

```
#include <sunperf.h>

void sggev(char jobvl, char jobvr, int n, float *a, int lda,
          float  *b,  int ldb, float *alphar, float *alphai,
          float *beta, float *vl, int ldvl, float  *vr,  int
          ldvr, int *info);
void sggev_64(char jobvl, char jobvr, long n, float *a, long
          lda,  float  *b,  long  ldb,  float *alphar, float
          *alphai, float *beta, float *vl, long ldvl,  float
          *vr, long ldvr, long *info);
```

PURPOSE

sggev computes for a pair of N-by-N real nonsymmetric matrices (A,B) the generalized eigenvalues, and optionally, the left and/or right generalized eigenvectors.

A generalized eigenvalue for a pair of matrices (A,B) is a scalar lambda or a ratio alpha/beta = lambda, such that A - lambda*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0, and even for both being zero.

The right eigenvector v(j) corresponding to the eigenvalue lambda(j) of (A,B) satisfies

A * v(j) = lambda(j) * B * v(j).

The left eigenvector u(j) corresponding to the eigenvalue lambda(j) of (A,B) satisfies

u(j)**H * A = lambda(j) * u(j)**H * B .

where u(j)**H is the conjugate-transpose of u(j).

ARGUMENTS

JOBVL (input)
= 'N': do not compute the left generalized eigenvectors;

= 'V': compute the left generalized eigenvectors.

JOBVR (input)
= 'N': do not compute the right generalized eigenvectors;
= 'V': compute the right generalized eigenvectors.

N (input) The order of the matrices A, B, VL, and VR. N >= 0.

A (input/output)
On entry, the matrix A in the pair (A,B). On exit, A has been overwritten.

LDA (input)
The leading dimension of A. LDA >= max(1,N).

B (input/output)
On entry, the matrix B in the pair (A,B). On exit, B has been overwritten.

LDB (input)
The leading dimension of B. LDB >= max(1,N).

ALPHAR (output)
On exit, (ALPHAR(j) + ALPHAI(j)*i)/BETA(j), j=1,...,N, will be the generalized eigenvalues. If ALPHAI(j) is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-st eigenvalues are a complex conjugate pair, with ALPHAI(j+1) negative.

Note: the quotients ALPHAR(j)/BETA(j) and ALPHAI(j)/BETA(j) may easily over- or underflow, and BETA(j) may even be zero. Thus, the user should avoid naively computing the ratio alpha/beta. However, ALPHAR and ALPHAI will be always less than and usually comparable with norm(A) in magnitude, and BETA always less than and usually comparable with norm(B).

ALPHAI (output)
See the description for ALPHAR.

BETA (output)
See the description for ALPHAR.

VL (input)
If JOBVL = 'V', the left eigenvectors u(j) are

stored one after another in the columns of VL, in the same order as their eigenvalues. If the j-th eigenvalue is real, then u(j) = VL(:,j), the j-th column of VL. If the j-th and (j+1)-th eigenvalues form a complex conjugate pair, then u(j) = VL(:,j)+i*VL(:,j+1) and u(j+1) = VL(:,j)-i*VL(:,j+1). Each eigenvector will be scaled so the largest component have abs(real part)+abs(imag. part)=1. Not referenced if JOBVL = 'N'.

LDVL (input)

The leading dimension of the matrix VL. LDVL >= 1, and if JOBVL = 'V', LDVL >= N.

VR (input)

If JOBVR = 'V', the right eigenvectors v(j) are stored one after another in the columns of VR, in the same order as their eigenvalues. If the j-th eigenvalue is real, then v(j) = VR(:,j), the j-th column of VR. If the j-th and (j+1)-th eigenvalues form a complex conjugate pair, then v(j) = VR(:,j)+i*VR(:,j+1) and v(j+1) = VR(:,j)-i*VR(:,j+1). Each eigenvector will be scaled so the largest component have abs(real part)+abs(imag. part)=1. Not referenced if JOBVR = 'N'.

LDVR (input)

The leading dimension of the matrix VR. LDVR >= 1, and if JOBVR = 'V', LDVR >= N.

WORK (workspace)

On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)

The dimension of the array WORK. LWORK >= max(1,8*N). For good performance, LWORK must generally be larger.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an ille-

gal value.
= 1,...,N: The QZ iteration failed. No eigenvectors have been calculated, but ALPHAR(j), ALPHAI(j), and BETA(j) should be correct for j=INFO+1,...,N. > N: =N+1: other than QZ iteration failed in SHGEQZ.
=N+2: error return from STGEVC.

Contents

NAME

sggevx - compute for a pair of N-by-N real nonsymmetric matrices (A,B)

SYNOPSIS

```
SUBROUTINE SGGEVX(BALANC, JOBVL, JOBVR, SENSE, N, A, LDA, B, LDB,
      ALPHAR, ALPHAI, BETA, VL, LDVL, VR, LDVR, ILO, IHI, LSCALE,
      RSCALE, ABNRM, BBNRM, RCONDE, RCONDV, WORK, LWORK, IWORK, BWORK,
      INFO)

CHARACTER * 1 BALANC, JOBVL, JOBVR, SENSE
INTEGER N, LDA, LDB, LDVL, LDVR, ILO, IHI, LWORK, INFO
INTEGER IWORK(*)
LOGICAL BWORK(*)
REAL ABNRM, BBNRM
REAL  A(LDA,*),  B(LDB,*),  ALPHAR(*),  ALPHAI(*),  BETA(*),
VL(LDVL,*),  VR(LDVR,*),  LSCALE(*),  RSCALE(*),  RCONDE(*),
RCONDV(*), WORK(*)

SUBROUTINE SGGEVX_64(BALANC, JOBVL, JOBVR, SENSE, N, A, LDA, B, LDB,
      ALPHAR, ALPHAI, BETA, VL, LDVL, VR, LDVR, ILO, IHI, LSCALE,
      RSCALE, ABNRM, BBNRM, RCONDE, RCONDV, WORK, LWORK, IWORK, BWORK,
      INFO)

CHARACTER * 1 BALANC, JOBVL, JOBVR, SENSE
INTEGER*8 N, LDA, LDB, LDVL, LDVR, ILO, IHI, LWORK, INFO
INTEGER*8 IWORK(*)
LOGICAL*8 BWORK(*)
REAL ABNRM, BBNRM
REAL  A(LDA,*),  B(LDB,*),  ALPHAR(*),  ALPHAI(*),  BETA(*),
VL(LDVL,*),  VR(LDVR,*),  LSCALE(*),  RSCALE(*),  RCONDE(*),
```

```
   RCONDV(*), WORK(*)
```

F95 INTERFACE

```
   SUBROUTINE GGEVX(BALANC, JOBVL, JOBVR, SENSE, [N], A, [LDA], B, [LDB],
          ALPHAR, ALPHAI, BETA, VL, [LDVL], VR, [LDVR], ILO, IHI, LSCALE,
          RSCALE, ABNRM, BBNRM, RCONDE, RCONDV, [WORK], [LWORK], [IWORK],
          [BWORK], [INFO])

   CHARACTER(LEN=1) :: BALANC, JOBVL, JOBVR, SENSE
   INTEGER :: N, LDA, LDB, LDVL, LDVR, ILO, IHI, LWORK, INFO
   INTEGER, DIMENSION(:) :: IWORK
   LOGICAL, DIMENSION(:) :: BWORK
   REAL :: ABNRM, BBNRM
   REAL, DIMENSION(:) :: ALPHAR, ALPHAI, BETA, LSCALE,  RSCALE,
   RCONDE, RCONDV, WORK
   REAL, DIMENSION(:,:) :: A, B, VL, VR
   SUBROUTINE GGEVX_64(BALANC, JOBVL, JOBVR, SENSE, [N], A, [LDA], B,
          [LDB], ALPHAR, ALPHAI, BETA, VL, [LDVL], VR, [LDVR], ILO, IHI,
          LSCALE, RSCALE, ABNRM, BBNRM, RCONDE, RCONDV, [WORK], [LWORK],
          [IWORK], [BWORK], [INFO])

   CHARACTER(LEN=1) :: BALANC, JOBVL, JOBVR, SENSE
   INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, ILO, IHI, LWORK, INFO
   INTEGER(8), DIMENSION(:) :: IWORK
   LOGICAL(8), DIMENSION(:) :: BWORK
   REAL :: ABNRM, BBNRM
   REAL, DIMENSION(:) :: ALPHAR, ALPHAI, BETA, LSCALE,  RSCALE,
   RCONDE, RCONDV, WORK
   REAL, DIMENSION(:,:) :: A, B, VL, VR
```

C INTERFACE

```
   #include <sunperf.h>

   void sggevx(char balanc, char jobvl, char jobvr, char sense,
             int n, float *a, int lda, float *b, int ldb, float
             *alphar, float *alphai, float  *beta,  float  *vl,
             int ldvl, float *vr, int ldvr, int *ilo, int *ihi,
             float *lscale, float *rscale, float *abnrm,  float
             *bbnrm, float *rconde, float *rcondv, int *info);

   void sggevx_64(char balanc, char  jobvl,  char  jobvr,  char
             sense,  long n, float *a, long lda, float *b, long
             ldb, float *alphar, float  *alphai,  float  *beta,
             float  *vl,  long ldvl, float *vr, long ldvr, long
             *ilo, long *ihi,  float  *lscale,  float  *rscale,
             float  *abnrm,  float *bbnrm, float *rconde, float
             *rcondv, long *info);
```

PURPOSE

sggevx computes for a pair of N-by-N real nonsymmetric matrices (A,B) the generalized eigenvalues, and optionally, the left and/or right generalized eigenvectors.

Optionally also, it computes a balancing transformation to improve the conditioning of the eigenvalues and eigenvectors (ILO, IHI, LSCALE, RSCALE, ABNRM, and BBNRM), reciprocal condition numbers for the eigenvalues (RCONDE), and reciprocal condition numbers for the right eigenvectors (RCONDV).

A generalized eigenvalue for a pair of matrices (A,B) is a scalar lambda or a ratio alpha/beta = lambda, such that A - lambda*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0, and even for both being zero.
The right eigenvector v(j) corresponding to the eigenvalue lambda(j) of (A,B) satisfies

A * v(j) = lambda(j) * B * v(j) .

The left eigenvector u(j) corresponding to the eigenvalue lambda(j) of (A,B) satisfies

u(j)**H * A = lambda(j) * u(j)**H * B.

where u(j)**H is the conjugate-transpose of u(j).

ARGUMENTS

BALANC (input)
Specifies the balance option to be performed. = 'N': do not diagonally scale or permute;
= 'P': permute only;
= 'S': scale only;
= 'B': both permute and scale. Computed reciprocal condition numbers will be for the matrices after permuting and/or balancing. Permuting does not change condition numbers (in exact arithmetic), but balancing does.

JOBVL (input)
= 'N': do not compute the left generalized eigenvectors;
= 'V': compute the left generalized eigenvectors.

JOBVR (input)
= 'N': do not compute the right generalized

eigenvectors;
= 'V': compute the right generalized eigenvectors.

SENSE (input)
Determines which reciprocal condition numbers are computed. = 'N': none are computed;
= 'E': computed for eigenvalues only;
= 'V': computed for eigenvectors only;
= 'B': computed for eigenvalues and eigenvectors.

N (input) The order of the matrices A, B, VL, and VR. N >= 0.

A (input/output)
On entry, the matrix A in the pair (A,B). On exit, A has been overwritten. If JOBVL='V' or JOBVR='V' or both, then A contains the first part of the real Schur form of the "balanced" versions of the input A and B.

LDA (input)
The leading dimension of A. LDA >= max(1,N).

B (input/output)
On entry, the matrix B in the pair (A,B). On exit, B has been overwritten. If JOBVL='V' or JOBVR='V' or both, then B contains the second part of the real Schur form of the "balanced" versions of the input A and B.

LDB (input)
The leading dimension of B. LDB >= max(1,N).

ALPHAR (output)
On exit, (ALPHAR(j) + ALPHAI(j)*i)/BETA(j), j=1,...,N, will be the generalized eigenvalues. If ALPHAI(j) is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-st eigenvalues are a complex conjugate pair, with ALPHAI(j+1) negative.

Note: the quotients ALPHAR(j)/BETA(j) and ALPHAI(j)/BETA(j) may easily over- or underflow, and BETA(j) may even be zero. Thus, the user should avoid naively computing the ratio ALPHA/BETA. However, ALPHAR and ALPHAI will be always less than and usually comparable with norm(A) in magnitude, and BETA always less than and usually comparable with norm(B).

ALPHAI (output)
See the description of ALPHAR.

BETA (output)
See the description of ALPHAR.

VL (output)
If JOBVL = 'V', the left eigenvectors u(j) are stored one after another in the columns of VL, in the same order as their eigenvalues. If the j-th eigenvalue is real, then u(j) = VL(:,j), the j-th column of VL. If the j-th and (j+1)-th eigenvalues form a complex conjugate pair, then u(j) = VL(:,j)+i*VL(:,j+1) and u(j+1) = VL(:,j)-i*VL(:,j+1). Each eigenvector will be scaled so the largest component have abs(real part) + abs(imag. part) = 1. Not referenced if JOBVL = 'N'.

LDVL (input)
The leading dimension of the matrix VL. LDVL >= 1, and if JOBVL = 'V', LDVL >= N.

VR (output)
If JOBVR = 'V', the right eigenvectors v(j) are stored one after another in the columns of VR, in the same order as their eigenvalues. If the j-th eigenvalue is real, then v(j) = VR(:,j), the j-th column of VR. If the j-th and (j+1)-th eigenvalues form a complex conjugate pair, then v(j) = VR(:,j)+i*VR(:,j+1) and v(j+1) = VR(:,j)-i*VR(:,j+1). Each eigenvector will be scaled so the largest component have abs(real part) + abs(imag. part) = 1. Not referenced if JOBVR = 'N'.

LDVR (input)
The leading dimension of the matrix VR. LDVR >= 1, and if JOBVR = 'V', LDVR >= N.

ILO (output)
ILO and IHI are integer values such that on exit A(i,j) = 0 and B(i,j) = 0 if i > j and j = 1,...,ILO-1 or i = IHI+1,...,N. If BALANC = 'N' or 'S', ILO = 1 and IHI = N.

IHI (output)
See the description of ILO.

LSCALE (output)

Details of the permutations and scaling factors applied to the left side of A and B. If PL(j) is the index of the row interchanged with row j, and DL(j) is the scaling factor applied to row j, then LSCALE(j) = PL(j) for j = 1,...,ILO-1 = DL(j) for j = ILO,...,IHI = PL(j) for j = IHI+1,...,N. The order in which the interchanges are made is N to IHI+1, then 1 to ILO-1.

RSCALE (output)

Details of the permutations and scaling factors applied to the right side of A and B. If PR(j) is the index of the column interchanged with column j, and DR(j) is the scaling factor applied to column j, then RSCALE(j) = PR(j) for j = 1,...,ILO-1 = DR(j) for j = ILO,...,IHI = PR(j) for j = IHI+1,...,N The order in which the interchanges are made is N to IHI+1, then 1 to ILO-1.

ABNRM (output)

The one-norm of the balanced matrix A.

BBNRM (output)

The one-norm of the balanced matrix B.

RCONDE (output)

If SENSE = 'E' or 'B', the reciprocal condition numbers of the selected eigenvalues, stored in consecutive elements of the array. For a complex conjugate pair of eigenvalues two consecutive elements of RCONDE are set to the same value. Thus RCONDE(j), RCONDV(j), and the j-th columns of VL and VR all correspond to the same eigenpair (but not in general the j-th eigenpair, unless all eigenpairs are selected). If SENSE = 'V', RCONDE is not referenced.

RCONDV (output)

If SENSE = 'V' or 'B', the estimated reciprocal condition numbers of the selected eigenvectors, stored in consecutive elements of the array. For a complex eigenvector two consecutive elements of RCONDV are set to the same value. If the eigenvalues cannot be reordered to compute RCONDV(j), RCONDV(j) is set to 0; this can only occur when the true value would be very small anyway. If SENSE = 'E', RCONDV is not referenced.

WORK (workspace)

On exit, if INFO = 0, WORK(1) returns the optimal

LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,6*N). If SENSE = 'E', LWORK >= 12*N. If SENSE = 'V' or 'B', LWORK >= 2*N*N+12*N+16.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace)
dimension(N+6) If SENSE = 'E', IWORK is not referenced.

BWORK (workspace)
dimension(N) If SENSE = 'N', BWORK is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
= 1,...,N: The QZ iteration failed. No eigenvectors have been calculated, but ALPHAR(j), ALPHAI(j), and BETA(j) should be correct for j=INFO+1,...,N. > N: =N+1: other than QZ iteration failed in SHGEQZ.
=N+2: error return from STGEVC.

FURTHER DETAILS

Balancing a matrix pair (A,B) includes, first, permuting rows and columns to isolate eigenvalues, second, applying diagonal similarity transformation to the rows and columns to make the rows and columns as close in norm as possible. The computed reciprocal condition numbers correspond to the balanced matrix. Permuting rows and columns will not change the condition numbers (in exact arithmetic) but diagonal scaling will. For further explanation of balancing, see section 4.11.1.2 of LAPACK Users' Guide.

An approximate error bound on the chordal distance between the i-th computed generalized eigenvalue w and the corresponding exact eigenvalue lambda is
hord(w, lambda) <= EPS * norm(ABNRM, BBNRM) / RCONDE(I)
An approximate error bound for the angle between the i-th computed eigenvector VL(i) or VR(i) is given by

PS * norm(ABNRM, BBNRM) / DIF(i).

For further explanation of the reciprocal condition numbers RCONDE and RCONDV, see section 4.11 of LAPACK User's Guide.

Contents

NAME

```
sggglm - solve a general  Gauss-Markov  linear  model  (GLM)
problem
```

SYNOPSIS

```
SUBROUTINE SGGGLM(N, M, P, A, LDA, B, LDB, D, X, Y, WORK, LDWORK,
      INFO)

INTEGER N, M, P, LDA, LDB, LDWORK, INFO
REAL A(LDA,*), B(LDB,*), D(*), X(*), Y(*), WORK(*)

SUBROUTINE SGGGLM_64(N, M, P, A, LDA, B, LDB, D, X, Y, WORK, LDWORK,
      INFO)

INTEGER*8 N, M, P, LDA, LDB, LDWORK, INFO
REAL A(LDA,*), B(LDB,*), D(*), X(*), Y(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGGLM([N], [M], [P], A, [LDA], B, [LDB], D, X, Y, [WORK],
       [LDWORK], [INFO])

INTEGER :: N, M, P, LDA, LDB, LDWORK, INFO
REAL, DIMENSION(:) :: D, X, Y, WORK
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE GGGLM_64([N], [M], [P], A, [LDA], B, [LDB], D, X, Y, [WORK],
       [LDWORK], [INFO])

INTEGER(8) :: N, M, P, LDA, LDB, LDWORK, INFO
REAL, DIMENSION(:) :: D, X, Y, WORK
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void sggglm(int n, int m, int p, float *a,  int  lda,  float
          *b,  int  ldb,  float  *d, float *x, float *y, int
          *info);

void sggglm_64(long n, long m, long p, float *a,  long  lda,
          float  *b, long ldb, float *d, float *x, float *y,
          long *info);
```

PURPOSE

```
sggglm solves a  general  Gauss-Markov  linear  model  (GLM)
problem:
        minimize || y ||_2   subject to   d = A*x + B*y
            x

where A is an N-by-M matrix, B is an N-by-P matrix, and d is
a given N-vector. It is assumed that M <= N <= M+P, and

           rank(A) = M    and    rank( A B ) = N.

Under these assumptions, the constrained equation is  always
consistent,  and  there is a unique solution x and a minimal
2-norm solution y, which is obtained using a generalized  QR
factorization of A and B.

In particular, if matrix B is square nonsingular,  then  the
problem  GLM  is equivalent to the following weighted linear
least squares problem

             minimize || inv(B)*(d-A*x) ||_2
                 x

where inv(B) denotes the inverse of B.
```

ARGUMENTS

```
N (input) The number of rows of the matrices A and B.  N  >=
          0.

M (input) The number of columns of the matrix A.  0 <= M  <=
          N.

P (input) The number of columns of the matrix B.  P >= N-M.
```

A (input/output)
On entry, the N-by-M matrix A. On exit, A is destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the N-by-P matrix B. On exit, B is destroyed.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

D (input/output)
On entry, D is the left hand side of the GLM equation. On exit, D is destroyed.

X (output)
On exit, X and Y are the solutions of the GLM problem.

Y (output)
See the description of X.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,N+M+P). For optimum performance, LDWORK >= M+min(N,P)+max(N,P)*NB, where NB is an upper bound for the optimal blocksizes for SGEQRF, SGERQF, SORMQR and SORMRQ.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

```
sgghrd - reduce a pair of real matrices (A,B) to generalized
upper  Hessenberg  form  using  orthogonal  transformations,
where A is a general matrix and B is upper triangular
```

SYNOPSIS

```
SUBROUTINE SGGHRD(COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB, Q, LDQ,
      Z, LDZ, INFO)

CHARACTER * 1 COMPQ, COMPZ
INTEGER N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO
REAL A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*)

SUBROUTINE SGGHRD_64(COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB, Q,
      LDQ, Z, LDZ, INFO)

CHARACTER * 1 COMPQ, COMPZ
INTEGER*8 N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO
REAL A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*)
```

F95 INTERFACE

```
SUBROUTINE GGHRD(COMPQ, COMPZ, [N], ILO, IHI, A, [LDA], B, [LDB], Q,
       [LDQ], Z, [LDZ], [INFO])

CHARACTER(LEN=1) :: COMPQ, COMPZ
INTEGER :: N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO
REAL, DIMENSION(:,:) :: A, B, Q, Z

SUBROUTINE GGHRD_64(COMPQ, COMPZ, [N], ILO, IHI, A, [LDA], B, [LDB],
       Q, [LDQ], Z, [LDZ], [INFO])
```

```
CHARACTER(LEN=1) :: COMPQ, COMPZ
INTEGER(8) :: N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO
REAL, DIMENSION(:,:) :: A, B, Q, Z
```

C INTERFACE

```
#include <sunperf.h>

void sgghrd(char compq, char compz, int n, int ilo, int ihi,
          float  *a,  int  lda, float *b, int ldb, float *q,
          int ldq, float *z, int ldz, int *info);

void sgghrd_64(char compq, char compz,  long  n,  long  ilo,
          long  ihi, float *a, long lda, float *b, long ldb,
          float *q, long  ldq,  float  *z,  long  ldz,  long
          *info);
```

PURPOSE

sgghrd reduces a pair of real matrices (A,B) to generalized upper Hessenberg form using orthogonal transformations, where A is a general matrix and B is upper triangular: Q' * A * Z = H and Q' * B * Z = T, where H is upper Hessenberg, T is upper triangular, and Q and Z are orthogonal, and ' means transpose.

The orthogonal matrices Q and Z are determined as products of Givens rotations. They may either be formed explicitly, or they may be postmultiplied into input matrices Q1 and Z1, so that
1 * A * Z1' = (Q1*Q) * H * (Z1*Z)'

ARGUMENTS

COMPQ (input)
= 'N': do not compute Q;
= 'I': Q is initialized to the unit matrix, and the orthogonal matrix Q is returned; = 'V': Q must contain an orthogonal matrix Q1 on entry, and the product Q1*Q is returned.

COMPZ (input)
= 'N': do not compute Z;
= 'I': Z is initialized to the unit matrix, and the orthogonal matrix Z is returned; = 'V': Z must contain an orthogonal matrix Z1 on entry, and the product Z1*Z is returned.

N (input) The order of the matrices A and B. N >= 0.

ILO (input)
It is assumed that A is already upper triangular in rows and columns 1:ILO-1 and IHI+1:N. ILO and IHI are normally set by a previous call to SGGBAL; otherwise they should be set to 1 and N respectively. 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

IHI (input)
See the description of ILO.

A (input/output)
On entry, the N-by-N general matrix to be reduced. On exit, the upper triangle and the first subdiagonal of A are overwritten with the upper Hessenberg matrix H, and the rest is set to zero.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the N-by-N upper triangular matrix B. On exit, the upper triangular matrix T = Q' B Z. The elements below the diagonal are set to zero.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

Q (input/output)
If COMPQ='N': Q is not referenced.
If COMPQ='I': on entry, Q need not be set, and on exit it contains the orthogonal matrix Q, where Q' is the product of the Givens transformations which are applied to A and B on the left. If COMPQ='V': on entry, Q must contain an orthogonal matrix Q1, and on exit this is overwritten by Q1*Q.

LDQ (input)
The leading dimension of the array Q. LDQ >= N if COMPQ='V' or 'I'; LDQ >= 1 otherwise.

Z (input/output)
If COMPZ='N': Z is not referenced.
If COMPZ='I': on entry, Z need not be set, and on exit it contains the orthogonal matrix Z, which is the product of the Givens transformations which

are applied to A and B on the right. If COMPZ='V': on entry, Z must contain an orthogonal matrix Z1, and on exit this is overwritten by Z1*Z.

LDZ (input)
The leading dimension of the array Z. LDZ >= N if COMPZ='V' or 'I'; LDZ >= 1 otherwise.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

This routine reduces A to Hessenberg and B to triangular form by an unblocked reduction, as described in _Matrix_Computations_, by Golub and Van Loan (Johns Hopkins Press.)

Contents

NAME

```
sgglse - solve the linear equality-constrained least squares
(LSE) problem
```

SYNOPSIS

```
SUBROUTINE SGGLSE(M, N, P, A, LDA, B, LDB, C, D, X, WORK, LDWORK,
      INFO)

INTEGER M, N, P, LDA, LDB, LDWORK, INFO
REAL A(LDA,*), B(LDB,*), C(*), D(*), X(*), WORK(*)

SUBROUTINE SGGLSE_64(M, N, P, A, LDA, B, LDB, C, D, X, WORK, LDWORK,
      INFO)

INTEGER*8 M, N, P, LDA, LDB, LDWORK, INFO
REAL A(LDA,*), B(LDB,*), C(*), D(*), X(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGLSE([M], [N], [P], A, [LDA], B, [LDB], C, D, X, [WORK],
       [LDWORK], [INFO])

INTEGER :: M, N, P, LDA, LDB, LDWORK, INFO
REAL, DIMENSION(:) :: C, D, X, WORK
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE GGLSE_64([M], [N], [P], A, [LDA], B, [LDB], C, D, X, [WORK],
       [LDWORK], [INFO])

INTEGER(8) :: M, N, P, LDA, LDB, LDWORK, INFO
REAL, DIMENSION(:) :: C, D, X, WORK
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void sgglse(int m, int n, int p, float *a,  int  lda,  float
          *b,  int  ldb,  float  *c, float *d, float *x, int
          *info);

void sgglse_64(long m, long n, long p, float *a,  long  lda,
          float  *b, long ldb, float *c, float *d, float *x,
          long *info);
```

PURPOSE

```
sgglse solves the linear equality-constrained least  squares
(LSE) problem:
        minimize || c - A*x ||_2   subject to   B*x = d

where A is an M-by-N matrix, B is a P-by-N matrix,  c  is  a
given  M-vector,  and  d  is a given P-vector. It is assumed
that
P <= N <= M+P, and

         rank(B) = P and  rank( ( A ) ) = N.
                              ( ( B ) )

These conditions ensure that the LSE problem  has  a  unique
solution, which is obtained using a GRQ factorization of the
matrices B and A.
```

ARGUMENTS

```
M (input) The number of rows of the matrix A.  M >= 0.

N (input) The number of columns of the matrices A and  B.  N
          >= 0.

P (input) The number of rows of the matrix B. 0 <= P <= N <=
          M+P.

A (input/output)
          On entry, the M-by-N matrix A.  On exit, A is des-
          troyed.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,M).
```

B (input/output)
On entry, the P-by-N matrix B. On exit, B is destroyed.

LDB (input)
The leading dimension of the array B. LDB >= max(1,P).

C (input/output)
On entry, C contains the right hand side vector for the least squares part of the LSE problem. On exit, the residual sum of squares for the solution is given by the sum of squares of elements N-P+1 to M of vector C.

D (input/output)
On entry, D contains the right hand side vector for the constrained equation. On exit, D is destroyed.

X (output)
On exit, X is the solution of the LSE problem.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,M+N+P). For optimum performance LDWORK >= P+min(M,N)+max(M,N)*NB, where NB is an upper bound for the optimal blocksizes for SGEQRF, SGERQF, SORMQR and SORMRQ.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

```
sggqrf - compute a generalized QR factorization of an N-by-M
matrix A and an N-by-P matrix B.
```

SYNOPSIS

```
SUBROUTINE SGGQRF(N, M, P, A, LDA, TAUA, B, LDB, TAUB, WORK, LWORK,
      INFO)

INTEGER N, M, P, LDA, LDB, LWORK, INFO
REAL A(LDA,*), TAUA(*), B(LDB,*), TAUB(*), WORK(*)

SUBROUTINE SGGQRF_64(N, M, P, A, LDA, TAUA, B, LDB, TAUB, WORK,
      LWORK, INFO)

INTEGER*8 N, M, P, LDA, LDB, LWORK, INFO
REAL A(LDA,*), TAUA(*), B(LDB,*), TAUB(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGQRF([N], [M], [P], A, [LDA], TAUA, B, [LDB], TAUB, [WORK],
       [LWORK], [INFO])

INTEGER :: N, M, P, LDA, LDB, LWORK, INFO
REAL, DIMENSION(:) :: TAUA, TAUB, WORK
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE GGQRF_64([N], [M], [P], A, [LDA], TAUA, B, [LDB], TAUB,
       [WORK], [LWORK], [INFO])

INTEGER(8) :: N, M, P, LDA, LDB, LWORK, INFO
REAL, DIMENSION(:) :: TAUA, TAUB, WORK
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void sggqrf(int n, int m, int p, float *a,  int  lda,  float
          *taua, float *b, int ldb, float *taub, int *info);

void sggqrf_64(long n, long m, long p, float *a,  long  lda,
          float *taua, float *b, long ldb, float *taub, long
          *info);
```

PURPOSE

```
sggqrf computes a generalized QR factorization of an  N-by-M
matrix A and an N-by-P matrix B:
            A = Q*R,        B = Q*T*Z,

where Q is an  N-by-N  orthogonal  matrix,  Z  is  a  P-by-P
orthogonal matrix, and R and T assume one of the forms:

if N >= M,  R = ( R11 ) M  ,   or if N < M,  R = ( R11   R12
) N,
                (  0  ) N-M                         N   M-N
                   M

where R11 is upper triangular, and

if N <= P,  T = ( 0  T12 ) N,   or if N > P,  T =  (  T11  )
N-P,
                 P-N  N                           ( T21 ) P
                                                     P

where T12 or T21 is upper triangular.

In particular, if B is square and nonsingular, the GQR  fac-
torization  of A and B implicitly gives the QR factorization
of inv(B)*A:

             inv(B)*A = Z'*(inv(T)*R)

where inv(B) denotes the inverse of the  matrix  B,  and  Z'
denotes the transpose of the matrix Z.
```

ARGUMENTS

```
N (input) The number of rows of the matrices A and B.  N  >=
          0.
```

M (input) The number of columns of the matrix A. M >= 0.

P (input) The number of columns of the matrix B. P >= 0.

A (input/output)
On entry, the N-by-M matrix A. On exit, the elements on and above the diagonal of the array contain the min(N,M)-by-M upper trapezoidal matrix R (R is upper triangular if N >= M); the elements below the diagonal, with the array TAUA, represent the orthogonal matrix Q as a product of min(N,M) elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

TAUA (output)
The scalar factors of the elementary reflectors which represent the orthogonal matrix Q (see Further Details).

B (input/output)
On entry, the N-by-P matrix B. On exit, if N <= P, the upper triangle of the subarray B(1:N,P-N+1:P) contains the N-by-N upper triangular matrix T; if N > P, the elements on and above the (N-P)-th subdiagonal contain the N-by-P upper trapezoidal matrix T; the remaining elements, with the array TAUB, represent the orthogonal matrix Z as a product of elementary reflectors (see Further Details).

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

TAUB (output)
The scalar factors of the elementary reflectors which represent the orthogonal matrix Z (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,N,M,P). For optimum performance LWORK >=

max(N,M,P)*max(NB1,NB2,NB3), where NB1 is the optimal blocksize for the QR factorization of an N-by-M matrix, NB2 is the optimal blocksize for the RQ factorization of an N-by-P matrix, and NB3 is the optimal blocksize for a call of SORMQR.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

```
The matrix Q is  represented  as  a  product  of  elementary
reflectors

   Q = H(1) H(2) . . . H(k), where k = min(n,m).

Each H(i) has the form

   H(i) = I - taua * v * v'

where taua is a real scalar, and v is a real vector with
v(1:i-1) = 0 and v(i) = 1; v(i+1:n) is  stored  on  exit  in
A(i+1:n,i), and taua in TAUA(i).
To form Q explicitly, use LAPACK subroutine SORGQR.
To use Q to update another  matrix,  use  LAPACK  subroutine
SORMQR.

The matrix Z is  represented  as  a  product  of  elementary
reflectors

   Z = H(1) H(2) . . . H(k), where k = min(n,p).

Each H(i) has the form

   H(i) = I - taub * v * v'

where taub is a real scalar, and v is a real vector with
v(p-k+i+1:p) = 0 and v(p-k+i) = 1; v(1:p-k+i-1) is stored on
exit in B(n-k+i,1:p-k+i-1), and taub in TAUB(i).
To form Z explicitly, use LAPACK subroutine SORGRQ.
To use Z to update another  matrix,  use  LAPACK  subroutine
SORMRQ.
```

Contents

NAME

```
sggrqf - compute a generalized RQ factorization of an M-by-N
matrix A and a P-by-N matrix B
```

SYNOPSIS

```
SUBROUTINE SGGRQF(M, P, N, A, LDA, TAUA, B, LDB, TAUB, WORK, LWORK,
      INFO)

INTEGER M, P, N, LDA, LDB, LWORK, INFO
REAL A(LDA,*), TAUA(*), B(LDB,*), TAUB(*), WORK(*)

SUBROUTINE SGGRQF_64(M, P, N, A, LDA, TAUA, B, LDB, TAUB, WORK,
      LWORK, INFO)

INTEGER*8 M, P, N, LDA, LDB, LWORK, INFO
REAL A(LDA,*), TAUA(*), B(LDB,*), TAUB(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGRQF([M], [P], [N], A, [LDA], TAUA, B, [LDB], TAUB, [WORK],
       [LWORK], [INFO])

INTEGER :: M, P, N, LDA, LDB, LWORK, INFO
REAL, DIMENSION(:) :: TAUA, TAUB, WORK
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE GGRQF_64([M], [P], [N], A, [LDA], TAUA, B, [LDB], TAUB,
       [WORK], [LWORK], [INFO])

INTEGER(8) :: M, P, N, LDA, LDB, LWORK, INFO
REAL, DIMENSION(:) :: TAUA, TAUB, WORK
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void sggrqf(int m, int p, int n, float *a,  int  lda,  float
          *taua, float *b, int ldb, float *taub, int *info);

void sggrqf_64(long m, long p, long n, float *a,  long  lda,
          float *taua, float *b, long ldb, float *taub, long
          *info);
```

PURPOSE

```
sggrqf computes a generalized RQ factorization of an  M-by-N
matrix A and a P-by-N matrix B:
            A = R*Q,        B = Z*T*Q,

where Q is an  N-by-N  orthogonal  matrix,  Z  is  a  P-by-P
orthogonal matrix, and R and T assume one of the forms:

if M <= N,  R = ( 0  R12 ) M,   or if M > N,  R =  (  R11  )
M-N,
                 N-M  M                            ( R21 ) N
                                                      N

where R12 or R21 is upper triangular, and

if P >= N,  T = ( T11 ) N  ,   or if P < N,  T = ( T11   T12
) P,
                (  0  ) P-N                          P   N-P
                   N

where T11 is upper triangular.

In particular, if B is square and nonsingular, the GRQ  fac-
torization  of A and B implicitly gives the RQ factorization
of A*inv(B):

             A*inv(B) = (R*inv(T))*Z'

where inv(B) denotes the inverse of the  matrix  B,  and  Z'
denotes the transpose of the matrix Z.
```

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

```
P (input) The number of rows of the matrix B.  P >= 0.

N (input) The number of columns of the matrices A and  B.  N
          >= 0.

A (input/output)
          On entry, the M-by-N matrix A.  On exit, if  M  <=
          N,  the  upper  triangle  of the subarray A(1:M,N-
          M+1:N) contains the M-by-M upper triangular matrix
          R;  if M > N, the elements on and above the (M-N)-
          th  subdiagonal  contain  the  M-by-N  upper  tra-
          pezoidal  matrix  R;  the remaining elements, with
          the array TAUA, represent the orthogonal matrix  Q
          as a product of elementary reflectors (see Further
          Details).
LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,M).

TAUA (output)
          The scalar factors of  the  elementary  reflectors
          which  represent  the  orthogonal  matrix  Q  (see
          Further Details).

B (input/output)
          On entry, the P-by-N matrix B.  On exit, the  ele-
          ments  on and above the diagonal of the array con-
          tain the min(P,N)-by-N upper trapezoidal matrix  T
          (T  is  upper  triangular if P >= N); the elements
          below the diagonal, with the array TAUB, represent
          the orthogonal matrix Z as a product of elementary
          reflectors (see Further Details).

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          max(1,P).

TAUB (output)
          The scalar factors of  the  elementary  reflectors
          which  represent  the  orthogonal  matrix  Z  (see
          Further Details).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The  dimension  of  the  array  WORK.   LWORK   >=
          max(1,N,M,P).   For  optimum  performance LWORK >=
          max(N,M,P)*max(NB1,NB2,NB3),  where  NB1  is   the
```

```
            optimal  blocksize  for the RQ factorization of an
            M-by-N matrix, NB2 is the  optimal  blocksize  for
            the  QR  factorization of a P-by-N matrix, and NB3
            is the optimal blocksize for a call of SORMRQ.

            If LWORK = -1, then a workspace query is  assumed;
            the  routine  only  calculates the optimal size of
            the WORK array, returns this value  as  the  first
            entry  of  the  WORK  array,  and no error message
            related to LWORK is issued by XERBLA.
  INFO (output)
            = 0:  successful exit
            < 0:  if INF0= -i, the i-th argument had an  ille-
            gal value.
```

FURTHER DETAILS

```
  The matrix Q is  represented  as  a  product  of  elementary
  reflectors

     Q = H(1) H(2) . . . H(k), where k = min(m,n).

  Each H(i) has the form

     H(i) = I - taua * v * v'

  where taua is a real scalar, and v is a real vector with
  v(n-k+i+1:n) = 0 and v(n-k+i) = 1; v(1:n-k+i-1) is stored on
  exit in A(m-k+i,1:n-k+i-1), and taua in TAUA(i).
  To form Q explicitly, use LAPACK subroutine SORGRQ.
  To use Q to update another  matrix,  use  LAPACK  subroutine
  SORMRQ.

  The matrix Z is  represented  as  a  product  of  elementary
  reflectors

     Z = H(1) H(2) . . . H(k), where k = min(p,n).

  Each H(i) has the form

     H(i) = I - taub * v * v'

  where taub is a real scalar, and v is a real vector with
  v(1:i-1) = 0 and v(i) = 1; v(i+1:p) is  stored  on  exit  in
  B(i+1:p,i), and taub in TAUB(i).
  To form Z explicitly, use LAPACK subroutine SORGQR.
  To use Z to update another  matrix,  use  LAPACK  subroutine
  SORMQR.
```

Contents

NAME

sggsvd - compute the generalized singular value decomposition (GSVD) of an M-by-N real matrix A and P-by-N real matrix B

SYNOPSIS

```
SUBROUTINE SGGSVD(JOBU, JOBV, JOBQ, M, N, P, K, L, A, LDA, B, LDB,
      ALPHA, BETA, U, LDU, V, LDV, Q, LDQ, WORK, IWORK3, INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
INTEGER M, N, P, K, L, LDA, LDB, LDU, LDV, LDQ, INFO
INTEGER IWORK3(*)
REAL  A(LDA,*),  B(LDB,*),  ALPHA(*),   BETA(*),   U(LDU,*),
V(LDV,*), Q(LDQ,*), WORK(*)

SUBROUTINE SGGSVD_64(JOBU, JOBV, JOBQ, M, N, P, K, L, A, LDA, B, LDB,
      ALPHA, BETA, U, LDU, V, LDV, Q, LDQ, WORK, IWORK3, INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
INTEGER*8 M, N, P, K, L, LDA, LDB, LDU, LDV, LDQ, INFO
INTEGER*8 IWORK3(*)
REAL  A(LDA,*),  B(LDB,*),  ALPHA(*),   BETA(*),   U(LDU,*),
V(LDV,*), Q(LDQ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGSVD(JOBU, JOBV, JOBQ, [M], [N], [P], K, L, A, [LDA], B,
       [LDB], ALPHA, BETA, U, [LDU], V, [LDV], Q, [LDQ], [WORK], IWORK3,
       [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
INTEGER :: M, N, P, K, L, LDA, LDB, LDU, LDV, LDQ, INFO
INTEGER, DIMENSION(:) :: IWORK3
```

```
REAL, DIMENSION(:) :: ALPHA, BETA, WORK
REAL, DIMENSION(:,:) :: A, B, U, V, Q

SUBROUTINE GGSVD_64(JOBU, JOBV, JOBQ, [M], [N], [P], K, L, A, [LDA],
       B, [LDB], ALPHA, BETA, U, [LDU], V, [LDV], Q, [LDQ], [WORK],
       IWORK3, [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
INTEGER(8) :: M, N, P, K, L, LDA, LDB, LDU, LDV, LDQ, INFO
INTEGER(8), DIMENSION(:) :: IWORK3
REAL, DIMENSION(:) :: ALPHA, BETA, WORK
REAL, DIMENSION(:,:) :: A, B, U, V, Q
```

C INTERFACE

```
#include <sunperf.h>
void sggsvd(char jobu, char jobv, char jobq, int m,  int  n,
          int  p,  int  *k, int *l, float *a, int lda, float
          *b, int ldb, float *alpha, float *beta, float  *u,
          int ldu, float *v, int ldv, float *q, int ldq, int
          *iwork3, int *info);

void sggsvd_64(char jobu, char jobv, char jobq, long m, long
          n,  long  p, long *k, long *l, float *a, long lda,
          float *b, long ldb,  float  *alpha,  float  *beta,
          float  *u, long ldu, float *v, long ldv, float *q,
          long ldq, long *iwork3, long *info);
```

PURPOSE

```
sggsvd computes the generalized singular value decomposition
(GSVD) of an M-by-N real matrix A and P-by-N real matrix B:

    U'*A*Q = D1*( 0 R ),    V'*B*Q = D2*( 0 R )

where U, V and Q are orthogonal  matrices,  and  Z'  is  the
transpose  of  Z.  Let K+L = the effective numerical rank of
the matrix (A',B')', then  R  is  a  K+L-by-K+L  nonsingular
upper  triangular matrix, D1 and D2 are M-by-(K+L) and P-by-
(K+L) "diagonal" matrices and of the  following  structures,
respectively:

If M-K-L >= 0,

                    K  L
       D1 =     K ( I  0 )
                L ( 0  C )
            M-K-L ( 0  0 )

                  K  L
```

```
       D2 =   L ( 0  S )
            P-L ( 0  0 )

                N-K-L  K    L
  ( 0 R ) = K (  0   R11  R12 )
            L (  0    0   R22 )
```

where

```
  C = diag( ALPHA(K+1), ... , ALPHA(K+L) ),
  S = diag( BETA(K+1),  ... , BETA(K+L) ),
  C**2 + S**2 = I.
```

R is stored in A(1:K+L,N-K-L+1:N) on exit.

If M-K-L < 0,

```
                  K M-K K+L-M
       D1 =   K ( I  0    0   )
            M-K ( 0  C    0   )

                    K M-K K+L-M
       D2 =   M-K ( 0  S    0  )
            K+L-M ( 0  0    I  )
              P-L ( 0  0    0  )

                   N-K-L  K   M-K  K+L-M
  ( 0 R ) =     K ( 0    R11  R12  R13  )
              M-K ( 0     0   R22  R23  )
            K+L-M ( 0     0    0   R33  )
```

where

```
  C = diag( ALPHA(K+1), ... , ALPHA(M) ),
  S = diag( BETA(K+1),  ... , BETA(M) ),
  C**2 + S**2 = I.
```

(R11 R12 R13) is stored in A(1:M, N-K-L+1:N), and R33 is stored
(0 R22 R23)
in B(M-K+1:L,N+M-K-L+1:N) on exit.

The routine computes C, S, R, and optionally the orthogonal transformation matrices U, V and Q.

In particular, if B is an N-by-N nonsingular matrix, then the GSVD of A and B implicitly gives the SVD of A*inv(B):
A*inv(B) = U*(D1*inv(D2))*V'.
If (A',B')' has orthonormal columns, then the GSVD of A and B is also equal to the CS decomposition of A and B. Further-

more, the GSVD can be used to derive the solution of the eigenvalue problem:

```
                      A'*A x = lambda* B'*B x.
```

In some literature, the GSVD of A and B is presented in the form

```
                  U'*A*X = ( 0 D1 ),   V'*B*X = ( 0 D2 )
```

where U and V are orthogonal and X is nonsingular, D1 and D2 are ``diagonal''. The former GSVD form can be converted to the latter form by taking the nonsingular matrix X as

```
                      X = Q*( I   0    )
                            ( 0 inv(R) ).
```

ARGUMENTS

JOBU (input)
  = 'U': Orthogonal matrix U is computed;
  = 'N': U is not computed.

JOBV (input)
  = 'V': Orthogonal matrix V is computed;
  = 'N': V is not computed.

JOBQ (input)
  = 'Q': Orthogonal matrix Q is computed;
  = 'N': Q is not computed.

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrices A and B. N >= 0.

P (input) The number of rows of the matrix B. P >= 0.

K (output)
  On exit, K and L specify the dimension of the sub-blocks described in the Purpose section. K + L = effective numerical rank of (A',B')'.

L (output)
  See the description of K.

A (input/output)
  On entry, the M-by-N matrix A. On exit, A contains the triangular matrix R, or part of R. See Purpose for details.

LDA (input)
  The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the P-by-N matrix B. On exit, B contains the triangular matrix R if M-K-L < 0. See Purpose for details.

LDB (input)
The leading dimension of the array B. LDA >= max(1,P).

ALPHA (output)
On exit, ALPHA and BETA contain the generalized singular value pairs of A and B; ALPHA(1:K) = 1,
BETA(1:K) = 0, and if M-K-L >= 0, ALPHA(K+1:K+L) = C,
BETA(K+1:K+L) = S, or if M-K-L < 0,
ALPHA(K+1:M)=C, ALPHA(M+1:K+L)=0
BETA(K+1:M) =S, BETA(M+1:K+L) =1 and
ALPHA(K+L+1:N) = 0
BETA(K+L+1:N) = 0

BETA (output)
See the description of ALPHA.

U (output)
If JOBU = 'U', U contains the M-by-M orthogonal matrix U. If JOBU = 'N', U is not referenced.

LDU (input)
The leading dimension of the array U. LDU >= max(1,M) if JOBU = 'U'; LDU >= 1 otherwise.

V (output)
If JOBV = 'V', V contains the P-by-P orthogonal matrix V. If JOBV = 'N', V is not referenced.

LDV (input)
The leading dimension of the array V. LDV >= max(1,P) if JOBV = 'V'; LDV >= 1 otherwise.

Q (output)
If JOBQ = 'Q', Q contains the N-by-N orthogonal matrix Q. If JOBQ = 'N', Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= max(1,N) if JOBQ = 'Q'; LDQ >= 1 otherwise.

WORK (workspace)
dimension (max(3*N,M,P)+N)

IWORK3 (output)
dimension(N) On exit, IWORK3 stores the sorting information. More precisely, the following loop will sort ALPHA for I = K+1, min(M,K+L) swap ALPHA(I) and ALPHA(IWORK3(I)) endfor such that ALPHA(1) >= ALPHA(2) >= ... >= ALPHA(N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = 1, the Jacobi-type procedure failed to converge. For further details, see subroutine STGSJA.

Contents

NAME

```
sggsvp - compute orthogonal matrices U, V and  Q  such  that
N-K-L K L  U'*A*Q = K ( 0 A12 A13 ) if M-K-L >= 0
```

SYNOPSIS

```
SUBROUTINE SGGSVP(JOBU, JOBV, JOBQ, M, P, N, A, LDA, B, LDB, TOLA,
      TOLB, K, L, U, LDU, V, LDV, Q, LDQ, IWORK, TAU, WORK, INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
INTEGER M, P, N, LDA, LDB, K, L, LDU, LDV, LDQ, INFO
INTEGER IWORK(*)
REAL TOLA, TOLB
REAL  A(LDA,*),  B(LDB,*),  U(LDU,*),  V(LDV,*),   Q(LDQ,*),
TAU(*), WORK(*)

SUBROUTINE SGGSVP_64(JOBU, JOBV, JOBQ, M, P, N, A, LDA, B, LDB, TOLA,
      TOLB, K, L, U, LDU, V, LDV, Q, LDQ, IWORK, TAU, WORK, INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
INTEGER*8 M, P, N, LDA, LDB, K, L, LDU, LDV, LDQ, INFO
INTEGER*8 IWORK(*)
REAL TOLA, TOLB
REAL  A(LDA,*),  B(LDB,*),  U(LDU,*),  V(LDV,*),   Q(LDQ,*),
TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGSVP(JOBU, JOBV, JOBQ, [M], [P], [N], A, [LDA], B, [LDB],
       TOLA, TOLB, K, L, U, [LDU], V, [LDV], Q, [LDQ], [IWORK], [TAU],
       [WORK], [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
```

```
INTEGER :: M, P, N, LDA, LDB, K, L, LDU, LDV, LDQ, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL :: TOLA, TOLB
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A, B, U, V, Q

SUBROUTINE GGSVP_64(JOBU, JOBV, JOBQ, [M], [P], [N], A, [LDA], B,
       [LDB], TOLA, TOLB, K, L, U, [LDU], V, [LDV], Q, [LDQ], [IWORK],
       [TAU], [WORK], [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
INTEGER(8) :: M, P, N, LDA, LDB, K, L, LDU, LDV, LDQ, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL :: TOLA, TOLB
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A, B, U, V, Q
```

C INTERFACE

```
#include <sunperf.h>

void sggsvp(char jobu, char jobv, char jobq, int m,  int  p,
          int n, float *a, int lda, float *b, int ldb, float
          tola, float tolb, int *k, int *l,  float  *u,  int
          ldu,  float  *v,  int  ldv, float *q, int ldq, int
          *info);

void sggsvp_64(char jobu, char jobv, char jobq, long m, long
          p, long n, float *a, long lda, float *b, long ldb,
          float tola, float tolb, long *k,  long  *l,  float
          *u,  long  ldu, float *v, long ldv, float *q, long
          ldq, long *info);
```

PURPOSE

```
sggsvp computes orthogonal matrices U, V and Q such that
              L ( 0     0   A23 )
          M-K-L ( 0     0    0  )

                 N-K-L  K    L
        =     K ( 0    A12  A13 )  if M-K-L < 0;
            M-K ( 0     0   A23 )

               N-K-L  K    L
 V'*B*Q =   L ( 0     0   B13 )
          P-L ( 0     0    0  )

where the K-by-K matrix A12 and L-by-L matrix B13  are  non-
singular upper triangular; A23 is L-by-L upper triangular if
M-K-L >= 0, otherwise A23 is (M-K)-by-L  upper  trapezoidal.
K+L  = the effective numerical rank of the (M+P)-by-N matrix
```

```
(A',B')'.  Z' denotes the transpose of Z.

This decomposition is the preprocessing step  for  computing
the  Generalized  Singular  Value  Decomposition (GSVD), see
subroutine SGGSVD.
```

ARGUMENTS

```
JOBU (input)
          = 'U':  Orthogonal matrix U is computed;
          = 'N':  U is not computed.

JOBV (input)
          = 'V':  Orthogonal matrix V is computed;
          = 'N':  V is not computed.
JOBQ (input)
          = 'Q':  Orthogonal matrix Q is computed;
          = 'N':  Q is not computed.

M (input) The number of rows of the matrix A.  M >= 0.

P (input) The number of rows of the matrix B.  P >= 0.

N (input) The number of columns of the matrices A and B.   N
          >= 0.

A (input/output)
          On entry, the M-by-N matrix A.  On  exit,  A  con-
          tains   the  triangular  (or  trapezoidal)  matrix
          described in the Purpose section.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,M).

B (input/output)
          On entry, the P-by-N matrix B.  On  exit,  B  con-
          tains  the triangular matrix described in the Pur-
          pose section.

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          max(1,P).

TOLA (input)
          TOLA and TOLB are the thresholds to determine  the
          effective  numerical  rank  of matrix B and a sub-
          block of A. Generally, they  are  set  to  TOLA  =
```

MAX(M,N)*norm(A)*MACHEPS, TOLB = MAX(P,N)*norm(B)*MACHEPS. The size of TOLA and TOLB may affect the size of backward errors of the decomposition.

TOLB (input)
See the description of TOLA.

K (output)
On exit, K and L specify the dimension of the sub-blocks described in Purpose. K + L = effective numerical rank of (A',B')'.

L (output)
See the description of K.

U (input) If JOBU = 'U', U contains the orthogonal matrix U.
If JOBU = 'N', U is not referenced.

LDU (input)
The leading dimension of the array U. LDU >= max(1,M) if JOBU = 'U'; LDU >= 1 otherwise.

V (input) If JOBV = 'V', V contains the orthogonal matrix V.
If JOBV = 'N', V is not referenced.

LDV (input)
The leading dimension of the array V. LDV >= max(1,P) if JOBV = 'V'; LDV >= 1 otherwise.

Q (input) If JOBQ = 'Q', Q contains the orthogonal matrix Q.
If JOBQ = 'N', Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= max(1,N) if JOBQ = 'Q'; LDQ >= 1 otherwise.

IWORK (workspace)
dimension(N)

TAU (workspace)
dimension(N)

WORK (workspace)
dimension(MAX(3*N,M,P))

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

The subroutine uses LAPACK subroutine SGEQPF for the QR factorization with column pivoting to detect the effective numerical rank of the a matrix. It may be replaced by a better rank determination strategy.

Contents

NAME

sgssco - General sparse solver condition number estimate.

SYNOPSIS

```
SUBROUTINE SGSSCO ( COND, HANDLE, IER )

INTEGER            IER
REAL               COND
DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

SGSSCO - Condition number estimate.

PARAMETERS

```
COND      - REAL
        On exit, an estimate of the condition number of the
        factored matrix.  Must be called after the numerical
        factorization subroutine, SGSSFA().

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -700 : Invalid calling sequence - need to call SGSSFA first.
```

-710 : Condition number estimate not available (not implemented
       for this HANDLE's matix type).

Contents

NAME

```
sgssda - Deallocate working storage for the general sparse
solver.
```

SYNOPSIS

```
 SUBROUTINE SGSSDA ( HANDLE, IER )

 INTEGER           IER
 DOUBLE PRECISION  HANDLE(150)
```

PURPOSE

SGSSDA - Deallocate dynamically allocated working storage.

PARAMETERS

```
HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         none
```

Contents

NAME

sgssfa - General sparse solver numeric factorization.

SYNOPSIS

```
 SUBROUTINE SGSSFA ( NEQNS, COLSTR, ROWIND, VALUES, HANDLE, IER )

 INTEGER            NEQNS, COLSTR(*), ROWIND(*), IER
 REAL               VALUES(*)
 DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

SGSSFA - Numeric factorization of a sparse matrix.

PARAMETERS

NEQNS   - INTEGER
        On entry, NEQNS specifies the number of equations in
        coefficient matrix. Unchanged on exit.

COLSTR(*) - INTEGER array
        On entry, *COLSTR*(*) is an array of size (NEQNS+1),
        containing the pointers of the matrix structure.
        Unchanged on exit.

ROWIND(*) - INTEGER array
        On entry, *ROWIND*(*) is an array of size
        COLSTR(NEQNS+1)-1, containing the indices of the
        matrix structure. Unchanged on exit.

VALUES(*) - REAL array
        On entry, *VALUES*(*) is an array of size
        COLSTR(NEQNS+1)-1, containing the numeric values of

```
        the sparse matrix to be factored.  Unchanged on
        exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER        - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -300 : Invalid calling sequence - need to call SGSSOR first.
         -301 : Failure to dynamically allocate memory.
         -666 : Internal error.
```

Contents

NAME

```
     sgssfs - General sparse solver one call interface.
```

SYNOPSIS

```
      SUBROUTINE SGSSFS ( MTXTYP, PIVOT , NEQNS, COLSTR, ROWIND,
                          VALUES, NRHS  , RHS   , LDRHS , ORDMTHD,
                          OUTUNT, MSGLVL, HANDLE, IER )

      CHARACTER*2        MTXTYP
      CHARACTER*1        PIVOT
      INTEGER            NEQNS, COLSTR(*), ROWIND(*), NRHS, LDRHS,
                         OUTUNT, MSGLVL, IER
      CHARACTER*3        ORDMTHD
      REAL               VALUES(*), RHS(*)
      DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

SGSSFS - General sparse solver one call interface.

PARAMETERS

```
     MTXTYP    - CHARACTER*2
             On entry, MTXTYP specifies the coefficient matrix
             type.  Specifically, the valid options are:

              'sp' or 'SP' - symmetric structure, positive-definite values
              'ss' or 'SS' - symmetric structure, symmetric values
              'su' or 'SU' - symmetric structure, unsymmetric values
              'uu' or 'UU' - unsymmetric structure, unsymmetric values

             Unchanged on exit.
```

```
PIVOT     - CHARACTER*1
        On entry, pivot specifies whether or not pivoting is
        used in the course of the numeric factorization.
        The valid options are:

         'n' or 'N' - no pivoting is used
         (Pivoting is not supported for this release).

        Unchanged on exit.

NEQNS     - INTEGER
        On entry, NEQNS specifies the number of equations in
        the coefficient matrix.  NEQNS must be at least one.
        Unchanged on exit.

COLSTR(*) - INTEGER array
        On entry, COLSTR(*) is an array of size (NEQNS+1),
        containing the pointers of the matrix structure.
        Unchanged on exit.
ROWIND(*) - INTEGER array
        On entry, ROWIND(*) is an array of size
        COLSTR(NEQNS+1)-1, containing the indices of the
        matrix structure.  Unchanged on exit.

VALUES(*) - REAL array
        On entry, VALUES(*) is an array of size
        COLSTR(NEQNS+1)-1, containing the non-zero numeric
        values of the sparse matrix to be factored.
        Unchanged on exit.

NRHS      - INTEGER
        On entry, NRHS specifies the number of right hand
        sides to solve for.  Unchanged on exit.

RHS(*)    - REAL array
        On entry, RHS(LDRHS,NRHS) contains the NRHS right
        hand sides.  On exit, it contains the solutions.

LDRHS     - INTEGER
        On entry, LDRHS specifies the leading dimension of
        the RHS array. Unchanged on exit.

ORDMTHD   - CHARACTER*3
        On entry, ORDMTHD specifies the fill-reducing
        ordering to be used by the sparse solver.
        Specifically, the valid options are:

         'nat' or 'NAT' - natural ordering (no ordering)
         'mmd' or 'MMD' - multiple minimum degree
         'gnd' or 'GND' - general nested dissection
```

```
          'uso' or 'USO' - user specified ordering (see SGSSUO)

         Unchanged on exit.

OUTUNT    - INTEGER
         Output unit.  Unchanged on exit.

MSGLVL    - INTEGER
         Message level.

          0 - no output from solver.
          (No messages supported for this release.)

         Unchanged on exit.

HANDLE(150)  - DOUBLE PRECISION array
         On entry, HANDLE(*) is an array of containing
         information needed by the solver, and must be passed
         unchanged to each sparse solver subroutine.
         Modified on exit.
IER       - INTEGER
         Error number.  If no error encountered, unchanged on
         exit.  If error encountered, it is set to a non-zero
         integer.  Error numbers set by this subroutine:

          -101 : Failure to dynamically allocate memory.
          -102 : Invalid matrix type.
          -103 : Invalid pivot option.
          -104 : Number of nonzeros is less than NEQNS.
          -105 : NEQNS < 1
          -201 : Failure to dynamically allocate memory.
          -301 : Failure to dynamically allocate memory.
          -401 : Failure to dynamically allocate memory.
          -402 : NRHS < 1
          -403 : NEQNS > LDRHS
          -666 : Internal error.
```

Contents

NAME

```
sgssin - Initialize the general sparse solver.
```

SYNOPSIS

```
 SUBROUTINE SGSSIN ( MTXTYP, PIVOT, NEQNS, COLSTR, ROWIND, OUTUNT,
                     MSGLVL, HANDLE, IER )

 CHARACTER*2       MTXTYP
 CHARACTER*1       PIVOT
 INTEGER           NEQNS, COLSTR(*), ROWIND(*), OUTUNT, MSGLVL, IER
 DOUBLE PRECISION  HANDLE(150)
```

PURPOSE

SGSSIN - Initialize the sparse solver and input the matrix structure.

PARAMETERS

```
MTXTYP    - CHARACTER*2
        On entry, MTXTYP specifies the coefficient matrix
        type.  Specifically, the valid options are:

         'sp' or 'SP' - symmetric structure, positive-definite values
         'ss' or 'SS' - symmetric structure, symmetric values
         'su' or 'SU' - symmetric structure, unsymmetric values
         'uu' or 'UU' - unsymmetric structure, unsymmetric values

        Unchanged on exit.

PIVOT     - CHARACTER*1
        On entry, PIVOT specifies whether or not pivoting is
        used in the course of the numeric factorization.
```

```
        The valid options are:

         'n' or 'N' - no pivoting is used
         (Pivoting is not supported for this release).

        Unchanged on exit.

NEQNS     - INTEGER
        On entry, NEQNS specifies the number of equations in
        the coefficient matrix.  NEQNS must be at least one.
        Unchanged on exit.

COLSTR(*) - INTEGER array
        On entry, COLSTR(*) is an array of size (NEQNS+1),
        containing the pointers of the matrix structure.
        Unchanged on exit.

ROWIND(*) - INTEGER array
        On entry, ROWIND(*) is an array of size
        COLSTR(NEQNS+1)-1, containing the indices of the
        matrix structure.  Unchanged on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

OUTUNT    - INTEGER
        Output unit.  Unchanged on exit.

MSGLVL    - INTEGER
        Message level.

         0 - no output from solver.
         (No messages supported for this release.)

        Unchanged on exit.

IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -101 : Failure to dynamically allocate memory.
         -102 : Invalid matrix type.
         -103 : Invalid pivot option.
         -104 : Number of nonzeros less than NEQNS.
         -105 : NEQNS < 1
```

Contents

NAME

```
sgssor - General sparse solver ordering and symbolic
factorization.
```

SYNOPSIS

```
 SUBROUTINE SGSSOR ( ORDMTHD, HANDLE, IER )

 CHARACTER*3       ORDMTHD
 INTEGER           IER
 DOUBLE PRECISION  HANDLE(150)
```

PURPOSE

SGSSOR - Orders and symbolically factors a sparse matrix.

PARAMETERS

```
ORDMTHD   - CHARACTER*3
        On entry, ORDMTHD specifies the fill-reducing
        ordering to be used by the sparse solver.
        Specifically, the valid options are:

         'nat' or 'NAT' - natural ordering (no ordering)
         'mmd' or 'MMD' - multiple minimum degree
         'gnd' or 'GND' - general nested dissection
         'uso' or 'USO' - user specified ordering (see SGSSUO)

        Unchanged on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
```

```
        Modified on exit.

IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -200 : Invalid calling sequence - need to call SGSSIN first.
         -201 : Failure to dynamically allocate memory.
         -666 : Internal error.
```

Contents

NAME

```
sgssps - Print general sparse solver statics.
```

SYNOPSIS

```
 SUBROUTINE SGSSPS ( HANDLE, IER )

 INTEGER            IER
 DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

SGSSPS - Print solver statistics.

PARAMETERS

```
HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -800 : Invalid calling sequence - need to call SGSSSL first.
         -899 : Printed solver statistics not supported this release.
```

Contents

NAME

sgssrp - Return permutation used by the general sparse solver.

SYNOPSIS

```
SUBROUTINE SGSSRP ( PERM, HANDLE, IER )

INTEGER            PERM(*), IER
DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

SGSSRP - Returns the permutation used by the solver for the fill-reducing ordering.

PARAMETERS

```
PERM(NEQNS)  - INTEGER array
        Undefined on entry.  PERM(NEQNS) is the permutation
        array used by the sparse solver for the fill-
        reducing ordering.  Modified on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:
```

-600 : Invalid calling sequence - need to call SGSSOR first.

Contents

NAME

```
sgsssl - Solve routine for the general sparse solver.
```

SYNOPSIS

```
 SUBROUTINE SGSSSL ( NRHS, RHS, LDRHS, HANDLE, IER )

 INTEGER            NRHS, LDRHS, IER
 REAL               RHS(LDRHS,NRHS)
 DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

SGSSSL - Triangular solve of a factored sparse matrix.

PARAMETERS

```
NRHS      - INTEGER
        On entry, NRHS specifies the number of right hand
        sides to solve for.  Unchanged on exit.

RHS(LDRHS,*) - REAL array
        On entry, RHS(LDRHS,NRHS) contains the NRHS right
        hand sides.  On exit, it contains the solutions.

LDRHS     - INTEGER
        On entry, LDRHS specifies the leading dimension of
        the RHS array. Unchanged on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.
```

IER - INTEGER
Error number. If no error encountered, unchanged on exit. If error encountered, it is set to a non-zero integer. Error numbers set by this subroutine:

-400 : Invalid calling sequence - need to call SGSSFA first.
-401 : Failure to dynamically allocate memory.
-402 : NRHS < 1
-403 : NEQNS > LDRHS

Contents

NAME

```
sgssuo - User supplied permutation for ordering used in the
general sparse solver.
```

SYNOPSIS

```
 SUBROUTINE SGSSUO ( PERM, HANDLE, IER )

 INTEGER            PERM(*), IER
 DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

SGSSUO - User supplied permutation for ordering. Must be
called after *SGSSIN*() (sparse solver initialization) and
before *SGSSOR*() (sparse solver ordering).

PARAMETERS

```
PERM(NEQNS)  - INTEGER array
        On entry, PERM(NEQNS) is a permutation array
        supplied by the user for the fill-reducing ordering.
        Unchanged on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER        - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:
```

-500 : Invalid calling sequence - need to call SGSSIN first.

Contents

NAME

```
sgtcon - estimate the reciprocal of the condition number  of
a  real  tridiagonal  matrix A using the LU factorization as
computed by SGTTRF
```

SYNOPSIS

```
SUBROUTINE SGTCON(NORM, N, LOW, DIAG, UP1, UP2, IPIVOT, ANORM, RCOND,
      WORK, IWORK2, INFO)

CHARACTER * 1 NORM
INTEGER N, INFO
INTEGER IPIVOT(*), IWORK2(*)
REAL ANORM, RCOND
REAL LOW(*), DIAG(*), UP1(*), UP2(*), WORK(*)

SUBROUTINE SGTCON_64(NORM, N, LOW, DIAG, UP1, UP2, IPIVOT, ANORM,
      RCOND, WORK, IWORK2, INFO)

CHARACTER * 1 NORM
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*), IWORK2(*)
REAL ANORM, RCOND
REAL LOW(*), DIAG(*), UP1(*), UP2(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GTCON(NORM, [N], LOW, DIAG, UP1, UP2, IPIVOT, ANORM,
       RCOND, [WORK], [IWORK2], [INFO])

CHARACTER(LEN=1) :: NORM
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT, IWORK2
REAL :: ANORM, RCOND
```

```
REAL, DIMENSION(:) :: LOW, DIAG, UP1, UP2, WORK

SUBROUTINE GTCON_64(NORM, [N], LOW, DIAG, UP1, UP2, IPIVOT, ANORM,
       RCOND, [WORK], [IWORK2], [INFO])

CHARACTER(LEN=1) :: NORM
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, IWORK2
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: LOW, DIAG, UP1, UP2, WORK
```

C INTERFACE

```
#include <sunperf.h>
void sgtcon(char norm, int n, float *low, float *diag, float
          *up1,  float *up2, int *ipivot, float anorm, float
          *rcond, int *info);

void sgtcon_64(char norm, long n, float *low,  float  *diag,
          float *up1, float *up2, long *ipivot, float anorm,
          float *rcond, long *info);
```

PURPOSE

sgtcon estimates the reciprocal of the condition number of a real tridiagonal matrix A using the LU factorization as computed by SGTTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

NORM (input)
Specifies whether the 1-norm condition number or the infinity-norm condition number is required:
= '1' or 'O': 1-norm;
= 'I':   Infinity-norm.

N (input) The order of the matrix A. N >= 0.

LOW (input)
The (n-1) multipliers that define the matrix L from the LU factorization of A as computed by SGTTRF.

DIAG (input)

The n diagonal elements of the upper triangular matrix U from the LU factorization of A.

UP1 (input)
The (n-1) elements of the first superdiagonal of U.

UP2 (input)
The (n-2) elements of the second superdiagonal of U.

IPIVOT (input)
The pivot indices; for 1 <= i <= n, row i of the matrix was interchanged with row IPIVOT(i). IPIVOT(i) will always be either i or i+1; IPIVOT(i) = i indicates a row interchange was not required.

ANORM (input)
If NORM = '1' or 'O', the 1-norm of the original matrix A. If NORM = 'I', the infinity-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(2*N)

IWORK2 (workspace)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
sgthr - Gathers specified elements from y into x.
```

SYNOPSIS

```
 SUBROUTINE SGTHR(NZ, Y, X, INDX)

 REAL Y(*), X(*)
 INTEGER NZ
 INTEGER INDX(*)

 SUBROUTINE SGTHR_64(NZ, Y, X, INDX)

 REAL Y(*), X(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 SUBROUTINE GTHR([NZ], Y, X, INDX)

 REAL, DIMENSION(:) :: Y, X
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 SUBROUTINE GTHR_64([NZ], Y, X, INDX)

 REAL, DIMENSION(:) :: Y, X
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
SGTHR - Gathers the specified elements from a vector y in
full storage form into a vector x in compressed form.  Only
```

the elements of y whose indices are listed in indx are referenced.

```
 do i = 1, n
   x(i) = y(indx(i))
 enddo
```

ARGUMENTS

NZ (input) - INTEGER
Number of elements in the compressed form. Unchanged on exit.

Y (input)
Vector in full storage form. Unchanged on exit.

X (output)
Vector in compressed form. Contains elements of y whose indices are listed in indx on exit.

INDX (input) - INTEGER
Vector containing the indices of the compressed form. It is assumed that the elements in INDX are distinct and greater than zero. Unchanged on exit.

Contents

NAME

```
sgthrz - Gather and zero.
```

SYNOPSIS

```
 SUBROUTINE SGTHRZ(NZ, Y, X, INDX)

 REAL Y(*), X(*)
 INTEGER NZ
 INTEGER INDX(*)

 SUBROUTINE SGTHRZ_64(NZ, Y, X, INDX)

 REAL Y(*), X(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 SUBROUTINE GTHRZ([NZ], Y, X, INDX)

 REAL, DIMENSION(:) :: Y, X
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 SUBROUTINE GTHRZ_64([NZ], Y, X, INDX)

 REAL, DIMENSION(:) :: Y, X
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
SGTHRZ - Gathers the specified elements from a vector y in
full storage form into a vector x in compressed form.  The
```

gathered elements of y are set to zero. Only the elements of y whose indices are listed in indx are referenced.

```
 do i = 1, n
   x(i) = y(indx(i))
   y(indx(i)) = 0
 enddo
```

ARGUMENTS

NZ (input) - INTEGER
Number of elements in the compressed form. Unchanged on exit.

Y (input/output)
Vector in full storage form. Gathered elements are set to zero.

X (output)
Vector in compressed form. Contains elements of y whose indices are listed in indx on exit.

INDX (input) - INTEGER
Vector containing the indices of the compressed form. It is assumed that the elements in INDX are distinct and greater than zero. Unchanged on exit.

Contents

NAME

sgtrfs - improve the computed solution to a system of linear equations when the coefficient matrix is tridiagonal, and provides error bounds and backward error estimates for the solution

SYNOPSIS

```
SUBROUTINE SGTRFS(TRANSA, N, NRHS, LOW, DIAG, UP, LOWF, DIAGF, UPF1,
      UPF2, IPIVOT, B, LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*), WORK2(*)
REAL LOW(*), DIAG(*),  UP(*),  LOWF(*),  DIAGF(*),  UPF1(*),
UPF2(*), B(LDB,*), X(LDX,*), FERR(*), BERR(*), WORK(*)

SUBROUTINE SGTRFS_64(TRANSA, N, NRHS, LOW, DIAG, UP, LOWF, DIAGF,
      UPF1, UPF2, IPIVOT, B, LDB, X, LDX, FERR, BERR, WORK, WORK2,
      INFO)

CHARACTER * 1 TRANSA
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
REAL LOW(*), DIAG(*),  UP(*),  LOWF(*),  DIAGF(*),  UPF1(*),
UPF2(*), B(LDB,*), X(LDX,*), FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GTRFS([TRANSA], [N], [NRHS], LOW, DIAG, UP, LOWF, DIAGF,
       UPF1, UPF2, IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: TRANSA
```

```
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
REAL, DIMENSION(:) :: LOW,  DIAG,  UP,  LOWF,  DIAGF,  UPF1,
UPF2, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: B, X

SUBROUTINE GTRFS_64([TRANSA], [N], [NRHS], LOW, DIAG, UP, LOWF,
       DIAGF, UPF1, UPF2, IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: TRANSA
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL, DIMENSION(:) :: LOW,  DIAG,  UP,  LOWF,  DIAGF,  UPF1,
UPF2, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: B, X
```

C INTERFACE

```
#include <sunperf.h>

void sgtrfs(char transa, int n, int nrhs, float *low,  float
          *diag, float *up, float *lowf, float *diagf, float
          *upf1, float *upf2, int  *ipivot,  float  *b,  int
          ldb,  float *x, int ldx, float *ferr, float *berr,
          int *info);

void sgtrfs_64(char transa, long n, long nrhs,  float  *low,
          float *diag, float *up, float *lowf, float *diagf,
          float *upf1, float *upf2, long *ipivot, float  *b,
          long  ldb,  float *x, long ldx, float *ferr, float
          *berr, long *info);
```

PURPOSE

sgtrfs improves the computed solution to a system of linear equations when the coefficient matrix is tridiagonal, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

```
TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose = Tran-
          spose)
```

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

LOW (input)
The (n-1) subdiagonal elements of A.

DIAG (input)
The diagonal elements of A.

UP (input)
The (n-1) superdiagonal elements of A.

LOWF (input)
The (n-1) multipliers that define the matrix L from the LU factorization of A as computed by SGTTRF.

DIAGF (input)
The n diagonal elements of the upper triangular matrix U from the LU factorization of A.

UPF1 (input)
The (n-1) elements of the first superdiagonal of U.

UPF2 (input)
The (n-2) elements of the second superdiagonal of U.

IPIVOT (input)
The pivot indices; for 1 <= i <= n, row i of the matrix was interchanged with row IPIVOT(i). IPIVOT(i) will always be either i or i+1; IPIVOT(i) = i indicates a row interchange was not required.

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by SGTTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
sgtsv - solve the equation   A*X = B,
```

SYNOPSIS

```
SUBROUTINE SGTSV(N, NRHS, LOW, DIAG, UP, B, LDB, INFO)

INTEGER N, NRHS, LDB, INFO
REAL LOW(*), DIAG(*), UP(*), B(LDB,*)

SUBROUTINE SGTSV_64(N, NRHS, LOW, DIAG, UP, B, LDB, INFO)

INTEGER*8 N, NRHS, LDB, INFO
REAL LOW(*), DIAG(*), UP(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE GTSV([N], [NRHS], LOW, DIAG, UP, B, [LDB], [INFO])

INTEGER :: N, NRHS, LDB, INFO
REAL, DIMENSION(:) :: LOW, DIAG, UP
REAL, DIMENSION(:,:) :: B

SUBROUTINE GTSV_64([N], [NRHS], LOW, DIAG, UP, B, [LDB], [INFO])

INTEGER(8) :: N, NRHS, LDB, INFO
REAL, DIMENSION(:) :: LOW, DIAG, UP
REAL, DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void sgtsv(int n, int nrhs, float *low, float  *diag,  float
          *up, float *b, int ldb, int *info);
```

```
void sgtsv_64(long n, long nrhs, float  *low,  float  *diag,
          float *up, float *b, long ldb, long *info);
```

PURPOSE

sgtsv solves the equation

where A is an n by n tridiagonal matrix, by Gaussian elimination with partial pivoting.

Note that the equation A'*X = B may be solved by interchanging the order of the arguments DU and DL.

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

LOW (input/output)
On entry, LOW must contain the (n-1) sub-diagonal elements of A.

On exit, LOW is overwritten by the (n-2) elements of the second super-diagonal of the upper triangular matrix U from the LU factorization of A, in LOW(1), ..., LOW(n-2).

DIAG (input/output)
On entry, DIAG must contain the diagonal elements of A.

On exit, DIAG is overwritten by the n diagonal elements of U.

UP (input/output)
On entry, UP must contain the (n-1) super-diagonal elements of A.

On exit, UP is overwritten by the (n-1) elements of the first super-diagonal of U.

B (input/output)
On entry, the N by NRHS matrix of right hand side

matrix B. On exit, if INFO = 0, the N by NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, U(i,i) is exactly zero, and the solution has not been computed. The factorization has not been completed unless i = N.

Contents

NAME

```
sgtsvx - use the LU factorization to compute the solution to
a real system of linear equations A * X = B or A**T * X = B,
```

SYNOPSIS

```
SUBROUTINE SGTSVX(FACT, TRANSA, N, NRHS, LOW, DIAG, UP, LOWF, DIAGF,
      UPF1, UPF2, IPIVOT, B, LDB, X, LDX, RCOND, FERR, BERR, WORK,
      WORK2, INFO)

CHARACTER * 1 FACT, TRANSA
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*), WORK2(*)
REAL RCOND
REAL LOW(*), DIAG(*),  UP(*),  LOWF(*),  DIAGF(*),  UPF1(*),
UPF2(*), B(LDB,*), X(LDX,*), FERR(*), BERR(*), WORK(*)

SUBROUTINE SGTSVX_64(FACT, TRANSA, N, NRHS, LOW, DIAG, UP, LOWF,
      DIAGF, UPF1, UPF2, IPIVOT, B, LDB, X, LDX, RCOND, FERR, BERR,
      WORK, WORK2, INFO)

CHARACTER * 1 FACT, TRANSA
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
REAL RCOND
REAL LOW(*), DIAG(*),  UP(*),  LOWF(*),  DIAGF(*),  UPF1(*),
UPF2(*), B(LDB,*), X(LDX,*), FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GTSVX(FACT, [TRANSA], [N], [NRHS], LOW, DIAG, UP, LOWF,
       DIAGF, UPF1, UPF2, IPIVOT, B, [LDB], X, [LDX], RCOND, FERR, BERR,
       [WORK], [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: FACT, TRANSA
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
REAL :: RCOND
REAL, DIMENSION(:) :: LOW,  DIAG,  UP,  LOWF,  DIAGF,  UPF1,
UPF2, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: B, X

SUBROUTINE GTSVX_64(FACT, [TRANSA], [N], [NRHS], LOW, DIAG, UP, LOWF,
       DIAGF, UPF1, UPF2, IPIVOT, B, [LDB], X, [LDX], RCOND, FERR, BERR,
       [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, TRANSA
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL :: RCOND
REAL, DIMENSION(:) :: LOW,  DIAG,  UP,  LOWF,  DIAGF,  UPF1,
UPF2, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: B, X
```

C INTERFACE

```
#include <sunperf.h>

void sgtsvx(char fact, char transa, int n, int  nrhs,  float
          *low,  float  *diag, float *up, float *lowf, float
          *diagf, float *upf1,  float  *upf2,  int  *ipivot,
          float  *b,  int  ldb,  float  *x,  int  ldx, float
          *rcond, float *ferr, float *berr, int *info);

void sgtsvx_64(char fact, char transa, long  n,  long  nrhs,
          float  *low,  float *diag, float *up, float *lowf,
          float  *diagf,  float  *upf1,  float  *upf2,  long
          *ipivot,  float  *b, long ldb, float *x, long ldx,
          float  *rcond,  float  *ferr,  float  *berr,  long
          *info);
```

PURPOSE

sgtsvx uses the LU factorization to compute the solution to a real system of linear equations A * X = B or A**T * X = B, where A is a tridiagonal matrix of order N and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'N', the LU decomposition is used to factor the

matrix A
as A = L * U, where L is a product of permutation and unit lower
bidiagonal matrices and U is upper triangular with nonzeros in
only the main diagonal and first two superdiagonals.

2. If some U(i,i)=0, so that U is exactly singular, then the routine
returns with INFO = i. Otherwise, the factored form of A is used
to estimate the condition number of the matrix A. If the
reciprocal of the condition number is less than machine precision,
INFO = N+1 is returned as a warning, but the routine still goes on
to solve for X and compute error bounds as described below.
3. The system of equations is solved for X using the factored form
of A.

4. Iterative refinement is applied to improve the computed solution
matrix and calculate error bounds and backward error estimates
for it.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of A has been supplied on entry. = 'F': LOWF, DIAGF, UPF1, UPF2, and IPIVOT contain the factored form of A; LOW, DIAG, UP, LOWF, DIAGF, UPF1, UPF2 and IPIVOT will not be modified. = 'N': The matrix will be copied to LOWF, DIAGF, and UPF1 and factored.

TRANSA (input)
Specifies the form of the system of equations:
= 'N': A * X = B (No transpose)
= 'T': A**T * X = B (Transpose)
= 'C': A**H * X = B (Conjugate transpose = Transpose)

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

LOW (input)
The (n-1) subdiagonal elements of A.

DIAG (input)
The n diagonal elements of A.

UP (input/output)
The (n-1) superdiagonal elements of A.

LOWF (input/output)
If FACT = 'F', then LOWF is an input argument and on entry contains the (n-1) multipliers that define the matrix L from the LU factorization of A as computed by SGTTRF.

If FACT = 'N', then LOWF is an output argument and on exit contains the (n-1) multipliers that define the matrix L from the LU factorization of A.

DIAGF (input/output)
If FACT = 'F', then DIAGF is an input argument and on entry contains the n diagonal elements of the upper triangular matrix U from the LU factorization of A.

If FACT = 'N', then DIAGF is an output argument and on exit contains the n diagonal elements of the upper triangular matrix U from the LU factorization of A.

UPF1 (input/output)
If FACT = 'F', then UPF1 is an input argument and on entry contains the (n-1) elements of the first superdiagonal of U.

If FACT = 'N', then UPF1 is an output argument and on exit contains the (n-1) elements of the first superdiagonal of U.

UPF2 (input/output)
If FACT = 'F', then UPF2 is an input argument and on entry contains the (n-2) elements of the second superdiagonal of U.

If FACT = 'N', then UPF2 is an output argument and on exit contains the (n-2) elements of the second superdiagonal of U.

IPIVOT (input/output)
If FACT = 'F', then IPIVOT is an input argument and on entry contains the pivot indices from the LU factorization of A as computed by SGTTRF.

If FACT = 'N', then IPIVOT is an output argument and on exit contains the pivot indices from the LU factorization of A; row i of the matrix was interchanged with row IPIVOT(i). IPIVOT(i) will always be either i or i+1; IPIVOT(i) = i indicates a row interchange was not required.

B (input) The N-by-NRHS right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A. If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each

solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: U(i,i) is exactly zero. The factorization has not been completed unless i = N, but the factor U is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

sgttrf - compute an LU factorization of a real tridiagonal matrix A using elimination with partial pivoting and row interchanges

SYNOPSIS

```
SUBROUTINE SGTTRF(N, LOW, DIAG, UP1, UP2, IPIVOT, INFO)

INTEGER N, INFO
INTEGER IPIVOT(*)
REAL LOW(*), DIAG(*), UP1(*), UP2(*)

SUBROUTINE SGTTRF_64(N, LOW, DIAG, UP1, UP2, IPIVOT, INFO)

INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
REAL LOW(*), DIAG(*), UP1(*), UP2(*)
```

F95 INTERFACE

```
SUBROUTINE GTTRF([N], LOW, DIAG, UP1, UP2, IPIVOT, [INFO])

INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: LOW, DIAG, UP1, UP2

SUBROUTINE GTTRF_64([N], LOW, DIAG, UP1, UP2, IPIVOT, [INFO])

INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: LOW, DIAG, UP1, UP2
```

C INTERFACE

```
#include <sunperf.h>

void sgttrf(int n, float  *low,  float  *diag,  float  *up1,
          float *up2, int *ipivot, int *info);

void sgttrf_64(long n, float *low, float *diag, float  *up1,
          float *up2, long *ipivot, long *info);
```

PURPOSE

sgttrf computes an LU factorization of a real tridiagonal matrix A using elimination with partial pivoting and row interchanges.

The factorization has the form
   A = L * U
where L is a product of permutation and unit lower bidiagonal matrices and U is upper triangular with nonzeros in only the main diagonal and first two superdiagonals.

ARGUMENTS

N (input) The order of the matrix A.

LOW (input/output)
On entry, LOW must contain the (n-1) sub-diagonal elements of A.

On exit, LOW is overwritten by the (n-1) multipliers that define the matrix L from the LU factorization of A.

DIAG (input/output)
On entry, DIAG must contain the diagonal elements of A.

On exit, DIAG is overwritten by the n diagonal elements of the upper triangular matrix U from the LU factorization of A.

UP1 (input/output)
On entry, UP1 must contain the (n-1) super-diagonal elements of A.

On exit, UP1 is overwritten by the (n-1) elements of the first super-diagonal of U.

UP2 (output)
On exit, UP2 is overwritten by the (n-2) elements of the second super-diagonal of U.

IPIVOT (output)
The pivot indices; for 1 <= i <= n, row i of the matrix was interchanged with row IPIVOT(i). IPIVOT(i) will always be either i or i+1; IPIVOT(i) = i indicates a row interchange was not required.

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, U(k,k) is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

Contents

NAME

```
sgttrs - solve one of the systems of equations  A*X =  B  or
A'*X = B,
```

SYNOPSIS

```
SUBROUTINE SGTTRS(TRANSA, N, NRHS, LOW, DIAG, UP1, UP2, IPIVOT, B,
      LDB, INFO)

CHARACTER * 1 TRANSA
INTEGER N, NRHS, LDB, INFO
INTEGER IPIVOT(*)
REAL LOW(*), DIAG(*), UP1(*), UP2(*), B(LDB,*)

SUBROUTINE SGTTRS_64(TRANSA, N, NRHS, LOW, DIAG, UP1, UP2, IPIVOT, B,
      LDB, INFO)

CHARACTER * 1 TRANSA
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIVOT(*)
REAL LOW(*), DIAG(*), UP1(*), UP2(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE GTTRS([TRANSA], [N], [NRHS], LOW, DIAG, UP1, UP2, IPIVOT,
       B, [LDB], [INFO])

CHARACTER(LEN=1) :: TRANSA
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: LOW, DIAG, UP1, UP2
REAL, DIMENSION(:,:) :: B

SUBROUTINE GTTRS_64([TRANSA], [N], [NRHS], LOW, DIAG, UP1, UP2,
```

```
          IPIVOT, B, [LDB], [INFO])

   CHARACTER(LEN=1) :: TRANSA
   INTEGER(8) :: N, NRHS, LDB, INFO
   INTEGER(8), DIMENSION(:) :: IPIVOT
   REAL, DIMENSION(:) :: LOW, DIAG, UP1, UP2
   REAL, DIMENSION(:,:) :: B
```

C INTERFACE

```
   #include <sunperf.h>

   void sgttrs(char transa, int n, int nrhs, float *low,  float
             *diag,  float *up1, float *up2, int *ipivot, float
             *b, int ldb, int *info);
   void sgttrs_64(char transa, long n, long nrhs,  float  *low,
             float *diag, float *up1, float *up2, long *ipivot,
             float *b, long ldb, long *info);
```

PURPOSE

```
   sgttrs solves one of the systems of equations
      A*X = B  or  A'*X = B, with a tridiagonal matrix A  using
   the LU factorization computed by SGTTRF.
```

ARGUMENTS

```
   TRANSA (input)
             Specifies the form of the system of equations.   =
             'N':  A * X = B  (No transpose)
             = 'T':  A'* X = B  (Transpose)
             = 'C':  A'* X = B  (Conjugate  transpose  =  Tran-
             spose)

             TRANSA is defaulted to 'N' for F95 INTERFACE.

   N (input) The order of the matrix A.

   NRHS (input)
             The number of right hand sides, i.e.,  the  number
             of columns of the matrix B.  NRHS >= 0.

   LOW (input)
             The (n-1) multipliers that  define  the  matrix  L
             from the LU factorization of A.

   DIAG (input)
             The n diagonal elements of  the  upper  triangular
```

matrix U from the LU factorization of A.

UP1 (input)
The (n-1) elements of the first super-diagonal of U.

UP2 (input)
The (n-2) elements of the second super-diagonal of U.

IPIVOT (input)
The pivot indices; for 1 <= i <= n, row i of the matrix was interchanged with row IPIVOT(i). IPIVOT(i) will always be either i or i+1; IPIVOT(i) = i indicates a row interchange was not required.

B (input/output)
On entry, the matrix of right hand side vectors B. On exit, B is overwritten by the solution vectors X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

shgeqz - implement a single-/double-shift version of the QZ method for finding the generalized eigenvalues w(j)=(ALPHAR(j) + i*ALPHAI(j))/BETAR(j) of the equation det(A-w(i) B) = 0 In addition, the pair A,B may be reduced to generalized Schur form

SYNOPSIS

```
SUBROUTINE SHGEQZ(JOB, COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB,
      ALPHAR, ALPHAI, BETA, Q, LDQ, Z, LDZ, WORK, LWORK, INFO)

CHARACTER * 1 JOB, COMPQ, COMPZ
INTEGER N, ILO, IHI, LDA, LDB, LDQ, LDZ, LWORK, INFO
REAL  A(LDA,*),  B(LDB,*),  ALPHAR(*),  ALPHAI(*),  BETA(*),
Q(LDQ,*), Z(LDZ,*), WORK(*)

SUBROUTINE SHGEQZ_64(JOB, COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB,
      ALPHAR, ALPHAI, BETA, Q, LDQ, Z, LDZ, WORK, LWORK, INFO)

CHARACTER * 1 JOB, COMPQ, COMPZ
INTEGER*8 N, ILO, IHI, LDA, LDB, LDQ, LDZ, LWORK, INFO
REAL  A(LDA,*),  B(LDB,*),  ALPHAR(*),  ALPHAI(*),  BETA(*),
Q(LDQ,*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE HGEQZ(JOB, COMPQ, COMPZ, [N], ILO, IHI, A, [LDA], B, [LDB],
       ALPHAR, ALPHAI, BETA, Q, [LDQ], Z, [LDZ], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: JOB, COMPQ, COMPZ
INTEGER :: N, ILO, IHI, LDA, LDB, LDQ, LDZ, LWORK, INFO
REAL, DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
```

```
REAL, DIMENSION(:,:) :: A, B, Q, Z

SUBROUTINE HGEQZ_64(JOB, COMPQ, COMPZ, [N], ILO, IHI, A, [LDA], B,
       [LDB], ALPHAR, ALPHAI, BETA, Q, [LDQ], Z, [LDZ], [WORK], [LWORK],
       [INFO])

CHARACTER(LEN=1) :: JOB, COMPQ, COMPZ
INTEGER(8) :: N, ILO, IHI, LDA, LDB, LDQ, LDZ, LWORK, INFO
REAL, DIMENSION(:) :: ALPHAR, ALPHAI, BETA, WORK
REAL, DIMENSION(:,:) :: A, B, Q, Z
```

C INTERFACE

```
#include <sunperf.h>

void shgeqz(char job, char compq, char  compz,  int  n,  int
          ilo,  int  ihi,  float  *a, int lda, float *b, int
          ldb, float *alphar, float  *alphai,  float  *beta,
          float *q, int ldq, float *z, int ldz, int *info);

void shgeqz_64(char job, char compq,  char  compz,  long  n,
          long  ilo, long ihi, float *a, long lda, float *b,
          long ldb,  float  *alphar,  float  *alphai,  float
          *beta,  float  *q,  long  ldq, float *z, long ldz,
          long *info);
```

PURPOSE

shgeqz implements a single-/double-shift version of the QZ method for finding the generalized eigenvalues B is upper triangular, and A is block upper triangular, where the diagonal blocks are either 1-by-1 or 2-by-2, the 2-by-2 blocks having complex generalized eigenvalues (see the description of the argument JOB.)

If JOB='S', then the pair (A,B) is simultaneously reduced to Schur form by applying one orthogonal tranformation (usually called Q) on the left and another (usually called Z) on the right. The 2-by-2 upper-triangular diagonal blocks of B corresponding to 2-by-2 blocks of A will be reduced to positive diagonal matrices. (I.e., if A(j+1,j) is non-zero, then B(j+1,j)=B(j,j+1)=0 and B(j,j) and B(j+1,j+1) will be positive.)

If JOB='E', then at each iteration, the same transformations are computed, but they are only applied to those parts of A and B which are needed to compute ALPHAR, ALPHAI, and BETAR.

If JOB='S' and COMPQ and COMPZ are 'V' or 'I', then the orthogonal transformations used to reduce (A,B) are accumu-

lated into the arrays Q and Z s.t.:
(in) A(in) Z(in)* = Q(out) A(out) Z(out)*

Ref: C.B. Moler & G.W. Stewart, "An Algorithm for Generalized Matrixigenvalue Problems", SIAM J. Numer. Anal., 10(1973),p. 241--256.

ARGUMENTS

JOB (input)
= 'E': compute only ALPHAR, ALPHAI, and BETA. A and B will not necessarily be put into generalized Schur form. = 'S': put A and B into generalized Schur form, as well as computing ALPHAR, ALPHAI, and BETA.

COMPQ (input)
= 'N': do not modify Q.
= 'V': multiply the array Q on the right by the transpose of the orthogonal tranformation that is applied to the left side of A and B to reduce them to Schur form. = 'I': like COMPQ='V', except that Q will be initialized to the identity first.

COMPZ (input)
= 'N': do not modify Z.
= 'V': multiply the array Z on the right by the orthogonal tranformation that is applied to the right side of A and B to reduce them to Schur form. = 'I': like COMPZ='V', except that Z will be initialized to the identity first.

N (input) The order of the matrices A, B, Q, and Z. N >= 0.

ILO (input)
It is assumed that A is already upper triangular in rows and columns 1:ILO-1 and IHI+1:N. 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

IHI (input)
See the description of ILO.

A (input) On entry, the N-by-N upper Hessenberg matrix A. Elements below the subdiagonal must be zero. If JOB='S', then on exit A and B will have been simultaneously reduced to generalized Schur form. If JOB='E', then on exit A will have been destroyed. The diagonal blocks will be correct, but the off-diagonal portion will be meaningless.

LDA (input)
The leading dimension of the array A. LDA >= max(1, N).

B (input) On entry, the N-by-N upper triangular matrix B. Elements below the diagonal must be zero. 2-by-2 blocks in B corresponding to 2-by-2 blocks in A will be reduced to positive diagonal form. (I.e., if A(j+1,j) is non-zero, then B(j+1,j)=B(j,j+1)=0 and B(j,j) and B(j+1,j+1) will be positive.) If JOB='S', then on exit A and B will have been simultaneously reduced to Schur form. If JOB='E', then on exit B will have been destroyed. Elements corresponding to diagonal blocks of A will be correct, but the off-diagonal portion will be meaningless.

LDB (input)
The leading dimension of the array B. LDB >= max(1, N).

ALPHAR (output)
ALPHAR(1:N) will be set to real parts of the diagonal elements of A that would result from reducing A and B to Schur form and then further reducing them both to triangular form using unitary transformations s.t. the diagonal of B was non-negative real. Thus, if A(j,j) is in a 1-by-1 block (i.e., A(j+1,j)=A(j,j+1)=0), then ALPHAR(j)=A(j,j). Note that the (real or complex) values (ALPHAR(j) + i*ALPHAI(j))/BETA(j), j=1,...,N, are the generalized eigenvalues of the matrix pencil A - wB.

ALPHAI (output)
ALPHAI(1:N) will be set to imaginary parts of the diagonal elements of A that would result from reducing A and B to Schur form and then further reducing them both to triangular form using unitary transformations s.t. the diagonal of B was non-negative real. Thus, if A(j,j) is in a 1-by-1 block (i.e., A(j+1,j)=A(j,j+1)=0), then ALPHAR(j)=0. Note that the (real or complex) values (ALPHAR(j) + i*ALPHAI(j))/BETA(j), j=1,...,N, are the generalized eigenvalues of the matrix pencil A - wB.

BETA (output)

BETA(1:N) will be set to the (real) diagonal elements of B that would result from reducing A and B to Schur form and then further reducing them both to triangular form using unitary transformations s.t. the diagonal of B was non-negative real. Thus, if A(j,j) is in a 1-by-1 block (i.e., A(j+1,j)=A(j,j+1)=0), then BETA(j)=B(j,j). Note that the (real or complex) values (ALPHAR(j) + i*ALPHAI(j))/BETA(j), j=1,...,N, are the generalized eigenvalues of the matrix pencil A - wB. (Note that BETA(1:N) will always be non-negative, and no BETAI is necessary.)

Q (input/output)

If COMPQ='N', then Q will not be referenced. If COMPQ='V' or 'I', then the transpose of the orthogonal transformations which are applied to A and B on the left will be applied to the array Q on the right.

LDQ (input)

The leading dimension of the array Q. LDQ >= 1. If COMPQ='V' or 'I', then LDQ >= N.

Z (input/output)

If COMPZ='N', then Z will not be referenced. If COMPZ='V' or 'I', then the orthogonal transformations which are applied to A and B on the right will be applied to the array Z on the right.

LDZ (input)

The leading dimension of the array Z. LDZ >= 1. If COMPZ='V' or 'I', then LDZ >= N.

WORK (workspace)

On exit, if INFO >= 0, WORK(1) returns the optimal LWORK.

LWORK (input)

The dimension of the array WORK. LWORK >= max(1,N).

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an ille-

gal value
= 1,...,N: the QZ iteration did not converge. (A,B) is not in Schur form, but ALPHAR(i), ALPHAI(i), and BETA(i), i=INFO+1,...,N should be correct. = N+1,...,2*N: the shift calculation failed. (A,B) is not in Schur form, but ALPHAR(i), ALPHAI(i), and BETA(i), i=INFO-N+1,...,N should be correct. > 2*N: various "impossible" errors.

FURTHER DETAILS

```
Iteration counters:

JITER  -- counts iterations.
IITER  -- counts iterations run since ILAST was last
          changed.  This is therefore reset only when  a  1-
by-1 or
          2-by-2 block deflates off the bottom.
```

Contents

NAME

```
shsein - use  inverse  iteration  to  find  specified  right
and/or left eigenvectors of a real upper Hessenberg matrix H
```

SYNOPSIS

```
SUBROUTINE SHSEIN(SIDE, EIGSRC, INITV, SELECT, N, H, LDH, WR, WI, VL,
      LDVL, VR, LDVR, MM, M, WORK, IFAILL, IFAILR, INFO)

CHARACTER * 1 SIDE, EIGSRC, INITV
INTEGER N, LDH, LDVL, LDVR, MM, M, INFO
INTEGER IFAILL(*), IFAILR(*)
LOGICAL SELECT(*)
REAL H(LDH,*), WR(*), WI(*), VL(LDVL,*), VR(LDVR,*), WORK(*)

SUBROUTINE SHSEIN_64(SIDE, EIGSRC, INITV, SELECT, N, H, LDH, WR, WI,
      VL, LDVL, VR, LDVR, MM, M, WORK, IFAILL, IFAILR, INFO)

CHARACTER * 1 SIDE, EIGSRC, INITV
INTEGER*8 N, LDH, LDVL, LDVR, MM, M, INFO
INTEGER*8 IFAILL(*), IFAILR(*)
LOGICAL*8 SELECT(*)
REAL H(LDH,*), WR(*), WI(*), VL(LDVL,*), VR(LDVR,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE HSEIN(SIDE, EIGSRC, INITV, SELECT, [N], H, [LDH], WR, WI,
       VL, [LDVL], VR, [LDVR], MM, M, [WORK], IFAILL, IFAILR, [INFO])

CHARACTER(LEN=1) :: SIDE, EIGSRC, INITV
INTEGER :: N, LDH, LDVL, LDVR, MM, M, INFO
INTEGER, DIMENSION(:) :: IFAILL, IFAILR
LOGICAL, DIMENSION(:) :: SELECT
```

```
REAL, DIMENSION(:) :: WR, WI, WORK
REAL, DIMENSION(:,:) :: H, VL, VR

SUBROUTINE HSEIN_64(SIDE, EIGSRC, INITV, SELECT, [N], H, [LDH], WR,
       WI, VL, [LDVL], VR, [LDVR], MM, M, [WORK], IFAILL, IFAILR, [INFO])

CHARACTER(LEN=1) :: SIDE, EIGSRC, INITV
INTEGER(8) :: N, LDH, LDVL, LDVR, MM, M, INFO
INTEGER(8), DIMENSION(:) :: IFAILL, IFAILR
LOGICAL(8), DIMENSION(:) :: SELECT
REAL, DIMENSION(:) :: WR, WI, WORK
REAL, DIMENSION(:,:) :: H, VL, VR
```

C INTERFACE

```
#include <sunperf.h>
void shsein(char side, char eigsrc, char initv, int *select,
          int  n,  float  *h, int ldh, float *wr, float *wi,
          float *vl, int ldvl, float *vr, int ldvr, int  mm,
          int *m, int *ifaill, int *ifailr, int *info);

void shsein_64(char side,  char  eigsrc,  char  initv,  long
          *select,  long  n,  float *h, long ldh, float *wr,
          float *wi, float *vl, long ldvl, float  *vr,  long
          ldvr,   long  mm,  long  *m,  long  *ifaill,  long
          *ifailr, long *info);
```

PURPOSE

shsein uses inverse iteration to find specified right and/or left eigenvectors of a real upper Hessenberg matrix H.

The right eigenvector x and the left eigenvector y of the matrix H corresponding to an eigenvalue w are defined by:

```
             H * x = w * x,     y**h * H = w * y**h
```

where y**h denotes the conjugate transpose of the vector y.

ARGUMENTS

```
SIDE (input)
          = 'R': compute right eigenvectors only;
          = 'L': compute left eigenvectors only;
          = 'B': compute both right and left eigenvectors.

EIGSRC (input)
          Specifies the source of  eigenvalues  supplied  in
```

(WR,WI):
= 'Q': the eigenvalues were found using SHSEQR; thus, if H has zero subdiagonal elements, and so is block-triangular, then the j-th eigenvalue can be assumed to be an eigenvalue of the block containing the j-th row/column. This property allows SHSEIN to perform inverse iteration on just one diagonal block. = 'N': no assumptions are made on the correspondence between eigenvalues and diagonal blocks. In this case, SHSEIN must always perform inverse iteration using the whole matrix H.

INITV (input)
= 'N': no initial vectors are supplied;
= 'U': user-supplied initial vectors are stored in the arrays VL and/or VR.

SELECT (input/output)
Specifies the eigenvectors to be computed. To select the real eigenvector corresponding to a real eigenvalue WR(j), SELECT(j) must be set to .TRUE.. To select the complex eigenvector corresponding to a complex eigenvalue (WR(j),WI(j)), with complex conjugate (WR(j+1),WI(j+1)), either SELECT(j) or SELECT(j+1) or both must be set to

N (input) The order of the matrix H. N >= 0.

H (input) The upper Hessenberg matrix H.

LDH (input)
The leading dimension of the array H. LDH >= max(1,N).

WR (input/output)
On entry, the real and imaginary parts of the eigenvalues of H; a complex conjugate pair of eigenvalues must be stored in consecutive elements of WR and WI. On exit, WR may have been altered since close eigenvalues are perturbed slightly in searching for independent eigenvectors.

WI (input)
See the description of WR.

VL (input/output)
On entry, if INITV = 'U' and SIDE = 'L' or 'B', VL must contain starting vectors for the inverse iteration for the left eigenvectors; the starting vector for each eigenvector must be in the same

column(s) in which the eigenvector will be stored. On exit, if SIDE = 'L' or 'B', the left eigenvectors specified by SELECT will be stored consecutively in the columns of VL, in the same order as their eigenvalues. A complex eigenvector corresponding to a complex eigenvalue is stored in two consecutive columns, the first holding the real part and the second the imaginary part. If SIDE = 'R', VL is not referenced.

LDVL (input)
The leading dimension of the array VL. LDVL >= max(1,N) if SIDE = 'L' or 'B'; LDVL >= 1 otherwise.

VR (input/output)
On entry, if INITV = 'U' and SIDE = 'R' or 'B', VR must contain starting vectors for the inverse iteration for the right eigenvectors; the starting vector for each eigenvector must be in the same column(s) in which the eigenvector will be stored. On exit, if SIDE = 'R' or 'B', the right eigenvectors specified by SELECT will be stored consecutively in the columns of VR, in the same order as their eigenvalues. A complex eigenvector corresponding to a complex eigenvalue is stored in two consecutive columns, the first holding the real part and the second the imaginary part. If SIDE = 'L', VR is not referenced.

LDVR (input)
The leading dimension of the array VR. LDVR >= max(1,N) if SIDE = 'R' or 'B'; LDVR >= 1 otherwise.

MM (input)
The number of columns in the arrays VL and/or VR. MM >= M.

M (output)
The number of columns in the arrays VL and/or VR required to store the eigenvectors; each selected real eigenvector occupies one column and each selected complex eigenvector occupies two columns.

WORK (workspace)
dimension((N+2)*N)

IFAILL (output)
If SIDE = 'L' or 'B', IFAILL(i) = j > 0 if the

left eigenvector in the i-th column of VL (corresponding to the eigenvalue w(j)) failed to converge; IFAILL(i) = 0 if the eigenvector converged satisfactorily. If the i-th and (i+1)th columns of VL hold a complex eigenvector, then IFAILL(i) and IFAILL(i+1) are set to the same value. If SIDE = 'R', IFAILL is not referenced.

IFAILR (output)

If SIDE = 'R' or 'B', IFAILR(i) = j > 0 if the right eigenvector in the i-th column of VR (corresponding to the eigenvalue w(j)) failed to converge; IFAILR(i) = 0 if the eigenvector converged satisfactorily. If the i-th and (i+1)th columns of VR hold a complex eigenvector, then IFAILR(i) and IFAILR(i+1) are set to the same value. If SIDE = 'L', IFAILR is not referenced.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, i is the number of eigenvectors which failed to converge; see IFAILL and IFAILR for further details.

FURTHER DETAILS

Each eigenvector is normalized so that the element of largest magnitude has magnitude 1; here the magnitude of a complex number (x,y) is taken to be |x|+|y|.

Contents

NAME

```
shseqr - compute the eigenvalues of a real upper  Hessenberg
matrix  H  and,  optionally,  the  matrices T and Z from the
Schur decomposition H = Z  T  Z**T,  where  T  is  an  upper
quasi-triangular  matrix  (the  Schur  form),  and  Z is the
orthogonal matrix of Schur vectors
```

SYNOPSIS

```
SUBROUTINE SHSEQR(JOB, COMPZ, N, ILO, IHI, H, LDH, WR, WI, Z, LDZ,
      WORK, LWORK, INFO)

CHARACTER * 1 JOB, COMPZ
INTEGER N, ILO, IHI, LDH, LDZ, LWORK, INFO
REAL H(LDH,*), WR(*), WI(*), Z(LDZ,*), WORK(*)

SUBROUTINE SHSEQR_64(JOB, COMPZ, N, ILO, IHI, H, LDH, WR, WI, Z, LDZ,
      WORK, LWORK, INFO)

CHARACTER * 1 JOB, COMPZ
INTEGER*8 N, ILO, IHI, LDH, LDZ, LWORK, INFO
REAL H(LDH,*), WR(*), WI(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE HSEQR(JOB, COMPZ, N, ILO, IHI, H, [LDH], WR, WI, Z, [LDZ],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: JOB, COMPZ
INTEGER :: N, ILO, IHI, LDH, LDZ, LWORK, INFO
REAL, DIMENSION(:) :: WR, WI, WORK
REAL, DIMENSION(:,:) :: H, Z

SUBROUTINE HSEQR_64(JOB, COMPZ, N, ILO, IHI, H, [LDH], WR, WI, Z,
```

```
        [LDZ], [WORK], [LWORK], [INFO])

    CHARACTER(LEN=1) :: JOB, COMPZ
    INTEGER(8) :: N, ILO, IHI, LDH, LDZ, LWORK, INFO
    REAL, DIMENSION(:) :: WR, WI, WORK
    REAL, DIMENSION(:,:) :: H, Z
```

C INTERFACE

```
    #include <sunperf.h>

    void shseqr(char job, char compz, int n, int ilo,  int  ihi,
              float *h, int ldh, float *wr, float *wi, float *z,
              int ldz, int *info);
    void shseqr_64(char job, char compz, long n, long ilo,  long
              ihi,  float  *h,  long  ldh, float *wr, float *wi,
              float *z, long ldz, long *info);
```

PURPOSE

shseqr computes the eigenvalues of a real upper Hessenberg matrix H and, optionally, the matrices T and Z from the Schur decomposition H = Z T Z**T, where T is an upper quasi-triangular matrix (the Schur form), and Z is the orthogonal matrix of Schur vectors.

Optionally Z may be postmultiplied into an input orthogonal matrix Q, so that this routine can give the Schur factorization of a matrix A which has been reduced to the Hessenberg form H by the orthogonal matrix Q: A = Q*H*Q**T = (QZ)*T*(QZ)**T.

ARGUMENTS

JOB (input)
= 'E': compute eigenvalues only;
= 'S': compute eigenvalues and the Schur form T.

COMPZ (input)
= 'N': no Schur vectors are computed;
= 'I': Z is initialized to the unit matrix and the matrix Z of Schur vectors of H is returned; = 'V': Z must contain an orthogonal matrix Q on entry, and the product Q*Z is returned.

N (input) The order of the matrix H. N >= 0.

ILO (input)

It is assumed that H is already upper triangular in rows and columns 1:ILO-1 and IHI+1:N. ILO and IHI are normally set by a previous call to SGEBAL, and then passed to SGEHRD when the matrix output by SGEBAL is reduced to Hessenberg form. Otherwise ILO and IHI should be set to 1 and N respectively. 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

IHI (input)
See the description of ILO.

H (input/output)
On entry, the upper Hessenberg matrix H. On exit, if JOB = 'S', H contains the upper quasi-triangular matrix T from the Schur decomposition (the Schur form); 2-by-2 diagonal blocks (corresponding to complex conjugate pairs of eigenvalues) are returned in standard form, with H(i,i) = H(i+1,i+1) and H(i+1,i)*H(i,i+1) < 0. If JOB = 'E', the contents of H are unspecified on exit.

LDH (input)
The leading dimension of the array H. LDH >= max(1,N).

WR (output)
The real and imaginary parts, respectively, of the computed eigenvalues. If two eigenvalues are computed as a complex conjugate pair, they are stored in consecutive elements of WR and WI, say the i-th and (i+1)th, with WI(i) > 0 and WI(i+1) < 0. If JOB = 'S', the eigenvalues are stored in the same order as on the diagonal of the Schur form returned in H, with WR(i) = H(i,i) and, if H(i:i+1,i:i+1) is a 2-by-2 diagonal block, WI(i) = sqrt(H(i+1,i)*H(i,i+1)) and WI(i+1) = -WI(i).

WI (output)
See the description of WR.

Z (input) If COMPZ = 'N': Z is not referenced.
If COMPZ = 'I': on entry, Z need not be set, and on exit, Z contains the orthogonal matrix Z of the Schur vectors of H. If COMPZ = 'V': on entry Z must contain an N-by-N matrix Q, which is assumed to be equal to the unit matrix except for the submatrix Z(ILO:IHI,ILO:IHI); on exit Z contains Q*Z. Normally Q is the orthogonal matrix generated by SORGHR after the call to SGEHRD which formed the

Hessenberg matrix H.

LDZ (input)
The leading dimension of the array Z. LDZ >= max(1,N) if COMPZ = 'I' or 'V'; LDZ >= 1 otherwise.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,N).

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, SHSEQR failed to compute all of the eigenvalues in a total of 30*(IHI-ILO+1) iterations; elements 1:ilo-1 and i+1:n of WR and WI contain those eigenvalues which have been successfully computed.

Contents

NAME

```
sinqb - synthesize a Fourier sequence from  its  representa-
tion  in  terms of a sine series with odd wave numbers.  The
SINQ operations are unnormalized inverses of themselves,  so
a  call  to  SINQF followed by a call to SINQB will multiply
the input sequence by 4 * N.
```

SYNOPSIS

```
SUBROUTINE SINQB(N, X, WSAVE)

INTEGER N
REAL X(*), WSAVE(*)

SUBROUTINE SINQB_64(N, X, WSAVE)

INTEGER*8 N
REAL X(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE SINQB(N, X, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: X, WSAVE

SUBROUTINE SINQB_64(N, X, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: X, WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void sinqb(int n, float *x, float *wsave);
```

```
void sinqb_64(long n, float *x, float *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. These subroutines are most efficient when N is a product of small primes. N >= 0.

X (input/output)
On entry, an array of length N containing the sequence to be transformed. On exit, the quarter-wave sine synthesis of the input.

WSAVE (input)
On entry, an array with dimension of at least (3 * N + 15) for scalar subroutines, initialized by SINQI.

Contents

NAME

```
sinqf - compute the Fourier coefficients in  a  sine  series
representation  with only odd wave numbers.  The SINQ opera-
tions are unnormalized inverses of themselves, so a call  to
SINQF  followed  by  a call to SINQB will multiply the input
sequence by 4 * N.
```

SYNOPSIS

```
SUBROUTINE SINQF(N, X, WSAVE)

INTEGER N
REAL X(*), WSAVE(*)

SUBROUTINE SINQF_64(N, X, WSAVE)

INTEGER*8 N
REAL X(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE SINQF(N, X, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: X, WSAVE

SUBROUTINE SINQF_64(N, X, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: X, WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void sinqf(int n, float *x, float *wsave);
```

```
void sinqf_64(long n, float *x, float *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence to be  transformed.   These
          subroutines are most efficient when N is a product
          of small primes.  N >= 0.

X (input/output)
          On entry, an array  of  length  N  containing  the
          sequence   to   be   transformed.   On  exit,  the
          quarter-wave sine transform of the input.
WSAVE (input)
          On entry, an array with dimension of at least   (3
          *  N  + 15) for scalar subroutines, initialized by
          SINQI.
```

Contents

NAME

```
sinqi - initialize the array xWSAVE, which is used  in  both
SINQF and SINQB.
```

SYNOPSIS

```
SUBROUTINE SINQI(N, WSAVE)

INTEGER N
REAL WSAVE(*)

SUBROUTINE SINQI_64(N, WSAVE)

INTEGER*8 N
REAL WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE SINQI(N, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: WSAVE

SUBROUTINE SINQI_64(N, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void sinqi(int n, float *wsave);

void sinqi_64(long n, float *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. The method is most efficient when N is a product of small primes.

WSAVE (input)
On entry, an array of dimension (3 * N + 15) or greater. SINQI needs to be called only once to initialize WSAVE before calling SINQF and/or SINQB if N and WSAVE remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
sint - compute the discrete Fourier sine transform of an odd
sequence.   The SINT transforms are unnormalized inverses of
themselves, so a call of SINT followed by  another  call  of
SINT will multiply the input sequence by 2 * (N+1).
```

SYNOPSIS

```
SUBROUTINE SINT(N, X, WSAVE)

INTEGER N
REAL X(*), WSAVE(*)

SUBROUTINE SINT_64(N, X, WSAVE)

INTEGER*8 N
REAL X(*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE SINT(N, X, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: X, WSAVE

SUBROUTINE SINT_64(N, X, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: X, WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void sint(int n, float *x, float *wsave);
```

```
void sint_64(long n, float *x, float *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence to be  transformed.   These
          subroutines  are most efficient when N+1 is a pro-
          duct of small primes.  N >= 0.

X (input/output)
          On entry, an array  of  length  N  containing  the
          sequence  to  be  transformed.   On exit, the sine
          transform of the input.
WSAVE (input/output)
          On entry, an array  with  dimension  of  at  least
          int(2.5 * N + 15) initialized by SINTI.
```

Contents

NAME

```
sinti - initialize the array WSAVE, which is used in subrou-
tine SINT.
```

SYNOPSIS

```
SUBROUTINE SINTI(N, WSAVE)

INTEGER N
REAL WSAVE(*)

SUBROUTINE SINTI_64(N, WSAVE)

INTEGER*8 N
REAL WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE SINTI(N, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: WSAVE

SUBROUTINE SINTI_64(N, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void sinti(int n, float *wsave);

void sinti_64(long n, float *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. N >= 0.

WSAVE (input/output)
On entry, an array of dimension (2N + N/2 + 15) or greater. SINTI is called once to initialize WSAVE before calling SINT and need not be called again between calls to SINT if N and WSAVE remain unchanged. Thus, subsequent transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

sjadmm - Jagged diagonal matrix-matrix multiply (modified Ellpack)

SYNOPSIS

```
 SUBROUTINE SJADMM( TRANSA, M, N, K, ALPHA, DESCRA,
*            VAL, INDX, PNTR, MAXNZ, IPERM,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER     TRANSA, M, N, K, DESCRA(5), MAXNZ,
*            LDB, LDC, LWORK
 INTEGER     INDX(NNZ), PNTR(MAXNZ+1), IPERM(M)
 REAL        ALPHA, BETA
 REAL        VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE SJADMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*            VAL, INDX, PNTR, MAXNZ, IPERM,
*            B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8   TRANSA, M, N, K, DESCRA(5), MAXNZ,
*            LDB, LDC, LWORK
 INTEGER*8   INDX(NNZ), PNTR(MAXNZ+1), IPERM(M)
 REAL        ALPHA, BETA
 REAL        VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

where NNZ=PNTR(MAXNZ+1)-PNTR(1)+1 is the number of non-zero elements
```

F95 INTERFACE

```
 SUBROUTINE JADMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*    PNTR, MAXNZ, IPERM, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, K, MAXNZ
 INTEGER, DIMENSION(:) ::  DESCRA, INDX, PNTR, IPERM
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) ::  VAL
```

```
 REAL, DIMENSION(:, :) ::  B, C

 SUBROUTINE JADMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*    PNTR, MAXNZ, IPERM, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8     TRANSA, M, K, MAXNZ
 INTEGER*8, DIMENSION(:) ::  DESCRA, INDX, PNTR, IPERM
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) ::  VAL
 REAL, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in jagged-diagonal format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
```

```
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

VAL()           array of length NNZ consisting of entries of A.
                VAL can be viewed as a column major ordering of a
                row permutation of the Ellpack representation of A,
                where the Ellpack representation is permuted so that
                the rows are non-increasing in the number of nonzero
                entries.  Values added for padding in Ellpack are
                not included in the Jagged-Diagonal format.

INDX()          array of length NNZ consisting of the column indices
                of the corresponding entries in VAL.
PNTR()          array of length MAXNZ+1, where PNTR(I)-PNTR(1)+1
                points to the location in VAL of the first element
                in the row-permuted Ellpack represenation of A.

MAXNZ           max number of nonzeros elements per row.

IPERM()         integer array of length M such that I = IPERM(I'),
                where row I in the original Ellpack representation
                corresponds to row I' in the permuted representation.
                If IPERM(1) = 0, it is assumed by convention that
                IPERM(I) = I. IPERM is used to determine the order
                in which rows of C are updated.

B()             rectangular array with first dimension LDB.

LDB             leading dimension of B

BETA            Scalar parameter

C()             rectangular array with first dimension LDC.

LDC             leading dimension of C

WORK()          scratch array of length LWORK. WORK is not
                referenced in the current version.

LWORK           length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

sjadrp - right permutation of a jagged diagonal matrix

SYNOPSIS

```
 SUBROUTINE SJADRP( TRANSP, M, K, VAL, INDX, PNTR, MAXNZ,
*                   IPERM, WORK, LWORK )
 INTEGER    TRANSP, M, K, MAXNZ, LWORK
 INTEGER    INDX(*), PNTR(MAXNZ+1), IPERM(K), WORK(LWORK)
 REAL       VAL(*)

 SUBROUTINE SJADRP_64( TRANSP, M, K, VAL, INDX, PNTR, MAXNZ,
*                   IPERM, WORK, LWORK )
 INTEGER*8  TRANSP, M, K, MAXNZ, LWORK
 INTEGER*8  INDX(*), PNTR(MAXNZ+1), IPERM(K), WORK(LWORK)
 REAL       VAL(*)
```

F95 INTERFACE

```
 SUBROUTINE JADRP( TRANSP, M, K, VAL, INDX, PNTR, MAXNZ,
*                   IPERM, [WORK], [LWORK] )
 INTEGER TRANSP, M, K,  MAXNZ
 INTEGER, DIMENSION(:) :: INDX, PNTR, IPERM
 REAL, DIMENSION(:) :: VAL

 SUBROUTINE JADRP_64( TRANSP, M, K, VAL, INDX, PNTR, MAXNZ,
*                   IPERM, [WORK], [LWORK] )
 INTEGER*8 TRANSP, M, K,  MAXNZ
 INTEGER*8, DIMENSION(:) :: INDX, PNTR, IPERM
 REAL, DIMENSION(:) :: VAL
```

DESCRIPTION

```
A <- A P
A <- A P'
                                   ( ' indicates matrix transpose)
```

where permutation P is represented by an integer vector IPERM, such that IPERM(I) is equal to the position of the only nonzero element in row I of permutation matrix P.

NOTE: In order to get a symetrically permuted jagged diagonal matrix P A P', one can explicitly permute the columns P A by calling

```
   SJADRP(0, M, M, VAL, INDX, PNTR, MAXNZ, IPERM, WORK, LWORK)
```

where parameters VAL, INDX, PNTR, MAXNZ, IPERM are the representation of A in the jagged diagonal format. The operation makes sense if the original matrix A is square.

ARGUMENTS

TRANSP — Indicates how to operate with the permutation matrix
  0 : operate with matrix
  1 : operate with transpose matrix

M — Number of rows in matrix A

K — Number of columns in matrix A

VAL() — array of length PNTR(MAXNZ+1)-PNTR(1) consisting of entries of A. VAL can be viewed as a column major ordering of a row permutation of the Ellpack representation of A, where the Ellpack representation is permuted so that the rows are non-increasing in the number of nonzero entries. Values added for padding in Ellpack are not included in the Jagged-Diagonal format.

INDX() — array of length PNTR(MAXNZ+1)-PNTR(1) consisting of the column indices of the corresponding entries in VAL.

PNTR() — array of length MAXNZ+1, where PNTR(I)-PNTR(1)+1 points to the location in VAL of the first element in the row-permuted Ellpack represenation of A.

MAXNZ — max number of nonzeros elements per row.

IPERM() — integer array of length K such that I = IPERM(I').

```
                 Array IPERM represents a permutation P, such that
                 IPERM(I) is equal to the position of the only nonzero
                 element in row I of permutation matrix P.
                 For example, if
                                | 0 0 1 |
                            P  =| 1 0 0 |
                                | 0 1 0 |
                 then IPERM = (3, 1, 2).

   WORK()        scratch array of length LWORK.  LWORK should be at
                 least K.

   LWORK         length of WORK array

                 If LWORK = -1, then a workspace query is assumed;
                 the routine only calculates the optimal size of the
                 WORK array, returns this value as the first entry of
                 the WORK array, and no error message related to LWORK
                 is issued by XERBLA.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

```
sjadsm - Jagged-diagonal format triangular solve
```

SYNOPSIS

```
 SUBROUTINE SJADSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, PNTR, MAXNZ, IPERM,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5), MAXNZ,
*           LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTR(MAXNZ+1), IPERM(M)
 REAL       ALPHA, BETA
 REAL       DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE SJADSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, PNTR, MAXNZ, IPERM,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5), MAXNZ,
*           LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTR(MAXNZ+1), IPERM(M)
 REAL       ALPHA, BETA
 REAL       DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

where NNZ=PNTR(MAXNZ+1)-PNTR(1)+1 is the number of non-zero elements
```

F95 INTERFACE

```
 SUBROUTINE JADSM(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*    PNTR, MAXNZ, IPERM, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, MAXNZ
 INTEGER, DIMENSION(:) ::  DESCRA, INDX, PNTR, IPERM
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) ::  VAL, DV
 REAL, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE JADSM_64(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*    PNTR, MAXNZ, IPERM, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, MAXNZ
 INTEGER*8, DIMENSION(:) ::  DESCRA, INDX, PNTR, IPERM
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) ::  VAL, DV
 REAL, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C
```

where ALPHA and BETA are scalar, C and B are m by n dense matrices, D is a diagonal scaling matrix, A is a unit, or non-unit, upper or lower triangular matrix represented in jagged-diagonal format and op(A) is one of

op(A) = inv(A) or op(A) = inv(A') or op(A) =inv(conjg(A'))
(inv denotes matrix inverse, ' indicates matrix transpose)

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                 0 : general
                 1 : symmetric (A=A')
                 2 : Hermitian (A= CONJG(A'))
                 3 : Triangular
                 4 : Skew(Anti)-Symmetric (A=-A')
                 5 : Diagonal
                 6 : Skew-Hermitian (A= -CONJG(A'))
               Note: For the routine, DESCRA(1)=3 is only supported.

               DESCRA(2) upper/lower triangular indicator
                 1 : lower
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices
VAL()          array of length NNZ consisting of entries of A.
               VAL can be viewed as a column major ordering of a
               row permutation of the Ellpack representation of A,
               where the Ellpack representation is permuted so that
               the rows are non-increasing in the number of nonzero
               entries.  Values added for padding in Ellpack are
               not included in the Jagged-Diagonal format.

INDX()         array of length NNZ consisting of the column indices
               of the corresponding entries in VAL.

PNTR()         array of length MAXNZ+1, where PNTR(I)-PNTR(1)+1
               points to the location in VAL of the first element
               in the row-permuted Ellpack represenation of A.

MAXNZ          max number of nonzeros elements per row.

IPERM()        integer array of length M such that I = IPERM(I'),
               where row I in the original Ellpack representation
               corresponds to row I' in the permuted representation.
               If IPERM(1)=0,   it's assumed by convention that
               IPERM(I)=I. IPERM is used to determine the order
               in which rows of C are updated.

B()            rectangular array with first dimension LDB.

LDB            leading dimension of B

BETA           Scalar parameter
```

```
C()            rectangular array with first dimension LDC.

LDC            leading dimension of C

WORK()         scratch array of length LWORK.
               On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.

LWORK          length of WORK array.  LWORK should be at least 2*M.

               For good performance, LWORK should generally be larger.
               For optimum performance on multiple processors, LWORK
               >=2*M*N_CPUS where N_CPUS is the maximum number of
               processors available to the program.

               If LWORK=0, the routine is to allocate workspace needed.

               If LWORK = -1, then a workspace query is assumed; the
               routine only calculates the optimum size of the WORK
               array, returns this value as the first entry of the WORK
               array, and no error message related to LWORK is issued
               by XERBLA.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the rows of A such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows have been scaled. UNITD=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the JAD representation of a sparse matrix. They are not used anyway in these cases. But if UNITD=4, the unit diagonal elements MUST be referenced in the JAD representation.

4. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

```
slagtf - factorize the matrix (T-lambda*I), where T is an  n
by  n  tridiagonal  matrix  and  lambda  is  a scalar, as T-
lambda*I = PLU
```

SYNOPSIS

```
SUBROUTINE SLAGTF(N, A, LAMBDA, B, C, TOL, D, IN, INFO)

INTEGER N, INFO
INTEGER IN(*)
REAL LAMBDA, TOL
REAL A(*), B(*), C(*), D(*)

SUBROUTINE SLAGTF_64(N, A, LAMBDA, B, C, TOL, D, IN, INFO)

INTEGER*8 N, INFO
INTEGER*8 IN(*)
REAL LAMBDA, TOL
REAL A(*), B(*), C(*), D(*)
```

F95 INTERFACE

```
SUBROUTINE LAGTF([N], A, LAMBDA, B, C, TOL, D, IN, [INFO])

INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IN
REAL :: LAMBDA, TOL
REAL, DIMENSION(:) :: A, B, C, D

SUBROUTINE LAGTF_64([N], A, LAMBDA, B, C, TOL, D, IN, [INFO])

INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IN
```

```
REAL :: LAMBDA, TOL
REAL, DIMENSION(:) :: A, B, C, D
```

C INTERFACE

```
#include <sunperf.h>

void slagtf(int n, float *a, float lambda, float  *b,  float
          *c, float tol, float *d, int *in, int *info);

void slagtf_64(long n, float *a,  float  lambda,  float  *b,
          float  *c,  float  tol,  float  *d, long *in, long
          *info);
```

PURPOSE

slagtf factorizes the matrix (T - lambda*I), where T is an n by n tridiagonal matrix and lambda is a scalar, as where P is a permutation matrix, L is a unit lower tridiagonal matrix with at most one non-zero sub-diagonal elements per column and U is an upper triangular matrix with at most two non-zero super-diagonal elements per column.

The factorization is obtained by Gaussian elimination with partial pivoting and implicit row scaling.

The parameter LAMBDA is included in the routine so that SLAGTF may be used, in conjunction with SLAGTS, to obtain eigenvectors of T by inverse iteration.

ARGUMENTS

N (input) The order of the matrix T.

A (input/output)
On entry, A must contain the diagonal elements of T.

On exit, A is overwritten by the n diagonal elements of the upper triangular matrix U of the factorization of T.

LAMBDA (input)
On entry, the scalar lambda.

B (input/output)
On entry, B must contain the (n-1) super-diagonal elements of T.

On exit, B is overwritten by the (n-1) super-diagonal elements of the matrix U of the factorization of T.

C (input/output)

On entry, C must contain the (n-1) sub-diagonal elements of T.

On exit, C is overwritten by the (n-1) sub-diagonal elements of the matrix L of the factorization of T.

TOL (input/output)

On entry, a relative tolerance used to indicate whether or not the matrix (T - lambda*I) is nearly singular. TOL should normally be chose as approximately the largest relative error in the elements of T. For example, if the elements of T are correct to about 4 significant figures, then TOL should be set to about 5*10**(-4). If TOL is supplied as less than eps, where eps is the relative machine precision, then the value eps is used in place of TOL.

D (output)

On exit, D is overwritten by the (n-2) second super-diagonal elements of the matrix U of the factorization of T.

IN (output)

On exit, IN contains details of the permutation matrix P. If an interchange occurred at the kth step of the elimination, then IN(k) = 1, otherwise IN(k) = 0. The element IN(n) returns the smallest positive integer j such that

abs(u(j,j)).le. norm((T - lambda*I)(j))*TOL,

where norm(A(j)) denotes the sum of the absolute values of the jth row of the matrix A. If no such j exists then IN(n) is returned as zero. If IN(n) is returned as positive, then a diagonal element of U is small, indicating that (T - lambda*I) is singular or nearly singular,

INFO (output)

= 0 : successful exit

Contents

NAME

```
slamrg - will create a permutation list which will merge the
elements of A (which is composed of two independently sorted
sets) into a single set which is sorted in ascending order
```

SYNOPSIS

```
SUBROUTINE SLAMRG(N1, N2, A, TRD1, TRD2, INDEX)

INTEGER N1, N2, TRD1, TRD2
INTEGER INDEX(*)
REAL A(*)

SUBROUTINE SLAMRG_64(N1, N2, A, TRD1, TRD2, INDEX)

INTEGER*8 N1, N2, TRD1, TRD2
INTEGER*8 INDEX(*)
REAL A(*)
```

F95 INTERFACE

```
SUBROUTINE LAMRG(N1, N2, A, TRD1, TRD2, INDEX)

INTEGER :: N1, N2, TRD1, TRD2
INTEGER, DIMENSION(:) :: INDEX
REAL, DIMENSION(:) :: A

SUBROUTINE LAMRG_64(N1, N2, A, TRD1, TRD2, INDEX)

INTEGER(8) :: N1, N2, TRD1, TRD2
INTEGER(8), DIMENSION(:) :: INDEX
REAL, DIMENSION(:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void slamrg(int n1, int n2, float *a, int  trd1,  int  trd2,
          int *index);

void slamrg_64(long n1, long n2, float *a, long  trd1,  long
          trd2, long *index);
```

PURPOSE

slamrg will create a permutation list which will merge the elements of A (which is composed of two independently sorted sets) into a single set which is sorted in ascending order.

ARGUMENTS

N1 (input)
Length of the first sequence to be merged.

N2 (input)
Length of the second sequence to be merged.

A (input) On entry, the first N1 elements of A contain a list of numbers which are sorted in either ascending or descending order. Likewise for the final N2 elements.

TRD1 (input)
Describes the stride to be taken through the array A for the first N1 elements.
= -1 subset is sorted in descending order.
= 1 subset is sorted in ascending order.

TRD2 (input)
Describes the stride to be taken through the array A for the first N1 elements.
= -1 subset is sorted in descending order.
= 1 subset is sorted in ascending order.

INDEX (output)
On exit this array will contain a permutation such that if B(I) = A(INDEX(I)) for I=1,N1+N2, then B will be sorted in ascending order.

Contents

NAME

slarz - applies a real elementary reflector H to a real M-by-N matrix C, from either the left or the right

SYNOPSIS

```
SUBROUTINE SLARZ(SIDE, M, N, L, V, INCV, TAU, C, LDC, WORK)

CHARACTER * 1 SIDE
INTEGER M, N, L, INCV, LDC
REAL TAU
REAL V(*), C(LDC,*), WORK(*)

SUBROUTINE SLARZ_64(SIDE, M, N, L, V, INCV, TAU, C, LDC, WORK)

CHARACTER * 1 SIDE
INTEGER*8 M, N, L, INCV, LDC
REAL TAU
REAL V(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE LARZ(SIDE, [M], [N], L, V, [INCV], TAU, C, [LDC], [WORK])

CHARACTER(LEN=1) :: SIDE
INTEGER :: M, N, L, INCV, LDC
REAL :: TAU
REAL, DIMENSION(:) :: V, WORK
REAL, DIMENSION(:,:) :: C

SUBROUTINE LARZ_64(SIDE, [M], [N], L, V, [INCV], TAU, C, [LDC], [WORK])

CHARACTER(LEN=1) :: SIDE
```

```
INTEGER(8) :: M, N, L, INCV, LDC
REAL :: TAU
REAL, DIMENSION(:) :: V, WORK
REAL, DIMENSION(:,:) :: C
```

C INTERFACE

```
#include <sunperf.h>

void slarz(char side, int m, int n, int  l,  float  *v,  int
          incv, float tau, float *c, int ldc);

void slarz_64(char side, long m, long n, long l,  float  *v,
          long incv, float tau, float *c, long ldc);
```

PURPOSE

slarz applies a real elementary reflector H to a real M-by-N matrix C, from either the left or the right. H is represented in the form

H = I - tau * v * v'

where tau is a real scalar and v is a real vector.

If tau = 0, then H is taken to be the unit matrix.

H is a product of k elementary reflectors as returned by STZRZF.

ARGUMENTS

SIDE (input)
= 'L': form H * C
= 'R': form C * H

M (input) The number of rows of the matrix C.

N (input) The number of columns of the matrix C.

L (input) The number of entries of the vector V containing the meaningful part of the Householder vectors. If SIDE = 'L', M >= L >= 0, if SIDE = 'R', N >= L >= 0.

V (input) The vector v in the representation of H as returned by STZRZF. V is not used if TAU = 0.

INCV (input)
The increment between elements of v. INCV <> 0.

TAU (input)
The value tau in the representation of H.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by the matrix H * C if SIDE = 'L', or C * H if SIDE = 'R'.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
(N) if SIDE = 'L' or (M) if SIDE = 'R'

FURTHER DETAILS

Based on contributions by
A. Petitet, Computer Science Dept., Univ. of Tenn., Knoxville, USA

Contents

NAME

```
slarzb - applies a real block reflector H or  its  transpose
H**T  to  a  real  distributed M-by-N C from the left or the
right
```

SYNOPSIS

```
SUBROUTINE SLARZB(SIDE, TRANS, DIRECT, STOREV, M, N, K, L, V, LDV, T,
      LDT, C, LDC, WORK, LDWORK)

CHARACTER * 1 SIDE, TRANS, DIRECT, STOREV
INTEGER M, N, K, L, LDV, LDT, LDC, LDWORK
REAL V(LDV,*), T(LDT,*), C(LDC,*), WORK(LDWORK,*)

SUBROUTINE SLARZB_64(SIDE, TRANS, DIRECT, STOREV, M, N, K, L, V, LDV,
      T, LDT, C, LDC, WORK, LDWORK)

CHARACTER * 1 SIDE, TRANS, DIRECT, STOREV
INTEGER*8 M, N, K, L, LDV, LDT, LDC, LDWORK
REAL V(LDV,*), T(LDT,*), C(LDC,*), WORK(LDWORK,*)
```

F95 INTERFACE

```
SUBROUTINE LARZB(SIDE, TRANS, DIRECT, STOREV, [M], [N], K, L, V, [LDV],
       T, [LDT], C, [LDC], [WORK], [LDWORK])

CHARACTER(LEN=1) :: SIDE, TRANS, DIRECT, STOREV
INTEGER :: M, N, K, L, LDV, LDT, LDC, LDWORK
REAL, DIMENSION(:,:) :: V, T, C, WORK

SUBROUTINE LARZB_64(SIDE, TRANS, DIRECT, STOREV, [M], [N], K, L, V,
       [LDV], T, [LDT], C, [LDC], [WORK], [LDWORK])
```

```
CHARACTER(LEN=1) :: SIDE, TRANS, DIRECT, STOREV
INTEGER(8) :: M, N, K, L, LDV, LDT, LDC, LDWORK
REAL, DIMENSION(:,:) :: V, T, C, WORK
```

C INTERFACE

```
#include <sunperf.h>

void slarzb(char side, char trans, char direct, char storev,
          int  m,  int  n,  int k, int l, float *v, int ldv,
          float *t, int ldt, float *c, int ldc, int ldwork);

void slarzb_64(char side,  char  trans,  char  direct,  char
          storev,  long m, long n, long k, long l, float *v,
          long ldv, float *t, long ldt, float *c, long  ldc,
          long ldwork);
```

PURPOSE

slarzb applies a real block reflector H or its transpose H**T to a real distributed M-by-N C from the left or the right.

Currently, only STOREV = 'R' and DIRECT = 'B' are supported.

ARGUMENTS

SIDE (input)
= 'L': apply H or H' from the Left
= 'R': apply H or H' from the Right

TRANS (input)
= 'N': apply H (No transpose)
= 'C': apply H' (Transpose)

DIRECT (input)
Indicates how H is formed from a product of elementary reflectors = 'F': H = H(1) H(2) . . . H(k) (Forward, not supported yet)
= 'B': H = H(k) . . . H(2) H(1) (Backward)

STOREV (input)
Indicates how the vectors which define the elementary reflectors are stored:
= 'C': Columnwise (not supported yet)
= 'R': Rowwise

M (input) The number of rows of the matrix C.

N (input) The number of columns of the matrix C.

K (input) The order of the matrix T (= the number of elementary reflectors whose product defines the block reflector).

L (input) The number of columns of the matrix V containing the meaningful part of the Householder reflectors. If SIDE = 'L', M >= L >= 0, if SIDE = 'R', N >= L >= 0.

V (input) If STOREV = 'C', NV = K; if STOREV = 'R', NV = L.

LDV (input)
The leading dimension of the array V. If STOREV = 'C', LDV >= L; if STOREV = 'R', LDV >= K.

T (input) The triangular K-by-K matrix T in the representation of the block reflector.

LDT (input)
The leading dimension of the array T. LDT >= K.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by H*C or H'*C or C*H or C*H'.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
dimension(MAX(M,N),K)

LDWORK (input)
The leading dimension of the array WORK. If SIDE = 'L', LDWORK >= max(1,N); if SIDE = 'R', LDWORK >= max(1,M).

FURTHER DETAILS

Based on contributions by
A. Petitet, Computer Science Dept., Univ. of Tenn., Knoxville, USA

Contents

NAME

slarzt - form the triangular factor T of a real block reflector H of order > n, which is defined as a product of k elementary reflectors

SYNOPSIS

```
SUBROUTINE SLARZT(DIRECT, STOREV, N, K, V, LDV, TAU, T, LDT)

CHARACTER * 1 DIRECT, STOREV
INTEGER N, K, LDV, LDT
REAL V(LDV,*), TAU(*), T(LDT,*)

SUBROUTINE SLARZT_64(DIRECT, STOREV, N, K, V, LDV, TAU, T, LDT)

CHARACTER * 1 DIRECT, STOREV
INTEGER*8 N, K, LDV, LDT
REAL V(LDV,*), TAU(*), T(LDT,*)
```

F95 INTERFACE

```
SUBROUTINE LARZT(DIRECT, STOREV, N, K, V, [LDV], TAU, T, [LDT])

CHARACTER(LEN=1) :: DIRECT, STOREV
INTEGER :: N, K, LDV, LDT
REAL, DIMENSION(:) :: TAU
REAL, DIMENSION(:,:) :: V, T

SUBROUTINE LARZT_64(DIRECT, STOREV, N, K, V, [LDV], TAU, T, [LDT])

CHARACTER(LEN=1) :: DIRECT, STOREV
INTEGER(8) :: N, K, LDV, LDT
REAL, DIMENSION(:) :: TAU
```

```
REAL, DIMENSION(:,:) :: V, T
```

C INTERFACE

```
#include <sunperf.h>

void slarzt(char direct, char storev, int n,  int  k,  float
          *v, int ldv, float *tau, float *t, int ldt);

void slarzt_64(char direct, char storev,  long  n,  long  k,
          float  *v,  long  ldv,  float *tau, float *t, long
          ldt);
```

PURPOSE

slarzt forms the triangular factor T of a real block reflector H of order > n, which is defined as a product of k elementary reflectors.

If DIRECT = 'F', H = H(1) H(2) . . . H(k) and T is upper triangular;

If DIRECT = 'B', H = H(k) . . . H(2) H(1) and T is lower triangular.

If STOREV = 'C', the vector which defines the elementary reflector H(i) is stored in the i-th column of the array V, and

H = I - V * T * V'

If STOREV = 'R', the vector which defines the elementary reflector H(i) is stored in the i-th row of the array V, and

H = I - V' * T * V

Currently, only STOREV = 'R' and DIRECT = 'B' are supported.

ARGUMENTS

DIRECT (input)
Specifies the order in which the elementary reflectors are multiplied to form the block reflector:
= 'F': H = H(1) H(2) . . . H(k) (Forward, not supported yet)
= 'B': H = H(k) . . . H(2) H(1) (Backward)

```
STOREV (input)
          Specifies how the vectors which define the elemen-
          tary  reflectors  are  stored  (see  also  Further
          Details):
          = 'R': rowwise

N (input) The order of the block reflector H. N >= 0.

K (input) The order of the triangular factor T (= the number
          of elementary reflectors). K >= 1.

V (input) (LDV,K) if STOREV = 'C' (LDV,N) if  STOREV  =  'R'
          The matrix V. See further details.

LDV (input)
          The leading dimension of the array V.  If STOREV =
          'C', LDV >= max(1,N); if STOREV = 'R', LDV >= K.

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i).

T (input) The k by  k  triangular  factor  T  of  the  block
          reflector.   If  DIRECT = 'F', T is upper triangu-
          lar; if DIRECT = 'B', T is lower  triangular.  The
          rest of the array is not used.

LDT (input)
          The leading dimension of the array T. LDT >= K.
```

FURTHER DETAILS

```
Based on contributions by
  A. Petitet, Computer Science Dept., Univ. of Tenn.,  Knox-
ville, USA

The shape of the matrix V and the  storage  of  the  vectors
which  define  the H(i) is best illustrated by the following
example with n = 5 and k = 3. The elements equal  to  1  are
not  stored;  the  corresponding array elements are modified
but restored on exit. The rest of the array is not used.

DIRECT = 'F' and STOREV =  'C':          DIRECT  =  'F'  and
STOREV = 'R':

                                                    ______V_____
       (    v1    v2    v3    )                                /
( v1 v2 v3 )                        ( v1 v1 v1 v1 v1 . . . . 1
)
   V = ( v1 v2 v3 )                        ( v2 v2 v2 v2 v2  .
```

```
. . 1   )
       ( v1 v2 v3 )                       ( v3 v3 v3 v3 v3  .
. 1     )
       ( v1 v2 v3 )
          .  .  .
          1  .  .
             1  .
                1

DIRECT = 'B' and STOREV =  'C':          DIRECT  =  'B'  and
STOREV = 'R':

                                                    ______V_____
          1                                              /
.  1                          ( 1 . . . . v1 v1 v1 v1 v1 )
          .  .  1                         ( . 1 . . .  v2  v2
v2 v2 v2 )
          .  .  .                         ( . . 1 . .  v3  v3
v3 v3 v3 )
          .  .  .
       ( v1 v2 v3 )
    V = ( v1 v2 v3 )
       ( v1 v2 v3 )
```

Contents

NAME

```
slasrt - the numbers in D in increasing order (if ID =  'I')
or in decreasing order (if ID = 'D' )
```

SYNOPSIS

```
SUBROUTINE SLASRT(ID, N, D, INFO)

CHARACTER * 1 ID
INTEGER N, INFO
REAL D(*)

SUBROUTINE SLASRT_64(ID, N, D, INFO)

CHARACTER * 1 ID
INTEGER*8 N, INFO
REAL D(*)
```

F95 INTERFACE

```
SUBROUTINE LASRT(ID, [N], D, [INFO])

CHARACTER(LEN=1) :: ID
INTEGER :: N, INFO
REAL, DIMENSION(:) :: D

SUBROUTINE LASRT_64(ID, [N], D, [INFO])

CHARACTER(LEN=1) :: ID
INTEGER(8) :: N, INFO
REAL, DIMENSION(:) :: D
```

C INTERFACE

```
#include <sunperf.h>
```

```
void slasrt(char id, int n, float *d, int *info);

void slasrt_64(char id, long n, float *d, long *info);
```

PURPOSE

```
slasrt the numbers in D in increasing order (if ID = 'I') or
in decreasing order (if ID = 'D' ).

Use Quick Sort, reverting to Insertion sort on arrays of
size <= 20. Dimension of STACK limits N to about 2**32.
```

ARGUMENTS

```
ID (input)
          = 'I': sort D in increasing order;
          = 'D': sort D in decreasing order.

N (input) The length of the array D.

D (input/output)
          On entry, the array to be sorted.  On exit, D  has
          been  sorted into increasing order (D(1) <= ... <=
          D(N) ) or into decreasing order (D(1)  >=  ...  >=
          D(N) ), depending on ID.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
slatzm - routine is deprecated and has been replaced by rou-
tine SORMRZ
```

SYNOPSIS

```
SUBROUTINE SLATZM(SIDE, M, N, V, INCV, TAU, C1, C2, LDC, WORK)

CHARACTER * 1 SIDE
INTEGER M, N, INCV, LDC
REAL TAU
REAL V(*), C1(LDC,*), C2(LDC,*), WORK(*)

SUBROUTINE SLATZM_64(SIDE, M, N, V, INCV, TAU, C1, C2, LDC, WORK)

CHARACTER * 1 SIDE
INTEGER*8 M, N, INCV, LDC
REAL TAU
REAL V(*), C1(LDC,*), C2(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE LATZM(SIDE, [M], [N], V, [INCV], TAU, C1, C2, [LDC], [WORK])

CHARACTER(LEN=1) :: SIDE
INTEGER :: M, N, INCV, LDC
REAL :: TAU
REAL, DIMENSION(:) :: V, WORK
REAL, DIMENSION(:,:) :: C1, C2

SUBROUTINE LATZM_64(SIDE, [M], [N], V, [INCV], TAU, C1, C2, [LDC],
       [WORK])

CHARACTER(LEN=1) :: SIDE
```

```
INTEGER(8) :: M, N, INCV, LDC
REAL :: TAU
REAL, DIMENSION(:) :: V, WORK
REAL, DIMENSION(:,:) :: C1, C2
```

C INTERFACE

```
#include <sunperf.h>

void slatzm(char side, int m, int n,  float  *v,  int  incv,
          float tau, float *c1, float *c2, int ldc);

void slatzm_64(char side, long m, long  n,  float  *v,  long
          incv, float tau, float *c1, float *c2, long ldc);
```

PURPOSE

```
slatzm routine is deprecated and has been replaced  by  rou-
tine SORMRZ.

SLATZM applies a Householder matrix generated by STZRQF to a
matrix.

Let P = I - tau*u*u',   u = ( 1 ),
                            ( v )
where v is an (m-1) vector if SIDE = 'L', or a (n-1)  vector
if SIDE = 'R'.

If SIDE equals 'L', let
       C = [ C1 ] 1
           [ C2 ] m-1
             n
Then C is overwritten by P*C.

If SIDE equals 'R', let
       C = [ C1, C2 ] m
              1  n-1
Then C is overwritten by C*P.
```

ARGUMENTS

```
SIDE (input)
          = 'L': form P * C
          = 'R': form C * P

M (input) The number of rows of the matrix C.

N (input) The number of columns of the matrix C.
```

V (input) (1 + (M-1)*abs(INCV)) if SIDE = 'L' (1 + (N-1)*abs(INCV)) if SIDE = 'R' The vector v in the representation of P. V is not used if TAU = 0.

INCV (input)
The increment between elements of v. INCV <> 0

TAU (input)
The value tau in the representation of P.

C1 (input/output)
(LDC,N) if SIDE = 'L' (M,1) if SIDE = 'R' On entry, the n-vector C1 if SIDE = 'L', or the m-vector C1 if SIDE = 'R'.

On exit, the first row of P*C if SIDE = 'L', or the first column of C*P if SIDE = 'R'.

C2 (input/output)
(LDC, N) if SIDE = 'L' (LDC, N-1) if SIDE = 'R' On entry, the (m - 1) x n matrix C2 if SIDE = 'L', or the m x (n - 1) matrix C2 if SIDE = 'R'.

On exit, rows 2:m of P*C if SIDE = 'L', or columns 2:m of C*P if SIDE = 'R'.

LDC (input)
The leading dimension of the arrays C1 and C2. LDC >= (1,M).

WORK (workspace)
(N) if SIDE = 'L' (M) if SIDE = 'R'

Contents

NAME

```
snrm2 - Return the Euclidian norm of a vector.
```

SYNOPSIS

```
REAL FUNCTION SNRM2(N, X, INCX)

INTEGER N, INCX
REAL X(*)

REAL FUNCTION SNRM2_64(N, X, INCX)

INTEGER*8 N, INCX
REAL X(*)
```

F95 INTERFACE

```
REAL FUNCTION NRM2([N], X, [INCX])

INTEGER :: N, INCX
REAL, DIMENSION(:) :: X

REAL FUNCTION NRM2_64([N], X, [INCX])

INTEGER(8) :: N, INCX
REAL, DIMENSION(:) :: X
```

C INTERFACE

```
#include <sunperf.h>

float snrm2(int n, float *x, int incx);

float snrm2_64(long n, float *x, long incx);
```

PURPOSE

snrm2 Return the Euclidian norm of a vector x where x is an n-vector.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must be positive. Unchanged on exit.

Contents

NAME

```
sopgtr - generate  a  real  orthogonal  matrix  Q  which  is
defined  as the product of n-1 elementary reflectors H(i) of
order n, as returned by SSPTRD using packed storage
```

SYNOPSIS

```
SUBROUTINE SOPGTR(UPLO, N, AP, TAU, Q, LDQ, WORK, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDQ, INFO
REAL AP(*), TAU(*), Q(LDQ,*), WORK(*)

SUBROUTINE SOPGTR_64(UPLO, N, AP, TAU, Q, LDQ, WORK, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDQ, INFO
REAL AP(*), TAU(*), Q(LDQ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE OPGTR(UPLO, [N], AP, TAU, Q, [LDQ], [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDQ, INFO
REAL, DIMENSION(:) :: AP, TAU, WORK
REAL, DIMENSION(:,:) :: Q

SUBROUTINE OPGTR_64(UPLO, [N], AP, TAU, Q, [LDQ], [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDQ, INFO
REAL, DIMENSION(:) :: AP, TAU, WORK
REAL, DIMENSION(:,:) :: Q
```

C INTERFACE

```
#include <sunperf.h>

void sopgtr(char uplo, int n, float *ap, float  *tau,  float
          *q, int ldq, int *info);

void sopgtr_64(char uplo, long n,  float  *ap,  float  *tau,
          float *q, long ldq, long *info);
```

PURPOSE

```
sopgtr generates a real orthogonal matrix Q which is defined
as the product of n-1 elementary reflectors H(i) of order n,
as returned by SSPTRD using packed storage:
if UPLO = 'U', Q = H(n-1) . . . H(2) H(1),

if UPLO = 'L', Q = H(1) H(2) . . . H(n-1).
```

ARGUMENTS

```
UPLO (input)
          = 'U': Upper triangular  packed  storage  used  in
          previous  call  to SSPTRD; = 'L': Lower triangular
          packed storage used in previous call to SSPTRD.

N (input) The order of the matrix Q. N >= 0.

AP (input)
          The vectors which define  the  elementary  reflec-
          tors, as returned by SSPTRD.

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i), as returned by SSPTRD.

Q (output)
          The N-by-N orthogonal matrix Q.

LDQ (input)
          The leading dimension  of  the  array  Q.  LDQ  >=
          max(1,N).

WORK (workspace)
          dimension(N-1)

INFO (output)
```

= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-
gal value

Contents

NAME

```
sopmtr - overwrite the general real  M-by-N  matrix  C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE SOPMTR(SIDE, UPLO, TRANS, M, N, AP, TAU, C, LDC, WORK,
      INFO)

CHARACTER * 1 SIDE, UPLO, TRANS
INTEGER M, N, LDC, INFO
REAL AP(*), TAU(*), C(LDC,*), WORK(*)

SUBROUTINE SOPMTR_64(SIDE, UPLO, TRANS, M, N, AP, TAU, C, LDC, WORK,
      INFO)

CHARACTER * 1 SIDE, UPLO, TRANS
INTEGER*8 M, N, LDC, INFO
REAL AP(*), TAU(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE OPMTR(SIDE, UPLO, [TRANS], [M], [N], AP, TAU, C, [LDC],
       [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANS
INTEGER :: M, N, LDC, INFO
REAL, DIMENSION(:) :: AP, TAU, WORK
REAL, DIMENSION(:,:) :: C

SUBROUTINE OPMTR_64(SIDE, UPLO, [TRANS], [M], [N], AP, TAU, C, [LDC],
       [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANS
```

```
INTEGER(8) :: M, N, LDC, INFO
REAL, DIMENSION(:) :: AP, TAU, WORK
REAL, DIMENSION(:,:) :: C
```

C INTERFACE

```
#include <sunperf.h>

void sopmtr(char side, char uplo, char trans, int m, int  n,
          float  *ap,  float  *tau,  float  *c, int ldc, int
          *info);

void sopmtr_64(char side, char uplo,  char  trans,  long  m,
          long n, float *ap, float *tau, float *c, long ldc,
          long *info);
```

PURPOSE

sopmtr overwrites the general real M-by-N matrix C with
TRANS = 'T':   Q**T * C   C * Q**T

where Q is a real orthogonal matrix of order nq, with nq = m if SIDE = 'L' and nq = n if SIDE = 'R'. Q is defined as the product of nq-1 elementary reflectors, as returned by SSPTRD using packed storage:

if UPLO = 'U', Q = H(nq-1) . . . H(2) H(1);

if UPLO = 'L', Q = H(1) H(2) . . . H(nq-1).

ARGUMENTS

SIDE (input)
= 'L': apply Q or Q**T from the Left;
= 'R': apply Q or Q**T from the Right.

UPLO (input)
= 'U': Upper triangular packed storage used in previous call to SSPTRD; = 'L': Lower triangular packed storage used in previous call to SSPTRD.

TRANS (input)
= 'N': No transpose, apply Q;
= 'T': Transpose, apply Q**T.

TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

AP (input)
(M*(M+1)/2) if SIDE = 'L' (N*(N+1)/2) if SIDE = 'R' The vectors which define the elementary reflectors, as returned by SSPTRD. AP is modified by the routine but restored on exit.

TAU (input)
or (N-1) if SIDE = 'R' TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SSPTRD.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by Q*C or Q**T*C or C*Q**T or C*Q.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
(N) if SIDE = 'L' (M) if SIDE = 'R'

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
sorg2l - generate an m by n real matrix Q  with  orthonormal
columns,
```

SYNOPSIS

```
SUBROUTINE SORG2L(M, N, K, A, LDA, TAU, WORK, INFO)

INTEGER M, N, K, LDA, INFO
REAL A(LDA,*), TAU(*), WORK(*)

SUBROUTINE SORG2L_64(M, N, K, A, LDA, TAU, WORK, INFO)

INTEGER*8 M, N, K, LDA, INFO
REAL A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORG2L([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

INTEGER :: M, N, K, LDA, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE ORG2L_64([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

INTEGER(8) :: M, N, K, LDA, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void sorg2l(int m, int n, int k, float *a,  int  lda,  float
```

```
          *tau, int *info);

void sorg2l_64(long m, long n, long k, float *a,  long  lda,
          float *tau, long *info);
```

PURPOSE

sorg2l L generates an m by n real matrix Q with orthonormal columns, which is defined as the last n columns of a product of k elementary reflectors of order m

```
      Q  =  H(k) . . . H(2) H(1)
```

as returned by SGEQLF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. M >= N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. N >= K >= 0.

A (input/output)
On entry, the (n-k+i)-th column must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by SGEQLF in the last k columns of its array argument A. On exit, the m by n matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGEQLF.

WORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
sorg2r - generate an m by n real matrix Q  with  orthonormal
columns,
```

SYNOPSIS

```
SUBROUTINE SORG2R(M, N, K, A, LDA, TAU, WORK, INFO)

INTEGER M, N, K, LDA, INFO
REAL A(LDA,*), TAU(*), WORK(*)

SUBROUTINE SORG2R_64(M, N, K, A, LDA, TAU, WORK, INFO)

INTEGER*8 M, N, K, LDA, INFO
REAL A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORG2R([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

INTEGER :: M, N, K, LDA, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE ORG2R_64([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

INTEGER(8) :: M, N, K, LDA, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void sorg2r(int m, int n, int k, float *a,  int  lda,  float
```

```
          *tau, int *info);

void sorg2r_64(long m, long n, long k, float *a,  long  lda,
          float *tau, long *info);
```

PURPOSE

sorg2r R generates an m by n real matrix Q with orthonormal columns, which is defined as the first n columns of a product of k elementary reflectors of order m

```
      Q  =  H(1) H(2) . . . H(k)
```

as returned by SGEQRF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. M >= N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. N >= K >= 0.

A (input/output)
On entry, the i-th column must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by SGEQRF in the first k columns of its array argument A. On exit, the m-by-n matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGEQRF.

WORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
sorgbr - generate one of the real orthogonal matrices  Q  or
P**T  determined  by SGEBRD when reducing a real matrix A to
bidiagonal form
```

SYNOPSIS

```
SUBROUTINE SORGBR(VECT, M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

CHARACTER * 1 VECT
INTEGER M, N, K, LDA, LWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)

SUBROUTINE SORGBR_64(VECT, M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

CHARACTER * 1 VECT
INTEGER*8 M, N, K, LDA, LWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORGBR(VECT, M, [N], K, A, [LDA], TAU, [WORK], [LWORK],
       [INFO])

CHARACTER(LEN=1) :: VECT
INTEGER :: M, N, K, LDA, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE ORGBR_64(VECT, M, [N], K, A, [LDA], TAU, [WORK], [LWORK],
       [INFO])

CHARACTER(LEN=1) :: VECT
INTEGER(8) :: M, N, K, LDA, LWORK, INFO
```

```
    REAL, DIMENSION(:) :: TAU, WORK
    REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
    #include <sunperf.h>

    void sorgbr(char vect, int m, int n, int k,  float  *a,  int
              lda, float *tau, int *info);

    void sorgbr_64(char vect, long m, long n, long k, float  *a,
              long lda, float *tau, long *info);
```

PURPOSE

```
    sorgbr generates one of the real orthogonal  matrices  Q  or
    P**T  determined  by SGEBRD when reducing a real matrix A to
    bidiagonal form: A = Q * B * P**T.  Q and P**T  are  defined
    as  products  of  elementary reflectors H(i) or G(i) respec-
    tively.

    If VECT = 'Q', A is assumed to have been an  M-by-K  matrix,
    and Q is of order M:
    if m >= k, Q = H(1) H(2) . . . H(k) and SORGBR  returns  the
    first n columns of Q, where m >= n >= k;
    if m < k, Q = H(1) H(2) . . . H(m-1) and SORGBR returns Q as
    an M-by-M matrix.

    If VECT = 'P', A is assumed to have been  a  K-by-N  matrix,
    and P**T is of order N:
    if k < n, P**T = G(k) . . . G(2) G(1) and SORGBR returns the
    first m rows of P**T, where n >= m >= k;
    if k >= n, P**T = G(n-1) . . . G(2) G(1) and SORGBR  returns
    P**T as an N-by-N matrix.
```

ARGUMENTS

```
    VECT (input)
              Specifies whether the matrix Q or the matrix  P**T
              is  required,  as  defined  in  the transformation
              applied by SGEBRD:
              = 'Q':  generate Q;
              = 'P':  generate P**T.

    M (input) The number of rows of the matrix Q or P**T  to  be
              returned.  M >= 0.

    N (input) The number of columns of the matrix Q or  P**T  to
```

be returned. N >= 0. If VECT = 'Q', M >= N >= min(M,K); if VECT = 'P', N >= M >= min(N,K).

K (input) If VECT = 'Q', the number of columns in the original M-by-K matrix reduced by SGEBRD. If VECT = 'P', the number of rows in the original K-by-N matrix reduced by SGEBRD. K >= 0.

A (input/output)
On entry, the vectors which define the elementary reflectors, as returned by SGEBRD. On exit, the M-by-N matrix Q or P**T.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

TAU (input)
(min(M,K)) if VECT = 'Q' (min(N,K)) if VECT = 'P' TAU(i) must contain the scalar factor of the elementary reflector H(i) or G(i), which determines Q or P**T, as returned by SGEBRD in its array argument TAUQ or TAUP.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,min(M,N)). For optimum performance LWORK >= min(M,N)*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

sorghr - generate a real orthogonal matrix Q which is defined as the product of IHI-ILO elementary reflectors of order N, as returned by SGEHRD

SYNOPSIS

```
SUBROUTINE SORGHR(N, ILO, IHI, A, LDA, TAU, WORK, LWORK, INFO)

INTEGER N, ILO, IHI, LDA, LWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)

SUBROUTINE SORGHR_64(N, ILO, IHI, A, LDA, TAU, WORK, LWORK, INFO)

INTEGER*8 N, ILO, IHI, LDA, LWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORGHR([N], ILO, IHI, A, [LDA], TAU, [WORK], [LWORK], [INFO])

INTEGER :: N, ILO, IHI, LDA, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE ORGHR_64([N], ILO, IHI, A, [LDA], TAU, [WORK], [LWORK],
       [INFO])

INTEGER(8) :: N, ILO, IHI, LDA, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void sorghr(int n, int ilo, int  ihi,  float  *a,  int  lda,
          float *tau, int *info);

void sorghr_64(long n, long ilo, long ihi,  float  *a,  long
          lda, float *tau, long *info);
```

PURPOSE

sorghr generates a real orthogonal matrix Q which is defined as the product of IHI-ILO elementary reflectors of order N, as returned by SGEHRD:

Q = H(ilo) H(ilo+1) . . . H(ihi-1).

ARGUMENTS

N (input) The order of the matrix Q. N >= 0.

ILO (input)
ILO and IHI must have the same values as in the previous call of SGEHRD. Q is equal to the unit matrix except in the submatrix Q(ilo+1:ihi,ilo+1:ihi). 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

IHI (input)
See the description of ILO.

A (input/output)
On entry, the vectors which define the elementary reflectors, as returned by SGEHRD. On exit, the N-by-N orthogonal matrix Q.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGEHRD.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)

The dimension of the array WORK. LWORK >= IHI-ILO. For optimum performance LWORK >= (IHI-ILO)*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
sorgl2 - generate an m by n real matrix Q  with  orthonormal
rows,
```

SYNOPSIS

```
SUBROUTINE SORGL2(M, N, K, A, LDA, TAU, WORK, INFO)

INTEGER M, N, K, LDA, INFO
REAL A(LDA,*), TAU(*), WORK(*)

SUBROUTINE SORGL2_64(M, N, K, A, LDA, TAU, WORK, INFO)

INTEGER*8 M, N, K, LDA, INFO
REAL A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORGL2([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

INTEGER :: M, N, K, LDA, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE ORGL2_64([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

INTEGER(8) :: M, N, K, LDA, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void sorgl2(int m, int n, int k, float *a,  int  lda,  float
```

```
          *tau, int *info);

void sorgl2_64(long m, long n, long k, float *a,  long  lda,
          float *tau, long *info);
```

PURPOSE

sorgl2 generates an m by n real matrix Q with orthonormal rows, which is defined as the first m rows of a product of k elementary reflectors of order n

      Q = H(k) . . . H(2) H(1)

as returned by SGELQF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. N >= M.

K (input) The number of elementary reflectors whose product defines the matrix Q. M >= K >= 0.

A (input/output)
On entry, the i-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by SGELQF in the first k rows of its array argument A. On exit, the m-by-n matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGELQF.

WORK (workspace)
dimension(M)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
sorglq - generate an M-by-N real matrix Q  with  orthonormal
rows,
```

SYNOPSIS

```
SUBROUTINE SORGLQ(M, N, K, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER M, N, K, LDA, LDWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)

SUBROUTINE SORGLQ_64(M, N, K, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER*8 M, N, K, LDA, LDWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORGLQ(M, [N], [K], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER :: M, N, K, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE ORGLQ_64(M, [N], [K], A, [LDA], TAU, [WORK], [LDWORK],
       [INFO])

INTEGER(8) :: M, N, K, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void sorglq(int m, int n, int k, float *a,  int  lda,  float
          *tau, int *info);

void sorglq_64(long m, long n, long k, float *a,  long  lda,
          float *tau, long *info);
```

PURPOSE

sorglq generates an M-by-N real matrix Q with orthonormal rows, which is defined as the first M rows of a product of K elementary reflectors of order N

```
      Q  =  H(k) . . . H(2) H(1)
```

as returned by SGELQF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. N >= M.

K (input) The number of elementary reflectors whose product defines the matrix Q. M >= K >= 0.

A (input/output)
On entry, the i-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by SGELQF in the first k rows of its array argument A. On exit, the M-by-N matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGELQF.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,M). For optimum performance LDWORK >= M*NB,

```
          where NB is the optimal blocksize.

          If LDWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LDWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument has an ille-
          gal value
```

Contents

NAME

```
sorgql - generate an M-by-N real matrix Q  with  orthonormal
columns,
```

SYNOPSIS

```
SUBROUTINE SORGQL(M, N, K, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER M, N, K, LDA, LDWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)

SUBROUTINE SORGQL_64(M, N, K, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER*8 M, N, K, LDA, LDWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORGQL(M, [N], [K], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER :: M, N, K, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE ORGQL_64(M, [N], [K], A, [LDA], TAU, [WORK], [LDWORK],
       [INFO])

INTEGER(8) :: M, N, K, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void sorgql(int m, int n, int k, float *a,  int  lda,  float
          *tau, int *info);

void sorgql_64(long m, long n, long k, float *a,  long  lda,
          float *tau, long *info);
```

PURPOSE

sorgql generates an M-by-N real matrix Q with orthonormal columns, which is defined as the last N columns of a product of K elementary reflectors of order M

      Q = H(k) . . . H(2) H(1)

as returned by SGEQLF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. M >= N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. N >= K >= 0.

A (input/output)
On entry, the (n-k+i)-th column must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by SGEQLF in the last k columns of its array argument A. On exit, the M-by-N matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGEQLF.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK   >=

max(1,N). For optimum performance LDWORK >= N*NB, where NB is the optimal blocksize.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
sorgqr - generate an M-by-N real matrix Q  with  orthonormal
columns,
```

SYNOPSIS

```
SUBROUTINE SORGQR(M, N, K, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER M, N, K, LDA, LDWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)

SUBROUTINE SORGQR_64(M, N, K, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER*8 M, N, K, LDA, LDWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORGQR(M, [N], [K], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER :: M, N, K, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE ORGQR_64(M, [N], [K], A, [LDA], TAU, [WORK], [LDWORK],
       [INFO])

INTEGER(8) :: M, N, K, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void sorgqr(int m, int n, int k, float *a,  int  lda,  float
          *tau, int *info);

void sorgqr_64(long m, long n, long k, float *a,  long  lda,
          float *tau, long *info);
```

PURPOSE

sorgqr generates an M-by-N real matrix Q with orthonormal columns, which is defined as the first N columns of a product of K elementary reflectors of order M

      Q = H(1) H(2) . . . H(k)

as returned by SGEQRF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. M >= N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. N >= K >= 0.

A (input/output)
On entry, the i-th column must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by SGEQRF in the first k columns of its array argument A. On exit, the M-by-N matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGEQRF.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK   >=

```
            max(1,N).  For optimum performance LDWORK >= N*NB,
            where NB is the optimal blocksize.

            If LDWORK = -1, then a workspace query is assumed;
            the  routine  only  calculates the optimal size of
            the WORK array, returns this value  as  the  first
            entry  of  the  WORK  array,  and no error message
            related to LDWORK is issued by XERBLA.

    INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument has an ille-
            gal value
```

Contents

NAME

```
sorgr2 - generate an m by n real matrix Q  with  orthonormal
rows,
```

SYNOPSIS

```
SUBROUTINE SORGR2(M, N, K, A, LDA, TAU, WORK, INFO)

INTEGER M, N, K, LDA, INFO
REAL A(LDA,*), TAU(*), WORK(*)

SUBROUTINE SORGR2_64(M, N, K, A, LDA, TAU, WORK, INFO)

INTEGER*8 M, N, K, LDA, INFO
REAL A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORGR2([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

INTEGER :: M, N, K, LDA, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE ORGR2_64([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

INTEGER(8) :: M, N, K, LDA, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void sorgr2(int m, int n, int k, float *a,  int  lda,  float
```

```
          *tau, int *info);

void sorgr2_64(long m, long n, long k, float *a,  long  lda,
          float *tau, long *info);
```

PURPOSE

sorgr2 generates an m by n real matrix Q with orthonormal rows, which is defined as the last m rows of a product of k elementary reflectors of order n

```
      Q  =  H(1) H(2) . . . H(k)
```

as returned by SGERQF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. N >= M.

K (input) The number of elementary reflectors whose product defines the matrix Q. M >= K >= 0.

A (input/output)
On entry, the (m-k+i)-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by SGERQF in the last k rows of its array argument A. On exit, the m by n matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGERQF.

WORK (workspace)
dimension(M)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
sorgrq - generate an M-by-N real matrix Q  with  orthonormal
rows,
```

SYNOPSIS

```
SUBROUTINE SORGRQ(M, N, K, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER M, N, K, LDA, LDWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)

SUBROUTINE SORGRQ_64(M, N, K, A, LDA, TAU, WORK, LDWORK, INFO)

INTEGER*8 M, N, K, LDA, LDWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORGRQ(M, [N], [K], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

INTEGER :: M, N, K, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE ORGRQ_64(M, [N], [K], A, [LDA], TAU, [WORK], [LDWORK],
       [INFO])

INTEGER(8) :: M, N, K, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void sorgrq(int m, int n, int k, float *a,  int  lda,  float
          *tau, int *info);

void sorgrq_64(long m, long n, long k, float *a,  long  lda,
          float *tau, long *info);
```

PURPOSE

sorgrq generates an M-by-N real matrix Q with orthonormal rows, which is defined as the last M rows of a product of K elementary reflectors of order N

      Q = H(1) H(2) . . . H(k)

as returned by SGERQF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. N >= M.

K (input) The number of elementary reflectors whose product defines the matrix Q. M >= K >= 0.

A (input/output)
On entry, the (m-k+i)-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by SGERQF in the last k rows of its array argument A. On exit, the M-by-N matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGERQF.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,M). For optimum performance LDWORK >= M*NB,

where NB is the optimal blocksize.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
sorgtr - generate  a  real  orthogonal  matrix  Q  which  is
defined as the product of n-1 elementary reflectors of order
N, as returned by SSYTRD
```

SYNOPSIS

```
SUBROUTINE SORGTR(UPLO, N, A, LDA, TAU, WORK, LWORK, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, LWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)

SUBROUTINE SORGTR_64(UPLO, N, A, LDA, TAU, WORK, LWORK, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, LWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORGTR(UPLO, [N], A, [LDA], TAU, [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE ORGTR_64(UPLO, [N], A, [LDA], TAU, [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void sorgtr(char uplo, int n, float *a, int lda, float *tau,
          int *info);

void sorgtr_64(char uplo, long n, float *a, long lda,  float
          *tau, long *info);
```

PURPOSE

```
sorgtr generates a real orthogonal matrix Q which is defined
as  the  product of n-1 elementary reflectors of order N, as
returned by SSYTRD:
if UPLO = 'U', Q = H(n-1) . . . H(2) H(1),

if UPLO = 'L', Q = H(1) H(2) . . . H(n-1).
```

ARGUMENTS

```
UPLO (input)
          = 'U': Upper triangle  of  A  contains  elementary
          reflectors from SSYTRD; = 'L': Lower triangle of A
          contains elementary reflectors from SSYTRD.

N (input) The order of the matrix Q. N >= 0.

A (input/output)
          On entry, the vectors which define the  elementary
          reflectors,  as  returned by SSYTRD.  On exit, the
          N-by-N orthogonal matrix Q.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,N).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i), as returned by SSYTRD.

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The  dimension  of  the  array  WORK.   LWORK   >=
```

max(1,N-1). For optimum performance LWORK >= (N-1)*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
sormbr - VECT = 'Q', SORMBR overwrites the general  real  M-
by-N matrix C with  SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE SORMBR(VECT, SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 VECT, SIDE, TRANS
INTEGER M, N, K, LDA, LDC, LWORK, INFO
REAL A(LDA,*), TAU(*), C(LDC,*), WORK(*)

SUBROUTINE SORMBR_64(VECT, SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 VECT, SIDE, TRANS
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
REAL A(LDA,*), TAU(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORMBR(VECT, SIDE, [TRANS], [M], [N], K, A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: VECT, SIDE, TRANS
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A, C

SUBROUTINE ORMBR_64(VECT, SIDE, [TRANS], [M], [N], K, A, [LDA], TAU,
       C, [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: VECT, SIDE, TRANS
```

```
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A, C
```

C INTERFACE

```
#include <sunperf.h>

void sormbr(char vect, char side, char trans, int m, int  n,
          int  k,  float  *a, int lda, float *tau, float *c,
          int ldc, int *info);

void sormbr_64(char vect, char side,  char  trans,  long  m,
          long  n,  long  k, float *a, long lda, float *tau,
          float *c, long ldc, long *info);
```

PURPOSE

```
sormbr VECT = 'Q', SORMBR overwrites the general real M-by-N
matrix C with
                SIDE = 'L'      SIDE  =  'R'  TRANS  =  'N':
Q  *  C           C * Q TRANS = 'T':      Q**T * C       C *
Q**T

If VECT = 'P', SORMBR overwrites  the  general  real  M-by-N
matrix C with
                SIDE = 'L'     SIDE = 'R'
TRANS = 'N':      P * C          C * P
TRANS = 'T':      P**T * C       C * P**T
```

Here Q and P**T are the orthogonal matrices determined by SGEBRD when reducing a real matrix A to bidiagonal form: A = Q * B * P**T. Q and P**T are defined as products of elementary reflectors H(i) and G(i) respectively.

Let nq = m if SIDE = 'L' and nq = n if SIDE = 'R'. Thus nq is the order of the orthogonal matrix Q or P**T that is applied.

```
If VECT = 'Q', A is assumed to have been an NQ-by-K  matrix:
if nq >= k, Q = H(1) H(2) . . . H(k);
if nq < k, Q = H(1) H(2) . . . H(nq-1).

If VECT = 'P', A is assumed to have been a  K-by-NQ  matrix:
if k < nq, P = G(1) G(2) . . . G(k);
if k >= nq, P = G(1) G(2) . . . G(nq-1).
```

ARGUMENTS

```
VECT (input)
          = 'Q': apply Q or Q**T;
          = 'P': apply P or P**T.

SIDE (input)
          = 'L': apply Q, Q**T, P or P**T from the Left;
          = 'R': apply Q, Q**T, P or P**T from the Right.

TRANS (input)
          = 'N':  No transpose, apply Q  or P;
          = 'T':  Transpose, apply Q**T or P**T.

          TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.
N (input) The number of columns of the matrix C. N >= 0.

K (input) If VECT = 'Q', the number of columns in the origi-
          nal  matrix reduced by SGEBRD.  If VECT = 'P', the
          number of rows in the original matrix  reduced  by
          SGEBRD.  K >= 0.

A (input) (LDA,min(nq,K)) if VECT = 'Q'  (LDA,nq)         if
          VECT = 'P' The vectors which define the elementary
          reflectors H(i) and G(i), whose products determine
          the matrices Q and P, as returned by SGEBRD.

LDA (input)
          The leading dimension of the array A.  If  VECT  =
          'Q',  LDA  >=  max(1,nq);  if  VECT  = 'P', LDA >=
          max(1,min(nq,K)).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary  reflector H(i) or G(i) which determines Q
          or P, as returned by SGEBRD in the array  argument
          TAUQ or TAUP.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten  by  Q*C or Q**T*C or C*Q**T or C*Q or
          P*C or P**T*C or C*P or C*P**T.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
```

LWORK.

LWORK (input)
The dimension of the array WORK. If SIDE = 'L', LWORK >= max(1,N); if SIDE = 'R', LWORK >= max(1,M). For optimum performance LWORK >= N*NB if SIDE = 'L', and LWORK >= M*NB if SIDE = 'R', where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
sormhr - overwrite the general real  M-by-N  matrix  C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE SORMHR(SIDE, TRANS, M, N, ILO, IHI, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER M, N, ILO, IHI, LDA, LDC, LWORK, INFO
REAL A(LDA,*), TAU(*), C(LDC,*), WORK(*)

SUBROUTINE SORMHR_64(SIDE, TRANS, M, N, ILO, IHI, A, LDA, TAU, C,
      LDC, WORK, LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER*8 M, N, ILO, IHI, LDA, LDC, LWORK, INFO
REAL A(LDA,*), TAU(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORMHR(SIDE, [TRANS], [M], [N], ILO, IHI, A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
INTEGER :: M, N, ILO, IHI, LDA, LDC, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A, C

SUBROUTINE ORMHR_64(SIDE, [TRANS], [M], [N], ILO, IHI, A, [LDA], TAU,
       C, [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
INTEGER(8) :: M, N, ILO, IHI, LDA, LDC, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A, C
```

C INTERFACE

```
#include <sunperf.h>

void sormhr(char side, char trans, int m, int  n,  int  ilo,
          int  ihi, float *a, int lda, float *tau, float *c,
          int ldc, int *info);

void sormhr_64(char side, char trans, long m, long  n,  long
          ilo,  long  ihi,  float  *a, long lda, float *tau,
          float *c, long ldc, long *info);
```

PURPOSE

sormhr overwrites the general real M-by-N matrix C with
TRANS = 'T':   Q**T * C   C * Q**T

where Q is a real orthogonal matrix of order nq, with nq = m if SIDE = 'L' and nq = n if SIDE = 'R'. Q is defined as the product of IHI-ILO elementary reflectors, as returned by SGEHRD:

Q = H(ilo) H(ilo+1) . . . H(ihi-1).

ARGUMENTS

SIDE (input)
- = 'L': apply Q or Q**T from the Left;
- = 'R': apply Q or Q**T from the Right.

TRANS (input)
- = 'N': No transpose, apply Q;
- = 'T': Transpose, apply Q**T.

TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

ILO (input)
ILO and IHI must have the same values as in the previous call of SGEHRD. Q is equal to the unit matrix except in the submatrix

Q(ilo+1:ihi,ilo+1:ihi). If SIDE = 'L', then 1 <= ILO <= IHI <= M, if M > 0, and ILO = 1 and IHI = 0, if M = 0; if SIDE = 'R', then 1 <= ILO <= IHI <= N, if N > 0, and ILO = 1 and IHI = 0, if N = 0.

IHI (input)
See the description of ILO.

A (input) (LDA,M) if SIDE = 'L' (LDA,N) if SIDE = 'R' The vectors which define the elementary reflectors, as returned by SGEHRD.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M) if SIDE = 'L'; LDA >= max(1,N) if SIDE = 'R'.

TAU (input)
(M-1) if SIDE = 'L' (N-1) if SIDE = 'R' TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGEHRD.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by Q*C or Q**T*C or C*Q**T or C*Q.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If SIDE = 'L', LWORK >= max(1,N); if SIDE = 'R', LWORK >= max(1,M). For optimum performance LWORK >= N*NB if SIDE = 'L', and LWORK >= M*NB if SIDE = 'R', where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-

gal value

Contents

NAME

```
sormlq - overwrite the general real  M-by-N  matrix  C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE SORMLQ(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER M, N, K, LDA, LDC, LWORK, INFO
REAL A(LDA,*), TAU(*), C(LDC,*), WORK(*)

SUBROUTINE SORMLQ_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
REAL A(LDA,*), TAU(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORMLQ(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A, C

SUBROUTINE ORMLQ_64(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A, C
```

C INTERFACE

```
#include <sunperf.h>

void sormlq(char side, char trans, int  m,  int  n,  int  k,
          float  *a, int lda, float *tau, float *c, int ldc,
          int *info);

void sormlq_64(char side, char trans, long m, long  n,  long
          k,  float *a, long lda, float *tau, float *c, long
          ldc, long *info);
```

PURPOSE

sormlq overwrites the general real M-by-N matrix C with
TRANS = 'T':   Q**T * C   C * Q**T

where Q is a real orthogonal matrix defined as the product of k elementary reflectors

      Q = H(k) . . . H(2) H(1)

as returned by SGELQF. Q is of order M if SIDE = 'L' and of order N if SIDE = 'R'.

ARGUMENTS

SIDE (input)
= 'L': apply Q or Q**T from the Left;
= 'R': apply Q or Q**T from the Right.

TRANS (input)
= 'N': No transpose, apply Q;
= 'T': Transpose, apply Q**T.

TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. If SIDE = 'L', M >= K >= 0; if SIDE = 'R', N >= K >= 0.

A (input) (LDA,M) if SIDE = 'L', (LDA,N) if SIDE = 'R' The i-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by SGELQF in the first k rows of its array argument A. A is modified by the routine but restored on exit.

LDA (input)
The leading dimension of the array A. LDA >= max(1,K).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGELQF.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by Q*C or Q**T*C or C*Q**T or C*Q.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If SIDE = 'L', LWORK >= max(1,N); if SIDE = 'R', LWORK >= max(1,M). For optimum performance LWORK >= N*NB if SIDE = 'L', and LWORK >= M*NB if SIDE = 'R', where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
sormql - overwrite the general real  M-by-N  matrix  C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE SORMQL(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER M, N, K, LDA, LDC, LWORK, INFO
REAL A(LDA,*), TAU(*), C(LDC,*), WORK(*)

SUBROUTINE SORMQL_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
REAL A(LDA,*), TAU(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORMQL(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A, C

SUBROUTINE ORMQL_64(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A, C
```

C INTERFACE

```
#include <sunperf.h>

void sormql(char side, char trans, int  m,  int  n,  int  k,
          float  *a, int lda, float *tau, float *c, int ldc,
          int *info);

void sormql_64(char side, char trans, long m, long  n,  long
          k,  float *a, long lda, float *tau, float *c, long
          ldc, long *info);
```

PURPOSE

```
sormql overwrites the general  real  M-by-N  matrix  C  with
TRANS = 'T':      Q**T * C       C * Q**T

where Q is a real orthogonal matrix defined as  the  product
of k elementary reflectors

      Q = H(k) . . . H(2) H(1)

as returned by SGEQLF. Q is of order M if SIDE = 'L' and  of
order N if SIDE = 'R'.
```

ARGUMENTS

```
SIDE (input)
          = 'L': apply Q or Q**T from the Left;
          = 'R': apply Q or Q**T from the Right.

TRANS (input)
          = 'N':  No transpose, apply Q;
          = 'T':  Transpose, apply Q**T.

          TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose  product
          defines the matrix Q.  If SIDE = 'L', M >= K >= 0;
          if SIDE = 'R', N >= K >= 0.
```

A (input) The i-th column must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by SGEQLF in the last k columns of its array argument A. A is modified by the routine but restored on exit.

LDA (input)
The leading dimension of the array A. If SIDE = 'L', LDA >= max(1,M); if SIDE = 'R', LDA >= max(1,N).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGEQLF.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by Q*C or Q**T*C or C*Q**T or C*Q.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If SIDE = 'L', LWORK >= max(1,N); if SIDE = 'R', LWORK >= max(1,M). For optimum performance LWORK >= N*NB if SIDE = 'L', and LWORK >= M*NB if SIDE = 'R', where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
sormqr - overwrite the general real  M-by-N  matrix  C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE SORMQR(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER M, N, K, LDA, LDC, LWORK, INFO
REAL A(LDA,*), TAU(*), C(LDC,*), WORK(*)

SUBROUTINE SORMQR_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
REAL A(LDA,*), TAU(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORMQR(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A, C

SUBROUTINE ORMQR_64(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A, C
```

C INTERFACE

```
#include <sunperf.h>

void sormqr(char side, char trans, int  m,  int  n,  int  k,
          float  *a, int lda, float *tau, float *c, int ldc,
          int *info);

void sormqr_64(char side, char trans, long m, long  n,  long
          k,  float *a, long lda, float *tau, float *c, long
          ldc, long *info);
```

PURPOSE

sormqr overwrites the general real M-by-N matrix C with
TRANS = 'T':   Q**T * C   C * Q**T

where Q is a real orthogonal matrix defined as the product of k elementary reflectors

      Q = H(1) H(2) . . . H(k)

as returned by SGEQRF. Q is of order M if SIDE = 'L' and of order N if SIDE = 'R'.

ARGUMENTS

SIDE (input)
= 'L': apply Q or Q**T from the Left;
= 'R': apply Q or Q**T from the Right.

TRANS (input)
= 'N': No transpose, apply Q;
= 'T': Transpose, apply Q**T.

TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. If SIDE = 'L', M >= K >= 0; if SIDE = 'R', N >= K >= 0.

```
A (input) The i-th column  must  contain  the  vector  which
          defines  the  elementary  reflector  H(i), for i =
          1,2,...,k, as returned by SGEQRF in  the  first  k
          columns of its array argument A.  A is modified by
          the routine but restored on exit.

LDA (input)
          The leading dimension of the array A.  If  SIDE  =
          'L',  LDA  >=  max(1,M);  if  SIDE  =  'R', LDA >=
          max(1,N).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i), as returned by SGEQRF.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten by Q*C or Q**T*C or C*Q**T or C*Q.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK.  If SIDE  =  'L',
          LWORK  >=  max(1,N);  if  SIDE  =  'R',  LWORK  >=
          max(1,M).  For optimum performance LWORK  >=  N*NB
          if  SIDE  =  'L', and LWORK >= M*NB if SIDE = 'R',
          where NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
sormrq - overwrite the general real  M-by-N  matrix  C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE SORMRQ(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER M, N, K, LDA, LDC, LWORK, INFO
REAL A(LDA,*), TAU(*), C(LDC,*), WORK(*)

SUBROUTINE SORMRQ_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
REAL A(LDA,*), TAU(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORMRQ(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A, C

SUBROUTINE ORMRQ_64(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A, C
```

C INTERFACE

```
#include <sunperf.h>

void sormrq(char side, char trans, int  m,  int  n,  int  k,
          float  *a, int lda, float *tau, float *c, int ldc,
          int *info);

void sormrq_64(char side, char trans, long m, long  n,  long
          k,  float *a, long lda, float *tau, float *c, long
          ldc, long *info);
```

PURPOSE

sormrq overwrites the general real M-by-N matrix C with
TRANS = 'T':   Q**T * C   C * Q**T

where Q is a real orthogonal matrix defined as the product
of k elementary reflectors

      Q = H(1) H(2) . . . H(k)

as returned by SGERQF. Q is of order M if SIDE = 'L' and of
order N if SIDE = 'R'.

ARGUMENTS

SIDE (input)
= 'L': apply Q or Q**T from the Left;
= 'R': apply Q or Q**T from the Right.

TRANS (input)
= 'N': No transpose, apply Q;
= 'T': Transpose, apply Q**T.

TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose product
defines the matrix Q. If SIDE = 'L', M >= K >= 0;
if SIDE = 'R', N >= K >= 0.

A (input) (LDA,M) if SIDE = 'L', (LDA,N) if SIDE = 'R' The i-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by SGERQF in the last k rows of its array argument A. A is modified by the routine but restored on exit.

LDA (input)
The leading dimension of the array A. LDA >= max(1,K).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SGERQF.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by Q*C or Q**T*C or C*Q**T or C*Q.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If SIDE = 'L', LWORK >= max(1,N); if SIDE = 'R', LWORK >= max(1,M). For optimum performance LWORK >= N*NB if SIDE = 'L', and LWORK >= M*NB if SIDE = 'R', where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
sormrz - overwrite the general real  M-by-N  matrix  C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE SORMRZ(SIDE, TRANS, M, N, K, L, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER M, N, K, L, LDA, LDC, LWORK, INFO
REAL A(LDA,*), TAU(*), C(LDC,*), WORK(*)

SUBROUTINE SORMRZ_64(SIDE, TRANS, M, N, K, L, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
INTEGER*8 M, N, K, L, LDA, LDC, LWORK, INFO
REAL A(LDA,*), TAU(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORMRZ(SIDE, TRANS, [M], [N], K, L, A, [LDA], TAU, C, [LDC],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
INTEGER :: M, N, K, L, LDA, LDC, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A, C

SUBROUTINE ORMRZ_64(SIDE, TRANS, [M], [N], K, L, A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])
```

```
CHARACTER(LEN=1) :: SIDE, TRANS
INTEGER(8) :: M, N, K, L, LDA, LDC, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A, C
```

C INTERFACE

```
#include <sunperf.h>

void sormrz(char side, char trans, int m, int n, int k,  int
          l,  float  *a,  int lda, float *tau, float *c, int
          ldc, int *info);

void sormrz_64(char side, char trans, long m, long  n,  long
          k,  long  l, float *a, long lda, float *tau, float
          *c, long ldc, long *info);
```

PURPOSE

```
sormrz overwrites the general  real  M-by-N  matrix  C  with
TRANS = 'T':      Q**T * C       C * Q**T

where Q is a real orthogonal matrix defined as  the  product
of k elementary reflectors

      Q = H(1) H(2) . . . H(k)

as returned by STZRZF. Q is of order M if SIDE = 'L' and  of
order N if SIDE = 'R'.
```

ARGUMENTS

```
SIDE (input)
          = 'L': apply Q or Q**T from the Left;
          = 'R': apply Q or Q**T from the Right.

TRANS (input)
          = 'N':  No transpose, apply Q;
          = 'T':  Transpose, apply Q**T.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose  product
          defines the matrix Q.  If SIDE = 'L', M >= K >= 0;
          if SIDE = 'R', N >= K >= 0.
```

L (input) The number of columns of the matrix A containing the meaningful part of the Householder reflectors. If SIDE = 'L', M >= L >= 0, if SIDE = 'R', N >= L >= 0.

A (input) (LDA,M) if SIDE = 'L', (LDA,N) if SIDE = 'R' The i-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by STZRZF in the last k rows of its array argument A. A is modified by the routine but restored on exit.

LDA (input)
The leading dimension of the array A. LDA >= max(1,K).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by STZRZF.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by Q*C or Q**H*C or C*Q**H or C*Q.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If SIDE = 'L', LWORK >= max(1,N); if SIDE = 'R', LWORK >= max(1,M). For optimum performance LWORK >= N*NB if SIDE = 'L', and LWORK >= M*NB if SIDE = 'R', where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

```
Based on contributions by
  A. Petitet, Computer Science Dept., Univ. of Tenn.,  Knox-
ville, USA
```

Contents

NAME

```
sormtr - overwrite the general real  M-by-N  matrix  C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE SORMTR(SIDE, UPLO, TRANS, M, N, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, UPLO, TRANS
INTEGER M, N, LDA, LDC, LWORK, INFO
REAL A(LDA,*), TAU(*), C(LDC,*), WORK(*)

SUBROUTINE SORMTR_64(SIDE, UPLO, TRANS, M, N, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 SIDE, UPLO, TRANS
INTEGER*8 M, N, LDA, LDC, LWORK, INFO
REAL A(LDA,*), TAU(*), C(LDC,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ORMTR(SIDE, UPLO, [TRANS], [M], [N], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANS
INTEGER :: M, N, LDA, LDC, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A, C

SUBROUTINE ORMTR_64(SIDE, UPLO, [TRANS], [M], [N], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANS
```

```
INTEGER(8) :: M, N, LDA, LDC, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A, C
```

C INTERFACE

```
#include <sunperf.h>

void sormtr(char side, char uplo, char trans, int m, int  n,
          float  *a, int lda, float *tau, float *c, int ldc,
          int *info);

void sormtr_64(char side, char uplo,  char  trans,  long  m,
          long  n, float *a, long lda, float *tau, float *c,
          long ldc, long *info);
```

PURPOSE

```
sormtr overwrites the general  real  M-by-N  matrix  C  with
TRANS = 'T':      Q**T * C       C * Q**T

where Q is a real orthogonal matrix of order nq, with nq = m
if  SIDE = 'L' and nq = n if SIDE = 'R'. Q is defined as the
product  of  nq-1  elementary  reflectors,  as  returned  by
SSYTRD:

if UPLO = 'U', Q = H(nq-1) . . . H(2) H(1);

if UPLO = 'L', Q = H(1) H(2) . . . H(nq-1).
```

ARGUMENTS

```
SIDE (input)
          = 'L': apply Q or Q**T from the Left;
          = 'R': apply Q or Q**T from the Right.

UPLO (input)
          = 'U': Upper triangle  of  A  contains  elementary
          reflectors from SSYTRD; = 'L': Lower triangle of A
          contains elementary reflectors from SSYTRD.

TRANS (input)
          = 'N':  No transpose, apply Q;
          = 'T':  Transpose, apply Q**T.

          TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.
```

N (input) The number of columns of the matrix C. N >= 0.

A (input) (LDA,M) if SIDE = 'L' (LDA,N) if SIDE = 'R' The vectors which define the elementary reflectors, as returned by SSYTRD.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M) if SIDE = 'L'; LDA >= max(1,N) if SIDE = 'R'.

TAU (input)
(M-1) if SIDE = 'L' (N-1) if SIDE = 'R' TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by SSYTRD.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by Q*C or Q**T*C or C*Q**T or C*Q.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If SIDE = 'L', LWORK >= max(1,N); if SIDE = 'R', LWORK >= max(1,M). For optimum performance LWORK >= N*NB if SIDE = 'L', and LWORK >= M*NB if SIDE = 'R', where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
spbcon - estimate the reciprocal of the condition number (in
the  1-norm)  of  a  real  symmetric  positive definite band
matrix using the Cholesky factorization A = U**T*U  or  A  =
L*L**T computed by SPBTRF
```

SYNOPSIS

```
SUBROUTINE SPBCON(UPLO, N, KD, A, LDA, ANORM, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 UPLO
INTEGER N, KD, LDA, INFO
INTEGER WORK2(*)
REAL ANORM, RCOND
REAL A(LDA,*), WORK(*)

SUBROUTINE SPBCON_64(UPLO, N, KD, A, LDA, ANORM, RCOND, WORK,
      WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, KD, LDA, INFO
INTEGER*8 WORK2(*)
REAL ANORM, RCOND
REAL A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PBCON(UPLO, [N], KD, A, [LDA], ANORM, RCOND, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, KD, LDA, INFO
INTEGER, DIMENSION(:) :: WORK2
```

```
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE PBCON_64(UPLO, [N], KD, A, [LDA], ANORM, RCOND, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, KD, LDA, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void spbcon(char uplo, int n, int kd,  float  *a,  int  lda,
          float anorm, float *rcond, int *info);

void spbcon_64(char uplo, long n, long kd,  float  *a,  long
          lda, float anorm, float *rcond, long *info);
```

PURPOSE

spbcon estimates the reciprocal of the condition number (in the 1-norm) of a real symmetric positive definite band matrix using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPBTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
= 'U': Upper triangular factor stored in A;
= 'L': Lower triangular factor stored in A.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T of the band matrix A, stored in the first KD+1 rows of the array. The j-th column of U or L is stored in the j-th column of the array A as follows: if UPLO ='U', A(kd+1+i-j,j) = U(i,j) for max(1,j-kd)<=i<=j; if UPLO ='L', A(1+i-j,j) = L(i,j) for j<=i<=min(n,j+kd).

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

ANORM (input)
The 1-norm (or infinity-norm) of the symmetric band matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

spbequ - compute row and column scalings intended to equilibrate a symmetric positive definite band matrix A and reduce its condition number (with respect to the two-norm)

SYNOPSIS

```
SUBROUTINE SPBEQU(UPLO, N, KD, A, LDA, SCALE, SCOND, AMAX, INFO)

CHARACTER * 1 UPLO
INTEGER N, KD, LDA, INFO
REAL SCOND, AMAX
REAL A(LDA,*), SCALE(*)

SUBROUTINE SPBEQU_64(UPLO, N, KD, A, LDA, SCALE, SCOND, AMAX,
      INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, KD, LDA, INFO
REAL SCOND, AMAX
REAL A(LDA,*), SCALE(*)
```

F95 INTERFACE

```
SUBROUTINE PBEQU(UPLO, [N], KD, A, [LDA], SCALE, SCOND, AMAX,
       [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, KD, LDA, INFO
REAL :: SCOND, AMAX
REAL, DIMENSION(:) :: SCALE
REAL, DIMENSION(:,:) :: A

SUBROUTINE PBEQU_64(UPLO, [N], KD, A, [LDA], SCALE, SCOND, AMAX,
```

```
        [INFO])

    CHARACTER(LEN=1) :: UPLO
    INTEGER(8) :: N, KD, LDA, INFO
    REAL :: SCOND, AMAX
    REAL, DIMENSION(:) :: SCALE
    REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
    #include <sunperf.h>

    void spbequ(char uplo, int n, int kd,  float  *a,  int  lda,
              float  *scale,  float  *scond,  float  *amax,  int
              *info);
    void spbequ_64(char uplo, long n, long kd,  float  *a,  long
              lda, float *scale, float *scond, float *amax, long
              *info);
```

PURPOSE

spbequ computes row and column scalings intended to equilibrate a symmetric positive definite band matrix A and reduce its condition number (with respect to the two-norm). S contains the scale factors, S(i) = 1/sqrt(A(i,i)), chosen so that the scaled matrix B with elements B(i,j) = S(i)*A(i,j)*S(j) has ones on the diagonal. This choice of S puts the condition number of B within a factor N of the smallest possible condition number over all possible diagonal scalings.

ARGUMENTS

```
    UPLO (input)
              = 'U':  Upper triangular of A is stored;
              = 'L':  Lower triangular of A is stored.

    N (input) The order of the matrix A.  N >= 0.

    KD (input)
              The number of superdiagonals of the  matrix  A  if
              UPLO  = 'U', or the number of subdiagonals if UPLO
              = 'L'.  KD >= 0.

    A (input) The upper or lower triangle of the symmetric  band
              matrix  A,  stored  in  the first KD+1 rows of the
              array.  The j-th column of A is stored in the j-th
              column  of the array A as follows:  if UPLO = 'U',
```

A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

SCALE (output)
If INFO = 0, SCALE contains the scale factors for A.

SCOND (output)
If INFO = 0, SCALE contains the ratio of the smallest SCALE(i) to the largest SCALE(i). If SCOND >= 0.1 and AMAX is neither too large nor too small, it is not worth scaling by SCALE.

AMAX (output)
Absolute value of largest matrix element. If AMAX is very close to overflow or very close to underflow, the matrix should be scaled.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, the i-th diagonal element is nonpositive.

Contents

NAME

```
spbrfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is symmetric positive
definite and banded, and provides error bounds and  backward
error estimates for the solution
```

SYNOPSIS

```
SUBROUTINE SPBRFS(UPLO, N, KD, NRHS, A, LDA, AF, LDAF, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER N, KD, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER WORK2(*)
REAL  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),  FERR(*),
BERR(*), WORK(*)

SUBROUTINE SPBRFS_64(UPLO, N, KD, NRHS, A, LDA, AF, LDAF, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, KD, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 WORK2(*)
REAL  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),  FERR(*),
BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PBRFS(UPLO, [N], KD, [NRHS], A, [LDA], AF, [LDAF], B,
       [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, KD, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: WORK2
```

```
REAL, DIMENSION(:) :: FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, AF, B, X

SUBROUTINE PBRFS_64(UPLO, [N], KD, [NRHS], A, [LDA], AF, [LDAF],
       B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, KD, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL, DIMENSION(:) :: FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
#include <sunperf.h>
void spbrfs(char uplo, int n, int kd, int  nrhs,  float  *a,
          int  lda,  float *af, int ldaf, float *b, int ldb,
          float *x, int ldx, float *ferr, float  *berr,  int
          *info);

void spbrfs_64(char uplo, long n, long kd, long nrhs,  float
          *a, long lda, float *af, long ldaf, float *b, long
          ldb, float *x, long ldx, float *ferr, float *berr,
          long *info);
```

PURPOSE

spbrfs improves the computed solution to a system of linear equations when the coefficient matrix is symmetric positive definite and banded, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  KD >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.
```

A (input) The upper or lower triangle of the symmetric band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

AF (input)
The triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T of the band matrix A as computed by SPBTRF, in the same storage format as A (see A).

LDAF (input)
The leading dimension of the array AF. LDAF >= KD+1.

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by SPBTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative

```
          change in any element of A or B that makes X(j) an
          exact solution).

WORK (workspace)
          dimension(3*N)
WORK2 (workspace)
          dimension(N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
spbstf - compute a split Cholesky factorization  of  a  real
symmetric positive definite band matrix A
```

SYNOPSIS

```
SUBROUTINE SPBSTF(UPLO, N, KD, AB, LDAB, INFO)

CHARACTER * 1 UPLO
INTEGER N, KD, LDAB, INFO
REAL AB(LDAB,*)

SUBROUTINE SPBSTF_64(UPLO, N, KD, AB, LDAB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, KD, LDAB, INFO
REAL AB(LDAB,*)
```

F95 INTERFACE

```
SUBROUTINE PBSTF(UPLO, [N], KD, AB, [LDAB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, KD, LDAB, INFO
REAL, DIMENSION(:,:) :: AB

SUBROUTINE PBSTF_64(UPLO, [N], KD, AB, [LDAB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, KD, LDAB, INFO
REAL, DIMENSION(:,:) :: AB
```

C INTERFACE

```
#include <sunperf.h>

void spbstf(char uplo, int n, int kd, float *ab,  int  ldab,
          int *info);

void spbstf_64(char uplo, long n, long kd, float  *ab,  long
          ldab, long *info);
```

PURPOSE

```
spbstf computes a split Cholesky  factorization  of  a  real
symmetric positive definite band matrix A.

This routine is designed to  be  used  in  conjunction  with
SSBGST.
The factorization has the form  A = S**T*S   where  S  is  a
band  matrix  of  the  same bandwidth as A and the following
structure:

  S = ( U    )
      ( M  L )

where U is upper triangular of order m = (n+kd)/2, and L  is
lower triangular of order n-m.
```

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  KD >= 0.

AB (input/output)
          On entry, the upper or lower triangle of the  sym-
          metric  band  matrix  A,  stored in the first kd+1
          rows of the array.  The j-th column of A is stored
          in the j-th column of the array AB as follows:  if
          UPLO = 'U', AB(kd+1+i-j,j) = A(i,j)  for  max(1,j-
          kd)<=i<=j;  if UPLO = 'L', AB(1+i-j,j)    = A(i,j)
          for j<=i<=min(n,j+kd).
```

On exit, if INFO = 0, the factor S from the split Cholesky factorization A = S**T*S. See Further Details.

LDAB (input)
The leading dimension of the array AB. LDAB >= KD+1.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the factorization could not be completed, because the updated element a(i,i) was negative; the matrix A is not positive definite.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when N = 7, KD = 2:

```
S = ( s11  s12  s13                     )
    (      s22  s23  s24                )
    (           s33  s34                )
    (                s44                )
    (           s53  s54  s55           )
    (                s64  s65  s66      )
    (                     s75  s76  s77 )
```

If UPLO = 'U', the array AB holds:

```
on entry:                          on exit:

 *    *   a13  a24  a35  a46  a57   *    *   s13   s24   s53
s64  s75
 *   a12  a23  a34  a45  a56  a67   *   s12  s23   s34   s54
s65   s76  a11   a22  a33  a44  a55  a66  a77  s11  s22  s33
s44  s55  s66  s77
```

If UPLO = 'L', the array AB holds:

```
on entry:                          on exit:

a11  a22  a33  a44  a55  a66  a77  s11  s22  s33   s44   s55
s66   s77  a21   a32  a43  a54  a65  a76   *   s12  s23  s34
s54  s65  s76   * a31  a42  a53  a64   a64    *     *    s13
s24  s53  s64  s75   *    *
```

Array elements marked * are not used by the routine.

Contents

NAME

```
spbsv - compute the solution to  a  real  system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE SPBSV(UPLO, N, NDIAG, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER N, NDIAG, NRHS, LDA, LDB, INFO
REAL A(LDA,*), B(LDB,*)

SUBROUTINE SPBSV_64(UPLO, N, NDIAG, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NDIAG, NRHS, LDA, LDB, INFO
REAL A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE PBSV(UPLO, [N], NDIAG, [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NDIAG, NRHS, LDA, LDB, INFO
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE PBSV_64(UPLO, [N], NDIAG, [NRHS], A, [LDA], B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NDIAG, NRHS, LDA, LDB, INFO
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void spbsv(char uplo, int n, int ndiag, int nrhs, float  *a,
          int lda, float *b, int ldb, int *info);

void spbsv_64(char uplo, long  n,  long  ndiag,  long  nrhs,
          float  *a,  long  lda,  float  *b,  long ldb, long
          *info);
```

PURPOSE

spbsv computes the solution to a real system of linear equations
   A * X = B, where A is an N-by-N symmetric positive definite band matrix and X and B are N-by-NRHS matrices.
The Cholesky decomposition is used to factor A as
   A = U**T * U, if UPLO = 'U', or
   A = L * L**T, if UPLO = 'L',
where U is an upper triangular band matrix, and L is a lower triangular band matrix, with the same number of superdiagonals or subdiagonals as A. The factored form of A is then used to solve the system of equations A * X = B.

ARGUMENTS

UPLO (input)
  = 'U': Upper triangle of A is stored;
  = 'L': Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NDIAG (input)
  The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. NDIAG >= 0.

NRHS (input)
  The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input/output)
  On entry, the upper or lower triangle of the symmetric band matrix A, stored in the first NDIAG+1 rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if

UPLO = 'U', A(NDIAG+1+i-j,j) = A(i,j) for max(1,j-NDIAG)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(N,j+NDIAG). See below for further details.

On exit, if INFO = 0, the triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T of the band matrix A, in the same storage format as A.

LDA (input)
The leading dimension of the array A. LDA >= NDIAG+1.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when N = 6, NDIAG = 2, and UPLO = 'U':

```
On entry:                       On exit:

    *    *   a13  a24  a35  a46      *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
```

Similarly, if UPLO = 'L' the format of A is as follows:

```
On entry:                       On exit:

   a11  a22  a33  a44  a55  a66     l11  l22  l33  l44   l55
```

```
l66
   a21  a32  a43  a54  a65   *      l21  l32  l43  l54   l65
*
   a31  a42  a53  a64   *    *      l31  l42  l53   l64    *
*
```

Array elements marked * are not used by the routine.

Contents

NAME

spbsvx - use the Cholesky factorization A = U**T*U or A = L*L**T to compute the solution to a real system of linear equations A * X = B,

SYNOPSIS

```
SUBROUTINE SPBSVX(FACT, UPLO, N, NDIAG, NRHS, A, LDA, AF, LDAF,
      EQUED, S, B, LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2,
      INFO)

CHARACTER * 1 FACT, UPLO, EQUED
INTEGER N, NDIAG, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER WORK2(*)
REAL RCOND
REAL  A(LDA,*),  AF(LDAF,*),   S(*),   B(LDB,*),   X(LDX,*),
FERR(*), BERR(*), WORK(*)

SUBROUTINE SPBSVX_64(FACT, UPLO, N, NDIAG, NRHS, A, LDA, AF, LDAF,
      EQUED, S, B, LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2,
      INFO)

CHARACTER * 1 FACT, UPLO, EQUED
INTEGER*8 N, NDIAG, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 WORK2(*)
REAL RCOND
REAL  A(LDA,*),  AF(LDAF,*),   S(*),   B(LDB,*),   X(LDX,*),
FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PBSVX(FACT, UPLO, [N], NDIAG, [NRHS], A, [LDA], AF, [LDAF],
       EQUED, S, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK],
```

```
       [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
INTEGER :: N, NDIAG, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL :: RCOND
REAL, DIMENSION(:) :: S, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, AF, B, X

SUBROUTINE PBSVX_64(FACT, UPLO, [N], NDIAG, [NRHS], A, [LDA], AF,
       [LDAF], EQUED, S, B, [LDB], X, [LDX], RCOND, FERR, BERR,
       [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
INTEGER(8) :: N, NDIAG, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL :: RCOND
REAL, DIMENSION(:) :: S, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
#include <sunperf.h>

void spbsvx(char fact, char uplo,  int  n,  int  ndiag,  int
          nrhs, float *a, int lda, float *af, int ldaf, char
          equed, float *s, float *b, int ldb, float *x,  int
          ldx,  float  *rcond, float *ferr, float *berr, int
          *info);

void spbsvx_64(char fact, char uplo,  long  n,  long  ndiag,
          long  nrhs,  float  *a,  long lda, float *af, long
          ldaf, char equed, float *s, float  *b,  long  ldb,
          float  *x,  long  ldx,  float *rcond, float *ferr,
          float *berr, long *info);
```

PURPOSE

spbsvx uses the Cholesky factorization A = U**T*U or A = L*L**T to compute the solution to a real system of linear equations
   A * X = B, where A is an N-by-N symmetric positive definite band matrix and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'E', real scaling factors are computed to

```
equilibrate
   the system:
      diag(S) * A * diag(S) * inv(diag(S)) * X = diag(S) * B
   Whether or not the system will be equilibrated depends on
the
   scaling of the matrix A, but if equilibration is used,  A
is
   overwritten by diag(S)*A*diag(S) and B by diag(S)*B.

2. If FACT = 'N' or 'E', the Cholesky decomposition is  used
to
   factor the matrix A (after equilibration if FACT  =  'E')
as
      A = U**T * U,  if UPLO = 'U', or
      A = L * L**T,  if UPLO = 'L',
   where U is an upper triangular band matrix, and  L  is  a
lower
   triangular band matrix.
3. If the leading i-by-i principal  minor  is  not  positive
definite,
   then the routine returns with INFO =  i.  Otherwise,  the
factored
   form of A is used to estimate the condition number of the
matrix
   A.  If the reciprocal of the  condition  number  is  less
than machine
   precision, INFO = N+1 is returned as a warning,  but  the
routine
   still goes on to solve for X and compute error bounds as
   described below.

4. The system of equations is solved for X  using  the  fac-
tored form
   of A.

5. Iterative refinement is applied to improve  the  computed
solution
   matrix and calculate  error  bounds  and  backward  error
estimates
   for it.

6. If equilibration was used, the matrix X is  premultiplied
by
   diag(S) so that it solves the original system before
   equilibration.
```

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored. = 'F': On entry, AF contains the factored form of A. If EQUED = 'Y', the matrix A has been equilibrated with scaling factors given by S. A and AF will not be modified. = 'N': The matrix A will be copied to AF and factored.
= 'E': The matrix A will be equilibrated if necessary, then copied to AF and factored.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NDIAG (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. NDIAG >= 0.

NRHS (input)
The number of right-hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input/output)
On entry, the upper or lower triangle of the symmetric band matrix A, stored in the first NDIAG+1 rows of the array, except if FACT = 'F' and EQUED = 'Y', then A must contain the equilibrated matrix diag(S)*A*diag(S). The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(NDIAG+1+i-j,j) = A(i,j) for max(1,j-NDIAG)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(N,j+NDIAG). See below for further details.

On exit, if FACT = 'E' and EQUED = 'Y', A is overwritten by diag(S)*A*diag(S).

LDA (input)
The leading dimension of the array A. LDA >= NDIAG+1.

AF (input/output)
If FACT = 'F', then AF is an input argument and on entry contains the triangular factor U or L from the Cholesky factorization A = U**T*U or A =

L*L**T of the band matrix A, in the same storage format as A (see A). If EQUED = 'Y', then AF is the factored form of the equilibrated matrix A.

If FACT = 'N', then AF is an output argument and on exit returns the triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T.

If FACT = 'E', then AF is an output argument and on exit returns the triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T of the equilibrated matrix A (see the description of A for the form of the equilibrated matrix).

LDAF (input)
The leading dimension of the array AF. LDAF >= NDIAG+1.

EQUED (input)
Specifies the form of equilibration that was done.
= 'N': No equilibration (always true if FACT = 'N').
= 'Y': Equilibration was done, i.e., A has been replaced by diag(S) * A * diag(S). EQUED is an input argument if FACT = 'F'; otherwise, it is an output argument.

S (input/output)
The scale factors for A; not accessed if EQUED = 'N'. S is an input argument if FACT = 'F'; otherwise, S is an output argument. If FACT = 'F' and EQUED = 'Y', each element of S must be positive.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if EQUED = 'N', B is not modified; if EQUED = 'Y', B is overwritten by diag(S) * B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X to the original system of equations. Note that if EQUED = 'Y', A and B are modified on exit, and the solution to the equilibrated system is inv(diag(S))*X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A after equilibration (if done). If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would

suggest.

FURTHER DETAILS

```
The band storage scheme  is  illustrated  by  the  following
example, when N = 6, NDIAG = 2, and UPLO = 'U':

Two-dimensional storage of the symmetric matrix A:
   a11  a12  a13
        a22  a23  a24
             a33  a34  a35
                  a44  a45  a46
                       a55  a56
   (aij=conjg(aji))         a66

Band storage of the upper triangle of A:

    *    *   a13  a24  a35  a46
    *   a12  a23  a34  a45  a56
   a11  a22  a33  a44  a55  a66

Similarly, if UPLO = 'L' the format of A is as follows:

   a11  a22  a33  a44  a55  a66
   a21  a32  a43  a54  a65   *
   a31  a42  a53  a64   *    *

Array elements marked * are not used by the routine.
```

Contents

NAME

```
spbtf2 - compute the Cholesky factorization of a  real  sym-
metric positive definite band matrix A
```

SYNOPSIS

```
SUBROUTINE SPBTF2(UPLO, N, KD, AB, LDAB, INFO)

CHARACTER * 1 UPLO
INTEGER N, KD, LDAB, INFO
REAL AB(LDAB,*)

SUBROUTINE SPBTF2_64(UPLO, N, KD, AB, LDAB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, KD, LDAB, INFO
REAL AB(LDAB,*)
```

F95 INTERFACE

```
SUBROUTINE PBTF2(UPLO, [N], KD, AB, [LDAB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, KD, LDAB, INFO
REAL, DIMENSION(:,:) :: AB

SUBROUTINE PBTF2_64(UPLO, [N], KD, AB, [LDAB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, KD, LDAB, INFO
REAL, DIMENSION(:,:) :: AB
```

C INTERFACE

```
#include <sunperf.h>

void spbtf2(char uplo, int n, int kd, float *ab,  int  ldab,
          int *info);

void spbtf2_64(char uplo, long n, long kd, float  *ab,  long
          ldab, long *info);
```

PURPOSE

spbtf2 computes the Cholesky factorization of a real symmetric positive definite band matrix A.

The factorization has the form

```
   A = U' * U ,  if UPLO = 'U', or
   A = L  * L',  if UPLO = 'L',
```

where U is an upper triangular matrix, U' is the transpose of U, and L is lower triangular.

This is the unblocked version of the algorithm, calling Level 2 BLAS.

ARGUMENTS

UPLO (input)
Specifies whether the upper or lower triangular part of the symmetric matrix A is stored:
= 'U': Upper triangular
= 'L': Lower triangular

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of super-diagonals of the matrix A if UPLO = 'U', or the number of sub-diagonals if UPLO = 'L'. KD >= 0.

AB (input/output)
On entry, the upper or lower triangle of the symmetric band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', AB(1+i-j,j)   = A(i,j) for j<=i<=min(n,j+kd).

On exit, if INFO = 0, the triangular factor U or L

from the Cholesky factorization A = U'*U or A = L*L' of the band matrix A, in the same storage format as A.

LDAB (input)
The leading dimension of the array AB. LDAB >= KD+1.

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, the leading minor of order k is not positive definite, and the factorization could not be completed.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when N = 6, KD = 2, and UPLO = 'U':

```
On entry:                       On exit:

    *    *   a13  a24  a35  a46      *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
```

Similarly, if UPLO = 'L' the format of A is as follows:

```
On entry:                       On exit:

   a11  a22  a33  a44  a55  a66     l11  l22  l33  l44   l55
l66
   a21  a32  a43  a54  a65   *      l21  l32  l43  l54   l65
*
   a31  a42  a53  a64   *    *      l31  l42  l53   l64    *
*
```

Array elements marked * are not used by the routine.

Contents

NAME

```
spbtrf - compute the Cholesky factorization of a  real  sym-
metric positive definite band matrix A
```

SYNOPSIS

```
SUBROUTINE SPBTRF(UPLO, N, KD, A, LDA, INFO)

CHARACTER * 1 UPLO
INTEGER N, KD, LDA, INFO
REAL A(LDA,*)

SUBROUTINE SPBTRF_64(UPLO, N, KD, A, LDA, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, KD, LDA, INFO
REAL A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE PBTRF(UPLO, [N], KD, A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, KD, LDA, INFO
REAL, DIMENSION(:,:) :: A

SUBROUTINE PBTRF_64(UPLO, [N], KD, A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, KD, LDA, INFO
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void spbtrf(char uplo, int n, int kd, float *a, int lda, int
          *info);

void spbtrf_64(char uplo, long n, long kd,  float  *a,  long
          lda, long *info);
```

PURPOSE

spbtrf computes the Cholesky factorization of a real symmetric positive definite band matrix A.

The factorization has the form

```
   A = U**T * U,  if UPLO = 'U', or
   A = L  * L**T,  if UPLO = 'L',
```

where U is an upper triangular matrix and L is lower triangular.

ARGUMENTS

UPLO (input)
: = 'U': Upper triangle of A is stored;
: = 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

KD (input)
: The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

A (input/output)
: On entry, the upper or lower triangle of the symmetric band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).

: On exit, if INFO = 0, the triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T of the band matrix A, in the same storage format as A.

LDA (input)

The leading dimension of the array A. LDA >= KD+1.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i is not positive definite, and the factorization could not be completed.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when N = 6, KD = 2, and UPLO = 'U':

```
On entry:                       On exit:

    *    *   a13  a24  a35  a46      *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
```

Similarly, if UPLO = 'L' the format of A is as follows:

```
On entry:                       On exit:

   a11  a22  a33  a44  a55  a66     l11  l22  l33  l44   l55
l66
   a21  a32  a43  a54  a65   *      l21  l32  l43  l54   l65
*
   a31  a42  a53  a64   *    *      l31  l42  l53   l64    *
*
```

Array elements marked * are not used by the routine.

Contributed by
Peter Mayes and Giuseppe Radicati, IBM ECSEC, Rome, March 23, 1989

Contents

NAME

```
spbtrs - solve a system of linear equations A*X = B  with  a
symmetric positive definite band matrix A using the Cholesky
factorization A = U**T*U or A = L*L**T computed by SPBTRF
```

SYNOPSIS

```
SUBROUTINE SPBTRS(UPLO, N, KD, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER N, KD, NRHS, LDA, LDB, INFO
REAL A(LDA,*), B(LDB,*)

SUBROUTINE SPBTRS_64(UPLO, N, KD, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, KD, NRHS, LDA, LDB, INFO
REAL A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE PBTRS(UPLO, [N], KD, [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, KD, NRHS, LDA, LDB, INFO
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE PBTRS_64(UPLO, [N], KD, [NRHS], A, [LDA], B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, KD, NRHS, LDA, LDB, INFO
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void spbtrs(char uplo, int n, int kd, int  nrhs,  float  *a,
          int lda, float *b, int ldb, int *info);

void spbtrs_64(char uplo, long n, long kd, long nrhs,  float
          *a, long lda, float *b, long ldb, long *info);
```

PURPOSE

spbtrs solves a system of linear equations A*X = B with a symmetric positive definite band matrix A using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPBTRF.

ARGUMENTS

UPLO (input)
= 'U': Upper triangular factor stored in A;
= 'L': Lower triangular factor stored in A.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T of the band matrix A, stored in the first KD+1 rows of the array. The j-th column of U or L is stored in the j-th column of the array A as follows: if UPLO ='U', A(kd+1+i-j,j) = U(i,j) for max(1,j-kd)<=i<=j; if UPLO ='L', A(1+i-j,j) = L(i,j) for j<=i<=min(n,j+kd).

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

B (input/output)
On entry, the right hand side matrix B. On exit,

the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
spocon - estimate the reciprocal of the condition number (in
the  1-norm)  of  a  real symmetric positive definite matrix
using the Cholesky factorization A = U**T*U or  A  =  L*L**T
computed by SPOTRF
```

SYNOPSIS

```
SUBROUTINE SPOCON(UPLO, N, A, LDA, ANORM, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, INFO
INTEGER WORK2(*)
REAL ANORM, RCOND
REAL A(LDA,*), WORK(*)

SUBROUTINE SPOCON_64(UPLO, N, A, LDA, ANORM, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, INFO
INTEGER*8 WORK2(*)
REAL ANORM, RCOND
REAL A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE POCON(UPLO, [N], A, [LDA], ANORM, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL :: ANORM, RCOND
```

```
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE POCON_64(UPLO, [N], A, [LDA], ANORM, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
void spocon(char uplo, int  n,  float  *a,  int  lda,  float
          anorm, float *rcond, int *info);

void spocon_64(char uplo, long n, float *a, long lda,  float
          anorm, float *rcond, long *info);
```

PURPOSE

spocon estimates the reciprocal of the condition number (in the 1-norm) of a real symmetric positive definite matrix using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPOTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T, as computed by SPOTRF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

ANORM (input)
The 1-norm (or infinity-norm) of the symmetric matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
spoequ - compute row and column scalings intended to equili-
brate  a symmetric positive definite matrix A and reduce its
condition number (with respect to the two-norm)
```

SYNOPSIS

```
SUBROUTINE SPOEQU(N, A, LDA, SCALE, SCOND, AMAX, INFO)

INTEGER N, LDA, INFO
REAL SCOND, AMAX
REAL A(LDA,*), SCALE(*)

SUBROUTINE SPOEQU_64(N, A, LDA, SCALE, SCOND, AMAX, INFO)

INTEGER*8 N, LDA, INFO
REAL SCOND, AMAX
REAL A(LDA,*), SCALE(*)
```

F95 INTERFACE

```
SUBROUTINE POEQU([N], A, [LDA], SCALE, SCOND, AMAX, [INFO])

INTEGER :: N, LDA, INFO
REAL :: SCOND, AMAX
REAL, DIMENSION(:) :: SCALE
REAL, DIMENSION(:,:) :: A

SUBROUTINE POEQU_64([N], A, [LDA], SCALE, SCOND, AMAX, [INFO])

INTEGER(8) :: N, LDA, INFO
REAL :: SCOND, AMAX
REAL, DIMENSION(:) :: SCALE
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void spoequ(int n, float *a, int lda,  float  *scale,  float
          *scond, float *amax, int *info);

void spoequ_64(long n, float *a,  long  lda,  float  *scale,
          float *scond, float *amax, long *info);
```

PURPOSE

spoequ computes row and column scalings intended to equilibrate a symmetric positive definite matrix A and reduce its condition number (with respect to the two-norm). S contains the scale factors, S(i) = 1/sqrt(A(i,i)), chosen so that the scaled matrix B with elements B(i,j) = S(i)*A(i,j)*S(j) has ones on the diagonal. This choice of S puts the condition number of B within a factor N of the smallest possible condition number over all possible diagonal scalings.

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

A (input) The N-by-N symmetric positive definite matrix whose scaling factors are to be computed. Only the diagonal elements of A are referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

SCALE (output)
If INFO = 0, SCALE contains the scale factors for A.

SCOND (output)
If INFO = 0, SCALE contains the ratio of the smallest SCALE(i) to the largest SCALE(i). If SCOND >= 0.1 and AMAX is neither too large nor too small, it is not worth scaling by SCALE.

AMAX (output)
Absolute value of largest matrix element. If AMAX is very close to overflow or very close to underflow, the matrix should be scaled.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the i-th diagonal element is nonpositive.

Contents

NAME

```
sporfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is symmetric positive
definite,
```

SYNOPSIS

```
SUBROUTINE SPORFS(UPLO, N, NRHS, A, LDA, AF, LDAF, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER WORK2(*)
REAL  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),  FERR(*),
BERR(*), WORK(*)

SUBROUTINE SPORFS_64(UPLO, N, NRHS, A, LDA, AF, LDAF, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 WORK2(*)
REAL  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),  FERR(*),
BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PORFS(UPLO, [N], [NRHS], A, [LDA], AF, [LDAF], B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL, DIMENSION(:) :: FERR, BERR, WORK
```

```
REAL, DIMENSION(:,:) :: A, AF, B, X

SUBROUTINE PORFS_64(UPLO, [N], [NRHS], A, [LDA], AF, [LDAF], B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL, DIMENSION(:) :: FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
#include <sunperf.h>
void sporfs(char uplo, int n, int nrhs, float *a,  int  lda,
          float  *af, int ldaf, float *b, int ldb, float *x,
          int ldx, float *ferr, float *berr, int *info);

void sporfs_64(char uplo, long n, long nrhs, float *a,  long
          lda,  float  *af,  long  ldaf, float *b, long ldb,
          float *x, long ldx, float *ferr, float *berr, long
          *info);
```

PURPOSE

sporfs improves the computed solution to a system of linear equations when the coefficient matrix is symmetric positive definite, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular

part of the matrix A, and the strictly upper tri-
angular part of A is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input)
The triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T, as computed by SPOTRF.

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by SPOTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)

```
            dimension(N)

INFO (output)
            = 0:  successful exit
            < 0:  if INFO =  -i,  the  i-th  argument  had  an
            illegal value
```

Contents

NAME

```
sposv - compute the solution to  a  real  system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE SPOSV(UPLO, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDA, LDB, INFO
REAL A(LDA,*), B(LDB,*)

SUBROUTINE SPOSV_64(UPLO, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDA, LDB, INFO
REAL A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE POSV(UPLO, [N], [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDA, LDB, INFO
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE POSV_64(UPLO, [N], [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>
```

```
void sposv(char uplo, int n, int nrhs, float  *a,  int  lda,
          float *b, int ldb, int *info);

void sposv_64(char uplo, long n, long nrhs, float  *a,  long
          lda, float *b, long ldb, long *info);
```

PURPOSE

sposv computes the solution to a real system of linear equations
   A * X = B, where A is an N-by-N symmetric positive definite matrix and X and B are N-by-NRHS matrices.

The Cholesky decomposition is used to factor A as
   A = U**T* U, if UPLO = 'U', or
   A = L * L**T, if UPLO = 'L',
where U is an upper triangular matrix and L is a lower triangular matrix. The factored form of A is then used to solve the system of equations A * X = B.

ARGUMENTS

UPLO (input)
          = 'U': Upper triangle of A is stored;
          = 'L': Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NRHS (input)
          The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input/output)
          On entry, the symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

          On exit, if INFO = 0, the factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed.

Contents

NAME

```
sposvx - use the Cholesky factorization A = U**T*U  or  A  =
L*L**T  to  compute  the solution to a real system of linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE SPOSVX(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, EQUED,
      S, B, LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO, EQUED
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER WORK2(*)
REAL RCOND
REAL  A(LDA,*),  AF(LDAF,*),   S(*),   B(LDB,*),   X(LDX,*),
FERR(*), BERR(*), WORK(*)

SUBROUTINE SPOSVX_64(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, EQUED,
      S, B, LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO, EQUED
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 WORK2(*)
REAL RCOND
REAL  A(LDA,*),  AF(LDAF,*),   S(*),   B(LDB,*),   X(LDX,*),
FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE POSVX(FACT, UPLO, [N], [NRHS], A, [LDA], AF, [LDAF],
       EQUED, S, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
```

```
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL :: RCOND
REAL, DIMENSION(:) :: S, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, AF, B, X

SUBROUTINE POSVX_64(FACT, UPLO, [N], [NRHS], A, [LDA], AF, [LDAF],
       EQUED, S, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL :: RCOND
REAL, DIMENSION(:) :: S, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
#include <sunperf.h>

void sposvx(char fact, char uplo, int n, int nrhs, float *a,
          int  lda,  float  *af, int ldaf, char equed, float
          *s, float *b, int ldb, float *x,  int  ldx,  float
          *rcond, float *ferr, float *berr, int *info);

void sposvx_64(char fact, char  uplo,  long  n,  long  nrhs,
          float  *a,  long  lda,  float *af, long ldaf, char
          equed, float *s, float *b,  long  ldb,  float  *x,
          long  ldx, float *rcond, float *ferr, float *berr,
          long *info);
```

PURPOSE

```
sposvx uses the Cholesky factorization A =  U**T*U  or  A  =
L*L**T  to  compute  the solution to a real system of linear
equations
   A * X = B, where A is an N-by-N symmetric positive defin-
ite matrix and X and B are N-by-NRHS matrices.

Error bounds on the solution and a  condition  estimate  are
also provided.

The following steps are performed:

1. If FACT = 'E',  real  scaling  factors  are  computed  to
equilibrate
   the system:
      diag(S) * A * diag(S) * inv(diag(S)) * X = diag(S) * B
   Whether or not the system will be equilibrated depends on
the
```

scaling of the matrix A, but if equilibration is used, A is
overwritten by diag(S)*A*diag(S) and B by diag(S)*B.

2. If FACT = 'N' or 'E', the Cholesky decomposition is used to
factor the matrix A (after equilibration if FACT = 'E') as
A = U**T* U, if UPLO = 'U', or
A = L * L**T, if UPLO = 'L',
where U is an upper triangular matrix and L is a lower triangular
matrix.

3. If the leading i-by-i principal minor is not positive definite,
then the routine returns with INFO = i. Otherwise, the factored
form of A is used to estimate the condition number of the matrix
A. If the reciprocal of the condition number is less than machine
precision, INFO = N+1 is returned as a warning, but the routine
still goes on to solve for X and compute error bounds as
described below.

4. The system of equations is solved for X using the factored form
of A.

5. Iterative refinement is applied to improve the computed solution
matrix and calculate error bounds and backward error estimates
for it.

6. If equilibration was used, the matrix X is premultiplied by
diag(S) so that it solves the original system before
equilibration.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is

```
            factored.   = 'F':  On entry, AF contains the fac-
            tored form of A.  If EQUED = 'Y', the matrix A has
            been equilibrated with scaling factors given by S.
            A and AF will not be modified.  = 'N':  The matrix
            A will be copied to AF and factored.
            = 'E':  The  matrix  A  will  be  equilibrated  if
            necessary, then copied to AF and factored.

UPLO (input)
            = 'U':  Upper triangle of A is stored;
            = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
            the matrix A.  N >= 0.

NRHS (input)
            The number of right hand sides, i.e.,  the  number
            of columns of the matrices B and X.  NRHS >= 0.
A (input/output)
            On entry, the symmetric matrix A, except if FACT =
            'F'  and  EQUED  =  'Y',  then  A must contain the
            equilibrated matrix diag(S)*A*diag(S).  If UPLO  =
            'U', the leading N-by-N upper triangular part of A
            contains the upper triangular part of  the  matrix
            A,  and the strictly lower triangular part of A is
            not referenced.  If UPLO = 'L', the leading N-by-N
            lower triangular part of A contains the lower tri-
            angular part of the matrix  A,  and  the  strictly
            upper  triangular  part of A is not referenced.  A
            is not modified if FACT = 'F' or 'N', or if FACT =
            'E' and EQUED = 'N' on exit.

            On exit, if FACT = 'E'  and  EQUED  =  'Y',  A  is
            overwritten by diag(S)*A*diag(S).

LDA (input)
            The leading dimension of  the  array  A.   LDA  >=
            max(1,N).

AF (input/output)
            If FACT = 'F', then AF is an input argument and on
            entry  contains  the triangular factor U or L from
            the Cholesky factorization  A  =  U**T*U  or  A  =
            L*L**T, in the same storage format as A.  If EQUED
            .ne. 'N', then AF is  the  factored  form  of  the
            equilibrated matrix diag(S)*A*diag(S).

            If FACT = 'N', then AF is an output  argument  and
            on  exit returns the triangular factor U or L from
            the Cholesky factorization  A  =  U**T*U  or  A  =
```

```
          L*L**T of the original matrix A.

          If FACT = 'E', then AF is an output  argument  and
          on  exit returns the triangular factor U or L from
          the Cholesky factorization  A  =  U**T*U  or  A  =
          L*L**T  of  the  equilibrated  matrix  A  (see the
          description of A for the form of the  equilibrated
          matrix).

LDAF (input)
          The leading dimension of the array  AF.   LDAF  >=
          max(1,N).

EQUED (input)
          Specifies the form of equilibration that was done.
          =  'N':   No  equilibration (always true if FACT =
          'N').
          = 'Y':  Equilibration was done, i.e., A  has  been
          replaced  by  diag(S)  * A * diag(S).  EQUED is an
          input argument if FACT = 'F'; otherwise, it is  an
          output argument.

S (input/output)
          The scale factors for A; not accessed if  EQUED  =
          'N'.  S is an input argument if FACT = 'F'; other-
          wise, S is an output argument.  If FACT = 'F'  and
          EQUED = 'Y', each element of S must be positive.

B (input/output)
          On entry, the N-by-NRHS right hand side matrix  B.
          On  exit,  if  EQUED  = 'N', B is not modified; if
          EQUED = 'Y', B is overwritten by diag(S) * B.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

X (output)
          If INFO = 0 or INFO = N+1, the N-by-NRHS  solution
          matrix  X  to  the  original  system of equations.
          Note that if EQUED = 'Y', A and B are modified  on
          exit,  and the solution to the equilibrated system
          is inv(diag(S))*X.

LDX (input)
          The leading dimension of  the  array  X.   LDX  >=
          max(1,N).

RCOND (output)
          The estimate of the reciprocal condition number of
```

the matrix A after equilibration (if done). If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
spotf2 - compute the Cholesky factorization of a  real  sym-
metric positive definite matrix A
```

SYNOPSIS

```
SUBROUTINE SPOTF2(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, INFO
REAL A(LDA,*)

SUBROUTINE SPOTF2_64(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, INFO
REAL A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE POTF2(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, INFO
REAL, DIMENSION(:,:) :: A

SUBROUTINE POTF2_64(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, INFO
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void spotf2(char uplo, int n, float *a, int lda, int *info);

void spotf2_64(char uplo, long n, float *a, long  lda,  long
          *info);
```

PURPOSE

spotf2 computes the Cholesky factorization of a real symmetric positive definite matrix A.

The factorization has the form
   A = U' * U , if UPLO = 'U', or
   A = L * L', if UPLO = 'L',
where U is an upper triangular matrix and L is lower triangular.

This is the unblocked version of the algorithm, calling Level 2 BLAS.

ARGUMENTS

UPLO (input)
Specifies whether the upper or lower triangular part of the symmetric matrix A is stored. = 'U': Upper triangular
= 'L': Lower triangular

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the symmetric matrix A. If UPLO = 'U', the leading n by n upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading n by n lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, if INFO = 0, the factor U or L from the Cholesky factorization A = U'*U or A = L*L'.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

```
INFO (output)
          = 0: successful exit
          < 0: if INFO = -k, the k-th argument had an  ille-
          gal value
          > 0: if INFO = k, the leading minor of order k  is
          not positive definite, and the factorization could
          not be completed.
```

Contents

NAME

spotrf - compute the Cholesky factorization of a real symmetric positive definite matrix A

SYNOPSIS

```
SUBROUTINE SPOTRF(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, INFO
REAL A(LDA,*)

SUBROUTINE SPOTRF_64(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, INFO
REAL A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE POTRF(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, INFO
REAL, DIMENSION(:,:) :: A

SUBROUTINE POTRF_64(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, INFO
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void spotrf(char uplo, int n, float *a, int lda, int *info);

void spotrf_64(char uplo, long n, float *a, long  lda,  long
          *info);
```

PURPOSE

```
spotrf computes the Cholesky factorization of  a  real  sym-
metric positive definite matrix A.

The factorization has the form
   A = U**T * U,  if UPLO = 'U', or
   A = L  * L**T,  if UPLO = 'L',
where U is  an  upper  triangular  matrix  and  L  is  lower
triangular.

This is the block version of the algorithm, calling Level  3
BLAS.
```

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the symmetric matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A,
          and the strictly lower triangular part of A is not
          referenced.  If UPLO =  'L',  the  leading  N-by-N
          lower triangular part of A contains the lower tri-
          angular part of the matrix  A,  and  the  strictly
          upper triangular part of A is not referenced.

          On exit, if INFO = 0, the factor U or L  from  the
          Cholesky factorization A = U**T*U or A = L*L**T.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

INFO (output)
          = 0:  successful exit
```

< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i is not positive definite, and the factorization could not be completed.

Contents

NAME

```
spotri - compute the inverse of a  real  symmetric  positive
definite  matrix  A  using  the  Cholesky  factorization A =
U**T*U or A = L*L**T computed by SPOTRF
```

SYNOPSIS

```
SUBROUTINE SPOTRI(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, INFO
REAL A(LDA,*)

SUBROUTINE SPOTRI_64(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, INFO
REAL A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE POTRI(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, INFO
REAL, DIMENSION(:,:) :: A

SUBROUTINE POTRI_64(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, INFO
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void spotri(char uplo, int n, float *a, int lda, int *info);

void spotri_64(char uplo, long n, float *a, long  lda,  long
          *info);
```

PURPOSE

spotri computes the inverse of a real symmetric positive definite matrix A using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPOTRF.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T, as computed by SPOTRF.   On exit, the upper or lower triangle of the (symmetric) inverse of A, overwriting the input factor U or L.

LDA (input)
The leading dimension of the array A.   LDA >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the (i,i) element of the factor U or L is zero, and the inverse could not be computed.

Contents

NAME

```
spotrs - solve a system of linear equations A*X = B  with  a
symmetric positive definite matrix A using the Cholesky fac-
torization A = U**T*U or A = L*L**T computed by SPOTRF
```

SYNOPSIS

```
SUBROUTINE SPOTRS(UPLO, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDA, LDB, INFO
REAL A(LDA,*), B(LDB,*)

SUBROUTINE SPOTRS_64(UPLO, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDA, LDB, INFO
REAL A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE POTRS(UPLO, [N], [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDA, LDB, INFO
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE POTRS_64(UPLO, [N], [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void spotrs(char uplo, int n, int nrhs, float *a,  int  lda,
          float *b, int ldb, int *info);

void spotrs_64(char uplo, long n, long nrhs, float *a,  long
          lda, float *b, long ldb, long *info);
```

PURPOSE

spotrs solves a system of linear equations A*X = B with a symmetric positive definite matrix A using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPOTRF.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T, as computed by SPOTRF.

LDA (input)
The leading dimension of the array A.   LDA >= max(1,N).

B (input/output)
On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B.   LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

sppcon - estimate the reciprocal of the condition number (in the 1-norm) of a real symmetric positive definite packed matrix using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPPTRF

SYNOPSIS

```
SUBROUTINE SPPCON(UPLO, N, A, ANORM, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER N, INFO
INTEGER WORK2(*)
REAL ANORM, RCOND
REAL A(*), WORK(*)

SUBROUTINE SPPCON_64(UPLO, N, A, ANORM, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, INFO
INTEGER*8 WORK2(*)
REAL ANORM, RCOND
REAL A(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PPCON(UPLO, N, A, ANORM, RCOND, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: A, WORK
```

```
SUBROUTINE PPCON_64(UPLO, N, A, ANORM, RCOND, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: A, WORK
```

C INTERFACE

```
#include <sunperf.h>

void sppcon(char uplo, int n, float *a, float  anorm,  float
          *rcond, int *info);
void sppcon_64(char uplo, long n,  float  *a,  float  anorm,
          float *rcond, long *info);
```

PURPOSE

sppcon estimates the reciprocal of the condition number (in the 1-norm) of a real symmetric positive definite packed matrix using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPPTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T, packed columnwise in a linear array. The j-th column of U or L is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = U(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = L(i,j) for j<=i<=n.

ANORM (input)
The 1-norm (or infinity-norm) of the symmetric matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

sppequ - compute row and column scalings intended to equilibrate a symmetric positive definite matrix A in packed storage and reduce its condition number (with respect to the two-norm)

SYNOPSIS

```
SUBROUTINE SPPEQU(UPLO, N, A, SCALE, SCOND, AMAX, INFO)

CHARACTER * 1 UPLO
INTEGER N, INFO
REAL SCOND, AMAX
REAL A(*), SCALE(*)

SUBROUTINE SPPEQU_64(UPLO, N, A, SCALE, SCOND, AMAX, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, INFO
REAL SCOND, AMAX
REAL A(*), SCALE(*)
```

F95 INTERFACE

```
SUBROUTINE PPEQU(UPLO, [N], A, SCALE, SCOND, AMAX, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INFO
REAL :: SCOND, AMAX
REAL, DIMENSION(:) :: A, SCALE

SUBROUTINE PPEQU_64(UPLO, [N], A, SCALE, SCOND, AMAX, [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
INTEGER(8) :: N, INFO
REAL :: SCOND, AMAX
REAL, DIMENSION(:) :: A, SCALE
```

C INTERFACE

```
#include <sunperf.h>

void sppequ(char uplo, int n, float *a, float *scale,  float
          *scond, float *amax, int *info);

void sppequ_64(char uplo, long n, float  *a,  float  *scale,
          float *scond, float *amax, long *info);
```

PURPOSE

sppequ computes row and column scalings intended to equilibrate a symmetric positive definite matrix A in packed storage and reduce its condition number (with respect to the two-norm). S contains the scale factors, S(i)=1/sqrt(A(i,i)), chosen so that the scaled matrix B with elements B(i,j)=S(i)*A(i,j)*S(j) has ones on the diagonal. This choice of S puts the condition number of B within a factor N of the smallest possible condition number over all possible diagonal scalings.

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

A (input) The upper  or  lower  triangle  of  the  symmetric
          matrix  A,  packed  columnwise  in a linear array.
          The j-th column of A is stored in the array  A  as
          follows:  if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j)
          for 1<=i<=j; if UPLO = 'L', A(i +  (j-1)*(2n-j)/2)
          = A(i,j) for j<=i<=n.

SCALE (output)
          If INFO = 0, SCALE contains the scale factors  for
          A.

SCOND (output)
          If INFO = 0, SCALE contains the ratio of the smal-
          lest  SCALE(i)  to the largest SCALE(i).  If SCOND
```

>= 0.1 and AMAX is neither too large nor too small, it is not worth scaling by SCALE.

AMAX (output)
Absolute value of largest matrix element. If AMAX is very close to overflow or very close to underflow, the matrix should be scaled.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the i-th diagonal element is nonpositive.

Contents

NAME

```
spprfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is symmetric positive
definite and packed, and provides error bounds and  backward
error estimates for the solution
```

SYNOPSIS

```
SUBROUTINE SPPRFS(UPLO, N, NRHS, A, AF, B, LDB, X, LDX, FERR, BERR,
      WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER WORK2(*)
REAL A(*),  AF(*),  B(LDB,*),  X(LDX,*),  FERR(*),  BERR(*),
WORK(*)

SUBROUTINE SPPRFS_64(UPLO, N, NRHS, A, AF, B, LDB, X, LDX, FERR,
      BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 WORK2(*)
REAL A(*),  AF(*),  B(LDB,*),  X(LDX,*),  FERR(*),  BERR(*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PPRFS(UPLO, N, [NRHS], A, AF, B, [LDB], X, [LDX], FERR,
       BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: WORK2
```

```
REAL, DIMENSION(:) :: A, AF, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: B, X

SUBROUTINE PPRFS_64(UPLO, N, [NRHS], A, AF, B, [LDB], X, [LDX], FERR,
       BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL, DIMENSION(:) :: A, AF, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: B, X
```

C INTERFACE

```
#include <sunperf.h>
void spprfs(char uplo, int n, int nrhs, float *a, float *af,
          float *b, int ldb, float *x, int ldx, float *ferr,
          float *berr, int *info);

void spprfs_64(char uplo, long n, long nrhs, float *a, float
          *af, float *b, long ldb, float *x, long ldx, float
          *ferr, float *berr, long *info);
```

PURPOSE

spprfs improves the computed solution to a system of linear equations when the coefficient matrix is symmetric positive definite and packed, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n.

AF (input)
The triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T, as computed by SPPTRF/CPPTRF, packed columnwise in a linear array in the same format as A (see A).

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by SPPTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
sppsv - compute the solution to  a  real  system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE SPPSV(UPLO, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDB, INFO
REAL A(*), B(LDB,*)

SUBROUTINE SPPSV_64(UPLO, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDB, INFO
REAL A(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE PPSV(UPLO, N, [NRHS], A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDB, INFO
REAL, DIMENSION(:) :: A
REAL, DIMENSION(:,:) :: B

SUBROUTINE PPSV_64(UPLO, N, [NRHS], A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDB, INFO
REAL, DIMENSION(:) :: A
REAL, DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void sppsv(char uplo, int n, int nrhs, float *a,  float  *b,
          int ldb, int *info);

void sppsv_64(char uplo, long n, long nrhs, float *a,  float
          *b, long ldb, long *info);
```

PURPOSE

```
sppsv computes the solution to a real system of linear equa-
tions
   A * X = B, where A is an N-by-N symmetric positive defin-
ite matrix stored in packed format and X and B are N-by-NRHS
matrices.

The Cholesky decomposition is used to factor A as
   A = U**T* U,  if UPLO = 'U', or
   A = L * L**T,  if UPLO = 'L',
where U is an upper triangular matrix and L is a lower  tri-
angular  matrix.   The  factored  form  of A is then used to
solve the system of equations A * X = B.
```

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

A (input/output)
          On entry, the upper or lower triangle of the  sym-
          metric  matrix  A,  packed  columnwise in a linear
          array.  The j-th column of  A  is  stored  in  the
          array  A  as  follows:   if  UPLO = 'U', A(i + (j-
          1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i +
          (j-1)*(2n-j)/2)  =  A(i,j) for j<=i<=n.  See below
          for further details.
```

On exit, if INFO = 0, the factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T, in the same storage format as A.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed.

FURTHER DETAILS

The packed storage scheme is illustrated by the following example when N = 4, UPLO = 'U':

Two-dimensional storage of the symmetric matrix A:

```
   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = conjg(aji))
               a44
```

Packed storage of the upper triangle of A:

A = [a11, a12, a22, a13, a23, a33, a14, a24, a34, a44]

Contents

NAME

```
sppsvx - use the Cholesky factorization A = U**T*U  or  A  =
L*L**T  to  compute  the solution to a real system of linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE SPPSVX(FACT, UPLO, N, NRHS, A, AF, EQUED, S, B, LDB,
      X, LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO, EQUED
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER WORK2(*)
REAL RCOND
REAL  A(*),  AF(*),  S(*),  B(LDB,*),   X(LDX,*),   FERR(*),
BERR(*), WORK(*)

SUBROUTINE SPPSVX_64(FACT, UPLO, N, NRHS, A, AF, EQUED, S, B,
      LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO, EQUED
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 WORK2(*)
REAL RCOND
REAL  A(*),  AF(*),  S(*),  B(LDB,*),   X(LDX,*),   FERR(*),
BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PPSVX(FACT, UPLO, [N], [NRHS], A, AF, EQUED, S, B,
       [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
```

```
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL :: RCOND
REAL, DIMENSION(:) :: A, AF, S, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: B, X

SUBROUTINE PPSVX_64(FACT, UPLO, [N], [NRHS], A, AF, EQUED, S, B,
       [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL :: RCOND
REAL, DIMENSION(:) :: A, AF, S, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: B, X
```

C INTERFACE

```
#include <sunperf.h>

void sppsvx(char fact, char uplo, int n, int nrhs, float *a,
          float  *af,  char  equed,  float *s, float *b, int
          ldb, float *x, int ldx, float *rcond, float *ferr,
          float *berr, int *info);

void sppsvx_64(char fact, char  uplo,  long  n,  long  nrhs,
          float  *a,  float *af, char equed, float *s, float
          *b, long ldb, float *x, long  ldx,  float  *rcond,
          float *ferr, float *berr, long *info);
```

PURPOSE

```
sppsvx uses the Cholesky factorization A =  U**T*U  or  A  =
L*L**T  to  compute  the solution to a real system of linear
equations
   A * X = B, where A is an N-by-N symmetric positive defin-
ite matrix stored in packed format and X and B are N-by-NRHS
matrices.

Error bounds on the solution and a  condition  estimate  are
also provided.

The following steps are performed:

1. If FACT = 'E',  real  scaling  factors  are  computed  to
equilibrate
   the system:
      diag(S) * A * diag(S) * inv(diag(S)) * X = diag(S) * B
   Whether or not the system will be equilibrated depends on
the
   scaling of the matrix A, but if equilibration is used,  A
```

is
overwritten by diag(S)*A*diag(S) and B by diag(S)*B.

2. If FACT = 'N' or 'E', the Cholesky decomposition is used to
factor the matrix A (after equilibration if FACT = 'E') as
A = U**T* U, if UPLO = 'U', or
A = L * L**T, if UPLO = 'L',
where U is an upper triangular matrix and L is a lower triangular
matrix.

3. If the leading i-by-i principal minor is not positive definite,
then the routine returns with INFO = i. Otherwise, the factored
form of A is used to estimate the condition number of the matrix
A. If the reciprocal of the condition number is less than machine
precision, INFO = N+1 is returned as a warning, but the routine
still goes on to solve for X and compute error bounds as
described below.

4. The system of equations is solved for X using the factored form
of A.

5. Iterative refinement is applied to improve the computed solution
matrix and calculate error bounds and backward error estimates
for it.

6. If equilibration was used, the matrix X is premultiplied by
diag(S) so that it solves the original system before
equilibration.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored. = 'F': On entry, AF contains the fac-

```
          tored form of A.  If EQUED = 'Y', the matrix A has
          been equilibrated with scaling factors given by S.
          A and AF will not be modified.  = 'N':  The matrix
          A will be copied to AF and factored.
          = 'E':  The  matrix  A  will  be  equilibrated  if
          necessary, then copied to AF and factored.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.
A (input/output)
          On entry, the upper or lower triangle of the  sym-
          metric  matrix  A,  packed  columnwise in a linear
          array, except if FACT = 'F' and EQUED = 'Y',  then
          A    must    contain   the   equilibrated   matrix
          diag(S)*A*diag(S).  The j-th column of A is stored
          in  the  array A as follows:  if UPLO = 'U', A(i +
          (j-1)*j/2) = A(i,j) for 1<=i<=j; if  UPLO  =  'L',
          A(i  +  (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n.  See
          below for further details.  A is not  modified  if
          FACT  =  'F'  or 'N', or if FACT = 'E' and EQUED =
          'N' on exit.

          On exit, if FACT = 'E'  and  EQUED  =  'Y',  A  is
          overwritten by diag(S)*A*diag(S).

AF (input/output)
          (N*(N+1)/2) If FACT = 'F', then  AF  is  an  input
          argument and on entry contains the triangular fac-
          tor U or L from the  Cholesky  factorization  A  =
          U'*U or A = L*L', in the same storage format as A.
          If EQUED .ne. 'N', then AF is the factored form of
          the equilibrated matrix A.

          If FACT = 'N', then AF is an output  argument  and
          on  exit returns the triangular factor U or L from
          the Cholesky factorization A = U'*U or A = L*L' of
          the original matrix A.

          If FACT = 'E', then AF is an output  argument  and
          on  exit returns the triangular factor U or L from
          the Cholesky factorization A = U'*U or A = L*L' of
          the  equilibrated matrix A (see the description of
```

A for the form of the equilibrated matrix).

EQUED (input)
Specifies the form of equilibration that was done.
= 'N': No equilibration (always true if FACT = 'N').
= 'Y': Equilibration was done, i.e., A has been replaced by diag(S) * A * diag(S). EQUED is an input argument if FACT = 'F'; otherwise, it is an output argument.

S (input/output)
The scale factors for A; not accessed if EQUED = 'N'. S is an input argument if FACT = 'F'; otherwise, S is an output argument. If FACT = 'F' and EQUED = 'Y', each element of S must be positive.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if EQUED = 'N', B is not modified; if EQUED = 'Y', B is overwritten by diag(S) * B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X to the original system of equations. Note that if EQUED = 'Y', A and B are modified on exit, and the solution to the equilibrated system is inv(diag(S))*X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A after equilibration (if done). If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest ele-

ment in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

FURTHER DETAILS

The packed storage scheme is illustrated by the following example when N = 4, UPLO = 'U':

Two-dimensional storage of the symmetric matrix A:

```
   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = conjg(aji))
               a44
```

Packed storage of the upper triangle of A:

A = [a11, a12, a22, a13, a23, a33, a14, a24, a34, a44]

Contents

NAME

```
spptrf - compute the Cholesky factorization of a  real  sym-
metric positive definite matrix A stored in packed format
```

SYNOPSIS

```
SUBROUTINE SPPTRF(UPLO, N, A, INFO)

CHARACTER * 1 UPLO
INTEGER N, INFO
REAL A(*)

SUBROUTINE SPPTRF_64(UPLO, N, A, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, INFO
REAL A(*)
```

F95 INTERFACE

```
SUBROUTINE PPTRF(UPLO, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INFO
REAL, DIMENSION(:) :: A

SUBROUTINE PPTRF_64(UPLO, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INFO
REAL, DIMENSION(:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void spptrf(char uplo, int n, float *a, int *info);

void spptrf_64(char uplo, long n, float *a, long *info);
```

PURPOSE

spptrf computes the Cholesky factorization of a real symmetric positive definite matrix A stored in packed format.

The factorization has the form
```
   A = U**T * U,  if UPLO = 'U', or
   A = L  * L**T,  if UPLO = 'L',
```
where U is an upper triangular matrix and L is lower triangular.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n. See below for further details.

On exit, if INFO = 0, the triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T, in the same storage format as A.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i is not positive definite, and the factorization could not be completed.

FURTHER DETAILS

```
The packed storage scheme is illustrated  by  the  following
example when N = 4, UPLO = 'U':

Two-dimensional storage of the symmetric matrix A:

   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = aji)
               a44

Packed storage of the upper triangle of A:

A = [ a11, a12, a22, a13, a23, a33, a14, a24, a34, a44 ]
```

Contents

NAME

sptri - compute the inverse of a real symmetric positive definite matrix A using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPPTRF

SYNOPSIS

```
SUBROUTINE SPPTRI(UPLO, N, A, INFO)

CHARACTER * 1 UPLO
INTEGER N, INFO
REAL A(*)

SUBROUTINE SPPTRI_64(UPLO, N, A, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, INFO
REAL A(*)
```

F95 INTERFACE

```
SUBROUTINE PPTRI(UPLO, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INFO
REAL, DIMENSION(:) :: A

SUBROUTINE PPTRI_64(UPLO, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INFO
REAL, DIMENSION(:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void spptri(char uplo, int n, float *a, int *info);

void spptri_64(char uplo, long n, float *a, long *info);
```

PURPOSE

spptri computes the inverse of a real symmetric positive definite matrix A using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPPTRF.

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangular factor is stored in A;
          = 'L':  Lower triangular factor is stored in A.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the triangular factor U or  L  from  the
          Cholesky  factorization  A = U**T*U or A = L*L**T,
          packed columnwise as a  linear  array.   The  j-th
          column  of U or L is stored in the array A as fol-
          lows:  if UPLO = 'U', A(i +  (j-1)*j/2)  =  U(i,j)
          for  1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2)
          = L(i,j) for j<=i<=n.

          On exit, the upper or lower triangle of the  (sym-
          metric) inverse of A, overwriting the input factor
          U or L.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, the (i,i) element of the factor
          U  or L is zero, and the inverse could not be com-
          puted.
```

Contents

NAME

```
spptrs - solve a system of linear equations A*X = B  with  a
symmetric positive definite matrix A in packed storage using
the Cholesky factorization A = U**T*U or A = L*L**T computed
by SPPTRF
```

SYNOPSIS

```
SUBROUTINE SPPTRS(UPLO, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDB, INFO
REAL A(*), B(LDB,*)

SUBROUTINE SPPTRS_64(UPLO, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDB, INFO
REAL A(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE PPTRS(UPLO, N, [NRHS], A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDB, INFO
REAL, DIMENSION(:) :: A
REAL, DIMENSION(:,:) :: B

SUBROUTINE PPTRS_64(UPLO, N, [NRHS], A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDB, INFO
REAL, DIMENSION(:) :: A
```

```
REAL, DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void spptrs(char uplo, int n, int nrhs, float *a, float  *b,
          int ldb, int *info);

void spptrs_64(char uplo, long n, long nrhs, float *a, float
          *b, long ldb, long *info);
```

PURPOSE

spptrs solves a system of linear equations A*X = B with a symmetric positive definite matrix A in packed storage using the Cholesky factorization A = U**T*U or A = L*L**T computed by SPPTRF.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**T*U or A = L*L**T, packed columnwise in a linear array. The j-th column of U or L is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = U(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = L(i,j) for j<=i<=n.

B (input/output)
On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B.   LDB >= max(1,N).

INFO (output)

= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
sptcon - compute the reciprocal of the condition number  (in
the  1-norm) of a real symmetric positive definite tridiago-
nal matrix using the factorization  A  =  L*D*L**T  or  A  =
U**T*D*U computed by SPTTRF
```

SYNOPSIS

```
SUBROUTINE SPTCON(N, DIAG, OFFD, ANORM, RCOND, WORK, INFO)

INTEGER N, INFO
REAL ANORM, RCOND
REAL DIAG(*), OFFD(*), WORK(*)

SUBROUTINE SPTCON_64(N, DIAG, OFFD, ANORM, RCOND, WORK, INFO)

INTEGER*8 N, INFO
REAL ANORM, RCOND
REAL DIAG(*), OFFD(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PTCON([N], DIAG, OFFD, ANORM, RCOND, [WORK], [INFO])

INTEGER :: N, INFO
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: DIAG, OFFD, WORK

SUBROUTINE PTCON_64([N], DIAG, OFFD, ANORM, RCOND, [WORK], [INFO])

INTEGER(8) :: N, INFO
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: DIAG, OFFD, WORK
```

C INTERFACE

```
#include <sunperf.h>

void sptcon(int n, float *diag, float  *offd,  float  anorm,
          float *rcond, int *info);

void sptcon_64(long  n,  float  *diag,  float  *offd,  float
          anorm, float *rcond, long *info);
```

PURPOSE

sptcon computes the reciprocal of the condition number (in the 1-norm) of a real symmetric positive definite tridiagonal matrix using the factorization A = L*D*L**T or A = U**T*D*U computed by SPTTRF.
Norm(inv(A)) is computed by a direct method, and the reciprocal of the condition number is computed as

RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

DIAG (input)
The n diagonal elements of the diagonal matrix DIAG from the factorization of A, as computed by SPTTRF.

OFFD (input)
The (n-1) off-diagonal elements of the unit bidiagonal factor U or L from the factorization of A, as computed by SPTTRF.

ANORM (input)
The 1-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(N)

INFO (output)

= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The method used is described in Nicholas J. Higham, "Efficient Algorithms for Computing the Condition Number of a Tridiagonal Matrix", SIAM J. Sci. Stat. Comput., Vol. 7, No. 1, January 1986.

Contents

NAME

```
spteqr - compute all eigenvalues and, optionally,  eigenvec-
tors  of a symmetric positive definite tridiagonal matrix by
first factoring the matrix using SPTTRF,  and  then  calling
SBDSQR to compute the singular values of the bidiagonal fac-
tor
```

SYNOPSIS

```
SUBROUTINE SPTEQR(COMPZ, N, D, E, Z, LDZ, WORK, INFO)

CHARACTER * 1 COMPZ
INTEGER N, LDZ, INFO
REAL D(*), E(*), Z(LDZ,*), WORK(*)

SUBROUTINE SPTEQR_64(COMPZ, N, D, E, Z, LDZ, WORK, INFO)

CHARACTER * 1 COMPZ
INTEGER*8 N, LDZ, INFO
REAL D(*), E(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PTEQR(COMPZ, [N], D, E, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
INTEGER :: N, LDZ, INFO
REAL, DIMENSION(:) :: D, E, WORK
REAL, DIMENSION(:,:) :: Z

SUBROUTINE PTEQR_64(COMPZ, [N], D, E, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
INTEGER(8) :: N, LDZ, INFO
```

```
REAL, DIMENSION(:) :: D, E, WORK
REAL, DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void spteqr(char compz, int n, float *d, float *e, float *z,
          int ldz, int *info);

void spteqr_64(char compz, long n, float *d, float *e, float
          *z, long ldz, long *info);
```

PURPOSE

spteqr computes all eigenvalues and, optionally, eigenvectors of a symmetric positive definite tridiagonal matrix by first factoring the matrix using SPTTRF, and then calling SBDSQR to compute the singular values of the bidiagonal factor.

This routine computes the eigenvalues of the positive definite tridiagonal matrix to high relative accuracy. This means that if the eigenvalues range over many orders of magnitude in size, then the small eigenvalues and corresponding eigenvectors will be computed more accurately than, for example, with the standard QR method.

The eigenvectors of a full or band symmetric positive definite matrix can also be found if SSYTRD, SSPTRD, or SSBTRD has been used to reduce this matrix to tridiagonal form. (The reduction to tridiagonal form, however, may preclude the possibility of obtaining high relative accuracy in the small eigenvalues of the original matrix, if these eigenvalues range over many orders of magnitude.)

ARGUMENTS

COMPZ (input)
= 'N': Compute eigenvalues only.
= 'V': Compute eigenvectors of original symmetric matrix also. Array Z contains the orthogonal matrix used to reduce the original matrix to tridiagonal form. = 'I': Compute eigenvectors of tridiagonal matrix also.

N (input) The order of the matrix. N >= 0.

D (input/output)
On entry, the n diagonal elements of the tridiagonal matrix. On normal exit, D contains the eigenvalues, in descending order.

E (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix. On exit, E has been destroyed.

Z (input) On entry, if COMPZ = 'V', the orthogonal matrix used in the reduction to tridiagonal form. On exit, if COMPZ = 'V', the orthonormal eigenvectors of the original symmetric matrix; if COMPZ = 'I', the orthonormal eigenvectors of the tridiagonal matrix. If INFO > 0 on exit, Z contains the eigenvectors associated with only the stored eigenvalues. If COMPZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if COMPZ = 'V' or 'I', LDZ >= max(1,N).

WORK (workspace)
dimension(4*N)

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, and i is: <= N the Cholesky factorization of the matrix could not be performed because the i-th principal minor was not positive definite. > N the SVD algorithm failed to converge; if INFO = N+i, i off-diagonal elements of the bidiagonal factor did not converge to zero.

Contents

NAME

sptrfs - improve the computed solution to a system of linear equations when the coefficient matrix is symmetric positive definite and tridiagonal, and provides error bounds and backward error estimates for the solution

SYNOPSIS

```
SUBROUTINE SPTRFS(N, NRHS, DIAG, OFFD, DIAGF, OFFDF, B, LDB, X, LDX,
      FERR, BERR, WORK, INFO)

INTEGER N, NRHS, LDB, LDX, INFO
REAL  DIAG(*),  OFFD(*),   DIAGF(*),   OFFDF(*),   B(LDB,*),
X(LDX,*), FERR(*), BERR(*), WORK(*)

SUBROUTINE SPTRFS_64(N, NRHS, DIAG, OFFD, DIAGF, OFFDF, B, LDB, X,
      LDX, FERR, BERR, WORK, INFO)

INTEGER*8 N, NRHS, LDB, LDX, INFO
REAL  DIAG(*),  OFFD(*),   DIAGF(*),   OFFDF(*),   B(LDB,*),
X(LDX,*), FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PTRFS([N], [NRHS], DIAG, OFFD, DIAGF, OFFDF, B, [LDB], X,
       [LDX], FERR, BERR, [WORK], [INFO])

INTEGER :: N, NRHS, LDB, LDX, INFO
REAL, DIMENSION(:) :: DIAG, OFFD, DIAGF, OFFDF, FERR,  BERR,
WORK
REAL, DIMENSION(:,:) :: B, X

SUBROUTINE PTRFS_64([N], [NRHS], DIAG, OFFD, DIAGF, OFFDF, B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [INFO])
```

```
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
REAL, DIMENSION(:) :: DIAG, OFFD, DIAGF, OFFDF, FERR,  BERR,
WORK
REAL, DIMENSION(:,:) :: B, X
```

C INTERFACE

```
#include <sunperf.h>

void sptrfs(int n, int nrhs, float *diag, float *offd, float
          *diagf, float *offdf, float *b, int ldb, float *x,
          int ldx, float *ferr, float *berr, int *info);

void sptrfs_64(long n, long nrhs, float *diag, float  *offd,
          float  *diagf,  float  *offdf, float *b, long ldb,
          float *x, long ldx, float *ferr, float *berr, long
          *info);
```

PURPOSE

sptrfs improves the computed solution to a system of linear equations when the coefficient matrix is symmetric positive definite and tridiagonal, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

DIAG (input)
The n diagonal elements of the tridiagonal matrix A.

OFFD (input)
The (n-1) subdiagonal elements of the tridiagonal matrix A.

DIAGF (input)
The n diagonal elements of the diagonal matrix DIAG from the factorization computed by SPTTRF.

OFFDF (input)
The (n-1) subdiagonal elements of the unit bidiag-

onal factor L from the factorization computed by SPTTRF.

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by SPTTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j).

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

sptsv - compute the solution to a real system of linear equations A*X = B, where A is an N-by-N symmetric positive definite tridiagonal matrix, and X and B are N-by-NRHS matrices.

SYNOPSIS

```
SUBROUTINE SPTSV(N, NRHS, DIAG, SUB, B, LDB, INFO)

INTEGER N, NRHS, LDB, INFO
REAL DIAG(*), SUB(*), B(LDB,*)

SUBROUTINE SPTSV_64(N, NRHS, DIAG, SUB, B, LDB, INFO)

INTEGER*8 N, NRHS, LDB, INFO
REAL DIAG(*), SUB(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE PTSV([N], [NRHS], DIAG, SUB, B, [LDB], [INFO])

INTEGER :: N, NRHS, LDB, INFO
REAL, DIMENSION(:) :: DIAG, SUB
REAL, DIMENSION(:,:) :: B

SUBROUTINE PTSV_64([N], [NRHS], DIAG, SUB, B, [LDB], [INFO])

INTEGER(8) :: N, NRHS, LDB, INFO
REAL, DIMENSION(:) :: DIAG, SUB
REAL, DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>
```

```
void sptsv(int n, int nrhs, float *diag, float  *sub,  float
          *b, int ldb, int *info);

void sptsv_64(long n, long nrhs, float  *diag,  float  *sub,
          float *b, long ldb, long *info);
```

PURPOSE

sptsv computes the solution to a real system of linear equations A*X = B, where A is an N-by-N symmetric positive definite tridiagonal matrix, and X and B are N-by-NRHS matrices.

A is factored as A = L*D*L**T, and the factored form of A is then used to solve the system of equations.

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

DIAG (input/output)
On entry, the n diagonal elements of the tridiagonal matrix A. On exit, the n diagonal elements of the diagonal matrix DIAG from the factorization A = L*DIAG*L**T.

SUB (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix A. On exit, the (n-1) subdiagonal elements of the unit bidiagonal factor L from the L*DIAG*L**T factorization of A. (SUB can also be regarded as the superdiagonal of the unit bidiagonal factor U from the U**T*DIAG*U factorization of A.)

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >=

max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i is not positive definite, and the solution has not been computed. The factorization has not been completed unless i = N.

Contents

NAME

```
sptsvx - use the factorization A = L*D*L**T to  compute  the
solution to a real system of linear equations A*X = B, where
A is  an  N-by-N  symmetric  positive  definite  tridiagonal
matrix and X and B are N-by-NRHS matrices
```

SYNOPSIS

```
SUBROUTINE SPTSVX(FACT, N, NRHS, DIAG, SUB, DIAGF, SUBF, B, LDB, X,
      LDX, RCOND, FERR, BERR, WORK, INFO)

CHARACTER * 1 FACT
INTEGER N, NRHS, LDB, LDX, INFO
REAL RCOND
REAL DIAG(*), SUB(*), DIAGF(*), SUBF(*), B(LDB,*), X(LDX,*),
FERR(*), BERR(*), WORK(*)

SUBROUTINE SPTSVX_64(FACT, N, NRHS, DIAG, SUB, DIAGF, SUBF, B, LDB,
      X, LDX, RCOND, FERR, BERR, WORK, INFO)

CHARACTER * 1 FACT
INTEGER*8 N, NRHS, LDB, LDX, INFO
REAL RCOND
REAL DIAG(*), SUB(*), DIAGF(*), SUBF(*), B(LDB,*), X(LDX,*),
FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PTSVX(FACT, [N], [NRHS], DIAG, SUB, DIAGF, SUBF, B, [LDB],
       X, [LDX], RCOND, FERR, BERR, [WORK], [INFO])

CHARACTER(LEN=1) :: FACT
INTEGER :: N, NRHS, LDB, LDX, INFO
REAL :: RCOND
```

```
REAL, DIMENSION(:) :: DIAG, SUB, DIAGF,  SUBF,  FERR,  BERR,
WORK
REAL, DIMENSION(:,:) :: B, X

SUBROUTINE PTSVX_64(FACT, [N], [NRHS], DIAG, SUB, DIAGF, SUBF, B,
       [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [INFO])

CHARACTER(LEN=1) :: FACT
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
REAL :: RCOND
REAL, DIMENSION(:) :: DIAG, SUB, DIAGF,  SUBF,  FERR,  BERR,
WORK
REAL, DIMENSION(:,:) :: B, X
```

C INTERFACE

```
#include <sunperf.h>

void sptsvx(char fact, int n, int nrhs, float  *diag,  float
          *sub,  float  *diagf,  float  *subf, float *b, int
          ldb, float *x, int ldx, float *rcond, float *ferr,
          float *berr, int *info);

void sptsvx_64(char fact, long n, long  nrhs,  float  *diag,
          float  *sub,  float *diagf, float *subf, float *b,
          long ldb, float *x, long ldx, float *rcond,  float
          *ferr, float *berr, long *info);
```

PURPOSE

```
sptsvx uses the factorization A = L*D*L**T  to  compute  the
solution to a real system of linear equations A*X = B, where
A is  an  N-by-N  symmetric  positive  definite  tridiagonal
matrix and X and B are N-by-NRHS matrices.

Error bounds on the solution and a  condition  estimate  are
also provided.

The following steps are performed:

1. If FACT = 'N', the matrix A is factored as A =  L*D*L**T,
where L
   is a unit lower bidiagonal matrix and D is diagonal.  The
   factorization can also be regarded as having the form
   A = U**T*D*U.

2. If the leading i-by-i principal  minor  is  not  positive
definite,
   then the routine returns with INFO =  i.  Otherwise,  the
factored
   form of A is used to estimate the condition number of the
```

```
matrix
   A.  If the reciprocal of the  condition  number  is  less
than machine
   precision, INFO = N+1 is returned as a warning,  but  the
routine
   still goes on to solve for X and compute error bounds as
   described below.

3. The system of equations is solved for X  using  the  fac-
tored form
   of A.

4. Iterative refinement is applied to improve  the  computed
solution
   matrix and calculate  error  bounds  and  backward  error
estimates
   for it.
```

ARGUMENTS

```
FACT (input)
          Specifies whether or not the factored  form  of  A
          has  been  supplied  on  entry.  = 'F':  On entry,
          DIAGF and SUBF contain the  factored  form  of  A.
          DIAG,  SUB,  DIAGF, and SUBF will not be modified.
          = 'N':  The matrix A will be copied to  DIAGF  and
          SUBF and factored.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

DIAG (input)
          The n diagonal elements of the tridiagonal  matrix
          A.

SUB (input)
          The (n-1) subdiagonal elements of the  tridiagonal
          matrix A.

DIAGF (input/output)
          If FACT = 'F', then DIAGF is an input argument and
          on  entry  contains the n diagonal elements of the
          diagonal matrix DIAG from the L*DIAG*L**T factori-
          zation of A.  If FACT = 'N', then DIAGF is an out-
          put argument and on exit contains the  n  diagonal
```

elements of the diagonal matrix DIAG from the L*DIAG*L**T factorization of A.

SUBF (input/output)
If FACT = 'F', then SUBF is an input argument and on entry contains the (n-1) subdiagonal elements of the unit bidiagonal factor L from the L*DIAG*L**T factorization of A. If FACT = 'N', then SUBF is an output argument and on exit contains the (n-1) subdiagonal elements of the unit bidiagonal factor L from the L*DIAG*L**T factorization of A.

B (input) The N-by-NRHS right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 of INFO = N+1, the N-by-NRHS solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The reciprocal condition number of the matrix A. If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j).

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, and i is

<= N: the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
spttrf - compute the L*D*L' factorization  of  a  real  sym-
metric positive definite tridiagonal matrix A
```

SYNOPSIS

```
SUBROUTINE SPTTRF(N, DIAG, OFFD, INFO)

INTEGER N, INFO
REAL DIAG(*), OFFD(*)

SUBROUTINE SPTTRF_64(N, DIAG, OFFD, INFO)

INTEGER*8 N, INFO
REAL DIAG(*), OFFD(*)
```

F95 INTERFACE

```
SUBROUTINE PTTRF([N], DIAG, OFFD, [INFO])

INTEGER :: N, INFO
REAL, DIMENSION(:) :: DIAG, OFFD

SUBROUTINE PTTRF_64([N], DIAG, OFFD, [INFO])

INTEGER(8) :: N, INFO
REAL, DIMENSION(:) :: DIAG, OFFD
```

C INTERFACE

```
#include <sunperf.h>

void spttrf(int n, float *diag, float *offd, int *info);

void  spttrf_64(long  n,  float  *diag,  float  *offd,  long
```

```
            *info);
```

PURPOSE

spttrf computes the L*D*L' factorization of a real symmetric positive definite tridiagonal matrix A. The factorization may also be regarded as having the form A = U'*D*U.

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

DIAG (input/output)
On entry, the n diagonal elements of the tridiagonal matrix A. On exit, the n diagonal elements of the diagonal matrix DIAG from the L*DIAG*L' factorization of A.

OFFD (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix A. On exit, the (n-1) subdiagonal elements of the unit bidiagonal factor L from the L*DIAG*L' factorization of A. OFFD can also be regarded as the superdiagonal of the unit bidiagonal factor U from the U'*DIAG*U factorization of A.

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, the leading minor of order k is not positive definite; if k < N, the factorization could not be completed, while if k = N, the factorization was completed, but DIAG(N) = 0.

Contents

NAME

```
spttrs - solve a tridiagonal system of the form  A * X  =  B
using the L*D*L' factorization of A computed by SPTTRF
```

SYNOPSIS

```
SUBROUTINE SPTTRS(N, NRHS, DIAG, OFFD, B, LDB, INFO)

INTEGER N, NRHS, LDB, INFO
REAL DIAG(*), OFFD(*), B(LDB,*)

SUBROUTINE SPTTRS_64(N, NRHS, DIAG, OFFD, B, LDB, INFO)

INTEGER*8 N, NRHS, LDB, INFO
REAL DIAG(*), OFFD(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE PTTRS([N], [NRHS], DIAG, OFFD, B, [LDB], [INFO])

INTEGER :: N, NRHS, LDB, INFO
REAL, DIMENSION(:) :: DIAG, OFFD
REAL, DIMENSION(:,:) :: B

SUBROUTINE PTTRS_64([N], [NRHS], DIAG, OFFD, B, [LDB], [INFO])

INTEGER(8) :: N, NRHS, LDB, INFO
REAL, DIMENSION(:) :: DIAG, OFFD
REAL, DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void spttrs(int n, int nrhs, float *diag, float *offd, float
```

```
          *b, int ldb, int *info);

void spttrs_64(long n, long nrhs, float *diag, float  *offd,
          float *b, long ldb, long *info);
```

PURPOSE

spttrs solves a tridiagonal system of the form
   A * X = B using the L*D*L' factorization of A computed by
SPTTRF. D is a diagonal matrix specified in the vector D, L
is a unit bidiagonal matrix whose subdiagonal is specified
in the vector E, and X and B are N by NRHS matrices.

ARGUMENTS

N (input) The order of the tridiagonal matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number
of columns of the matrix B. NRHS >= 0.

DIAG (input)
The n diagonal elements of the diagonal matrix
DIAG from the L*DIAG*L' factorization of A.

OFFD (input/output)
The (n-1) subdiagonal elements of the unit bidiag-
onal factor L from the L*DIAG*L' factorization of
A. OFFD can also be regarded as the superdiagonal
of the unit bidiagonal factor U from the factori-
zation A = U'*DIAG*U.

B (input/output)
On entry, the right hand side vectors B for the
system of linear equations. On exit, the solution
vectors, X.

LDB (input)
The leading dimension of the array B.   LDB >=
max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an ille-
gal value

Contents

NAME

```
sptts2 - solve a tridiagonal system of the form  A * X  =  B
using the L*D*L' factorization of A computed by SPTTRF
```

SYNOPSIS

```
SUBROUTINE SPTTS2(N, NRHS, D, E, B, LDB)

INTEGER N, NRHS, LDB
REAL D(*), E(*), B(LDB,*)

SUBROUTINE SPTTS2_64(N, NRHS, D, E, B, LDB)

INTEGER*8 N, NRHS, LDB
REAL D(*), E(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE SPTTS2(N, NRHS, D, E, B, LDB)

INTEGER :: N, NRHS, LDB
REAL, DIMENSION(:) :: D, E
REAL, DIMENSION(:,:) :: B

SUBROUTINE SPTTS2_64(N, NRHS, D, E, B, LDB)

INTEGER(8) :: N, NRHS, LDB
REAL, DIMENSION(:) :: D, E
REAL, DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void sptts2(int n, int nrhs, float *d, float *e,  float  *b,
```

```
          int ldb);

void sptts2_64(long n, long nrhs, float *d, float *e,  float
          *b, long ldb);
```

PURPOSE

sptts2 solves a tridiagonal system of the form
A * X = B using the L*D*L' factorization of A computed by SPTTRF. D is a diagonal matrix specified in the vector D, L is a unit bidiagonal matrix whose subdiagonal is specified in the vector E, and X and B are N by NRHS matrices.

ARGUMENTS

N (input) The order of the tridiagonal matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

D (input) The n diagonal elements of the diagonal matrix D from the L*D*L' factorization of A.

E (input) The (n-1) subdiagonal elements of the unit bidiagonal factor L from the L*D*L' factorization of A. E can also be regarded as the superdiagonal of the unit bidiagonal factor U from the factorization A = U'*D*U.

B (input/output)
On entry, the right hand side vectors B for the system of linear equations. On exit, the solution vectors, X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

Contents

NAME

```
srot - Apply a Given's rotation constructed by SROTG.
```

SYNOPSIS

```
SUBROUTINE SROT(N, X, INCX, Y, INCY, C, S)

INTEGER N, INCX, INCY
REAL C, S
REAL X(*), Y(*)

SUBROUTINE SROT_64(N, X, INCX, Y, INCY, C, S)

INTEGER*8 N, INCX, INCY
REAL C, S
REAL X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE ROT([N], X, [INCX], Y, [INCY], C, S)

INTEGER :: N, INCX, INCY
REAL :: C, S
REAL, DIMENSION(:) :: X, Y

SUBROUTINE ROT_64([N], X, [INCX], Y, [INCY], C, S)

INTEGER(8) :: N, INCX, INCY
REAL :: C, S
REAL, DIMENSION(:) :: X, Y
```

C INTERFACE

```
#include <sunperf.h>
```

```
void srot(int n, float *x, int incx,  float  *y,  int  incy,
          float c, float s);

void srot_64(long n, float *x, long  incx,  float  *y,  long
          incy, float c, float s);
```

PURPOSE

srot Apply a Given's rotation constructed by SROTG.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input/output)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

C (input) On entry, the C rotation value constructed by SROTG. Unchanged on exit.

S (input) On entry, the S rotation value constructed by SROTG. Unchanged on exit.

Contents

NAME

```
srotg - Construct a Given's plane rotation
```

SYNOPSIS

```
SUBROUTINE SROTG(A, B, C, S)

REAL A, B, C, S

SUBROUTINE SROTG_64(A, B, C, S)

REAL A, B, C, S
```

F95 INTERFACE

```
SUBROUTINE ROTG(A, B, C, S)

REAL :: A, B, C, S

SUBROUTINE ROTG_64(A, B, C, S)

REAL :: A, B, C, S
```

C INTERFACE

```
#include <sunperf.h>

void srotg(float *a, float *b, float *c, float *s);

void srotg_64(float *a, float *b, float *c, float *s);
```

PURPOSE

srotg Construct a Given's plane rotation that will annihilate an element of a vector.

ARGUMENTS

A (input/output)
On entry, A contains the entry in the first vector that corresponds to the element to be annihilated in the second vector. On exit, contains the nonzero element of the rotated vector.

B (input/output)
On entry, B contains the entry to be annihilated in the second vector. On exit, contains either S or 1/C depending on which of the input values of A and B is larger.

C (output)
On exit, C and S are the elements of the rotation matrix that will be applied to annihilate B.

S (output)
See the description of C.

Contents

NAME

```
sroti -  Apply an indexed Givens rotation.
```

SYNOPSIS

```
 SUBROUTINE SROTI(NZ, X, INDX, Y, C, S)

 INTEGER NZ
 INTEGER INDX(*)
 REAL C, S
 REAL X(*), Y(*)

 SUBROUTINE SROTI_64(NZ, X, INDX, Y, C, S)

 INTEGER*8 NZ
 INTEGER*8 INDX(*)
 REAL C, S
 REAL X(*), Y(*)

F95 INTERFACE
 SUBROUTINE ROTI([NZ], X, INDX, Y, C, S)

 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX
 REAL :: C, S
 REAL, DIMENSION(:) :: X, Y

 SUBROUTINE ROTI_64([NZ], X, INDX, Y, C, S)

 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
 REAL :: C, S
 REAL, DIMENSION(:) :: X, Y
```

PURPOSE

```
SROTI - Applies a Givens rotation to a sparse vector x
stored in compressed form and another vector y in full
storage form

 do i = 1, n
   temp = -s * x(i) + c * y(indx(i))
   x(i) = c * x(i) + s * y(indx(i))
   y(indx(i)) = temp
 enddo
```

ARGUMENTS

```
NZ (input) - INTEGER
        Number of elements in the compressed form.
        Unchanged on exit.
X (input)
        Vector containing the values of the compressed form.

INDX (input) - INTEGER
        Vector containing the indices of the compressed
        form.  It is assumed that the elements in INDX are
        distinct and greater than zero.  Unchanged on exit.

Y (input/output)
        Vector on input which contains the vector Y in full
        storage form.  On exit, only the elements
        corresponding to the indices in INDX have been
        modified.

C (input)
        Scalar defining the Givens rotation

S (input)
        Scalar defining the Givens rotation
```

Contents

NAME

```
srotm - Apply a Gentleman's modified Given's  rotation  con-
structed by SROTMG.
```

SYNOPSIS

```
SUBROUTINE SROTM(N, X, INCX, Y, INCY, PARAM)

INTEGER N, INCX, INCY
REAL X(*), Y(*), PARAM(*)

SUBROUTINE SROTM_64(N, X, INCX, Y, INCY, PARAM)

INTEGER*8 N, INCX, INCY
REAL X(*), Y(*), PARAM(*)
```

F95 INTERFACE

```
SUBROUTINE ROTM([N], X, [INCX], Y, [INCY], PARAM)

INTEGER :: N, INCX, INCY
REAL, DIMENSION(:) :: X, Y, PARAM

SUBROUTINE ROTM_64([N], X, [INCX], Y, [INCY], PARAM)

INTEGER(8) :: N, INCX, INCY
REAL, DIMENSION(:) :: X, Y, PARAM
```

C INTERFACE

```
#include <sunperf.h>

void srotm(int n, float *x, int incx, float  *y,  int  incy,
          float *param);
```

```
void srotm_64(long n, float *x, long incx,  float  *y,  long
          incy, float *param);
```

PURPOSE

srotm Apply a Given's rotation constructed by SROTMG.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input/output)
(1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. On exit, X is overwritten by the updated vector x.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

PARAM (input)
On entry, the rotation values constructed by SROTMG. Unchanged on exit.

Contents

NAME

```
srotmg - Construct  a  Gentleman's  modified  Given's  plane
rotation
```

SYNOPSIS

```
SUBROUTINE SROTMG(D1, D2, B1, B2, PARAM)

REAL D1, D2, B1, B2
REAL PARAM(*)

SUBROUTINE SROTMG_64(D1, D2, B1, B2, PARAM)

REAL D1, D2, B1, B2
REAL PARAM(*)
```

F95 INTERFACE

```
SUBROUTINE ROTMG(D1, D2, B1, B2, PARAM)

REAL :: D1, D2, B1, B2
REAL, DIMENSION(:) :: PARAM

SUBROUTINE ROTMG_64(D1, D2, B1, B2, PARAM)

REAL :: D1, D2, B1, B2
REAL, DIMENSION(:) :: PARAM
```

C INTERFACE

```
#include <sunperf.h>

void srotmg(float d1, float d2, float b1,  float  b2,  float
          *param);
```

```
void srotmg_64(float d1, float d2, float b1, float b2, float
          *param);
```

PURPOSE

srotmg Construct Gentleman's modified a Given's plane rotation that will annihilate an element of a vector.

ARGUMENTS

D1 (input/output)
On entry, the first diagonal entry in the H matrix. On exit, changed to reflect the effect of the transformation.

D2 (input/output)
On entry, the second diagonal entry in the H matrix. On exit, changed to reflect the effect of the transformation.

B1 (input/output)
On entry, the first element of the vector to which the H matrix is applied. On exit, changed to reflect the effect of the transformation.

B2 (input)
On entry, the second element of the vector to which the H matrix is applied. Unchanged on exit.

PARAM (output)
On exit, PARAM(1) describes the form of the rotation matrix H, and PARAM(2..5) contain the H matrix.

If PARAM(1) = -2 then H = I and no elements of PARAM are modified.

If PARAM(1) = -1 then PARAM(2) = h11, PARAM(3) = h21, PARAM(4) = h12, and PARAM(5) = h22.

If PARAM(1) = 0 then h11 = h22 = 1, PARAM(3) = h21, and PARAM(4) = h12.

If PARAM(1) = 1 then h12 = 1, h21 = -1, PARAM(2) = h11, and PARAM(5) = h22.

Contents

NAME

```
ssbev - compute all the eigenvalues and, optionally,  eigen-
vectors of a real symmetric band matrix A
```

SYNOPSIS

```
SUBROUTINE SSBEV(JOBZ, UPLO, N, KD, A, LDA, W, Z, LDZ, WORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER N, KD, LDA, LDZ, INFO
REAL A(LDA,*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSBEV_64(JOBZ, UPLO, N, KD, A, LDA, W, Z, LDZ, WORK,
      INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 N, KD, LDA, LDZ, INFO
REAL A(LDA,*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SBEV(JOBZ, UPLO, [N], KD, A, [LDA], W, Z, [LDZ], [WORK],
       [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: N, KD, LDA, LDZ, INFO
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: A, Z

SUBROUTINE SBEV_64(JOBZ, UPLO, [N], KD, A, [LDA], W, Z, [LDZ],
       [WORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: N, KD, LDA, LDZ, INFO
```

```
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: A, Z
```

C INTERFACE

```
#include <sunperf.h>

void ssbev(char jobz, char uplo, int n, int  kd,  float  *a,
          int lda, float *w, float *z, int ldz, int *info);

void ssbev_64(char jobz, char uplo, long n, long  kd,  float
          *a,  long  lda, float *w, float *z, long ldz, long
          *info);
```

PURPOSE

ssbev computes all the eigenvalues and, optionally, eigenvectors of a real symmetric band matrix A.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

A (input/output)
On entry, the upper or lower triangle of the symmetric band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).

On exit, A is overwritten by values generated during the reduction to tridiagonal form. If UPLO = 'U', the first superdiagonal and the diagonal of the tridiagonal matrix T are returned in rows KD and KD+1 of A, and if UPLO = 'L', the diagonal and first subdiagonal of T are returned in the first two rows of A.

LDA (input)
The leading dimension of the array A. LDA >= KD + 1.

W (output)
If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, Z contains the orthonormal eigenvectors of the matrix A, with the i-th column of Z holding the eigenvector associated with W(i). If JOBZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
dimension(MAX(1,3*N-2))

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

Contents

NAME

```
ssbevd - compute all the eigenvalues and, optionally, eigen-
vectors of a real symmetric band matrix A
```

SYNOPSIS

```
SUBROUTINE SSBEVD(JOBZ, UPLO, N, KD, AB, LDAB, W, Z, LDZ, WORK,
      LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER N, KD, LDAB, LDZ, LWORK, LIWORK, INFO
INTEGER IWORK(*)
REAL AB(LDAB,*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSBEVD_64(JOBZ, UPLO, N, KD, AB, LDAB, W, Z, LDZ, WORK,
      LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 N, KD, LDAB, LDZ, LWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
REAL AB(LDAB,*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SBEVD(JOBZ, UPLO, [N], KD, AB, [LDAB], W, Z, [LDZ], [WORK],
       [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: N, KD, LDAB, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: AB, Z

SUBROUTINE SBEVD_64(JOBZ, UPLO, [N], KD, AB, [LDAB], W, Z, [LDZ],
```

```
       [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: N, KD, LDAB, LDZ, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: AB, Z
```

C INTERFACE

```
#include <sunperf.h>

void ssbevd(char jobz, char uplo, int n, int kd, float  *ab,
          int ldab, float *w, float *z, int ldz, int *info);
void ssbevd_64(char jobz, char uplo, long n, long kd,  float
          *ab, long ldab, float *w, float *z, long ldz, long
          *info);
```

PURPOSE

ssbevd computes all the eigenvalues and, optionally, eigen- vectors of a real symmetric band matrix A. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  KD >= 0.
```

AB (input/output)

On entry, the upper or lower triangle of the symmetric band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).

On exit, AB is overwritten by values generated during the reduction to tridiagonal form. If UPLO = 'U', the first superdiagonal and the diagonal of the tridiagonal matrix T are returned in rows KD and KD+1 of AB, and if UPLO = 'L', the diagonal and first subdiagonal of T are returned in the first two rows of AB.

LDAB (input)

The leading dimension of the array AB. LDAB >= KD + 1.

W (output)

If INFO = 0, the eigenvalues in ascending order.

Z (output)

If JOBZ = 'V', then if INFO = 0, Z contains the orthonormal eigenvectors of the matrix A, with the i-th column of Z holding the eigenvector associated with W(i). If JOBZ = 'N', then Z is not referenced.

LDZ (input)

The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)

dimension (LWORK) On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)

The dimension of the array WORK. If N <= 1, LWORK must be at least 1. If JOBZ = 'N' and N > 2, LWORK must be at least 2*N. If JOBZ = 'V' and N > 2, LWORK must be at least (1 + 5*N + 2*N**2).

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first

```
            entry  of  the  WORK  array,  and no error message
            related to LWORK is issued by XERBLA.

IWORK (workspace/output)
            On exit, if INFO = 0, IWORK(1) returns the optimal
            LIWORK.
LIWORK (input)
            The dimension of the array LIWORK.  If JOBZ  = 'N'
            or  N <= 1, LIWORK must be at least 1.  If JOBZ  =
            'V' and N > 2, LIWORK must be at least 3 + 5*N.

            If LIWORK = -1, then a workspace query is assumed;
            the  routine  only  calculates the optimal size of
            the IWORK array, returns this value as  the  first
            entry  of  the  IWORK  array, and no error message
            related to LIWORK is issued by XERBLA.

INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value
            > 0:  if INFO = i, the algorithm  failed  to  con-
            verge;  i off-diagonal elements of an intermediate
            tridiagonal form did not converge to zero.
```

Contents

NAME

ssbevx - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric band matrix A

SYNOPSIS

```
SUBROUTINE SSBEVX(JOBZ, RANGE, UPLO, N, KD, A, LDA, Q, LDQ, VL,
      VU, IL, IU, ABTOL, NFOUND, W, Z, LDZ, WORK, IWORK2, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER N, KD, LDA, LDQ, IL, IU, NFOUND, LDZ, INFO
INTEGER IWORK2(*), IFAIL(*)
REAL VL, VU, ABTOL
REAL A(LDA,*), Q(LDQ,*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSBEVX_64(JOBZ, RANGE, UPLO, N, KD, A, LDA, Q, LDQ, VL,
      VU, IL, IU, ABTOL, NFOUND, W, Z, LDZ, WORK, IWORK2, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER*8 N, KD, LDA, LDQ, IL, IU, NFOUND, LDZ, INFO
INTEGER*8 IWORK2(*), IFAIL(*)
REAL VL, VU, ABTOL
REAL A(LDA,*), Q(LDQ,*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SBEVX(JOBZ, RANGE, UPLO, [N], KD, A, [LDA], Q, [LDQ],
       VL, VU, IL, IU, ABTOL, NFOUND, W, Z, [LDZ], [WORK], [IWORK2],
       IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER :: N, KD, LDA, LDQ, IL, IU, NFOUND, LDZ, INFO
INTEGER, DIMENSION(:) :: IWORK2, IFAIL
REAL :: VL, VU, ABTOL
```

```
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: A, Q, Z

SUBROUTINE SBEVX_64(JOBZ, RANGE, UPLO, [N], KD, A, [LDA], Q, [LDQ],
       VL, VU, IL, IU, ABTOL, NFOUND, W, Z, [LDZ], [WORK], [IWORK2],
       IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER(8) :: N, KD, LDA, LDQ, IL, IU, NFOUND, LDZ, INFO
INTEGER(8), DIMENSION(:) :: IWORK2, IFAIL
REAL :: VL, VU, ABTOL
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: A, Q, Z
```

C INTERFACE

```
#include <sunperf.h>

void ssbevx(char jobz, char range, char uplo, int n, int kd,
          float  *a,  int  lda, float *q, int ldq, float vl,
          float  vu,  int  il,  int  iu,  float  abtol,  int
          *nfound,  float *w, float *z, int ldz, int *ifail,
          int *info);

void ssbevx_64(char jobz, char range,  char  uplo,  long  n,
          long  kd,  float *a, long lda, float *q, long ldq,
          float vl, float vu, long il, long iu, float abtol,
          long  *nfound,  float *w, float *z, long ldz, long
          *ifail, long *info);
```

PURPOSE

ssbevx computes selected eigenvalues and, optionally, eigenvectors of a real symmetric band matrix A. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

RANGE (input)
          = 'A': all eigenvalues will be found;
          = 'V': all eigenvalues in the  half-open  interval
          (VL,VU]  will  be  found; = 'I': the IL-th through
          IU-th eigenvalues will be found.
```

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  KD >= 0.

A (input/output)
          On entry, the upper or lower triangle of the  sym-
          metric  band  matrix  A,  stored in the first KD+1
          rows of the array.  The j-th column of A is stored
          in  the j-th column of the array A as follows:  if
          UPLO = 'U', A(kd+1+i-j,j) =  A(i,j)  for  max(1,j-
          kd)<=i<=j;  if  UPLO = 'L', A(1+i-j,j)    = A(i,j)
          for j<=i<=min(n,j+kd).

          On exit, A is overwritten by values generated dur-
          ing  the reduction to tridiagonal form.  If UPLO =
          'U', the first superdiagonal and the  diagonal  of
          the  tridiagonal  matrix T are returned in rows KD
          and KD+1 of A, and if UPLO = 'L', the diagonal and
          first  subdiagonal  of T are returned in the first
          two rows of A.

LDA (input)
          The leading dimension of the array A.  LDA >= KD +
          1.

Q (output)
          If JOBZ = 'V', the N-by-N orthogonal  matrix  used
          in  the  reduction to tridiagonal form.  If JOBZ =
          'N', the array Q is not referenced.

LDQ (input)
          The leading dimension of the array Q.  If  JOBZ  =
          'V', then LDQ >= max(1,N).

VL (input)
          If RANGE='V', the lower and upper  bounds  of  the
          interval  to be searched for eigenvalues. VL < VU.
          Not referenced if RANGE = 'A' or 'I'.

VU (input)
          See the description of VL.

IL (input)
```

If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
See the description of IL.

ABTOL (input)
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing A to tridiagonal form.

Eigenvalues will be computed most accurately when ABTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABTOL to 2*SLAMCH('S').

See "Computing Small Singular Values of Bidiagonal Matrices with Guaranteed High Relative Accuracy," by Demmel and Kahan, LAPACK Working Note #3.

NFOUND (output)
The total number of eigenvalues found. 0 <= NFOUND <= N. If RANGE = 'A', NFOUND = N, and if RANGE = 'I', NFOUND = IU-IL+1.

W (output)
The first NFOUND elements contain the selected eigenvalues in ascending order.

Z (input)
If JOBZ = 'V', then if INFO = 0, the first NFOUND columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). If an eigenvector fails to converge, then that column of Z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in

```
            IFAIL.  If JOBZ = 'N', then Z is  not  referenced.
            Note:   the   user   must  ensure  that  at  least
            max(1,NFOUND) columns are supplied in the array Z;
            if  RANGE  = 'V', the exact value of NFOUND is not
            known in advance and an upper bound must be used.
LDZ (input)
            The leading dimension of the array Z.  LDZ  >=  1,
            and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
            dimension(7*N)

IWORK2 (workspace)

IFAIL (output)
            If JOBZ = 'V', then if INFO = 0, the first  NFOUND
            elements  of  IFAIL  are  zero.  If INFO > 0, then
            IFAIL contains the  indices  of  the  eigenvectors
            that  failed  to  converge.   If  JOBZ = 'N', then
            IFAIL is not referenced.

INFO (output)
            = 0:  successful exit.
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value.
            > 0:  if INFO = i, then i eigenvectors  failed  to
            converge.   Their  indices  are  stored  in  array
            IFAIL.
```

Contents

NAME

ssbgst - reduce a real symmetric-definite banded generalized eigenproblem A*x = lambda*B*x to standard form C*y = lambda*y,

SYNOPSIS

```
SUBROUTINE SSBGST(VECT, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, X, LDX,
      WORK, INFO)

CHARACTER * 1 VECT, UPLO
INTEGER N, KA, KB, LDAB, LDBB, LDX, INFO
REAL AB(LDAB,*), BB(LDBB,*), X(LDX,*), WORK(*)

SUBROUTINE SSBGST_64(VECT, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, X,
      LDX, WORK, INFO)

CHARACTER * 1 VECT, UPLO
INTEGER*8 N, KA, KB, LDAB, LDBB, LDX, INFO
REAL AB(LDAB,*), BB(LDBB,*), X(LDX,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SBGST(VECT, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB], X,
       [LDX], [WORK], [INFO])

CHARACTER(LEN=1) :: VECT, UPLO
INTEGER :: N, KA, KB, LDAB, LDBB, LDX, INFO
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: AB, BB, X

SUBROUTINE SBGST_64(VECT, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB],
       X, [LDX], [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: VECT, UPLO
INTEGER(8) :: N, KA, KB, LDAB, LDBB, LDX, INFO
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: AB, BB, X
```

C INTERFACE

```
#include <sunperf.h>

void ssbgst(char vect, char uplo, int n,  int  ka,  int  kb,
          float  *ab,  int  ldab, float *bb, int ldbb, float
          *x, int ldx, int *info);

void ssbgst_64(char vect, char uplo, long n, long  ka,  long
          kb,  float  *ab,  long ldab, float *bb, long ldbb,
          float *x, long ldx, long *info);
```

PURPOSE

ssbgst reduces a real symmetric-definite banded generalized eigenproblem A*x = lambda*B*x to standard form C*y = lambda*y, such that C has the same bandwidth as A.

B must have been previously factorized as S**T*S by SPBSTF, using a split Cholesky factorization. A is overwritten by C = X**T*A*X, where X = S**(-1)*Q and Q is an orthogonal matrix chosen to preserve the bandwidth of A.

ARGUMENTS

```
VECT (input)
          = 'N':  do not form the transformation matrix X;
          = 'V':  form X.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrices A and B.  N >= 0.

KA (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  KA >= 0.

KB (input)
          The number of superdiagonals of the  matrix  B  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
```

= 'L'. KA >= KB >= 0.

AB (input/output)
On entry, the upper or lower triangle of the symmetric band matrix A, stored in the first ka+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(ka+1+i-j,j) = A(i,j) for max(1,j-ka)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+ka).

On exit, the transformed matrix X**T*A*X, stored in the same format as A.

LDAB (input)
The leading dimension of the array AB. LDAB >= KA+1.

BB (input)
The banded factor S from the split Cholesky factorization of B, as returned by SPBSTF, stored in the first KB+1 rows of the array.

LDBB (input)
The leading dimension of the array BB. LDBB >= KB+1.

X (output)
If VECT = 'V', the n-by-n matrix X. If VECT = 'N', the array X is not referenced.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N) if VECT = 'V'; LDX >= 1 otherwise.

WORK (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

```
ssbgv - compute all the  eigenvalues,  and  optionally,  the
eigenvectors of a real generalized symmetric-definite banded
eigenproblem, of the form A*x=(lambda)*B*x
```

SYNOPSIS

```
SUBROUTINE SSBGV(JOBZ, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, W, Z,
      LDZ, WORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER N, KA, KB, LDAB, LDBB, LDZ, INFO
REAL AB(LDAB,*), BB(LDBB,*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSBGV_64(JOBZ, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, W, Z,
      LDZ, WORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 N, KA, KB, LDAB, LDBB, LDZ, INFO
REAL AB(LDAB,*), BB(LDBB,*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SBGV(JOBZ, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB], W,
       Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: N, KA, KB, LDAB, LDBB, LDZ, INFO
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: AB, BB, Z

SUBROUTINE SBGV_64(JOBZ, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB],
       W, Z, [LDZ], [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: N, KA, KB, LDAB, LDBB, LDZ, INFO
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: AB, BB, Z
```

C INTERFACE

```
#include <sunperf.h>

void ssbgv(char jobz, char uplo, int  n,  int  ka,  int  kb,
          float  *ab,  int  ldab, float *bb, int ldbb, float
          *w, float *z, int ldz, int *info);

void ssbgv_64(char jobz, char uplo, long n,  long  ka,  long
          kb,  float  *ab,  long ldab, float *bb, long ldbb,
          float *w, float *z, long ldz, long *info);
```

PURPOSE

ssbgv computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form A*x=(lambda)*B*x. Here A and B are assumed to be symmetric and banded, and B is also positive definite.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

UPLO (input)
= 'U': Upper triangles of A and B are stored;
= 'L': Lower triangles of A and B are stored.

N (input) The order of the matrices A and B. N >= 0.

KA (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KA >= 0.

KB (input)
The number of superdiagonals of the matrix B if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KB >= 0.

AB (input/output)

On entry, the upper or lower triangle of the symmetric band matrix A, stored in the first ka+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(ka+1+i-j,j) = A(i,j) for max(1,j-ka)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+ka).

On exit, the contents of AB are destroyed.

LDAB (input)
The leading dimension of the array AB. LDAB >= KA+1.

BB (input/output)
On entry, the upper or lower triangle of the symmetric band matrix B, stored in the first kb+1 rows of the array. The j-th column of B is stored in the j-th column of the array BB as follows: if UPLO = 'U', BB(kb+1+i-j,j) = B(i,j) for max(1,j-kb)<=i<=j; if UPLO = 'L', BB(1+i-j,j) = B(i,j) for j<=i<=min(n,j+kb).

On exit, the factor S from the split Cholesky factorization B = S**T*S, as returned by SPBSTF.

LDBB (input)
The leading dimension of the array BB. LDBB >= KB+1.

W (output)
If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, Z contains the matrix Z of eigenvectors, with the i-th column of Z holding the eigenvector associated with W(i). The eigenvectors are normalized so that Z**T*B*Z = I. If JOBZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= N.

WORK (workspace)
dimension(3*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is:

<= N: the algorithm failed to converge: i off-diagonal elements of an intermediate tridiagonal form did not converge to zero; > N: if INFO = N + i, for 1 <= i <= N, then SPBSTF
returned INFO = i: B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

Contents

NAME

```
ssbgvd - compute all the eigenvalues,  and  optionally,  the
eigenvectors of a real generalized symmetric-definite banded
eigenproblem, of the form A*x=(lambda)*B*x
```

SYNOPSIS

```
SUBROUTINE SSBGVD(JOBZ, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, W, Z,
      LDZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER N, KA, KB, LDAB, LDBB, LDZ, LWORK, LIWORK, INFO
INTEGER IWORK(*)
REAL AB(LDAB,*), BB(LDBB,*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSBGVD_64(JOBZ, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, W, Z,
      LDZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 N, KA, KB, LDAB, LDBB, LDZ, LWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
REAL AB(LDAB,*), BB(LDBB,*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SBGVD(JOBZ, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB], W,
       Z, [LDZ], [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: N, KA, KB, LDAB, LDBB, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: AB, BB, Z
```

```
SUBROUTINE SBGVD_64(JOBZ, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB],
       W, Z, [LDZ], [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: N, KA, KB, LDAB,  LDBB,  LDZ,  LWORK,  LIWORK,
INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: AB, BB, Z
```

C INTERFACE

```
#include <sunperf.h>

void ssbgvd(char jobz, char uplo, int n,  int  ka,  int  kb,
          float  *ab,  int  ldab, float *bb, int ldbb, float
          *w, float *z, int ldz, int *info);

void ssbgvd_64(char jobz, char uplo, long n, long  ka,  long
          kb,  float  *ab,  long ldab, float *bb, long ldbb,
          float *w, float *z, long ldz, long *info);
```

PURPOSE

ssbgvd computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form A*x=(lambda)*B*x. Here A and B are assumed to be symmetric and banded, and B is also positive definite. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
```

```
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.

KA (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  KA >= 0.

KB (input)
          The number of superdiagonals of the  matrix  B  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  KB >= 0.

AB (input/output)
          On entry, the upper or lower triangle of the  sym-
          metric  band  matrix  A,  stored in the first ka+1
          rows of the array.  The j-th column of A is stored
          in the j-th column of the array AB as follows:  if
          UPLO = 'U', AB(ka+1+i-j,j) = A(i,j)  for  max(1,j-
          ka)<=i<=j;  if UPLO = 'L', AB(1+i-j,j)    = A(i,j)
          for j<=i<=min(n,j+ka).

          On exit, the contents of AB are destroyed.

LDAB (input)
          The leading dimension of the array  AB.   LDAB  >=
          KA+1.

BB (input/output)
          On entry, the upper or lower triangle of the  sym-
          metric  band  matrix  B,  stored in the first kb+1
          rows of the array.  The j-th column of B is stored
          in the j-th column of the array BB as follows:  if
          UPLO = 'U', BB(ka+1+i-j,j) = B(i,j)  for  max(1,j-
          kb)<=i<=j;  if UPLO = 'L', BB(1+i-j,j)    = B(i,j)
          for j<=i<=min(n,j+kb).

          On exit, the factor S from the split Cholesky fac-
          torization B = S**T*S, as returned by SPBSTF.

LDBB (input)
          The leading dimension of the array  BB.   LDBB  >=
          KB+1.

W (output)
          If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0,  Z  contains  the
          matrix  Z of eigenvectors, with the i-th column of
```

Z holding the eigenvector associated with W(i). The eigenvectors are normalized so Z**T*B*Z = I. If JOBZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If N <= 1, LWORK >= 1. If JOBZ = 'N' and N > 1, LWORK >= 3*N. If JOBZ = 'V' and N > 1, LWORK >= 1 + 5*N + 2*N**2.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if LIWORK > 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of the array IWORK. If JOBZ = 'N' or N <= 1, LIWORK >= 1. If JOBZ = 'V' and N > 1, LIWORK >= 3 + 5*N.

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is:
<= N: the algorithm failed to converge: i off-diagonal elements of an intermediate tridiagonal form did not converge to zero; > N: if INFO = N + i, for 1 <= i <= N, then SPBSTF
returned INFO = i: B is not positive definite. The factorization of B could not be completed and

no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
   Mark Fahey, Department of Mathematics, Univ. of Kentucky,
USA

Contents

NAME

ssbgvx - compute selected eigenvalues, and optionally, eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form A*x=(lambda)*B*x

SYNOPSIS

```
SUBROUTINE SSBGVX(JOBZ, RANGE, UPLO, N, KA, KB, AB, LDAB, BB, LDBB,
      Q, LDQ, VL, VU, IL, IU, ABSTOL, M, W, Z, LDZ, WORK, IWORK, IFAIL,
      INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER N, KA, KB, LDAB, LDBB, LDQ, IL, IU, M, LDZ, INFO
INTEGER IWORK(*), IFAIL(*)
REAL VL, VU, ABSTOL
REAL  AB(LDAB,*),  BB(LDBB,*),  Q(LDQ,*),  W(*),   Z(LDZ,*),
WORK(*)

SUBROUTINE SSBGVX_64(JOBZ, RANGE, UPLO, N, KA, KB, AB, LDAB, BB,
      LDBB, Q, LDQ, VL, VU, IL, IU, ABSTOL, M, W, Z, LDZ, WORK, IWORK,
      IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER*8 N, KA, KB, LDAB, LDBB, LDQ, IL, IU, M, LDZ, INFO
INTEGER*8 IWORK(*), IFAIL(*)
REAL VL, VU, ABSTOL
REAL  AB(LDAB,*),  BB(LDBB,*),  Q(LDQ,*),  W(*),   Z(LDZ,*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SBGVX(JOBZ, RANGE, UPLO, [N], KA, KB, AB, [LDAB], BB,
       [LDBB], Q, [LDQ], VL, VU, IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK],
```

```
       [IWORK], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER :: N, KA, KB, LDAB, LDBB, LDQ, IL, IU, M, LDZ, INFO
INTEGER, DIMENSION(:) :: IWORK, IFAIL
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: AB, BB, Q, Z

SUBROUTINE SBGVX_64(JOBZ, RANGE, UPLO, [N], KA, KB, AB, [LDAB], BB,
       [LDBB], Q, [LDQ], VL, VU, IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK],
       [IWORK], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER(8) :: N, KA, KB, LDAB, LDBB, LDQ, IL,  IU,  M,  LDZ,
INFO
INTEGER(8), DIMENSION(:) :: IWORK, IFAIL
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: AB, BB, Q, Z
```

C INTERFACE

```
#include <sunperf.h>

void ssbgvx(char jobz, char range, char uplo, int n, int ka,
          int  kb, float *ab, int ldab, float *bb, int ldbb,
          float *q, int ldq, float vl, float vu, int il, int
          iu,  float abstol, int *m, float *w, float *z, int
          ldz, int *ifail, int *info);

void ssbgvx_64(char jobz, char range,  char  uplo,  long  n,
          long ka, long kb, float *ab, long ldab, float *bb,
          long ldbb, float *q, long ldq, float vl, float vu,
          long il, long iu, float abstol, long *m, float *w,
          float *z, long ldz, long *ifail, long *info);
```

PURPOSE

ssbgvx computes selected eigenvalues, and optionally, eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form A*x=(lambda)*B*x. Here A and B are assumed to be symmetric and banded, and B is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either all eigenvalues, a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

RANGE (input)
= 'A': all eigenvalues will be found.
= 'V': all eigenvalues in the half-open interval (VL,VU] will be found. = 'I': the IL-th through IU-th eigenvalues will be found.

UPLO (input)
= 'U': Upper triangles of A and B are stored;
= 'L': Lower triangles of A and B are stored.

N (input) The order of the matrices A and B. N >= 0.

KA (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KA >= 0.

KB (input)
The number of superdiagonals of the matrix B if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KB >= 0.

AB (input/output)
On entry, the upper or lower triangle of the symmetric band matrix A, stored in the first ka+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(ka+1+i-j,j) = A(i,j) for max(1,j-ka)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+ka).

On exit, the contents of AB are destroyed.

LDAB (input)
The leading dimension of the array AB. LDAB >= KA+1.

BB (input/output)
On entry, the upper or lower triangle of the symmetric band matrix B, stored in the first kb+1 rows of the array. The j-th column of B is stored in the j-th column of the array BB as follows: if UPLO = 'U', BB(ka+1+i-j,j) = B(i,j) for max(1,j-kb)<=i<=j; if UPLO = 'L', BB(1+i-j,j) = B(i,j) for j<=i<=min(n,j+kb).

On exit, the factor S from the split Cholesky factorization B = S**T*S, as returned by SPBSTF.

LDBB (input)
The leading dimension of the array BB. LDBB >= KB+1.

Q (output)
If JOBZ = 'V', the n-by-n matrix used in the reduction of A*x = (lambda)*B*x to standard form, i.e. C*x = (lambda)*x, and consequently C to tridiagonal form. If JOBZ = 'N', the array Q is not referenced.

LDQ (input)
The leading dimension of the array Q. If JOBZ = 'N', LDQ >= 1. If JOBZ = 'V', LDQ >= max(1,N).

VL (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)
See the description of VL.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
See the description of IL.

ABSTOL (input)
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABSTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABSTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing A to tridiagonal form.

Eigenvalues will be computed most accurately when ABSTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABSTOL to 2*SLAMCH('S').

M (output)
The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)
If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, Z contains the matrix Z of eigenvectors, with the i-th column of Z holding the eigenvector associated with W(i). The eigenvectors are normalized so Z**T*B*Z = I. If JOBZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
dimension(7*N)

IWORK (workspace/output)
dimension(5*N)

IFAIL (output)
If JOBZ = 'V', then if INFO = 0, the first M elements of IFAIL are zero. If INFO > 0, then IFAIL contains the indices of the eigenvalues that failed to converge. If JOBZ = 'N', then IFAIL is not referenced.

INFO (output)
= 0 : successful exit
< 0 : if INFO = -i, the i-th argument had an illegal value
<= N: if INFO = i, then i eigenvectors failed to converge. Their indices are stored in IFAIL. > N : SPBSTF returned an error code; i.e., if INFO = N + i, for 1 <= i <= N, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
   Mark Fahey, Department of Mathematics, Univ. of Kentucky,
USA

Contents

NAME

ssbmv - perform the matrix-vector operation   y := alpha*A*x + beta*y

SYNOPSIS

```
SUBROUTINE SSBMV(UPLO, N, K, ALPHA, A, LDA, X, INCX, BETA, Y,
      INCY)

CHARACTER * 1 UPLO
INTEGER N, K, LDA, INCX, INCY
REAL ALPHA, BETA
REAL A(LDA,*), X(*), Y(*)

SUBROUTINE SSBMV_64(UPLO, N, K, ALPHA, A, LDA, X, INCX, BETA, Y,
      INCY)

CHARACTER * 1 UPLO
INTEGER*8 N, K, LDA, INCX, INCY
REAL ALPHA, BETA
REAL A(LDA,*), X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE SBMV(UPLO, [N], K, ALPHA, A, [LDA], X, [INCX], BETA,
       Y, [INCY])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, K, LDA, INCX, INCY
REAL :: ALPHA, BETA
REAL, DIMENSION(:) :: X, Y
REAL, DIMENSION(:,:) :: A

SUBROUTINE SBMV_64(UPLO, [N], K, ALPHA, A, [LDA], X, [INCX],
```

```
        BETA, Y, [INCY])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, K, LDA, INCX, INCY
REAL :: ALPHA, BETA
REAL, DIMENSION(:) :: X, Y
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void ssbmv(char uplo, int n, int k, float alpha,  float  *a,
          int lda, float *x, int incx, float beta, float *y,
          int incy);
void ssbmv_64(char uplo, long n, long k, float alpha,  float
          *a,  long  lda,  float  *x, long incx, float beta,
          float *y, long incy);
```

PURPOSE

ssbmv performs the matrix-vector operation y := alpha*A*x + beta*y, where alpha and beta are scalars, x and y are n element vectors and A is an n by n symmetric band matrix, with k super-diagonals.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the band matrix A is being supplied as follows:

UPLO = 'U' or 'u'   The upper triangular part of A is being supplied.

UPLO = 'L' or 'l'   The lower triangular part of A is being supplied.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

K (input)
On entry, K specifies the number of super-diagonals of the matrix A. K >= 0. Unchanged on

exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
Before entry with UPLO = 'U' or 'u', the leading (k + 1) by n part of the array A must contain the upper triangular band part of the symmetric matrix, supplied column by column, with the leading diagonal of the matrix in row (k + 1) of the array, the first super-diagonal starting at position 2 in row k, and so on. The top left k by k triangle of the array A is not referenced. The following program segment will transfer the upper triangular part of a symmetric band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = K + 1 - J
     DO 10, I = MAX( 1, J - K ), J
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Before entry with UPLO = 'L' or 'l', the leading (k + 1) by n part of the array A must contain the lower triangular band part of the symmetric matrix, supplied column by column, with the leading diagonal of the matrix in row 1 of the array, the first sub-diagonal starting at position 1 in row 2, and so on. The bottom right k by k triangle of the array A is not referenced. The following program segment will transfer the lower triangular part of a symmetric band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = 1 - J
     DO 10, I = J, MIN( N, J + K )
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= (

k + 1). Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
ssbtrd - reduce a real symmetric band matrix A to  symmetric
tridiagonal  form  T by an orthogonal similarity transforma-
tion
```

SYNOPSIS

```
SUBROUTINE SSBTRD(VECT, UPLO, N, KD, AB, LDAB, D, E, Q, LDQ, WORK,
      INFO)

CHARACTER * 1 VECT, UPLO
INTEGER N, KD, LDAB, LDQ, INFO
REAL AB(LDAB,*), D(*), E(*), Q(LDQ,*), WORK(*)

SUBROUTINE SSBTRD_64(VECT, UPLO, N, KD, AB, LDAB, D, E, Q, LDQ, WORK,
      INFO)

CHARACTER * 1 VECT, UPLO
INTEGER*8 N, KD, LDAB, LDQ, INFO
REAL AB(LDAB,*), D(*), E(*), Q(LDQ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SBTRD(VECT, UPLO, [N], KD, AB, [LDAB], D, E, Q, [LDQ],
       [WORK], [INFO])

CHARACTER(LEN=1) :: VECT, UPLO
INTEGER :: N, KD, LDAB, LDQ, INFO
REAL, DIMENSION(:) :: D, E, WORK
REAL, DIMENSION(:,:) :: AB, Q

SUBROUTINE SBTRD_64(VECT, UPLO, [N], KD, AB, [LDAB], D, E, Q, [LDQ],
       [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: VECT, UPLO
INTEGER(8) :: N, KD, LDAB, LDQ, INFO
REAL, DIMENSION(:) :: D, E, WORK
REAL, DIMENSION(:,:) :: AB, Q
```

C INTERFACE

```
#include <sunperf.h>

void ssbtrd(char vect, char uplo, int n, int kd, float  *ab,
          int  ldab,  float *d, float *e, float *q, int ldq,
          int *info);

void ssbtrd_64(char vect, char uplo, long n, long kd,  float
          *ab, long ldab, float *d, float *e, float *q, long
          ldq, long *info);
```

PURPOSE

ssbtrd reduces a real symmetric band matrix A to symmetric tridiagonal form T by an orthogonal similarity transformation: Q**T * A * Q = T.

ARGUMENTS

```
VECT (input)
          = 'N':  do not form Q;
          = 'V':  form Q;
          = 'U':  update a matrix X, by forming X*Q.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  KD >= 0.

AB (input/output)
          On entry, the upper or lower triangle of the  sym-
          metric  band  matrix  A,  stored in the first KD+1
          rows of the array.  The j-th column of A is stored
          in the j-th column of the array AB as follows:  if
          UPLO = 'U', AB(kd+1+i-j,j) = A(i,j)  for  max(1,j-
```

kd)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd). On exit, the diagonal elements of AB are overwritten by the diagonal elements of the tridiagonal matrix T; if KD > 0, the elements on the first superdiagonal (if UPLO = 'U') or the first subdiagonal (if UPLO = 'L') are overwritten by the off-diagonal elements of T; the rest of AB is overwritten by values generated during the reduction.

LDAB (input)
The leading dimension of the array AB. LDAB >= KD+1.

D (output)
The diagonal elements of the tridiagonal matrix T.

E (output)
The off-diagonal elements of the tridiagonal matrix T: E(i) = T(i,i+1) if UPLO = 'U'; E(i) = T(i+1,i) if UPLO = 'L'.

Q (input/output)
On entry, if VECT = 'U', then Q must contain an N-by-N matrix X; if VECT = 'N' or 'V', then Q need not be set.

On exit: if VECT = 'V', Q contains the N-by-N orthogonal matrix Q; if VECT = 'U', Q contains the product X*Q; if VECT = 'N', the array Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= 1, and LDQ >= N if VECT = 'V' or 'U'.

WORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

Modified by Linda Kaufman, Bell Labs.

Contents

NAME

```
sscal - Compute y := alpha * y
```

SYNOPSIS

```
SUBROUTINE SSCAL(N, ALPHA, Y, INCY)

INTEGER N, INCY
REAL ALPHA
REAL Y(*)

SUBROUTINE SSCAL_64(N, ALPHA, Y, INCY)

INTEGER*8 N, INCY
REAL ALPHA
REAL Y(*)
```

F95 INTERFACE

```
SUBROUTINE SCAL([N], ALPHA, Y, [INCY])

INTEGER :: N, INCY
REAL :: ALPHA
REAL, DIMENSION(:) :: Y

SUBROUTINE SCAL_64([N], ALPHA, Y, [INCY])

INTEGER(8) :: N, INCY
REAL :: ALPHA
REAL, DIMENSION(:) :: Y
```

C INTERFACE

```
#include <sunperf.h>
```

```
void sscal(int n, float alpha, float *y, int incy);

void sscal_64(long n, float alpha, float *y, long incy);
```

PURPOSE

sscal Compute y := alpha * y where alpha is a scalar and y is an n-vector.

ARGUMENTS

N (input)
: On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

ALPHA (input)
: On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

Y (input/output)
: (1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
: On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
ssctr - Scatters elements from x into y.
```

SYNOPSIS

```
 SUBROUTINE SSCTR(NZ, X, INDX, Y)

 REAL X(*), Y(*)
 INTEGER NZ
 INTEGER INDX(*)

 SUBROUTINE SSCTR_64(NZ, X, INDX, Y)

 REAL X(*), Y(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 SUBROUTINE SCTR([NZ], X, INDX, Y)

 REAL, DIMENSION(:) :: X, Y
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 SUBROUTINE SCTR_64([NZ], X, INDX, Y)

 REAL, DIMENSION(:) :: X, Y
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
SSCTR - Scatters the components of a sparse vector x stored
in compressed form into specified components of a vector y
```

in full storage form.

```
 do i = 1, n
   y(indx(i)) = x(i)
 enddo
```

ARGUMENTS

NZ (input) - INTEGER
Number of elements in the compressed form.
Unchanged on exit.

X (input)
Vector containing the values to be scattered from compressed form into full storage form. Unchanged on exit.

INDX (input) - INTEGER
Vector containing the indices of the compressed form. It is assumed that the elements in INDX are distinct and greater than zero. Unchanged on exit.

Y (output)
Vector whose elements specified by indx have been set to the corresponding entries of x. Only the elements corresponding to the indices in indx have been modified.

Contents

NAME

sskymm - Skyline format matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE SSKYMM( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, PNTR,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, K, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER    PNTR(*),
 REAL       ALPHA, BETA
 REAL       VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE SSKYMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, PNTR,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, K, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER*8  PNTR(*),
 REAL       ALPHA, BETA
 REAL       VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTR(K+1)-PNTR(1) (upper triangular)
       NNZ = PNTR(M+1)-PNTR(1) (lower triangular)
       PNTR() size = (K+1) (upper triangular)
       PNTR() size = (M+1) (lower triangular)
```

F95 INTERFACE

```
 SUBROUTINE SKYMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL,
*    PNTR,  B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, K
 INTEGER, DIMENSION(:) ::  DESCRA,  PNTR
```

```
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) ::  VAL
 REAL, DIMENSION(:, :) ::  B, C

 SUBROUTINE SKYMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL,
*    PNTR,  B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, K
 INTEGER*8, DIMENSION(:) ::  DESCRA,  PNTR
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) ::  VAL
 REAL, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C
where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in skyline format and
op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general (NOT SUPPORTED)
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
```

```
                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

 VAL()          array contain the nonzeros of A in skyline profile form.
                Row-oriented if DESCRA(2) = 1 (lower triangular),
                column oriented if DESCRA(2) = 2 (upper triangular).
 PNTR()         integer array of length M+1 (lower triangular) or
                K+1 (upper triangular) such that PNTR(I)-PNTR(1)+1
                points to the location in VAL of the first element of
                the skyline profile in row (column) I.

 B()            rectangular array with first dimension LDB.

 LDB            leading dimension of B

 BETA           Scalar parameter

 C()            rectangular array with first dimension LDC.

 LDC            leading dimension of C

 WORK()         scratch array of length LWORK. WORK is not
                referenced in the current version.

 LWORK          length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS)
Standard", University of Tennessee, Knoxville, Tennessee,
1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

The SKY data structure is not supported for a general matrix structure (*DESCRA*(1)=0).

Also not supported:

1. lower triangular matrix A of size m by n where m > n
2. upper triangular matrix A of size m by n where m < n

Contents

NAME

```
sskysm - Skyline format triangular solve
```

SYNOPSIS

```
 SUBROUTINE SSKYSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, PNTR,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER    PNTR(*),
 REAL       ALPHA, BETA
 REAL       DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE SSKYSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, PNTR,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER*8  PNTR(*),
 REAL       ALPHA, BETA
 REAL       DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTR(M+1)-PNTR(1) (upper triangular)
       NNZ = PNTR(K+1)-PNTR(1) (lower triangular)
       PNTR() size = (M+1) (upper triangular)
       PNTR() size = (K+1) (lower triangular)
```

F95 INTERFACE

```
 SUBROUTINE SKYSM( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
*    PNTR,  B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, UNITD
 INTEGER, DIMENSION(:) ::  DESCRA,  PNTR
```

```
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) ::  VAL, DV
 REAL, DIMENSION(:, :) ::  B, C

 SUBROUTINE SKYSM_64( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA,
*    VAL, PNTR,  B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, UNITD
 INTEGER*8, DIMENSION(:) ::  DESCRA,  PNTR
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) ::  VAL, DV
 REAL, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C
where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a diagonal scaling matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in skyline format and
op( A )  is one  of

 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' )).
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row or column scaling (see section
                    NOTES for further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter
```

```
DESCRA()        Descriptor argument.  Five element integer array
                DESCRA(1) matrix structure
                  0 : general
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))

                Note: For the routine, DESCRA(1)=3 is only supported.
                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

VAL()           array contain the nonzeros of A in skyline profile form.
                Row-oriented if DESCRA(2) = 1 (lower triangular),
                column oriented if DESCRA(2) = 2 (upper triangular).

PNTR()          integer array of length M+1 (lower triangular) or
                K+1 (upper triangular) such that PNTR(I)-PNTR(1)+1
                points to the location in VAL of the first element of
                the skyline profile in row (column) I.

B()             rectangular array with first dimension LDB.

LDB             leading dimension of B

BETA            Scalar parameter

C()             rectangular array with first dimension LDC.

LDC             leading dimension of C

WORK()          scratch array of length LWORK.
                On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.

LWORK           length of WORK array.  LWORK should be at least M.

                For good performance, LWORK should generally be larger.
```

For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. Also not supported:
   a. lower triangular matrix A of size m by n where m > n
   b. upper triangular matrix A of size m by n where m < n

2. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

3. If UNITD =4, the routine scales the rows of A if *DESCRA*(2)=1 and the columns of A if *DESCRA*(2)=2 such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in this particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows (columns) have been scaled. UNITD=2 if *DESCRA*(2)=1 and UNITD=3 if *DESCRA*(2)=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row (column) number which 2-norm is exactly zero.

4. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements

might or might not be referenced in the SKY representation of a sparse matrix. They are not used anyway in these cases. But if UNITD=4, the unit diagonal elements MUST be referenced in the SKY representation.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

```
sspcon - estimate the reciprocal of the condition number (in
the  1-norm)  of  a real symmetric packed matrix A using the
factorization A = U*D*U**T  or  A  =  L*D*L**T  computed  by
SSPTRF
```

SYNOPSIS

```
SUBROUTINE SSPCON(UPLO, N, AP, IPIVOT, ANORM, RCOND, WORK, IWORK2,
      INFO)

CHARACTER * 1 UPLO
INTEGER N, INFO
INTEGER IPIVOT(*), IWORK2(*)
REAL ANORM, RCOND
REAL AP(*), WORK(*)

SUBROUTINE SSPCON_64(UPLO, N, AP, IPIVOT, ANORM, RCOND, WORK, IWORK2,
      INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*), IWORK2(*)
REAL ANORM, RCOND
REAL AP(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPCON(UPLO, N, AP, IPIVOT, ANORM, RCOND, [WORK], [IWORK2],
       [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT, IWORK2
```

```
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: AP, WORK

SUBROUTINE SPCON_64(UPLO, N, AP, IPIVOT, ANORM, RCOND, [WORK],
       [IWORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, IWORK2
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: AP, WORK
```

C INTERFACE

```
#include <sunperf.h>
void sspcon(char uplo, int n, float *ap, int *ipivot,  float
          anorm, float *rcond, int *info);

void sspcon_64(char uplo, long n, float *ap,  long  *ipivot,
          float anorm, float *rcond, long *info);
```

PURPOSE

sspcon estimates the reciprocal of the condition number (in the 1-norm) of a real symmetric packed matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by SSPTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

AP (input)
Real array, dimension (N*(N+1)/2) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by SSPTRF, stored as a packed triangular matrix.

IPIVOT (input)
Integer array, dimension (N) Details of the interchanges and the block structure of D as determined by SSPTRF.

ANORM (input)
The 1-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
Real array, dimension(2*N)

IWORK2 (workspace)
Integer array, dimension (2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
sspev - compute all the eigenvalues and, optionally,  eigen-
vectors of a real symmetric matrix A in packed storage
```

SYNOPSIS

```
SUBROUTINE SSPEV(JOBZ, UPLO, N, AP, W, Z, LDZ, WORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER N, LDZ, INFO
REAL AP(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSPEV_64(JOBZ, UPLO, N, AP, W, Z, LDZ, WORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 N, LDZ, INFO
REAL AP(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPEV(JOBZ, UPLO, N, AP, W, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: N, LDZ, INFO
REAL, DIMENSION(:) :: AP, W, WORK
REAL, DIMENSION(:,:) :: Z

SUBROUTINE SPEV_64(JOBZ, UPLO, N, AP, W, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: N, LDZ, INFO
REAL, DIMENSION(:) :: AP, W, WORK
REAL, DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void sspev(char jobz, char uplo, int n, float *ap, float *w,
          float *z, int ldz, int *info);

void sspev_64(char jobz, char uplo, long n, float *ap, float
          *w, float *z, long ldz, long *info);
```

PURPOSE

sspev computes all the eigenvalues and, optionally, eigenvectors of a real symmetric matrix A in packed storage.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

AP (input/output)
          Real array, dimension (N*(N+1)/2)  On  entry,  the
          upper or lower triangle of the symmetric matrix A,
          packed columnwise in a  linear  array.   The  j-th
          column  of A is stored in the array AP as follows:
          if UPLO = 'U', AP(i  +  (j-1)*j/2)  =  A(i,j)  for
          1<=i<=j;  if UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) =
          A(i,j) for j<=i<=n.

          On exit, AP is  overwritten  by  values  generated
          during the reduction to tridiagonal form.  If UPLO
          = 'U', the diagonal and first superdiagonal of the
          tridiagonal  matrix  T overwrite the corresponding
          elements of A, and if UPLO = 'L', the diagonal and
          first subdiagonal of T overwrite the corresponding
          elements of A.

W (output)
          Real array, dimension (N) If INFO = 0, the  eigen-
          values in ascending order.
```

```
Z (output)
          Real array, dimension (LDZ, N) If JOBZ = 'V', then
          if  INFO = 0, Z contains the orthonormal eigenvec-
          tors of the matrix A, with the i-th  column  of  Z
          holding  the eigenvector associated with W(i).  If
          JOBZ = 'N', then Z is not referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
          Real array, dimension(3*N)
INFO (output)
          = 0:  successful exit.
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value.
          > 0:  if INFO = i, the algorithm  failed  to  con-
          verge;  i off-diagonal elements of an intermediate
          tridiagonal form did not converge to zero.
```

Contents

NAME

```
sspevd - compute all the eigenvalues and, optionally, eigen-
vectors of a real symmetric matrix A in packed storage
```

SYNOPSIS

```
SUBROUTINE SSPEVD(JOBZ, UPLO, N, AP, W, Z, LDZ, WORK, LWORK, IWORK,
      LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER N, LDZ, LWORK, LIWORK, INFO
INTEGER IWORK(*)
REAL AP(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSPEVD_64(JOBZ, UPLO, N, AP, W, Z, LDZ, WORK, LWORK,
      IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 N, LDZ, LWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
REAL AP(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPEVD(JOBZ, UPLO, N, AP, W, Z, [LDZ], [WORK], [LWORK],
       [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: N, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: AP, W, WORK
REAL, DIMENSION(:,:) :: Z

SUBROUTINE SPEVD_64(JOBZ, UPLO, N, AP, W, Z, [LDZ], [WORK], [LWORK],
```

```
       [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: N, LDZ, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: AP, W, WORK
REAL, DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void sspevd(char jobz, char uplo, int n,  float  *ap,  float
          *w, float *z, int ldz, int *info);
void sspevd_64(char jobz, char  uplo,  long  n,  float  *ap,
          float *w, float *z, long ldz, long *info);
```

PURPOSE

sspevd computes all the eigenvalues and, optionally, eigenvectors of a real symmetric matrix A in packed storage. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

JOBZ (input)
          = 'N': Compute eigenvalues only;
          = 'V': Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U': Upper triangle of A is stored;
          = 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

AP (input/output)
          Real array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array.   The j-th

```
          column  of A is stored in the array AP as follows:
          if UPLO = 'U', AP(i  +  (j-1)*j/2)  =  A(i,j)  for
          1<=i<=j;  if UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) =
          A(i,j) for j<=i<=n.

          On exit, AP is  overwritten  by  values  generated
          during the reduction to tridiagonal form.  If UPLO
          = 'U', the diagonal and first superdiagonal of the
          tridiagonal  matrix  T overwrite the corresponding
          elements of A, and if UPLO = 'L', the diagonal and
          first subdiagonal of T overwrite the corresponding
          elements of A.
W (output)
          Real array, dimension (N) If INFO = 0, the  eigen-
          values in ascending order.

Z (input) Real array, dimension (LDZ, N) If JOBZ = 'V', then
          if  INFO = 0, Z contains the orthonormal eigenvec-
          tors of the matrix A, with the i-th  column  of  Z
          holding  the eigenvector associated with W(i).  If
          JOBZ = 'N', then Z is not referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
          Real array, dimension (LWORK) On exit, if  INFO  =
          0, WORK(1) returns the optimal LWORK.

LWORK (input)
          The dimension of the  array  WORK.   If  N  <=  1,
          LWORK  must  be at least 1.  If JOBZ = 'N' and N >
          1, LWORK must be at least 2*N.  If JOBZ = 'V'  and
          N > 1, LWORK must be at least 1 + 6*N + N**2.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

IWORK (workspace/output)
          Integer array, dimension (LIWORK) On exit, if INFO
          = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
          The dimension of the array IWORK.  If JOBZ  =  'N'
          or  N <= 1, LIWORK must be at least 1.  If JOBZ  =
          'V' and N > 1, LIWORK must be at least 3 + 5*N.
```

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value.

> 0: if INFO = i, the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

Contents

NAME

```
sspevx  -  compute  selected  eigenvalues  and,  optionally,
eigenvectors of a real symmetric matrix A in packed storage
```

SYNOPSIS

```
SUBROUTINE SSPEVX(JOBZ, RANGE, UPLO, N, AP, VL, VU, IL, IU, ABTOL,
      NFOUND, W, Z, LDZ, WORK, IWORK2, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER N, IL, IU, NFOUND, LDZ, INFO
INTEGER IWORK2(*), IFAIL(*)
REAL VL, VU, ABTOL
REAL AP(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSPEVX_64(JOBZ, RANGE, UPLO, N, AP, VL, VU, IL, IU, ABTOL,
      NFOUND, W, Z, LDZ, WORK, IWORK2, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER*8 N, IL, IU, NFOUND, LDZ, INFO
INTEGER*8 IWORK2(*), IFAIL(*)
REAL VL, VU, ABTOL
REAL AP(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPEVX(JOBZ, RANGE, UPLO, N, AP, VL, VU, IL, IU, ABTOL,
       [NFOUND], W, Z, [LDZ], [WORK], [IWORK2], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER :: N, IL, IU, NFOUND, LDZ, INFO
INTEGER, DIMENSION(:) :: IWORK2, IFAIL
REAL :: VL, VU, ABTOL
REAL, DIMENSION(:) :: AP, W, WORK
```

```
REAL, DIMENSION(:,:) :: Z

SUBROUTINE SPEVX_64(JOBZ, RANGE, UPLO, N, AP, VL, VU, IL, IU, ABTOL,
       [NFOUND], W, Z, [LDZ], [WORK], [IWORK2], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER(8) :: N, IL, IU, NFOUND, LDZ, INFO
INTEGER(8), DIMENSION(:) :: IWORK2, IFAIL
REAL :: VL, VU, ABTOL
REAL, DIMENSION(:) :: AP, W, WORK
REAL, DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>
void sspevx(char jobz, char range, char uplo, int  n,  float
          *ap,  float  vl,  float  vu, int il, int iu, float
          abtol, int *nfound, float *w, float *z,  int  ldz,
          int *ifail, int *info);

void sspevx_64(char jobz, char range,  char  uplo,  long  n,
          float  *ap,  float vl, float vu, long il, long iu,
          float abtol, long *nfound,  float  *w,  float  *z,
          long ldz, long *ifail, long *info);
```

PURPOSE

sspevx computes selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix A in packed storage. Eigenvalues/vectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

RANGE (input)
= 'A': all eigenvalues will be found;
= 'V': all eigenvalues in the half-open interval (VL,VU] will be found; = 'I': the IL-th through IU-th eigenvalues will be found.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

AP (input/output)

Real array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

On exit, AP is overwritten by values generated during the reduction to tridiagonal form. If UPLO = 'U', the diagonal and first superdiagonal of the tridiagonal matrix T overwrite the corresponding elements of A, and if UPLO = 'L', the diagonal and first subdiagonal of T overwrite the corresponding elements of A.

VL (input)

If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)

See the description of VL.

IL (input)

If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)

See the description of IL.

ABTOL (input)

The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing AP to tridiagonal form.

Eigenvalues will be computed most accurately when ABTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABTOL to 2*SLAMCH('S').

See "Computing Small Singular Values of Bidiagonal Matrices with Guaranteed High Relative Accuracy," by Demmel and Kahan, LAPACK Working Note #3.

NFOUND (output)

The total number of eigenvalues found. 0 <= NFOUND <= N. If RANGE = 'A', NFOUND = N, and if RANGE = 'I', NFOUND = IU-IL+1.

W (output)

Real array, dimension (N) If INFO = 0, the selected eigenvalues in ascending order.

Z (output)

Real array, dimension (LDZ, max(1,M)) If JOBZ = 'V', then if INFO = 0, the first NFOUND columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). If an eigenvector fails to converge, then that column of Z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in IFAIL. If JOBZ = 'N', then Z is not referenced. Note: the user must ensure that at least max(1,NFOUND) columns are supplied in the array Z; if RANGE = 'V', the exact value of NFOUND is not known in advance and an upper bound must be used.

LDZ (input)

The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)

Real array, dimension(8*N)

IWORK2 (workspace)

Integer array, dimension (5*N)

IFAIL (output)

Integer array, dimension (N) If JOBZ = 'V', then if INFO = 0, the first NFOUND elements of IFAIL are zero. If INFO > 0, then IFAIL contains the indices of the eigenvectors that failed to converge. If JOBZ = 'N', then IFAIL is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, then i eigenvectors failed to converge. Their indices are stored in array IFAIL.

Contents

NAME

```
sspgst - reduce a real symmetric-definite generalized eigen-
problem to standard form, using packed storage
```

SYNOPSIS

```
SUBROUTINE SSPGST(ITYPE, UPLO, N, AP, BP, INFO)

CHARACTER * 1 UPLO
INTEGER ITYPE, N, INFO
REAL AP(*), BP(*)

SUBROUTINE SSPGST_64(ITYPE, UPLO, N, AP, BP, INFO)

CHARACTER * 1 UPLO
INTEGER*8 ITYPE, N, INFO
REAL AP(*), BP(*)
```

F95 INTERFACE

```
SUBROUTINE SPGST(ITYPE, UPLO, N, AP, BP, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: ITYPE, N, INFO
REAL, DIMENSION(:) :: AP, BP

SUBROUTINE SPGST_64(ITYPE, UPLO, N, AP, BP, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: ITYPE, N, INFO
REAL, DIMENSION(:) :: AP, BP
```

C INTERFACE

```
#include <sunperf.h>
```

```
void sspgst(int itype, char uplo, int n,  float  *ap,  float
          *bp, int *info);

void sspgst_64(long itype, char uplo,  long  n,  float  *ap,
          float *bp, long *info);
```

PURPOSE

sspgst reduces a real symmetric-definite generalized eigen-problem to standard form, using packed storage.

If ITYPE = 1, the problem is A*x = lambda*B*x,
and A is overwritten by inv(U**T)*A*inv(U) or inv(L)*A*inv(L**T)
If ITYPE = 2 or 3, the problem is A*B*x = lambda*x or
B*A*x = lambda*x, and A is overwritten by U*A*U**T or L**T*A*L.

B must have been previously factorized as U**T*U or L*L**T by SPPTRF.

ARGUMENTS

ITYPE (input)
= 1: compute inv(U**T)*A*inv(U) or inv(L)*A*inv(L**T);
= 2 or 3: compute U*A*U**T or L**T*A*L.

UPLO (input)
= 'U': Upper triangle of A is stored and B is factored as U**T*U; = 'L': Lower triangle of A is stored and B is factored as L*L**T.

N (input) The order of the matrices A and B. N >= 0.

AP (input/output)
Real array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n.

On exit, if INFO = 0, the transformed matrix, stored in the same format as A.

BP (input)
Real array, dimension (N*(N+1)/2) The triangular factor from the Cholesky factorization of B, stored in the same format as A, as returned by SPPTRF.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
sspgv - compute all the  eigenvalues  and,  optionally,  the
eigenvectors of a real generalized symmetric-definite eigen-
problem, of the form A*x=(lambda)*B*x,  A*Bx=(lambda)*x,  or
B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE SSPGV(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, LDZ, WORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER ITYPE, N, LDZ, INFO
REAL AP(*), BP(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSPGV_64(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, LDZ, WORK,
      INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 ITYPE, N, LDZ, INFO
REAL AP(*), BP(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPGV(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, [LDZ], [WORK],
       [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: ITYPE, N, LDZ, INFO
REAL, DIMENSION(:) :: AP, BP, W, WORK
REAL, DIMENSION(:,:) :: Z

SUBROUTINE SPGV_64(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, [LDZ], [WORK],
       [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: ITYPE, N, LDZ, INFO
REAL, DIMENSION(:) :: AP, BP, W, WORK
REAL, DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void sspgv(int itype, char jobz, char  uplo,  int  n,  float
          *ap,  float  *bp, float *w, float *z, int ldz, int
          *info);

void sspgv_64(long itype, char  jobz,  char  uplo,  long  n,
          float  *ap,  float  *bp,  float *w, float *z, long
          ldz, long *info);
```

PURPOSE

sspgv computes all the eigenvalues and, optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be symmetric, stored in packed format, and B is also positive definite.

ARGUMENTS

```
ITYPE (input)
          Specifies the problem type to be solved:
          = 1:  A*x = (lambda)*B*x
          = 2:  A*B*x = (lambda)*x
          = 3:  B*A*x = (lambda)*x

JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.

AP (input/output)
          Real array, dimension (N*(N+1)/2)  On  entry,  the
          upper or lower triangle of the symmetric matrix A,
          packed columnwise in a  linear  array.   The  j-th
          column  of A is stored in the array AP as follows:
```

if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

On exit, the contents of AP are destroyed.

BP (input/output)

Real array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix B, packed columnwise in a linear array. The j-th column of B is stored in the array BP as follows: if UPLO = 'U', BP(i + (j-1)*j/2) = B(i,j) for 1<=i<=j; if UPLO = 'L', BP(i + (j-1)*(2*n-j)/2) = B(i,j) for j<=i<=n.

On exit, the triangular factor U or L from the Cholesky factorization B = U**T*U or B = L*L**T, in the same storage format as B.

W (output)

Real array, dimension (N) If INFO = 0, the eigenvalues in ascending order.

Z (output)

Real array, dimension (LDZ, N) If JOBZ = 'V', then if INFO = 0, Z contains the matrix Z of eigenvectors. The eigenvectors are normalized as follows: if ITYPE = 1 or 2, Z**T*B*Z = I; if ITYPE = 3, Z**T*inv(B)*Z = I. If JOBZ = 'N', then Z is not referenced.

LDZ (input)

The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)

Real array, dimension(3*N)

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: SPPTRF or SSPEV returned an error code:

<= N: if INFO = i, SSPEV failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero. > N: if INFO = n + i, for 1 <= i <= n, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

Contents

NAME

```
sspgvd - compute all the eigenvalues,  and  optionally,  the
eigenvectors of a real generalized symmetric-definite eigen-
problem, of the form A*x=(lambda)*B*x,  A*Bx=(lambda)*x,  or
B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE SSPGVD(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, LDZ, WORK,
      LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER ITYPE, N, LDZ, LWORK, LIWORK, INFO
INTEGER IWORK(*)
REAL AP(*), BP(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSPGVD_64(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, LDZ, WORK,
      LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 ITYPE, N, LDZ, LWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
REAL AP(*), BP(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPGVD(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, [LDZ], [WORK],
       [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: ITYPE, N, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: AP, BP, W, WORK
```

```
REAL, DIMENSION(:,:) :: Z

SUBROUTINE SPGVD_64(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, [LDZ],
       [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: ITYPE, N, LDZ, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: AP, BP, W, WORK
REAL, DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void sspgvd(int itype, char jobz, char uplo,  int  n,  float
          *ap,  float  *bp, float *w, float *z, int ldz, int
          *info);

void sspgvd_64(long itype, char jobz,  char  uplo,  long  n,
          float  *ap,  float  *bp,  float *w, float *z, long
          ldz, long *info);
```

PURPOSE

sspgvd computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be symmetric, stored in packed format, and B is also positive definite. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

```
ITYPE (input)
          Specifies the problem type to be solved:
          = 1:  A*x = (lambda)*B*x
          = 2:  A*B*x = (lambda)*x
          = 3:  B*A*x = (lambda)*x
```

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.

AP (input/output)
          Real array, dimension (N*(N+1)/2)  On  entry,  the
          upper or lower triangle of the symmetric matrix A,
          packed columnwise in a  linear  array.   The  j-th
          column  of A is stored in the array AP as follows:
          if UPLO = 'U', AP(i  +  (j-1)*j/2)  =  A(i,j)  for
          1<=i<=j;  if UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) =
          A(i,j) for j<=i<=n.

          On exit, the contents of AP are destroyed.

BP (input/output)
          Real array, dimension (N*(N+1)/2)  On  entry,  the
          upper or lower triangle of the symmetric matrix B,
          packed columnwise in a  linear  array.   The  j-th
          column  of B is stored in the array BP as follows:
          if UPLO = 'U', BP(i  +  (j-1)*j/2)  =  B(i,j)  for
          1<=i<=j;  if UPLO = 'L', BP(i + (j-1)*(2*n-j)/2) =
          B(i,j) for j<=i<=n.

          On exit, the triangular factor U  or  L  from  the
          Cholesky  factorization  B = U**T*U or B = L*L**T,
          in the same storage format as B.

W (output)
          Real array, dimension (N) If INFO = 0, the  eigen-
          values in ascending order.

Z (output)
          Real array, dimension (LDZ, N) If JOBZ = 'V', then
          if  INFO = 0, Z contains the matrix Z of eigenvec-
          tors.  The eigenvectors are normalized as follows:
          if  ITYPE  =  1  or 2, Z**T*B*Z = I; if ITYPE = 3,
          Z**T*inv(B)*Z = I.  If JOBZ = 'N', then Z  is  not
          referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).
```

WORK (workspace/output)
Real array, dimension (LWORK) On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If N <= 1, LWORK >= 1. If JOBZ = 'N' and N > 1, LWORK >= 2*N. If JOBZ = 'V' and N > 1, LWORK >= 1 + 6*N + 2*N**2.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace/output)
Integer array, dimension (LIWORK) On exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of the array IWORK. If JOBZ = 'N' or N <= 1, LIWORK >= 1. If JOBZ = 'V' and N > 1, LIWORK >= 3 + 5*N.

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: SPPTRF or SSPEVD returned an error code:
<= N: if INFO = i, SSPEVD failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero; > N: if INFO = N + i, for 1 <= i <= N, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
Mark Fahey, Department of Mathematics, Univ. of Kentucky, USA

Contents

NAME

```
sspgvx  -  compute  selected  eigenvalues,  and  optionally,
eigenvectors of a real generalized symmetric-definite eigen-
problem, of the form A*x=(lambda)*B*x,  A*Bx=(lambda)*x,  or
B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE SSPGVX(ITYPE, JOBZ, RANGE, UPLO, N, AP, BP, VL, VU, IL,
      IU, ABSTOL, M, W, Z, LDZ, WORK, IWORK, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER ITYPE, N, IL, IU, M, LDZ, INFO
INTEGER IWORK(*), IFAIL(*)
REAL VL, VU, ABSTOL
REAL AP(*), BP(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSPGVX_64(ITYPE, JOBZ, RANGE, UPLO, N, AP, BP, VL, VU, IL,
      IU, ABSTOL, M, W, Z, LDZ, WORK, IWORK, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER*8 ITYPE, N, IL, IU, M, LDZ, INFO
INTEGER*8 IWORK(*), IFAIL(*)
REAL VL, VU, ABSTOL
REAL AP(*), BP(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPGVX(ITYPE, JOBZ, RANGE, UPLO, N, AP, BP, VL, VU, IL,
       IU, ABSTOL, M, W, Z, [LDZ], [WORK], [IWORK], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER :: ITYPE, N, IL, IU, M, LDZ, INFO
```

```
INTEGER, DIMENSION(:) :: IWORK, IFAIL
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: AP, BP, W, WORK
REAL, DIMENSION(:,:) :: Z

SUBROUTINE SPGVX_64(ITYPE, JOBZ, RANGE, UPLO, N, AP, BP, VL, VU,
       IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK], [IWORK], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER(8) :: ITYPE, N, IL, IU, M, LDZ, INFO
INTEGER(8), DIMENSION(:) :: IWORK, IFAIL
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: AP, BP, W, WORK
REAL, DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void sspgvx(int itype, char jobz, char range, char uplo, int
          n,  float  *ap, float *bp, float vl, float vu, int
          il, int iu, float abstol, int *m, float *w,  float
          *z, int ldz, int *ifail, int *info);

void sspgvx_64(long itype, char jobz, char range, char uplo,
          long  n, float *ap, float *bp, float vl, float vu,
          long il, long iu, float abstol, long *m, float *w,
          float *z, long ldz, long *ifail, long *info);
```

PURPOSE

sspgvx computes selected eigenvalues, and optionally, eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be symmetric, stored in packed storage, and B is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

```
ITYPE (input)
          Specifies the problem type to be solved:
          = 1:  A*x = (lambda)*B*x
          = 2:  A*B*x = (lambda)*x
          = 3:  B*A*x = (lambda)*x

JOBZ (input)
```

```
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

RANGE (input)
          = 'A': all eigenvalues will be found.
          = 'V': all eigenvalues in the  half-open  interval
          (VL,VU]  will  be found.  = 'I': the IL-th through
          IU-th eigenvalues will be found.

UPLO (input)
          = 'U':  Upper triangle of A and B are stored;
          = 'L':  Lower triangle of A and B are stored.

N (input) The order of the matrix pencil (A,B).  N >= 0.
AP (input/output)
          Real array, dimension (N*(N+1)/2)  On  entry,  the
          upper or lower triangle of the symmetric matrix A,
          packed columnwise in a  linear  array.   The  j-th
          column  of A is stored in the array AP as follows:
          if UPLO = 'U', AP(i  +  (j-1)*j/2)  =  A(i,j)  for
          1<=i<=j;  if UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) =
          A(i,j) for j<=i<=n.

          On exit, the contents of AP are destroyed.

BP (input/output)
          Real array, dimension (N*(N+1)/2)  On  entry,  the
          upper or lower triangle of the symmetric matrix B,
          packed columnwise in a  linear  array.   The  j-th
          column  of B is stored in the array BP as follows:
          if UPLO = 'U', BP(i  +  (j-1)*j/2)  =  B(i,j)  for
          1<=i<=j;  if UPLO = 'L', BP(i + (j-1)*(2*n-j)/2) =
          B(i,j) for j<=i<=n.

          On exit, the triangular factor U  or  L  from  the
          Cholesky  factorization  B = U**T*U or B = L*L**T,
          in the same storage format as B.

VL (input)
          If RANGE='V', the lower and upper  bounds  of  the
          interval  to be searched for eigenvalues. VL < VU.
          Not referenced if RANGE = 'A' or 'I'.

VU (input)
          See the description of VL.

IL (input)
          If RANGE='I', the indices (in ascending order)  of
          the   smallest   and  largest  eigenvalues  to  be
          returned.  1 <= IL <= IU <= N, if N > 0;  IL  =  1
```

and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)

See the description of IL.

ABSTOL (input)

The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABSTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABSTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing A to tri- diagonal form.

Eigenvalues will be computed most accurately when ABSTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABSTOL to 2*SLAMCH('S').

M (output)

The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)

Real array, dimension (N) On normal exit, the first M elements contain the selected eigenvalues in ascending order.

Z (output)

Real array, dimension (LDZ, max(1,M)) If JOBZ = 'N', then Z is not referenced. If JOBZ = 'V', then if INFO = 0, the first M columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector asso- ciated with W(i). The eigenvectors are normalized as follows: if ITYPE = 1 or 2, Z**T*B*Z = I; if ITYPE = 3, Z**T*inv(B)*Z = I.

If an eigenvector fails to converge, then that column of Z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in IFAIL. Note: the user must ensure that at least max(1,M) columns are supplied in the array Z; if RANGE = 'V', the exact value of M is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
Real array, dimension(8*N)

IWORK (workspace)
INTEGER array, dimension(5*N)

IFAIL (output)
INTEGER array, dimension (N) If JOBZ = 'V', then if INFO = 0, the first M elements of IFAIL are zero. If INFO > 0, then IFAIL contains the indices of the eigenvectors that failed to converge. If JOBZ = 'N', then IFAIL is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: SPPTRF or SSPEVX returned an error code:
<= N: if INFO = i, SSPEVX failed to converge; i eigenvectors failed to converge. Their indices are stored in array IFAIL. > N: if INFO = N + i, for 1 <= i <= N, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
Mark Fahey, Department of Mathematics, Univ. of Kentucky, USA

Contents

NAME

```
sspmv - perform the matrix-vector operation   y := alpha*A*x
+ beta*y
```

SYNOPSIS

```
SUBROUTINE SSPMV(UPLO, N, ALPHA, A, X, INCX, BETA, Y, INCY)

CHARACTER * 1 UPLO
INTEGER N, INCX, INCY
REAL ALPHA, BETA
REAL A(*), X(*), Y(*)

SUBROUTINE SSPMV_64(UPLO, N, ALPHA, A, X, INCX, BETA, Y, INCY)

CHARACTER * 1 UPLO
INTEGER*8 N, INCX, INCY
REAL ALPHA, BETA
REAL A(*), X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE SPMV(UPLO, N, ALPHA, A, X, [INCX], BETA, Y, [INCY])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INCX, INCY
REAL :: ALPHA, BETA
REAL, DIMENSION(:) :: A, X, Y

SUBROUTINE SPMV_64(UPLO, N, ALPHA, A, X, [INCX], BETA, Y, [INCY])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INCX, INCY
REAL :: ALPHA, BETA
```

```
REAL, DIMENSION(:) :: A, X, Y
```

C INTERFACE

```
#include <sunperf.h>

void sspmv(char uplo, int n, float alpha,  float  *a,  float
          *x, int incx, float beta, float *y, int incy);

void sspmv_64(char uplo, long  n,  float  alpha,  float  *a,
          float  *x,  long  incx, float beta, float *y, long
          incy);
```

PURPOSE

sspmv performs the matrix-vector operation y := alpha*A*x + beta*y, where alpha and beta are scalars, x and y are n element vectors and A is an n by n symmetric matrix, supplied in packed form.

ARGUMENTS

UPLO (input)

On entry, UPLO specifies whether the upper or lower triangular part of the matrix A is supplied in the packed array A as follows:

UPLO = 'U' or 'u'   The upper triangular part of A is supplied in A.

UPLO = 'L' or 'l'   The lower triangular part of A is supplied in A.

Unchanged on exit.

N (input)

On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)

On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)

((n*(n + 1))/2). Before entry with UPLO = 'U' or 'u', the array A must contain the upper triangular part of the symmetric matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(1, 2) and a(2, 2) respectively, and so on. Before entry with UPLO = 'L' or 'l', the array A must contain the lower triangular part of the symmetric matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(2, 1) and a(3, 1) respectively, and so on. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. When BETA is supplied as zero then Y need not be set on input. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

sspr - perform the symmetric rank 1 operation A := alpha*x*x' + A

SYNOPSIS

```
SUBROUTINE SSPR(UPLO, N, ALPHA, X, INCX, A)

CHARACTER * 1 UPLO
INTEGER N, INCX
REAL ALPHA
REAL X(*), A(*)

SUBROUTINE SSPR_64(UPLO, N, ALPHA, X, INCX, A)

CHARACTER * 1 UPLO
INTEGER*8 N, INCX
REAL ALPHA
REAL X(*), A(*)
```

F95 INTERFACE

```
SUBROUTINE SPR(UPLO, N, ALPHA, X, [INCX], A)

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INCX
REAL :: ALPHA
REAL, DIMENSION(:) :: X, A

SUBROUTINE SPR_64(UPLO, N, ALPHA, X, [INCX], A)

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INCX
REAL :: ALPHA
```

```
REAL, DIMENSION(:) :: X, A
```

C INTERFACE

```
#include <sunperf.h>

void sspr(char uplo, int n, float alpha, float *x, int incx,
          float *a);

void sspr_64(char uplo, long n, float alpha, float *x,  long
          incx, float *a);
```

PURPOSE

sspr performs the symmetric rank 1 operation A := alpha*x*x' + A, where alpha is a real scalar, x is an n element vector and A is an n by n symmetric matrix, supplied in packed form.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the matrix A is supplied in the packed array A as follows:

UPLO = 'U' or 'u'   The upper triangular part of A is supplied in A.

UPLO = 'L' or 'l'   The lower triangular part of A is supplied in A.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)

On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

A (input/output)

((n*(n + 1))/2). Before entry with UPLO = 'U' or 'u', the array A must contain the upper triangular part of the symmetric matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(1, 2) and a(2, 2) respectively, and so on. On exit, the array A is overwritten by the upper triangular part of the updated matrix. Before entry with UPLO = 'L' or 'l', the array A must contain the lower triangular part of the symmetric matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(2, 1) and a(3, 1) respectively, and so on. On exit, the array A is overwritten by the lower triangular part of the updated matrix.

Contents

NAME

sspr2 - perform the symmetric rank 2 operation A := alpha*x*y' + alpha*y*x' + A

SYNOPSIS

```
SUBROUTINE SSPR2(UPLO, N, ALPHA, X, INCX, Y, INCY, AP)

CHARACTER * 1 UPLO
INTEGER N, INCX, INCY
REAL ALPHA
REAL X(*), Y(*), AP(*)

SUBROUTINE SSPR2_64(UPLO, N, ALPHA, X, INCX, Y, INCY, AP)

CHARACTER * 1 UPLO
INTEGER*8 N, INCX, INCY
REAL ALPHA
REAL X(*), Y(*), AP(*)
```

F95 INTERFACE

```
SUBROUTINE SPR2(UPLO, [N], ALPHA, X, [INCX], Y, [INCY], AP)

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INCX, INCY
REAL :: ALPHA
REAL, DIMENSION(:) :: X, Y, AP

SUBROUTINE SPR2_64(UPLO, [N], ALPHA, X, [INCX], Y, [INCY], AP)

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INCX, INCY
REAL :: ALPHA
```

```
REAL, DIMENSION(:) :: X, Y, AP
```

C INTERFACE

```
#include <sunperf.h>

void sspr2(char uplo, int n,  float  alpha,  float  *x,  int
          incx, float *y, int incy, float *ap);

void sspr2_64(char uplo, long n, float alpha, float *x, long
          incx, float *y, long incy, float *ap);
```

PURPOSE

sspr2 performs the symmetric rank 2 operation A := alpha*x*y' + alpha*y*x' + A, where alpha is a scalar, x and y are n element vectors and A is an n by n symmetric matrix, supplied in packed form.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the matrix A is supplied in the packed array A as follows:

UPLO = 'U' or 'u'   The upper triangular part of A is supplied in AP.

UPLO = 'L' or 'l'   The lower triangular part of A is supplied in AP.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
Real array, dimension (1 + (n - 1)*abs(INCX)) Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)

On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

Y (input)
Real array, dimension (1 + (n - 1)*abs(INCY)) Before entry, the incremented array Y must contain the n element vector y. Unchanged on exit.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

AP (input/output)
Real array, dimension ((n*(n + 1))/2) Before entry with UPLO = 'U' or 'u', the array AP must contain the upper triangular part of the symmetric matrix packed sequentially, column by column, so that AP(1) contains a(1, 1), AP(2) and AP(3) contain a(1, 2) and a(2, 2) respectively, and so on. On exit, the array A is overwritten by the upper triangular part of the updated matrix. Before entry with UPLO = 'L' or 'l', the array AP must contain the lower triangular part of the symmetric matrix packed sequentially, column by column, so that AP(1) contains a(1, 1), AP(2) and AP(3) contain a(2, 1) and a(3, 1) respectively, and so on. On exit, the array AP is overwritten by the lower triangular part of the updated matrix.

Contents

NAME

```
ssprfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is symmetric indefin-
ite and packed, and provides error bounds and backward error
estimates for the solution
```

SYNOPSIS

```
SUBROUTINE SSPRFS(UPLO, N, NRHS, AP, AF, IPIVOT, B, LDB, X, LDX, FERR,
      BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*), WORK2(*)
REAL AP(*), AF(*),  B(LDB,*),  X(LDX,*),  FERR(*),  BERR(*),
WORK(*)

SUBROUTINE SSPRFS_64(UPLO, N, NRHS, AP, AF, IPIVOT, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
REAL AP(*), AF(*),  B(LDB,*),  X(LDX,*),  FERR(*),  BERR(*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPRFS(UPLO, N, [NRHS], AP, AF, IPIVOT, B, [LDB], X, [LDX],
       FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
```

```
REAL, DIMENSION(:) :: AP, AF, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: B, X

SUBROUTINE SPRFS_64(UPLO, N, [NRHS], AP, AF, IPIVOT, B, [LDB], X, [LDX],
       FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL, DIMENSION(:) :: AP, AF, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: B, X
```

C INTERFACE

```
#include <sunperf.h>
void ssprfs(char uplo, int n, int  nrhs,  float  *ap,  float
          *af, int *ipivot, float *b, int ldb, float *x, int
          ldx, float *ferr, float *berr, int *info);

void ssprfs_64(char uplo, long  n,  long  nrhs,  float  *ap,
          float *af, long *ipivot, float *b, long ldb, float
          *x, long ldx,  float  *ferr,  float  *berr,  float
          *work, long *iwork, long *info);
```

PURPOSE

ssprfs improves the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite and packed, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) Real array, dimension (N*(N+1)/2) The upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if

UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

AF (input)
Real array, dimension (N*(N+1)/2) The factored form of the matrix A. AF contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by SSPTRF, stored as a packed triangular matrix.

IPIVOT (input)
Integer array, dimension (N) Details of the interchanges and the block structure of D as determined by SSPTRF.

B (input) Real array, dimension (LDB,NRHS) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
Real array, dimension (LDX,NRHS) On entry, the solution matrix X, as computed by SSPTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
Real array, dimension (NRHS) The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
Real array, dimension (NRHS) The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

```
WORK (workspace)
          Real array, dimension(3*N)

WORK2 (workspace)
          Integer array, dimension(N)
INFO (output) Integer
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
sspsv - compute the solution to  a  real  system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE SSPSV(UPLO, N, NRHS, AP, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDB, INFO
INTEGER IPIVOT(*)
REAL AP(*), B(LDB,*)

SUBROUTINE SSPSV_64(UPLO, N, NRHS, AP, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIVOT(*)
REAL AP(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE SPSV(UPLO, N, [NRHS], AP, IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: AP
REAL, DIMENSION(:,:) :: B

SUBROUTINE SPSV_64(UPLO, N, [NRHS], AP, IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: AP
REAL, DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void sspsv(char uplo,  int  n,  int  nrhs,  float  *ap,  int
          *ipivot, float *b, int ldb, int *info);

void sspsv_64(char uplo, long n, long nrhs, float *ap,  long
          *ipivot, float *b, long ldb, long *info);
```

PURPOSE

sspsv computes the solution to a real system of linear equations
   A * X = B, where A is an N-by-N symmetric matrix stored in packed format and X and B are N-by-NRHS matrices.

The diagonal pivoting method is used to factor A as
   A = U * D * U**T, if UPLO = 'U', or
   A = L * D * L**T, if UPLO = 'L',
where U (or L) is a product of permutation and unit upper (lower) triangular matrices, D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks. The factored form of A is then used to solve the system of equations A * X = B.

ARGUMENTS

UPLO (input)
  = 'U': Upper triangle of A is stored;
  = 'L': Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NRHS (input)
  The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

AP (input/output)
  Real array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th

column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n. See below for further details.

On exit, the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by SSPTRF, stored as a packed triangular matrix in the same storage format as A.

IPIVOT (output)

Integer array, dimension (N) Details of the interchanges and the block structure of D, as determined by SSPTRF. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged, and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

B (input/output)

Real array, dimension (LDB,NRHS) On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)

The leading dimension of the array B. LDB >= max(1,N).

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, so the solution could not be computed.

FURTHER DETAILS

The packed storage scheme is illustrated by the following example when N = 4, UPLO = 'U':

Two-dimensional storage of the symmetric matrix A:

```
   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = aji)
               a44

Packed storage of the upper triangle of A:

AP = [ a11, a12, a22, a13, a23, a33, a14, a24, a34, a44 ]
```

Contents

NAME

```
sspsvx  -  use  the  diagonal  pivoting  factorization  A  =
U*D*U**T  or  A = L*D*L**T to compute the solution to a real
system of linear equations A * X = B, where A is  an  N-by-N
symmetric  matrix stored in packed format and X and B are N-
by-NRHS matrices
```

SYNOPSIS

```
SUBROUTINE SSPSVX(FACT, UPLO, N, NRHS, AP, AF, IPIVOT, B, LDB, X, LDX,
      RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*), WORK2(*)
REAL RCOND
REAL AP(*), AF(*),  B(LDB,*),  X(LDX,*),  FERR(*),  BERR(*),
WORK(*)

SUBROUTINE SSPSVX_64(FACT, UPLO, N, NRHS, AP, AF, IPIVOT, B, LDB, X,
      LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
REAL RCOND
REAL AP(*), AF(*),  B(LDB,*),  X(LDX,*),  FERR(*),  BERR(*),
WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPSVX(FACT, UPLO, N, [NRHS], AP, AF, IPIVOT, B, [LDB], X,
       [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: FACT, UPLO
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
REAL :: RCOND
REAL, DIMENSION(:) :: AP, AF, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: B, X

SUBROUTINE SPSVX_64(FACT, UPLO, N, [NRHS], AP, AF, IPIVOT, B, [LDB], X,
       [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL :: RCOND
REAL, DIMENSION(:) :: AP, AF, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: B, X
```

C INTERFACE

```
#include <sunperf.h>

void sspsvx(char fact, char uplo, int  n,  int  nrhs,  float
          *ap,  float  *af,  int *ipivot, float *b, int ldb,
          float *x, int  ldx,  float  *rcond,  float  *ferr,
          float *berr, int *info);

void sspsvx_64(char fact, char  uplo,  long  n,  long  nrhs,
          float *ap, float *af, long *ipivot, float *b, long
          ldb, float  *x,  long  ldx,  float  *rcond,  float
          *ferr, float *berr, long *info);
```

PURPOSE

SSPSVX uses the diagonal pivoting factorization A = U*D*U**T or A = L*D*L**T to compute the solution to a real system of linear equations A * X = B, where A is an N-by-N symmetric matrix stored in packed format and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'N', the diagonal pivoting method is used to factor A as

```
      A = U * D * U**T,  if UPLO = 'U', or
      A = L * D * L**T,  if UPLO = 'L',
```

where U (or L) is a product of permutation and unit upper (lower)

triangular matrices and D is symmetric and block diagonal with
1-by-1 and 2-by-2 diagonal blocks.

2. If some D(i,i)=0, so that D is exactly singular, then the routine
returns with INFO = i. Otherwise, the factored form of A is used
to estimate the condition number of the matrix A. If the
reciprocal of the condition number is less than machine precision,
INFO = N+1 is returned as a warning, but the routine still goes on
to solve for X and compute error bounds as described below.

3. The system of equations is solved for X using the factored form
of A.
4. Iterative refinement is applied to improve the computed solution
matrix and calculate error bounds and backward error estimates
for it.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of A has been supplied on entry. = 'F': On entry, AF and IPIVOT contain the factored form of A. AP, AF and IPIVOT will not be modified. = 'N': The matrix A will be copied to AF and factored.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

AP (input)
Real array, dimension (N*(N+1)/2) The upper or lower triangle of the symmetric matrix A, packed

columnwise in a linear array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n. See below for further details.

AF (input/output)
Real array, dimension (N*(N+1)/2) If FACT = 'F', then AF is an input argument and on entry contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by SSPTRF, stored as a packed triangular matrix in the same storage format as A.

If FACT = 'N', then AF is an output argument and on exit contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by SSPTRF, stored as a packed triangular matrix in the same storage format as A.

IPIVOT (input or output)
Integer array, dimension (N) If FACT = 'F', then IPIVOT is an input argument and on entry contains details of the interchanges and the block structure of D, as determined by SSPTRF. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

If FACT = 'N', then IPIVOT is an output argument and on exit contains details of the interchanges and the block structure of D, as determined by SSPTRF.

B (input) Real array, dimension (LDB,NRHS) The N-by-NRHS right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)

Real array, dimension (LDX,NRHS) If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A. If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
Real array, dimension (NRHS) The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
Real array, dimension (NRHS) The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
Real array, dimension(3*N)

WORK2 (workspace)
Integer array, dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: D(i,i) is exactly zero. The factorization has been completed but the factor D is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: D is nonsingular, but RCOND is less than machine

precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

FURTHER DETAILS

The packed storage scheme is illustrated by the following example when N = 4, UPLO = 'U':
Two-dimensional storage of the symmetric matrix A:

```
   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = aji)
               a44
```

Packed storage of the upper triangle of A:

AP = [a11, a12, a22, a13, a23, a33, a14, a24, a34, a44]

Contents

NAME

```
ssptrd - reduce a real symmetric matrix A stored  in  packed
form  to symmetric tridiagonal form T by an orthogonal simi-
larity transformation
```

SYNOPSIS

```
SUBROUTINE SSPTRD(UPLO, N, AP, D, E, TAU, INFO)

CHARACTER * 1 UPLO
INTEGER N, INFO
REAL AP(*), D(*), E(*), TAU(*)

SUBROUTINE SSPTRD_64(UPLO, N, AP, D, E, TAU, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, INFO
REAL AP(*), D(*), E(*), TAU(*)
```

F95 INTERFACE

```
SUBROUTINE SPTRD(UPLO, N, AP, D, E, TAU, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INFO
REAL, DIMENSION(:) :: AP, D, E, TAU

SUBROUTINE SPTRD_64(UPLO, N, AP, D, E, TAU, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INFO
REAL, DIMENSION(:) :: AP, D, E, TAU
```

C INTERFACE

```
#include <sunperf.h>

void ssptrd(char uplo, int n, float *ap, float *d, float *e,
          float *tau, int *info);

void ssptrd_64(char uplo, long n, float *ap, float *d, float
          *e, float *tau, long *info);
```

PURPOSE

ssptrd reduces a real symmetric matrix A stored in packed form to symmetric tridiagonal form T by an orthogonal similarity transformation: Q**T * A * Q = T.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

AP (input)
Real array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n. On exit, if UPLO = 'U', the diagonal and first superdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements above the first superdiagonal, with the array TAU, represent the orthogonal matrix Q as a product of elementary reflectors; if UPLO = 'L', the diagonal and first subdiagonal of A are over- written by the corresponding elements of the tridiagonal matrix T, and the elements below the first subdiagonal, with the array TAU, represent the orthogonal matrix Q as a product of elementary reflectors. See Further Details.

D (output)
Real array, dimension (N) The diagonal elements of the tridiagonal matrix T: D(i) = A(i,i).

E (output)
Real array, dimension (N-1) The off-diagonal elements of the tridiagonal matrix T: E(i) = A(i,i+1) if UPLO = 'U', E(i) = A(i+1,i) if UPLO = 'L'.

TAU (output)
Real array, dimension (N-1) The scalar factors of the elementary reflectors (see Further Details).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

If UPLO = 'U', the matrix Q is represented as a product of elementary reflectors

```
   Q = H(n-1) . . . H(2) H(1).
```

Each H(i) has the form

```
   H(i) = I - tau * v * v'
```

where tau is a real scalar, and v is a real vector with v(i+1:n) = 0 and v(i) = 1; v(1:i-1) is stored on exit in AP, overwriting A(1:i-1,i+1), and tau is stored in TAU(i).

If UPLO = 'L', the matrix Q is represented as a product of elementary reflectors

```
   Q = H(1) H(2) . . . H(n-1).
```

Each H(i) has the form

```
   H(i) = I - tau * v * v'
```

where tau is a real scalar, and v is a real vector with v(1:i) = 0 and v(i+1) = 1; v(i+2:n) is stored on exit in AP, overwriting A(i+2:n,i), and tau is stored in TAU(i).

Contents

NAME

```
ssptrf - compute  the  factorization  of  a  real  symmetric
matrix  A  stored  in  packed format using the Bunch-Kaufman
diagonal pivoting method
```

SYNOPSIS

```
SUBROUTINE SSPTRF(UPLO, N, AP, IPIVOT, INFO)

CHARACTER * 1 UPLO
INTEGER N, INFO
INTEGER IPIVOT(*)
REAL AP(*)

SUBROUTINE SSPTRF_64(UPLO, N, AP, IPIVOT, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
REAL AP(*)
```

F95 INTERFACE

```
SUBROUTINE SPTRF(UPLO, N, AP, IPIVOT, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: AP

SUBROUTINE SPTRF_64(UPLO, N, AP, IPIVOT, [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: AP
```

C INTERFACE

```
#include <sunperf.h>

void ssptrf(char uplo, int n, float *ap,  int  *ipivot,  int
          *info);

void ssptrf_64(char uplo, long n, float *ap,  long  *ipivot,
          long *info);
```

PURPOSE

ssptrf computes the factorization of a real symmetric matrix A stored in packed format using the Bunch-Kaufman diagonal pivoting method:

```
A = U*D*U**T  or  A = L*D*L**T
```

where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

AP (input/output)
Real array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array.   The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n.

On exit, the block diagonal matrix D and the multipliers used to obtain the factor U or L, stored as a packed triangular matrix overwriting A (see below for further details).

IPIVOT (output)
Integer array, dimension (N) Details of the interchanges and the block structure of D. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

```
5-96 - Based on modifications by J. Lewis,  Boeing  Computer
Services
       Company

If UPLO = 'U', then A = U*D*U', where
   U = P(n)*U(n)* ... *P(k)U(k)* ...,
i.e., U is a product of terms P(k)*U(k), where  k  decreases
from  n  to  1 in steps of 1 or 2, and D is a block diagonal
matrix with 1-by-1 and 2-by-2 diagonal blocks D(k).  P(k) is
a  permutation matrix as defined by IPIVOT(k), and U(k) is a
unit upper triangular matrix,  such  that  if  the  diagonal
block D(k) is of order s (s = 1 or 2), then

           (   I    v    0   )   k-s
   U(k) =  (   0    I    0   )   s
           (   0    0    I   )   n-k
              k-s   s   n-k

If s = 1, D(k) overwrites A(k,k), and  v  overwrites  A(1:k-
1,k).   If s = 2, the upper triangle of D(k) overwrites A(k-
1,k-1), A(k-1,k), and A(k,k), and  v  overwrites  A(1:k-2,k-
1:k).

If UPLO = 'L', then A = L*D*L', where
   L = P(1)*L(1)* ... *P(k)*L(k)* ...,
```

i.e., L is a product of terms P(k)*L(k), where k increases from 1 to n in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIVOT(k), and L(k) is a unit lower triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    0     0   )  k-1
   L(k) =  (   0    I     0   )  s
           (   0    v     I   )  n-k-s+1
              k-1   s  n-k-s+1
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(k+1:n,k). If s = 2, the lower triangle of D(k) overwrites A(k,k), A(k+1,k), and A(k+1,k+1), and v overwrites A(k+2:n,k:k+1).

Contents

NAME

ssptri - compute the inverse of a real symmetric indefinite matrix A in packed storage using the factorization A = U*D*U**T or A = L*D*L**T computed by SSPTRF

SYNOPSIS

```
SUBROUTINE SSPTRI(UPLO, N, AP, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
INTEGER N, INFO
INTEGER IPIVOT(*)
REAL AP(*), WORK(*)

SUBROUTINE SSPTRI_64(UPLO, N, AP, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
REAL AP(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SPTRI(UPLO, N, AP, IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: AP, WORK

SUBROUTINE SPTRI_64(UPLO, N, AP, IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INFO
```

```
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: AP, WORK
```

C INTERFACE

```
#include <sunperf.h>

void ssptri(char uplo, int n, float  *a,  int  *ipivot,  int
          *info);

void ssptri_64(char uplo, long n, float  *a,  long  *ipivot,
          long *info);
```

PURPOSE

ssptri computes the inverse of a real symmetric indefinite matrix A in packed storage using the factorization A = U*D*U**T or A = L*D*L**T computed by SSPTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

AP (input/output)
Real array, dimension (N*(N+1)/2) On entry, the block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by SSPTRF, stored as a packed triangular matrix.

On exit, if INFO = 0, the (symmetric) inverse of the original matrix, stored as a packed triangular matrix. The j-th column of inv(A) is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = inv(A)(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2n-j)/2) = inv(A)(i,j) for j<=i<=n.

IPIVOT (input)
Integer array, dimension (N) Details of the interchanges and the block structure of D as determined by SSPTRF.

WORK (workspace)

```
          Real array, dimension(N)

INFO (output)
          = 0: successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
          gal value
          > 0: if INFO = i, D(i,i) = 0; the matrix is singu-
          lar and its inverse could not be computed.
```

Contents

NAME

SSPTRS - solve a system of linear equations A*X = B with a real symmetric matrix A stored in packed format using the factorization A = U*D*U**T or A = L*D*L**T computed by SSPTRF

SYNOPSIS

```
SUBROUTINE SSPTRS(UPLO, N, NRHS, AP, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDB, INFO
INTEGER IPIVOT(*)
REAL AP(*), B(LDB,*)

SUBROUTINE SSPTRS_64(UPLO, N, NRHS, AP, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIVOT(*)
REAL AP(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE SPTRS(UPLO, N, [NRHS], AP, IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: AP
REAL, DIMENSION(:,:) :: B

SUBROUTINE SPTRS_64(UPLO, N, [NRHS], AP, IPIVOT, B, [LDB], [INFO])
```

```
CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: AP
REAL, DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void ssptrs(char uplo, int  n,  int  nrhs,  float  *ap,  int
          *ipivot, float *b, int ldb, int *info);

void ssptrs_64(char uplo, long n, long nrhs, float *ap, long
          *ipivot, float *b, long ldb, long *info);
```

PURPOSE

ssptrs solves a system of linear equations A*X = B with a real symmetric matrix A stored in packed format using the factorization A = U*D*U**T or A = L*D*L**T computed by SSPTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

AP (input)
Real array, dimension (N*(N+1)/2) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by SSPTRF, stored as a packed triangular matrix.

IPIVOT (input)
Integer array, dimension (N) Details of the interchanges and the block structure of D as determined by SSPTRF.

B (input/output)
Real array, dimension (LDB, NRHS) On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

sstebz - compute the eigenvalues of a symmetric tridiagonal matrix T

SYNOPSIS

```
SUBROUTINE SSTEBZ(RANGE, ORDER, N, VL, VU, IL, IU, ABSTOL, D, E, M,
      NSPLIT, W, IBLOCK, ISPLIT, WORK, IWORK, INFO)

CHARACTER * 1 RANGE, ORDER
INTEGER N, IL, IU, M, NSPLIT, INFO
INTEGER IBLOCK(*), ISPLIT(*), IWORK(*)
REAL VL, VU, ABSTOL
REAL D(*), E(*), W(*), WORK(*)

SUBROUTINE SSTEBZ_64(RANGE, ORDER, N, VL, VU, IL, IU, ABSTOL, D, E,
      M, NSPLIT, W, IBLOCK, ISPLIT, WORK, IWORK, INFO)

CHARACTER * 1 RANGE, ORDER
INTEGER*8 N, IL, IU, M, NSPLIT, INFO
INTEGER*8 IBLOCK(*), ISPLIT(*), IWORK(*)
REAL VL, VU, ABSTOL
REAL D(*), E(*), W(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEBZ(RANGE, ORDER, N, VL, VU, IL, IU, ABSTOL, D, E, M,
       NSPLIT, W, IBLOCK, ISPLIT, [WORK], [IWORK], [INFO])

CHARACTER(LEN=1) :: RANGE, ORDER
INTEGER :: N, IL, IU, M, NSPLIT, INFO
INTEGER, DIMENSION(:) :: IBLOCK, ISPLIT, IWORK
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: D, E, W, WORK
```

```
SUBROUTINE STEBZ_64(RANGE, ORDER, N, VL, VU, IL, IU, ABSTOL, D, E, M,
       NSPLIT, W, IBLOCK, ISPLIT, [WORK], [IWORK], [INFO])

CHARACTER(LEN=1) :: RANGE, ORDER
INTEGER(8) :: N, IL, IU, M, NSPLIT, INFO
INTEGER(8), DIMENSION(:) :: IBLOCK, ISPLIT, IWORK
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: D, E, W, WORK
```

C INTERFACE

```
#include <sunperf.h>

void sstebz(char range, char order, int n, float  vl,  float
          vu,  int il, int iu, float abstol, float *d, float
          *e, int *m, int *nsplit, float  *w,  int  *iblock,
          int *isplit, int *info);

void sstebz_64(char range, char order,  long  n,  float  vl,
          float  vu,  long  il, long iu, float abstol, float
          *d, float *e, long *m,  long  *nsplit,  float  *w,
          long *iblock, long *isplit, long *info);
```

PURPOSE

sstebz computes the eigenvalues of a symmetric tridiagonal matrix T. The user may ask for all eigenvalues, all eigenvalues in the half-open interval (VL, VU], or the IL-th through IU-th eigenvalues.

To avoid overflow, the matrix must be scaled so that its largest element is no greater than overflow**(1/2) * underflow**(1/4) in absolute value, and for greatest accuracy, it should not be much smaller than that.

See W. Kahan "Accurate Eigenvalues of a Symmetric Tridiagonal Matrix", Report CS41, Computer Science Dept., Stanford University, July 21, 1966.

ARGUMENTS

RANGE (input)
= 'A': ("All")   all eigenvalues will be found.
= 'V': ("Value") all eigenvalues in the half-open interval (VL, VU] will be found. = 'I': ("Index") the IL-th through IU-th eigenvalues (of the entire matrix) will be found.

ORDER (input)
= 'B': ("By Block") the eigenvalues will be grouped by split-off block (see IBLOCK, ISPLIT) and ordered from smallest to largest within the block. = 'E': ("Entire matrix") the eigenvalues for the entire matrix will be ordered from smallest to largest.

N (input) The order of the tridiagonal matrix T. N >= 0.

VL (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. Eigenvalues less than or equal to VL, or greater than VU, will not be returned. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)
See the description of VL.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
See the description of IL.

ABSTOL (input)
The absolute tolerance for the eigenvalues. An eigenvalue (or cluster) is considered to be located if it has been determined to lie in an interval whose width is ABSTOL or less. If ABSTOL is less than or equal to zero, then ULP*|T| will be used, where |T| means the 1-norm of T.

Eigenvalues will be computed most accurately when ABSTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero.

D (input) The n diagonal elements of the tridiagonal matrix T.

E (input) The (n-1) off-diagonal elements of the tridiagonal matrix T.

M (output)

The actual number of eigenvalues found. 0 <= M <= N. (See also the description of INFO=2,3.)

NSPLIT (output)
The number of diagonal blocks in the matrix T. 1 <= NSPLIT <= N.

W (output)
On exit, the first M elements of W will contain the eigenvalues. (SSTEBZ may use the remaining N-M elements as workspace.)

IBLOCK (output)
At each row/column j where E(j) is zero or small, the matrix T is considered to split into a block diagonal matrix. On exit, if INFO = 0, IBLOCK(i) specifies to which block (from 1 to the number of blocks) the eigenvalue W(i) belongs. (SSTEBZ may use the remaining N-M elements as workspace.)

ISPLIT (output)
The splitting points, at which T breaks up into submatrices. The first submatrix consists of rows/columns 1 to ISPLIT(1), the second of rows/columns ISPLIT(1)+1 through ISPLIT(2), etc., and the NSPLIT-th consists of rows/columns ISPLIT(NSPLIT-1)+1 through ISPLIT(NSPLIT)=N. (Only the first NSPLIT elements will actually be used, but since the user cannot know a priori what value NSPLIT will have, N words must be reserved for ISPLIT.)

WORK (workspace)
dimension(4*N)

IWORK (workspace)
dimension(3*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: some or all of the eigenvalues failed to converge or
were not computed:
=1 or 3: Bisection failed to converge for some eigenvalues; these eigenvalues are flagged by a negative block number. The effect is that the eigenvalues may not be as accurate as the absolute and relative tolerances. This is generally caused

by unexpectedly inaccurate arithmetic. =2 or 3: RANGE='I' only: Not all of the eigenvalues IL:IU were found.
Effect: M < IU+1-IL
Cause: non-monotonic arithmetic, causing the Sturm sequence to be non-monotonic. Cure: recalculate, using RANGE='A', and pick
out eigenvalues IL:IU. = 4: RANGE='I', and the Gershgorin interval initially used was too small. No eigenvalues were computed.

Contents

NAME

```
sstedc - compute all eigenvalues and, optionally,  eigenvec-
tors  of a symmetric tridiagonal matrix using the divide and
conquer method
```

SYNOPSIS

```
SUBROUTINE SSTEDC(COMPZ, N, D, E, Z, LDZ, WORK, LWORK, IWORK, LIWORK,
      INFO)

CHARACTER * 1 COMPZ
INTEGER N, LDZ, LWORK, LIWORK, INFO
INTEGER IWORK(*)
REAL D(*), E(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSTEDC_64(COMPZ, N, D, E, Z, LDZ, WORK, LWORK, IWORK,
      LIWORK, INFO)

CHARACTER * 1 COMPZ
INTEGER*8 N, LDZ, LWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
REAL D(*), E(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEDC(COMPZ, N, D, E, Z, [LDZ], [WORK], [LWORK], [IWORK],
       [LIWORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
INTEGER :: N, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: D, E, WORK
REAL, DIMENSION(:,:) :: Z
```

```
SUBROUTINE STEDC_64(COMPZ, N, D, E, Z, [LDZ], [WORK], [LWORK], [IWORK],
       [LIWORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
INTEGER(8) :: N, LDZ, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: D, E, WORK
REAL, DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void sstedc(char compz, int n, float *d, float *e, float *z,
          int ldz, int *info);
void sstedc_64(char compz, long n, float *d, float *e, float
          *z, long ldz, long *info);
```

PURPOSE

sstedc computes all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the divide and conquer method. The eigenvectors of a full or band real symmetric matrix can also be found if SSYTRD or SSPTRD or SSBTRD has been used to reduce this matrix to tridiagonal form.

This code makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none. See SLAED3 for details.

ARGUMENTS

COMPZ (input)
= 'N': Compute eigenvalues only.
= 'I': Compute eigenvectors of tridiagonal matrix also.
= 'V': Compute eigenvectors of original dense symmetric matrix also. On entry, Z contains the orthogonal matrix used to reduce the original matrix to tridiagonal form.

N (input) The dimension of the symmetric tridiagonal matrix.

```
          N >= 0.

D (input/output)
          On entry, the diagonal elements of the tridiagonal
          matrix.   On exit, if INFO = 0, the eigenvalues in
          ascending order.

E (input/output)
          On entry, the subdiagonal elements of the tridiag-
          onal matrix.  On exit, E has been destroyed.

Z (input) On entry, if COMPZ =  'V',  then  Z  contains  the
          orthogonal  matrix used in the reduction to tridi-
          agonal form.  On exit, if INFO = 0, then if  COMPZ
          =  'V', Z contains the orthonormal eigenvectors of
          the original symmetric matrix, and if COMPZ = 'I',
          Z  contains  the  orthonormal  eigenvectors of the
          symmetric tridiagonal matrix.  If   COMPZ  =  'N',
          then Z is not referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1.
          If eigenvectors are desired, then LDZ >= max(1,N).

WORK (workspace)
          dimension (LWORK) On exit, if INFO  =  0,  WORK(1)
          returns the optimal LWORK.

LWORK (input)
          The dimension of the array WORK.  If COMPZ  =  'N'
          or N <= 1 then LWORK must be at least 1.  If COMPZ
          = 'V' and N > 1 then LWORK must be at least ( 1  +
          3*N  +  2*N*lg N + 3*N**2 ), where lg( N ) = smal-
          lest integer k such that 2**k >= N.   If  COMPZ  =
          'I'  and  N  > 1 then LWORK must be at least ( 1 +
          4*N + N**2 ).

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

IWORK (workspace/output)
          On exit, if INFO = 0, IWORK(1) returns the optimal
          LIWORK.

LIWORK (input)
          The dimension of the array IWORK.  If COMPZ =  'N'
          or  N  <=  1  then  LIWORK must be at least 1.  If
```

```
           COMPZ = 'V' and N > 1 then LIWORK must be at least
           (  6 + 6*N + 5*N*lg N ).  If COMPZ = 'I' and N > 1
           then LIWORK must be at least ( 3 + 5*N ).

           If LIWORK = -1, then a workspace query is assumed;
           the  routine  only  calculates the optimal size of
           the IWORK array, returns this value as  the  first
           entry  of  the  IWORK  array, and no error message
           related to LIWORK is issued by XERBLA.

 INFO (output)
           = 0:  successful exit.
           < 0:  if INFO = -i, the i-th argument had an ille-
           gal value.
           > 0:  The algorithm failed to  compute  an  eigen-
           value while working on the submatrix lying in rows
           and columns INFO/(N+1) through mod(INFO,N+1).
```

FURTHER DETAILS

```
 Based on contributions by
    Jeff Rutter, Computer  Science  Division,  University  of
 California
    at Berkeley, USA
 Modified by Francoise Tisseur, University of Tennessee.
```

Contents

NAME

```
sstegr - (a) Compute T-sigma_i = L_i D_i  L_i^T,  such  that
L_i D_i L_i^T  is a relatively robust representation
```

SYNOPSIS

```
SUBROUTINE SSTEGR(JOBZ, RANGE, N, D, E, VL, VU, IL, IU, ABSTOL, M, W,
      Z, LDZ, ISUPPZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE
INTEGER N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER ISUPPZ(*), IWORK(*)
REAL VL, VU, ABSTOL
REAL D(*), E(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSTEGR_64(JOBZ, RANGE, N, D, E, VL, VU, IL, IU, ABSTOL, M,
      W, Z, LDZ, ISUPPZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE
INTEGER*8 N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER*8 ISUPPZ(*), IWORK(*)
REAL VL, VU, ABSTOL
REAL D(*), E(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEGR(JOBZ, RANGE, [N], D, E, VL, VU, IL, IU, ABSTOL, M,
       W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE
INTEGER :: N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: ISUPPZ, IWORK
REAL :: VL, VU, ABSTOL
```

```
REAL, DIMENSION(:) :: D, E, W, WORK
REAL, DIMENSION(:,:) :: Z

SUBROUTINE STEGR_64(JOBZ, RANGE, [N], D, E, VL, VU, IL, IU, ABSTOL,
       M, W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE
INTEGER(8) :: N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: ISUPPZ, IWORK
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: D, E, W, WORK
REAL, DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>
void sstegr(char jobz, char range, int n,  float  *d,  float
          *e,  float  vl,  float  vu,  int il, int iu, float
          abstol, int *m, float *w, float *z, int  ldz,  int
          *isuppz, int *info);

void sstegr_64(char jobz, char  range,  long  n,  float  *d,
          float  *e,  float  vl, float vu, long il, long iu,
          float abstol, long *m, float *w,  float  *z,  long
          ldz, long *isuppz, long *info);
```

PURPOSE

```
sstegr b) Compute the  eigenvalues,  lambda_j,  of  L_i  D_i
L_i^T to high
       relative accuracy by the dqds algorithm,
   (c) If there is a cluster of close eigenvalues,  "choose"
sigma_i
       close to the cluster, and go to step (a),
   (d) Given the approximate eigenvalue lambda_j of L_i  D_i
L_i^T,
       compute the corresponding eigenvector by forming a
       rank-revealing twisted factorization.
The desired accuracy of the output can be specified  by  the
input parameter ABSTOL.

For more details, see "A new O(n^2) algorithm for  the  sym-
metric   tridiagonal   eigenvalue/eigenvector  problem",  by
Inderjit Dhillon, Computer Science Division Technical Report
No. UCB/CSD-97-971, UC Berkeley, May 1997.

Note 1 : Currently SSTEGR is only set up to find ALL  the  n
eigenvalues and eigenvectors of T in O(n^2) time
Note 2 : Currently the routine  SSTEIN  is  called  when  an
appropriate  sigma_i  cannot  be  chosen  in step (c) above.
```

SSTEIN invokes modified Gram-Schmidt when eigenvalues are close.
Note 3 : SSTEGR works only on machines which follow ieee-754 floating-point standard in their handling of infinities and NaNs. Normal execution of SSTEGR may create NaNs and infinities and hence may abort due to a floating point exception in environments which do not conform to the ieee standard.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

RANGE (input)
= 'A': all eigenvalues will be found.
= 'V': all eigenvalues in the half-open interval (VL,VU] will be found. = 'I': the IL-th through IU-th eigenvalues will be found.

N (input) The order of the matrix. N >= 0.

D (input/output)
On entry, the n diagonal elements of the tridiagonal matrix T. On exit, D is overwritten.

E (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix T in elements 1 to N-1 of E; E(N) need not be set. On exit, E is overwritten.

VL (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)
See the description of VL.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
See the description of IL.

ABSTOL (input)
The absolute error tolerance for the eigenvalues/eigenvectors. IF JOBZ = 'V', the eigenvalues and eigenvectors output have residual norms bounded by ABSTOL, and the dot products between different eigenvectors are bounded by ABSTOL. If ABSTOL is less than N*EPS*|T|, then N*EPS*|T| will be used in its place, where EPS is the machine precision and |T| is the 1-norm of the tridiagonal matrix. The eigenvalues are computed to an accuracy of EPS*|T| irrespective of ABSTOL. If high relative accuracy is important, set ABSTOL to DLAMCH('Safe minimum'). See Barlow and Demmel "Computing Accurate Eigensystems of Scaled Diagonally Dominant Matrices", LAPACK Working Note #7 for a discussion of which matrices define their eigenvalues to high relative accuracy.

M (output)
The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)
The first M elements contain the selected eigenvalues in ascending order.

Z (input)
If JOBZ = 'V', then if INFO = 0, the first M columns of Z contain the orthonormal eigenvectors of the matrix T corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). If JOBZ = 'N', then Z is not referenced. Note: the user must ensure that at least max(1,M) columns are supplied in the array Z; if RANGE = 'V', the exact value of M is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

ISUPPZ (output)
The support of the eigenvectors in Z, i.e., the indices indicating the nonzero elements in Z. The i-th eigenvector is nonzero only in elements ISUPPZ(2*i-1) through ISUPPZ(2*i).

WORK (workspace)

On exit, if INFO = 0, WORK(1) returns the optimal (and minimal) LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,18*N)
If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of the array IWORK. LIWORK >= max(1,10*N)

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = 1, internal error in SLARRE, if INFO = 2, internal error in SLARRV.

FURTHER DETAILS

Based on contributions by
Inderjit Dhillon, IBM Almaden, USA
Osni Marques, LBNL/NERSC, USA

Contents

NAME

```
sstein - compute the eigenvectors of a real symmetric tridi-
agonal  matrix  T  corresponding  to  specified eigenvalues,
using inverse iteration
```

SYNOPSIS

```
SUBROUTINE SSTEIN(N, D, E, M, W, IBLOCK, ISPLIT, Z, LDZ, WORK, IWORK,
      IFAIL, INFO)

INTEGER N, M, LDZ, INFO
INTEGER IBLOCK(*), ISPLIT(*), IWORK(*), IFAIL(*)
REAL D(*), E(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSTEIN_64(N, D, E, M, W, IBLOCK, ISPLIT, Z, LDZ, WORK,
      IWORK, IFAIL, INFO)

INTEGER*8 N, M, LDZ, INFO
INTEGER*8 IBLOCK(*), ISPLIT(*), IWORK(*), IFAIL(*)
REAL D(*), E(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEIN(N, D, E, M, W, IBLOCK, ISPLIT, Z, [LDZ], [WORK],
       [IWORK], IFAIL, [INFO])

INTEGER :: N, M, LDZ, INFO
INTEGER, DIMENSION(:) :: IBLOCK, ISPLIT, IWORK, IFAIL
REAL, DIMENSION(:) :: D, E, W, WORK
REAL, DIMENSION(:,:) :: Z

SUBROUTINE STEIN_64(N, D, E, M, W, IBLOCK, ISPLIT, Z, [LDZ], [WORK],
       [IWORK], IFAIL, [INFO])
```

```
INTEGER(8) :: N, M, LDZ, INFO
INTEGER(8), DIMENSION(:) :: IBLOCK, ISPLIT, IWORK, IFAIL
REAL, DIMENSION(:) :: D, E, W, WORK
REAL, DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void sstein(int n, float *d, float *e, int m, float *w,  int
          *iblock,  int  *isplit,  float  *z,  int  ldz, int
          *ifail, int *info);

void sstein_64(long n, float *d, float *e, long m, float *w,
          long  *iblock,  long  *isplit, float *z, long ldz,
          long *ifail, long *info);
```

PURPOSE

sstein computes the eigenvectors of a real symmetric tridiagonal matrix T corresponding to specified eigenvalues, using inverse iteration.

The maximum number of iterations allowed for each eigenvector is specified by an internal parameter MAXITS (currently set to 5).

ARGUMENTS

N (input) The order of the matrix. N >= 0.

D (input) The n diagonal elements of the tridiagonal matrix T.

E (input) The (n-1) subdiagonal elements of the tridiagonal matrix T, in elements 1 to N-1. E(N) need not be set.

M (input) The number of eigenvectors to be found. 0 <= M <= N.

W (input) The first M elements of W contain the eigenvalues for which eigenvectors are to be computed. The eigenvalues should be grouped by split-off block and ordered from smallest to largest within the block. (The output array W from SSTEBZ with ORDER = 'B' is expected here.)

IBLOCK (input)
The submatrix indices associated with the corresponding eigenvalues in W; IBLOCK(i)=1 if eigenvalue W(i) belongs to the first submatrix from the top, =2 if W(i) belongs to the second submatrix, etc. (The output array IBLOCK from SSTEBZ is expected here.)

ISPLIT (input)
The splitting points, at which T breaks up into submatrices. The first submatrix consists of rows/columns 1 to ISPLIT(1), the second of rows/columns ISPLIT(1)+1 through ISPLIT(2), etc. (The output array ISPLIT from SSTEBZ is expected here.)

Z (output)
The computed eigenvectors. The eigenvector associated with the eigenvalue W(i) is stored in the i-th column of Z. Any vector which fails to converge is set to its current iterate after MAXITS iterations.

LDZ (input)
The leading dimension of the array Z. LDZ >= max(1,N).

WORK (workspace)
dimension(5*N)

IWORK (workspace)
dimension(N)

IFAIL (output)
On normal exit, all elements of IFAIL are zero. If one or more eigenvectors fail to converge after MAXITS iterations, then their indices are stored in array IFAIL.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, then i eigenvectors failed to converge in MAXITS iterations. Their indices are stored in array IFAIL.

Contents

NAME

```
ssteqr - compute all eigenvalues and, optionally,  eigenvec-
tors of a symmetric tridiagonal matrix using the implicit QL
or QR method
```

SYNOPSIS

```
SUBROUTINE SSTEQR(COMPZ, N, D, E, Z, LDZ, WORK, INFO)

CHARACTER * 1 COMPZ
INTEGER N, LDZ, INFO
REAL D(*), E(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSTEQR_64(COMPZ, N, D, E, Z, LDZ, WORK, INFO)

CHARACTER * 1 COMPZ
INTEGER*8 N, LDZ, INFO
REAL D(*), E(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEQR(COMPZ, N, D, E, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
INTEGER :: N, LDZ, INFO
REAL, DIMENSION(:) :: D, E, WORK
REAL, DIMENSION(:,:) :: Z

SUBROUTINE STEQR_64(COMPZ, N, D, E, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
INTEGER(8) :: N, LDZ, INFO
REAL, DIMENSION(:) :: D, E, WORK
REAL, DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void ssteqr(char compz, int n, float *d, float *e, float *z,
          int ldz, int *info);

void ssteqr_64(char compz, long n, float *d, float *e, float
          *z, long ldz, long *info);
```

PURPOSE

ssteqr computes all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the implicit QL or QR method. The eigenvectors of a full or band symmetric matrix can also be found if SSYTRD or SSPTRD or SSBTRD has been used to reduce this matrix to tridiagonal form.

ARGUMENTS

COMPZ (input)
= 'N': Compute eigenvalues only.
= 'V': Compute eigenvalues and eigenvectors of the original symmetric matrix. On entry, Z must contain the orthogonal matrix used to reduce the original matrix to tridiagonal form. = 'I': Compute eigenvalues and eigenvectors of the tridiagonal matrix. Z is initialized to the identity matrix.

N (input) The order of the matrix. N >= 0.

D (input/output)
On entry, the diagonal elements of the tridiagonal matrix. On exit, if INFO = 0, the eigenvalues in ascending order.

E (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix. On exit, E has been destroyed.

Z (input) On entry, if COMPZ = 'V', then Z contains the orthogonal matrix used in the reduction to tridiagonal form. On exit, if INFO = 0, then if COMPZ = 'V', Z contains the orthonormal eigenvectors of the original symmetric matrix, and if COMPZ = 'I', Z contains the orthonormal eigenvectors of the symmetric tridiagonal matrix. If COMPZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if eigenvectors are desired, then LDZ >= max(1,N).

WORK (workspace)
dimension(max(1,2*N-2)) If COMPZ = 'N', then WORK is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille- gal value
> 0: the algorithm has failed to find all the eigenvalues in a total of 30*N iterations; if INFO = i, then i elements of E have not converged to zero; on exit, D and E contain the elements of a symmetric tridiagonal matrix which is orthogonally similar to the original matrix.

Contents

NAME

```
ssterf - compute all eigenvalues of a symmetric  tridiagonal
matrix  using  the  Pal-Walker-Kahan variant of the QL or QR
algorithm
```

SYNOPSIS

```
SUBROUTINE SSTERF(N, D, E, INFO)

INTEGER N, INFO
REAL D(*), E(*)

SUBROUTINE SSTERF_64(N, D, E, INFO)

INTEGER*8 N, INFO
REAL D(*), E(*)
```

F95 INTERFACE

```
SUBROUTINE STERF([N], D, E, [INFO])

INTEGER :: N, INFO
REAL, DIMENSION(:) :: D, E

SUBROUTINE STERF_64([N], D, E, [INFO])

INTEGER(8) :: N, INFO
REAL, DIMENSION(:) :: D, E
```

C INTERFACE

```
#include <sunperf.h>

void ssterf(int n, float *d, float *e, int *info);
```

```
void ssterf_64(long n, float *d, float *e, long *info);
```

PURPOSE

```
ssterf computes all eigenvalues of a  symmetric  tridiagonal
matrix  using  the  Pal-Walker-Kahan variant of the QL or QR
algorithm.
```

ARGUMENTS

```
N (input) The order of the matrix.  N >= 0.

D (input/output)
          On entry, the n diagonal elements of the tridiago-
          nal matrix.  On exit, if INFO = 0, the eigenvalues
          in ascending order.

E (input/output)
          On entry, the (n-1) subdiagonal  elements  of  the
          tridiagonal  matrix.   On  exit,  E  has been des-
          troyed.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  the algorithm failed  to  find  all  of  the
          eigenvalues in a total of 30*N iterations; if INFO
          = i, then i elements of E have  not  converged  to
          zero.
```

Contents

NAME

```
sstev - compute all eigenvalues and,  optionally,  eigenvec-
tors of a real symmetric tridiagonal matrix A
```

SYNOPSIS

```
SUBROUTINE SSTEV(JOBZ, N, DIAG, OFFD, Z, LDZ, WORK, INFO)

CHARACTER * 1 JOBZ
INTEGER N, LDZ, INFO
REAL DIAG(*), OFFD(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSTEV_64(JOBZ, N, DIAG, OFFD, Z, LDZ, WORK, INFO)

CHARACTER * 1 JOBZ
INTEGER*8 N, LDZ, INFO
REAL DIAG(*), OFFD(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEV(JOBZ, N, DIAG, OFFD, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: JOBZ
INTEGER :: N, LDZ, INFO
REAL, DIMENSION(:) :: DIAG, OFFD, WORK
REAL, DIMENSION(:,:) :: Z

SUBROUTINE STEV_64(JOBZ, N, DIAG, OFFD, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: JOBZ
INTEGER(8) :: N, LDZ, INFO
REAL, DIMENSION(:) :: DIAG, OFFD, WORK
REAL, DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void sstev(char jobz, int n, float *diag, float *offd, float
          *z, int ldz, int *info);

void sstev_64(char jobz, long n, float *diag,  float  *offd,
          float *z, long ldz, long *info);
```

PURPOSE

sstev computes all eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix A.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

N (input) The order of the matrix.  N >= 0.

DIAG (input/output)
          On entry, the n diagonal elements of the tridiago-
          nal  matrix  A.   On exit, if INFO = 0, the eigen-
          values in ascending order.

OFFD (input/output)
          On entry, the (n-1) subdiagonal  elements  of  the
          tridiagonal  matrix A, stored in elements 1 to N-1
          of OFFD; OFFD(N) need not be set, but is  used  by
          the  routine.   On  exit, the contents of OFFD are
          destroyed.

Z (input) If JOBZ = 'V', then if INFO = 0,  Z  contains  the
          orthonormal eigenvectors of the matrix A, with the
          i-th column of Z holding the  eigenvector  associ-
          ated  with  DIAG(i).  If JOBZ = 'N', then Z is not
          referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
          If JOBZ = 'N', WORK is not referenced.
```

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the algorithm failed to converge; i off-diagonal elements of OFFD did not converge to zero.

Contents

NAME

```
sstevd - compute all eigenvalues and, optionally,  eigenvec-
tors of a real symmetric tridiagonal matrix
```

SYNOPSIS

```
SUBROUTINE SSTEVD(JOBZ, N, D, E, Z, LDZ, WORK, LWORK, IWORK, LIWORK,
      INFO)

CHARACTER * 1 JOBZ
INTEGER N, LDZ, LWORK, LIWORK, INFO
INTEGER IWORK(*)
REAL D(*), E(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSTEVD_64(JOBZ, N, D, E, Z, LDZ, WORK, LWORK, IWORK,
      LIWORK, INFO)

CHARACTER * 1 JOBZ
INTEGER*8 N, LDZ, LWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
REAL D(*), E(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEVD(JOBZ, N, D, E, Z, [LDZ], [WORK], [LWORK], [IWORK],
       [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ
INTEGER :: N, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: D, E, WORK
REAL, DIMENSION(:,:) :: Z

SUBROUTINE STEVD_64(JOBZ, N, D, E, Z, [LDZ], [WORK], [LWORK], [IWORK],
```

```
       [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ
INTEGER(8) :: N, LDZ, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: D, E, WORK
REAL, DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>

void sstevd(char jobz, int n, float *d, float *e, float  *z,
          int ldz, int *info);
void sstevd_64(char jobz, long n, float *d, float *e,  float
          *z, long ldz, long *info);
```

PURPOSE

sstevd computes all eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

N (input) The order of the matrix. N >= 0.

D (input/output)
On entry, the n diagonal elements of the tridiagonal matrix A. On exit, if INFO = 0, the eigenvalues in ascending order.

E (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix A, stored in elements 1 to N-1 of E; E(N) need not be set, but is used by the

```
          routine.   On  exit,  the  contents  of E are des-
          troyed.

Z (input) If JOBZ = 'V', then if INFO = 0,  Z  contains  the
          orthonormal eigenvectors of the matrix A, with the
          i-th column of Z holding the  eigenvector  associ-
          ated  with  D(i).   If  JOBZ  = 'N', then Z is not
          referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).
WORK (workspace)
          dimension (LWORK) On exit, if INFO  =  0,  WORK(1)
          returns the optimal LWORK.

LWORK (input)
          The dimension of the array WORK.  If JOBZ   =  'N'
          or  N <= 1 then LWORK must be at least 1.  If JOBZ
          = 'V' and N > 1 then LWORK must be at least ( 1  +
          4*N + N**2 ).

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

IWORK (workspace/output)
          On exit, if INFO = 0, IWORK(1) returns the optimal
          LIWORK.

LIWORK (input)
          The dimension of the array IWORK.  If JOBZ  =  'N'
          or N <= 1 then LIWORK must be at least 1.  If JOBZ
          = 'V' and N > 1  then  LIWORK  must  be  at  least
          3+5*N.

          If LIWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the IWORK array, returns this value as  the  first
          entry  of  the  IWORK  array, and no error message
          related to LIWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, the algorithm  failed  to  con-
          verge;  i  off-diagonal elements of E did not con-
```

verge to zero.

Contents

NAME

sstevr - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix T

SYNOPSIS

```
SUBROUTINE SSTEVR(JOBZ, RANGE, N, D, E, VL, VU, IL, IU, ABSTOL, M, W,
      Z, LDZ, ISUPPZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE
INTEGER N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER ISUPPZ(*), IWORK(*)
REAL VL, VU, ABSTOL
REAL D(*), E(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSTEVR_64(JOBZ, RANGE, N, D, E, VL, VU, IL, IU, ABSTOL, M,
      W, Z, LDZ, ISUPPZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE
INTEGER*8 N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER*8 ISUPPZ(*), IWORK(*)
REAL VL, VU, ABSTOL
REAL D(*), E(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEVR(JOBZ, RANGE, [N], D, E, VL, VU, IL, IU, ABSTOL, M,
       W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE
INTEGER :: N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: ISUPPZ, IWORK
REAL :: VL, VU, ABSTOL
```

```
REAL, DIMENSION(:) :: D, E, W, WORK
REAL, DIMENSION(:,:) :: Z

SUBROUTINE STEVR_64(JOBZ, RANGE, [N], D, E, VL, VU, IL, IU, ABSTOL,
       M, W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE
INTEGER(8) :: N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: ISUPPZ, IWORK
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: D, E, W, WORK
REAL, DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>
void sstevr(char jobz, char range, int n,  float  *d,  float
          *e,  float  vl,  float  vu,  int il, int iu, float
          abstol, int *m, float *w, float *z, int  ldz,  int
          *isuppz, int *info);

void sstevr_64(char jobz, char  range,  long  n,  float  *d,
          float  *e,  float  vl, float vu, long il, long iu,
          float abstol, long *m, float *w,  float  *z,  long
          ldz, long *isuppz, long *info);
```

PURPOSE

sstevr computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix T. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Whenever possible, SSTEVR calls SSTEGR to compute the eigenspectrum using Relatively Robust Representations. SSTEGR computes eigenvalues by the dqds algorithm, while orthogonal eigenvectors are computed from various "good" L D L^T representations (also known as Relatively Robust Representations). Gram-Schmidt orthogonalization is avoided as far as possible. More specifically, the various steps of the algorithm are as follows. For the i-th unreduced block of T,

```
   (a) Compute T - sigma_i = L_i D_i L_i^T,  such  that  L_i
D_i L_i^T
        is a relatively robust representation,
   (b) Compute the eigenvalues, lambda_j, of L_i  D_i  L_i^T
to high
       relative accuracy by the dqds algorithm,
   (c) If there is a cluster of close eigenvalues,  "choose"
```

sigma_i
close to the cluster, and go to step (a),
(d) Given the approximate eigenvalue lambda_j of L_i D_i L_i^T,
compute the corresponding eigenvector by forming a
rank-revealing twisted factorization.
The desired accuracy of the output can be specified by the input parameter ABSTOL.

For more details, see "A new O(n^2) algorithm for the symmetric tridiagonal eigenvalue/eigenvector problem", by Inderjit Dhillon, Computer Science Division Technical Report No. UCB//CSD-97-971, UC Berkeley, May 1997.

Note 1 : SSTEVR calls SSTEGR when the full spectrum is requested on machines which conform to the ieee-754 floating point standard. SSTEVR calls SSTEBZ and SSTEIN on non-ieee machines and
when partial spectrum requests are made.

Normal execution of SSTEGR may create NaNs and infinities and hence may abort due to a floating point exception in environments which do not handle NaNs and infinities in the ieee standard default manner.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

RANGE (input)
= 'A': all eigenvalues will be found.
= 'V': all eigenvalues in the half-open interval (VL,VU] will be found. = 'I': the IL-th through IU-th eigenvalues will be found.

N (input) The order of the matrix. N >= 0.

D (input/output)
On entry, the n diagonal elements of the tridiagonal matrix A. On exit, D may be multiplied by a constant factor chosen to avoid over/underflow in computing the eigenvalues.

E (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix A in elements 1 to N-1 of E;

E(N) need not be set. On exit, E may be multiplied by a constant factor chosen to avoid over/underflow in computing the eigenvalues.

VL (input)

If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)

See the description of VL.

IL (input)

If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)

See the description of IL.

ABSTOL (input)

The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABSTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABSTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing A to tridiagonal form.

See "Computing Small Singular Values of Bidiagonal Matrices with Guaranteed High Relative Accuracy," by Demmel and Kahan, LAPACK Working Note #3.

If high relative accuracy is important, set ABSTOL to SLAMCH('Safe minimum'). Doing so will guarantee that eigenvalues are computed to high relative accuracy when possible in future releases. The current code does not make any guarantees about high relative accuracy, but future releases will. See J. Barlow and J. Demmel, "Computing Accurate Eigensystems of Scaled Diagonally Dominant Matrices", LAPACK Working Note #7, for a discussion of which matrices define their eigenvalues to high relative accuracy.

M (output)
The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)
The first M elements contain the selected eigenvalues in ascending order.

Z (input)
If JOBZ = 'V', then if INFO = 0, the first M columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). Note: the user must ensure that at least max(1,M) columns are supplied in the array Z; if RANGE = 'V', the exact value of M is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

ISUPPZ (output)
The support of the eigenvectors in Z, i.e., the indices indicating the nonzero elements in Z. The i-th eigenvector is nonzero only in elements ISUPPZ(2*i-1) through ISUPPZ(2*i).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal (and minimal) LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 20*N.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if INFO = 0, IWORK(1) returns the optimal (and minimal) LIWORK.

LIWORK (input)
The dimension of the array IWORK. LIWORK >= 10*N.

```
            If LIWORK = -1, then a workspace query is assumed;
            the  routine  only  calculates the optimal size of
            the IWORK array, returns this value as  the  first
            entry  of  the  IWORK  array, and no error message
            related to LIWORK is issued by XERBLA.
  INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value
            > 0:  Internal error
```

FURTHER DETAILS

```
  Based on contributions by
     Inderjit Dhillon, IBM Almaden, USA
     Osni Marques, LBNL/NERSC, USA
     Ken Stanley, Computer Science Division, University of
       California at Berkeley, USA
```

Contents

NAME

sstevx - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix A

SYNOPSIS

```
SUBROUTINE SSTEVX(JOBZ, RANGE, N, DIAG, OFFD, VL, VU, IL, IU, ABTOL,
      NFOUND, W, Z, LDZ, WORK, IWORK2, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE
INTEGER N, IL, IU, NFOUND, LDZ, INFO
INTEGER IWORK2(*), IFAIL(*)
REAL VL, VU, ABTOL
REAL DIAG(*), OFFD(*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSTEVX_64(JOBZ, RANGE, N, DIAG, OFFD, VL, VU, IL, IU,
      ABTOL, NFOUND, W, Z, LDZ, WORK, IWORK2, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE
INTEGER*8 N, IL, IU, NFOUND, LDZ, INFO
INTEGER*8 IWORK2(*), IFAIL(*)
REAL VL, VU, ABTOL
REAL DIAG(*), OFFD(*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEVX(JOBZ, RANGE, N, DIAG, OFFD, VL, VU, IL, IU, ABTOL,
       NFOUND, W, Z, [LDZ], [WORK], [IWORK2], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE
INTEGER :: N, IL, IU, NFOUND, LDZ, INFO
INTEGER, DIMENSION(:) :: IWORK2, IFAIL
REAL :: VL, VU, ABTOL
REAL, DIMENSION(:) :: DIAG, OFFD, W, WORK
```

```
REAL, DIMENSION(:,:) :: Z

SUBROUTINE STEVX_64(JOBZ, RANGE, N, DIAG, OFFD, VL, VU, IL, IU,
       ABTOL, NFOUND, W, Z, [LDZ], [WORK], [IWORK2], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE
INTEGER(8) :: N, IL, IU, NFOUND, LDZ, INFO
INTEGER(8), DIMENSION(:) :: IWORK2, IFAIL
REAL :: VL, VU, ABTOL
REAL, DIMENSION(:) :: DIAG, OFFD, W, WORK
REAL, DIMENSION(:,:) :: Z
```

C INTERFACE

```
#include <sunperf.h>
void sstevx(char jobz, char range, int n, float *diag, float
          *offd,  float  vl, float vu, int il, int iu, float
          abtol, int *nfound, float *w, float *z,  int  ldz,
          int *ifail, int *info);

void sstevx_64(char jobz, char range, long n,  float  *diag,
          float *offd, float vl, float vu, long il, long iu,
          float abtol, long *nfound,  float  *w,  float  *z,
          long ldz, long *ifail, long *info);
```

PURPOSE

sstevx computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix A. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

RANGE (input)
= 'A': all eigenvalues will be found.
= 'V': all eigenvalues in the half-open interval (VL,VU] will be found. = 'I': the IL-th through IU-th eigenvalues will be found.

N (input) The order of the matrix. N >= 0.

DIAG (input/output)

On entry, the n diagonal elements of the tridiagonal matrix A. On exit, DIAG may be multiplied by a constant factor chosen to avoid over/underflow in computing the eigenvalues.

OFFD (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix A in elements 1 to N-1 of OFFD; OFFD(N) need not be set. On exit, OFFD may be multiplied by a constant factor chosen to avoid over/underflow in computing the eigenvalues.

VL (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)
See the description of VL.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
See the description of IL.

ABTOL (input)
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix.

Eigenvalues will be computed most accurately when ABTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABTOL to 2*SLAMCH('S').

See "Computing Small Singular Values of Bidiagonal

Matrices with Guaranteed High Relative Accuracy," by Demmel and Kahan, LAPACK Working Note #3.

NFOUND (output)
: The total number of eigenvalues found. 0 <= NFOUND <= N. If RANGE = 'A', NFOUND = N, and if RANGE = 'I', NFOUND = IU-IL+1.

W (output)
: The first NFOUND elements contain the selected eigenvalues in ascending order.

Z (input)
: If JOBZ = 'V', then if INFO = 0, the first NFOUND columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). If an eigenvector fails to converge (INFO > 0), then that column of Z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in IFAIL. If JOBZ = 'N', then Z is not referenced. Note: the user must ensure that at least max(1,NFOUND) columns are supplied in the array Z; if RANGE = 'V', the exact value of NFOUND is not known in advance and an upper bound must be used.

LDZ (input)
: The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
: dimension(5*N)

IWORK2 (workspace)

IFAIL (output)
: If JOBZ = 'V', then if INFO = 0, the first NFOUND elements of IFAIL are zero. If INFO > 0, then IFAIL contains the indices of the eigenvectors that failed to converge. If JOBZ = 'N', then IFAIL is not referenced.

INFO (output)
: = 0: successful exit
: < 0: if INFO = -i, the i-th argument had an illegal value
: > 0: if INFO = i, then i eigenvectors failed to converge. Their indices are stored in array IFAIL.

Contents

NAME

```
sstsv - compute the solution to a system of linear equations
A * X = B where A is a symmetric tridiagonal matrix
```

SYNOPSIS

```
SUBROUTINE SSTSV(N, NRHS, L, D, SUBL, B, LDB, IPIV, INFO)

INTEGER N, NRHS, LDB, INFO
INTEGER IPIV(*)
REAL L(*), D(*), SUBL(*), B(LDB,*)

SUBROUTINE SSTSV_64(N, NRHS, L, D, SUBL, B, LDB, IPIV, INFO)

INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIV(*)
REAL L(*), D(*), SUBL(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE STSV(N, NRHS, L, D, SUBL, B, [LDB], IPIV, [INFO])

INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIV
REAL, DIMENSION(:) :: L, D, SUBL
REAL, DIMENSION(:,:) :: B

SUBROUTINE STSV_64(N, NRHS, L, D, SUBL, B, [LDB], IPIV, [INFO])

INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIV
REAL, DIMENSION(:) :: L, D, SUBL
REAL, DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void sstsv(int n, int nrhs, float *l, float *d, float *subl,
          float *b, int ldb, int *ipiv, int *info);

void sstsv_64(long n, long nrhs, float *l, float  *d,  float
          *subl,  float  *b,  long  ldb,  long  *ipiv,  long
          *info);
```

PURPOSE

sstsv computes the solution to a system of linear equations A * X = B where A is a symmetric tridiagonal matrix.

ARGUMENTS

N (input) INTEGER
The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides in B.

L (input/output)
REAL array, dimension (N)
On entry, the n-1 subdiagonal elements of the tridiagonal matrix A. On exit, part of the factorization of A.

D (input/output)
REAL array, dimension (N)
On entry, the n diagonal elements of the tridiagonal matrix A. On exit, the n diagonal elements of the diagonal matrix D from the factorization of A.

SUBL (output)
REAL array, dimension (N)
On exit, part of the factorization of A.

B (input/output)
The columns of B contain the right hand sides.

LDB (input)
The leading dimension of B as specified in a type or DIMENSION statement.

IPIV (output)

INTEGER array, dimension (N)
On exit, the pivot indices of the factorization.

INFO (output)
INTEGER
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(k,k) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular and division by zero will occur if it is used to solve a system of equations.

Contents

NAME

```
ssttrf - compute the factorization of a symmetric  tridiago-
nal matrix A
```

SYNOPSIS

```
SUBROUTINE SSTTRF(N, L, D, SUBL, IPIV, INFO)

INTEGER N, INFO
INTEGER IPIV(*)
REAL L(*), D(*), SUBL(*)

SUBROUTINE SSTTRF_64(N, L, D, SUBL, IPIV, INFO)

INTEGER*8 N, INFO
INTEGER*8 IPIV(*)
REAL L(*), D(*), SUBL(*)
```

F95 INTERFACE

```
SUBROUTINE STTRF([N], L, D, SUBL, IPIV, [INFO])

INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIV
REAL, DIMENSION(:) :: L, D, SUBL

SUBROUTINE STTRF_64([N], L, D, SUBL, IPIV, [INFO])

INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIV
REAL, DIMENSION(:) :: L, D, SUBL
```

C INTERFACE

```
#include <sunperf.h>
```

```
void ssttrf(int n, float *l,  float  *d,  float  *subl,  int
          *ipiv, int *info);

void ssttrf_64(long n, float *l, float *d, float *subl, long
          *ipiv, long *info);
```

PURPOSE

ssttrf computes the factorization of a complex Hermitian tridiagonal matrix A.

ARGUMENTS

N (input) INTEGER
The order of the matrix A. N >= 0.

L (input/output)
REAL array, dimension (N)
On entry, the n-1 subdiagonal elements of the tri- diagonal matrix A. On exit, part of the factori- zation of A.

D (input/output)
REAL array, dimension (N)
On entry, the n diagonal elements of the tridiago- nal matrix A. On exit, the n diagonal elements of the diagonal matrix D from the L*D*L**H factoriza- tion of A.

SUBL (output)
REAL array, dimension (N)
On exit, part of the factorization of A.

IPIV (output)
INTEGER array, dimension (N)
On exit, the pivot indices of the factorization.

INFO (output)
INTEGER
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille- gal value
> 0: if INFO = i, D(k,k) is exactly zero.   The factorization has been completed, but the block diagonal matrix D is exactly singular and division by zero will occur if it is used to solve a system

of equations.

Contents

NAME

```
ssttrs - computes the solution to a real  system  of  linear
equations A * X = B
```

SYNOPSIS

```
SUBROUTINE SSTTRS(N, NRHS, L, D, SUBL, B, LDB, IPIV, INFO)

INTEGER N, NRHS, LDB, INFO
INTEGER IPIV(*)
REAL L(*), D(*), SUBL(*), B(LDB,*)

SUBROUTINE SSTTRS_64(N, NRHS, L, D, SUBL, B, LDB, IPIV, INFO)

INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIV(*)
REAL L(*), D(*), SUBL(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE STTRS(N, NRHS, L, D, SUBL, B, [LDB], IPIV, [INFO])

INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIV
REAL, DIMENSION(:) :: L, D, SUBL
REAL, DIMENSION(:,:) :: B

SUBROUTINE STTRS_64(N, NRHS, L, D, SUBL, B, [LDB], IPIV, [INFO])

INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIV
REAL, DIMENSION(:) :: L, D, SUBL
REAL, DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void ssttrs(int n, int  nrhs,  float  *l,  float  *d,  float
          *subl, float *b, int ldb, int *ipiv, int *info);

void ssttrs_64(long n, long nrhs, float *l, float *d,  float
          *subl,  float  *b,  long  ldb,  long  *ipiv,  long
          *info);
```

PURPOSE

ssttrs computes the solution to a real system of linear equations A * X = B, where A is an N-by-N symmetric tridiagonal matrix and X and B are N-by-NRHS matrices.

ARGUMENTS

N (input) INTEGER
The order of the matrix A. N >= 0.

NRHS (input)
INTEGER
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

L (input) REAL array, dimension (N-1)
On entry, the subdiagonal elements of LL and DD.

D (input) REAL array, dimension (N)
On entry, the diagonal elements of DD.

SUBL (input)
REAL array, dimension (N-2)
On entry, the second subdiagonal elements of LL.

B (input/output)
REAL array, dimension
(LDB, NRHS) On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
INTEGER
The leading dimension of the array B. LDB >= max(1, N)

```
IPIV (output)
           INTEGER array, dimension (N)
          Details of the interchanges and block  pivot.   If
          IPIV(K)  > 0, 1 by 1 pivot, and if IPIV(K) = K + 1
          an interchange done;  If  IPIV(K)  <  0,  2  by  2
          pivot, no interchange required.

INFO (output)
           INTEGER
          = 0:  successful exit
          < 0:  if INFO = -k, the k-th argument had an ille-
          gal value
```

Contents

NAME

```
sswap - Exchange vectors x and y.
```

SYNOPSIS

```
SUBROUTINE SSWAP(N, X, INCX, Y, INCY)

INTEGER N, INCX, INCY
REAL X(*), Y(*)

SUBROUTINE SSWAP_64(N, X, INCX, Y, INCY)

INTEGER*8 N, INCX, INCY
REAL X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE SWAP([N], X, [INCX], Y, [INCY])

INTEGER :: N, INCX, INCY
REAL, DIMENSION(:) :: X, Y

SUBROUTINE SWAP_64([N], X, [INCX], Y, [INCY])

INTEGER(8) :: N, INCX, INCY
REAL, DIMENSION(:) :: X, Y
```

C INTERFACE

```
#include <sunperf.h>

void sswap(int n, float *x, int incx, float *y, int incy);

void sswap_64(long n, float *x, long incx,  float  *y,  long
          incy);
```

PURPOSE

sswap Exchange x and y where x and y are n-vectors.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

X (input/output)
(1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. On exit, the y vector.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. On exit, the x vector.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

ssycon - estimate the reciprocal of the condition number (in the 1-norm) of a real symmetric matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by SSYTRF

SYNOPSIS

```
SUBROUTINE SSYCON(UPLO, N, A, LDA, IPIVOT, ANORM, RCOND, WORK,
      IWORK2, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, INFO
INTEGER IPIVOT(*), IWORK2(*)
REAL ANORM, RCOND
REAL A(LDA,*), WORK(*)

SUBROUTINE SSYCON_64(UPLO, N, A, LDA, IPIVOT, ANORM, RCOND, WORK,
      IWORK2, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, INFO
INTEGER*8 IPIVOT(*), IWORK2(*)
REAL ANORM, RCOND
REAL A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYCON(UPLO, N, A, [LDA], IPIVOT, ANORM, RCOND, [WORK],
       [IWORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT, IWORK2
REAL :: ANORM, RCOND
```

```
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE SYCON_64(UPLO, N, A, [LDA], IPIVOT, ANORM, RCOND, [WORK],
       [IWORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, IWORK2
REAL :: ANORM, RCOND
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
void ssycon(char  uplo,  int  n,  float  *a,  int  lda,  int
          *ipivot, float anorm, float *rcond, int *info);

void ssycon_64(char uplo, long n, float *a, long  lda,  long
          *ipivot, float anorm, float *rcond, long *info);
```

PURPOSE

```
ssycon estimates the reciprocal of the condition number  (in
the  1-norm) of a real symmetric matrix A using the factori-
zation A = U*D*U**T or A = L*D*L**T computed by SSYTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal
of  the condition number is computed as RCOND = 1 / (ANORM *
norm(inv(A))).
```

ARGUMENTS

```
UPLO (input)
          Specifies whether the details of the factorization
          are stored as an upper or lower triangular matrix.
          = 'U':  Upper triangular, form is A = U*D*U**T;
          = 'L':  Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A.  N >= 0.

A (input) The block diagonal matrix D  and  the  multipliers
          used  to  obtain  the factor U or L as computed by
          SSYTRF.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
```

max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by SSYTRF.

ANORM (input)
The 1-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(2*N)

IWORK2 (workspace)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
ssyev - compute all eigenvalues and,  optionally,  eigenvec-
tors of a real symmetric matrix A
```

SYNOPSIS

```
SUBROUTINE SSYEV(JOBZ, UPLO, N, A, LDA, W, WORK, LDWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER N, LDA, LDWORK, INFO
REAL A(LDA,*), W(*), WORK(*)

SUBROUTINE SSYEV_64(JOBZ, UPLO, N, A, LDA, W, WORK, LDWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 N, LDA, LDWORK, INFO
REAL A(LDA,*), W(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYEV(JOBZ, UPLO, N, A, [LDA], W, [WORK], [LDWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: N, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE SYEV_64(JOBZ, UPLO, N, A, [LDA], W, [WORK], [LDWORK],
       [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: N, LDA, LDWORK, INFO
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void ssyev(char jobz, char uplo, int n, float *a,  int  lda,
          float *w, int *info);

void ssyev_64(char jobz, char uplo, long n, float  *a,  long
          lda, float *w, long *info);
```

PURPOSE

```
ssyev computes all eigenvalues and, optionally, eigenvectors
of a real symmetric matrix A.
```

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the symmetric matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of A contains the lower  triangular  part  of
          the  matrix  A.   On  exit, if JOBZ = 'V', then if
          INFO = 0, A contains the orthonormal  eigenvectors
          of  the matrix A.  If JOBZ = 'N', then on exit the
          lower triangle (if UPLO='L') or the upper triangle
          (if  UPLO='U')  of  A,  including the diagonal, is
          destroyed.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

W (output)
          If INFO = 0, the eigenvalues in ascending order.
```

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of the array WORK. LDWORK >= max(1,3*N-1). For optimal efficiency, LDWORK >= (NB+2)*N, where NB is the blocksize for SSYTRD returned by ILAENV.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

Contents

NAME

```
ssyevd - compute all eigenvalues and, optionally,  eigenvec-
tors of a real symmetric matrix A
```

SYNOPSIS

```
SUBROUTINE SSYEVD(JOBZ, UPLO, N, A, LDA, W, WORK, LWORK, IWORK,
      LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER N, LDA, LWORK, LIWORK, INFO
INTEGER IWORK(*)
REAL A(LDA,*), W(*), WORK(*)

SUBROUTINE SSYEVD_64(JOBZ, UPLO, N, A, LDA, W, WORK, LWORK, IWORK,
      LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 N, LDA, LWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
REAL A(LDA,*), W(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYEVD(JOBZ, UPLO, N, A, [LDA], W, [WORK], [LWORK], [IWORK],
       [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: N, LDA, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: A
```

```
SUBROUTINE SYEVD_64(JOBZ, UPLO, N, A, [LDA], W, [WORK], [LWORK],
       [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: N, LDA, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void ssyevd(char jobz, char uplo, int n, float *a, int  lda,
          float *w, int *info);
void ssyevd_64(char jobz, char uplo, long n, float *a,  long
          lda, float *w, long *info);
```

PURPOSE

ssyevd computes all eigenvalues and, optionally, eigenvectors of a real symmetric matrix A. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

Because of large use of BLAS of level 3, SSYEVD needs N**2 more workspace than SSYEVX.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
```

On entry, the symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A. On exit, if JOBZ = 'V', then if INFO = 0, A contains the orthonormal eigenvectors of the matrix A. If JOBZ = 'N', then on exit the lower triangle (if UPLO='L') or the upper triangle (if UPLO='U') of A, including the diagonal, is destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

W (output)
If INFO = 0, the eigenvalues in ascending order.

WORK (workspace)
dimension (LWORK) On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If N <= 1, LWORK must be at least 1. If JOBZ = 'N' and N > 1, LWORK must be at least 2*N+1. If JOBZ = 'V' and N > 1, LWORK must be at least 1 + 6*N + 2*N**2.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of the array IWORK. If N <= 1, LIWORK must be at least 1. If JOBZ = 'N' and N > 1, LIWORK must be at least 1. If JOBZ = 'V' and N > 1, LIWORK must be at least 3 + 5*N.

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message

```
            related to LIWORK is issued by XERBLA.

INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value
            > 0:  if INFO = i, the algorithm  failed  to  con-
            verge;  i off-diagonal elements of an intermediate
            tridiagonal form did not converge to zero.
```

FURTHER DETAILS

```
Based on contributions by
   Jeff Rutter, Computer  Science  Division,  University  of
California
   at Berkeley, USA
Modified by Francoise Tisseur, University of Tennessee.
```

Contents

NAME

ssyevr - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix T

SYNOPSIS

```
SUBROUTINE SSYEVR(JOBZ, RANGE, UPLO, N, A, LDA, VL, VU, IL, IU,
      ABSTOL, M, W, Z, LDZ, ISUPPZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER N, LDA, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER ISUPPZ(*), IWORK(*)
REAL VL, VU, ABSTOL
REAL A(LDA,*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSYEVR_64(JOBZ, RANGE, UPLO, N, A, LDA, VL, VU, IL, IU,
      ABSTOL, M, W, Z, LDZ, ISUPPZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER*8 N, LDA, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER*8 ISUPPZ(*), IWORK(*)
REAL VL, VU, ABSTOL
REAL A(LDA,*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYEVR(JOBZ, RANGE, UPLO, [N], A, [LDA], VL, VU, IL, IU,
       ABSTOL, M, W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [IWORK], [LIWORK],
       [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER :: N, LDA, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: ISUPPZ, IWORK
```

```
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: A, Z

SUBROUTINE SYEVR_64(JOBZ, RANGE, UPLO, [N], A, [LDA], VL, VU, IL, IU,
       ABSTOL, M, W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [IWORK], [LIWORK],
       [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER(8) :: N, LDA, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: ISUPPZ, IWORK
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: A, Z
```

C INTERFACE

```
#include <sunperf.h>

void ssyevr(char jobz, char range, char uplo, int  n,  float
          *a,  int  lda, float vl, float vu, int il, int iu,
          float abstol, int *m, float *w, float *z, int ldz,
          int *isuppz, int *info);

void ssyevr_64(char jobz, char range,  char  uplo,  long  n,
          float  *a,  long lda, float vl, float vu, long il,
          long iu, float abstol, long *m,  float  *w,  float
          *z, long ldz, long *isuppz, long *info);
```

PURPOSE

ssyevr computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix T. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Whenever possible, SSYEVR calls SSTEGR to compute the eigenspectrum using Relatively Robust Representations. SSTEGR computes eigenvalues by the dqds algorithm, while orthogonal eigenvectors are computed from various "good" L D L^T representations (also known as Relatively Robust Representations). Gram-Schmidt orthogonalization is avoided as far as possible. More specifically, the various steps of the algorithm are as follows. For the i-th unreduced block of T,

(a) Compute T - sigma_i = L_i D_i L_i^T, such that L_i D_i L_i^T is a relatively robust representation,

(b) Compute the eigenvalues, lambda_j, of L_i D_i L_i^T to high

relative accuracy by the dqds algorithm,
(c) If there is a cluster of close eigenvalues, "choose" sigma_i
close to the cluster, and go to step (a),
(d) Given the approximate eigenvalue lambda_j of L_i D_i L_i^T,
compute the corresponding eigenvector by forming a
rank-revealing twisted factorization.
The desired accuracy of the output can be specified by the input parameter ABSTOL.

For more details, see "A new O(n^2) algorithm for the symmetric tridiagonal eigenvalue/eigenvector problem", by Inderjit Dhillon, Computer Science Division Technical Report No. UCB//CSD-97-971, UC Berkeley, May 1997.
Note 1 : SSYEVR calls SSTEGR when the full spectrum is requested on machines which conform to the ieee-754 floating point standard. SSYEVR calls SSTEBZ and SSTEIN on non-ieee machines and
when partial spectrum requests are made.

Normal execution of SSTEGR may create NaNs and infinities and hence may abort due to a floating point exception in environments which do not handle NaNs and infinities in the ieee standard default manner.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

RANGE (input)
= 'A': all eigenvalues will be found.
= 'V': all eigenvalues in the half-open interval (VL,VU] will be found. = 'I': the IL-th through IU-th eigenvalues will be found.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A.

If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A. On exit, the lower triangle (if UPLO='L') or the upper triangle (if UPLO='U') of A, including the diagonal, is destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

VL (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)
See the description of VL.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
See the description of IL.

ABSTOL (input)
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABSTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABSTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing A to tri-diagonal form.

See "Computing Small Singular Values of Bidiagonal Matrices with Guaranteed High Relative Accuracy," by Demmel and Kahan, LAPACK Working Note #3.

If high relative accuracy is important, set ABSTOL to SLAMCH('Safe minimum'). Doing so will guarantee that eigenvalues are computed to high relative accuracy when possible in future releases. The current code does not make any guarantees about high relative accuracy, but furu- tre releases will. See J. Barlow and J. Demmel, "Computing Accurate Eigensystems of Scaled Diago- nally Dominant Matrices", LAPACK Working Note #7, for a discussion of which matrices define their eigenvalues to high relative accuracy.

M (output)
The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)
The first M elements contain the selected eigen- values in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, the first M columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). If JOBZ = 'N', then Z is not referenced. Note: the user must ensure that at least max(1,M) columns are supplied in the array Z; if RANGE = 'V', the exact value of M is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

ISUPPZ (output)
The support of the eigenvectors in Z, i.e., the indices indicating the nonzero elements in Z. The i-th eigenvector is nonzero only in elements ISUPPZ(2*i-1) through ISUPPZ(2*i).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,26*N). For optimal efficiency, LWORK >= (NB+6)*N, where NB is the max of the blocksize for SSYTRD and SORMTR returned by ILAENV.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of

```
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.
IWORK (workspace/output)
          On exit, if INFO = 0, IWORK(1) returns the optimal
          LWORK.

LIWORK (input)
          The dimension  of  the  array  IWORK.   LIWORK  >=
          max(1,10*N).

          If LIWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the IWORK array, returns this value as  the  first
          entry  of  the  IWORK  array, and no error message
          related to LIWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  Internal error
```

FURTHER DETAILS

```
Based on contributions by
   Inderjit Dhillon, IBM Almaden, USA
   Osni Marques, LBNL/NERSC, USA
   Ken Stanley, Computer Science Division, University of
     California at Berkeley, USA
```

Contents

NAME

ssyevx - compute selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix A

SYNOPSIS

```
SUBROUTINE SSYEVX(JOBZ, RANGE, UPLO, N, A, LDA, VL, VU, IL, IU,
      ABTOL, NFOUND, W, Z, LDZ, WORK, LDWORK, IWORK2, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER N, LDA, IL, IU, NFOUND, LDZ, LDWORK, INFO
INTEGER IWORK2(*), IFAIL(*)
REAL VL, VU, ABTOL
REAL A(LDA,*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSYEVX_64(JOBZ, RANGE, UPLO, N, A, LDA, VL, VU, IL, IU,
      ABTOL, NFOUND, W, Z, LDZ, WORK, LDWORK, IWORK2, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER*8 N, LDA, IL, IU, NFOUND, LDZ, LDWORK, INFO
INTEGER*8 IWORK2(*), IFAIL(*)
REAL VL, VU, ABTOL
REAL A(LDA,*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYEVX(JOBZ, RANGE, UPLO, N, A, [LDA], VL, VU, IL, IU,
       ABTOL, NFOUND, W, Z, [LDZ], [WORK], [LDWORK], [IWORK2], IFAIL,
       [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER :: N, LDA, IL, IU, NFOUND, LDZ, LDWORK, INFO
INTEGER, DIMENSION(:) :: IWORK2, IFAIL
REAL :: VL, VU, ABTOL
```

```
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: A, Z

SUBROUTINE SYEVX_64(JOBZ, RANGE, UPLO, N, A, [LDA], VL, VU, IL, IU,
       ABTOL, NFOUND, W, Z, [LDZ], [WORK], [LDWORK], [IWORK2], IFAIL,
       [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER(8) :: N, LDA, IL, IU, NFOUND, LDZ, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK2, IFAIL
REAL :: VL, VU, ABTOL
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: A, Z
```

C INTERFACE

```
#include <sunperf.h>

void ssyevx(char jobz, char range, char uplo, int  n,  float
          *a,  int  lda, float vl, float vu, int il, int iu,
          float abtol, int *nfound, float *w, float *z,  int
          ldz, int *ifail, int *info);

void ssyevx_64(char jobz, char range,  char  uplo,  long  n,
          float  *a,  long lda, float vl, float vu, long il,
          long iu, float  abtol,  long  *nfound,  float  *w,
          float *z, long ldz, long *ifail, long *info);
```

PURPOSE

ssyevx computes selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix A. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

RANGE (input)
          = 'A': all eigenvalues will be found.
          = 'V': all eigenvalues in the  half-open  interval
          (VL,VU]  will  be found.  = 'I': the IL-th through
          IU-th eigenvalues will be found.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
```

= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A. On exit, the lower triangle (if UPLO='L') or the upper triangle (if UPLO='U') of A, including the diagonal, is destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

VL (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)
See the description of VL.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
See the description of IL.

ABTOL (input)
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing A to tridiagonal form.

Eigenvalues will be computed most accurately when

```
          ABTOL  is  set  to  twice  the underflow threshold
          2*SLAMCH('S'), not zero.  If this routine  returns
          with INFO>0, indicating that some eigenvectors did
          not converge, try setting ABTOL to 2*SLAMCH('S').

          See "Computing Small Singular Values of Bidiagonal
          Matrices  with Guaranteed High Relative Accuracy,"
          by Demmel and Kahan, LAPACK Working Note #3.
NFOUND (output)
          The total  number  of  eigenvalues  found.   0  <=
          NFOUND  <=  N.  If RANGE = 'A', NFOUND = N, and if
          RANGE = 'I', NFOUND = IU-IL+1.

W (output)
          On normal exit, the first NFOUND elements  contain
          the selected eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, the first  NFOUND
          columns  of Z contain the orthonormal eigenvectors
          of the matrix  A  corresponding  to  the  selected
          eigenvalues, with the i-th column of Z holding the
          eigenvector associated with W(i).  If an eigenvec-
          tor  fails to converge, then that column of Z con-
          tains the latest approximation to the eigenvector,
          and  the  index  of the eigenvector is returned in
          IFAIL.  If JOBZ = 'N', then Z is  not  referenced.
          Note:   the   user   must  ensure  that  at  least
          max(1,NFOUND) columns are supplied in the array Z;
          if  RANGE  = 'V', the exact value of NFOUND is not
          known in advance and an upper bound must be used.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LDWORK.

LDWORK (input)
          The  length  of  the  array   WORK.   LDWORK   >=
          max(1,8*N).   For  optimal  efficiency,  LDWORK >=
          (NB+3)*N, where NB is the max of the blocksize for
          SSYTRD and SORMTR returned by ILAENV.

          If LDWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LDWORK is issued by XERBLA.
```

IWORK2 (workspace)
IFAIL (output)
If JOBZ = 'V', then if INFO = 0, the first NFOUND elements of IFAIL are zero. If INFO > 0, then IFAIL contains the indices of the eigenvectors that failed to converge. If JOBZ = 'N', then IFAIL is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, then i eigenvectors failed to converge. Their indices are stored in array IFAIL.

Contents

NAME

```
ssygs2 - reduce a real symmetric-definite generalized eigen-
problem to standard form
```

SYNOPSIS

```
SUBROUTINE SSYGS2(ITYPE, UPLO, N, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER ITYPE, N, LDA, LDB, INFO
REAL A(LDA,*), B(LDB,*)

SUBROUTINE SSYGS2_64(ITYPE, UPLO, N, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 ITYPE, N, LDA, LDB, INFO
REAL A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE SYGS2(ITYPE, UPLO, N, A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: ITYPE, N, LDA, LDB, INFO
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE SYGS2_64(ITYPE, UPLO, N, A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: ITYPE, N, LDA, LDB, INFO
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>
```

```
void ssygs2(int itype, char uplo, int n, float *a, int  lda,
          float *b, int ldb, int *info);

void ssygs2_64(long itype, char uplo, long n, float *a, long
          lda, float *b, long ldb, long *info);
```

PURPOSE

ssygs2 reduces a real symmetric-definite generalized eigen-problem to standard form.

If ITYPE = 1, the problem is A*x = lambda*B*x,
and A is overwritten by inv(U')*A*inv(U) or inv(L)*A*inv(L')
If ITYPE = 2 or 3, the problem is A*B*x = lambda*x or
B*A*x = lambda*x, and A is overwritten by U*A*U` or L'*A*L.

B must have been previously factorized as U'*U or L*L' by SPOTRF.

ARGUMENTS

ITYPE (input)
= 1: compute inv(U')*A*inv(U) or inv(L)*A*inv(L');
= 2 or 3: compute U*A*U' or L'*A*L.

UPLO (input)
Specifies whether the upper or lower triangular part of the symmetric matrix A is stored, and how B has been factorized. = 'U': Upper triangular
= 'L': Lower triangular

N (input) The order of the matrices A and B. N >= 0.

A (input/output)
On entry, the symmetric matrix A. If UPLO = 'U', the leading n by n upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading n by n lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, if INFO = 0, the transformed matrix, stored in the same format as A.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input) The triangular factor from the Cholesky factorization of B, as returned by SPOTRF.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

```
ssygst - reduce a real symmetric-definite generalized eigen-
problem to standard form
```

SYNOPSIS

```
SUBROUTINE SSYGST(ITYPE, UPLO, N, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER ITYPE, N, LDA, LDB, INFO
REAL A(LDA,*), B(LDB,*)

SUBROUTINE SSYGST_64(ITYPE, UPLO, N, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 ITYPE, N, LDA, LDB, INFO
REAL A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE SYGST(ITYPE, UPLO, N, A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: ITYPE, N, LDA, LDB, INFO
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE SYGST_64(ITYPE, UPLO, N, A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: ITYPE, N, LDA, LDB, INFO
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>
```

```
void ssygst(int itype, char uplo, int n, float *a, int  lda,
          float *b, int ldb, int *info);

void ssygst_64(long itype, char uplo, long n, float *a, long
          lda, float *b, long ldb, long *info);
```

PURPOSE

ssygst reduces a real symmetric-definite generalized eigen-problem to standard form.

If ITYPE = 1, the problem is A*x = lambda*B*x,
and A is overwritten by inv(U**T)*A*inv(U) or inv(L)*A*inv(L**T)
If ITYPE = 2 or 3, the problem is A*B*x = lambda*x or
B*A*x = lambda*x, and A is overwritten by U*A*U**T or L**T*A*L.

B must have been previously factorized as U**T*U or L*L**T by SPOTRF.

ARGUMENTS

ITYPE (input)
= 1: compute inv(U**T)*A*inv(U) or inv(L)*A*inv(L**T);
= 2 or 3: compute U*A*U**T or L**T*A*L.

UPLO (input)
= 'U': Upper triangle of A is stored and B is factored as U**T*U; = 'L': Lower triangle of A is stored and B is factored as L*L**T.

N (input) The order of the matrices A and B. N >= 0.

A (input/output)
On entry, the symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, if INFO = 0, the transformed matrix,

```
          stored in the same format as A.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

B (input) The triangular factor from the Cholesky factoriza-
          tion of B, as returned by SPOTRF.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
ssygv - compute all the  eigenvalues,  and  optionally,  the
eigenvectors of a real generalized symmetric-definite eigen-
problem, of the form A*x=(lambda)*B*x,  A*Bx=(lambda)*x,  or
B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE SSYGV(ITYPE, JOBZ, UPLO, N, A, LDA, B, LDB, W, WORK,
      LDWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER ITYPE, N, LDA, LDB, LDWORK, INFO
REAL A(LDA,*), B(LDB,*), W(*), WORK(*)

SUBROUTINE SSYGV_64(ITYPE, JOBZ, UPLO, N, A, LDA, B, LDB, W, WORK,
      LDWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 ITYPE, N, LDA, LDB, LDWORK, INFO
REAL A(LDA,*), B(LDB,*), W(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYGV(ITYPE, JOBZ, UPLO, N, A, [LDA], B, [LDB], W, [WORK],
       [LDWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: ITYPE, N, LDA, LDB, LDWORK, INFO
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE SYGV_64(ITYPE, JOBZ, UPLO, N, A, [LDA], B, [LDB], W, [WORK],
       [LDWORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: ITYPE, N, LDA, LDB, LDWORK, INFO
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void ssygv(int itype, char jobz, char uplo, int n, float *a,
          int lda, float *b, int ldb, float *w, int *info);

void ssygv_64(long itype, char  jobz,  char  uplo,  long  n,
          float  *a, long lda, float *b, long ldb, float *w,
          long *info);
```

PURPOSE

ssygv computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be symmetric and B is also
positive definite.

ARGUMENTS

```
ITYPE (input)
          Specifies the problem type to be solved:
          = 1:  A*x = (lambda)*B*x
          = 2:  A*B*x = (lambda)*x
          = 3:  B*A*x = (lambda)*x

JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.

A (input/output)
          On entry, the symmetric matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A.
```

If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A.

On exit, if JOBZ = 'V', then if INFO = 0, A contains the matrix Z of eigenvectors. The eigenvectors are normalized as follows: if ITYPE = 1 or 2, Z**T*B*Z = I; if ITYPE = 3, Z**T*inv(B)*Z = I. If JOBZ = 'N', then on exit the upper triangle (if UPLO='U') or the lower triangle (if UPLO='L') of A, including the diagonal, is destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the symmetric positive definite matrix B. If UPLO = 'U', the leading N-by-N upper triangular part of B contains the upper triangular part of the matrix B. If UPLO = 'L', the leading N-by-N lower triangular part of B contains the lower triangular part of the matrix B.

On exit, if INFO <= N, the part of B containing the matrix is overwritten by the triangular factor U or L from the Cholesky factorization B = U**T*U or B = L*L**T.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

W (output)
If INFO = 0, the eigenvalues in ascending order.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of the array WORK. LDWORK >= max(1,3*N-1). For optimal efficiency, LDWORK >= (NB+2)*N, where NB is the blocksize for SSYTRD returned by ILAENV.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: SPOTRF or SSYEV returned an error code:
<= N: if INFO = i, SSYEV failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero; > N: if INFO = N + i, for 1 <= i <= N, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

Contents

NAME

```
ssygvd - compute all the eigenvalues,  and  optionally,  the
eigenvectors of a real generalized symmetric-definite eigen-
problem, of the form A*x=(lambda)*B*x,  A*Bx=(lambda)*x,  or
B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE SSYGVD(ITYPE, JOBZ, UPLO, N, A, LDA, B, LDB, W, WORK,
      LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER ITYPE, N, LDA, LDB, LWORK, LIWORK, INFO
INTEGER IWORK(*)
REAL A(LDA,*), B(LDB,*), W(*), WORK(*)

SUBROUTINE SSYGVD_64(ITYPE, JOBZ, UPLO, N, A, LDA, B, LDB, W, WORK,
      LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
INTEGER*8 ITYPE, N, LDA, LDB, LWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
REAL A(LDA,*), B(LDB,*), W(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYGVD(ITYPE, JOBZ, UPLO, [N], A, [LDA], B, [LDB], W, [WORK],
       [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER :: ITYPE, N, LDA, LDB, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: W, WORK
```

```
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE SYGVD_64(ITYPE, JOBZ, UPLO, [N], A, [LDA], B, [LDB], W,
       [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
INTEGER(8) :: ITYPE, N, LDA, LDB, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void ssygvd(int itype, char jobz, char uplo,  int  n,  float
          *a,  int  lda,  float  *b,  int ldb, float *w, int
          *info);

void ssygvd_64(long itype, char jobz,  char  uplo,  long  n,
          float  *a, long lda, float *b, long ldb, float *w,
          long *info);
```

PURPOSE

ssygvd computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigen-problem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be symmetric and B is also positive definite. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

ITYPE (input)

Specifies the problem type to be solved:
= 1: A*x = (lambda)*B*x
= 2: A*B*x = (lambda)*x
= 3: B*A*x = (lambda)*x

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

UPLO (input)
= 'U': Upper triangles of A and B are stored;
= 'L': Lower triangles of A and B are stored.

N (input) The order of the matrices A and B. N >= 0.

A (input/output)
On entry, the symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A.

On exit, if JOBZ = 'V', then if INFO = 0, A contains the matrix Z of eigenvectors. The eigenvectors are normalized as follows: if ITYPE = 1 or 2, Z**T*B*Z = I; if ITYPE = 3, Z**T*inv(B)*Z = I. If JOBZ = 'N', then on exit the upper triangle (if UPLO='U') or the lower triangle (if UPLO='L') of A, including the diagonal, is destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the symmetric matrix B. If UPLO = 'U', the leading N-by-N upper triangular part of B contains the upper triangular part of the matrix B. If UPLO = 'L', the leading N-by-N lower triangular part of B contains the lower triangular part of the matrix B.

On exit, if INFO <= N, the part of B containing the matrix is overwritten by the triangular factor U or L from the Cholesky factorization B = U**T*U or B = L*L**T.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

W (output)
If INFO = 0, the eigenvalues in ascending order.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If N <= 1, LWORK >= 1. If JOBZ = 'N' and N > 1, LWORK >= 2*N+1. If JOBZ = 'V' and N > 1, LWORK >= 1 + 6*N + 2*N**2.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of the array IWORK. If N <= 1, LIWORK >= 1. If JOBZ = 'N' and N > 1, LIWORK >= 1. If JOBZ = 'V' and N > 1, LIWORK >= 3 + 5*N.

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: SPOTRF or SSYEVD returned an error code:
<= N: if INFO = i, SSYEVD failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero; > N: if INFO = N + i, for 1 <= i <= N, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
Mark Fahey, Department of Mathematics, Univ. of Kentucky, USA

Contents

NAME

```
ssygvx  -  compute  selected  eigenvalues,  and  optionally,
eigenvectors of a real generalized symmetric-definite eigen-
problem, of the form A*x=(lambda)*B*x,  A*Bx=(lambda)*x,  or
B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE SSYGVX(ITYPE, JOBZ, RANGE, UPLO, N, A, LDA, B, LDB, VL,
      VU, IL, IU, ABSTOL, M, W, Z, LDZ, WORK, LWORK, IWORK, IFAIL,
      INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER ITYPE, N, LDA, LDB, IL, IU, M, LDZ, LWORK, INFO
INTEGER IWORK(*), IFAIL(*)
REAL VL, VU, ABSTOL
REAL A(LDA,*), B(LDB,*), W(*), Z(LDZ,*), WORK(*)

SUBROUTINE SSYGVX_64(ITYPE, JOBZ, RANGE, UPLO, N, A, LDA, B, LDB, VL,
      VU, IL, IU, ABSTOL, M, W, Z, LDZ, WORK, LWORK, IWORK, IFAIL,
      INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
INTEGER*8 ITYPE, N, LDA, LDB, IL, IU, M, LDZ, LWORK, INFO
INTEGER*8 IWORK(*), IFAIL(*)
REAL VL, VU, ABSTOL
REAL A(LDA,*), B(LDB,*), W(*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYGVX(ITYPE, JOBZ, RANGE, UPLO, [N], A, [LDA], B, [LDB],
       VL, VU, IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK], [LWORK], [IWORK],
       IFAIL, [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER :: ITYPE, N, LDA, LDB, IL, IU, M, LDZ, LWORK, INFO
INTEGER, DIMENSION(:) :: IWORK, IFAIL
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: A, B, Z

SUBROUTINE SYGVX_64(ITYPE, JOBZ, RANGE, UPLO, [N], A, [LDA], B, [LDB],
       VL, VU, IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK], [LWORK], [IWORK],
       IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
INTEGER(8) :: ITYPE, N, LDA, LDB, IL,  IU,  M,  LDZ,  LWORK,
INFO
INTEGER(8), DIMENSION(:) :: IWORK, IFAIL
REAL :: VL, VU, ABSTOL
REAL, DIMENSION(:) :: W, WORK
REAL, DIMENSION(:,:) :: A, B, Z
```

C INTERFACE

```
#include <sunperf.h>

void ssygvx(int itype, char jobz, char range, char uplo, int
          n, float *a, int lda, float *b, int ldb, float vl,
          float vu, int il, int iu, float  abstol,  int  *m,
          float  *w,  float  *z,  int  ldz,  int *ifail, int
          *info);

void ssygvx_64(long itype, char jobz, char range, char uplo,
          long  n,  float  *a, long lda, float *b, long ldb,
          float vl,  float  vu,  long  il,  long  iu,  float
          abstol,  long  *m,  float  *w, float *z, long ldz,
          long *ifail, long *info);
```

PURPOSE

ssygvx computes selected eigenvalues, and optionally, eigenvectors of a real generalized symmetric-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be symmetric and B is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

```
ITYPE (input)
          Specifies the problem type to be solved:
          = 1:  A*x = (lambda)*B*x
          = 2:  A*B*x = (lambda)*x
          = 3:  B*A*x = (lambda)*x

JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

RANGE (input)
          = 'A': all eigenvalues will be found.
          = 'V': all eigenvalues in the  half-open  interval
          (VL,VU]  will  be found.  = 'I': the IL-th through
          IU-th eigenvalues will be found.

UPLO (input)
          = 'U':  Upper triangle of A and B are stored;
          = 'L':  Lower triangle of A and B are stored.

N (input) The order of the matrix pencil (A,B).  N >= 0.

A (input/output)
          On entry, the symmetric matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of A contains the lower  triangular  part  of
          the matrix A.

          On exit, the lower triangle (if UPLO='L')  or  the
          upper  triangle  (if UPLO='U') of A, including the
          diagonal, is destroyed.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

B (input/output)
          On entry, the symmetric matrix B.  If UPLO =  'U',
          the leading N-by-N upper triangular part of B con-
          tains the upper triangular part of the  matrix  B.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of B contains the lower  triangular  part  of
          the matrix B.

          On exit, if INFO <= N, the part  of  B  containing
          the matrix is overwritten by the triangular factor
          U or L from the Cholesky factorization B =  U**T*U
          or B = L*L**T.
```

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

VL (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)
See the description of VL.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
See the description of IL.

ABSTOL (input)
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABSTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABSTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing A to tri-diagonal form.

Eigenvalues will be computed most accurately when ABSTOL is set to twice the underflow threshold 2*DLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABSTOL to 2*SLAMCH('S').

M (output)
The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)
On normal exit, the first M elements contain the selected eigenvalues in ascending order.

```
Z (input) If JOBZ = 'N', then Z is not referenced.  If  JOBZ
          =  'V', then if INFO = 0, the first M columns of Z
          contain the orthonormal eigenvectors of the matrix
          A  corresponding to the selected eigenvalues, with
          the i-th column of Z holding the eigenvector asso-
          ciated with W(i).  The eigenvectors are normalized
          as follows:  if ITYPE = 1 or 2, Z**T*B*Z =  I;  if
          ITYPE = 3, Z**T*inv(B)*Z = I.
          If an eigenvector fails  to  converge,  then  that
          column  of  Z contains the latest approximation to
          the eigenvector, and the index of the  eigenvector
          is  returned in IFAIL.  Note: the user must ensure
          that at least max(1,M) columns are supplied in the
          array  Z;  if RANGE = 'V', the exact value of M is
          not known in advance and an upper  bound  must  be
          used.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The  length  of  the   array   WORK.    LWORK   >=
          max(1,8*N).   For  optimal  efficiency,  LWORK  >=
          (NB+3)*N, where NB is  the  blocksize  for  SSYTRD
          returned by ILAENV.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

IWORK (workspace)
          dimension(5*N)

IFAIL (output)
          If JOBZ = 'V', then if INFO = 0, the first M  ele-
          ments  of IFAIL are zero.  If INFO > 0, then IFAIL
          contains the  indices  of  the  eigenvectors  that
          failed  to converge.  If JOBZ = 'N', then IFAIL is
          not referenced.

INFO (output)
          = 0:  successful exit
```

< 0: if INFO = -i, the i-th argument had an illegal value
> 0: SPOTRF or SSYEVX returned an error code:
<= N: if INFO = i, SSYEVX failed to converge; i eigenvectors failed to converge. Their indices are stored in array IFAIL. > N: if INFO = N + i, for 1 <= i <= N, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
Mark Fahey, Department of Mathematics, Univ. of Kentucky, USA

Contents

NAME

```
ssymm - perform one of the matrix-matrix operations    C  :=
alpha*A*B + beta*C or C := alpha*B*A + beta*C
```

SYNOPSIS

```
SUBROUTINE SSYMM(SIDE, UPLO, M, N, ALPHA, A, LDA, B, LDB, BETA, C,
      LDC)

CHARACTER * 1 SIDE, UPLO
INTEGER M, N, LDA, LDB, LDC
REAL ALPHA, BETA
REAL A(LDA,*), B(LDB,*), C(LDC,*)

SUBROUTINE SSYMM_64(SIDE, UPLO, M, N, ALPHA, A, LDA, B, LDB, BETA, C,
      LDC)

CHARACTER * 1 SIDE, UPLO
INTEGER*8 M, N, LDA, LDB, LDC
REAL ALPHA, BETA
REAL A(LDA,*), B(LDB,*), C(LDC,*)
```

F95 INTERFACE

```
SUBROUTINE SYMM(SIDE, UPLO, [M], [N], ALPHA, A, [LDA], B, [LDB],
       BETA, C, [LDC])

CHARACTER(LEN=1) :: SIDE, UPLO
INTEGER :: M, N, LDA, LDB, LDC
REAL :: ALPHA, BETA
REAL, DIMENSION(:,:) :: A, B, C

SUBROUTINE SYMM_64(SIDE, UPLO, [M], [N], ALPHA, A, [LDA], B, [LDB],
       BETA, C, [LDC])
```

```
CHARACTER(LEN=1) :: SIDE, UPLO
INTEGER(8) :: M, N, LDA, LDB, LDC
REAL :: ALPHA, BETA
REAL, DIMENSION(:,:) :: A, B, C
```

C INTERFACE

```
#include <sunperf.h>

void ssymm(char side, char uplo, int m, int n, float  alpha,
          float  *a, int lda, float *b, int ldb, float beta,
          float *c, int ldc);

void ssymm_64(char side, char uplo, long m,  long  n,  float
          alpha,  float  *a,  long  lda, float *b, long ldb,
          float beta, float *c, long ldc);
```

PURPOSE

ssymm performs one of the matrix-matrix operations C := alpha*A*B + beta*C or C := alpha*B*A + beta*C where alpha and beta are scalars, A is a symmetric matrix and B and C are m by n matrices.

ARGUMENTS

SIDE (input)
On entry, SIDE specifies whether the symmetric matrix A appears on the left or right in the operation as follows:

SIDE = 'L' or 'l'   C := alpha*A*B + beta*C,

SIDE = 'R' or 'r'   C := alpha*B*A + beta*C,

Unchanged on exit.

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the symmetric matrix A is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of the symmetric matrix is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of the symmetric matrix is to be referenced.

```
           Unchanged on exit.

  M (input)
           On entry,  M  specifies the number of rows of  the
           matrix  C.  M >= 0.  Unchanged on exit.

  N (input)
           On entry, N specifies the number of columns of the
           matrix C.  N >= 0.  Unchanged on exit.

  ALPHA (input)
           On  entry,  ALPHA  specifies  the  scalar   alpha.
           Unchanged on exit.
  A (input)
           REAL array of DIMENSION ( LDA, ka ), where ka is m
           when  SIDE = 'L' or 'l'  and is  n otherwise.

           Before entry  with  SIDE = 'L' or 'l',  the  m  by
           m   part  of  the array  A  must contain the  sym-
           metric matrix,  such that when  UPLO = 'U' or 'u',
           the  leading  m  by m upper triangular part of the
           array  A  must contain the upper  triangular  part
           of  the  symmetric matrix and the  strictly  lower
           triangular part of  A   is  not  referenced,   and
           when   UPLO  =  'L'  or  'l',  the leading  m by m
           lower triangular part  of the  array  A must  con-
           tain   the   lower  triangular  part  of the  sym-
           metric matrix and the  strictly  upper  triangular
           part of  A  is not referenced.

           Before entry  with  SIDE = 'R' or 'r',  the  n  by
           n   part  of  the array  A  must contain the  sym-
           metric matrix,  such that when  UPLO = 'U' or 'u',
           the  leading  n  by n upper triangular part of the
           array  A  must contain the upper  triangular  part
           of  the  symmetric matrix and the  strictly  lower
           triangular part of  A   is  not  referenced,   and
           when   UPLO  =  'L'  or  'l',  the leading  n by n
           lower triangular part  of the  array  A must  con-
           tain   the   lower  triangular  part  of the  sym-
           metric matrix and the  strictly  upper  triangular
           part of  A  is not referenced.

           Unchanged on exit.

  LDA (input)
           On entry, LDA specifies the first dimension  of  A
           as  declared  in  the calling (sub) program.  When
           SIDE = 'L' or 'l'  then LDA >= max( 1, m ), other-
```

wise LDA >= max(1, n). Unchanged on exit.

B (input)
REAL array of DIMENSION (LDB, n). Before entry, the leading m by n part of the array B must contain the matrix B. Unchanged on exit.

LDB (input)
On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. LDB >= max(1, m). Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. When BETA is supplied as zero then C need not be set on input. Unchanged on exit.

C (input/output)
REAL array of DIMENSION (LDC, n). Before entry, the leading m by n part of the array C must contain the matrix C, except when beta is zero, in which case C need not be set on entry. On exit, the array C is overwritten by the m by n updated matrix.

LDC (input)
On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC >= max(1, m). Unchanged on exit.

Contents

NAME

```
ssymv - perform the matrix-vector operation   y := alpha*A*x
+ beta*y
```

SYNOPSIS

```
SUBROUTINE SSYMV(UPLO, N, ALPHA, A, LDA, X, INCX, BETA, Y, INCY)

CHARACTER * 1 UPLO
INTEGER N, LDA, INCX, INCY
REAL ALPHA, BETA
REAL A(LDA,*), X(*), Y(*)

SUBROUTINE SSYMV_64(UPLO, N, ALPHA, A, LDA, X, INCX, BETA, Y, INCY)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, INCX, INCY
REAL ALPHA, BETA
REAL A(LDA,*), X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE SYMV(UPLO, [N], ALPHA, A, [LDA], X, [INCX], BETA, Y, [INCY])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, INCX, INCY
REAL :: ALPHA, BETA
REAL, DIMENSION(:) :: X, Y
REAL, DIMENSION(:,:) :: A

SUBROUTINE SYMV_64(UPLO, [N], ALPHA, A, [LDA], X, [INCX], BETA, Y,
       [INCY])

CHARACTER(LEN=1) :: UPLO
```

```
INTEGER(8) :: N, LDA, INCX, INCY
REAL :: ALPHA, BETA
REAL, DIMENSION(:) :: X, Y
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void ssymv(char uplo, int n, float alpha, float *a, int lda,
          float  *x,  int  incx,  float  beta, float *y, int
          incy);

void ssymv_64(char uplo, long n, float alpha, float *a, long
          lda,  float  *x,  long incx, float beta, float *y,
          long incy);
```

PURPOSE

ssymv performs the matrix-vector operation y := alpha*A*x + beta*y, where alpha and beta are scalars, x and y are n element vectors and A is an n by n symmetric matrix.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array A is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of A is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of A is to be referenced.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
Before entry with UPLO = 'U' or 'u', the leading

n by n upper triangular part of the array A must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of A is not referenced. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of A is not referenced. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. When BETA is supplied as zero then Y need not be set on input. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
ssyr - perform the symmetric rank 1 operation   A :=
alpha*x*x' + A
```

SYNOPSIS

```
SUBROUTINE SSYR(UPLO, N, ALPHA, X, INCX, A, LDA)

CHARACTER * 1 UPLO
INTEGER N, INCX, LDA
REAL ALPHA
REAL X(*), A(LDA,*)

SUBROUTINE SSYR_64(UPLO, N, ALPHA, X, INCX, A, LDA)

CHARACTER * 1 UPLO
INTEGER*8 N, INCX, LDA
REAL ALPHA
REAL X(*), A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE SYR(UPLO, [N], ALPHA, X, [INCX], A, [LDA])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INCX, LDA
REAL :: ALPHA
REAL, DIMENSION(:) :: X
REAL, DIMENSION(:,:) :: A

SUBROUTINE SYR_64(UPLO, [N], ALPHA, X, [INCX], A, [LDA])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INCX, LDA
```

```
REAL :: ALPHA
REAL, DIMENSION(:) :: X
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void ssyr(char uplo, int n, float alpha, float *x, int incx,
          float *a, int lda);

void ssyr_64(char uplo, long n, float alpha, float *x,  long
          incx, float *a, long lda);
```

PURPOSE

ssyr performs the symmetric rank 1 operation A := alpha*x*x' + A, where alpha is a real scalar, x is an n element vector and A is an n by n symmetric matrix.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array A is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of A is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of A is to be referenced.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)

On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

A (input/output)

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of A is not referenced. On exit, the upper triangular part of the array A is overwritten by the upper triangular part of the updated matrix. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of A is not referenced. On exit, the lower triangular part of the array A is overwritten by the lower triangular part of the updated matrix.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

Contents

NAME

ssyr2 - perform the symmetric rank 2 operation A := alpha*x*y' + alpha*y*x' + A

SYNOPSIS

```
SUBROUTINE SSYR2(UPLO, N, ALPHA, X, INCX, Y, INCY, A, LDA)

CHARACTER * 1 UPLO
INTEGER N, INCX, INCY, LDA
REAL ALPHA
REAL X(*), Y(*), A(LDA,*)

SUBROUTINE SSYR2_64(UPLO, N, ALPHA, X, INCX, Y, INCY, A, LDA)

CHARACTER * 1 UPLO
INTEGER*8 N, INCX, INCY, LDA
REAL ALPHA
REAL X(*), Y(*), A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE SYR2(UPLO, [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, INCX, INCY, LDA
REAL :: ALPHA
REAL, DIMENSION(:) :: X, Y
REAL, DIMENSION(:,:) :: A

SUBROUTINE SYR2_64(UPLO, [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, INCX, INCY, LDA
```

```
REAL :: ALPHA
REAL, DIMENSION(:) :: X, Y
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void ssyr2(char uplo, int n,  float  alpha,  float  *x,  int
          incx, float *y, int incy, float *a, int lda);

void ssyr2_64(char uplo, long n, float alpha, float *x, long
          incx, float *y, long incy, float *a, long lda);
```

PURPOSE

ssyr2 performs the symmetric rank 2 operation A := alpha*x*y' + alpha*y*x' + A, where alpha is a scalar, x and y are n element vectors and A is an n by n symmetric matrix.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array A is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of A is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of A is to be referenced.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)

On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

Y (input)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. Unchanged on exit.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

A (input/output)

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of A is not referenced. On exit, the upper triangular part of the array A is overwritten by the upper triangular part of the updated matrix. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of A is not referenced. On exit, the lower triangular part of the array A is overwritten by the lower triangular part of the updated matrix.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

Contents

NAME

```
ssyr2k - perform one of the symmetric rank 2k operations   C
:=  alpha*A*B'  +  alpha*B*A'  + beta*C or C := alpha*A'*B +
alpha*B'*A + beta*C
```

SYNOPSIS

```
SUBROUTINE SSYR2K(UPLO, TRANSA, N, K, ALPHA, A, LDA, B, LDB, BETA, C,
      LDC)

CHARACTER * 1 UPLO, TRANSA
INTEGER N, K, LDA, LDB, LDC
REAL ALPHA, BETA
REAL A(LDA,*), B(LDB,*), C(LDC,*)

SUBROUTINE SSYR2K_64(UPLO, TRANSA, N, K, ALPHA, A, LDA, B, LDB, BETA,
      C, LDC)

CHARACTER * 1 UPLO, TRANSA
INTEGER*8 N, K, LDA, LDB, LDC
REAL ALPHA, BETA
REAL A(LDA,*), B(LDB,*), C(LDC,*)
```

F95 INTERFACE

```
SUBROUTINE SYR2K(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], B, [LDB],
       BETA, C, [LDC])

CHARACTER(LEN=1) :: UPLO, TRANSA
INTEGER :: N, K, LDA, LDB, LDC
REAL :: ALPHA, BETA
REAL, DIMENSION(:,:) :: A, B, C

SUBROUTINE SYR2K_64(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], B,
```

```
          [LDB], BETA, C, [LDC])

   CHARACTER(LEN=1) :: UPLO, TRANSA
   INTEGER(8) :: N, K, LDA, LDB, LDC
   REAL :: ALPHA, BETA
   REAL, DIMENSION(:,:) :: A, B, C
```

C INTERFACE

```
   #include <sunperf.h>

   void ssyr2k(char uplo, char transa,  int  n,  int  k,  float
             alpha, float *a, int lda, float *b, int ldb, float
             beta, float *c, int ldc);
   void ssyr2k_64(char uplo, char transa, long n, long k, float
             alpha,  float  *a,  long  lda, float *b, long ldb,
             float beta, float *c, long ldc);
```

PURPOSE

ssyr2k performs one of the symmetric rank 2k operations C := alpha*A*B' + alpha*B*A' + beta*C or C := alpha*A'*B + alpha*B'*A + beta*C where alpha and beta are scalars, C is an n by n symmetric matrix and A and B are n by k matrices in the first case and k by n matrices in the second case.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array C is to be referenced as follows:

UPLO = 'U' or 'u' Only the upper triangular part of C is to be referenced.

UPLO = 'L' or 'l' Only the lower triangular part of C is to be referenced.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n' C := alpha*A*B' + alpha*B*A' + beta*C.

TRANSA = 'T' or 't'   C := alpha*A'*B + alpha*B'*A + beta*C.

TRANSA = 'C' or 'c'   C := alpha*A'*B + alpha*B'*A + beta*C.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input)

On entry, N specifies the order of the matrix C. N must be at least zero. Unchanged on exit.

K (input)

On entry with TRANSA = 'N' or 'n', K specifies the number of columns of the matrices A and B, and on entry with TRANSA = 'T' or 't' or 'C' or 'c', K specifies the number of rows of the matrices A and B. K must be at least zero. Unchanged on exit.

ALPHA (input)

On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)

REAL array of DIMENSION (LDA, ka), where ka is k when TRANSA = 'N' or 'n', and is n otherwise. Before entry with TRANSA = 'N' or 'n', the leading n by k part of the array A must contain the matrix A, otherwise the leading k by n part of the array A must contain the matrix A. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDA must be at least max(1, n), otherwise LDA must be at least max(1, k). Unchanged on exit.

B (input)

REAL array of DIMENSION (LDB, kb), where kb is k when TRANSA = 'N' or 'n', and is n otherwise. Before entry with TRANSA = 'N' or 'n', the leading n by k part of the array B must contain the matrix B, otherwise the leading k by n part of the array B must contain the matrix B. Unchanged on exit.

LDB (input)

On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDB must be at least max(1, n), otherwise LDB must be at least max(1, k). Unchanged on exit.

BETA (input)

On entry, BETA specifies the scalar beta. Unchanged on exit.

C (input/output)

REAL array of DIMENSION (LDC, n).

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array C must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of C is not referenced. On exit, the upper triangular part of the array C is overwritten by the upper triangular part of the updated matrix.

Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array C must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of C is not referenced. On exit, the lower triangular part of the array C is overwritten by the lower triangular part of the updated matrix.

LDC (input)

On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC must be at least max(1, n). Unchanged on exit.

Contents

NAME

```
ssyrfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is symmetric indefin-
ite, and provides error bounds and backward error  estimates
for the solution
```

SYNOPSIS

```
SUBROUTINE SSYRFS(UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*), WORK2(*)
REAL  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),  FERR(*),
BERR(*), WORK(*)

SUBROUTINE SSYRFS_64(UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
REAL  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),  FERR(*),
BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYRFS(UPLO, N, NRHS, A, [LDA], AF, [LDAF], IPIVOT, B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT, WORK2
```

```
REAL, DIMENSION(:) :: FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, AF, B, X

SUBROUTINE SYRFS_64(UPLO, N, NRHS, A, [LDA], AF, [LDAF], IPIVOT, B,
       [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
REAL, DIMENSION(:) :: FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
#include <sunperf.h>
void ssyrfs(char uplo, int n, int nrhs, float *a,  int  lda,
          float  *af,  int  ldaf, int *ipivot, float *b, int
          ldb, float *x, int ldx, float *ferr, float  *berr,
          int *info);

void ssyrfs_64(char uplo, long n, long nrhs, float *a,  long
          lda, float *af, long ldaf, long *ipivot, float *b,
          long ldb, float *x, long ldx, float  *ferr,  float
          *berr, long *info);
```

PURPOSE

ssyrfs improves the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not refer-

enced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input)
The factored form of the matrix A. AF contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by SSYTRF.

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by SSYTRF.

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by SSYTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)

The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

ssyrk - perform one of the symmetric rank k operations C := alpha*A*A' + beta*C or C := alpha*A'*A + beta*C

SYNOPSIS

```
SUBROUTINE SSYRK(UPLO, TRANSA, N, K, ALPHA, A, LDA, BETA, C, LDC)

CHARACTER * 1 UPLO, TRANSA
INTEGER N, K, LDA, LDC
REAL ALPHA, BETA
REAL A(LDA,*), C(LDC,*)

SUBROUTINE SSYRK_64(UPLO, TRANSA, N, K, ALPHA, A, LDA, BETA, C, LDC)

CHARACTER * 1 UPLO, TRANSA
INTEGER*8 N, K, LDA, LDC
REAL ALPHA, BETA
REAL A(LDA,*), C(LDC,*)
```

F95 INTERFACE

```
SUBROUTINE SYRK(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], BETA, C,
       [LDC])

CHARACTER(LEN=1) :: UPLO, TRANSA
INTEGER :: N, K, LDA, LDC
REAL :: ALPHA, BETA
REAL, DIMENSION(:,:) :: A, C

SUBROUTINE SYRK_64(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], BETA,
       C, [LDC])

CHARACTER(LEN=1) :: UPLO, TRANSA
```

```
INTEGER(8) :: N, K, LDA, LDC
REAL :: ALPHA, BETA
REAL, DIMENSION(:,:) :: A, C
```

C INTERFACE

```
#include <sunperf.h>

void ssyrk(char uplo, char  transa,  int  n,  int  k,  float
          alpha,  float  *a,  int lda, float beta, float *c,
          int ldc);

void ssyrk_64(char uplo, char transa, long n, long k,  float
          alpha,  float  *a, long lda, float beta, float *c,
          long ldc);
```

PURPOSE

ssyrk performs one of the symmetric rank k operations C := alpha*A*A' + beta*C or C := alpha*A'*A + beta*C where alpha and beta are scalars, C is an n by n symmetric matrix and A is an n by k matrix in the first case and a k by n matrix in the second case.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array C is to be referenced as follows:

UPLO = 'U' or 'u' Only the upper triangular part of C is to be referenced.

UPLO = 'L' or 'l' Only the lower triangular part of C is to be referenced.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n' C := alpha*A*A' + beta*C.

TRANSA = 'T' or 't' C := alpha*A'*A + beta*C.

TRANSA = 'C' or 'c' C := alpha*A'*A + beta*C.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input)
On entry, N specifies the order of the matrix C. N must be at least zero. Unchanged on exit.

K (input)
On entry with TRANSA = 'N' or 'n', K specifies the number of columns of the matrix A, and on entry with TRANSA = 'T' or 't' or 'C' or 'c', K specifies the number of rows of the matrix A. K must be at least zero. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
REAL array of DIMENSION (LDA, ka), where ka is k when TRANSA = 'N' or 'n', and is n otherwise. Before entry with TRANSA = 'N' or 'n', the leading n by k part of the array A must contain the matrix A, otherwise the leading k by n part of the array A must contain the matrix A. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDA must be at least max(1, n), otherwise LDA must be at least max(1, k). Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. Unchanged on exit.

C (input/output)
REAL array of DIMENSION (LDC, n).

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array C must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of C is not referenced. On exit, the upper triangular part of the array C is overwritten by the upper triangular part of the updated

matrix.

Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array C must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of C is not referenced. On exit, the lower triangular part of the array C is overwritten by the lower triangular part of the updated matrix.

LDC (input)

On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC must be at least max(1, n). Unchanged on exit.

Contents

NAME

ssysv - compute the solution to a real system of linear equations A * X = B,

SYNOPSIS

```
SUBROUTINE SSYSV(UPLO, N, NRHS, A, LDA, IPIV, B, LDB, WORK, LWORK,
      INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDA, LDB, LWORK, INFO
INTEGER IPIV(*)
REAL A(LDA,*), B(LDB,*), WORK(*)

SUBROUTINE SSYSV_64(UPLO, N, NRHS, A, LDA, IPIV, B, LDB, WORK,
      LWORK, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDA, LDB, LWORK, INFO
INTEGER*8 IPIV(*)
REAL A(LDA,*), B(LDB,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYSV(UPLO, [N], [NRHS], A, [LDA], IPIV, B, [LDB], [WORK],
       [LWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDA, LDB, LWORK, INFO
INTEGER, DIMENSION(:) :: IPIV
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE SYSV_64(UPLO, [N], [NRHS], A, [LDA], IPIV, B, [LDB], [WORK],
```

```
          [LWORK], [INFO])

   CHARACTER(LEN=1) :: UPLO
   INTEGER(8) :: N, NRHS, LDA, LDB, LWORK, INFO
   INTEGER(8), DIMENSION(:) :: IPIV
   REAL, DIMENSION(:) :: WORK
   REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
   #include <sunperf.h>

   void ssysv(char uplo, int n, int nrhs, float  *a,  int  lda,
             int *ipivot, float *b, int ldb, int *info);
   void ssysv_64(char uplo, long n, long nrhs, float  *a,  long
             lda,  long  *ipivot,  float  *b,  long  ldb,  long
             *info);
```

PURPOSE

```
   ssysv computes the solution to a real system of linear equa-
   tions
      A * X = B, where A is an N-by-N symmetric  matrix  and  X
   and B are N-by-NRHS matrices.

   The diagonal pivoting method is used to factor A as
      A = U * D * U**T,  if UPLO = 'U', or
      A = L * D * L**T,  if UPLO = 'L',
   where U (or L) is a product of permutation  and  unit  upper
   (lower)  triangular  matrices,  and D is symmetric and block
   diagonal with 1-by-1 and 2-by-2 diagonal blocks.   The  fac-
   tored  form  of  A is then used to solve the system of equa-
   tions A * X = B.
```

ARGUMENTS

```
   UPLO (input)
             = 'U':  Upper triangle of A is stored;
             = 'L':  Lower triangle of A is stored.

   N (input) The number of linear equations, i.e., the order of
             the matrix A.  N >= 0.

   NRHS (input)
             The number of right hand sides, i.e.,  the  number
             of columns of the matrix B.  NRHS >= 0.

   A (input/output)
```

On entry, the symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, if INFO = 0, the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by SSYTRF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIV (output)
Details of the interchanges and the block structure of D, as determined by SSYTRF. If IPIV(k) > 0, then rows and columns k and IPIV(k) were interchanged, and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIV(k) = IPIV(k-1) < 0, then rows and columns k-1 and -IPIV(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIV(k) = IPIV(k+1) < 0, then rows and columns k+1 and -IPIV(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The length of WORK. LWORK >= 1, and for best performance LWORK >= N*NB, where NB is the optimal blocksize for SSYTRF.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of

the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, so the solution could not be computed.

Contents

NAME

```
ssysvx - use the diagonal pivoting factorization to  compute
the solution to a real system of linear equations A * X = B,
```

SYNOPSIS

```
SUBROUTINE SSYSVX(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B,
      LDB, X, LDX, RCOND, FERR, BERR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER IPIVOT(*), WORK2(*)
REAL RCOND
REAL  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),  FERR(*),
BERR(*), WORK(*)

SUBROUTINE SSYSVX_64(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT,
      B, LDB, X, LDX, RCOND, FERR, BERR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER*8 IPIVOT(*), WORK2(*)
REAL RCOND
REAL  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),  FERR(*),
BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYSVX(FACT, UPLO, N, NRHS, A, [LDA], AF, [LDAF], IPIVOT,
       B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [LDWORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: FACT, UPLO
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
```

```
    INTEGER, DIMENSION(:) :: IPIVOT, WORK2
    REAL :: RCOND
    REAL, DIMENSION(:) :: FERR, BERR, WORK
    REAL, DIMENSION(:,:) :: A, AF, B, X

    SUBROUTINE SYSVX_64(FACT, UPLO, N, NRHS, A, [LDA], AF, [LDAF],
           IPIVOT, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [LDWORK],
           [WORK2], [INFO])

    CHARACTER(LEN=1) :: FACT, UPLO
    INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
    INTEGER(8), DIMENSION(:) :: IPIVOT, WORK2
    REAL :: RCOND
    REAL, DIMENSION(:) :: FERR, BERR, WORK
    REAL, DIMENSION(:,:) :: A, AF, B, X
```

C INTERFACE

```
    #include <sunperf.h>

    void ssysvx(char fact, char uplo, int n, int nrhs, float *a,
              int  lda,  float *af, int ldaf, int *ipivot, float
              *b, int ldb, float  *x,  int  ldx,  float  *rcond,
              float *ferr, float *berr, int *info);

    void ssysvx_64(char fact, char  uplo,  long  n,  long  nrhs,
              float  *a,  long  lda,  float *af, long ldaf, long
              *ipivot, float *b, long ldb, float *x,  long  ldx,
              float  *rcond,  float  *ferr,  float  *berr,  long
              *info);
```

PURPOSE

```
    ssysvx uses the diagonal pivoting factorization  to  compute
    the solution to a real system of linear equations A * X = B,
    where A is an N-by-N symmetric matrix and X and B are  N-by-
    NRHS matrices.

    Error bounds on the solution and a  condition  estimate  are
    also provided.

    The following steps are performed:

    1. If FACT = 'N', the diagonal pivoting method  is  used  to
    factor A.
       The form of the factorization is
          A = U * D * U**T,  if UPLO = 'U', or
          A = L * D * L**T,  if UPLO = 'L',
       where U (or L) is a product of permutation and unit upper
    (lower)
       triangular matrices, and D is symmetric and block  diago-
```

nal with
1-by-1 and 2-by-2 diagonal blocks.

2. If some D(i,i)=0, so that D is exactly singular, then the routine returns with INFO = i. Otherwise, the factored form of A is used to estimate the condition number of the matrix A. If the reciprocal of the condition number is less than machine precision, INFO = N+1 is returned as a warning, but the routine still goes on to solve for X and compute error bounds as described below.

3. The system of equations is solved for X using the factored form of A.

4. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of A has been supplied on entry. = 'F': On entry, AF and IPIVOT contain the factored form of A. AF and IPIVOT will not be modified. = 'N': The matrix A will be copied to AF and factored.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not refer-

enced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input/output)
If FACT = 'F', then AF is an input argument and on entry contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by SSYTRF.
If FACT = 'N', then AF is an output argument and on exit returns the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T.

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

IPIVOT (input or output)
If FACT = 'F', then IPIVOT is an input argument and on entry contains details of the interchanges and the block structure of D, as determined by SSYTRF. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

If FACT = 'N', then IPIVOT is an output argument and on exit contains details of the interchanges and the block structure of D, as determined by SSYTRF.

B (input) The N-by-NRHS right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A. If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of WORK. LDWORK >= 3*N, and for best performance LDWORK >= N*NB, where NB is the optimal blocksize for SSYTRF.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: D(i,i) is exactly zero. The factorization has been completed but the factor D is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: D is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
ssytd2 - reduce a real symmetric matrix A to symmetric  tri-
diagonal form T by an orthogonal similarity transformation
```

SYNOPSIS

```
SUBROUTINE SSYTD2(UPLO, N, A, LDA, D, E, TAU, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, INFO
REAL A(LDA,*), D(*), E(*), TAU(*)

SUBROUTINE SSYTD2_64(UPLO, N, A, LDA, D, E, TAU, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, INFO
REAL A(LDA,*), D(*), E(*), TAU(*)
```

F95 INTERFACE

```
SUBROUTINE SYTD2(UPLO, N, A, [LDA], D, E, TAU, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, INFO
REAL, DIMENSION(:) :: D, E, TAU
REAL, DIMENSION(:,:) :: A

SUBROUTINE SYTD2_64(UPLO, N, A, [LDA], D, E, TAU, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, INFO
REAL, DIMENSION(:) :: D, E, TAU
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void ssytd2(char uplo, int n, float *a, int lda,  float  *d,
          float *e, float *tau, int *info);

void ssytd2_64(char uplo, long n, float *a, long lda,  float
          *d, float *e, float *tau, long *info);
```

PURPOSE

ssytd2 reduces a real symmetric matrix A to symmetric tridiagonal form T by an orthogonal similarity transformation: Q' * A * Q = T.

ARGUMENTS

UPLO (input)
Specifies whether the upper or lower triangular part of the symmetric matrix A is stored:
= 'U': Upper triangular
= 'L': Lower triangular

N (input) The order of the matrix A. N >= 0.

A (input) On entry, the symmetric matrix A. If UPLO = 'U', the leading n-by-n upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced. On exit, if UPLO = 'U', the diagonal and first superdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements above the first superdiagonal, with the array TAU, represent the orthogonal matrix Q as a product of elementary reflectors; if UPLO = 'L', the diagonal and first subdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements below the first subdiagonal, with the array TAU, represent the orthogonal matrix Q as a product of elementary reflectors. See Further Details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

D (output)
The diagonal elements of the tridiagonal matrix T: D(i) = A(i,i).

E (output)
The off-diagonal elements of the tridiagonal matrix T: E(i) = A(i,i+1) if UPLO = 'U', E(i) = A(i+1,i) if UPLO = 'L'.

TAU (output)
The scalar factors of the elementary reflectors (see Further Details).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

If UPLO = 'U', the matrix Q is represented as a product of elementary reflectors

```
   Q = H(n-1) . . . H(2) H(1).
```

Each H(i) has the form

```
   H(i) = I - tau * v * v'
```

where tau is a real scalar, and v is a real vector with v(i+1:n) = 0 and v(i) = 1; v(1:i-1) is stored on exit in A(1:i-1,i+1), and tau in TAU(i).

If UPLO = 'L', the matrix Q is represented as a product of elementary reflectors

```
   Q = H(1) H(2) . . . H(n-1).
```

Each H(i) has the form

```
   H(i) = I - tau * v * v'
```

where tau is a real scalar, and v is a real vector with v(1:i) = 0 and v(i+1) = 1; v(i+2:n) is stored on exit in A(i+2:n,i), and tau in TAU(i).

The contents of A on exit are illustrated by the following examples with n = 5:

```
if UPLO = 'U':                       if UPLO = 'L':

  (  d   e   v2  v3  v4 )              (  d
)
  (      d   e   v3  v4 )              (  e   d
)
  (          d   e   v4 )              (  v1  e   d
)
  (              d   e  )              (  v1  v2  e   d
)
  (                  d  )              (  v1  v2  v3  e   d
)
```

where d and e denote diagonal and off-diagonal elements of T, and vi denotes an element of the vector defining H(i).

Contents

NAME

ssytf2 - compute the factorization of a real symmetric matrix A using the Bunch-Kaufman diagonal pivoting method

SYNOPSIS

```
SUBROUTINE SSYTF2(UPLO, N, A, LDA, IPIV, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, INFO
INTEGER IPIV(*)
REAL A(LDA,*)

SUBROUTINE SSYTF2_64(UPLO, N, A, LDA, IPIV, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, INFO
INTEGER*8 IPIV(*)
REAL A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE SYTF2(UPLO, [N], A, [LDA], IPIV, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIV
REAL, DIMENSION(:,:) :: A

SUBROUTINE SYTF2_64(UPLO, [N], A, [LDA], IPIV, [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, INFO
```

```
INTEGER(8), DIMENSION(:) :: IPIV
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void ssytf2(char uplo, int n, float *a, int lda, int  *ipiv,
          int *info);

void ssytf2_64(char uplo, long n, float *a, long  lda,  long
          *ipiv, long *info);
```

PURPOSE

ssytf2 computes the factorization of a real symmetric matrix A using the Bunch-Kaufman diagonal pivoting method:

```
   A = U*D*U'  or  A = L*D*L'
```

where U (or L) is a product of permutation and unit upper (lower) triangular matrices, U' is the transpose of U, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

This is the unblocked version of the algorithm, calling Level 2 BLAS.

ARGUMENTS

UPLO (input)
Specifies whether the upper or lower triangular part of the symmetric matrix A is stored:
= 'U': Upper triangular
= 'L': Lower triangular

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the symmetric matrix A. If UPLO = 'U', the leading n-by-n upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, the block diagonal matrix D and the mul-

```
          tipliers  used  to  obtain  the factor U or L (see
          below for further details).

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

IPIV (output)
          Details of the interchanges and the  block  struc-
          ture  of D.  If IPIV(k) > 0, then rows and columns
          k and IPIV(k) were interchanged and  D(k,k)  is  a
          1-by-1  diagonal block.  If UPLO = 'U' and IPIV(k)
          = IPIV(k-1) < 0, then rows  and  columns  k-1  and
          -IPIV(k) were interchanged and D(k-1:k,k-1:k) is a
          2-by-2 diagonal block.  If UPLO = 'L' and  IPIV(k)
          =  IPIV(k+1)  <  0,  then rows and columns k+1 and
          -IPIV(k) were interchanged and D(k:k+1,k:k+1) is a
          2-by-2 diagonal block.

INFO (output)
          = 0: successful exit
          < 0: if INFO = -k, the k-th argument had an  ille-
          gal value
          > 0: if INFO = k, D(k,k)  is  exactly  zero.   The
          factorization  has  been  completed, but the block
          diagonal matrix D is exactly singular,  and  divi-
          sion  by  zero will occur if it is used to solve a
          system of equations.
```

FURTHER DETAILS

```
1-96 - Based on modifications by J. Lewis,  Boeing  Computer
Services
       Company

If UPLO = 'U', then A = U*D*U', where
   U = P(n)*U(n)* ... *P(k)U(k)* ...,
i.e., U is a product of terms P(k)*U(k), where  k  decreases
from  n  to  1 in steps of 1 or 2, and D is a block diagonal
matrix with 1-by-1 and 2-by-2 diagonal blocks D(k).  P(k) is
a  permutation  matrix  as defined by IPIV(k), and U(k) is a
unit upper triangular matrix,  such  that  if  the  diagonal
block D(k) is of order s (s = 1 or 2), then

           (   I    v    0   )   k-s
   U(k) =  (   0    I    0   )   s
           (   0    0    I   )   n-k
              k-s   s   n-k

If s = 1, D(k) overwrites A(k,k), and  v  overwrites  A(1:k-
```

1,k).   If s = 2, the upper triangle of D(k) overwrites A(k-1,k-1), A(k-1,k), and A(k,k), and v overwrites A(1:k-2,k-1:k).

If UPLO = 'L', then A = L*D*L', where

```
   L = P(1)*L(1)* ... *P(k)*L(k)* ...,
```

i.e., L is a product of terms P(k)*L(k), where k increases from 1 to n in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIV(k), and L(k) is a unit lower triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    0     0   )  k-1
   L(k) =  (   0    I     0   )  s
           (   0    v     I   )  n-k-s+1
              k-1   s  n-k-s+1
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(k+1:n,k). If s = 2, the lower triangle of D(k) overwrites A(k,k), A(k+1,k), and A(k+1,k+1), and v overwrites A(k+2:n,k:k+1).

Contents

NAME

```
ssytrd - reduce a real symmetric matrix A to real  symmetric
tridiagonal  form  T by an orthogonal similarity transforma-
tion
```

SYNOPSIS

```
SUBROUTINE SSYTRD(UPLO, N, A, LDA, D, E, TAU, WORK, LWORK, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, LWORK, INFO
REAL A(LDA,*), D(*), E(*), TAU(*), WORK(*)

SUBROUTINE SSYTRD_64(UPLO, N, A, LDA, D, E, TAU, WORK, LWORK, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, LWORK, INFO
REAL A(LDA,*), D(*), E(*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYTRD(UPLO, N, A, [LDA], D, E, TAU, [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, LWORK, INFO
REAL, DIMENSION(:) :: D, E, TAU, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE SYTRD_64(UPLO, N, A, [LDA], D, E, TAU, [WORK], [LWORK],
       [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, LWORK, INFO
```

```
REAL, DIMENSION(:) :: D, E, TAU, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void ssytrd(char uplo, int n, float *a, int lda,  float  *d,
          float *e, float *tau, int *info);

void ssytrd_64(char uplo, long n, float *a, long lda,  float
          *d, float *e, float *tau, long *info);
```

PURPOSE

ssytrd reduces a real symmetric matrix A to real symmetric tridiagonal form T by an orthogonal similarity transformation: Q**T * A * Q = T.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input) On entry, the symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced. On exit, if UPLO = 'U', the diagonal and first superdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements above the first superdiagonal, with the array TAU, represent the orthogonal matrix Q as a product of elementary reflectors; if UPLO = 'L', the diagonal and first subdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements below the first subdiagonal, with the array TAU, represent the orthogonal matrix Q as a product of elementary reflectors. See Further Details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

D (output)
The diagonal elements of the tridiagonal matrix T: D(i) = A(i,i).

E (output)
The off-diagonal elements of the tridiagonal matrix T: E(i) = A(i,i+1) if UPLO = 'U', E(i) = A(i+1,i) if UPLO = 'L'.

TAU (output)
The scalar factors of the elementary reflectors (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 1. For optimum performance LWORK >= N*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

If UPLO = 'U', the matrix Q is represented as a product of elementary reflectors

```
Q = H(n-1) . . . H(2) H(1).
```

Each H(i) has the form

```
H(i) = I - tau * v * v'
```

where tau is a real scalar, and v is a real vector with v(i+1:n) = 0 and v(i) = 1; v(1:i-1) is stored on exit in

```
A(1:i-1,i+1), and tau in TAU(i).

If UPLO = 'L', the matrix Q is represented as a  product  of
elementary reflectors

   Q = H(1) H(2) . . . H(n-1).

Each H(i) has the form

   H(i) = I - tau * v * v'

where tau is a real scalar, and v is a real vector with
v(1:i) = 0 and v(i+1) = 1; v(i+2:n) is  stored  on  exit  in
A(i+2:n,i), and tau in TAU(i).
The contents of A on exit are illustrated by  the  following
examples with n = 5:

if UPLO = 'U':                       if UPLO = 'L':

  (  d   e   v2  v3  v4 )              (  d
)
  (      d   e   v3  v4 )              (  e   d
)
  (          d   e   v4 )              (  v1  e   d
)
  (              d   e  )              (  v1  v2  e   d
)
  (                  d  )              (  v1  v2  v3  e   d
)

where d and e denote diagonal and off-diagonal  elements  of
T, and vi denotes an element of the vector defining H(i).
```

Contents

NAME

```
ssytrf - compute  the  factorization  of  a  real  symmetric
matrix A using the Bunch-Kaufman diagonal pivoting method
```

SYNOPSIS

```
SUBROUTINE SSYTRF(UPLO, N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, LDWORK, INFO
INTEGER IPIVOT(*)
REAL A(LDA,*), WORK(*)

SUBROUTINE SSYTRF_64(UPLO, N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, LDWORK, INFO
INTEGER*8 IPIVOT(*)
REAL A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYTRF(UPLO, N, A, [LDA], IPIVOT, [WORK], [LDWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, LDWORK, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE SYTRF_64(UPLO, N, A, [LDA], IPIVOT, [WORK], [LDWORK],
       [INFO])
```

```
CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, LDA, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void ssytrf(char  uplo,  int  n,  float  *a,  int  lda,  int
          *ipivot, int *info);

void ssytrf_64(char uplo, long n, float *a, long  lda,  long
          *ipivot, long *info);
```

PURPOSE

ssytrf computes the factorization of a real symmetric matrix A using the Bunch-Kaufman diagonal pivoting method. The form of the factorization is

```
   A = U*D*U**T  or  A = L*D*L**T
```

where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

This is the blocked version of the algorithm, calling Level 3 BLAS.

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the symmetric matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A,
          and the strictly lower triangular part of A is not
          referenced.  If UPLO =  'L',  the  leading  N-by-N
          lower triangular part of A contains the lower tri-
          angular part of the matrix  A,  and  the  strictly
          upper triangular part of A is not referenced.
```

On exit, the block diagonal matrix D and the multipliers used to obtain the factor U or L (see below for further details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (output)
Details of the interchanges and the block structure of D. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of WORK. LDWORK >=1. For best performance LDWORK >= N*NB, where NB is the block size returned by ILAENV.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

If UPLO = 'U', then A = U*D*U', where

```
   U = P(n)*U(n)* ... *P(k)U(k)* ...,
```

i.e., U is a product of terms P(k)*U(k), where k decreases from n to 1 in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIVOT(k), and U(k) is a unit upper triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    v    0   )   k-s
   U(k) =  (   0    I    0   )   s
           (   0    0    I   )   n-k
              k-s   s   n-k
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(1:k-1,k). If s = 2, the upper triangle of D(k) overwrites A(k-1,k-1), A(k-1,k), and A(k,k), and v overwrites A(1:k-2,k-1:k).

If UPLO = 'L', then A = L*D*L', where

```
   L = P(1)*L(1)* ... *P(k)*L(k)* ...,
```

i.e., L is a product of terms P(k)*L(k), where k increases from 1 to n in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIVOT(k), and L(k) is a unit lower triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    0     0   )  k-1
   L(k) =  (   0    I     0   )  s
           (   0    v     I   )  n-k-s+1
              k-1   s  n-k-s+1
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(k+1:n,k). If s = 2, the lower triangle of D(k) overwrites A(k,k), A(k+1,k), and A(k+1,k+1), and v overwrites A(k+2:n,k:k+1).

Contents

NAME

```
ssytri - compute the inverse of a real symmetric  indefinite
matrix  A  using  the  factorization  A  =  U*D*U**T  or A =
L*D*L**T computed by SSYTRF
```

SYNOPSIS

```
SUBROUTINE SSYTRI(UPLO, N, A, LDA, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
INTEGER N, LDA, INFO
INTEGER IPIVOT(*)
REAL A(LDA,*), WORK(*)

SUBROUTINE SSYTRI_64(UPLO, N, A, LDA, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, LDA, INFO
INTEGER*8 IPIVOT(*)
REAL A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE SYTRI(UPLO, N, A, [LDA], IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE SYTRI_64(UPLO, N, A, [LDA], IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void ssytri(char  uplo,  int  n,  float  *a,  int  lda,  int
          *ipivot, int *info);

void ssytri_64(char uplo, long n, float *a, long  lda,  long
          *ipivot, long *info);
```

PURPOSE

ssytri computes the inverse of a real symmetric indefinite matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by SSYTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by SSYTRF.

On exit, if INFO = 0, the (symmetric) inverse of the original matrix. If UPLO = 'U', the upper triangular part of the inverse is formed and the part of A below the diagonal is not referenced; if UPLO = 'L' the lower triangular part of the inverse is formed and the part of A above the diagonal is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

```
IPIVOT (input)
          Details of the interchanges and the  block  struc-
          ture of D as determined by SSYTRF.

WORK (workspace)
          dimension(N)

INFO (output)
          = 0: successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
          gal value
          > 0: if INFO = i, D(i,i) = 0; the matrix is singu-
          lar and its inverse could not be computed.
```

Contents

NAME

```
ssytrs - solve a system of linear equations A*X = B  with  a
real symmetric matrix A using the factorization A = U*D*U**T
or A = L*D*L**T computed by SSYTRF
```

SYNOPSIS

```
SUBROUTINE SSYTRS(UPLO, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER N, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)
REAL A(LDA,*), B(LDB,*)

SUBROUTINE SSYTRS_64(UPLO, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
INTEGER*8 N, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
REAL A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE SYTRS(UPLO, N, NRHS, A, [LDA], IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER :: N, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE SYTRS_64(UPLO, N, NRHS, A, [LDA], IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
```

```
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void ssytrs(char uplo, int n, int nrhs, float *a,  int  lda,
          int *ipivot, float *b, int ldb, int *info);

void ssytrs_64(char uplo, long n, long nrhs, float *a,  long
          lda,  long  *ipivot,  float  *b,  long  ldb,  long
          *info);
```

PURPOSE

ssytrs solves a system of linear equations A*X = B with a real symmetric matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by SSYTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by SSYTRF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by SSYTRF.

B (input/output)
On entry, the right hand side matrix B. On exit,

the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

stbcon - estimate the reciprocal of the condition number of a triangular band matrix A, in either the 1-norm or the infinity-norm

SYNOPSIS

```
SUBROUTINE STBCON(NORM, UPLO, DIAG, N, KD, A, LDA, RCOND, WORK,
      WORK2, INFO)

CHARACTER * 1 NORM, UPLO, DIAG
INTEGER N, KD, LDA, INFO
INTEGER WORK2(*)
REAL RCOND
REAL A(LDA,*), WORK(*)

SUBROUTINE STBCON_64(NORM, UPLO, DIAG, N, KD, A, LDA, RCOND, WORK,
      WORK2, INFO)

CHARACTER * 1 NORM, UPLO, DIAG
INTEGER*8 N, KD, LDA, INFO
INTEGER*8 WORK2(*)
REAL RCOND
REAL A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TBCON(NORM, UPLO, DIAG, N, KD, A, [LDA], RCOND, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
INTEGER :: N, KD, LDA, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL :: RCOND
```

```
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE TBCON_64(NORM, UPLO, DIAG, N, KD, A, [LDA], RCOND,
       [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
INTEGER(8) :: N, KD, LDA, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL :: RCOND
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
void stbcon(char norm, char uplo, char diag, int n, int  kd,
          float *a, int lda, float *rcond, int *info);

void stbcon_64(char norm, char uplo, char diag, long n, long
          kd, float *a, long lda, float *rcond, long *info);
```

PURPOSE

stbcon estimates the reciprocal of the condition number of a triangular band matrix A, in either the 1-norm or the infinity-norm.

The norm of A is computed and an estimate is obtained for norm(inv(A)), then the reciprocal of the condition number is computed as

```
   RCOND = 1 / ( norm(A) * norm(inv(A)) ).
```

ARGUMENTS

```
NORM (input)
          Specifies whether the 1-norm condition  number  or
          the infinity-norm condition number is required:
          = '1' or 'O':  1-norm;
          = 'I':         Infinity-norm.

UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.
```

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals or subdiagonals of the triangular band matrix A. KD >= 0.

A (input) The upper or lower triangular band matrix A, stored in the first kd+1 rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd). If DIAG = 'U', the diagonal elements of A are not referenced and are assumed to be 1.

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(norm(A) * norm(inv(A))).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
stbmv - perform one of the matrix-vector operations    x  :=
A*x, or x := A'*x
```

SYNOPSIS

```
SUBROUTINE STBMV(UPLO, TRANSA, DIAG, N, K, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, K, LDA, INCY
REAL A(LDA,*), Y(*)

SUBROUTINE STBMV_64(UPLO, TRANSA, DIAG, N, K, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, K, LDA, INCY
REAL A(LDA,*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE TBMV(UPLO, [TRANSA], DIAG, [N], K, A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, K, LDA, INCY
REAL, DIMENSION(:) :: Y
REAL, DIMENSION(:,:) :: A

SUBROUTINE TBMV_64(UPLO, [TRANSA], DIAG, [N], K, A, [LDA], Y,
       [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, K, LDA, INCY
REAL, DIMENSION(:) :: Y
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void stbmv(char uplo, char transa, char diag, int n, int  k,
          float *a, int lda, float *y, int incy);

void stbmv_64(char uplo, char transa,  char  diag,  long  n,
          long k, float *a, long lda, float *y, long incy);
```

PURPOSE

stbmv performs one of the matrix-vector operations x := A*x, or x := A'*x, where x is an n element vector and A is an n by n unit, or non-unit, upper or lower triangular band matrix, with (k + 1) diagonals.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   x := A*x.

TRANSA = 'T' or 't'   x := A'*x.

TRANSA = 'C' or 'c'   x := A'*x.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
On entry, DIAG specifies whether or not A is unit

triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit tri-angular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

N (input)

On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

K (input)

On entry with UPLO = 'U' or 'u', K specifies the number of super-diagonals of the matrix A. On entry with UPLO = 'L' or 'l', K specifies the number of sub-diagonals of the matrix A. K >= 0. Unchanged on exit.

A (input)

Before entry with UPLO = 'U' or 'u', the leading (k + 1) by n part of the array A must contain the upper triangular band part of the matrix of coef-ficients, supplied column by column, with the leading diagonal of the matrix in row (k + 1) of the array, the first super-diagonal starting at position 2 in row k, and so on. The top left k by k triangle of the array A is not referenced. The following program segment will transfer an upper triangular band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = K + 1 - J
     DO 10, I = MAX( 1, J - K ), J
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Before entry with UPLO = 'L' or 'l', the leading (k + 1) by n part of the array A must contain the lower triangular band part of the matrix of coef-ficients, supplied column by column, with the leading diagonal of the matrix in row 1 of the array, the first sub-diagonal starting at position 1 in row 2, and so on. The bottom right k by k triangle of the array A is not referenced. The following program segment will transfer a lower

triangular band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = 1 - J
     DO 10, I = J, MIN( N, J + K )
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Note that when DIAG = 'U' or 'u' the elements of the array A corresponding to the diagonal elements of the matrix are not referenced, but are assumed to be unity. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= (k + 1). Unchanged on exit.

Y (input/output)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector x. On exit, Y is overwritten with the tran- formed vector x.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
stbrfs - provide error bounds and backward  error  estimates
for the solution to a system of linear equations with a tri-
angular band coefficient matrix
```

SYNOPSIS

```
SUBROUTINE STBRFS(UPLO, TRANSA, DIAG, N, KD, NRHS, A, LDA, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, KD, NRHS, LDA, LDB, LDX, INFO
INTEGER WORK2(*)
REAL A(LDA,*), B(LDB,*), X(LDX,*), FERR(*), BERR(*), WORK(*)

SUBROUTINE STBRFS_64(UPLO, TRANSA, DIAG, N, KD, NRHS, A, LDA, B,
      LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, KD, NRHS, LDA, LDB, LDX, INFO
INTEGER*8 WORK2(*)
REAL A(LDA,*), B(LDB,*), X(LDX,*), FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TBRFS(UPLO, [TRANSA], DIAG, N, KD, NRHS, A, [LDA], B,
       [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, KD, NRHS, LDA, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL, DIMENSION(:) :: FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, B, X
```

```
SUBROUTINE TBRFS_64(UPLO, [TRANSA], DIAG, N, KD, NRHS, A, [LDA],
       B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, KD, NRHS, LDA, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL, DIMENSION(:) :: FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, B, X
```

C INTERFACE

```
#include <sunperf.h>

void stbrfs(char uplo, char transa, char diag,  int  n,  int
          kd,  int  nrhs,  float  *a, int lda, float *b, int
          ldb, float *x, int ldx, float *ferr, float  *berr,
          int *info);

void stbrfs_64(char uplo, char transa, char  diag,  long  n,
          long  kd, long nrhs, float *a, long lda, float *b,
          long ldb, float *x, long ldx, float  *ferr,  float
          *berr, long *info);
```

PURPOSE

stbrfs provides error bounds and backward error estimates for the solution to a system of linear equations with a triangular band coefficient matrix.

The solution matrix X must be computed by STBTRS or some other means before entering this routine. STBRFS does not do iterative refinement because doing so cannot improve the backward error.

ARGUMENTS

UPLO (input)
  = 'U': A is upper triangular;
  = 'L': A is lower triangular.

TRANSA (input)
  Specifies the form of the system of equations:
  = 'N': A * X = B (No transpose)
  = 'T': A**T * X = B (Transpose)
  = 'C': A**H * X = B (Conjugate transpose = Transpose)

  TRANSA is defaulted to 'N' for F95 INTERFACE.

```
DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals  or  subdiagonals  of
          the triangular band matrix A.  KD >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.
A (input) The upper  or  lower  triangular  band  matrix  A,
          stored  in  the  first kd+1 rows of the array. The
          j-th column of A is stored in the j-th  column  of
          the  array A as follows:  if UPLO = 'U', A(kd+1+i-
          j,j) = A(i,j) for  max(1,j-kd)<=i<=j;  if  UPLO  =
          'L', A(1+i-j,j)    = A(i,j) for j<=i<=min(n,j+kd).
          If DIAG = 'U', the diagonal elements of A are  not
          referenced and are assumed to be 1.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          KD+1.

B (input) The right hand side matrix B.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

X (input) The solution matrix X.

LDX (input)
          The leading dimension of  the  array  X.   LDX  >=
          max(1,N).

FERR (output)
          The estimated forward error bound for  each  solu-
          tion  vector X(j) (the j-th column of the solution
          matrix  X).   If  XTRUE  is  the   true   solution
          corresponding  to  X(j),  FERR(j)  is an estimated
          upper bound for the magnitude of the largest  ele-
          ment in (X(j) - XTRUE) divided by the magnitude of
          the largest element in X(j).  The estimate  is  as
          reliable  as the estimate for RCOND, and is almost
          always a slight overestimate of the true error.
```

```
BERR (output)
          The componentwise relative backward error of  each
          solution  vector X(j) (i.e., the smallest relative
          change in any element of A or B that makes X(j) an
          exact solution).

WORK (workspace)
          dimension(3*N)
WORK2 (workspace)
          dimension(N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
stbsv - solve one of the systems of equations A*x  =  b,  or
A'*x = b
```

SYNOPSIS

```
SUBROUTINE STBSV(UPLO, TRANSA, DIAG, N, K, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, K, LDA, INCY
REAL A(LDA,*), Y(*)

SUBROUTINE STBSV_64(UPLO, TRANSA, DIAG, N, K, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, K, LDA, INCY
REAL A(LDA,*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE TBSV(UPLO, [TRANSA], DIAG, [N], K, A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, K, LDA, INCY
REAL, DIMENSION(:) :: Y
REAL, DIMENSION(:,:) :: A

SUBROUTINE TBSV_64(UPLO, [TRANSA], DIAG, [N], K, A, [LDA], Y,
       [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, K, LDA, INCY
REAL, DIMENSION(:) :: Y
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void stbsv(char uplo, char transa, char diag, int n, int  k,
          float *a, int lda, float *y, int incy);

void stbsv_64(char uplo, char transa,  char  diag,  long  n,
          long k, float *a, long lda, float *y, long incy);
```

PURPOSE

stbsv solves one of the systems of equations A*x = b, or A'*x = b, where b and x are n element vectors and A is an n by n unit, or non-unit, upper or lower triangular band matrix, with (k + 1) diagonals.

No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the equations to be solved as follows:

TRANSA = 'N' or 'n'   A*x = b.

TRANSA = 'T' or 't'   A'*x = b.

TRANSA = 'C' or 'c'   A'*x = b.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit triangular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

K (input)
On entry with UPLO = 'U' or 'u', K specifies the number of super-diagonals of the matrix A. On entry with UPLO = 'L' or 'l', K specifies the number of sub-diagonals of the matrix A. K >= 0. Unchanged on exit.

A (input)
Before entry with UPLO = 'U' or 'u', the leading (k + 1) by n part of the array A must contain the upper triangular band part of the matrix of coefficients, supplied column by column, with the leading diagonal of the matrix in row (k + 1) of the array, the first super-diagonal starting at position 2 in row k, and so on. The top left k by k triangle of the array A is not referenced. The following program segment will transfer an upper triangular band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = K + 1 - J
     DO 10, I = MAX( 1, J - K ), J
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Before entry with UPLO = 'L' or 'l', the leading (k + 1) by n part of the array A must contain the lower triangular band part of the matrix of coefficients, supplied column by column, with the leading diagonal of the matrix in row 1 of the array, the first sub-diagonal starting at position 1 in row 2, and so on. The bottom right k by k triangle of the array A is not referenced. The following program segment will transfer a lower triangular band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = 1 - J
     DO 10, I = J, MIN( N, J + K )
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Note that when DIAG = 'U' or 'u' the elements of the array A corresponding to the diagonal elements of the matrix are not referenced, but are assumed to be unity. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= (k + 1). Unchanged on exit.

Y (input/output)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element right-hand side vector b. On exit, Y is overwritten with the solution vector x.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
stbtrs - solve a triangular system of the form   A * X  =  B
or A**T * X = B,
```

SYNOPSIS

```
SUBROUTINE STBTRS(UPLO, TRANSA, DIAG, N, KD, NRHS, A, LDA, B, LDB,
      INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, KD, NRHS, LDA, LDB, INFO
REAL A(LDA,*), B(LDB,*)

SUBROUTINE STBTRS_64(UPLO, TRANSA, DIAG, N, KD, NRHS, A, LDA, B,
      LDB, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, KD, NRHS, LDA, LDB, INFO
REAL A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE TBTRS(UPLO, TRANSA, DIAG, N, KD, NRHS, A, [LDA], B,
       [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, KD, NRHS, LDA, LDB, INFO
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE TBTRS_64(UPLO, TRANSA, DIAG, N, KD, NRHS, A, [LDA], B,
       [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, KD, NRHS, LDA, LDB, INFO
```

```
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void stbtrs(char uplo, char transa, char diag,  int  n,  int
          kd,  int  nrhs,  float  *a, int lda, float *b, int
          ldb, int *info);

void stbtrs_64(char uplo, char transa, char  diag,  long  n,
          long  kd, long nrhs, float *a, long lda, float *b,
          long ldb, long *info);
```

PURPOSE

stbtrs solves a triangular system of the form

where A is a triangular band matrix of order N, and B is an N-by NRHS matrix. A check is made to verify that A is non-singular.

ARGUMENTS

UPLO (input)
- = 'U': A is upper triangular;
- = 'L': A is lower triangular.

TRANSA (input)
Specifies the form the system of equations:
- = 'N': A * X = B (No transpose)
- = 'T': A**T * X = B (Transpose)
- = 'C': A**H * X = B (Conjugate transpose = Transpose)

DIAG (input)
- = 'N': A is non-unit triangular;
- = 'U': A is unit triangular.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals or subdiagonals of the triangular band matrix A. KD >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

```
A (input) The upper  or  lower  triangular  band  matrix  A,
          stored  in  the  first  kd+1  rows of A.  The j-th
          column of A is stored in the j-th  column  of  the
          array  A as follows:  if UPLO = 'U', A(kd+1+i-j,j)
          = A(i,j) for max(1,j-kd)<=i<=j;  if  UPLO  =  'L',
          A(1+i-j,j)     = A(i,j) for j<=i<=min(n,j+kd).  If
          DIAG = 'U', the diagonal elements  of  A  are  not
          referenced and are assumed to be 1.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          KD+1.

B (input/output)
          On entry, the right hand side matrix B.  On  exit,
          if INFO = 0, the solution matrix X.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, the i-th diagonal element of  A
          is  zero,  indicating  that the matrix is singular
          and the solutions X have not been computed.
```

Contents

NAME

```
stgevc - compute some or all of the right and/or  left  gen-
eralized  eigenvectors  of  a  pair of real upper triangular
matrices (A,B)
```

SYNOPSIS

```
SUBROUTINE STGEVC(SIDE, HOWMNY, SELECT, N, A, LDA, B, LDB, VL, LDVL,
      VR, LDVR, MM, M, WORK, INFO)

CHARACTER * 1 SIDE, HOWMNY
INTEGER N, LDA, LDB, LDVL, LDVR, MM, M, INFO
LOGICAL SELECT(*)
REAL A(LDA,*), B(LDB,*), VL(LDVL,*), VR(LDVR,*), WORK(*)

SUBROUTINE STGEVC_64(SIDE, HOWMNY, SELECT, N, A, LDA, B, LDB, VL,
      LDVL, VR, LDVR, MM, M, WORK, INFO)

CHARACTER * 1 SIDE, HOWMNY
INTEGER*8 N, LDA, LDB, LDVL, LDVR, MM, M, INFO
LOGICAL*8 SELECT(*)
REAL A(LDA,*), B(LDB,*), VL(LDVL,*), VR(LDVR,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TGEVC(SIDE, HOWMNY, SELECT, N, A, [LDA], B, [LDB], VL,
       [LDVL], VR, [LDVR], MM, M, [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, HOWMNY
INTEGER :: N, LDA, LDB, LDVL, LDVR, MM, M, INFO
LOGICAL, DIMENSION(:) :: SELECT
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A, B, VL, VR
```

```
SUBROUTINE TGEVC_64(SIDE, HOWMNY, SELECT, N, A, [LDA], B, [LDB], VL,
       [LDVL], VR, [LDVR], MM, M, [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, HOWMNY
INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, MM, M, INFO
LOGICAL(8), DIMENSION(:) :: SELECT
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A, B, VL, VR
```

C INTERFACE

```
#include <sunperf.h>

void stgevc(char side, char  howmny,  int  *select,  int  n,
          float  *a,  int lda, float *b, int ldb, float *vl,
          int ldvl, float *vr, int ldvr, int mm, int *m, int
          *info);

void stgevc_64(char side, char howmny, long *select, long n,
          float *a, long lda, float *b, long ldb, float *vl,
          long ldvl, float *vr, long ldvr, long mm, long *m,
          long *info);
```

PURPOSE

stgevc computes some or all of the right and/or left generalized eigenvectors of a pair of real upper triangular matrices (A,B).

The right generalized eigenvector x and the left generalized eigenvector y of (A,B) corresponding to a generalized eigenvalue w are defined by:

```
        (A - wB) * x = 0  and  y**H * (A - wB) = 0
```

where y**H denotes the conjugate tranpose of y.

If an eigenvalue w is determined by zero diagonal elements of both A and B, a unit vector is returned as the corresponding eigenvector.

If all eigenvectors are requested, the routine may either return the matrices X and/or Y of right or left eigenvectors of (A,B), or the products Z*X and/or Q*Y, where Z and Q are input orthogonal matrices. If (A,B) was obtained from the generalized real-Schur factorization of an original pair of matrices
   (A0,B0) = (Q*A*Z**H,Q*B*Z**H),
then Z*X and Q*Y are the matrices of right or left eigenvec-

tors of A.

A must be block upper triangular, with 1-by-1 and 2-by-2 diagonal blocks. Corresponding to each 2-by-2 diagonal block is a complex conjugate pair of eigenvalues and eigen- vectors; only one
eigenvector of the pair is computed, namely the one corresponding to the eigenvalue with positive imaginary part.

ARGUMENTS

SIDE (input)
= 'R': compute right eigenvectors only;
= 'L': compute left eigenvectors only;
= 'B': compute both right and left eigenvectors.

HOWMNY (input)
= 'A': compute all right and/or left eigenvectors;
= 'B': compute all right and/or left eigenvectors, and backtransform them using the input matrices supplied in VR and/or VL; = 'S': compute selected right and/or left eigenvectors, specified by the logical array SELECT.

SELECT (input)
If HOWMNY='S', SELECT specifies the eigenvectors to be computed. If HOWMNY='A' or 'B', SELECT is not referenced. To select the real eigenvector corresponding to the real eigenvalue w(j), SELECT(j) must be set to .TRUE. To select the complex eigenvector corresponding to a complex conjugate pair w(j) and w(j+1), either SELECT(j) or SELECT(j+1) must be set to .TRUE..

N (input) The order of the matrices A and B. N >= 0.

A (input) The upper quasi-triangular matrix A.

LDA (input)
The leading dimension of array A. LDA >= max(1, N).

B (input) The upper triangular matrix B. If A has a 2-by-2 diagonal block, then the corresponding 2-by-2 block of B must be diagonal with positive ele- ments.

LDB (input)

The leading dimension of array B. LDB >= max(1,N).

VL (input/output)

On entry, if SIDE = 'L' or 'B' and HOWMNY = 'B', VL must contain an N-by-N matrix Q (usually the orthogonal matrix Q of left Schur vectors returned by SHGEQZ). On exit, if SIDE = 'L' or 'B', VL contains: if HOWMNY = 'A', the matrix Y of left eigenvectors of (A,B); if HOWMNY = 'B', the matrix Q*Y; if HOWMNY = 'S', the left eigenvectors of (A,B) specified by SELECT, stored consecutively in the columns of VL, in the same order as their eigenvalues. If SIDE = 'R', VL is not referenced.

A complex eigenvector corresponding to a complex eigenvalue is stored in two consecutive columns, the first holding the real part, and the second the imaginary part.

LDVL (input)

The leading dimension of array VL. LDVL >= max(1,N) if SIDE = 'L' or 'B'; LDVL >= 1 otherwise.

VR (input/output)

On entry, if SIDE = 'R' or 'B' and HOWMNY = 'B', VR must contain an N-by-N matrix Q (usually the orthogonal matrix Z of right Schur vectors returned by SHGEQZ). On exit, if SIDE = 'R' or 'B', VR contains: if HOWMNY = 'A', the matrix X of right eigenvectors of (A,B); if HOWMNY = 'B', the matrix Z*X; if HOWMNY = 'S', the right eigenvectors of (A,B) specified by SELECT, stored consecutively in the columns of VR, in the same order as their eigenvalues. If SIDE = 'L', VR is not referenced.

A complex eigenvector corresponding to a complex eigenvalue is stored in two consecutive columns, the first holding the real part and the second the imaginary part.

LDVR (input)

The leading dimension of the array VR. LDVR >= max(1,N) if SIDE = 'R' or 'B'; LDVR >= 1 otherwise.

MM (input)

The number of columns in the arrays VL and/or VR.

```
          MM >= M.

M (output)
          The number of columns in the arrays VL  and/or  VR
          actually  used  to  store  the  eigenvectors.   If
          HOWMNY = 'A' or 'B', M is set to N.  Each selected
          real  eigenvector  occupies  one  column  and each
          selected complex eigenvector occupies two columns.
WORK (workspace)
          dimension(6*N)

INFO (output)
          = 0:  successful exit.
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value.
          > 0:  the 2-by-2 block (INFO:INFO+1) does not have
          a complex eigenvalue.
```

FURTHER DETAILS

```
Allocation of workspace:
---------- -- ---------

   WORK( j ) = 1-norm of j-th column of A, above the  diago-
nal
   WORK( N+j ) = 1-norm of j-th column of B, above the diag-
onal
   WORK( 2*N+1:3*N ) = real part of eigenvector
   WORK( 3*N+1:4*N ) = imaginary part of eigenvector
   WORK( 4*N+1:5*N ) = real part of back-transformed  eigen-
vector
   WORK( 5*N+1:6*N ) = imaginary  part  of  back-transformed
eigenvector

Rowwise vs. columnwise solution methods:
------- --  ---------- -------- -------

Finding a  generalized  eigenvector  consists  basically  of
solving the singular triangular system

 (A - w B) x = 0     (for right) or:   (A - w B)**H  y  =  0
(for left)

Consider finding the  i-th  right  eigenvector  (assume  all
eigenvalues are real). The equation to be solved is:
0 = sum  C(j,k) v(k)  = sum  C(j,k) v(k)     for j =  i,.  .
.,1
    k=j                 k=j

where  C = (A - w B)  (The components v(i+1:n) are 0.)
```

The "rowwise" method is:

```
(1)  v(i) := 1
for j = i-1,. . .,1:
                        i
    (2) compute  s = - sum C(j,k) v(k)   and
                      k=j+1

    (3) v(j) := s / C(j,j)
```

Step 2 is sometimes called the "dot product" step, since it is an inner product between the j-th row and the portion of the eigenvector that has been computed so far.

The "columnwise" method consists basically in doing the sums for all the rows in parallel. As each v(j) is computed, the contribution of v(j) times the j-th column of C is added to the partial sums. Since FORTRAN arrays are stored column-wise, this has the advantage that at each step, the elements of C that are accessed are adjacent to one another, whereas with the rowwise method, the elements accessed at a step are spaced LDA (and LDB) words apart.

When finding left eigenvectors, the matrix in question is the transpose of the one in storage, so the rowwise method then actually accesses columns of A and B at each step, and so is the preferred method.

Contents

NAME

```
stgexc - reorder the generalized real Schur decomposition of
a  real  matrix  pair  (A,B) using an orthogonal equivalence
transformation   (A, B) = Q * (A, B) * Z',
```

SYNOPSIS

```
SUBROUTINE STGEXC(WANTQ, WANTZ, N, A, LDA, B, LDB, Q, LDQ, Z, LDZ,
      IFST, ILST, WORK, LWORK, INFO)

INTEGER N, LDA, LDB, LDQ, LDZ, IFST, ILST, LWORK, INFO
LOGICAL WANTQ, WANTZ
REAL A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*), WORK(*)

SUBROUTINE STGEXC_64(WANTQ, WANTZ, N, A, LDA, B, LDB, Q, LDQ, Z, LDZ,
      IFST, ILST, WORK, LWORK, INFO)

INTEGER*8 N, LDA, LDB, LDQ, LDZ, IFST, ILST, LWORK, INFO
LOGICAL*8 WANTQ, WANTZ
REAL A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TGEXC(WANTQ, WANTZ, N, A, [LDA], B, [LDB], Q, [LDQ], Z,
       [LDZ], IFST, ILST, [WORK], [LWORK], [INFO])

INTEGER :: N, LDA, LDB, LDQ, LDZ, IFST, ILST, LWORK, INFO
LOGICAL :: WANTQ, WANTZ
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A, B, Q, Z

SUBROUTINE TGEXC_64(WANTQ, WANTZ, N, A, [LDA], B, [LDB], Q, [LDQ], Z,
       [LDZ], IFST, ILST, [WORK], [LWORK], [INFO])
```

```
INTEGER(8) :: N, LDA, LDB, LDQ, LDZ, IFST, ILST, LWORK, INFO
LOGICAL(8) :: WANTQ, WANTZ
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A, B, Q, Z
```

C INTERFACE

```
#include <sunperf.h>

void stgexc(int wantq, int wantz, int n, float *a, int  lda,
          float  *b,  int  ldb, float *q, int ldq, float *z,
          int ldz, int *ifst, int *ilst, int *info);

void stgexc_64(long wantq, long wantz,  long  n,  float  *a,
          long  lda, float *b, long ldb, float *q, long ldq,
          float *z, long ldz, long *ifst, long  *ilst,  long
          *info);
```

PURPOSE

stgexc reorders the generalized real Schur decomposition of a real matrix pair (A,B) using an orthogonal equivalence transformation

so that the diagonal block of (A, B) with row index IFST is moved to row ILST.

(A, B) must be in generalized real Schur canonical form (as returned by SGGES), i.e. A is block upper triangular with 1-by-1 and 2-by-2 diagonal blocks. B is upper triangular.

Optionally, the matrices Q and Z of generalized Schur vectors are updated.

```
Q(in) * A(in) * Z(in)' = Q(out) * A(out) * Z(out)'
Q(in) * B(in) * Z(in)' = Q(out) * B(out) * Z(out)'
```

ARGUMENTS

WANTQ (input)

WANTZ (input)

N (input) The order of the matrices A and B. N >= 0.

A (input/output)
On entry, the matrix A in generalized real Schur

canonical form. On exit, the updated matrix A, again in generalized real Schur canonical form.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the matrix B in generalized real Schur canonical form (A,B). On exit, the updated matrix B, again in generalized real Schur canonical form (A,B).

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

Q (input/output)
On entry, if WANTQ = .TRUE., the orthogonal matrix Q. On exit, the updated matrix Q. If WANTQ = .FALSE., Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= 1. If WANTQ = .TRUE., LDQ >= N.

Z (input/output)
On entry, if WANTZ = .TRUE., the orthogonal matrix Z. On exit, the updated matrix Z. If WANTZ = .FALSE., Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1. If WANTZ = .TRUE., LDZ >= N.

IFST (input/output)
Specify the reordering of the diagonal blocks of (A, B). The block with row index IFST is moved to row ILST, by a sequence of swapping between adjacent blocks. On exit, if IFST pointed on entry to the second row of a 2-by-2 block, it is changed to point to the first row; ILST always points to the first row of the block in its final position (which may differ from its input value by +1 or -1). 1 <= IFST, ILST <= N.

ILST (input/output)
See the description of IFST.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal

LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 4*N + 16.
If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
=0: successful exit.
<0: if INFO = -i, the i-th argument had an illegal value.
=1: The transformed matrix pair (A, B) would be too far from generalized Schur form; the problem is ill- conditioned. (A, B) may have been partially reordered, and ILST points to the first row of the current position of the block being moved.

FURTHER DETAILS

Based on contributions by
Bo Kagstrom and Peter Poromaa, Department of Computing Science,
Umea University, S-901 87 Umea, Sweden.

[1] B. Kagstrom; A Direct Method for Reordering Eigenvalues in the
Generalized Real Schur Form of a Regular Matrix Pair (A, B), in
M.S. Moonen et al (eds), Linear Algebra for Large Scale and
Real-Time Applications, Kluwer Academic Publ. 1993, pp 195-218.

Contents

NAME

```
stgsen - reorder the generalized real Schur decomposition of
a  real  matrix  pair  (A,  B)  (in  terms of an orthonormal
equivalence trans- formation Q' * (A, B) *  Z),  so  that  a
selected cluster of eigenvalues appears in the leading diag-
onal blocks of the upper quasi-triangular matrix A  and  the
upper triangular B
```

SYNOPSIS

```
SUBROUTINE STGSEN(IJOB, WANTQ, WANTZ, SELECT, N, A, LDA, B, LDB,
      ALPHAR, ALPHAI, BETA, Q, LDQ, Z, LDZ, M, PL, PR, DIF, WORK,
      LWORK, IWORK, LIWORK, INFO)

INTEGER IJOB, N, LDA, LDB, LDQ, LDZ, M, LWORK, LIWORK, INFO
INTEGER IWORK(*)
LOGICAL WANTQ, WANTZ
LOGICAL SELECT(*)
REAL PL, PR
REAL  A(LDA,*),  B(LDB,*),  ALPHAR(*),  ALPHAI(*),  BETA(*),
Q(LDQ,*), Z(LDZ,*), DIF(*), WORK(*)

SUBROUTINE STGSEN_64(IJOB, WANTQ, WANTZ, SELECT, N, A, LDA, B, LDB,
      ALPHAR, ALPHAI, BETA, Q, LDQ, Z, LDZ, M, PL, PR, DIF, WORK,
      LWORK, IWORK, LIWORK, INFO)

INTEGER*8 IJOB, N, LDA, LDB, LDQ,  LDZ,  M,  LWORK,  LIWORK,
INFO
INTEGER*8 IWORK(*)
LOGICAL*8 WANTQ, WANTZ
LOGICAL*8 SELECT(*)
REAL PL, PR
```

```
REAL  A(LDA,*),  B(LDB,*),  ALPHAR(*),  ALPHAI(*),  BETA(*),
Q(LDQ,*), Z(LDZ,*), DIF(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TGSEN(IJOB, WANTQ, WANTZ, SELECT, N, A, [LDA], B, [LDB],
       ALPHAR, ALPHAI, BETA, Q, [LDQ], Z, [LDZ], M, PL, PR, DIF, [WORK],
       [LWORK], [IWORK], [LIWORK], [INFO])

INTEGER :: IJOB, N, LDA, LDB, LDQ, LDZ,  M,  LWORK,  LIWORK,
INFO
INTEGER, DIMENSION(:) :: IWORK
LOGICAL :: WANTQ, WANTZ
LOGICAL, DIMENSION(:) :: SELECT
REAL :: PL, PR
REAL, DIMENSION(:) :: ALPHAR, ALPHAI, BETA, DIF, WORK
REAL, DIMENSION(:,:) :: A, B, Q, Z
SUBROUTINE TGSEN_64(IJOB, WANTQ, WANTZ, SELECT, N, A, [LDA], B, [LDB],
       ALPHAR, ALPHAI, BETA, Q, [LDQ], Z, [LDZ], M, PL, PR, DIF, [WORK],
       [LWORK], [IWORK], [LIWORK], [INFO])

INTEGER(8) :: IJOB, N, LDA, LDB, LDQ, LDZ, M, LWORK, LIWORK,
INFO
INTEGER(8), DIMENSION(:) :: IWORK
LOGICAL(8) :: WANTQ, WANTZ
LOGICAL(8), DIMENSION(:) :: SELECT
REAL :: PL, PR
REAL, DIMENSION(:) :: ALPHAR, ALPHAI, BETA, DIF, WORK
REAL, DIMENSION(:,:) :: A, B, Q, Z
```

C INTERFACE

```
#include <sunperf.h>

void stgsen(int ijob, int wantq, int wantz, int *select, int
          n,  float  *a,  int  lda, float *b, int ldb, float
          *alphar, float *alphai, float *beta, float *q, int
          ldq,  float  *z, int ldz, int *m, float *pl, float
          *pr, float *dif, int *info);

void stgsen_64(long  ijob,  long  wantq,  long  wantz,  long
          *select,  long  n,  float  *a, long lda, float *b,
          long ldb,  float  *alphar,  float  *alphai,  float
          *beta,  float  *q,  long  ldq, float *z, long ldz,
          long *m, float *pl, float *pr,  float  *dif,  long
          *info);
```

PURPOSE

stgsen reorders the generalized real Schur decomposition of a real matrix pair (A, B) (in terms of an orthonormal equivalence trans- formation Q' * (A, B) * Z), so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper quasi-triangular matrix A and the upper triangular B. The leading columns of Q and Z form orthonormal bases of the corresponding left and right eigenspaces (deflating subspaces). (A, B) must be in generalized real Schur canonical form (as returned by SGGES), i.e. A is block upper triangular with 1-by-1 and 2-by-2 diagonal blocks. B is upper triangular.

STGSEN also computes the generalized eigenvalues

w(j) = (ALPHAR(j) + i*ALPHAI(j))/BETA(j)

of the reordered matrix pair (A, B).

Optionally, STGSEN computes the estimates of reciprocal condition numbers for eigenvalues and eigenspaces. These are Difu[(A11,B11), (A22,B22)] and Difl[(A11,B11), (A22,B22)], i.e. the separation(s) between the matrix pairs (A11, B11) and (A22,B22) that correspond to the selected cluster and the eigenvalues outside the cluster, resp., and norms of "projections" onto left and right eigenspaces w.r.t. the selected cluster in the (1,1)-block.

ARGUMENTS

IJOB (input)
Specifies whether condition numbers are required for the cluster of eigenvalues (PL and PR) or the deflating subspaces (Difu and Difl):
=0: Only reorder w.r.t. SELECT. No extras.
=1: Reciprocal of norms of "projections" onto left and right eigenspaces w.r.t. the selected cluster (PL and PR). =2: Upper bounds on Difu and Difl. F-norm-based estimate
(DIF(1:2)).
=3: Estimate of Difu and Difl. 1-norm-based estimate
(DIF(1:2)). About 5 times as expensive as IJOB = 2. =4: Compute PL, PR and DIF (i.e. 0, 1 and 2 above): Economic version to get it all. =5: Compute PL, PR and DIF (i.e. 0, 1 and 3 above)

WANTQ (input)

WANTZ (input)

SELECT (input)
SELECT specifies the eigenvalues in the selected cluster. To select a real eigenvalue w(j), SELECT(j) must be set to w(j) and w(j+1), corresponding to a 2-by-2 diagonal block, either SELECT(j) or SELECT(j+1) or both must be set to either both included in the cluster or both excluded.

N (input) The order of the matrices A and B. N >= 0.

A (input/output)
On entry, the upper quasi-triangular matrix A, with (A, B) in generalized real Schur canonical form. On exit, A is overwritten by the reordered matrix A.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the upper triangular matrix B, with (A, B) in generalized real Schur canonical form. On exit, B is overwritten by the reordered matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

ALPHAR (output)
On exit, (ALPHAR(j) + ALPHAI(j)*i)/BETA(j), j=1,...,N, will be the generalized eigenvalues. ALPHAR(j) + ALPHAI(j)*i and BETA(j),j=1,...,N are the diagonals of the complex Schur form (S,T) that would result if the 2-by-2 diagonal blocks of the real generalized Schur form of (A,B) were further reduced to triangular form using complex unitary transformations. If ALPHAI(j) is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-st eigenvalues are a complex conjugate pair, with ALPHAI(j+1) negative.

ALPHAI (output)
See the description of ALPHAR.

BETA (output)
See the description of ALPHAR.

Q (input/output)

On entry, if WANTQ = .TRUE., Q is an N-by-N matrix. On exit, Q has been postmultiplied by the left orthogonal transformation matrix which reorder (A, B); The leading M columns of Q form orthonormal bases for the specified pair of left eigenspaces (deflating subspaces). If WANTQ = .FALSE., Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= 1; and if WANTQ = .TRUE., LDQ >= N.

Z (input/output)
On entry, if WANTZ = .TRUE., Z is an N-by-N matrix. On exit, Z has been postmultiplied by the left orthogonal transformation matrix which reorder (A, B); The leading M columns of Z form orthonormal bases for the specified pair of left eigenspaces (deflating subspaces). If WANTZ = .FALSE., Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1; If WANTZ = .TRUE., LDZ >= N.

M (output)
The dimension of the specified pair of left and right eigen- spaces (deflating subspaces). 0 <= M <= N.

PL (output)
If IJOB = 1, 4 or 5, PL, PR are lower bounds on the reciprocal of the norm of "projections" onto left and right eigenspaces with respect to the selected cluster. 0 < PL, PR <= 1. If M = 0 or M = N, PL = PR = 1. If IJOB = 0, 2 or 3, PL and PR are not referenced.

PR (output)
See the description of PL.

DIF (output)
If IJOB >= 2, DIF(1:2) store the estimates of Difu and Difl.
If IJOB = 2 or 4, DIF(1:2) are F-norm-based upper bounds on
Difu and Difl. If IJOB = 3 or 5, DIF(1:2) are 1-norm-based estimates of Difu and Difl. If M = 0 or N, DIF(1:2) = F-norm([A, B]). If IJOB = 0 or 1, DIF is not referenced.

WORK (workspace)
If IJOB = 0, WORK is not referenced. Otherwise, on exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 4*N+16. If IJOB = 1, 2 or 4, LWORK >= MAX(4*N+16, 2*M*(N-M)). If IJOB = 3 or 5, LWORK >= MAX(4*N+16, 4*M*(N-M)).

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace/output)
If IJOB = 0, IWORK is not referenced. Otherwise, on exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of the array IWORK. LIWORK >= 1. If IJOB = 1, 2 or 4, LIWORK >= N+6. If IJOB = 3 or 5, LIWORK >= MAX(2*M*(N-M), N+6).

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
=0: Successful exit.
<0: If INFO = -i, the i-th argument had an illegal value.
=1: Reordering of (A, B) failed because the transformed matrix pair (A, B) would be too far from generalized Schur form; the problem is very ill-conditioned. (A, B) may have been partially reordered. If requested, 0 is returned in DIF(*), PL and PR.

FURTHER DETAILS

STGSEN first collects the selected eigenvalues by computing orthogonal U and W that move them to the top left corner of (A, B). In other words, the selected eigenvalues are the eigenvalues of (A11, B11) in:

```
              U'*(A, B)*W = (A11 A12) (B11 B12) n1
                            ( 0  A22),( 0  B22) n2
                              n1  n2    n1  n2
```

where N = n1+n2 and U' means the transpose of U. The first n1 columns of U and W span the specified pair of left and right eigenspaces (deflating subspaces) of (A, B).

If (A, B) has been obtained from the generalized real Schur decomposition of a matrix pair (C, D) = Q*(A, B)*Z', then the reordered generalized real Schur form of (C, D) is given by

```
         (C, D) = (Q*U)*(U'*(A, B)*W)*(Z*W)',
```

and the first n1 columns of Q*U and Z*W span the corresponding deflating subspaces of (C, D) (Q and Z store Q*U and Z*W, resp.).

Note that if the selected eigenvalue is sufficiently ill-conditioned, then its value may differ significantly from its value before reordering.

The reciprocal condition numbers of the left and right eigenspaces spanned by the first n1 columns of U and W (or Q*U and Z*W) may be returned in DIF(1:2), corresponding to Difu and Difl, resp.

The Difu and Difl are defined as:
ifu[(A11, B11), (A22, B22)] = sigma-min(Zu)
and

where sigma-min(Zu) is the smallest singular value of the (2*n1*n2)-by-(2*n1*n2) matrix

```
u = [ kron(In2, A11)  -kron(A22', In1) ]
          [ kron(In2, B11)  -kron(B22', In1) ].
```

Here, Inx is the identity matrix of size nx and A22' is the transpose of A22. kron(X, Y) is the Kronecker product between the matrices X and Y.

When DIF(2) is small, small changes in (A, B) can cause large changes in the deflating subspace. An approximate (asymptotic) bound on the maximum angular error in the computed deflating subspaces is PS * norm((A, B)) / DIF(2),

where EPS is the machine precision.

The reciprocal norm of the projectors on the left and right eigenspaces associated with (A11, B11) may be returned in PL and PR. They are computed as follows. First we compute L and R so that P*(A, B)*Q is block diagonal, where

```
 = ( I -L ) n1           Q = ( I R ) n1
        ( 0  I ) n2    and        ( 0 I ) n2
          n1 n2                    n1 n2
```

and (L, R) is the solution to the generalized Sylvester equation 11*R - L*A22 = -A12

Then PL = (F-norm(L)**2+1)**(-1/2) and PR = (F-norm(R)**2+1)**(-1/2). An approximate (asymptotic) bound on the average absolute error of the selected eigenvalues is
PS * norm((A, B)) / PL.

There are also global error bounds which valid for perturbations up to a certain restriction: A lower bound (x) on the smallest F-norm(E,F) for which an eigenvalue of (A11, B11) may move and coalesce with an eigenvalue of (A22, B22) under perturbation (E,F), (i.e. (A + E, B + F), is

x = min(Difu,Difl)/((1/(PL*PL)+1/(PR*PR))**(1/2)+2*max(1/PL,1/PR)).

An approximate bound on x can be computed from DIF(1:2), PL and PR.

If y = (F-norm(E,F) / x) <= 1, the angles between the perturbed (L', R') and unperturbed (L, R) left and right deflating subspaces associated with the selected cluster in the (1,1)-blocks can be bounded as

max-angle(L, L') <= arctan(y * PL / (1 - y * (1 - PL * PL)**(1/2))
max-angle(R, R') <= arctan(y * PR / (1 - y * (1 - PR * PR)**(1/2))

See LAPACK User's Guide section 4.11 or the following references for more information.

Note that if the default method for computing the Frobenius-norm- based estimate DIF is not wanted (see SLATDF), then the parameter IDIFJB (see below) should be changed from 3 to 4 (routine SLATDF (IJOB = 2 will be used)). See STGSYL for more details.

Based on contributions by
Bo Kagstrom and Peter Poromaa, Department of Computing Science,

Umea University, S-901 87 Umea, Sweden.

References

[1] B. Kagstrom; A Direct Method for Reordering Eigenvalues in the Generalized Real Schur Form of a Regular Matrix Pair (A, B), in M.S. Moonen et al (eds), Linear Algebra for Large Scale and Real-Time Applications, Kluwer Academic Publ. 1993, pp 195-218.

[2] B. Kagstrom and P. Poromaa; Computing Eigenspaces with Specified Eigenvalues of a Regular Matrix Pair (A, B) and Condition Estimation: Theory, Algorithms and Software, Report UMINF - 94.04, Department of Computing Science, Umea University, S-901 87 Umea, Sweden, 1994. Also as LAPACK Working Note 87. To appear in Numerical Algorithms, 1996.

[3] B. Kagstrom and P. Poromaa, LAPACK-Style Algorithms and Software for Solving the Generalized Sylvester Equation and Estimating the Separation between Regular Matrix Pairs, Report UMINF - 93.23, Department of Computing Science, Umea University, S-901 87 Umea, Sweden, December 1993, Revised April 1994, Also as LAPACK Working Note 75. To appear in ACM Trans. on Math. Software, Vol 22, No 1, 1996.

Contents

NAME

```
stgsja - compute the generalized singular  value  decomposi-
tion  (GSVD)  of  two real upper triangular (or trapezoidal)
matrices A and B
```

SYNOPSIS

```
SUBROUTINE STGSJA(JOBU, JOBV, JOBQ, M, P, N, K, L, A, LDA, B, LDB,
      TOLA, TOLB, ALPHA, BETA, U, LDU, V, LDV, Q, LDQ, WORK, NCYCLE,
      INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
INTEGER M, P, N, K, L, LDA, LDB, LDU, LDV, LDQ, NCYCLE, INFO
REAL TOLA, TOLB
REAL  A(LDA,*),  B(LDB,*),  ALPHA(*),   BETA(*),   U(LDU,*),
V(LDV,*), Q(LDQ,*), WORK(*)

SUBROUTINE STGSJA_64(JOBU, JOBV, JOBQ, M, P, N, K, L, A, LDA, B, LDB,
      TOLA, TOLB, ALPHA, BETA, U, LDU, V, LDV, Q, LDQ, WORK, NCYCLE,
      INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
INTEGER*8 M, P, N, K, L, LDA, LDB, LDU,  LDV,  LDQ,  NCYCLE,
INFO
REAL TOLA, TOLB
REAL  A(LDA,*),  B(LDB,*),  ALPHA(*),   BETA(*),   U(LDU,*),
V(LDV,*), Q(LDQ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TGSJA(JOBU, JOBV, JOBQ, M, P, N, K, L, A, [LDA], B, [LDB],
       TOLA, TOLB, ALPHA, BETA, U, [LDU], V, [LDV], Q, [LDQ], [WORK],
       NCYCLE, [INFO])
```

```
CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
INTEGER :: M, P, N, K, L, LDA, LDB, LDU, LDV,  LDQ,  NCYCLE,
INFO
REAL :: TOLA, TOLB
REAL, DIMENSION(:) :: ALPHA, BETA, WORK
REAL, DIMENSION(:,:) :: A, B, U, V, Q

SUBROUTINE TGSJA_64(JOBU, JOBV, JOBQ, M, P, N, K, L, A, [LDA], B,
       [LDB], TOLA, TOLB, ALPHA, BETA, U, [LDU], V, [LDV], Q, [LDQ],
       [WORK], NCYCLE, [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
INTEGER(8) :: M, P, N, K, L, LDA, LDB, LDU, LDV,  LDQ,  NCY-
CLE, INFO
REAL :: TOLA, TOLB
REAL, DIMENSION(:) :: ALPHA, BETA, WORK
REAL, DIMENSION(:,:) :: A, B, U, V, Q
```

C INTERFACE

```
#include <sunperf.h>

void stgsja(char jobu, char jobv, char jobq, int m,  int  p,
          int  n, int k, int l, float *a, int lda, float *b,
          int ldb, float tola,  float  tolb,  float  *alpha,
          float *beta, float *u, int ldu, float *v, int ldv,
          float *q, int ldq, int *ncycle, int *info);

void stgsja_64(char jobu, char jobv, char jobq, long m, long
          p,  long  n,  long  k, long l, float *a, long lda,
          float *b, long ldb, float tola, float tolb,  float
          *alpha, float *beta, float *u, long ldu, float *v,
          long ldv, float *q, long ldq, long  *ncycle,  long
          *info);
```

PURPOSE

stgsja computes the generalized singular value decomposition (GSVD) of two real upper triangular (or trapezoidal) matrices A and B.

On entry, it is assumed that matrices A and B have the following forms, which may be obtained by the preprocessing subroutine SGGSVP from a general M-by-N matrix A and P-by-N matrix B:

```
             N-K-L  K    L
   A =    K ( 0    A12  A13 ) if M-K-L >= 0;
          L ( 0     0   A23 )
      M-K-L ( 0     0    0  )
```

```
           N-K-L  K    L
   A =  K ( 0    A12  A13 ) if M-K-L < 0;
      M-K ( 0     0   A23 )

           N-K-L  K    L
   B =  L ( 0     0   B13 )
      P-L ( 0     0    0  )
```

where the K-by-K matrix A12 and L-by-L matrix B13 are non-singular upper triangular; A23 is L-by-L upper triangular if M-K-L >= 0, otherwise A23 is (M-K)-by-L upper trapezoidal.

On exit,

```
            U'*A*Q = D1*( 0 R ),    V'*B*Q = D2*( 0 R ),
```

where U, V and Q are orthogonal matrices, Z' denotes the transpose of Z, R is a nonsingular upper triangular matrix, and D1 and D2 are ``diagonal'' matrices, which are of the following structures:
If M-K-L >= 0,

```
                    K  L
       D1 =     K ( I  0 )
                L ( 0  C )
            M-K-L ( 0  0 )

                  K  L
       D2 = L   ( 0  S )
            P-L ( 0  0 )

               N-K-L  K    L
  ( 0 R ) = K (  0   R11  R12 ) K
            L (  0    0   R22 ) L
```

where

```
  C = diag( ALPHA(K+1), ... , ALPHA(K+L) ),
  S = diag( BETA(K+1),  ... , BETA(K+L) ),
  C**2 + S**2 = I.

  R is stored in A(1:K+L,N-K-L+1:N) on exit.
```

If M-K-L < 0,

```
               K M-K K+L-M
    D1 =   K ( I  0    0   )
         M-K ( 0  C    0   )

                 K M-K K+L-M
```

```
    D2 =   M-K ( 0  S    0   )
         K+L-M ( 0  0    I   )
           P-L ( 0  0    0   )

               N-K-L  K   M-K  K+L-M

          M-K ( 0     0   R22  R23  )
        K+L-M ( 0     0    0   R33  )
```

where
C = diag(ALPHA(K+1), ... , ALPHA(M)),
S = diag(BETA(K+1), ... , BETA(M)),
C**2 + S**2 = I.

R = (R11 R12 R13) is stored in A(1:M, N-K-L+1:N) and R33 is stored
(0 R22 R23)
in B(M-K+1:L,N+M-K-L+1:N) on exit.

The computation of the orthogonal transformation matrices U, V or Q is optional. These matrices may either be formed explicitly, or they may be postmultiplied into input matrices U1, V1, or Q1.
STGSJA essentially uses a variant of Kogbetliantz algorithm to reduce min(L,M-K)-by-L triangular (or trapezoidal) matrix A23 and L-by-L matrix B13 to the form:
U1'*A13*Q1 = C1*R1; V1'*B13*Q1 = S1*R1,
where U1, V1 and Q1 are orthogonal matrix, and Z' is the transpose of Z. C1 and S1 are diagonal matrices satisfying
C1**2 + S1**2 = I,
and R1 is an L-by-L nonsingular upper triangular matrix.

ARGUMENTS

JOBU (input)
= 'U': U must contain an orthogonal matrix U1 on entry, and the product U1*U is returned; = 'I': U is initialized to the unit matrix, and the orthogonal matrix U is returned; = 'N': U is not computed.

JOBV (input)
= 'V': V must contain an orthogonal matrix V1 on entry, and the product V1*V is returned; = 'I': V is initialized to the unit matrix, and the orthogonal matrix V is returned; = 'N': V is not computed.

JOBQ (input)

= 'Q': Q must contain an orthogonal matrix Q1 on entry, and the product Q1*Q is returned; = 'I': Q is initialized to the unit matrix, and the orthogonal matrix Q is returned; = 'N': Q is not computed.

M (input) The number of rows of the matrix A. M >= 0.

P (input) The number of rows of the matrix B. P >= 0.

N (input) The number of columns of the matrices A and B. N >= 0.

K (input) K and L specify the subblocks in the input matrices A and B:
A23 = A(K+1:MIN(K+L,M),N-L+1:N) and B13 = B(1:L,N-L+1:N) of A and B, whose GSVD is going to be computed by STGSJA. See Further details.
L (input) See the description of K.

A (input/output)
On entry, the M-by-N matrix A. On exit, A(N-K+1:N,1:MIN(K+L,M)) contains the triangular matrix R or part of R. See Purpose for details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the P-by-N matrix B. On exit, if necessary, B(M-K+1:L,N+M-K-L+1:N) contains a part of R. See Purpose for details.

LDB (input)
The leading dimension of the array B. LDB >= max(1,P).

TOLA (input)
TOLA and TOLB are the convergence criteria for the Jacobi- Kogbetliantz iteration procedure. Generally, they are the same as used in the preprocessing step, say TOLA = max(M,N)*norm(A)*MACHEPS, TOLB = max(P,N)*norm(B)*MACHEPS.

TOLB (input)
See the description of TOLA.

ALPHA (output)
On exit, ALPHA and BETA contain the generalized

```
           singular value pairs of A and B; ALPHA(1:K) = 1,
           BETA(1:K)  = 0, and if M-K-L >= 0,  ALPHA(K+1:K+L)
           = diag(C),
           BETA(K+1:K+L)   =  diag(S),  or  if  M-K-L  <   0,
           ALPHA(K+1:M)= C, ALPHA(M+1:K+L)= 0
           BETA(K+1:M) = S, BETA(M+1:K+L) = 1.   Furthermore,
           if K+L < N, ALPHA(K+L+1:N) = 0 and
           BETA(K+L+1:N)  = 0.

 BETA (output)
           See the description of ALPHA.
 U (input) On entry, if JOBU = 'U', U must contain  a  matrix
           U1  (usually  the  orthogonal  matrix  returned by
           SGGSVP).  On exit, if JOBU = 'I', U  contains  the
           orthogonal matrix U; if JOBU = 'U', U contains the
           product U1*U.  If JOBU = 'N', U is not referenced.

 LDU (input)
           The leading dimension  of  the  array  U.  LDU  >=
           max(1,M) if JOBU = 'U'; LDU >= 1 otherwise.

 V (input) On entry, if JOBV = 'V', V must contain  a  matrix
           V1  (usually  the  orthogonal  matrix  returned by
           SGGSVP).  On exit, if JOBV = 'I', V  contains  the
           orthogonal matrix V; if JOBV = 'V', V contains the
           product V1*V.  If JOBV = 'N', V is not referenced.

 LDV (input)
           The leading dimension  of  the  array  V.  LDV  >=
           max(1,P) if JOBV = 'V'; LDV >= 1 otherwise.

 Q (input) On entry, if JOBQ = 'Q', Q must contain  a  matrix
           Q1  (usually  the  orthogonal  matrix  returned by
           SGGSVP).  On exit, if JOBQ = 'I', Q  contains  the
           orthogonal matrix Q; if JOBQ = 'Q', Q contains the
           product Q1*Q.  If JOBQ = 'N', Q is not referenced.

 LDQ (input)
           The leading dimension  of  the  array  Q.  LDQ  >=
           max(1,N) if JOBQ = 'Q'; LDQ >= 1 otherwise.

 WORK (workspace)
           dimension(2*N)

 NCYCLE (output)
           The number of cycles required for convergence.

 INFO (output)
           = 0:  successful exit
           < 0:  if INFO = -i, the i-th argument had an ille-
```

gal value.
= 1: the procedure does not converge after MAXIT cycles.

Contents

NAME

```
stgsna - estimate reciprocal condition numbers for specified
eigenvalues  and/or  eigenvectors of a matrix pair (A, B) in
generalized real Schur canonical form (or of any matrix pair
(Q*A*Z',  Q*B*Z') with orthogonal matrices Q and Z, where Z'
denotes the transpose of Z
```

SYNOPSIS

```
SUBROUTINE STGSNA(JOB, HOWMNT, SELECT, N, A, LDA, B, LDB, VL, LDVL,
      VR, LDVR, S, DIF, MM, M, WORK, LWORK, IWORK, INFO)

CHARACTER * 1 JOB, HOWMNT
INTEGER N, LDA, LDB, LDVL, LDVR, MM, M, LWORK, INFO
INTEGER IWORK(*)
LOGICAL SELECT(*)
REAL  A(LDA,*),  B(LDB,*),  VL(LDVL,*),  VR(LDVR,*),   S(*),
DIF(*), WORK(*)

SUBROUTINE STGSNA_64(JOB, HOWMNT, SELECT, N, A, LDA, B, LDB, VL,
      LDVL, VR, LDVR, S, DIF, MM, M, WORK, LWORK, IWORK, INFO)

CHARACTER * 1 JOB, HOWMNT
INTEGER*8 N, LDA, LDB, LDVL, LDVR, MM, M, LWORK, INFO
INTEGER*8 IWORK(*)
LOGICAL*8 SELECT(*)
REAL  A(LDA,*),  B(LDB,*),  VL(LDVL,*),  VR(LDVR,*),   S(*),
DIF(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TGSNA(JOB, HOWMNT, SELECT, [N], A, [LDA], B, [LDB], VL,
       [LDVL], VR, [LDVR], S, DIF, MM, M, [WORK], [LWORK], [IWORK],
```

```
          [INFO])

     CHARACTER(LEN=1) :: JOB, HOWMNT
     INTEGER :: N, LDA, LDB, LDVL, LDVR, MM, M, LWORK, INFO
     INTEGER, DIMENSION(:) :: IWORK
     LOGICAL, DIMENSION(:) :: SELECT
     REAL, DIMENSION(:) :: S, DIF, WORK
     REAL, DIMENSION(:,:) :: A, B, VL, VR

     SUBROUTINE TGSNA_64(JOB, HOWMNT, SELECT, [N], A, [LDA], B, [LDB], VL,
            [LDVL], VR, [LDVR], S, DIF, MM, M, [WORK], [LWORK], [IWORK],
            [INFO])

     CHARACTER(LEN=1) :: JOB, HOWMNT
     INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, MM, M, LWORK, INFO
     INTEGER(8), DIMENSION(:) :: IWORK
     LOGICAL(8), DIMENSION(:) :: SELECT
     REAL, DIMENSION(:) :: S, DIF, WORK
     REAL, DIMENSION(:,:) :: A, B, VL, VR
```

C INTERFACE

```
     #include <sunperf.h>

     void stgsna(char job, char howmnt, int *select, int n, float
               *a,  int  lda,  float  *b, int ldb, float *vl, int
               ldvl, float *vr, int ldvr, float *s,  float  *dif,
               int mm, int *m, int *info);

     void stgsna_64(char job, char howmnt, long *select, long  n,
               float *a, long lda, float *b, long ldb, float *vl,
               long ldvl, float *vr, long ldvr, float  *s,  float
               *dif, long mm, long *m, long *info);
```

PURPOSE

stgsna estimates reciprocal condition numbers for specified eigenvalues and/or eigenvectors of a matrix pair (A, B) in generalized real Schur canonical form (or of any matrix pair (Q*A*Z', Q*B*Z') with orthogonal matrices Q and Z, where Z' denotes the transpose of Z.

(A, B) must be in generalized real Schur form (as returned by SGGES), i.e. A is block upper triangular with 1-by-1 and 2-by-2 diagonal blocks. B is upper triangular.

ARGUMENTS

JOB (input)
Specifies whether condition numbers are required for eigenvalues (S) or eigenvectors (DIF):
= 'E': for eigenvalues only (S);
= 'V': for eigenvectors only (DIF);
= 'B': for both eigenvalues and eigenvectors (S and DIF).

HOWMNT (input)
= 'A': compute condition numbers for all eigenpairs;
= 'S': compute condition numbers for selected eigenpairs specified by the array SELECT.

SELECT (input)
If HOWMNT = 'S', SELECT specifies the eigenpairs for which condition numbers are required. To select condition numbers for the eigenpair corresponding to a real eigenvalue w(j), SELECT(j) must be set to .TRUE.. To select condition numbers corresponding to a complex conjugate pair of eigenvalues w(j) and w(j+1), either SELECT(j) or SELECT(j+1) or both, must be set to .TRUE.. If HOWMNT = 'A', SELECT is not referenced.

N (input) The order of the square matrix pair (A, B). N >= 0.

A (input) The upper quasi-triangular matrix A in the pair (A,B).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input) The upper triangular matrix B in the pair (A,B).

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

VL (input)
If JOB = 'E' or 'B', VL must contain left eigenvectors of (A, B), corresponding to the eigenpairs specified by HOWMNT and SELECT. The eigenvectors must be stored in consecutive columns of VL, as returned by STGEVC. If JOB = 'V', VL is not referenced.

LDVL (input)

The leading dimension of the array VL. LDVL >= 1.
If JOB = 'E' or 'B', LDVL >= N.

VR (input)
If JOB = 'E' or 'B', VR must contain right eigen- vectors of (A, B), corresponding to the eigenpairs specified by HOWMNT and SELECT. The eigenvectors must be stored in consecutive columns ov VR, as returned by STGEVC. If JOB = 'V', VR is not referenced.

LDVR (input)
The leading dimension of the array VR. LDVR >= 1.
If JOB = 'E' or 'B', LDVR >= N.

S (output)
If JOB = 'E' or 'B', the reciprocal condition numbers of the selected eigenvalues, stored in consecutive elements of the array. For a complex conjugate pair of eigenvalues two consecutive ele- ments of S are set to the same value. Thus S(j), DIF(j), and the j-th columns of VL and VR all correspond to the same eigenpair (but not in gen- eral the j-th eigenpair, unless all eigenpairs are selected). If JOB = 'V', S is not referenced.

DIF (output)
If JOB = 'V' or 'B', the estimated reciprocal con- dition numbers of the selected eigenvectors, stored in consecutive elements of the array. For a complex eigenvector two consecutive elements of DIF are set to the same value. If the eigenvalues cannot be reordered to compute DIF(j), DIF(j) is set to 0; this can only occur when the true value would be very small anyway. If JOB = 'E', DIF is not referenced.

MM (input)
The number of elements in the arrays S and DIF. MM >= M.

M (output)
The number of elements of the arrays S and DIF used to store the specified condition numbers; for each selected real eigenvalue one element is used, and for each selected complex conjugate pair of eigenvalues, two elements are used. If HOWMNT = 'A', M is set to N.

WORK (workspace)
If JOB = 'E', WORK is not referenced. Otherwise,

on exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= N. If JOB = 'V' or 'B' LWORK >= 2*N*(N+2)+16.
If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace)
dimension(N+6) If JOB = 'E', IWORK is not referenced.

INFO (output)
=0: Successful exit
<0: If INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The reciprocal of the condition number of a generalized eigenvalue w = (a, b) is defined as
(w) = (|u'Av|**2 + |u'Bv|**2)**(1/2) / (norm(u)*norm(v))

where u and v are the left and right eigenvectors of (A, B) corresponding to w; |z| denotes the absolute value of the complex number, and norm(u) denotes the 2-norm of the vector u.
The pair (a, b) corresponds to an eigenvalue w = a/b (= u'Av/u'Bv) of the matrix pair (A, B). If both a and b equal zero, then (A B) is singular and S(I) = -1 is returned.

An approximate error bound on the chordal distance between the i-th computed generalized eigenvalue w and the corresponding exact eigenvalue lambda is
hord(w, lambda) <= EPS * norm(A, B) / S(I)

where EPS is the machine precision.

The reciprocal of the condition number DIF(i) of right eigenvector u and left eigenvector v corresponding to the generalized eigenvalue w is defined as follows:

a) If the i-th eigenvalue w = (a,b) is real

Suppose U and V are orthogonal transformations such that

```
              U'*(A, B)*V  = (S, T) = ( a   *  ) ( b   *   )
1
                                        ( 0  S22 ),( 0  T22  )
n-1
                                          1  n-1     1 n-1

   Then the reciprocal condition number DIF(i) is
              Difl((a, b), (S22, T22)) = sigma-min( Zl ),

   where sigma-min(Zl) denotes the smallest  singular  value
of the
   2(n-1)-by-2(n-1) matrix

       Zl = [ kron(a, In-1)  -kron(1, S22) ]
            [ kron(b, In-1)  -kron(1, T22) ] .

   Here In-1 is the identity matrix of size n-1. kron(X,  Y)
is the
   Kronecker product between the matrices X and Y.

   Note that if the default method for computing  DIF(i)  is
wanted
   (see SLATDF),  then  the  parameter  DIFDRI  (see  below)
should be
   changed from 3 to 4 (routine  SLATDF(IJOB  =  2  will  be
used)).
   See STGSYL for more details.

b) If the i-th and (i+1)-th eigenvalues are  complex  conju-
gate pair,

   Suppose U and V are orthogonal transformations such that

              U'*(A, B)*V = (S, T) = ( S11  *   ) (  T11   *
)  2
                                       (  0     S22   ),(   0
T22) n-2
                                         2    n-2     2    n-2

   and (S11,  T11)  corresponds  to  the  complex  conjugate
eigenvalue
   pair (w, conjg(w)). There exist unitary matrices  U1  and
V1 such
   that

       U1'*S11*V1 = ( s11 s12 )   and U1'*T11*V1 = ( t11 t12
)
                    (  0  s22 )                    (  0  t22
)
```

where the generalized eigenvalues w = s11/t11 and
conjg(w) = s22/t22.

Then the reciprocal condition number DIF(i) is bounded by

min(d1, max(1, |real(s11)/real(s22)|)*d2)

where, d1 = Difl((s11, t11), (s22, t22)) = sigma-min(Z1), where
Z1 is the complex 2-by-2 matrix

```
Z1 =  [ s11  -s22 ]
      [ t11  -t22 ],
```

This is done by computing (using real arithmetic) the roots of the characteristical polynomial det(Z1' * Z1 - lambda I),
where Z1' denotes the conjugate transpose of Z1 and det(X) denotes
the determinant of X.

and d2 is an upper bound on Difl((S11, T11), (S22, T22)), i.e. an
upper bound on sigma-min(Z2), where Z2 is (2n-2)-by-(2n-2)

```
Z2 = [ kron(S11', In-2)  -kron(I2, S22) ]
     [ kron(T11', In-2)  -kron(I2, T22) ]
```

Note that if the default method for computing DIF is wanted (see
SLATDF), then the parameter DIFDRI (see below) should be changed
from 3 to 4 (routine SLATDF(IJOB = 2 will be used)). See STGSYL
for more details.

For each eigenvalue/vector specified by SELECT, DIF stores a Frobenius norm-based estimate of Difl.

An approximate error bound for the i-th computed eigenvector VL(i) or VR(i) is given by

EPS * norm(A, B) / DIF(i).

See ref. [2-3] for more details and further references.

Based on contributions by
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Umea University, S-901 87 Umea, Sweden.

References

[1] B. Kagstrom; A Direct Method for Reordering Eigenvalues in the Generalized Real Schur Form of a Regular Matrix Pair (A, B), in M.S. Moonen et al (eds), Linear Algebra for Large Scale and Real-Time Applications, Kluwer Academic Publ. 1993, pp 195-218.

[2] B. Kagstrom and P. Poromaa; Computing Eigenspaces with Specified Eigenvalues of a Regular Matrix Pair (A, B) and Condi- tion Estimation: Theory, Algorithms and Software, Report UMINF - 94.04, Department of Computing Science, Umea University, S-901 87 Umea, Sweden, 1994. Also as LAPACK Working Note 87. To appear in Numerical Algorithms, 1996.

[3] B. Kagstrom and P. Poromaa, LAPACK-Style Algorithms and Software for Solving the Generalized Sylvester Equation and Estimating the Separation between Regular Matrix Pairs, Report UMINF - 93.23, Department of Computing Science, Umea University, S-901 87 Umea, Sweden, December 1993, Revised April 1994, Also as LAPACK Working Note 75. To appear in ACM Trans. on Math. Software, Vol 22, No 1, 1996.

Contents

NAME

```
stgsyl - solve the generalized Sylvester equation
```

SYNOPSIS

```
SUBROUTINE STGSYL(TRANS, IJOB, M, N, A, LDA, B, LDB, C, LDC, D, LDD,
      E, LDE, F, LDF, SCALE, DIF, WORK, LWORK, IWORK, INFO)

CHARACTER * 1 TRANS
INTEGER IJOB, M, N, LDA, LDB, LDC,  LDD,  LDE,  LDF,  LWORK,
INFO
INTEGER IWORK(*)
REAL SCALE, DIF
REAL  A(LDA,*),  B(LDB,*),  C(LDC,*),  D(LDD,*),   E(LDE,*),
F(LDF,*), WORK(*)

SUBROUTINE STGSYL_64(TRANS, IJOB, M, N, A, LDA, B, LDB, C, LDC, D,
      LDD, E, LDE, F, LDF, SCALE, DIF, WORK, LWORK, IWORK, INFO)

CHARACTER * 1 TRANS
INTEGER*8 IJOB, M, N, LDA, LDB, LDC, LDD, LDE,  LDF,  LWORK,
INFO
INTEGER*8 IWORK(*)
REAL SCALE, DIF
REAL  A(LDA,*),  B(LDB,*),  C(LDC,*),  D(LDD,*),   E(LDE,*),
F(LDF,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TGSYL(TRANS, IJOB, [M], [N], A, [LDA], B, [LDB], C, [LDC],
       D, [LDD], E, [LDE], F, [LDF], SCALE, DIF, [WORK], [LWORK], [IWORK],
       [INFO])
```

```
CHARACTER(LEN=1) :: TRANS
INTEGER :: IJOB, M, N, LDA, LDB, LDC, LDD, LDE, LDF,  LWORK,
INFO
INTEGER, DIMENSION(:) :: IWORK
REAL :: SCALE, DIF
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A, B, C, D, E, F

SUBROUTINE TGSYL_64(TRANS, IJOB, [M], [N], A, [LDA], B, [LDB], C,
       [LDC], D, [LDD], E, [LDE], F, [LDF], SCALE, DIF, [WORK], [LWORK],
       [IWORK], [INFO])

CHARACTER(LEN=1) :: TRANS
INTEGER(8) :: IJOB, M, N, LDA,  LDB,  LDC,  LDD,  LDE,  LDF,
LWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL :: SCALE, DIF
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A, B, C, D, E, F
```

C INTERFACE

```
#include <sunperf.h>

void stgsyl(char trans, int ijob, int m, int  n,  float  *a,
          int  lda,  float  *b,  int ldb, float *c, int ldc,
          float *d, int ldd, float *e, int  lde,  float  *f,
          int ldf, float *scale, float *dif, int *info);

void stgsyl_64(char trans, long ijob, long m, long n,  float
          *a,  long  lda, float *b, long ldb, float *c, long
          ldc, float *d, long ldd, float *e, long lde, float
          *f,  long  ldf,  float  *scale,  float  *dif, long
          *info);
```

PURPOSE

stgsyl solves the generalized Sylvester equation:

```
            A * R - L * B = scale * C                 (1)
            D * R - L * E = scale * F
```

where R and L are unknown m-by-n matrices, (A, D), (B, E) and (C, F) are given matrix pairs of size m-by-m, n-by-n and m-by-n, respectively, with real entries. (A, D) and (B, E) must be in generalized (real) Schur canonical form, i.e. A, B are upper quasi triangular and D, E are upper triangular.

The solution (R, L) overwrites (C, F). 0 <= SCALE <= 1 is an output scaling factor chosen to avoid overflow.

In matrix notation (1) is equivalent to solve Zx = scale b, where Z is defined as

```
           Z = [ kron(In, A)  -kron(B', Im) ]         (2)
               [ kron(In, D)  -kron(E', Im) ].
```

Here Ik is the identity matrix of size k and X' is the transpose of X. kron(X, Y) is the Kronecker product between the matrices X and Y.

If TRANS = 'T', STGSYL solves the transposed system Z'*y = scale*b, which is equivalent to solve for R and L in

```
            A' * R  + D' * L   = scale *  C           (3)
            R  * B' + L  * E'  = scale * (-F)
```

This case (TRANS = 'T') is used to compute an one-norm-based estimate of Dif[(A,D), (B,E)], the separation between the matrix pairs (A,D) and (B,E), using SLACON.

If IJOB >= 1, STGSYL computes a Frobenius norm-based estimate of Dif[(A,D),(B,E)]. That is, the reciprocal of a lower bound on the reciprocal of the smallest singular value of Z. See [1-2] for more information.

This is a level 3 BLAS algorithm.

ARGUMENTS

TRANS (input)
= 'N', solve the generalized Sylvester equation (1). = 'T', solve the 'transposed' system (3).

IJOB (input)
Specifies what kind of functionality to be performed. =0: solve (1) only.
=1: The functionality of 0 and 3.
=2: The functionality of 0 and 4.
=3: Only an estimate of Dif[(A,D), (B,E)] is computed. (look ahead strategy IJOB = 1 is used).
=4: Only an estimate of Dif[(A,D), (B,E)] is computed. (SGECON on sub-systems is used). Not referenced if TRANS = 'T'.

M (input) The order of the matrices A and D, and the row dimension of the matrices C, F, R and L.

N (input) The order of the matrices B and E, and the column dimension of the matrices C, F, R and L.

A (input) The upper quasi triangular matrix A.

LDA (input)
The leading dimension of the array A. LDA >= max(1, M).

B (input) The upper quasi triangular matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1, N).

C (input/output)
On entry, C contains the right-hand-side of the first matrix equation in (1) or (3). On exit, if IJOB = 0, 1 or 2, C has been overwritten by the solution R. If IJOB = 3 or 4 and TRANS = 'N', C holds R, the solution achieved during the computation of the Dif-estimate.

LDC (input)
The leading dimension of the array C. LDC >= max(1, M).

D (input) The upper triangular matrix D.

LDD (input)
The leading dimension of the array D. LDD >= max(1, M).

E (input) The upper triangular matrix E.

LDE (input)
The leading dimension of the array E. LDE >= max(1, N).

F (input/output)
On entry, F contains the right-hand-side of the second matrix equation in (1) or (3). On exit, if IJOB = 0, 1 or 2, F has been overwritten by the solution L. If IJOB = 3 or 4 and TRANS = 'N', F holds L, the solution achieved during the computation of the Dif-estimate.

LDF (input)
The leading dimension of the array F. LDF >= max(1, M).

DIF (output)
On exit SCALE is the reciprocal of a lower bound of the reciprocal of the Dif-function, i.e. SCALE is an upper bound of Dif[(A,D), (B,E)] = sigma_min(Z), where Z as in (2). If IJOB = 0 or TRANS = 'T', SCALE is not touched.

SCALE (output)
On exit SCALE is the reciprocal of a lower bound of the reciprocal of the Dif-function, i.e. SCALE is an upper bound of Dif[(A,D), (B,E)] = sigma_min(Z), where Z as in (2). If IJOB = 0 or TRANS = 'T', SCALE is not touched.

WORK (workspace)
If IJOB = 0, WORK is not referenced. Otherwise, on exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK > = 1. If IJOB = 1 or 2 and TRANS = 'N', LWORK >= 2*M*N.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace)
dimension(M+N+2)

INFO (output)
=0: successful exit
<0: If INFO = -i, the i-th argument had an illegal value.
>0: (A, D) and (B, E) have common or close eigen- values.

FURTHER DETAILS

Based on contributions by
Bo Kagstrom and Peter Poromaa, Department of Computing Science,
Umea University, S-901 87 Umea, Sweden.

[1] B. Kagstrom and P. Poromaa, LAPACK-Style Algorithms and Software
for Solving the Generalized Sylvester Equation and Estimating the
Separation between Regular Matrix Pairs, Report UMINF -

93.23,
Department of Computing Science, Umea University, S-901 87 Umea,
Sweden, December 1993, Revised April 1994, Also as LAPACK Working
Note 75. To appear in ACM Trans. on Math. Software, Vol 22,
No 1, 1996.

[2] B. Kagstrom, A Perturbation Analysis of the Generalized Sylvester
Equation (AR - LB, DR - LE) = (C, F), SIAM J. Matrix Anal.
Appl., 15(4):1045-1060, 1994

[3] B. Kagstrom and L. Westin, Generalized Schur Methods with
Condition Estimators for Solving the Generalized Sylves-ter
Equation, IEEE Transactions on Automatic Control, Vol. 34, No. 7,
July 1989, pp 745-751.

Contents

NAME

```
stpcon - estimate the reciprocal of the condition number  of
a  packed  triangular  matrix A, in either the 1-norm or the
infinity-norm
```

SYNOPSIS

```
SUBROUTINE STPCON(NORM, UPLO, DIAG, N, A, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 NORM, UPLO, DIAG
INTEGER N, INFO
INTEGER WORK2(*)
REAL RCOND
REAL A(*), WORK(*)

SUBROUTINE STPCON_64(NORM, UPLO, DIAG, N, A, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 NORM, UPLO, DIAG
INTEGER*8 N, INFO
INTEGER*8 WORK2(*)
REAL RCOND
REAL A(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TPCON(NORM, UPLO, DIAG, N, A, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL :: RCOND
REAL, DIMENSION(:) :: A, WORK
```

```
SUBROUTINE TPCON_64(NORM, UPLO, DIAG, N, A, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL :: RCOND
REAL, DIMENSION(:) :: A, WORK
```

C INTERFACE

```
#include <sunperf.h>

void stpcon(char norm, char uplo, char diag,  int  n,  float
          *a, float *rcond, int *info);
void stpcon_64(char norm, char  uplo,  char  diag,  long  n,
          float *a, float *rcond, long *info);
```

PURPOSE

stpcon estimates the reciprocal of the condition number of a packed triangular matrix A, in either the 1-norm or the infinity-norm.

The norm of A is computed and an estimate is obtained for norm(inv(A)), then the reciprocal of the condition number is computed as

RCOND = 1 / (norm(A) * norm(inv(A))).

ARGUMENTS

```
NORM (input)
          Specifies whether the 1-norm condition  number  or
          the infinity-norm condition number is required:
          = '1' or 'O':  1-norm;
          = 'I':         Infinity-norm.

UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.
```

A (input) The upper or lower triangular matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n. If DIAG = 'U', the diagonal elements of A are not referenced and are assumed to be 1.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(norm(A) * norm(inv(A))).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
stpmv - perform one of the  matrix-vector  operations  x  :=
A*x, or x := A'*x
```

SYNOPSIS

```
SUBROUTINE STPMV(UPLO, TRANSA, DIAG, N, A, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, INCY
REAL A(*), Y(*)

SUBROUTINE STPMV_64(UPLO, TRANSA, DIAG, N, A, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, INCY
REAL A(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE TPMV(UPLO, [TRANSA], DIAG, [N], A, Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, INCY
REAL, DIMENSION(:) :: A, Y

SUBROUTINE TPMV_64(UPLO, [TRANSA], DIAG, [N], A, Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, INCY
REAL, DIMENSION(:) :: A, Y
```

C INTERFACE

```
#include <sunperf.h>
```

```
void stpmv(char uplo, char transa, char diag, int  n,  float
          *a, float *y, int incy);

void stpmv_64(char uplo, char transa,  char  diag,  long  n,
          float *a, float *y, long incy);
```

PURPOSE

stpmv performs one of the matrix-vector operations x := A*x, or x := A'*x, where x is an n element vector and A is an n by n unit, or non-unit, upper or lower triangular matrix, supplied in packed form.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   x := A*x.

TRANSA = 'T' or 't'   x := A'*x.

TRANSA = 'C' or 'c'   x := A'*x.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit tri-

angular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

N (input)

On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

A (input)

((n*(n + 1))/2). Before entry with UPLO = 'U' or 'u', the array A must contain the upper triangular matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(1, 2) and a(2, 2) respectively, and so on. Before entry with UPLO = 'L' or 'l', the array A must contain the lower triangular matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(2, 1) and a(3, 1) respectively, and so on. Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced, but are assumed to be unity. Unchanged on exit.

Y (input/output)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector x. On exit, Y is overwritten with the tranformed vector x.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
stprfs - provide error bounds and backward  error  estimates
for the solution to a system of linear equations with a tri-
angular packed coefficient matrix
```

SYNOPSIS

```
SUBROUTINE STPRFS(UPLO, TRANSA, DIAG, N, NRHS, A, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER WORK2(*)
REAL A(*), B(LDB,*), X(LDX,*), FERR(*), BERR(*), WORK(*)

SUBROUTINE STPRFS_64(UPLO, TRANSA, DIAG, N, NRHS, A, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 WORK2(*)
REAL A(*), B(LDB,*), X(LDX,*), FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TPRFS(UPLO, [TRANSA], DIAG, N, NRHS, A, B, [LDB], X, [LDX],
       FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL, DIMENSION(:) :: A, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: B, X
```

```
SUBROUTINE TPRFS_64(UPLO, [TRANSA], DIAG, N, NRHS, A, B, [LDB], X,
       [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL, DIMENSION(:) :: A, FERR, BERR, WORK
REAL, DIMENSION(:,:) :: B, X
```

C INTERFACE

```
#include <sunperf.h>

void stprfs(char uplo, char transa, char diag,  int  n,  int
          nrhs,  float  *a, float *b, int ldb, float *x, int
          ldx, float *ferr, float *berr, int *info);
void stprfs_64(char uplo, char transa, char  diag,  long  n,
          long nrhs, float *a, float *b, long ldb, float *x,
          long ldx, float *ferr, float *berr, long *info);
```

PURPOSE

stprfs provides error bounds and backward error estimates for the solution to a system of linear equations with a triangular packed coefficient matrix.

The solution matrix X must be computed by STPTRS or some other means before entering this routine. STPRFS does not do iterative refinement because doing so cannot improve the backward error.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B  (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose = Tran-
          spose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
          = 'N':  A is non-unit triangular;
```

```
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input) The upper or lower  triangular  matrix  A,  packed
          columnwise  in a linear array.  The j-th column of
          A is stored in the array A as follows:  if UPLO  =
          'U',  A(i  +  (j-1)*j/2)  = A(i,j) for 1<=i<=j; if
          UPLO = 'L', A(i + (j-1)*(2*n-j)/2)  =  A(i,j)  for
          j<=i<=n.   If DIAG = 'U', the diagonal elements of
          A are not referenced and are assumed to be 1.

B (input) The right hand side matrix B.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

X (input) The solution matrix X.

LDX (input)
          The leading dimension of  the  array  X.   LDX  >=
          max(1,N).

FERR (output)
          The estimated forward error bound for  each  solu-
          tion  vector X(j) (the j-th column of the solution
          matrix  X).   If  XTRUE  is  the   true   solution
          corresponding  to  X(j),  FERR(j)  is an estimated
          upper bound for the magnitude of the largest  ele-
          ment in (X(j) - XTRUE) divided by the magnitude of
          the largest element in X(j).  The estimate  is  as
          reliable  as the estimate for RCOND, and is almost
          always a slight overestimate of the true error.

BERR (output)
          The componentwise relative backward error of  each
          solution  vector X(j) (i.e., the smallest relative
          change in any element of A or B that makes X(j) an
          exact solution).

WORK (workspace)
          dimension(3*N)

WORK2 (workspace)
          dimension(N)
```

```
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
stpsv - solve one of the systems of equations A*x  =  b,  or
A'*x = b
```

SYNOPSIS

```
SUBROUTINE STPSV(UPLO, TRANSA, DIAG, N, A, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, INCY
REAL A(*), Y(*)

SUBROUTINE STPSV_64(UPLO, TRANSA, DIAG, N, A, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, INCY
REAL A(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE TPSV(UPLO, [TRANSA], DIAG, [N], A, Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, INCY
REAL, DIMENSION(:) :: A, Y

SUBROUTINE TPSV_64(UPLO, [TRANSA], DIAG, [N], A, Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, INCY
REAL, DIMENSION(:) :: A, Y
```

C INTERFACE

```
#include <sunperf.h>
```

```
void stpsv(char uplo, char transa, char diag, int  n,  float
          *a, float *y, int incy);

void stpsv_64(char uplo, char transa,  char  diag,  long  n,
          float *a, float *y, long incy);
```

PURPOSE

stpsv solves one of the systems of equations A*x = b, or A'*x = b, where b and x are n element vectors and A is an n by n unit, or non-unit, upper or lower triangular matrix, supplied in packed form.

No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the equations to be solved as follows:

TRANSA = 'N' or 'n'   A*x = b.

TRANSA = 'T' or 't'   A'*x = b.

TRANSA = 'C' or 'c'   A'*x = b.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)

On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u' A is assumed to be unit triangular.

DIAG = 'N' or 'n' A is not assumed to be unit triangular.

Unchanged on exit.

N (input)

On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

A (input)

((n*(n + 1))/2). Before entry with UPLO = 'U' or 'u', the array A must contain the upper triangular matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(1, 2) and a(2, 2) respectively, and so on. Before entry with UPLO = 'L' or 'l', the array A must contain the lower triangular matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(2, 1) and a(3, 1) respectively, and so on. Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced, but are assumed to be unity. Unchanged on exit.

Y (input/output)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element right-hand side vector b. On exit, Y is overwritten with the solution vector x.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
stptri - compute the inverse of a real upper or  lower  tri-
angular matrix A stored in packed format
```

SYNOPSIS

```
SUBROUTINE STPTRI(UPLO, DIAG, N, A, INFO)

CHARACTER * 1 UPLO, DIAG
INTEGER N, INFO
REAL A(*)

SUBROUTINE STPTRI_64(UPLO, DIAG, N, A, INFO)

CHARACTER * 1 UPLO, DIAG
INTEGER*8 N, INFO
REAL A(*)
```

F95 INTERFACE

```
SUBROUTINE TPTRI(UPLO, DIAG, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
INTEGER :: N, INFO
REAL, DIMENSION(:) :: A

SUBROUTINE TPTRI_64(UPLO, DIAG, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
INTEGER(8) :: N, INFO
REAL, DIMENSION(:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void stptri(char uplo, char  diag,  int  n,  float  *a,  int
          *info);

void stptri_64(char uplo, char diag, long n, float *a,  long
          *info);
```

PURPOSE

stptri computes the inverse of a real upper or lower triangular matrix A stored in packed format.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the upper or lower triangular matrix  A,
          stored  columnwise  in  a  linear array.  The j-th
          column of A is stored in the array A  as  follows:
          if  UPLO  =  'U',  A(i  +  (j-1)*j/2) = A(i,j) for
          1<=i<=j; if UPLO = 'L', A(i + (j-1)*((2*n-j)/2)  =
          A(i,j)   for   j<=i<=n.   See  below  for  further
          details.  On exit, the (triangular) inverse of the
          original  matrix,  in the same packed storage for-
          mat.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, A(i,i) is  exactly  zero.   The
          triangular  matrix is singular and its inverse can
          not be computed.
```

FURTHER DETAILS

A triangular matrix A can be transferred to packed storage using one of the following program segments:

UPLO = 'U':

```
      JC = 1
      DO 2 J = 1, N
         DO 1 I = 1, J
            A(JC+I-1)  =  A(I,J)
    1    CONTINUE
         JC = JC + J
    2 CONTINUE
```

UPLO = 'L':

```
      JC = 1
      DO 2 J = 1, N
         DO 1 I = J, N
            A(JC+I-J)   =  A(I,J)
    1    CONTINUE
         JC = JC + N - J  + 1
    2 CONTINUE
```

Contents

NAME

```
stptrs - solve a triangular system of the form   A * X  =  B
or A**T * X = B,
```

SYNOPSIS

```
SUBROUTINE STPTRS(UPLO, TRANSA, DIAG, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, NRHS, LDB, INFO
REAL A(*), B(LDB,*)

SUBROUTINE STPTRS_64(UPLO, TRANSA, DIAG, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, NRHS, LDB, INFO
REAL A(*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE TPTRS(UPLO, TRANSA, DIAG, N, NRHS, A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, NRHS, LDB, INFO
REAL, DIMENSION(:) :: A
REAL, DIMENSION(:,:) :: B

SUBROUTINE TPTRS_64(UPLO, TRANSA, DIAG, N, NRHS, A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, NRHS, LDB, INFO
REAL, DIMENSION(:) :: A
REAL, DIMENSION(:,:) :: B
```

C INTERFACE

```
#include <sunperf.h>

void stptrs(char uplo, char transa, char diag,  int  n,  int
          nrhs, float *a, float *b, int ldb, int *info);

void stptrs_64(char uplo, char transa, char  diag,  long  n,
          long  nrhs,  float  *a,  float  *b, long ldb, long
          *info);
```

PURPOSE

stptrs solves a triangular system of the form

where A is a triangular matrix of order N stored in packed format, and B is an N-by-NRHS matrix. A check is made to verify that A is nonsingular.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B  (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose = Tran-
          spose)

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

A (input) The upper or lower  triangular  matrix  A,  packed
          columnwise  in a linear array.  The j-th column of
          A is stored in the array A as follows:  if UPLO  =
          'U',  A(i  +  (j-1)*j/2)  = A(i,j) for 1<=i<=j; if
          UPLO = 'L', A(i + (j-1)*(2*n-j)/2)  =  A(i,j)  for
          j<=i<=n.
```

B (input/output)
On entry, the right hand side matrix B. On exit, if INFO = 0, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the i-th diagonal element of A is zero, indicating that the matrix is singular and the solutions X have not been computed.

Contents

NAME

```
strans - transpose and scale source matrix
```

SYNOPSIS

```
SUBROUTINE STRANS(PLACE, SCALE, SOURCE, M, N, DEST)

CHARACTER * 1 PLACE
INTEGER M, N
REAL SCALE
REAL SOURCE(*), DEST(*)

SUBROUTINE STRANS_64(PLACE, SCALE, SOURCE, M, N, DEST)

CHARACTER * 1 PLACE
INTEGER*8 M, N
REAL SCALE
REAL SOURCE(*), DEST(*)
```

F95 INTERFACE

```
SUBROUTINE TRANS([PLACE], SCALE, SOURCE, M, N, [DEST])

CHARACTER(LEN=1) :: PLACE
INTEGER :: M, N
REAL :: SCALE
REAL, DIMENSION(:) :: SOURCE, DEST

SUBROUTINE TRANS_64([PLACE], SCALE, SOURCE, M, N, [DEST])

CHARACTER(LEN=1) :: PLACE
INTEGER(8) :: M, N
REAL :: SCALE
REAL, DIMENSION(:) :: SOURCE, DEST
```

C INTERFACE

```
#include <sunperf.h>

void strans(char place, float scale, float *source,  int  m,
          int n, float *dest);

void strans_64(char place, float scale, float *source,  long
          m, long n, float *dest);
```

PURPOSE

strans scales and transposes the source matrix. The N2 x N1 result is written into SOURCE when PLACE = 'I' or 'i', and DEST when PLACE = 'O' or 'o'.

PLACE = 'I' or 'i': SOURCE = SCALE * SOURCE'

PLACE = 'O' or 'o': DEST = SCALE * SOURCE'

ARGUMENTS

PLACE (input)
Type of transpose. 'I' or 'i' for in-place, 'O' or 'o' for out-of-place. 'I' is default.

SCALE (input)
Scale factor on the SOURCE matrix.

SOURCE (input/output)
(M, N) on input. Array of (N, M) on output if in-place transpose.

M (input)
Number of rows in the SOURCE matrix on input.

N (input)
Number of columns in the SOURCE matrix on input.

DEST (output)
Scaled and transposed SOURCE matrix if out-of-place transpose. Not referenced if in-place transpose.

Contents

NAME

```
strcon - estimate the reciprocal of the condition number  of
a   triangular  matrix  A,  in  either  the  1-norm  or  the
infinity-norm
```

SYNOPSIS

```
SUBROUTINE STRCON(NORM, UPLO, DIAG, N, A, LDA, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 NORM, UPLO, DIAG
INTEGER N, LDA, INFO
INTEGER WORK2(*)
REAL RCOND
REAL A(LDA,*), WORK(*)

SUBROUTINE STRCON_64(NORM, UPLO, DIAG, N, A, LDA, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 NORM, UPLO, DIAG
INTEGER*8 N, LDA, INFO
INTEGER*8 WORK2(*)
REAL RCOND
REAL A(LDA,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TRCON(NORM, UPLO, DIAG, N, A, [LDA], RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL :: RCOND
```

```
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE TRCON_64(NORM, UPLO, DIAG, N, A, [LDA], RCOND, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL :: RCOND
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
void strcon(char norm, char uplo, char diag,  int  n,  float
          *a, int lda, float *rcond, int *info);

void strcon_64(char norm, char  uplo,  char  diag,  long  n,
          float *a, long lda, float *rcond, long *info);
```

PURPOSE

strcon estimates the reciprocal of the condition number of a triangular matrix A, in either the 1-norm or the infinity-norm.

The norm of A is computed and an estimate is obtained for norm(inv(A)), then the reciprocal of the condition number is computed as

RCOND = 1 / (norm(A) * norm(inv(A))).

ARGUMENTS

```
NORM (input)
          Specifies whether the 1-norm condition  number  or
          the infinity-norm condition number is required:
          = '1' or 'O':  1-norm;
          = 'I':         Infinity-norm.

UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.
```

N (input) The order of the matrix A. N >= 0.

A (input) The triangular matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of the array A contains the upper triangular matrix, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of the array A contains the lower triangular matrix, and the strictly upper triangular part of A is not referenced. If DIAG = 'U', the diagonal elements of A are also not referenced and are assumed to be 1.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(norm(A) * norm(inv(A))).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
strevc - compute some or all of the right and/or left eigen-
vectors of a real upper quasi-triangular matrix T
```

SYNOPSIS

```
SUBROUTINE STREVC(SIDE, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR,
      LDVR, MM, M, WORK, INFO)

CHARACTER * 1 SIDE, HOWMNY
INTEGER N, LDT, LDVL, LDVR, MM, M, INFO
LOGICAL SELECT(*)
REAL T(LDT,*), VL(LDVL,*), VR(LDVR,*), WORK(*)

SUBROUTINE STREVC_64(SIDE, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR,
      LDVR, MM, M, WORK, INFO)

CHARACTER * 1 SIDE, HOWMNY
INTEGER*8 N, LDT, LDVL, LDVR, MM, M, INFO
LOGICAL*8 SELECT(*)
REAL T(LDT,*), VL(LDVL,*), VR(LDVR,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TREVC(SIDE, HOWMNY, SELECT, N, T, [LDT], VL, [LDVL], VR,
       [LDVR], MM, M, [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, HOWMNY
INTEGER :: N, LDT, LDVL, LDVR, MM, M, INFO
LOGICAL, DIMENSION(:) :: SELECT
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: T, VL, VR
```

```
SUBROUTINE TREVC_64(SIDE, HOWMNY, SELECT, N, T, [LDT], VL, [LDVL],
       VR, [LDVR], MM, M, [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, HOWMNY
INTEGER(8) :: N, LDT, LDVL, LDVR, MM, M, INFO
LOGICAL(8), DIMENSION(:) :: SELECT
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: T, VL, VR
```

C INTERFACE

```
#include <sunperf.h>

void strevc(char side, char  howmny,  int  *select,  int  n,
          float *t, int ldt, float *vl, int ldvl, float *vr,
          int ldvr, int mm, int *m, int *info);
void strevc_64(char side, char howmny, long *select, long n,
          float  *t,  long  ldt, float *vl, long ldvl, float
          *vr, long ldvr, long mm, long *m, long *info);
```

PURPOSE

strevc computes some or all of the right and/or left eigen-vectors of a real upper quasi-triangular matrix T.

The right eigenvector x and the left eigenvector y of T corresponding to an eigenvalue w are defined by:

```
T*x = w*x,     y'*T = w*y'
```

where y' denotes the conjugate transpose of the vector y.

If all eigenvectors are requested, the routine may either return the matrices X and/or Y of right or left eigenvectors of T, or the products Q*X and/or Q*Y, where Q is an input orthogonal
matrix. If T was obtained from the real-Schur factorization of an original matrix A = Q*T*Q', then Q*X and Q*Y are the matrices of right or left eigenvectors of A.

T must be in Schur canonical form (as returned by SHSEQR), that is, block upper triangular with 1-by-1 and 2-by-2 diag-onal blocks; each 2-by-2 diagonal block has its diagonal elements equal and its off-diagonal elements of opposite sign. Corresponding to each 2-by-2 diagonal block is a com-plex conjugate pair of eigenvalues and eigenvectors; only one eigenvector of the pair is computed, namely the one corresponding to the eigenvalue with positive imaginary part.

ARGUMENTS

```
SIDE (input)
          = 'R':  compute right eigenvectors only;
          = 'L':  compute left eigenvectors only;
          = 'B':  compute both right and left eigenvectors.

HOWMNY (input)
          = 'A':  compute all right  and/or  left  eigenvec-
          tors;
          = 'B':  compute all right  and/or  left  eigenvec-
          tors,  and  backtransform  them  using  the  input
          matrices supplied in VR and/or VL; = 'S':  compute
          selected right and/or left eigenvectors, specified
          by the logical array SELECT.
SELECT (input/output)
          If HOWMNY = 'S', SELECT specifies the eigenvectors
          to be computed.  If HOWMNY = 'A' or 'B', SELECT is
          not referenced.  To select  the  real  eigenvector
          corresponding to a real eigenvalue w(j), SELECT(j)
          must be set to  .TRUE..   To  select  the  complex
          eigenvector  corresponding  to a complex conjugate
          pair  w(j)  and  w(j+1),   either   SELECT(j)   or
          SELECT(j+1)  must  be  set to .TRUE.; then on exit
          SELECT(j) is .TRUE. and SELECT(j+1) is .FALSE..

N (input) The order of the matrix T. N >= 0.

T (input/output)
          The  upper  quasi-triangular  matrix  T  in  Schur
          canonical form.

LDT (input)
          The leading dimension  of  the  array  T.  LDT  >=
          max(1,N).

VL (input/output)
          On entry, if SIDE = 'L' or 'B' and HOWMNY  =  'B',
          VL  must  contain  an N-by-N matrix Q (usually the
          orthogonal matrix Q of Schur vectors  returned  by
          SHSEQR).   On  exit, if SIDE = 'L' or 'B', VL con-
          tains:  if HOWMNY = 'A',  the  matrix  Y  of  left
          eigenvectors  of  T;  VL  has the same quasi-lower
          triangular form as T'. If T(i,i) is a real  eigen-
          value,  then  the  i-th column VL(i) of VL  is its
          corresponding eigenvector. If T(i:i+1,i:i+1) is  a
          2-by-2   block   whose  eigenvalues  are  complex-
          conjugate  eigenvalues  of  T,  then  VL(i)+sqrt(-
```

1)*VL(i+1) is the complex eigenvector corresponding to the eigenvalue with positive real part. if HOWMNY = 'B', the matrix Q*Y; if HOWMNY = 'S', the left eigenvectors of T specified by SELECT, stored consecutively in the columns of VL, in the same order as their eigenvalues. A complex eigenvector corresponding to a complex eigenvalue is stored in two consecutive columns, the first holding the real part, and the second the imaginary part. If SIDE = 'R', VL is not referenced.

LDVL (input)
The leading dimension of the array VL. LDVL >= max(1,N) if SIDE = 'L' or 'B'; LDVL >= 1 otherwise.

VR (input/output)
On entry, if SIDE = 'R' or 'B' and HOWMNY = 'B', VR must contain an N-by-N matrix Q (usually the orthogonal matrix Q of Schur vectors returned by SHSEQR). On exit, if SIDE = 'R' or 'B', VR contains: if HOWMNY = 'A', the matrix X of right eigenvectors of T; VR has the same quasi-upper triangular form as T. If T(i,i) is a real eigenvalue, then the i-th column VR(i) of VR is its corresponding eigenvector. If T(i:i+1,i:i+1) is a 2-by-2 block whose eigenvalues are complex-conjugate eigenvalues of T, then VR(i)+sqrt(-1)*VR(i+1) is the complex eigenvector corresponding to the eigenvalue with positive real part. if HOWMNY = 'B', the matrix Q*X; if HOWMNY = 'S', the right eigenvectors of T specified by SELECT, stored consecutively in the columns of VR, in the same order as their eigenvalues. A complex eigenvector corresponding to a complex eigenvalue is stored in two consecutive columns, the first holding the real part and the second the imaginary part. If SIDE = 'L', VR is not referenced.

LDVR (input)
The leading dimension of the array VR. LDVR >= max(1,N) if SIDE = 'R' or 'B'; LDVR >= 1 otherwise.

MM (input)
The number of columns in the arrays VL and/or VR. MM >= M.

M (output)
The number of columns in the arrays VL and/or VR

actually used to store the eigenvectors. If HOWMNY = 'A' or 'B', M is set to N. Each selected real eigenvector occupies one column and each selected complex eigenvector occupies two columns.

WORK (workspace)
dimension(3*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The algorithm used in this program is basically backward (forward) substitution, with scaling to make the the code robust against possible overflow.

Each eigenvector is normalized so that the element of largest magnitude has magnitude 1; here the magnitude of a complex number (x,y) is taken to be |x| + |y|.

Contents

NAME

```
strexc - reorder the real  Schur  factorization  of  a  real
matrix  A  =  Q*T*Q**T, so that the diagonal block of T with
row index IFST is moved to row ILST
```

SYNOPSIS

```
SUBROUTINE STREXC(COMPQ, N, T, LDT, Q, LDQ, IFST, ILST, WORK, INFO)

CHARACTER * 1 COMPQ
INTEGER N, LDT, LDQ, IFST, ILST, INFO
REAL T(LDT,*), Q(LDQ,*), WORK(*)

SUBROUTINE STREXC_64(COMPQ, N, T, LDT, Q, LDQ, IFST, ILST, WORK,
      INFO)

CHARACTER * 1 COMPQ
INTEGER*8 N, LDT, LDQ, IFST, ILST, INFO
REAL T(LDT,*), Q(LDQ,*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TREXC(COMPQ, N, T, [LDT], Q, [LDQ], IFST, ILST, [WORK],
       [INFO])

CHARACTER(LEN=1) :: COMPQ
INTEGER :: N, LDT, LDQ, IFST, ILST, INFO
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: T, Q

SUBROUTINE TREXC_64(COMPQ, N, T, [LDT], Q, [LDQ], IFST, ILST, [WORK],
       [INFO])

CHARACTER(LEN=1) :: COMPQ
```

```
INTEGER(8) :: N, LDT, LDQ, IFST, ILST, INFO
REAL, DIMENSION(:) :: WORK
REAL, DIMENSION(:,:) :: T, Q
```

C INTERFACE

```
#include <sunperf.h>

void strexc(char compq, int n, float *t, int ldt, float  *q,
          int ldq, int *ifst, int *ilst, int *info);

void strexc_64(char compq, long n, float *t, long ldt, float
          *q, long ldq, long *ifst, long *ilst, long *info);
```

PURPOSE

strexc reorders the real Schur factorization of a real matrix A = Q*T*Q**T, so that the diagonal block of T with row index IFST is moved to row ILST.

The real Schur form T is reordered by an orthogonal similarity transformation Z**T*T*Z, and optionally the matrix Q of Schur vectors is updated by postmultiplying it with Z.

T must be in Schur canonical form (as returned by SHSEQR), that is, block upper triangular with 1-by-1 and 2-by-2 diagonal blocks; each 2-by-2 diagonal block has its diagonal elements equal and its off-diagonal elements of opposite sign.

ARGUMENTS

COMPQ (input)
= 'V': update the matrix Q of Schur vectors;
= 'N': do not update Q.

N (input) The order of the matrix T. N >= 0.

T (input/output)
On entry, the upper quasi-triangular matrix T, in Schur Schur canonical form. On exit, the reordered upper quasi-triangular matrix, again in Schur canonical form.

LDT (input)
The leading dimension of the array T. LDT >= max(1,N).

Q (input) On entry, if COMPQ = 'V', the matrix Q of Schur vectors. On exit, if COMPQ = 'V', Q has been postmultiplied by the orthogonal transformation matrix Z which reorders T. If COMPQ = 'N', Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= max(1,N).

IFST (input/output)
Specify the reordering of the diagonal blocks of T. The block with row index IFST is moved to row ILST, by a sequence of transpositions between adjacent blocks. On exit, if IFST pointed on entry to the second row of a 2-by-2 block, it is changed to point to the first row; ILST always points to the first row of the block in its final position (which may differ from its input value by +1 or -1). 1 <= IFST <= N; 1 <= ILST <= N.

ILST (input/output)
See the description of IFST.

WORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
= 1: two adjacent blocks were too close to swap (the problem is very ill-conditioned); T may have been partially reordered, and ILST points to the first row of the current position of the block being moved.

Contents

NAME

```
strmm - perform one of the matrix-matrix operations    B  :=
alpha*op( A )*B, or B := alpha*B*op( A )
```

SYNOPSIS

```
SUBROUTINE STRMM(SIDE, UPLO, TRANSA, DIAG, M, N, ALPHA, A, LDA, B,
      LDB)

CHARACTER * 1 SIDE, UPLO, TRANSA, DIAG
INTEGER M, N, LDA, LDB
REAL ALPHA
REAL A(LDA,*), B(LDB,*)

SUBROUTINE STRMM_64(SIDE, UPLO, TRANSA, DIAG, M, N, ALPHA, A, LDA, B,
      LDB)

CHARACTER * 1 SIDE, UPLO, TRANSA, DIAG
INTEGER*8 M, N, LDA, LDB
REAL ALPHA
REAL A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE TRMM(SIDE, UPLO, [TRANSA], DIAG, [M], [N], ALPHA, A, [LDA],
       B, [LDB])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANSA, DIAG
INTEGER :: M, N, LDA, LDB
REAL :: ALPHA
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE TRMM_64(SIDE, UPLO, [TRANSA], DIAG, [M], [N], ALPHA, A,
       [LDA], B, [LDB])
```

```
CHARACTER(LEN=1) :: SIDE, UPLO, TRANSA, DIAG
INTEGER(8) :: M, N, LDA, LDB
REAL :: ALPHA
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void strmm(char side, char uplo, char transa, char diag, int
          m,  int  n,  float alpha, float *a, int lda, float
          *b, int ldb);

void strmm_64(char side, char uplo, char transa, char  diag,
          long  m,  long n, float alpha, float *a, long lda,
          float *b, long ldb);
```

PURPOSE

strmm performs one of the matrix-matrix operations B := alpha*op(A)*B, or B := alpha*B*op(A) where alpha is a scalar, B is an m by n matrix, A is a unit, or non-unit, upper or lower triangular matrix and op(A) is one of

op(A) = A   or   op(A) = A'.

ARGUMENTS

SIDE (input)
On entry, SIDE specifies whether op(A) multiplies B from the left or right as follows:

SIDE = 'L' or 'l'   B := alpha*op(A)*B.

SIDE = 'R' or 'r'   B := alpha*B*op(A).

Unchanged on exit.

UPLO (input)
On entry, UPLO specifies whether the matrix A is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the form of op(A) to be used in the matrix multiplication as follows:

TRANSA = 'N' or 'n'   op(A) = A.

TRANSA = 'T' or 't'   op(A) = A'.

TRANSA = 'C' or 'c'   op(A) = A'.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit triangular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

M (input)
On entry, M specifies the number of rows of B. M >= 0. Unchanged on exit.

N (input)
On entry, N specifies the number of columns of B. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. When alpha is zero then A is not referenced and B need not be set before entry. Unchanged on exit.

A (input)
REAL array of DIMENSION (LDA, k), where k is m when SIDE = 'L' or 'l' and is n when SIDE = 'R' or 'r'.

Before entry with UPLO = 'U' or 'u', the leading k by k upper triangular part of the array A must contain the upper triangular matrix and the strictly lower triangular part of A is not refer-

enced.

Before entry with UPLO = 'L' or 'l', the leading k by k lower triangular part of the array A must contain the lower triangular matrix and the strictly upper triangular part of A is not referenced.

Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced either, but are assumed to be one. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When SIDE = 'L' or 'l' then LDA >= max(1, m), when SIDE = 'R' or 'r' then LDA >= max(1, n). Unchanged on exit.

B (input/output)

REAL array of DIMENSION (LDB, n). Before entry, the leading m by n part of the array B must contain the matrix B, and on exit is overwritten by the transformed matrix.

LDB (input)

On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. LDB >= max(1, m). Unchanged on exit.

Contents

NAME

```
strmv - perform one of the  matrix-vector  operations  x  :=
A*x, or x := A'*x
```

SYNOPSIS

```
SUBROUTINE STRMV(UPLO, TRANSA, DIAG, N, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, LDA, INCY
REAL A(LDA,*), Y(*)

SUBROUTINE STRMV_64(UPLO, TRANSA, DIAG, N, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, LDA, INCY
REAL A(LDA,*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE TRMV(UPLO, [TRANSA], DIAG, [N], A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, LDA, INCY
REAL, DIMENSION(:) :: Y
REAL, DIMENSION(:,:) :: A

SUBROUTINE TRMV_64(UPLO, [TRANSA], DIAG, [N], A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, LDA, INCY
REAL, DIMENSION(:) :: Y
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void strmv(char uplo, char transa, char diag, int  n,  float
          *a, int lda, float *y, int incy);

void strmv_64(char uplo, char transa,  char  diag,  long  n,
          float *a, long lda, float *y, long incy);
```

PURPOSE

strmv performs one of the matrix-vector operations x := A*x, or x := A'*x, where x is an n element vector and A is an n by n unit, or non-unit, upper or lower triangular matrix.

ARGUMENTS

```
UPLO (input)
          On entry, UPLO specifies whether the matrix is  an
          upper or lower triangular matrix as follows:

          UPLO = 'U' or  'u'    A  is  an  upper  triangular
          matrix.

          UPLO = 'L'  or  'l'    A  is  a  lower  triangular
          matrix.

          Unchanged on exit.

TRANSA (input)
          On entry, TRANSA specifies  the  operation  to  be
          performed as follows:

          TRANSA = 'N' or 'n'   x := A*x.

          TRANSA = 'T' or 't'   x := A'*x.

          TRANSA = 'C' or 'c'   x := A'*x.

          Unchanged on exit.

          TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
          On entry, DIAG specifies whether or not A is  unit
          triangular as follows:
```

DIAG = 'U' or 'u'   A is assumed to be unit triangular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

N (input)

On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

A (input)

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular matrix and the strictly lower triangular part of A is not referenced. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular matrix and the strictly upper triangular part of A is not referenced. Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced either, but are assumed to be unity. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

Y (input/output)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector x. On exit, Y is overwritten with the tranformed vector x.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

strrfs - provide error bounds and backward error estimates for the solution to a system of linear equations with a triangular coefficient matrix

SYNOPSIS

```
SUBROUTINE STRRFS(UPLO, TRANSA, DIAG, N, NRHS, A, LDA, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, NRHS, LDA, LDB, LDX, INFO
INTEGER WORK2(*)
REAL A(LDA,*), B(LDB,*), X(LDX,*), FERR(*), BERR(*), WORK(*)

SUBROUTINE STRRFS_64(UPLO, TRANSA, DIAG, N, NRHS, A, LDA, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, NRHS, LDA, LDB, LDX, INFO
INTEGER*8 WORK2(*)
REAL A(LDA,*), B(LDB,*), X(LDX,*), FERR(*), BERR(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TRRFS(UPLO, [TRANSA], DIAG, N, NRHS, A, [LDA], B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, NRHS, LDA, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: WORK2
REAL, DIMENSION(:) :: FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, B, X
```

```
SUBROUTINE TRRFS_64(UPLO, [TRANSA], DIAG, N, NRHS, A, [LDA], B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, NRHS, LDA, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: WORK2
REAL, DIMENSION(:) :: FERR, BERR, WORK
REAL, DIMENSION(:,:) :: A, B, X
```

C INTERFACE

```
#include <sunperf.h>

void strrfs(char uplo, char transa, char diag,  int  n,  int
          nrhs,  float *a, int lda, float *b, int ldb, float
          *x, int ldx, float *ferr, float *berr, int *info);
void strrfs_64(char uplo, char transa, char  diag,  long  n,
          long nrhs, float *a, long lda, float *b, long ldb,
          float *x, long ldx, float *ferr, float *berr, long
          *info);
```

PURPOSE

strrfs provides error bounds and backward error estimates for the solution to a system of linear equations with a triangular coefficient matrix.

The solution matrix X must be computed by STRTRS or some other means before entering this routine. STRRFS does not do iterative refinement because doing so cannot improve the backward error.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B  (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose = Tran-
          spose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
```

```
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input) The triangular matrix A.  If UPLO = 'U', the lead-
          ing  N-by-N  upper  triangular part of the array A
          contains the  upper  triangular  matrix,  and  the
          strictly  lower triangular part of A is not refer-
          enced.  If UPLO = 'L', the  leading  N-by-N  lower
          triangular  part of the array A contains the lower
          triangular matrix, and the strictly upper triangu-
          lar  part  of A is not referenced.  If DIAG = 'U',
          the diagonal elements of A are also not referenced
          and are assumed to be 1.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

B (input) The right hand side matrix B.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

X (input) The solution matrix X.

LDX (input)
          The leading dimension of  the  array  X.   LDX  >=
          max(1,N).

FERR (output)
          The estimated forward error bound for  each  solu-
          tion  vector X(j) (the j-th column of the solution
          matrix  X).   If  XTRUE  is  the   true   solution
          corresponding  to  X(j),  FERR(j)  is an estimated
          upper bound for the magnitude of the largest  ele-
          ment in (X(j) - XTRUE) divided by the magnitude of
          the largest element in X(j).  The estimate  is  as
          reliable  as the estimate for RCOND, and is almost
          always a slight overestimate of the true error.

BERR (output)
          The componentwise relative backward error of  each
          solution  vector X(j) (i.e., the smallest relative
```

change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(3*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
strsen - reorder the real  Schur  factorization  of  a  real
matrix  A  =  Q*T*Q**T, so that a selected cluster of eigen-
values appears in the leading diagonal blocks of  the  upper
quasi-triangular matrix T,
```

SYNOPSIS

```
SUBROUTINE STRSEN(JOB, COMPQ, SELECT, N, T, LDT, Q, LDQ, WR, WI, M,
      S, SEP, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOB, COMPQ
INTEGER N, LDT, LDQ, M, LWORK, LIWORK, INFO
INTEGER IWORK(*)
LOGICAL SELECT(*)
REAL S, SEP
REAL T(LDT,*), Q(LDQ,*), WR(*), WI(*), WORK(*)

SUBROUTINE STRSEN_64(JOB, COMPQ, SELECT, N, T, LDT, Q, LDQ, WR, WI,
      M, S, SEP, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOB, COMPQ
INTEGER*8 N, LDT, LDQ, M, LWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
LOGICAL*8 SELECT(*)
REAL S, SEP
REAL T(LDT,*), Q(LDQ,*), WR(*), WI(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TRSEN(JOB, COMPQ, SELECT, N, T, [LDT], Q, [LDQ], WR, WI,
       M, S, SEP, [WORK], [LWORK], [IWORK], [LIWORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOB, COMPQ
INTEGER :: N, LDT, LDQ, M, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
LOGICAL, DIMENSION(:) :: SELECT
REAL :: S, SEP
REAL, DIMENSION(:) :: WR, WI, WORK
REAL, DIMENSION(:,:) :: T, Q

SUBROUTINE TRSEN_64(JOB, COMPQ, SELECT, N, T, [LDT], Q, [LDQ], WR,
       WI, M, S, SEP, [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOB, COMPQ
INTEGER(8) :: N, LDT, LDQ, M, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
LOGICAL(8), DIMENSION(:) :: SELECT
REAL :: S, SEP
REAL, DIMENSION(:) :: WR, WI, WORK
REAL, DIMENSION(:,:) :: T, Q
```

C INTERFACE

```
#include <sunperf.h>

void strsen(char job, char compq, int *select, int n,  float
          *t,  int  ldt, float *q, int ldq, float *wr, float
          *wi, int *m, float *s, float *sep, int *info);

void strsen_64(char job, char compq, long *select,  long  n,
          float *t, long ldt, float *q, long ldq, float *wr,
          float *wi, long *m, float  *s,  float  *sep,  long
          *info);
```

PURPOSE

strsen reorders the real Schur factorization of a real matrix A = Q*T*Q**T, so that a selected cluster of eigen-values appears in the leading diagonal blocks of the upper quasi-triangular matrix T, and the leading columns of Q form an orthonormal basis of the corresponding right invariant subspace.

Optionally the routine computes the reciprocal condition numbers of the cluster of eigenvalues and/or the invariant subspace.

T must be in Schur canonical form (as returned by SHSEQR), that is, block upper triangular with 1-by-1 and 2-by-2 diag-onal blocks; each 2-by-2 diagonal block has its diagonal elemnts equal and its off-diagonal elements of opposite sign.

ARGUMENTS

JOB (input)
Specifies whether condition numbers are required for the cluster of eigenvalues (S) or the invariant subspace (SEP):
= 'N': none;
= 'E': for eigenvalues only (S);
= 'V': for invariant subspace only (SEP);
= 'B': for both eigenvalues and invariant subspace (S and SEP).

COMPQ (input)
= 'V': update the matrix Q of Schur vectors;
= 'N': do not update Q.

SELECT (input)
SELECT specifies the eigenvalues in the selected cluster. To select a real eigenvalue w(j), SELECT(j) must be set to w(j) and w(j+1), corresponding to a 2-by-2 diagonal block, either SELECT(j) or SELECT(j+1) or both must be set to either both included in the cluster or both excluded.

N (input) The order of the matrix T. N >= 0.

T (input/output)
On entry, the upper quasi-triangular matrix T, in Schur canonical form. On exit, T is overwritten by the reordered matrix T, again in Schur canonical form, with the selected eigenvalues in the leading diagonal blocks.

LDT (input)
The leading dimension of the array T. LDT >= max(1,N).

Q (input) On entry, if COMPQ = 'V', the matrix Q of Schur vectors. On exit, if COMPQ = 'V', Q has been postmultiplied by the orthogonal transformation matrix which reorders T; the leading M columns of Q form an orthonormal basis for the specified invariant subspace. If COMPQ = 'N', Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= 1;

```
           and if COMPQ = 'V', LDQ >= N.

 WR (output)
           The real and imaginary parts, respectively, of the
           reordered  eigenvalues  of  T. The eigenvalues are
           stored in the same order as on the diagonal of  T,
           with  WR(i)  =  T(i,i) and, if T(i:i+1,i:i+1) is a
           2-by-2 diagonal block, WI(i) >  0  and  WI(i+1)  =
           -WI(i).  Note that if a complex eigenvalue is suf-
           ficiently  ill-conditioned,  then  its  value  may
           differ  significantly from its value before reord-
           ering.

 WI (output)
           See the description of WR.
 M (output)
           The dimension of the specified invariant subspace.
           0 < = M <= N.

 S (output)
           If JOB = 'E' or 'B', S is a  lower  bound  on  the
           reciprocal condition number for the selected clus-
           ter of eigenvalues.  S  cannot  underestimate  the
           true  reciprocal  condition  number by more than a
           factor of sqrt(N). If M = 0 or N, S = 1.  If JOB =
           'N' or 'V', S is not referenced.

 SEP (output)
           If JOB = 'V' or 'B', SEP is the estimated recipro-
           cal  condition  number  of the specified invariant
           subspace. If M = 0 or N, SEP = norm(T).  If JOB  =
           'N' or 'E', SEP is not referenced.

 WORK (workspace)
           On exit, if INFO = 0, WORK(1) returns the  optimal
           LWORK.

 LWORK (input)
           The dimension of the array WORK.  If  JOB  =  'N',
           LWORK >= max(1,N); if JOB = 'E', LWORK >= M*(N-M);
           if JOB = 'V' or 'B', LWORK >= 2*M*(N-M).

           If LWORK = -1, then a workspace query is  assumed;
           the  routine  only  calculates the optimal size of
           the WORK array, returns this value  as  the  first
           entry  of  the  WORK  array,  and no error message
           related to LWORK is issued by XERBLA.

 IWORK (workspace/output)
           If JOB = 'N' or 'E', IWORK is not referenced.
```

```
LIWORK (input)
          The dimension of the array IWORK.  If JOB = 'N' or
          'E',  LIWORK  >= 1; if JOB = 'V' or 'B', LIWORK >=
          M*(N-M).

          If LIWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the IWORK array, returns this value as  the  first
          entry  of  the  IWORK  array, and no error message
          related to LIWORK is issued by XERBLA.
INFO (output)
          = 0: successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
          gal value
          = 1: reordering of T failed  because  some  eigen-
          values  are  too close to separate (the problem is
          very ill-conditioned); T may have  been  partially
          reordered,  and  WR and WI contain the eigenvalues
          in  the  same  order  as  in  T;  S  and  SEP  (if
          requested) are set to zero.
```

FURTHER DETAILS

```
STRSEN first collects the selected eigenvalues by  computing
an  orthogonal transformation Z to move them to the top left
corner of T.  In other words, the selected  eigenvalues  are
the eigenvalues of T11 in:

              Z'*T*Z = ( T11 T12 ) n1
                       (  0  T22 ) n2
                          n1  n2

where N = n1+n2 and Z' means the transpose of Z.  The  first
n1 columns of Z span the specified invariant subspace of T.

If T has been obtained from the real Schur factorization  of
a  matrix A = Q*T*Q', then the reordered real Schur factori-
zation of A is given by A = (Q*Z)*(Z'*T*Z)*(Q*Z)',  and  the
first  n1  columns  of  Q*Z span the corresponding invariant
subspace of A.

The reciprocal condition number of the average of the eigen-
values  of  T11 may be returned in S. S lies between 0 (very
badly conditioned) and 1 (very well conditioned). It is com-
puted as follows. First we compute R so that

                       P = ( I  R ) n1
                           ( 0  0 ) n2
                             n1 n2
```

is the projector on the invariant subspace associated with T11. R is the solution of the Sylvester equation:

T11*R - R*T22 = T12.

Let F-norm(M) denote the Frobenius-norm of M and 2-norm(M) denote the two-norm of M. Then S is computed as the lower bound

(1 + F-norm(R)**2)**(-1/2)

on the reciprocal of 2-norm(P), the true reciprocal condition number. S cannot underestimate 1 / 2-norm(P) by more than a factor of sqrt(N).

An approximate error bound for the computed average of the eigenvalues of T11 is

EPS * norm(T) / S

where EPS is the machine precision.

The reciprocal condition number of the right invariant subspace spanned by the first n1 columns of Z (or of Q*Z) is returned in SEP. SEP is defined as the separation of T11 and T22:

sep(T11, T22) = sigma-min(C)

where sigma-min(C) is the smallest singular value of the n1*n2-by-n1*n2 matrix

C = kprod(I(n2), T11) - kprod(transpose(T22), I(n1))

I(m) is an m by m identity matrix, and kprod denotes the Kronecker product. We estimate sigma-min(C) by the reciprocal of an estimate of the 1-norm of inverse(C). The true reciprocal 1-norm of inverse(C) cannot differ from sigma-min(C) by more than a factor of sqrt(n1*n2).

When SEP is small, small changes in T can cause large changes in the invariant subspace. An approximate bound on the maximum angular error in the computed right invariant subspace is

EPS * norm(T) / SEP

Contents

NAME

strsm - solve one of the matrix equations op(A)*X = alpha*B, or X*op(A) = alpha*B

SYNOPSIS

```
SUBROUTINE STRSM(SIDE, UPLO, TRANSA, DIAG, M, N, ALPHA, A, LDA, B,
      LDB)

CHARACTER * 1 SIDE, UPLO, TRANSA, DIAG
INTEGER M, N, LDA, LDB
REAL ALPHA
REAL A(LDA,*), B(LDB,*)

SUBROUTINE STRSM_64(SIDE, UPLO, TRANSA, DIAG, M, N, ALPHA, A, LDA, B,
      LDB)

CHARACTER * 1 SIDE, UPLO, TRANSA, DIAG
INTEGER*8 M, N, LDA, LDB
REAL ALPHA
REAL A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE TRSM(SIDE, UPLO, [TRANSA], DIAG, [M], [N], ALPHA, A, [LDA],
       B, [LDB])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANSA, DIAG
INTEGER :: M, N, LDA, LDB
REAL :: ALPHA
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE TRSM_64(SIDE, UPLO, [TRANSA], DIAG, [M], [N], ALPHA, A,
       [LDA], B, [LDB])
```

```
CHARACTER(LEN=1) :: SIDE, UPLO, TRANSA, DIAG
INTEGER(8) :: M, N, LDA, LDB
REAL :: ALPHA
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void strsm(char side, char uplo, char transa, char diag, int
          m,  int  n,  float alpha, float *a, int lda, float
          *b, int ldb);

void strsm_64(char side, char uplo, char transa, char  diag,
          long  m,  long n, float alpha, float *a, long lda,
          float *b, long ldb);
```

PURPOSE

strsm solves one of the matrix equations op(A)*X = alpha*B, or X*op(A) = alpha*B where alpha is a scalar, X and B are m by n matrices, A is a unit, or non-unit, upper or lower triangular matrix and op(A) is one of

op(A) = A or op(A) = A'.

The matrix X is overwritten on B.

ARGUMENTS

SIDE (input)
On entry, SIDE specifies whether op(A) appears on the left or right of X as follows:

SIDE = 'L' or 'l'   op(A)*X = alpha*B.

SIDE = 'R' or 'r'   X*op(A) = alpha*B.

Unchanged on exit.

UPLO (input)
On entry, UPLO specifies whether the matrix A is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l' A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)

On entry, TRANSA specifies the form of op(A) to be used in the matrix multiplication as follows:

TRANSA = 'N' or 'n' op(A) = A.

TRANSA = 'T' or 't' op(A) = A'.

TRANSA = 'C' or 'c' op(A) = A'.

Unchanged on exit.
TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)

On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u' A is assumed to be unit tri- angular.

DIAG = 'N' or 'n' A is not assumed to be unit triangular.

Unchanged on exit.

M (input)

On entry, M specifies the number of rows of B. M >= 0. Unchanged on exit.

N (input)

On entry, N specifies the number of columns of B. N >= 0. Unchanged on exit.

ALPHA (input)

On entry, ALPHA specifies the scalar alpha. When alpha is zero then A is not referenced and B need not be set before entry. Unchanged on exit.

A (input)

REAL array of DIMENSION (LDA, k), where k is m when SIDE = 'L' or 'l' and is n when SIDE = 'R' or 'r'.
Before entry with UPLO = 'U' or 'u', the lead- ing k by k upper triangular part of the array A must contain the upper triangular matrix and the

strictly lower triangular part of A is not referenced.
Before entry with UPLO = 'L' or 'l', the leading k by k lower triangular part of the array A must contain the lower triangular matrix and the strictly upper triangular part of A is not referenced.
Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced either, but are assumed to be one. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When SIDE = 'L' or 'l' then LDA >= max(1, m), when SIDE = 'R' or 'r' then LDA >= max(1, n). Unchanged on exit.

B (input/output)
REAL array of DIMENSION (LDB, n). Before entry, the leading m by n part of the array B must contain the right-hand side matrix B, and on exit is overwritten by the solution matrix X.

LDB (input)
On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. LDB >= max(1, m). Unchanged on exit.

Contents

NAME

strsna - estimate reciprocal condition numbers for specified eigenvalues and/or right eigenvectors of a real upper quasi-triangular matrix T (or of any matrix Q*T*Q**T with Q orthogonal)

SYNOPSIS

```
SUBROUTINE STRSNA(JOB, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR, LDVR,
      S, SEP, MM, M, WORK, LDWORK, WORK1, INFO)

CHARACTER * 1 JOB, HOWMNY
INTEGER N, LDT, LDVL, LDVR, MM, M, LDWORK, INFO
INTEGER WORK1(*)
LOGICAL SELECT(*)
REAL  T(LDT,*),  VL(LDVL,*),   VR(LDVR,*),   S(*),   SEP(*),
WORK(LDWORK,*)

SUBROUTINE STRSNA_64(JOB, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR,
      LDVR, S, SEP, MM, M, WORK, LDWORK, WORK1, INFO)

CHARACTER * 1 JOB, HOWMNY
INTEGER*8 N, LDT, LDVL, LDVR, MM, M, LDWORK, INFO
INTEGER*8 WORK1(*)
LOGICAL*8 SELECT(*)
REAL  T(LDT,*),  VL(LDVL,*),   VR(LDVR,*),   S(*),   SEP(*),
WORK(LDWORK,*)
```

F95 INTERFACE

```
SUBROUTINE TRSNA(JOB, HOWMNY, SELECT, N, T, [LDT], VL, [LDVL], VR,
       [LDVR], S, SEP, MM, M, [WORK], [LDWORK], [WORK1], [INFO])
```

```
CHARACTER(LEN=1) :: JOB, HOWMNY
INTEGER :: N, LDT, LDVL, LDVR, MM, M, LDWORK, INFO
INTEGER, DIMENSION(:) :: WORK1
LOGICAL, DIMENSION(:) :: SELECT
REAL, DIMENSION(:) :: S, SEP
REAL, DIMENSION(:,:) :: T, VL, VR, WORK

SUBROUTINE TRSNA_64(JOB, HOWMNY, SELECT, N, T, [LDT], VL, [LDVL], VR,
       [LDVR], S, SEP, MM, M, [WORK], [LDWORK], [WORK1], [INFO])

CHARACTER(LEN=1) :: JOB, HOWMNY
INTEGER(8) :: N, LDT, LDVL, LDVR, MM, M, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: WORK1
LOGICAL(8), DIMENSION(:) :: SELECT
REAL, DIMENSION(:) :: S, SEP
REAL, DIMENSION(:,:) :: T, VL, VR, WORK
```

C INTERFACE

```
#include <sunperf.h>

void strsna(char job, char howmny, int *select, int n, float
          *t,  int  ldt, float *vl, int ldvl, float *vr, int
          ldvr, float *s, float *sep, int mm,  int  *m,  int
          ldwork, int *info);

void strsna_64(char job, char howmny, long *select, long  n,
          float  *t,  long  ldt, float *vl, long ldvl, float
          *vr, long ldvr, float *s,  float  *sep,  long  mm,
          long *m, long ldwork, long *info);
```

PURPOSE

strsna estimates reciprocal condition numbers for specified eigenvalues and/or right eigenvectors of a real upper quasi-triangular matrix T (or of any matrix Q*T*Q**T with Q orthogonal).

T must be in Schur canonical form (as returned by SHSEQR), that is, block upper triangular with 1-by-1 and 2-by-2 diagonal blocks; each 2-by-2 diagonal block has its diagonal elements equal and its off-diagonal elements of opposite sign.

ARGUMENTS

JOB (input)
Specifies whether condition numbers are required for eigenvalues (S) or eigenvectors (SEP):

= 'E': for eigenvalues only (S);
= 'V': for eigenvectors only (SEP);
= 'B': for both eigenvalues and eigenvectors (S and SEP).

HOWMNY (input)
= 'A': compute condition numbers for all eigenpairs;
= 'S': compute condition numbers for selected eigenpairs specified by the array SELECT.

SELECT (input)
If HOWMNY = 'S', SELECT specifies the eigenpairs for which condition numbers are required. To select condition numbers for the eigenpair corresponding to a real eigenvalue w(j), SELECT(j) must be set to .TRUE.. To select condition numbers corresponding to a complex conjugate pair of eigenvalues w(j) and w(j+1), either SELECT(j) or SELECT(j+1) or both, must be set to .TRUE.. If HOWMNY = 'A', SELECT is not referenced.

N (input) The order of the matrix T. N >= 0.

T (input) The upper quasi-triangular matrix T, in Schur canonical form.

LDT (input)
The leading dimension of the array T. LDT >= max(1,N).

VL (input)
If JOB = 'E' or 'B', VL must contain left eigenvectors of T (or of any Q*T*Q**T with Q orthogonal), corresponding to the eigenpairs specified by HOWMNY and SELECT. The eigenvectors must be stored in consecutive columns of VL, as returned by SHSEIN or STREVC. If JOB = 'V', VL is not referenced.

LDVL (input)
The leading dimension of the array VL. LDVL >= 1; and if JOB = 'E' or 'B', LDVL >= N.

VR (input)
If JOB = 'E' or 'B', VR must contain right eigenvectors of T (or of any Q*T*Q**T with Q orthogonal), corresponding to the eigenpairs specified by HOWMNY and SELECT. The eigenvectors must be stored in consecutive columns of VR, as returned by

SHSEIN or STREVC. If JOB = 'V', VR is not referenced.

LDVR (input)
The leading dimension of the array VR. LDVR >= 1; and if JOB = 'E' or 'B', LDVR >= N.

S (output)
If JOB = 'E' or 'B', the reciprocal condition numbers of the selected eigenvalues, stored in consecutive elements of the array. For a complex conjugate pair of eigenvalues two consecutive elements of S are set to the same value. Thus S(j), SEP(j), and the j-th columns of VL and VR all correspond to the same eigenpair (but not in general the j-th eigenpair, unless all eigenpairs are selected). If JOB = 'V', S is not referenced.

SEP (output)
If JOB = 'V' or 'B', the estimated reciprocal condition numbers of the selected eigenvectors, stored in consecutive elements of the array. For a complex eigenvector two consecutive elements of SEP are set to the same value. If the eigenvalues cannot be reordered to compute SEP(j), SEP(j) is set to 0; this can only occur when the true value would be very small anyway. If JOB = 'E', SEP is not referenced.

MM (input)
The number of elements in the arrays S (if JOB = 'E' or 'B') and/or SEP (if JOB = 'V' or 'B'). MM >= M.

M (output)
The number of elements of the arrays S and/or SEP actually used to store the estimated condition numbers. If HOWMNY = 'A', M is set to N.

WORK (workspace)
dimension(LDWORK,N+1) If JOB = 'E', WORK is not referenced.

LDWORK (input)
The leading dimension of the array WORK. LDWORK >= 1; and if JOB = 'V' or 'B', LDWORK >= N.

WORK1 (workspace)
dimension(N) If JOB = 'E', WORK1 is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The reciprocal of the condition number of an eigenvalue lambda is defined as

S(lambda) = |v'*u| / (norm(u)*norm(v))

where u and v are the right and left eigenvectors of T corresponding to lambda; v' denotes the conjugate-transpose of v, and norm(u) denotes the Euclidean norm. These reciprocal condition numbers always lie between zero (very badly conditioned) and one (very well conditioned). If n = 1, S(lambda) is defined to be 1.

An approximate error bound for a computed eigenvalue W(i) is given by

EPS * norm(T) / S(i)

where EPS is the machine precision.

The reciprocal of the condition number of the right eigenvector u corresponding to lambda is defined as follows. Suppose

```
T = ( lambda  c  )
    (   0    T22 )
```

Then the reciprocal condition number is

SEP(lambda, T22) = sigma-min(T22 - lambda*I)

where sigma-min denotes the smallest singular value. We approximate the smallest singular value by the reciprocal of an estimate of the one-norm of the inverse of T22 - lambda*I. If n = 1, SEP(1) is defined to be abs(T(1,1)).

An approximate error bound for a computed right eigenvector VR(i) is given by

EPS * norm(T) / SEP(i)

Contents

NAME

strsv - solve one of the systems of equations A*x = b, or
A'*x = b

SYNOPSIS

```
SUBROUTINE STRSV(UPLO, TRANSA, DIAG, N, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, LDA, INCY
REAL A(LDA,*), Y(*)

SUBROUTINE STRSV_64(UPLO, TRANSA, DIAG, N, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, LDA, INCY
REAL A(LDA,*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE TRSV(UPLO, [TRANSA], DIAG, [N], A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, LDA, INCY
REAL, DIMENSION(:) :: Y
REAL, DIMENSION(:,:) :: A

SUBROUTINE TRSV_64(UPLO, [TRANSA], DIAG, [N], A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, LDA, INCY
REAL, DIMENSION(:) :: Y
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>

void strsv(char uplo, char transa, char diag, int  n,  float
          *a, int lda, float *y, int incy);

void strsv_64(char uplo, char transa,  char  diag,  long  n,
          float *a, long lda, float *y, long incy);
```

PURPOSE

strsv solves one of the systems of equations A*x = b, or A'*x = b, where b and x are n element vectors and A is an n by n unit, or non-unit, upper or lower triangular matrix.
No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the equations to be solved as follows:

TRANSA = 'N' or 'n'   A*x = b.

TRANSA = 'T' or 't'   A'*x = b.

TRANSA = 'C' or 'c'   A'*x = b.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)

On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit triangular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

N (input)

On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

A (input)

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular matrix and the strictly lower triangular part of A is not referenced. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular matrix and the strictly upper triangular part of A is not referenced. Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced either, but are assumed to be unity. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

Y (input/output)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element right-hand side vector b. On exit, Y is overwritten with the solution vector x.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

strsyl - solve the real Sylvester matrix equation

SYNOPSIS

```
SUBROUTINE STRSYL(TRANA, TRANB, ISGN, M, N, A, LDA, B, LDB, C, LDC,
      SCALE, INFO)

CHARACTER * 1 TRANA, TRANB
INTEGER ISGN, M, N, LDA, LDB, LDC, INFO
REAL SCALE
REAL A(LDA,*), B(LDB,*), C(LDC,*)

SUBROUTINE STRSYL_64(TRANA, TRANB, ISGN, M, N, A, LDA, B, LDB, C,
      LDC, SCALE, INFO)

CHARACTER * 1 TRANA, TRANB
INTEGER*8 ISGN, M, N, LDA, LDB, LDC, INFO
REAL SCALE
REAL A(LDA,*), B(LDB,*), C(LDC,*)
```

F95 INTERFACE

```
SUBROUTINE TRSYL(TRANA, TRANB, ISGN, M, N, A, [LDA], B, [LDB], C,
       [LDC], SCALE, [INFO])

CHARACTER(LEN=1) :: TRANA, TRANB
INTEGER :: ISGN, M, N, LDA, LDB, LDC, INFO
REAL :: SCALE
REAL, DIMENSION(:,:) :: A, B, C

SUBROUTINE TRSYL_64(TRANA, TRANB, ISGN, M, N, A, [LDA], B, [LDB], C,
       [LDC], SCALE, [INFO])
```

```
CHARACTER(LEN=1) :: TRANA, TRANB
INTEGER(8) :: ISGN, M, N, LDA, LDB, LDC, INFO
REAL :: SCALE
REAL, DIMENSION(:,:) :: A, B, C
```

C INTERFACE

```
#include <sunperf.h>

void strsyl(char trana, char tranb, int isgn, int m, int  n,
          float  *a,  int  lda, float *b, int ldb, float *c,
          int ldc, float *scale, int *info);

void strsyl_64(char trana, char tranb, long  isgn,  long  m,
          long  n,  float  *a, long lda, float *b, long ldb,
          float *c, long ldc, float *scale, long *info);
```

PURPOSE

strsyl solves the real Sylvester matrix equation:

```
op(A)*X + X*op(B) = scale*C or
op(A)*X - X*op(B) = scale*C,
```

where op(A) = A or A**T, and A and B are both upper quasi-triangular. A is M-by-M and B is N-by-N; the right hand side C and the solution X are M-by-N; and scale is an output scale factor, set <= 1 to avoid overflow in X.

A and B must be in Schur canonical form (as returned by SHSEQR), that is, block upper triangular with 1-by-1 and 2-by-2 diagonal blocks; each 2-by-2 diagonal block has its diagonal elements equal and its off-diagonal elements of opposite sign.

ARGUMENTS

```
TRANA (input)
          Specifies the option op(A):
          = 'N': op(A) = A    (No transpose)
          = 'T': op(A) = A**T (Transpose)
          = 'C': op(A) = A**H (Conjugate transpose  =  Tran-
          spose)

TRANB (input)
          Specifies the option op(B):
          = 'N': op(B) = B    (No transpose)
          = 'T': op(B) = B**T (Transpose)
```

```
          = 'C': op(B) = B**H (Conjugate transpose  =  Tran-
          spose)

ISGN (input)
          Specifies the sign in the equation:
          = +1: solve op(A)*X + X*op(B) = scale*C
          = -1: solve op(A)*X - X*op(B) = scale*C

M (input) The order of the matrix A, and the number of  rows
          in the matrices X and C. M >= 0.

N (input) The order of the  matrix  B,  and  the  number  of
          columns in the matrices X and C. N >= 0.

A (input) The upper  quasi-triangular  matrix  A,  in  Schur
          canonical form.
LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,M).

B (input) The upper  quasi-triangular  matrix  B,  in  Schur
          canonical form.

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          max(1,N).

C (input/output)
          On entry, the M-by-N right hand side matrix C.  On
          exit, C is overwritten by the solution matrix X.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M)

SCALE (output)
          The scale factor, scale, set <= 1 to  avoid  over-
          flow in X.

INFO (output)
          = 0: successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
          gal value
          = 1: A and B have  common  or  very  close  eigen-
          values;  perturbed  values  were used to solve the
          equation (but the matrices A and B are unchanged).
```

Contents

NAME

```
strti2 - compute the inverse of a real upper or  lower  tri-
angular matrix
```

SYNOPSIS

```
SUBROUTINE STRTI2(UPLO, DIAG, N, A, LDA, INFO)

CHARACTER * 1 UPLO, DIAG
INTEGER N, LDA, INFO
REAL A(LDA,*)

SUBROUTINE STRTI2_64(UPLO, DIAG, N, A, LDA, INFO)

CHARACTER * 1 UPLO, DIAG
INTEGER*8 N, LDA, INFO
REAL A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE TRTI2(UPLO, DIAG, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
INTEGER :: N, LDA, INFO
REAL, DIMENSION(:,:) :: A

SUBROUTINE TRTI2_64(UPLO, DIAG, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
INTEGER(8) :: N, LDA, INFO
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void strti2(char uplo, char diag, int n, float *a, int  lda,
          int *info);

void strti2_64(char uplo, char diag, long n, float *a,  long
          lda, long *info);
```

PURPOSE

strti2 computes the inverse of a real upper or lower triangular matrix.

This is the Level 2 BLAS version of the algorithm.

ARGUMENTS

UPLO (input)
Specifies whether the matrix A is upper or lower triangular. = 'U': Upper triangular
= 'L': Lower triangular

DIAG (input)
Specifies whether or not the matrix A is unit triangular. = 'N': Non-unit triangular
= 'U': Unit triangular

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the triangular matrix A. If UPLO = 'U', the leading n by n upper triangular part of the array A contains the upper triangular matrix, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading n by n lower triangular part of the array A contains the lower triangular matrix, and the strictly upper triangular part of A is not referenced. If DIAG = 'U', the diagonal elements of A are also not referenced and are assumed to be 1.

On exit, the (triangular) inverse of the original matrix, in the same storage format.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value

Contents

NAME

```
strtri - compute the inverse of a real upper or  lower  tri-
angular matrix A
```

SYNOPSIS

```
SUBROUTINE STRTRI(UPLO, DIAG, N, A, LDA, INFO)

CHARACTER * 1 UPLO, DIAG
INTEGER N, LDA, INFO
REAL A(LDA,*)

SUBROUTINE STRTRI_64(UPLO, DIAG, N, A, LDA, INFO)

CHARACTER * 1 UPLO, DIAG
INTEGER*8 N, LDA, INFO
REAL A(LDA,*)
```

F95 INTERFACE

```
SUBROUTINE TRTRI(UPLO, DIAG, N, A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
INTEGER :: N, LDA, INFO
REAL, DIMENSION(:,:) :: A

SUBROUTINE TRTRI_64(UPLO, DIAG, N, A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
INTEGER(8) :: N, LDA, INFO
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void strtri(char uplo, char diag, int n, float *a, int  lda,
          int *info);

void strtri_64(char uplo, char diag, long n, float *a,  long
          lda, long *info);
```

PURPOSE

strtri computes the inverse of a real upper or lower triangular matrix A.

This is the Level 3 BLAS version of the algorithm.

ARGUMENTS

UPLO (input)
= 'U': A is upper triangular;
= 'L': A is lower triangular.

DIAG (input)
= 'N': A is non-unit triangular;
= 'U': A is unit triangular.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the triangular matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of the array A contains the upper triangular matrix, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of the array A contains the lower triangular matrix, and the strictly upper triangular part of A is not referenced. If DIAG = 'U', the diagonal elements of A are also not referenced and are assumed to be 1. On exit, the (triangular) inverse of the original matrix, in the same storage format.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-

gal value
> 0: if INFO = i, A(i,i) is exactly zero. The triangular matrix is singular and its inverse can not be computed.

Contents

NAME

```
strtrs - solve a triangular system of the form   A * X  =  B
or A**T * X = B,
```

SYNOPSIS

```
SUBROUTINE STRTRS(UPLO, TRANSA, DIAG, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER N, NRHS, LDA, LDB, INFO
REAL A(LDA,*), B(LDB,*)

SUBROUTINE STRTRS_64(UPLO, TRANSA, DIAG, N, NRHS, A, LDA, B, LDB,
      INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
INTEGER*8 N, NRHS, LDA, LDB, INFO
REAL A(LDA,*), B(LDB,*)
```

F95 INTERFACE

```
SUBROUTINE TRTRS(UPLO, [TRANSA], DIAG, N, NRHS, A, [LDA], B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER :: N, NRHS, LDA, LDB, INFO
REAL, DIMENSION(:,:) :: A, B

SUBROUTINE TRTRS_64(UPLO, [TRANSA], DIAG, N, NRHS, A, [LDA], B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
REAL, DIMENSION(:,:) :: A, B
```

C INTERFACE

```
#include <sunperf.h>

void strtrs(char uplo, char transa, char diag,  int  n,  int
          nrhs,  float  *a,  int lda, float *b, int ldb, int
          *info);

void strtrs_64(char uplo, char transa, char  diag,  long  n,
          long nrhs, float *a, long lda, float *b, long ldb,
          long *info);
```

PURPOSE

strtrs solves a triangular system of the form where A is a triangular matrix of order N, and B is an N-by-NRHS matrix. A check is made to verify that A is non-singular.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B  (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose = Tran-
          spose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

A (input) The triangular matrix A.  If UPLO = 'U', the lead-
          ing  N-by-N  upper  triangular part of the array A
          contains the  upper  triangular  matrix,  and  the
```

strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of the array A contains the lower triangular matrix, and the strictly upper triangular part of A is not referenced. If DIAG = 'U', the diagonal elements of A are also not referenced and are assumed to be 1.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the right hand side matrix B. On exit, if INFO = 0, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the i-th diagonal element of A is zero, indicating that the matrix is singular and the solutions X have not been computed.

Contents

NAME

```
stzrqf - routine is deprecated and has been replaced by rou-
tine STZRZF
```

SYNOPSIS

```
SUBROUTINE STZRQF(M, N, A, LDA, TAU, INFO)

INTEGER M, N, LDA, INFO
REAL A(LDA,*), TAU(*)

SUBROUTINE STZRQF_64(M, N, A, LDA, TAU, INFO)

INTEGER*8 M, N, LDA, INFO
REAL A(LDA,*), TAU(*)
```

F95 INTERFACE

```
SUBROUTINE TZRQF(M, N, A, [LDA], TAU, [INFO])

INTEGER :: M, N, LDA, INFO
REAL, DIMENSION(:) :: TAU
REAL, DIMENSION(:,:) :: A

SUBROUTINE TZRQF_64(M, N, A, [LDA], TAU, [INFO])

INTEGER(8) :: M, N, LDA, INFO
REAL, DIMENSION(:) :: TAU
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void stzrqf(int m, int n, float *a, int lda, float *tau, int
          *info);

void stzrqf_64(long m, long n, float  *a,  long  lda,  float
          *tau, long *info);
```

PURPOSE

stzrqf routine is deprecated and has been replaced by routine STZRZF.

STZRQF reduces the M-by-N (M<=N) real upper trapezoidal matrix A to upper triangular form by means of orthogonal transformations.

The upper trapezoidal matrix A is factored as
   A = (R 0) * Z,

where Z is an N-by-N orthogonal matrix and R is an M-by-M upper triangular matrix.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= M.

A (input/output)
On entry, the leading M-by-N upper trapezoidal part of the array A must contain the matrix to be factorized. On exit, the leading M-by-M upper triangular part of A contains the upper triangular matrix R, and elements M+1 to N of the first M rows of A, with the array TAU, represent the orthogonal matrix Z as a product of M elementary reflectors.

LDA (input)
The leading dimension of the array A.   LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-

gal value

FURTHER DETAILS

The factorization is obtained by Householder's method. The kth transformation matrix, Z(k), which is used to introduce zeros into the (m - k + 1)th row of A, is given in the form

```
   Z( k ) = ( I     0   ),
            ( 0  T( k ) )
```

where

```
   T( k ) = I - tau*u( k )*u( k )',   u( k ) = (   1    ),
                                               (   0    )
                                               ( z( k ) )
```

tau is a scalar and z(k) is an (n - m) element vector. tau and z(k) are chosen to annihilate the elements of the kth row of X.

The scalar tau is returned in the kth element of TAU and the vector u(k) in the kth row of A, such that the elements of z(k) are in a(k, m + 1), ..., a(k, n). The elements of R are returned in the upper triangular part of A.

Z is given by

```
   Z =  Z( 1 ) * Z( 2 ) * ... * Z( m ).
```

Contents

NAME

```
stzrzf - reduce the M-by-N ( M<=N ) real  upper  trapezoidal
matrix  A  to  upper  triangular form by means of orthogonal
transformations
```

SYNOPSIS

```
SUBROUTINE STZRZF(M, N, A, LDA, TAU, WORK, LWORK, INFO)

INTEGER M, N, LDA, LWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)

SUBROUTINE STZRZF_64(M, N, A, LDA, TAU, WORK, LWORK, INFO)

INTEGER*8 M, N, LDA, LWORK, INFO
REAL A(LDA,*), TAU(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE TZRZF([M], [N], A, [LDA], TAU, [WORK], [LWORK], [INFO])

INTEGER :: M, N, LDA, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A

SUBROUTINE TZRZF_64([M], [N], A, [LDA], TAU, [WORK], [LWORK], [INFO])

INTEGER(8) :: M, N, LDA, LWORK, INFO
REAL, DIMENSION(:) :: TAU, WORK
REAL, DIMENSION(:,:) :: A
```

C INTERFACE

```
#include <sunperf.h>
```

```
void stzrzf(int m, int n, float *a, int lda, float *tau, int
          *info);

void stzrzf_64(long m, long n, float  *a,  long  lda,  float
          *tau, long *info);
```

PURPOSE

stzrzf reduces the M-by-N (M<=N) real upper trapezoidal matrix A to upper triangular form by means of orthogonal transformations.

The upper trapezoidal matrix A is factored as

```
   A = ( R  0 ) * Z,
```

where Z is an N-by-N orthogonal matrix and R is an M-by-M upper triangular matrix.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the leading M-by-N upper trapezoidal part of the array A must contain the matrix to be factorized. On exit, the leading M-by-M upper triangular part of A contains the upper triangular matrix R, and elements M+1 to N of the first M rows of A, with the array TAU, represent the orthogonal matrix Z as a product of M elementary reflectors.

LDA (input)
The leading dimension of the array A.   LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)

The dimension of the array WORK. LWORK >= max(1,M). For optimum performance LWORK >= M*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

Based on contributions by
A. Petitet, Computer Science Dept., Univ. of Tenn., Knoxville, USA

The factorization is obtained by Householder's method. The kth transformation matrix, Z(k), which is used to introduce zeros into the (m - k + 1)th row of A, is given in the form

```
   Z( k ) = ( I     0   ),
            ( 0  T( k ) )
```

where

```
   T( k ) = I - tau*u( k )*u( k )',   u( k ) = (   1    ),
                                               (   0    )
                                               ( z( k ) )
```

tau is a scalar and z(k) is an (n - m) element vector. tau and z(k) are chosen to annihilate the elements of the kth row of X.

The scalar tau is returned in the kth element of TAU and the vector u(k) in the kth row of A, such that the elements of z(k) are in a(k, m + 1), ..., a(k, n). The elements of R are returned in the upper triangular part of A.

Z is given by

```
   Z =  Z( 1 ) * Z( 2 ) * ... * Z( m ).
```

Contents

NAME

```
     sunperf_version - gets library information
```

F95 INTERFACE

C INTERFACE

```
     #include <sunperf.h>

     The C version of sunperf_version also return  a  pointer  to
               the version string.

     char  *sunperf_version(int   *version,   int   *patch,   int
               *update);

     char *sunperf_version_64(long *version,  long  *patch,  long
               *update);
```

ARGUMENTS

```
     VERSION (output)
               Version number of library

     PATCH (output)
               Patch number of library

               Update number of library
```

Contents

NAME

svbrmm - variable block sparse row format matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE SVBRMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, KB, DESCRA(5), LDB, LDC, LWORK
 INTEGER    INDX(*), BINDX(*), RPNTR(MB+1), CPNTR(KB+1),
*           BPNTRB(MB), BPNTRE(MB)
 REAL       ALPHA, BETA
 REAL       VAL(*), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE SVBRMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), LDB, LDC, LWORK
 INTEGER*8  INDX(*), BINDX(*), RPNTR(MB+1), CPNTR(KB+1),
*           BPNTRB(MB), BPNTRE(MB)
 REAL       ALPHA, BETA
 REAL       VAL(*), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE VBRMM(TRANSA, MB, [N], KB, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, KB
 INTEGER, DIMENSION(:) ::  DESCRA, INDX, BINDX
 INTEGER, DIMENSION(:) ::  RPNTR, CPNTR, BPNTRB, BPNTRE
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL
```

```
 REAL, DIMENSION(:, :) ::  B, C

 SUBROUTINE VBRMM_64(TRANSA, MB, [N], KB, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, KB
 INTEGER*8, DIMENSION(:) ::  DESCRA, INDX, BINDX
 INTEGER*8, DIMENSION(:) ::  RPNTR, CPNTR, BPNTRB, BPNTRE
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL
 REAL, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C
where ALPHA and BETA are scalar, C and B are  matrices,
A is a matrix represented in variable block sparse row  format
and op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if the matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
```

```
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
                0 : non-unit
                1 : unit
              DESCRA(4) Array base  (NOT IMPLEMENTED)
                0 : C/C++ compatible
                1 : Fortran compatible
              DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                0 : unknown
                1 : no repeated indices

VAL()         scalar array of length NNZ consisting of the block entries
              of A where each block entry is a dense rectangular matrix
              stored column by column.
              NNZ is the total number of point entries in all nonzero
              block  entries of a matrix A.
INDX()        integer array of length BNNZ+1 where BNNZ is the number of
              block entries of a matrix A such that the I-th element of
              INDX[] points to the location in VAL of the (1,1) element
              of the I-th block entry.

BINDX()       integer array of length BNNZ consisting of the block
              column indices of the block entries of A where BNNZ is
              the number block entries of a matrix A.

RPNTR()       integer array of length MB+1 such that RPNTR(I)-RPNTR(1)+1
              is the row index of the first point row in the I-th block
              row.
              RPNTR(MB+1) is set to M+RPNTR(1) where M is the number of
              rows in matrix A.
              Thus, the number of point rows in the I-th block row is
              RPNTR(I+1)-RPNTR(I).

CPNTR()       integer array of length KB+1 such that CPNTR(J)-CPNTR(1)+1
              is the column index of the first point column in the J-th
              block column. CPNTR(KB+1) is set to K+CPNTR(1) where K is
              the number of columns in matrix A.
              Thus, the number of point columns in the J-th block column
              is CPNTR(J+1)-CPNTR(J).

BPNTRB()      integer array of length MB such that BPNTRB(I)-BPNTRB(1)+1
              points to location in BINDX of the first block entry of
              the I-th block row of A.

BPNTRE()      integer array of length MB such that BPNTRE(I)-BPNTRB(1)
              points to location in BINDX of the last block entry of
              the I-th block row of A.
```

B()	rectangular array with first dimension LDB.
LDB	leading dimension of B
BETA	Scalar parameter
C()	rectangular array with first dimension LDC.
LDC	leading dimension of C
WORK()	scratch array of length LWORK. WORK is not referenced in the current version.
LWORK	length of WORK array. LWORK is not referenced in the current version.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. For a general matrix (*DESCRA*(1)=0), array CPNTR can be different from RPNTR. For all other matrix types, RPNTR must equal CPNTR and a single array can be passed for both arguments.

2. It is known that there exists another representation of the variable block sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of six array instead of the seven used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block row in the array BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of variable block sparse row format the following calling sequence should be used

```
  SUBROUTINE SVBRMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *           VAL, INDX, BINDX, RPNTR, CPNTR, IA, IA(2),
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

svbrsm - variable block sparse row format triangular solve

SYNOPSIS

```
 SUBROUTINE SVBRSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), LDB, LDC, LWORK
 INTEGER    INDX(*), BINDX(*), RPNTR(MB+1), CPNTR(MB+1),
*           BPNTRB(MB), BPNTRE(MB)
 REAL       ALPHA, BETA
 REAL       DV(*), VAL(*), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE SVBRSM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK )
 INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), LDB, LDC, LWORK
 INTEGER*8  INDX(*), BINDX(*), RPNTR(MB+1), CPNTR(MB+1),
*           BPNTRB(MB), BPNTRE(MB)
 REAL       ALPHA, BETA
 REAL       DV(*), VAL(*), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE VBRSM(TRANSA, MB, [N], UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, UNITD
 INTEGER, DIMENSION(:) ::  DESCRA, INDX, BINDX
 INTEGER, DIMENSION(:) ::  RPNTR, CPNTR, BPNTRB, BPNTRE
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL, DV
 REAL, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE VBRSM_64(TRANSA, MB, [N], UNITD, DV, ALPHA, DESCRA,
*            VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*            B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, UNITD
 INTEGER*8, DIMENSION(:) ::  DESCRA, INDX, BINDX
 INTEGER*8, DIMENSION(:) ::  RPNTR, CPNTR, BPNTRB, BPNTRE
 REAL    ALPHA, BETA
 REAL, DIMENSION(:) :: VAL, DV
 REAL, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C
where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in variable block sparse row
format and  op( A )  is one  of

 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row block scaling)
                3 : Scale on right (column block scaling)

DV()          Array containing the block entries of the block
              diagonal matrix D.  The size of the J-th block is
              RPNTR(J+1)-RPNTR(J) and each block contains matrix
              entries stored column-major.  The total length of
              array DV is given by the formula:

              sum over J from 1 to MB:
```

```
                 ((RPNTR(J+1)-RPNTR(J))*(RPNTR(J+1)-RPNTR(J)))

ALPHA           Scalar parameter

DESCRA()        Descriptor argument.  Five element integer array

                DESCRA(1) matrix structure
                  0 : general
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))
                Note: For the routine, DESCRA(1)=3 is only supported.
                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-identity blocks on the main diagonal
                  1 : identity diagonal block
                  2 : diagonal blocks are dense matrices
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

VAL()           scalar array of length NNZ consisting of the block entries
                of A where each block entry is a dense rectangular matrix
                stored column by column.
                NNZ is the total number of point entries in all nonzero
                block  entries of a matrix A.

INDX()          integer array of length BNNZ+1 where BNNZ is the number
                block entries of a matrix A such that the I-th element of
                INDX[] points to the location in VAL of the (1,1) element
                of the I-th block entry.

BINDX()         integer array of length BNNZ consisting of the block
                column indices of the block entries of A where BNNZ is
                the number block entries of a matrix A.   Block column
                indices MUST be sorted in increasing order for each block
                row.

RPNTR()         integer array of length MB+1 such that RPNTR(I)-RPNTR(1)+1
                is the row index of the first point row in the I-th block
                row.
                RPNTR(MB+1) is set to M+RPNTR(1) where M is the number
```

of rows in square triangular matrix A.
Thus, the number of point rows in the I-th block row is RPNTR(I+1)-RPNTR(I).

NOTE: For the current version CPNTR must equal RPNTR and a single array can be passed for both arguments

CPNTR() integer array of length MB+1 such that CPNTR(J)-CPNTR(1)+1 is the column index of the first point column in the J-th block column. CPNTR(MB+1) is set to M+CPNTR(1).
Thus, the number of point columns in the J-th block column is CPNTR(J+1)-CPNTR(J).

NOTE: For the current version CPNTR must equal RPNTR and a single array can be passed for both arguments

BPNTRB() integer array of length MB such that BPNTRB(I)-BPNTRB(1)+1 points to location in BINDX of the first block entry of the I-th block row of A.

BPNTRE() integer array of length MB such that BPNTRE(I)-BPNTRB(1) points to location in BINDX of the last block entry of the I-th block row of A.

B() rectangular array with first dimension LDB.

LDB leading dimension of B

BETA Scalar parameter

C() rectangular array with first dimension LDC.

LDC leading dimension of C

WORK() scratch array of length LWORK.
On exit, if LWORK= -1, WORK(1) returns the optimum size of LWORK.

LWORK length of WORK array. LWORK should be at least M = RPNTR(MB+1)-RPNTR(1).

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If *DESCRA*(3)=0,the lower or upper triangular part of each diagonal block is used by the routine depending on *DESCRA*(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might not be referenced in the VBR representation of a sparse matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense matrices and the LU factorization with partial pivoting is used by the routine. *WORK*(1)=0 on return if the factorization for all diagonal blocks has been completed successfully, otherwise *WORK*(1) = -i where i is the block number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3.

6. It is known that there exists another representation of the variable block sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of six array instead of the seven used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block row in the array BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of variable block sparse row

format the following calling sequence should be used

```
  SUBROUTINE SVBRSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
 *            VAL, INDX, BINDX, RPNTR, CPNTR, IA, IA(2),
 *            B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

```
swiener - perform Wiener deconvolution of two signals
```

SYNOPSIS

```
SUBROUTINE SWIENER(N_POINTS, ACOR, XCOR, FLTR, EROP, ISW, IERR)

INTEGER N_POINTS, ISW, IERR
REAL ACOR(*), XCOR(*), FLTR(*), EROP(*)

SUBROUTINE SWIENER_64(N_POINTS, ACOR, XCOR, FLTR, EROP, ISW, IERR)

INTEGER*8 N_POINTS, ISW, IERR
REAL ACOR(*), XCOR(*), FLTR(*), EROP(*)
```

F95 INTERFACE

```
SUBROUTINE WIENER(N_POINTS, ACOR, XCOR, FLTR, EROP, ISW, IERR)

INTEGER :: N_POINTS, ISW, IERR
REAL, DIMENSION(:) :: ACOR, XCOR, FLTR, EROP

SUBROUTINE WIENER_64(N_POINTS, ACOR, XCOR, FLTR, EROP, ISW, IERR)

INTEGER(8) :: N_POINTS, ISW, IERR
REAL, DIMENSION(:) :: ACOR, XCOR, FLTR, EROP
```

C INTERFACE

```
#include <sunperf.h>

void swiener(int n_points, float *acor, float  *xcor,  float
          *fltr, float *erop, int *isw, int *ierr);

void swiener_64(long n_points,  float  *acor,  float  *xcor,
```

```
            float *fltr, float *erop, long *isw, long *ierr);
```

PURPOSE

swiener performs Wiener deconvolution of two signals.

ARGUMENTS

N_POINTS (input)
On entry, the number of points in the input correlations. Unchanged on exit.

ACOR (input)
On entry, autocorrelation coefficients. Unchanged on exit.

XCOR (input)
On entry, cross-correlation coefficients. Unchanged on exit.

FLTR (output)
On exit, filter coefficients.

EROP (output)
On exit, the prediction error.

ISW (input)
On entry, if ISW .EQ. 0 then perform spiking deconvolution, otherwise perform general deconvolution. Unchanged on exit.

IERR (output)
On exit, the deconvolution was successful iff IERR .EQ. 0, otherwise there was an error.

Contents

NAME

```
use_threads - set the upper bound on the number  of  threads
that the calling thread wants used
```

SYNOPSIS

```
SUBROUTINE USE_THREADS(NTHREADS)

INTEGER NTHREADS

SUBROUTINE USE_THREADS_64(NTHREADS)

INTEGER*8 NTHREADS
```

F95 INTERFACE

```
SUBROUTINE USE_THREADS(NTHREADS)

INTEGER :: NTHREADS

SUBROUTINE USE_THREADS_64(NTHREADS)

INTEGER(8) :: NTHREADS
```

C INTERFACE

```
#include <sunperf.h>

void use_threads(int nthreads);

void use_threads_64(long nthreads);
```

PURPOSE

use_threads THREADS sets an upper bound on the number of threads that the calling thread wants used. Subsequent calls to this routine result in replacement of the previous Use number for the calling thread. This counts all threads working on the callers behalf, so if it passes 2 for NTHREADS and then calls some subroutine, there will be at most 1 additional thread started to do the computation. There is no restriction that the sum of all NTHREADS from USE_THREADS calls may not exceed the number of CPUs in a system.

Contents

NAME

```
using_threads - returns the current Use number  set  by  the
USE_THREADS subroutine
```

SYNOPSIS

```
INTEGER FUNCTION USING_THREADS()

INTEGER*8 FUNCTION USING_THREADS_64()
```

F95 INTERFACE

```
INTEGER FUNCTION USING_THREADS()

INTEGER(8) FUNCTION USING_THREADS_64()
```

C INTERFACE

```
#include <sunperf.h>

int using_threads();

long using_threads_64();
```

PURPOSE

```
using_threads THREADS will return  the  current  Use  number
from the USE_THREADS subroutine for the calling thread.
```

Contents

NAME

vcfftb - compute a periodic sequence from its Fourier coefficients. The VCFFT operations are normalized, so a call of VCFFTF followed by a call of VCFFTB will return the original sequence.

SYNOPSIS

```
SUBROUTINE VCFFTB(M, N, X, XT, MDIMX, ROWCOL, WSAVE)

CHARACTER * 1 ROWCOL
COMPLEX X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
INTEGER M, N, MDIMX

SUBROUTINE VCFFTB_64(M, N, X, XT, MDIMX, ROWCOL, WSAVE)

CHARACTER * 1 ROWCOL
COMPLEX X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
INTEGER*8 M, N, MDIMX
```

F95 INTERFACE

```
SUBROUTINE FFTB([M], [N], X, XT, [MDIMX], ROWCOL, WSAVE)

CHARACTER(LEN=1) :: ROWCOL
COMPLEX, DIMENSION(:) :: WSAVE
COMPLEX, DIMENSION(:,:) :: X, XT
INTEGER :: M, N, MDIMX

SUBROUTINE FFTB_64([M], [N], X, XT, [MDIMX], ROWCOL, WSAVE)

CHARACTER(LEN=1) :: ROWCOL
COMPLEX, DIMENSION(:) :: WSAVE
COMPLEX, DIMENSION(:,:) :: X, XT
INTEGER(8) :: M, N, MDIMX
```

C INTERFACE

```
#include <sunperf.h>

void vcfftb(int m, int  n,  complex  *x,  complex  *xt,  int
          mdimx, char rowcol, complex *wsave);

void vcfftb_64(long m, long n, complex *x, complex *xt, long
          mdimx, char rowcol, complex *wsave);
```

ARGUMENTS

M (input) If ROWCOL = 'R' or 'r', M is the number of sequences to be transformed. Otherwise, M is the length of the sequences to be transformed. M >= 0.

N (input) If ROWCOL = 'R' or 'r', N is the length of the sequences to be transformed. Otherwise, N is the number of sequences to be transformed. N >= 0.

X (input) On entry, if ROWCOL = 'R' or 'r' X(MDIMX,N) is an array whose first M rows contain the sequences to be transformed. Otherwise, X(MDIMX,N) contains data sequences of length M stored in N columns of X.

XT (input)
A work array. The size of this workspace depends on the number of threads that are used to execute this routine. There are various functions that can be used to determine the number of threads available (get_env, available_threads, etc). The appropriate amount, which is (number of threads * length of data sequences), can then be dynamically allocated for XT from the driver routine. If XT can only be allocated statically, then the size of XT should be (length of data sequences * number of sequences).

MDIMX (input)
Leading dimension of the arrays X and XT as specified in a dimension or type statement. MDIMX >= M.

ROWCOL (input)
Indicates whether to transform rows ('R' or 'r') or columns ('C' or 'c').

WSAVE (input/output)

On entry, an array of dimension (L2+15) or greater, where L2 = 2*M if ROWCOL = ('R' or 'r'). Otherwise, L2 = 2*N. WSAVE is initialized by VCFFTI.

Contents

NAME

```
vcfftf - compute the  Fourier  coefficients  of  a  periodic
sequence.  The VCFFT operations are normalized, so a call of
VCFFTF followed by a call of VCFFTB will return the original
sequence.
```

SYNOPSIS

```
SUBROUTINE VCFFTF(M, N, X, XT, MDIMX, ROWCOL, WSAVE)

CHARACTER * 1 ROWCOL
COMPLEX X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
INTEGER M, N, MDIMX

SUBROUTINE VCFFTF_64(M, N, X, XT, MDIMX, ROWCOL, WSAVE)

CHARACTER * 1 ROWCOL
COMPLEX X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
INTEGER*8 M, N, MDIMX
```

F95 INTERFACE

```
SUBROUTINE FFTF([M], [N], X, XT, [MDIMX], ROWCOL, WSAVE)

CHARACTER(LEN=1) :: ROWCOL
COMPLEX, DIMENSION(:) :: WSAVE
COMPLEX, DIMENSION(:,:) :: X, XT
INTEGER :: M, N, MDIMX

SUBROUTINE FFTF_64([M], [N], X, XT, [MDIMX], ROWCOL, WSAVE)

CHARACTER(LEN=1) :: ROWCOL
COMPLEX, DIMENSION(:) :: WSAVE
COMPLEX, DIMENSION(:,:) :: X, XT
INTEGER(8) :: M, N, MDIMX
```

C INTERFACE

```
#include <sunperf.h>

void vcfftf(int m, int  n,  complex  *x,  complex  *xt,  int
          mdimx, char rowcol, complex *wsave);

void vcfftf_64(long m, long n, complex *x, complex *xt, long
          mdimx, char rowcol, complex *wsave);
```

ARGUMENTS

M (input) If ROWCOL = 'R' or 'r', M is the number of sequences to be transformed. Otherwise, M is the length of the sequences to be transformed. M >= 0.

N (input) If ROWCOL = 'R' or 'r', N is the length of the sequences to be transformed. Otherwise, N is the number of sequences to be transformed. N >= 0.

X (input) On entry, if ROWCOL = 'R' or 'r' X(MDIMX,N) is an array whose first M rows contain the sequences to be transformed. Otherwise, X(MDIMX,N) contains data sequences of length M stored in N columns of X.

XT (input)
A work array. The size of this workspace depends on the number of threads that are used to execute this routine. There are various functions that can be used to determine the number of threads available (get_env, available_threads, etc). The appropriate amount, which is (number of threads * length of data sequences), can then be dynamically allocated for XT from the driver routine. If XT can only be allocated statically, then the size of XT should be (length of data sequences * number of sequences).

MDIMX (input)
Leading dimension of the arrays X. MDIMX >= M.

ROWCOL (input)
Indicates whether data sequences in X are stored row-wise ('R' or 'r') or column-wise ('C' or 'c').

WSAVE (input/output)
On entry, an array of dimension (L2+15) or

greater, where L2 = 2*M if ROWCOL = ('R' or 'r'). Otherwise, L2 = 2*N. WSAVE is initialized by VCFFTI.

Contents

NAME

```
vcffti - initialize the array WSAVE, which is used  in  both
VCFFTF and VCFFTB.
```

SYNOPSIS

```
SUBROUTINE VCFFTI(N, WSAVE)

COMPLEX WSAVE(*)
INTEGER N

SUBROUTINE VCFFTI_64(N, WSAVE)

COMPLEX WSAVE(*)
INTEGER*8 N
```

F95 INTERFACE

```
SUBROUTINE VFFTI(N, WSAVE)

COMPLEX, DIMENSION(:) :: WSAVE
INTEGER :: N

SUBROUTINE VFFTI_64(N, WSAVE)

COMPLEX, DIMENSION(:) :: WSAVE
INTEGER(8) :: N
```

C INTERFACE

```
#include <sunperf.h>

void vcffti(int n, complex *wsave);

void vcffti_64(long n, complex *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. N >= 0.

WSAVE (input)
On entry, an array of dimension (2*N + 15) or greater. VCFFTI needs to be called only once to initialize WSAVE before calling VCFFTF and/or VCFFTB if N and WSAVE remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
vcosqb - synthesize a Fourier sequence from its  representa-
tion in terms of a cosine series with odd wave numbers.  The
VCOSQ operations are normalized, so a call  of  VCOSQF  fol-
lowed by a call of VCOSQB will return the original sequence.
```

SYNOPSIS

```
SUBROUTINE VCOSQB(M, N, X, XT, MDIMX, WSAVE)

INTEGER M, N, MDIMX
REAL X(MDIMX,*), XT(MDIMX,*), WSAVE(*)

SUBROUTINE VCOSQB_64(M, N, X, XT, MDIMX, WSAVE)

INTEGER*8 M, N, MDIMX
REAL X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE COSQB([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER :: M, N, MDIMX
REAL, DIMENSION(:) :: WSAVE
REAL, DIMENSION(:,:) :: X, XT

SUBROUTINE COSQB_64([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER(8) :: M, N, MDIMX
REAL, DIMENSION(:) :: WSAVE
REAL, DIMENSION(:,:) :: X, XT
```

C INTERFACE

```
#include <sunperf.h>
```

```
void vcosqb(int m, int n, float *x, float  *xt,  int  mdimx,
          float *wsave);

void vcosqb_64(long m, long n, float  *x,  float  *xt,  long
          mdimx, float *wsave);
```

ARGUMENTS

M (input) The number of sequences to be transformed. M >= 0.

N (input) Length of the sequence to be transformed. These subroutines are most efficient when N is a product of small primes. N >= 0.

X (input/output)
On entry, the rows contain the sequences to be transformed. On exit, the quarter-wave cosine synthesis of the input.

XT (input)
A work array.

MDIMX (input)
Leading dimension of the arrays X and XT as specified in a dimension or type statement. MDIMX >= M.

WSAVE (input)
On entry, an array of dimension (2 * N + 15) or greater initialized by VCOSQI.

Contents

NAME

vcosqf - compute the Fourier coefficients in a cosine series representation with only odd wave numbers. The VCOSQ operations are normalized, so a call of VCOSQF followed by a call of VCOSQB will return the original sequence.

SYNOPSIS

```
SUBROUTINE VCOSQF(M, N, X, XT, MDIMX, WSAVE)

INTEGER M, N, MDIMX
REAL X(MDIMX,*), XT(MDIMX,*), WSAVE(*)

SUBROUTINE VCOSQF_64(M, N, X, XT, MDIMX, WSAVE)

INTEGER*8 M, N, MDIMX
REAL X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE COSQF([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER :: M, N, MDIMX
REAL, DIMENSION(:) :: WSAVE
REAL, DIMENSION(:,:) :: X, XT

SUBROUTINE COSQF_64([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER(8) :: M, N, MDIMX
REAL, DIMENSION(:) :: WSAVE
REAL, DIMENSION(:,:) :: X, XT
```

C INTERFACE

```
#include <sunperf.h>
```

```
void vcosqf(int m, int n, float *x, float  *xt,  int  mdimx,
          float *wsave);

void vcosqf_64(long m, long n, float  *x,  float  *xt,  long
          mdimx, float *wsave);
```

ARGUMENTS

M (input)
The number of sequences to be transformed. M >= 0.

N (input) Length of the sequence to be transformed. These subroutines are most efficient when N is a product of small primes. N >= 0.

X (input/output)
On entry, an array of length N containing the sequence to be transformed. For VCOSQF, a real two-dimensional array with dimensions of (MDIMX x N) whose rows contain the sequences to be transformed. On exit, the quarter-wave cosine transform of the input.

XT (input)
A real two-dimensional work array with dimensions of (MDIMX x N).

MDIMX (input)
Leading dimension of the arrays X and XT as specified in a dimension or type statement. MDIMX >= M.

WSAVE (input)
On entry, an array of dimension (2 * N + 15) or greater initialized by VCOSTI.

Contents

NAME

```
vcosqi - initialize the array WSAVE, which is used  in  both
VCOSQF and VCOSQB.
```

SYNOPSIS

```
SUBROUTINE VCOSQI(N, WSAVE)

INTEGER N
REAL WSAVE(*)

SUBROUTINE VCOSQI_64(N, WSAVE)

INTEGER*8 N
REAL WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE VCOSQI(N, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: WSAVE

SUBROUTINE VCOSQI_64(N, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void vcosqi(int n, float *wsave);

void vcosqi_64(long n, float *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. The method is most efficient when N is a product of small primes.

WSAVE (input)
On entry, an array of dimension (2 * N + 15) or greater. VCOSQI needs to be called only once to initialize WSAVE before calling VCOSQF and/or VCOSQB if N and WSAVE remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
vcost - compute the discrete Fourier cosine transform of  an
even sequence.  The VCOST transform is normalized, so a call
of VCOST followed by a call of VCOST will return the  origi-
nal sequence.
```

SYNOPSIS

```
SUBROUTINE VCOST(M, N, X, XT, MDIMX, WSAVE)

INTEGER M, N, MDIMX
REAL X(MDIMX,*), XT(MDIMX,*), WSAVE(*)

SUBROUTINE VCOST_64(M, N, X, XT, MDIMX, WSAVE)

INTEGER*8 M, N, MDIMX
REAL X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE COST([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER :: M, N, MDIMX
REAL, DIMENSION(:) :: WSAVE
REAL, DIMENSION(:,:) :: X, XT

SUBROUTINE COST_64([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER(8) :: M, N, MDIMX
REAL, DIMENSION(:) :: WSAVE
REAL, DIMENSION(:,:) :: X, XT
```

C INTERFACE

```
#include <sunperf.h>
```

```
void vcost(int m, int n, float *x,  float  *xt,  int  mdimx,
          float *wsave);

void vcost_64(long m, long n,  float  *x,  float  *xt,  long
          mdimx, float *wsave);
```

ARGUMENTS

M (input)
The number of sequences to be transformed. M >= 0.

N (input) Length of the sequence to be transformed. These subroutines are most efficient when N - 1 is a product of small primes. N >= 2.

X (input/output)
On entry, an array of length N containing the sequence to be transformed. For VCOST, a real two- dimensional array with dimensions of (MDIMX x (N+1)) whose rows contain the sequences to be transformed. On exit, the cosine transform of the input.

XT (input)
A real two-dimensional work array with dimensions of (MDIMX x (N-1)).

MDIMX (input)
Leading dimension of the arrays X and XT as specified in a dimension or type statement. MDIMX >= M.

WSAVE (input)
On entry, an array of dimension (2 * N + 15) or greater initialized by VCOSTI.

Contents

NAME

```
vcosti - initialize the array WSAVE, which is used in VCOST.
```

SYNOPSIS

```
SUBROUTINE VCOSTI(N, WSAVE)

INTEGER N
REAL WSAVE(*)

SUBROUTINE VCOSTI_64(N, WSAVE)

INTEGER*8 N
REAL WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE VCOSTI(N, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: WSAVE

SUBROUTINE VCOSTI_64(N, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void vcosti(int n, float *wsave);

void vcosti_64(long n, float *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. The method is most efficient when N - 1 is a product of small primes. N >= 2.

WSAVE (input)
On entry, an array of dimension (2 * N + 15) or greater. VCOSTI is called once to initialize WSAVE before calling VCOST and need not be called again between calls to VCOST if N and WSAVE remain unchanged. Thus, subsequent transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
vdcosqb - synthesize a Fourier sequence from its representa-
tion in terms of a cosine series with odd wave numbers.  The
VCOSQ operations are normalized, so a call  of  VCOSQF  fol-
lowed by a call of VCOSQB will return the original sequence.
```

SYNOPSIS

```
SUBROUTINE VDCOSQB(M, N, X, XT, MDIMX, WSAVE)

INTEGER M, N, MDIMX
DOUBLE PRECISION X(MDIMX,*), XT(MDIMX,*), WSAVE(*)

SUBROUTINE VDCOSQB_64(M, N, X, XT, MDIMX, WSAVE)

INTEGER*8 M, N, MDIMX
DOUBLE PRECISION X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE COSQB([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER :: M, N, MDIMX
REAL(8), DIMENSION(:) :: WSAVE
REAL(8), DIMENSION(:,:) :: X, XT

SUBROUTINE COSQB_64([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER(8) :: M, N, MDIMX
REAL(8), DIMENSION(:) :: WSAVE
REAL(8), DIMENSION(:,:) :: X, XT
```

C INTERFACE

```
#include <sunperf.h>
```

```
void vdcosqb(int m, int n, double *x, double *xt, int mdimx,
          double *wsave);

void vdcosqb_64(long m, long n, double *x, double *xt,  long
          mdimx, double *wsave);
```

ARGUMENTS

M (input) The number of sequences to be transformed. M >= 0.

N (input) Length of the sequence to be transformed. These subroutines are most efficient when N is a product of small primes. N >= 0.

X (input/output)
On entry, the rows contain the sequences to be transformed. On exit, the quarter-wave cosine synthesis of the input.

XT (input)
A work array.

MDIMX (input)
Leading dimension of the arrays X and XT as specified in a dimension or type statement. MDIMX >= M.

WSAVE (input)
On entry, an array of dimension (2 * N + 15) or greater initialized by VCOSQI.

Contents

NAME

vdcosqf - compute the Fourier coefficients in a cosine series representation with only odd wave numbers. The VCOSQ operations are normalized, so a call of VCOSQF followed by a call of VCOSQB will return the original sequence.

SYNOPSIS

```
SUBROUTINE VDCOSQF(M, N, X, XT, MDIMX, WSAVE)

INTEGER M, N, MDIMX
DOUBLE PRECISION X(MDIMX,*), XT(MDIMX,*), WSAVE(*)

SUBROUTINE VDCOSQF_64(M, N, X, XT, MDIMX, WSAVE)

INTEGER*8 M, N, MDIMX
DOUBLE PRECISION X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE COSQF([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER :: M, N, MDIMX
REAL(8), DIMENSION(:) :: WSAVE
REAL(8), DIMENSION(:,:) :: X, XT

SUBROUTINE COSQF_64([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER(8) :: M, N, MDIMX
REAL(8), DIMENSION(:) :: WSAVE
REAL(8), DIMENSION(:,:) :: X, XT
```

C INTERFACE

```
#include <sunperf.h>
```

```
void vdcosqf(int m, int n, double *x, double *xt, int mdimx,
          double *wsave);

void vdcosqf_64(long m, long n, double *x, double *xt,  long
          mdimx, double *wsave);
```

ARGUMENTS

```
M (input)
          The number of sequences to be transformed.   M  >=
          0.

N (input) Length of the sequence to be  transformed.   These
          subroutines are most efficient when N is a product
          of small primes.  N >= 0.

X (input/output)
          On entry, an array  of  length  N  containing  the
          sequence  to  be  transformed.  For VCOSQF, a real
          two-dimensional array with dimensions of (MDIMX  x
          N)   whose   rows  contain  the  sequences  to  be
          transformed.  On  exit,  the  quarter-wave  cosine
          transform of the input.

XT (input)
          A real two-dimensional work array with  dimensions
          of (MDIMX x N).

MDIMX (input)
          Leading dimension of the arrays X and XT as speci-
          fied  in  a dimension or type statement.  MDIMX >=
          M.

WSAVE (input)
          On entry, an array of dimension (2 * N  +  15)  or
          greater initialized by VCOSQI.
```

Contents

NAME

```
vdcosqi - initialize the array WSAVE, which is used in  both
VCOSQF and VCOSQB.
```

SYNOPSIS

```
SUBROUTINE VDCOSQI(N, WSAVE)

INTEGER N
DOUBLE PRECISION WSAVE(*)

SUBROUTINE VDCOSQI_64(N, WSAVE)

INTEGER*8 N
DOUBLE PRECISION WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE VCOSQI(N, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: WSAVE

SUBROUTINE VCOSQI_64(N, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void vdcosqi(int n, double *wsave);

void vdcosqi_64(long n, double *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. The method is most efficient when N is a product of small primes.

WSAVE (input)
On entry, an array of dimension (2 * N + 15) or greater. VDCOSQI needs to be called only once to initialize WSAVE before calling VDCOSQF and/or VDCOSQB if N and WSAVE remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
vdcost - compute the discrete Fourier cosine transform of an
even sequence.  The VCOST transform is normalized, so a call
of VCOST followed by a call of VCOST will return the  origi-
nal sequence.
```

SYNOPSIS

```
SUBROUTINE VDCOST(M, N, X, XT, MDIMX, WSAVE)

INTEGER M, N, MDIMX
DOUBLE PRECISION X(MDIMX,*), XT(MDIMX,*), WSAVE(*)

SUBROUTINE VDCOST_64(M, N, X, XT, MDIMX, WSAVE)

INTEGER*8 M, N, MDIMX
DOUBLE PRECISION X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE COST([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER :: M, N, MDIMX
REAL(8), DIMENSION(:) :: WSAVE
REAL(8), DIMENSION(:,:) :: X, XT

SUBROUTINE COST_64([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER(8) :: M, N, MDIMX
REAL(8), DIMENSION(:) :: WSAVE
REAL(8), DIMENSION(:,:) :: X, XT
```

C INTERFACE

```
#include <sunperf.h>
```

```
void vdcost(int m, int n, double *x, double *xt, int  mdimx,
          double *wsave);

void vdcost_64(long m, long n, double *x, double  *xt,  long
          mdimx, double *wsave);
```

ARGUMENTS

M (input)
The number of sequences to be transformed.   M >= 0.

N (input) Length of the sequence to be transformed.   These subroutines are most efficient when N - 1 is a product of small primes. N >= 2.

X (input/output)
On entry, an array of length N containing the sequence to be transformed.   For VCOST, a real two- dimensional array with dimensions of (MDIMX x (N+1)) whose rows contain the sequences to be transformed. On exit, the cosine transform of the input.

XT (input)
A real two-dimensional work array with dimensions of (MDIMX x (N-1)).

MDIMX (input)
Leading dimension of the arrays X and XT as specified in a dimension or type statement. MDIMX >= M.

WSAVE (input)
On entry, an array of dimension (2 * N + 15) or greater initialized by VDCOSTI.

Contents

NAME

```
vdcosti - initialize the  array  WSAVE,  which  is  used  in
VCOST.
```

SYNOPSIS

```
SUBROUTINE VDCOSTI(N, WSAVE)

INTEGER N
DOUBLE PRECISION WSAVE(*)

SUBROUTINE VDCOSTI_64(N, WSAVE)

INTEGER*8 N
DOUBLE PRECISION WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE VCOSTI(N, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: WSAVE

SUBROUTINE VCOSTI_64(N, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void vdcosti(int n, double *wsave);

void vdcosti_64(long n, double *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. The method is most efficient when N - 1 is a product of small primes. N >= 2.

WSAVE (input)
On entry, an array of dimension (2 * N + 15) or greater. VDCOSTI is called once to initialize WSAVE before calling VDCOST and need not be called again between calls to VDCOST if N and WSAVE remain unchanged. Thus, subsequent transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
vdfftb - compute a periodic sequence from its Fourier  coef-
ficients.  The VRFFT operations are normalized, so a call of
VRFFTF followed by a call of VRFFTB will return the original
sequence.
```

SYNOPSIS

```
SUBROUTINE VDFFTB(M, N, X, XT, MDIMX, WSAVE)

INTEGER M, N, MDIMX
DOUBLE PRECISION X(MDIMX,*), XT(MDIMX,*), WSAVE(*)

SUBROUTINE VDFFTB_64(M, N, X, XT, MDIMX, WSAVE)

INTEGER*8 M, N, MDIMX
DOUBLE PRECISION X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE FFTB([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER :: M, N, MDIMX
REAL(8), DIMENSION(:) :: WSAVE
REAL(8), DIMENSION(:,:) :: X, XT

SUBROUTINE FFTB_64([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER(8) :: M, N, MDIMX
REAL(8), DIMENSION(:) :: WSAVE
REAL(8), DIMENSION(:,:) :: X, XT
```

C INTERFACE

```
#include <sunperf.h>
```

```
void vdfftb(int m, int n, double *x, double *xt, int  mdimx,
          double *wsave);

void vdfftb_64(long m, long n, double *x, double  *xt,  long
          mdimx, double *wsave);
```

ARGUMENTS

M (input) The number of sequences to be transformed. M >= 0.

N (input) Length of the sequence to be transformed. These subroutines are most efficient when N is a product of small primes. N >= 0.

X (input) On entry, an array of length N containing the sequence to be transformed. For VRFFTF, a real two-dimensional array X(M,N) whose rows contain the sequences to be transformed.

XT (input)
A real two-dimensional work array with dimensions of (MDIMX x N).

MDIMX (input)
Leading dimension of the arrays X and XT as specified in a dimension or type statement. MDIMX >= M.

WSAVE (input)
On entry, an array of dimension (N+15) or greater initialized by VRFFTI.

Contents

NAME

vdfftf - compute the Fourier coefficients of a periodic sequence. The VRFFT operations are normalized, so a call of VRFFTF followed by a call of VRFFTB will return the original sequence.

SYNOPSIS

```
SUBROUTINE VDFFTF(M, N, X, XT, MDIMX, WSAVE)

INTEGER M, N, MDIMX
DOUBLE PRECISION X(MDIMX,*), XT(MDIMX,*), WSAVE(*)

SUBROUTINE VDFFTF_64(M, N, X, XT, MDIMX, WSAVE)

INTEGER*8 M, N, MDIMX
DOUBLE PRECISION X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE FFTF([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER :: M, N, MDIMX
REAL(8), DIMENSION(:) :: WSAVE
REAL(8), DIMENSION(:,:) :: X, XT

SUBROUTINE FFTF_64([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER(8) :: M, N, MDIMX
REAL(8), DIMENSION(:) :: WSAVE
REAL(8), DIMENSION(:,:) :: X, XT
```

C INTERFACE

```
#include <sunperf.h>
```

```
void vdfftf(int m, int n, double *x, double *xt, int  mdimx,
          double *wsave);

void vdfftf_64(long m, long n, double *x, double  *xt,  long
          mdimx, double *wsave);
```

ARGUMENTS

M (input)
The number of sequences to be transformed. M >= 0.

N (input) Length of the sequence to be transformed. These subroutines are most efficient when N is a product of small primes. N >= 0.

X (input) On entry, an array of length N containing the sequence to be transformed. For VRFFTF, a real two- dimensional array X(M,N) whose rows contain the sequences to be transformed.

XT (input)
A real two-dimensional work array with dimensions of (MDIMX x N).

MDIMX (input)
Leading dimension of the arrays X and XT as specified in a dimension or type statement. MDIMX >= M.

WSAVE (input)
On entry, an array of dimension (N+15) or greater initialized by VRFFTI.

Contents

NAME

```
vdffti - initialize the array WSAVE, which is used  in  both
VRFFTF and VRFFTB.
```

SYNOPSIS

```
SUBROUTINE VDFFTI(N, WSAVE)

INTEGER N
DOUBLE PRECISION WSAVE(*)

SUBROUTINE VDFFTI_64(N, WSAVE)

INTEGER*8 N
DOUBLE PRECISION WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE VFFTI(N, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: WSAVE

SUBROUTINE VFFTI_64(N, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void vdffti(int n, double *wsave);

void vdffti_64(long n, double *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. N >= 0.

WSAVE (input)
On entry, an array of dimension (N + 15) or greater. VRFFTI needs to be called only once to initialize WSAVE before calling VRFFTF and/or VRFFTB if N and WSAVE remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

vdsinqb - synthesize a Fourier sequence from its representation in terms of a sine series with odd wave numbers. The VSINQ operations are normalized, so a call of VSINQF followed by a call of VSINQB will return the original sequence.

SYNOPSIS

```
SUBROUTINE VDSINQB(M, N, X, XT, MDIMX, WSAVE)

INTEGER M, N, MDIMX
DOUBLE PRECISION X(MDIMX,*), XT(MDIMX,*), WSAVE(*)

SUBROUTINE VDSINQB_64(M, N, X, XT, MDIMX, WSAVE)

INTEGER*8 M, N, MDIMX
DOUBLE PRECISION X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE SINQB([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER :: M, N, MDIMX
REAL(8), DIMENSION(:) :: WSAVE
REAL(8), DIMENSION(:,:) :: X, XT

SUBROUTINE SINQB_64([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER(8) :: M, N, MDIMX
REAL(8), DIMENSION(:) :: WSAVE
REAL(8), DIMENSION(:,:) :: X, XT
```

C INTERFACE

```
#include <sunperf.h>
```

```
void vdsinqb(int m, int n, double *x, double *xt, int mdimx,
          double *wsave);

void vdsinqb_64(long m, long n, double *x, double *xt,  long
          mdimx, double *wsave);
```

ARGUMENTS

```
M (input)
          The number of sequences to be transformed.   M  >=
          0.

N (input) Length of the sequence to be  transformed.   These
          subroutines are most efficient when N is a product
          of small primes.  N >= 0.

X (input/output)
          On entry,  a  real  two-  dimensional  array  with
          dimensions  of  (MDIMX x N) whose rows contain the
          sequences  to  be  transformed.   On   exit,   the
          quarter-wave sine synthesis of the input.

XT (input)
          A real two-dimensional work array with  dimensions
          of (MDIMX x N).

MDIMX (input)
          Leading dimension of the arrays X and XT as speci-
          fied  in  a dimension or type statement.  MDIMX >=
          M.

WSAVE (input)
          On entry, an array with dimension of at least (2 *
          N  +  15)  for  vector subroutines, initialized by
          VSINQI.
```

Contents

NAME

vdsinqf - compute the Fourier coefficients in a sine series representation with only odd wave numbers. The VSINQ operations are normalized, so a call of VSINQF followed by a call of VSINQB will return the original sequence.

SYNOPSIS

```
SUBROUTINE VDSINQF(M, N, X, XT, MDIMX, WSAVE)

INTEGER M, N, MDIMX
DOUBLE PRECISION X(MDIMX,*), XT(MDIMX,*), WSAVE(*)

SUBROUTINE VDSINQF_64(M, N, X, XT, MDIMX, WSAVE)

INTEGER*8 M, N, MDIMX
DOUBLE PRECISION X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE SINQF([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER :: M, N, MDIMX
REAL(8), DIMENSION(:) :: WSAVE
REAL(8), DIMENSION(:,:) :: X, XT

SUBROUTINE SINQF_64([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER(8) :: M, N, MDIMX
REAL(8), DIMENSION(:) :: WSAVE
REAL(8), DIMENSION(:,:) :: X, XT
```

C INTERFACE

```
#include <sunperf.h>
```

```
void vdsinqf(int m, int n, double *x, double *xt, int mdimx,
          double *wsave);

void vdsinqf_64(long m, long n, double *x, double *xt,  long
          mdimx, double *wsave);
```

ARGUMENTS

M (input)
The number of sequences to be transformed.   M >= 0.

N (input) Length of the sequence to be transformed.   These subroutines are most efficient when N is a product of small primes. N >= 0.

X (input/output)
On entry, an array of length N containing the sequence to be transformed. For VSINQF, a real two-dimensional array with dimensions of (MDIMX x N)   whose   rows contain the sequences to be transformed.   On exit, the quarter-wave   sine transform of the input.

XT (input)
A real two-dimensional work array with dimensions of (MDIMX x N).

MDIMX (input)
Leading dimension of the arrays X and XT as specified in a dimension or type statement. MDIMX >= M.

WSAVE (input)
On entry, an array with dimension of at least (2 * N + 15), initialized by VSINQI.

Contents

NAME

```
vdsinqi - initialize the array WSAVE, which is used in  both
VSINQF and VSINQB.
```

SYNOPSIS

```
SUBROUTINE VDSINQI(N, WSAVE)

INTEGER N
DOUBLE PRECISION WSAVE(*)

SUBROUTINE VDSINQI_64(N, WSAVE)

INTEGER*8 N
DOUBLE PRECISION WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE VSINQI(N, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: WSAVE

SUBROUTINE VSINQI_64(N, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void vdsinqi(int n, double *wsave);

void vdsinqi_64(long n, double *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. The method is most efficient when N is a product of small primes.

WSAVE (input)
On entry, an array with a dimension of at least (2 * N + 15). The same work array can be used for both VSINQF and VSINQB as long as N remains unchanged. Different WSAVE arrays are required for different values of N. This initialization does not have to be repeated between calls to VSINQF or VSINQB as long as N and WSAVE remain unchanged, thus subsequent transforms can be obtained faster than the first.

Contents

NAME

```
vdsint - compute the discrete Fourier sine transform  of  an
odd   sequence.    The  VSINT  transforms  are  unnormalized
inverses of themselves, so  a  call  of  VSINT  followed  by
another  call of VSINT will multiply the input sequence by 2
* (N+1).  The VSINT transforms are normalized, so a call  of
VSINT  followed  by a call of VSINT will return the original
sequence.
```

SYNOPSIS

```
SUBROUTINE VDSINT(M, N, X, XT, MDIMX, WSAVE)

INTEGER M, N, MDIMX
DOUBLE PRECISION X(MDIMX,*), XT(MDIMX,*), WSAVE(*)

SUBROUTINE VDSINT_64(M, N, X, XT, MDIMX, WSAVE)

INTEGER*8 M, N, MDIMX
DOUBLE PRECISION X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE SINT([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER :: M, N, MDIMX
REAL(8), DIMENSION(:) :: WSAVE
REAL(8), DIMENSION(:,:) :: X, XT

SUBROUTINE SINT_64([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER(8) :: M, N, MDIMX
REAL(8), DIMENSION(:) :: WSAVE
REAL(8), DIMENSION(:,:) :: X, XT
```

C INTERFACE

```
#include <sunperf.h>

void vdsint(int m, int n, double *x, double *xt, int  mdimx,
          double *wsave);

void vdsint_64(long m, long n, double *x, double  *xt,  long
          mdimx, double *wsave);
```

ARGUMENTS

```
M (input)
          The number of sequences to be transformed.   M  >=
          0.
N (input) Length of the sequence to be  transformed.   These
          subroutines  are most efficient when N+1 is a pro-
          duct of small primes.  N >= 0.

X (input/output)
          On entry, a real two-dimensional array with dimen-
          sions  of  (MDIMX  x (N+1)) whose rows contain the
          sequences to be transformed.  On  exit,  the  sine
          transform of the input.

XT (input/output)
          A real two-dimensional work array with  dimensions
          of (MDIMX x (N+1)).

MDIMX (input)
          Leading dimension of the arrays X and XT as speci-
          fied  in  a dimension or type statement.  MDIMX >=
          M.

WSAVE (input)
          On entry, an array  with  dimension  of  at  least
          int(2.5 * N + 15) initialized by VSINTI.
```

Contents

NAME

```
vdsinti - initialize the array WSAVE, which is used in  sub-
routine VSINT.
```

SYNOPSIS

```
SUBROUTINE VDSINTI(N, WSAVE)

INTEGER N
DOUBLE PRECISION WSAVE(*)

SUBROUTINE VDSINTI_64(N, WSAVE)

INTEGER*8 N
DOUBLE PRECISION WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE VSINTI(N, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: WSAVE

SUBROUTINE VSINTI_64(N, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void vdsinti(int n, double *wsave);

void vdsinti_64(long n, double *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. N >= 0.

WSAVE (input)
On entry, an array of dimension (2N + N/2 + 15) or greater. VSINTI is called once to initialize WSAVE before calling VSINT and need not be called again between calls to VSINT if N and WSAVE remain unchanged. Thus, subsequent transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
vrfftb - compute a periodic sequence from its Fourier  coef-
ficients.  The VRFFT operations are normalized, so a call of
VRFFTF followed by a call of VRFFTB will return the original
sequence.
```

SYNOPSIS

```
SUBROUTINE VRFFTB(M, N, X, XT, MDIMX, WSAVE)

INTEGER M, N, MDIMX
REAL X(MDIMX,*), XT(MDIMX,*), WSAVE(*)

SUBROUTINE VRFFTB_64(M, N, X, XT, MDIMX, WSAVE)

INTEGER*8 M, N, MDIMX
REAL X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE FFTB([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER :: M, N, MDIMX
REAL, DIMENSION(:) :: WSAVE
REAL, DIMENSION(:,:) :: X, XT

SUBROUTINE FFTB_64([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER(8) :: M, N, MDIMX
REAL, DIMENSION(:) :: WSAVE
REAL, DIMENSION(:,:) :: X, XT
```

C INTERFACE

```
#include <sunperf.h>
```

```
void vrfftb(int m, int n, float *x, float  *xt,  int  mdimx,
          float *wsave);

void vrfftb_64(long m, long n, float  *x,  float  *xt,  long
          mdimx, float *wsave);
```

ARGUMENTS

M (input) The number of sequences to be transformed. M >= 0.

N (input) Length of the sequence to be transformed. These subroutines are most efficient when N is a product of small primes. N >= 0.

X (input) On entry, an array of length N containing the sequence to be transformed. For VRFFTF, a real two-dimensional array X(M,N) whose rows contain the sequences to be transformed.

XT (input)
A real two-dimensional work array with dimensions of (MDIMX x N).

MDIMX (input)
Leading dimension of the arrays X and XT as specified in a dimension or type statement. MDIMX >= M.

WSAVE (input)
On entry, an array of dimension (N+15) or greater initialized by VRFFTI.

Contents

NAME

```
vrfftf - compute the  Fourier  coefficients  of  a  periodic
sequence.  The VRFFT operations are normalized, so a call of
VRFFTF followed by a call of VRFFTB will return the original
sequence.
```

SYNOPSIS

```
SUBROUTINE VRFFTF(M, N, X, XT, MDIMX, WSAVE)

INTEGER M, N, MDIMX
REAL X(MDIMX,*), XT(MDIMX,*), WSAVE(*)

SUBROUTINE VRFFTF_64(M, N, X, XT, MDIMX, WSAVE)

INTEGER*8 M, N, MDIMX
REAL X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE FFTF([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER :: M, N, MDIMX
REAL, DIMENSION(:) :: WSAVE
REAL, DIMENSION(:,:) :: X, XT

SUBROUTINE FFTF_64([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER(8) :: M, N, MDIMX
REAL, DIMENSION(:) :: WSAVE
REAL, DIMENSION(:,:) :: X, XT
```

C INTERFACE

```
#include <sunperf.h>
```

```
void vrfftf(int m, int n, float *x, float  *xt,  int  mdimx,
          float *wsave);

void vrfftf_64(long m, long n, float  *x,  float  *xt,  long
          mdimx, float *wsave);
```

ARGUMENTS

```
M (input)
          The number of sequences to be transformed.   M  >=
          0.

N (input) Length of the sequence to be  transformed.   These
          subroutines are most efficient when N is a product
          of small primes.  N >= 0.

X (input) On entry, an array  of  length  N  containing  the
          sequence  to  be  transformed.  For VRFFTF, a real
          two- dimensional array X(M,N) whose  rows  contain
          the sequences to be transformed.

XT (input)
          A real two-dimensional work array with  dimensions
          of (MDIMX x N).

MDIMX (input)
          Leading dimension of the arrays X and XT as speci-
          fied  in  a dimension or type statement.  MDIMX >=
          M.

WSAVE (input)
          On entry, an array of dimension (N+15) or  greater
          initialized by VRFFTI.
```

Contents

NAME

```
vrffti - initialize the array WSAVE, which is used  in  both
VRFFTF and VRFFTB.
```

SYNOPSIS

```
SUBROUTINE VRFFTI(N, WSAVE)

INTEGER N
REAL WSAVE(*)

SUBROUTINE VRFFTI_64(N, WSAVE)

INTEGER*8 N
REAL WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE VFFTI(N, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: WSAVE

SUBROUTINE VFFTI_64(N, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void vrffti(int n, float *wsave);

void vrffti_64(long n, float *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. N >= 0.

WSAVE (input)
On entry, an array of dimension (N + 15) or greater. VRFFTI needs to be called only once to initialize WSAVE before calling VRFFTF and/or VRFFTB if N and WSAVE remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
vsinqb - synthesize a Fourier sequence from its  representa-
tion  in  terms of a sine series with odd wave numbers.  The
VSINQ operations are normalized, so a call  of  VSINQF  fol-
lowed by a call of VSINQB will return the original sequence.
```

SYNOPSIS

```
SUBROUTINE VSINQB(M, N, X, XT, MDIMX, WSAVE)

INTEGER M, N, MDIMX
REAL X(MDIMX,*), XT(MDIMX,*), WSAVE(*)

SUBROUTINE VSINQB_64(M, N, X, XT, MDIMX, WSAVE)

INTEGER*8 M, N, MDIMX
REAL X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE SINQB([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER :: M, N, MDIMX
REAL, DIMENSION(:) :: WSAVE
REAL, DIMENSION(:,:) :: X, XT

SUBROUTINE SINQB_64([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER(8) :: M, N, MDIMX
REAL, DIMENSION(:) :: WSAVE
REAL, DIMENSION(:,:) :: X, XT
```

C INTERFACE

```
#include <sunperf.h>
```

```
void vsinqb(int m, int n, float *x, float  *xt,  int  mdimx,
          float *wsave);

void vsinqb_64(long m, long n, float  *x,  float  *xt,  long
          mdimx, float *wsave);
```

ARGUMENTS

M (input)
The number of sequences to be transformed. M >= 0.

N (input) Length of the sequence to be transformed. These subroutines are most efficient when N is a product of small primes. N >= 0.

X (input/output)
On entry, a real two- dimensional array with dimensions of (MDIMX x N) whose rows contain the sequences to be transformed. On exit, the quarter-wave sine synthesis of the input.

XT (input)
A real two-dimensional work array with dimensions of (MDIMX x N).

MDIMX (input)
Leading dimension of the arrays X and XT as specified in a dimension or type statement. MDIMX >= M.

WSAVE (input)
On entry, an array with dimension of at least (2 * N + 15) for vector subroutines, initialized by VSINQI.

Contents

NAME

vsinqf - compute the Fourier coefficients in a sine series representation with only odd wave numbers. The VSINQ operations are normalized, so a call of VSINQF followed by a call of VSINQB will return the original sequence.

SYNOPSIS

```
SUBROUTINE VSINQF(M, N, X, XT, MDIMX, WSAVE)

INTEGER M, N, MDIMX
REAL X(MDIMX,*), XT(MDIMX,*), WSAVE(*)

SUBROUTINE VSINQF_64(M, N, X, XT, MDIMX, WSAVE)

INTEGER*8 M, N, MDIMX
REAL X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE SINQF([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER :: M, N, MDIMX
REAL, DIMENSION(:) :: WSAVE
REAL, DIMENSION(:,:) :: X, XT

SUBROUTINE SINQF_64([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER(8) :: M, N, MDIMX
REAL, DIMENSION(:) :: WSAVE
REAL, DIMENSION(:,:) :: X, XT
```

C INTERFACE

```
#include <sunperf.h>
```

```
void vsinqf(int m, int n, float *x, float  *xt,  int  mdimx,
          float *wsave);

void vsinqf_64(long m, long n, float  *x,  float  *xt,  long
          mdimx, float *wsave);
```

ARGUMENTS

M (input)
The number of sequences to be transformed. M >= 0.

N (input)
Length of the sequence to be transformed. These subroutines are most efficient when N is a product of small primes. N >= 0.

X (input/output)
On entry, an array of length N containing the sequence to be transformed. For VSINQF, a real two-dimensional array with dimensions of (MDIMX x N) whose rows contain the sequences to be transformed. On exit, the quarter-wave sine transform of the input.

XT (input)
A real two-dimensional work array with dimensions of (MDIMX x N).

MDIMX (input)
Leading dimension of the arrays X and XT as specified in a dimension or type statement. MDIMX >= M.

WSAVE (input)
On entry, an array with dimension of at least (2 * N + 15), initialized by VSINQI.

Contents

NAME

```
vsinqi - initialize the array WSAVE, which is used  in  both
VSINQF and VSINQB.
```

SYNOPSIS

```
SUBROUTINE VSINQI(N, WSAVE)

INTEGER N
REAL WSAVE(*)

SUBROUTINE VSINQI_64(N, WSAVE)

INTEGER*8 N
REAL WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE VSINQI(N, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: WSAVE

SUBROUTINE VSINQI_64(N, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void vsinqi(int n, float *wsave);

void vsinqi_64(long n, float *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. The method is most efficient when N is a product of small primes.

WSAVE (input)
On entry, an array with a dimension of at least (2 * N + 15). The same work array can be used for both VSINQF and VSINQB as long as N remains unchanged. Different WSAVE arrays are required for different values of N. This initialization does not have to be repeated between calls to VSINQF or VSINQB as long as N and WSAVE remain unchanged, thus subsequent transforms can be obtained faster than the first.

Contents

NAME

```
vsint - compute the discrete Fourier sine  transform  of  an
odd   sequence.    The  VSINT  transforms  are  unnormalized
inverses of themselves, so  a  call  of  VSINT  followed  by
another  call of VSINT will multiply the input sequence by 2
* (N+1).  The VSINT transforms are normalized, so a call  of
VSINT  followed  by a call of VSINT will return the original
sequence.
```

SYNOPSIS

```
SUBROUTINE VSINT(M, N, X, XT, MDIMX, WSAVE)

INTEGER M, N, MDIMX
REAL X(MDIMX,*), XT(MDIMX,*), WSAVE(*)

SUBROUTINE VSINT_64(M, N, X, XT, MDIMX, WSAVE)

INTEGER*8 M, N, MDIMX
REAL X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE SINT([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER :: M, N, MDIMX
REAL, DIMENSION(:) :: WSAVE
REAL, DIMENSION(:,:) :: X, XT

SUBROUTINE SINT_64([M], [N], X, XT, [MDIMX], WSAVE)

INTEGER(8) :: M, N, MDIMX
REAL, DIMENSION(:) :: WSAVE
REAL, DIMENSION(:,:) :: X, XT
```

C INTERFACE

```
#include <sunperf.h>

void vsint(int m, int n, float *x,  float  *xt,  int  mdimx,
          float *wsave);

void vsint_64(long m, long n,  float  *x,  float  *xt,  long
          mdimx, float *wsave);
```

ARGUMENTS

```
M (input)
          The number of sequences to be transformed.   M  >=
          0.
N (input) Length of the sequence to be  transformed.   These
          subroutines  are most efficient when N+1 is a pro-
          duct of small primes.  N >= 0.

X (input/output)
          On entry, a real two-dimensional array with dimen-
          sions  of  (MDIMX  x (N+1)) whose rows contain the
          sequences to be transformed.  On  exit,  the  sine
          transform of the input.

XT (input/output)
          A real two-dimensional work array with  dimensions
          of (MDIMX x (N+1)).

MDIMX (input)
          Leading dimension of the arrays X and XT as speci-
          fied  in  a dimension or type statement.  MDIMX >=
          M.

WSAVE (input)
          On entry, an array  with  dimension  of  at  least
          int(2.5 * N + 15) initialized by VSINTI.
```

Contents

NAME

```
vsinti - initialize the array WSAVE, which is used  in  sub-
routine VSINT.
```

SYNOPSIS

```
SUBROUTINE VSINTI(N, WSAVE)

INTEGER N
REAL WSAVE(*)

SUBROUTINE VSINTI_64(N, WSAVE)

INTEGER*8 N
REAL WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE VSINTI(N, WSAVE)

INTEGER :: N
REAL, DIMENSION(:) :: WSAVE

SUBROUTINE VSINTI_64(N, WSAVE)

INTEGER(8) :: N
REAL, DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void vsinti(int n, float *wsave);

void vsinti_64(long n, float *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. N >= 0.

WSAVE (input)
On entry, an array of dimension (2N + N/2 + 15) or greater. VSINTI is called once to initialize WSAVE before calling VSINT and need not be called again between calls to VSINT if N and WSAVE remain unchanged. Thus, subsequent transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

vzfftb - compute a periodic sequence from its Fourier coefficients. The VZFFT operations are normalized, so a call of VZFFTF followed by a call of VZFFTB will return the original sequence.

SYNOPSIS

```
SUBROUTINE VZFFTB(M, N, X, XT, MDIMX, ROWCOL, WSAVE)

CHARACTER * 1 ROWCOL
DOUBLE COMPLEX X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
INTEGER M, N, MDIMX

SUBROUTINE VZFFTB_64(M, N, X, XT, MDIMX, ROWCOL, WSAVE)

CHARACTER * 1 ROWCOL
DOUBLE COMPLEX X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
INTEGER*8 M, N, MDIMX
```

F95 INTERFACE

```
SUBROUTINE FFTB([M], [N], X, XT, [MDIMX], ROWCOL, WSAVE)

CHARACTER(LEN=1) :: ROWCOL
COMPLEX(8), DIMENSION(:) :: WSAVE
COMPLEX(8), DIMENSION(:,:) :: X, XT
INTEGER :: M, N, MDIMX

SUBROUTINE FFTB_64([M], [N], X, XT, [MDIMX], ROWCOL, WSAVE)

CHARACTER(LEN=1) :: ROWCOL
COMPLEX(8), DIMENSION(:) :: WSAVE
COMPLEX(8), DIMENSION(:,:) :: X, XT
INTEGER(8) :: M, N, MDIMX
```

C INTERFACE

```
#include <sunperf.h>

void vzfftb(int m, int n,  doublecomplex  *x,  doublecomplex
          *xt,   int   mdimx,   char  rowcol,  doublecomplex
          *wsave);

void vzfftb_64(long m, long n, doublecomplex *x,  doublecom-
          plex  *xt,  long mdimx, char rowcol, doublecomplex
          *wsave);
```

ARGUMENTS

M (input) The number of sequences to be transformed. M >= 0.

N (input) Length of the sequence to be transformed. These subroutines are most efficient when N is a product of small primes. N >= 0.

X (input) On entry, the rows contain the sequences to be transformed.

XT (input)
A work array.

MDIMX (input)
Leading dimension of the arrays X and XT as specified in a dimension or type statement. MDIMX >= M.

ROWCOL (input)
Indicates whether to transform rows ('R' or 'r') or columns ('C' or 'c').

WSAVE (input/output)
On entry, an array of dimension (K+15) or greater, where K = M if ROWCOL = ('R' or 'r'). Otherwise, K = N. WSAVE is initialized by VZFFTI.

Contents

NAME

```
vzfftf - compute the  Fourier  coefficients  of  a  periodic
sequence.  The VZFFT operations are normalized, so a call of
VZFFTF followed by a call of VZFFTB will return the original
sequence.
```

SYNOPSIS

```
SUBROUTINE VZFFTF(M, N, X, XT, MDIMX, ROWCOL, WSAVE)

CHARACTER * 1 ROWCOL
DOUBLE COMPLEX X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
INTEGER M, N, MDIMX

SUBROUTINE VZFFTF_64(M, N, X, XT, MDIMX, ROWCOL, WSAVE)

CHARACTER * 1 ROWCOL
DOUBLE COMPLEX X(MDIMX,*), XT(MDIMX,*), WSAVE(*)
INTEGER*8 M, N, MDIMX
```

F95 INTERFACE

```
SUBROUTINE FFTF([M], [N], X, XT, [MDIMX], ROWCOL, WSAVE)

CHARACTER(LEN=1) :: ROWCOL
COMPLEX(8), DIMENSION(:) :: WSAVE
COMPLEX(8), DIMENSION(:,:) :: X, XT
INTEGER :: M, N, MDIMX

SUBROUTINE FFTF_64([M], [N], X, XT, [MDIMX], ROWCOL, WSAVE)

CHARACTER(LEN=1) :: ROWCOL
COMPLEX(8), DIMENSION(:) :: WSAVE
COMPLEX(8), DIMENSION(:,:) :: X, XT
INTEGER(8) :: M, N, MDIMX
```

C INTERFACE

```
#include <sunperf.h>

void vzfftf(int m, int n,  doublecomplex  *x,  doublecomplex
          *xt,   int   mdimx,   char  rowcol,  doublecomplex
          *wsave);

void vzfftf_64(long m, long n, doublecomplex *x,  doublecom-
          plex  *xt,  long mdimx, char rowcol, doublecomplex
          *wsave);
```

ARGUMENTS

M (input) The number of sequences to be transformed. M >= 0.

N (input) Length of the sequence to be transformed. These subroutines are most efficient when N is a product of small primes. N >= 0.

X (input) On entry, an array X(M,N) whose rows contain the sequences to be transformed.

XT (input)
A work array.

MDIMX (input)
Leading dimension of the arrays X and XT as specified in a dimension or type statement. MDIMX >= M.

ROWCOL (input)
Indicates whether to transform rows ('R' or 'r') or columns ('C' or 'c').

WSAVE (input/output)
On entry, an array of dimension (K+15) or greater, where K = M if ROWCOL = ('R' or 'r'). Otherwise, K = N. WSAVE is initialized by VZFFTI.

Contents

NAME

```
vzffti - initialize the array WSAVE, which is used  in  both
VZFFTF and VZFFTB.
```

SYNOPSIS

```
SUBROUTINE VZFFTI(N, WSAVE)

DOUBLE COMPLEX WSAVE(*)
INTEGER N

SUBROUTINE VZFFTI_64(N, WSAVE)

DOUBLE COMPLEX WSAVE(*)
INTEGER*8 N
```

F95 INTERFACE

```
SUBROUTINE VFFTI(N, WSAVE)

COMPLEX(8), DIMENSION(:) :: WSAVE
INTEGER :: N

SUBROUTINE VFFTI_64(N, WSAVE)

COMPLEX(8), DIMENSION(:) :: WSAVE
INTEGER(8) :: N
```

C INTERFACE

```
#include <sunperf.h>

void vzffti(int n, doublecomplex *wsave);

void vzffti_64(long n, doublecomplex *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. N >= 0.

WSAVE (input)
On entry, an array of dimension (N + 15) or greater. VZFFTI needs to be called only once to initialize WSAVE before calling VZFFTF and/or VZFFTB if N and WSAVE remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
zaxpy - compute y := alpha * x + y
```

SYNOPSIS

```
SUBROUTINE ZAXPY(N, ALPHA, X, INCX, Y, INCY)

DOUBLE COMPLEX ALPHA
DOUBLE COMPLEX X(*), Y(*)
INTEGER N, INCX, INCY

SUBROUTINE ZAXPY_64(N, ALPHA, X, INCX, Y, INCY)

DOUBLE COMPLEX ALPHA
DOUBLE COMPLEX X(*), Y(*)
INTEGER*8 N, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE AXPY([N], ALPHA, X, [INCX], Y, [INCY])

COMPLEX(8) :: ALPHA
COMPLEX(8), DIMENSION(:) :: X, Y
INTEGER :: N, INCX, INCY

SUBROUTINE AXPY_64([N], ALPHA, X, [INCX], Y, [INCY])

COMPLEX(8) :: ALPHA
COMPLEX(8), DIMENSION(:) :: X, Y
INTEGER(8) :: N, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zaxpy(int n, doublecomplex  *alpha,  doublecomplex  *x,
          int incx, doublecomplex *y, int incy);

void zaxpy_64(long n,  doublecomplex  *alpha,  doublecomplex
          *x, long incx, doublecomplex *y, long incy);
```

PURPOSE

zaxpy compute y := alpha * x + y where alpha is a scalar and x and y are n-vectors.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
array of DIMENSION at least (1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input/output)
array of DIMENSION at least (1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
zaxpyi - Compute y := alpha * x + y
```

SYNOPSIS

```
 SUBROUTINE ZAXPYI(NZ, A, X, INDX, Y)

 DOUBLE COMPLEX A
 DOUBLE COMPLEX X(*), Y(*)
 INTEGER NZ
 INTEGER INDX(*)

 SUBROUTINE ZAXPYI_64(NZ, A, X, INDX, Y)

 DOUBLE COMPLEX A
 DOUBLE COMPLEX X(*), Y(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 SUBROUTINE AXPYI([NZ], [A], X, INDX, Y)

 COMPLEX(8) :: A
 COMPLEX(8), DIMENSION(:) :: X, Y
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 SUBROUTINE AXPYI_64([NZ], [A], X, INDX, Y)

 COMPLEX(8) :: A
 COMPLEX(8), DIMENSION(:) :: X, Y
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
ZAXPYI Compute y := alpha * x + y where alpha is a scalar, x
is a sparse vector, and y is a vector in full storage form

 do i = 1, n
   y(indx(i)) = alpha * x(i) + y(indx(i))
 enddo
```

ARGUMENTS

```
NZ (input) - INTEGER
        Number of elements in the compressed form.
        Unchanged on exit.

A (input)
        On entry, A(LPHA) specifies the scaling value.
        Unchanged on exit.  A is defaulted to (1.0D0,0.0D0)
        for F95 INTERFACE.
X (input)
        Vector containing the values of the compressed form.
        Unchanged on exit.

INDX (input) - INTEGER
        Vector containing the indices of the compressed
        form.  It is assumed that the elements in INDX are
        distinct and greater than zero.  Unchanged on exit.

Y (output)
        Vector on input which contains the vector Y in full
        storage form.  On exit, only the elements
        corresponding to the indices in INDX have been
        modified.
```

Contents

NAME

zbcomm - block coordinate matrix-matrix multiply

SYNOPSIS

```
  SUBROUTINE ZBCOMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *            VAL, BINDX, BJNDX, BNNZ, LB,
 *            B, LDB, BETA, C, LDC, WORK, LWORK)
  INTEGER    TRANSA, MB, N, KB, DESCRA(5), BNNZ, LB,
 *            LDB, LDC, LWORK
  INTEGER    BINDX(BNNZ), BJNDX(BNNZ)
  DOUBLE COMPLEX ALPHA, BETA
  DOUBLE COMPLEX VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

  SUBROUTINE ZBCOMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *            VAL, BINDX, BJNDX, BNNZ, LB,
 *            B, LDB, BETA, C, LDC, WORK, LWORK)
  INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), BNNZ, LB,
 *            LDB, LDC, LWORK
  INTEGER*8  BINDX(BNNZ), BJNDX(BNNZ)
  DOUBLE COMPLEX ALPHA, BETA
  DOUBLE COMPLEX VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
   SUBROUTINE BCOMM(TRANSA,MB,N,KB,ALPHA,DESCRA,VAL,BINDX, BJNDX,
 *   BNNZ, LB, B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
  INTEGER    TRANSA, MB, N, KB, BNNZ, LB
  INTEGER, DIMENSION(:) ::    DESCRA, BINDX, BJNDX
  DOUBLE COMPLEX    ALPHA, BETA
  DOUBLE COMPLEX, DIMENSION(:) :: VAL
  DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C

   SUBROUTINE BCOMM_64(TRANSA,MB,N,KB,ALPHA,DESCRA,VAL,BINDX, BJNDX,
```

```
*   BNNZ, LB, B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, N, KB, BNNZ, LB
 INTEGER*8, DIMENSION(:) ::   DESCRA, BINDX, BJNDX
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
         C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block coordinate format and
op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if the matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
```

```
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices

 VAL()         scalar array of length LB*LB*BNNZ consisting of
               the non-zero block entries of A, in any order.
               Each block is stored in standard column-major form.

 BINDX()       integer array of length BNNZ consisting of the
               block row indices of the block entries of A.

 BJNDX()       integer array of length BNNZ consisting of the
               block column indices of the block entries of A.

 BNNZ          number of block entries

 LB            dimension of dense blocks composing A.

 B()           rectangular array with first dimension LDB.
 LDB           leading dimension of B

 BETA          Scalar parameter

 C()           rectangular array with first dimension LDC.

 LDC           leading dimension of C

 WORK()        scratch array of length LWORK. WORK is not
               referenced in the current version.

 LWORK         length of WORK array. LWORK is not referenced
               in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS)
Standard", University of Tennessee, Knoxville, Tennessee,
1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

```
zbdimm  - block diagonal format matrix-matrix multiply
```

SYNOPSIS

```
 SUBROUTINE ZBDIMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BLDA, IBDIAG, NBDIAG, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, MB, N, KB, DESCRA(5), BLDA, NBDIAG, LB,
*           LDB, LDC, LWORK
 INTEGER    IBDIAG(NBDIAG)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(LB*LB*BLDA*NBDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE ZBDIMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BLDA, IBDIAG, NBDIAG, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), BLDA, NBDIAG, LB,
*           LDB, LDC, LWORK
 INTEGER*8  IBDIAG(NBDIAG)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(LB*LB*BLDA*NBDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
  SUBROUTINE BDIMM(TRANSA,MB, [N], KB, ALPHA, DESCRA, VAL, BLDA,
*    IBDIAG, NBDIAG, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, KB, BLDA, NBDIAG, LB
 INTEGER, DIMENSION(:) ::    DESCRA, IBDIAG
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C

  SUBROUTINE BDIMM_64(TRANSA,MB, [N], KB, ALPHA, DESCRA, VAL, BLDA,
```

```
*    IBDIAG, NBDIAG, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, KB, BLDA, NBDIAG, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, IBDIAG
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
         C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block diagonal format and
op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix  is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
```

```
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices

 VAL()         two-dimensional LB*LB*BLDA-by-NBDIAG scalar array
               consisting of the NBDIAG nonzero block diagonal in
               any order. Each dense block is stored in standard
               column-major form.

 BLDA          leading block dimension of VAL().

 IBDIAG()      integer array of length NBDIAG consisting of the
               corresponding diagonal offsets of the non-zero
               block diagonals of A in VAL.  Lower triangular
               block diagonals have negative offsets, the main
               block diagonal has offset 0, and upper triangular
               block diagonals have positive offset.

 NBDIAG        the number of non-zero block diagonals in A.
 LB            dimension of dense blocks composing A.

 B()           rectangular array with first dimension LDB.

 LDB           leading dimension of B

 BETA          Scalar parameter

 C()           rectangular array with first dimension LDC.

 LDC           leading dimension of C

 WORK()        scratch array of length LWORK. WORK is not
               referenced in the current version.

 LWORK         length of WORK array. LWORK is not referenced
               in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

```
zbdism -  block diagonal format triangular solve
```

SYNOPSIS

```
 SUBROUTINE ZBDISM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, BLDA, IBDIAG, NBDIAG, LB,
*            B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), BLDA, NBDIAG, LB,
*           LDB, LDC, LWORK
 INTEGER    IBDIAG(NBDIAG)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX DV(MB*LB*LB), VAL(LB*LB*BLDA, NBDIAG), B(LDB,*), C(LDC,*),
*           WORK(LWORK)

 SUBROUTINE ZBDISM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, BLDA, IBDIAG, NBDIAG, LB,
*            B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), BLDA, NBDIAG, LB,
*           LDB, LDC, LWORK
 INTEGER*8  IBDIAG(NBDIAG)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX DV(MB*LB*LB), VAL(LB*LB*BLDA, NBDIAG), B(LDB,*), C(LDC,*),
*           WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE BDISM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BLDA,
*   IBDIAG, NBDIAG, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, N, UNITD, BLDA, NBDIAG, LB
 INTEGER, DIMENSION(:) ::    DESCRA, IBDIAG
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL, DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::   B, C
```

```
 SUBROUTINE BDISM_64(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BLDA,
*   IBDIAG, NBDIAG, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, N, UNITD, BLDA, NBDIAG, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, IBDIAG
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL, DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in block diagonal format
and  op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)

DV()          Array of length MB*LB*LB containing the elements of
              the diagonal blocks of the matrix D.  The size of each
              square block is LB-by-LB and each block
              is stored in standard column-major form.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                  0 : general
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))
                Note: For the routine, DESCRA(1)=3 is only supported.

                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-identity blocks on the main diagonal
                  1 : identity diagonal blocks
                  2 : diagonal blocks are dense matrices
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices
VAL()           Two-dimensional LB*LB*BLDA-by-NBDIAG scalar array
                consisting of the NBDIAG non-zero block diagonal.
                Each dense block is stored in standard column-major form.

BLDA            Leading block dimension of VAL().  Should be greater
                than or equal to MB.

IBDIAG()        integer array of length NBDIAG consisting of the
                corresponding diagonal offsets of the non-zero block
                diagonals of A in VAL.  Lower triangular block diagonals
                have negative offsets, the main block diagonal has offset
                0, and upper triangular block diagonals have positive offset.
                Elements of IBDIAG  MUST be sorted in  increasing order.

NBDIAG          The number of non-zero block diagonals in A.

LB              Dimension of dense blocks composing A.

B()             Rectangular array with first dimension LDB.

LDB             Leading dimension of B.

BETA            Scalar parameter.

C()             Rectangular array with first dimension LDC.

LDC             Leading dimension of C.
```

```
WORK()        scratch array of length LWORK.
              On exit, if LWORK= -1, WORK(1) returns the optimum  size
              of LWORK.

LWORK        length of WORK array. LWORK should be at least
             MB*LB.

             For good performance, LWORK should generally be larger.
             For optimum performance on multiple processors, LWORK
             >=MB*LB*N_CPUS where N_CPUS is the maximum number of
             processors available to the program.

             If LWORK=0, the routine is to allocate workspace needed.

             If LWORK = -1, then a workspace query is assumed; the
             routine only calculates the optimum size of the WORK array,
             returns this value as the first entry of the WORK array,
             and no error message related to LWORK is issued by XERBLA.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If *DESCRA*(3)=0,the lower or upper triangular part of each diagonal block is used by the routine depending on *DESCRA*(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might not be referenced in the BDI representation of a sparse matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense matrices and the LU factorization with partial pivoting is used by the routine. *WORK*(1)=0 on return if the factorization for all diagonal blocks has been completed successfully, otherwise *WORK*(1) = -i where i is the block

number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

```
zbdsqr - compute the singular value decomposition (SVD) of a
real N-by-N (upper or lower) bidiagonal matrix B.
```

SYNOPSIS

```
SUBROUTINE ZBDSQR(UPLO, N, NCVT, NRU, NCC, D, E, VT, LDVT, U, LDU, C,
      LDC, WORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX VT(LDVT,*), U(LDU,*), C(LDC,*)
INTEGER N, NCVT, NRU, NCC, LDVT, LDU, LDC, INFO
DOUBLE PRECISION D(*), E(*), WORK(*)

SUBROUTINE ZBDSQR_64(UPLO, N, NCVT, NRU, NCC, D, E, VT, LDVT, U, LDU,
      C, LDC, WORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX VT(LDVT,*), U(LDU,*), C(LDC,*)
INTEGER*8 N, NCVT, NRU, NCC, LDVT, LDU, LDC, INFO
DOUBLE PRECISION D(*), E(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE BDSQR(UPLO, [N], [NCVT], [NRU], [NCC], D, E, VT, [LDVT],
       U, [LDU], C, [LDC], [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: VT, U, C
INTEGER :: N, NCVT, NRU, NCC, LDVT, LDU, LDC, INFO
REAL(8), DIMENSION(:) :: D, E, WORK

SUBROUTINE BDSQR_64(UPLO, [N], [NCVT], [NRU], [NCC], D, E, VT, [LDVT],
       U, [LDU], C, [LDC], [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: VT, U, C
INTEGER(8) :: N, NCVT, NRU, NCC, LDVT, LDU, LDC, INFO
REAL(8), DIMENSION(:) :: D, E, WORK
```

C INTERFACE

```
#include <sunperf.h>

void zbdsqr(char uplo, int n, int ncvt, int  nru,  int  ncc,
          double *d, double *e, doublecomplex *vt, int ldvt,
          doublecomplex *u, int ldu, doublecomplex  *c,  int
          ldc, int *info);
void zbdsqr_64(char uplo, long n, long ncvt, long nru,  long
          ncc, double *d, double *e, doublecomplex *vt, long
          ldvt, doublecomplex *u,  long  ldu,  doublecomplex
          *c, long ldc, long *info);
```

PURPOSE

zbdsqr computes the singular value decomposition (SVD) of a real N-by-N (upper or lower) bidiagonal matrix B: B = Q * S * P' (P' denotes the transpose of P), where S is a diagonal matrix with non-negative diagonal elements (the singular values of B), and Q and P are orthogonal matrices.

The routine computes S, and optionally computes U * Q, P' * VT, or Q' * C, for given complex input matrices U, VT, and C.

See "Computing Small Singular Values of Bidiagonal Matrices With Guaranteed High Relative Accuracy," by J. Demmel and W. Kahan, LAPACK Working Note #3 (or SIAM J. Sci. Statist. Comput. vol. 11, no. 5, pp. 873-912, Sept 1990) and "Accurate singular values and differential qd algorithms," by B. Parlett and V. Fernando, Technical Report CPAM-554, Mathematics Department, University of California at Berkeley, July 1992 for a detailed description of the algorithm.

ARGUMENTS

UPLO (input)
= 'U': B is upper bidiagonal;
= 'L': B is lower bidiagonal.

N (input) The order of the matrix B. N >= 0.

NCVT (input)
The number of columns of the matrix VT. NCVT >= 0.

NRU (input)
The number of rows of the matrix U. NRU >= 0.

NCC (input)
The number of columns of the matrix C. NCC >= 0.

D (input/output)
On entry, the n diagonal elements of the bidiagonal matrix B. On exit, if INFO=0, the singular values of B in decreasing order.

E (input/output)
On entry, the elements of E contain the offdiagonal elements of of the bidiagonal matrix whose SVD is desired. On normal exit (INFO = 0), E is destroyed. If the algorithm does not converge (INFO > 0), D and E will contain the diagonal and superdiagonal elements of a bidiagonal matrix orthogonally equivalent to the one given as input. E(N) is used for workspace.

VT (input/output)
On entry, an N-by-NCVT matrix VT. On exit, VT is overwritten by P' * VT. VT is not referenced if NCVT = 0.

LDVT (input)
The leading dimension of the array VT. LDVT >= max(1,N) if NCVT > 0; LDVT >= 1 if NCVT = 0.

U (input/output)
On entry, an NRU-by-N matrix U. On exit, U is overwritten by U * Q. U is not referenced if NRU = 0.

LDU (input)
The leading dimension of the array U. LDU >= max(1,NRU).

C (input/output)
On entry, an N-by-NCC matrix C. On exit, C is overwritten by Q' * C. C is not referenced if NCC = 0.

LDC (input)
The leading dimension of the array C. LDC >= max(1,N) if NCC > 0; LDC >=1 if NCC = 0.

WORK (workspace)
dimension (4*N)

INFO (output)
= 0: successful exit
< 0: If INFO = -i, the i-th argument had an illegal value
> 0: the algorithm did not converge; D and E contain the elements of a bidiagonal matrix which is orthogonally similar to the input matrix B; if INFO = i, i elements of E have not converged to zero.

Contents

NAME

zbelmm - block Ellpack format matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE ZBELMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BLDA, MAXBNZ, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, MB, N, KB, DESCRA(5), BLDA, MAXBNZ, LB,
*           LDB, LDC, LWORK
 INTEGER    BINDX(BLDA,MAXBNZ)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(LB*LB*BLDA*MAXBNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE ZBELMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BLDA, MAXBNZ, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), BLDA, MAXBNZ, LB,
*           LDB, LDC, LWORK
 INTEGER*8  BINDX(BLDA,MAXBNZ)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(LB*LB*BLDA*MAXBNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE BELMM( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
*           BLDA, MAXBNZ, LB, B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, KB, BLDA, MAXBNZ, LB
 INTEGER, DIMENSION(:) ::    DESCRA, BINDX
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C

 SUBROUTINE BELMM_64( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
```

```
*           BLDA, MAXBNZ, LB, B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, KB, BLDA, MAXBNZ, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block Ellpack format and
op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
```

```
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

 VAL()          scalar array of length LB*LB*BLDA*MAXBNZ containing
                matrix entries, stored column-major within each dense
                block.

 BINDX()        two-dimensional integer BLDA-by-MAXBNZ array such
                BINDX(i,:) consists of the block column indices of the
                nonzero blocks in block row i, padded by the integer
                value i if the number of nonzero blocks is less than
                MAXBNZ.

 BLDA           leading dimension of BINDX(:,:).

 MAXBNZ         max number of nonzeros blocks per row.
 LB             row and column dimension of the dense blocks composing
                VAL.

 B()            rectangular array with first dimension LDB.

 LDB            leading dimension of B

 BETA           Scalar parameter

 C()            rectangular array with first dimension LDC.

 LDC            leading dimension of C

 WORK()         scratch array of length LWORK. WORK is not
                referenced in the current version.

 LWORK          length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS)

Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

zbelsm - block Ellpack format triangular solve

SYNOPSIS

```
 SUBROUTINE ZBELSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, BINDX, BLDA, MAXBNZ, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), BLDA, MAXBNZ, LB,
*           LDB, LDC, LWORK
 INTEGER    BINDX(BLDA,MAXBNZ)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX DV(MB*LB*LB), VAL(LB*LB*BLDA*MAXBNZ), B(LDB,*), C(LDC,*),
*           WORK(LWORK)

 SUBROUTINE ZBELSM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, BINDX, BLDA, MAXBNZ, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), BLDA, MAXBNZ, LB,
*           LDB, LDC, LWORK
 INTEGER*8  BINDX(BLDA,MAXBNZ)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX DV(MB*LB*LB), VAL(LB*LB*BLDA*MAXBNZ), B(LDB,*), C(LDC,*),
*           WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE BELSM( TRANSA, MB, [N], UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*   BLDA, MAXBNZ, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, UNITD,  BLDA, MAXBNZ, LB
 INTEGER, DIMENSION(:) ::    DESCRA,  BINDX
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL, DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::   B, C
```

```
 SUBROUTINE BELSM_64( TRANSA, MB, [N], UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*   BLDA, MAXBNZ, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8  TRANSA, MB, UNITD,  BLDA, MAXBNZ, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA,  BINDX
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL, DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::   B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in block Ellpack format and
op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)

DV()          Array of the length MB*LB*LB consisting of the block
              entries of block diagonal matrix D where each
              block is stored in standard column-major form.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
```

```
                 1 : symmetric (A=A')
                 2 : Hermitian (A= CONJG(A'))
                 3 : Triangular
                 4 : Skew(Anti)-Symmetric (A=-A')
                 5 : Diagonal
                 6 : Skew-Hermitian (A= -CONJG(A'))
               Note: For the routine, DESCRA(1)=3 is only supported.

               DESCRA(2) upper/lower triangular indicator
                 1 : lower
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-identity blocks on the main diagonal
                 1 : identity diagonal blocks
                 2 : diagonal blocks are dense matrices
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices
VAL()          scalar array of length LB*LB*BLDA*MAXBNZ containing
               matrix entries, stored column-major within each dense
               block.

BINDX()        two-dimensional integer BLDA-by-MAXBNZ array such
               BINDX(i,:) consists of the block column indices of the
               nonzero blocks in block row i, padded by the integer
               value i if the number of nonzero blocks is less than
               MAXBNZ.  The block column indices MUST be sorted
               in increasing order for each block row.

BLDA           leading dimension of BINDX(:,:).

MAXBNZ         max number of nonzeros blocks per row.

LB             row and column dimension of the dense blocks composing A.

B()            rectangular array with first dimension LDB.

LDB            leading dimension of B

BETA           Scalar parameter

C()            rectangular array with first dimension LDC.

LDC            leading dimension of C

WORK()         scratch array of length LWORK.
               On exit, if LWORK= -1, WORK(1) returns the minimum
```

size of LWORK.

LWORK length of WORK array. LWORK should be at least MB*LB.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=MB*LB*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at: http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If *DESCRA*(3)=0,the lower or upper triangular part of each diagonal block is used by the routine depending on *DESCRA*(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might not be referenced in the BEL representation of a sparse matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense matrices and the LU factorization with partial pivoting is used by the routine. *WORK*(1)=0 on return if the factorization for all diagonal blocks has been completed successfully, otherwise *WORK*(1) = -i where i is the block number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

zbscmm - block sparse column matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE ZBSCMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BPNTRB, BPNTRE, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, MB, N, KB, DESCRA(5), LB,
*           LDB, LDC, LWORK
 INTEGER    BINDX(BNNZ), BPNTRB(KB), BPNTRE(KB)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE ZBSCMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BPNTRB, BPNTRE, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), LB,
*           LDB, LDC, LWORK
 INTEGER*8  BINDX(BNNZ), BPNTRB(KB), BPNTRE(KB)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where: BNNZ = BPNTRE(KB)-BPNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE BSCMM( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, KB, LB
 INTEGER, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE BSCMM_64( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB,  KB, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block sparse column format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
```

```
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

VAL()           scalar array of length LB*LB*BNNZ consisting of
                the block entries stored column-major within each
                dense block.

BINDX()         integer array of length BNNZ consisting of the
                block row indices of the block entries of A.

BPNTRB()        integer array of length KB such that
                BPNTRB(J)-BPNTRB(1)+1 points to location in BINDX
                of the first block entry of the J-th block column
                of A.
BPNTRE()        integer array of length KB such that
                BPNTRE(J)-BPNTRB(1) points to location in BINDX
                of the last block entry of the J-th block column
                of A.

LB              dimension of dense blocks composing A.

B()             rectangular array with first dimension LDB.

LDB             leading dimension of B

BETA            Scalar parameter

C()             rectangular array with first dimension LDC.

LDC             leading dimension of C

WORK()          scratch array of length LWORK. WORK is not
                referenced in the current version.

LWORK           length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

It is known that there exists another representation of the block sparse column format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block column in the arrays VAL and BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of block sparse column format the following calling sequence should be used

```
  CALL ZBSCMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *           VAL, BINDX, IA, IA(2), LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

zbscsm - block sparse column format triangular solve

SYNOPSIS

```
       SUBROUTINE ZBSCSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
      *           VAL, BINDX, BPNTRB, BPNTRE, LB,
      *           B, LDB, BETA, C, LDC, WORK, LWORK)
       INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), LB,
      *           LDB, LDC, LWORK
       INTEGER    BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
       DOUBLE COMPLEX ALPHA, BETA
       DOUBLE COMPLEX DV(MB*LB*LB), VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK
(LWORK)

       SUBROUTINE ZBSCSM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
      *           VAL, BINDX, BPNTRB, BPNTRE, LB,
      *           B, LDB, BETA, C, LDC, WORK, LWORK)
       INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), LB,
      *           LDB, LDC, LWORK
       INTEGER*8  BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
       DOUBLE COMPLEX ALPHA, BETA
       DOUBLE COMPLEX DV(MB*LB*LB), VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK
(LWORK)

       where: BNNZ = BPNTRE(MB)- BPNTRB(1)
```

F95 INTERFACE

```
       SUBROUTINE BSCSM(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
      *     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
       INTEGER    TRANSA, MB, N, UNITD, LB
       INTEGER, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
       DOUBLE COMPLEX    ALPHA, BETA
```

```
 DOUBLE COMPLEX, DIMENSION(:) :: VAL, DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C

 SUBROUTINE BSCSM_64(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, N, UNITD, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL, DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in block sparse column format
and  op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row block scaling)
                3 : Scale on right (column block scaling)

DV()          Array of the length MB*LB*LB consisting of the block
              entries of block diagonal matrix D where each
              block is stored in standard column-major form.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
```

```
                  DESCRA(1) matrix structure
                    0 : general
                    1 : symmetric (A=A')
                    2 : Hermitian (A= CONJG(A'))
                    3 : Triangular
                    4 : Skew(Anti)-Symmetric (A=-A')
                    5 : Diagonal
                    6 : Skew-Hermitian (A= -CONJG(A'))

                  Note: For the routine, DESCRA(1)=3 is only supported.

                  DESCRA(2) upper/lower triangular indicator
                    1 : lower
                    2 : upper
                  DESCRA(3) main diagonal type
                    0 : non-identity blocks on the main diagonal
                    1 : identity diagonal blocks
                    2 : diagonal blocks are dense matrices
                  DESCRA(4) Array base  (NOT IMPLEMENTED)
                    0 : C/C++ compatible
                    1 : Fortran compatible
                  DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                    0 : unknown
                    1 : no repeated indices
VAL()             scalar array of length LB*LB*BNNZ consisting of the block
                  entries stored column-major within each dense block.

BINDX()           integer array of length BNNZ consisting of the
                  block row indices of the block entries of A.
                  The block row indices MUST be sorted
                  in increasing order for each block column.

BPNTRB()          integer array of length MB such that
                  BPNTRB(J)-BPNTRB(1)+1 points to location in BINDX
                  of the first block entry of the J-th block column of A.

BPNTRE()          integer array of length MB such that
                  BPNTRE(J)-BPNTRB(1) points to location in BINDX
                  of the last block entry of the J-th block column of A.

LB                dimension of dense blocks composing A.

B()               rectangular array with first dimension LDB.

LDB               leading dimension of B

BETA              Scalar parameter

C()               rectangular array with first dimension LDC.
```

LDC   leading dimension of C

WORK()   scratch array of length LWORK.
On exit, if LWORK= -1, WORK(1) returns the optimum
size of LWORK.

LWORK   length of WORK array. LWORK should be at least
MB*LB.

For good performance, LWORK should generally be larger.
For optimum performance on multiple processors, LWORK
>=MB*LB*N_CPUS where N_CPUS is the maximum number of
processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the
routine only calculates the optimum size of the WORK
array, returns this value as the first entry of the WORK
array, and no error message related to LWORK is issued
by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS)
Standard", University of Tennessee, Knoxville, Tennessee,
1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included
in this routine. Such tests must be performed before calling
this routine.

2. If *DESCRA*(3)=0,the lower or upper triangular part of each
diagonal block is used by the routine depending on
DESCRA(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might
not be referenced in the BSC representation of a sparse
matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense
matrices and the LU factorization with partial pivoting is

used by the routine. *WORK*(1)=0 on return if the factorization for all diagonal blocks has been completed successfully, otherwise *WORK*(1) = -i where i is the block number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3 in this case.

6. It is known that there exists another representation of the block sparse column format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block column in the arrays VAL and BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of block sparse column format the following calling sequence should be used

```
  CALL ZBSCSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, BINDX, IA, IA(2), LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

zbsrmm - block sparse row format matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE ZBSRMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BPNTRB, BPNTRE, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, MB, N, KB, DESCRA(5), LB,
*           LDB, LDC, LWORK
 INTEGER    BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE ZBSRMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, BINDX, BPNTRB, BPNTRE, LB,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), LB,
*           LDB, LDC, LWORK
 INTEGER*8  BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where: BNNZ = BPNTRE(MB)-BPNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE BSRMM( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, KB, LB
 INTEGER, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE BSRMM_64( TRANSA, MB, [N], KB, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB,  KB, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in block sparse row format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix A is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
```

```
                 DESCRA(3) main diagonal type
                   0 : non-unit
                   1 : unit
                 DESCRA(4) Array base  (NOT IMPLEMENTED)
                   0 : C/C++ compatible
                   1 : Fortran compatible
                 DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                   0 : unknown
                   1 : no repeated indices

   VAL()         scalar array of length LB*LB*BNNZ consisting
                 of the block  entries stored column-major within
                 each dense block.

   BINDX()       integer array of length BNNZ consisting of the
                 block column indices of the block entries of A.

   BPNTRB()      integer array of length MB such that
                 BPNTRB(J)-BPNTRB(1)+1 points to location in BINDX
                 of the first block entry of the J-th block row of A.
   BPNTRE()      integer array of length MB such that
                 BPNTRE(J)-BPNTRB(1) points to location in BINDX
                 of the last block entry of the J-th block row of A.

   LB            dimension of dense blocks composing A.

   B()           rectangular array with first dimension LDB.

   LDB           leading dimension of B

   BETA          Scalar parameter

   C()           rectangular array with first dimension LDC.

   LDC           leading dimension of C

   WORK()        scratch array of length LWORK. WORK is not
                 referenced in the current version.

   LWORK         length of WORK array. LWORK is not referenced
                 in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

It is known that there exists another representation of the block sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block row in the arrays VAL and BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of block sparse row format the following calling sequence should be used

```
  CALL ZBSRMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *           VAL, BINDX, IA, IA(2), LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

```
zbsrsm - block sparse row format triangular solve
```

SYNOPSIS

```
       SUBROUTINE ZBSRSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
      *           VAL, BINDX, BPNTRB, BPNTRE, LB,
      *           B, LDB, BETA, C, LDC, WORK, LWORK)
       INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), LB,
      *           LDB, LDC, LWORK
       INTEGER    BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
       DOUBLE COMPLEX ALPHA, BETA
       DOUBLE COMPLEX DV(MB*LB*LB), VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK
(LWORK)

       SUBROUTINE ZBSRSM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
      *           VAL, BINDX, BPNTRB, BPNTRE, LB,
      *           B, LDB, BETA, C, LDC, WORK, LWORK)
       INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), LB,
      *           LDB, LDC, LWORK
       INTEGER*8  BINDX(BNNZ), BPNTRB(MB), BPNTRE(MB)
       DOUBLE COMPLEX ALPHA, BETA
       DOUBLE COMPLEX DV(MB*LB*LB), VAL(LB*LB*BNNZ), B(LDB,*), C(LDC,*), WORK
(LWORK)

       where: BNNZ = BPNTRE(MB)-BPNTRB(1)
```

F95 INTERFACE

```
       SUBROUTINE BSRSM(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
      *     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
       INTEGER    TRANSA, MB, N, UNITD, LB
       INTEGER, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
       DOUBLE COMPLEX    ALPHA, BETA
```

```
 DOUBLE COMPLEX, DIMENSION(:) :: VAL, DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C

 SUBROUTINE BSRSM_64(TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA, VAL, BINDX,
*     BPNTRB, BPNTRE, LB, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, N, UNITD, LB
 INTEGER*8, DIMENSION(:) ::    DESCRA, BINDX, BPNTRB, BPNTRE
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL, DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in block sparse row format
format and  op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row block scaling)
                3 : Scale on right (column block scaling)

DV()          Array of the length MB*LB*LB consisting of the block
              entries of block diagonal matrix D where each
              block is stored in standard column-major form.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
```

```
               DESCRA(1) matrix structure
                 0 : general
                 1 : symmetric (A=A')
                 2 : Hermitian (A= CONJG(A'))
                 3 : Triangular
                 4 : Skew(Anti)-Symmetric (A=-A')
                 5 : Diagonal
                 6 : Skew-Hermitian (A= -CONJG(A'))

               Note: For the routine, DESCRA(1)=3 is only supported.

               DESCRA(2) upper/lower triangular indicator
                 1 : lower
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-identity blocks on the main diagonal
                 1 : identity diagonal blocks
                 2 : diagonal blocks are dense matrices
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices
VAL()          scalar array of length LB*LB*BNNZ consisting of the block
               entries stored column-major within each dense block.

BINDX()        integer array of length BNNZ consisting of the
               block column indices of the block entries of A.
               The block column indices MUST be sorted
               in increasing order for each block row.

BPNTRB()       integer array of length MB such that
               BPNTRB(J)-BPNTRB(1)+1 points to location in BINDX
               of the first block entry of the J-th block row of A.

BPNTRE()       integer array of length MB such that
               BPNTRE(J)-BPNTRB(1) points to location in BINDX
               of the last block entry of the J-th block row of A.

LB             dimension of dense blocks composing A.

B()            rectangular array with first dimension LDB.

LDB            leading dimension of B

BETA           Scalar parameter

C()            rectangular array with first dimension LDC.
```

LDC leading dimension of C

WORK() scratch array of length LWORK.
On exit, if LWORK= -1, WORK(1) returns the optimum size of LWORK.

LWORK length of WORK array. LWORK should be at least MB*LB.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=MB*LB*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If *DESCRA*(3)=0,the lower or upper triangular part of each diagonal block is used by the routine depending on *DESCRA*(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might not be referenced in the BSC representation of a sparse matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense matrices and the LU factorization with partial pivoting is

used by the routine. *WORK*(1)=0 on return if the factorization for all diagonal blocks has been completed successfully, otherwise *WORK*(1) = -i where i is the block number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3 in this case.

6. It is known that there exists another representation of the block sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block row in the arrays VAL and BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of block sparse row format the following calling sequence should be used

```
  CALL ZBSRSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, BINDX, IA, IA(2), LB,
 *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

```
zcnvcor - compute the convolution or correlation of  complex
vectors
```

SYNOPSIS

```
SUBROUTINE ZCNVCOR(CNVCOR, FOUR, NX, X, IFX, INCX, NY, NPRE, M, Y,
      IFY, INC1Y, INC2Y, NZ, K, Z, IFZ, INC1Z, INC2Z, WORK, LWORK)

CHARACTER * 1 CNVCOR, FOUR
DOUBLE COMPLEX X(*), Y(*), Z(*), WORK(*)
INTEGER NX, IFX, INCX, NY, NPRE, M, IFY, INC1Y,  INC2Y,  NZ,
K, IFZ, INC1Z, INC2Z, LWORK

SUBROUTINE ZCNVCOR_64(CNVCOR, FOUR, NX, X, IFX, INCX, NY, NPRE, M, Y,
      IFY, INC1Y, INC2Y, NZ, K, Z, IFZ, INC1Z, INC2Z, WORK, LWORK)

CHARACTER * 1 CNVCOR, FOUR
DOUBLE COMPLEX X(*), Y(*), Z(*), WORK(*)
INTEGER*8 NX, IFX, INCX, NY, NPRE, M, IFY, INC1Y, INC2Y, NZ,
K, IFZ, INC1Z, INC2Z, LWORK
```

F95 INTERFACE

```
SUBROUTINE CNVCOR(CNVCOR, FOUR, [NX], X, IFX, [INCX], NY, NPRE, M, Y,
       IFY, INC1Y, INC2Y, NZ, K, Z, IFZ, INC1Z, INC2Z, WORK, [LWORK])

CHARACTER(LEN=1) :: CNVCOR, FOUR
COMPLEX(8), DIMENSION(:) :: X, Y, Z, WORK
INTEGER :: NX, IFX, INCX, NY, NPRE, M,  IFY,  INC1Y,  INC2Y,
NZ, K, IFZ, INC1Z, INC2Z, LWORK

SUBROUTINE CNVCOR_64(CNVCOR, FOUR, [NX], X, IFX, [INCX], NY, NPRE, M,
       Y, IFY, INC1Y, INC2Y, NZ, K, Z, IFZ, INC1Z, INC2Z, WORK, [LWORK])
```

```
CHARACTER(LEN=1) :: CNVCOR, FOUR
COMPLEX(8), DIMENSION(:) :: X, Y, Z, WORK
INTEGER(8) :: NX, IFX, INCX, NY, NPRE, M, IFY, INC1Y, INC2Y,
NZ, K, IFZ, INC1Z, INC2Z, LWORK
```

C INTERFACE

```
#include <sunperf.h>

void zcnvcor(char cnvcor, char four, int  nx,  doublecomplex
          *x,  int  ifx,  int incx, int ny, int npre, int m,
          doublecomplex *y, int ify, int inc1y,  int  inc2y,
          int  nz,  int  k,  doublecomplex  *z, int ifz, int
          inc1z, int inc2z, doublecomplex *work, int lwork);
void zcnvcor_64(char cnvcor, char four, long nx,  doublecom-
          plex  *x, long ifx, long incx, long ny, long npre,
          long m, doublecomplex *y, long  ify,  long  inc1y,
          long  inc2y,  long  nz,  long k, doublecomplex *z,
          long ifz, long inc1z,  long  inc2z,  doublecomplex
          *work, long lwork);
```

PURPOSE

zcnvcor computes the convolution or correlation of complex vectors.

ARGUMENTS

CNVCOR (input)
'V' or 'v' if convolution is desired, 'R' or 'r' if correlation is desired.

FOUR (input)
'T' or 't' if the Fourier transform method is to be used, 'D' or 'd' if the computation should be done directly from the definition. The Fourier transform method is generally faster, but it may introduce noticeable errors into certain results, notably when both the real and imaginary parts of the filter and data vectors consist entirely of integers or vectors where elements of either the filter vector or a given data vector differ significantly in magnitude from the 1-norm of the vector.

NX (input)
Length of the filter vector. NX >= 0. ZCNVCOR

will return immediately if NX = 0.

X (input) dimension(*)
Filter vector.

IFX (input)
Index of the first element of X. NX >= IFX >= 1.

INCX (input)
Stride between elements of the filter vector in X. INCX > 0.

NY (input)
Length of the input vectors. NY >= 0. ZCNVCOR will return immediately if NY = 0.

NPRE (input)
The number of implicit zeros prepended to the Y vectors. NPRE >= 0.

M (input)
Number of input vectors. M >= 0. ZCNVCOR will return immediately if M = 0.

Y (input) dimension(*)
Input vectors.

IFY (input)
Index of the first element of Y. NY >= IFY >= 1.

INC1Y (input)
Stride between elements of the input vectors in Y. INC1Y > 0.

INC2Y (input)
Stride between the input vectors in Y. INC2Y > 0.

NZ (input)
Length of the output vectors. NZ >= 0. ZCNVCOR will return immediately if NZ = 0. See the Notes section below for information about how this argument interacts with NX and NY to control circular versus end-off shifting.

K (input)
Number of Z vectors. K >= 0. If K = 0 then ZCNVCOR will return immediately. If K < M then only the first K input vectors will be processed. If K > M then M input vectors will be processed.

Z (output)
dimension(*)
Result vectors.

IFZ (input)
Index of the first element of Z. NZ >= IFZ >= 1.

INC1Z (input)
Stride between elements of the output vectors in Z. INC1Z > 0.

INC2Z (input)
Stride between the output vectors in Z. INC2Z > 0.

WORK (input/output)
(input/scratch) dimension(LWORK)
Scratch space. Before the first call to ZCNVCOR with particular values of the integer arguments the first element of WORK must be set to zero. If WORK is written between calls to ZCNVCOR or if ZCNVCOR is called with different values of the integer arguments then the first element of WORK must again be set to zero before each call. If WORK has not been written and the same values of the integer arguments are used then the first element of WORK to zero. This can avoid certain initializations that store their results into WORK, and avoiding the initialization can make ZCNVCOR run faster.

LWORK (input)
Length of WORK. LWORK >= 2*MAX(NX,NY,NZ)+8.

Contents

NAME

```
zcnvcor2 - compute the convolution or correlation of complex
matrices
```

SYNOPSIS

```
SUBROUTINE ZCNVCOR2(CNVCOR, METHOD, TRANSX, SCRATCHX, TRANSY,
      SCRATCHY, MX, NX, X, LDX, MY, NY, MPRE, NPRE, Y, LDY, MZ, NZ, Z,
      LDZ, WORKIN, LWORK)

CHARACTER * 1  CNVCOR,  METHOD,  TRANSX,  SCRATCHX,  TRANSY,
SCRATCHY
DOUBLE COMPLEX X(LDX,*), Y(LDY,*), Z(LDZ,*), WORKIN(*)
INTEGER MX, NX, LDX, MY, NY, MPRE, NPRE, LDY, MZ,  NZ,  LDZ,
LWORK

SUBROUTINE ZCNVCOR2_64(CNVCOR, METHOD, TRANSX, SCRATCHX, TRANSY,
      SCRATCHY, MX, NX, X, LDX, MY, NY, MPRE, NPRE, Y, LDY, MZ, NZ, Z,
      LDZ, WORKIN, LWORK)

CHARACTER * 1  CNVCOR,  METHOD,  TRANSX,  SCRATCHX,  TRANSY,
SCRATCHY
DOUBLE COMPLEX X(LDX,*), Y(LDY,*), Z(LDZ,*), WORKIN(*)
INTEGER*8 MX, NX, LDX, MY, NY, MPRE, NPRE, LDY, MZ, NZ, LDZ,
LWORK
```

F95 INTERFACE

```
SUBROUTINE CNVCOR2(CNVCOR, METHOD, TRANSX, SCRATCHX, TRANSY,
       SCRATCHY, [MX], [NX], X, [LDX], [MY], [NY], MPRE, NPRE, Y, [LDY],
       [MZ], [NZ], Z, [LDZ], WORKIN, [LWORK])

CHARACTER(LEN=1)  ::  CNVCOR,  METHOD,   TRANSX,   SCRATCHX,
TRANSY, SCRATCHY
```

```
COMPLEX(8), DIMENSION(:) :: WORKIN
COMPLEX(8), DIMENSION(:,:) :: X, Y, Z
INTEGER :: MX, NX, LDX, MY, NY, MPRE,  NPRE,  LDY,  MZ,  NZ,
LDZ, LWORK

SUBROUTINE CNVCOR2_64(CNVCOR, METHOD, TRANSX, SCRATCHX, TRANSY,
       SCRATCHY, [MX], [NX], X, [LDX], [MY], [NY], MPRE, NPRE, Y, [LDY],
       [MZ], [NZ], Z, [LDZ], WORKIN, [LWORK])

CHARACTER(LEN=1)  ::  CNVCOR,  METHOD,   TRANSX,   SCRATCHX,
TRANSY, SCRATCHY
COMPLEX(8), DIMENSION(:) :: WORKIN
COMPLEX(8), DIMENSION(:,:) :: X, Y, Z
INTEGER(8) :: MX, NX, LDX, MY, NY, MPRE, NPRE, LDY, MZ,  NZ,
LDZ, LWORK
```

C INTERFACE

```
#include <sunperf.h>

void zcnvcor2(char cnvcor, char method,  char  transx,  char
          scratchx,  char transy, char scratchy, int mx, int
          nx, doublecomplex *x, int ldx, int my, int ny, int
          mpre, int npre, doublecomplex *y, int ldy, int mz,
          int nz, doublecomplex *z, int  ldz,  doublecomplex
          *workin, int lwork);

void zcnvcor2_64(char cnvcor, char method, char transx, char
          scratchx,  char  transy,  char  scratchy, long mx,
          long nx, doublecomplex *x, long ldx, long my, long
          ny,  long  mpre, long npre, doublecomplex *y, long
          ldy, long mz, long nz, doublecomplex *z, long ldz,
          doublecomplex *workin, long lwork);
```

PURPOSE

zcnvcor2 computes the convolution or correlation of complex matrices.

ARGUMENTS

CNVCOR (input)
'V' or 'v' to compute convolution, 'R' or 'r' to compute correlation.

METHOD (input)
'T' or 't' if the Fourier transform method is to be used, 'D' or 'd' to compute directly from the definition.

TRANSX (input)
'N' or 'n' if X is the filter matrix, 'T' or 't' if transpose(X) is the filter matrix.

SCRATCHX (input)
'N' or 'n' if X must be preserved, 'S' or 's' if X can be used as scratch space. The contents of X are undefined after returning from a call in which X is allowed to be used for scratch.

TRANSY (input)
'N' or 'n' if Y is the input matrix, 'T' or 't' if transpose(Y) is the input matrix.

SCRATCHY (input)
'N' or 'n' if Y must be preserved, 'S' or 's' if Y can be used as scratch space. The contents of Y are undefined after returning from a call in which Y is allowed to be used for scratch.

MX (input)
Number of rows in the filter matrix. MX >= 0.

NX (input)
Number of columns in the filter matrix. NX >= 0.

X (input) dimension(LDX,NX)
On entry, the filter matrix. Unchanged on exit if SCRATCHX is 'N' or 'n', undefined on exit if SCRATCHX is 'S' or 's'.

LDX (input)
Leading dimension of the array that contains the filter matrix.

MY (input)
Number of rows in the input matrix. MY >= 0.

NY (input)
Number of columns in the input matrix. NY >= 0.

MPRE (input)
Number of implicit zeros to prepend to each row of the input matrix. MPRE >= 0.

NPRE (input)
Number of implicit zeros to prepend to each column of the input matrix. NPRE >= 0.

Y (input) dimension(LDY,*)

Input matrix. Unchanged on exit if SCRATCHY is 'N' or 'n', undefined on exit if SCRATCHY is 'S' or 's'.

LDY (input)
Leading dimension of the array that contains the input matrix.

MZ (input)
Number of rows in the output matrix. MZ >= 0. ZCNVCOR2 will return immediately if MZ = 0.

NZ (input)
Number of columns in the output matrix. NZ >= 0. ZCNVCOR2 will return immediately if NZ = 0.

Z (output)
dimension(LDZ,*)
Result matrix.

LDZ (input)
Leading dimension of the array that contains the result matrix. LDZ >= MAX(1,MZ).

WORKIN (input/output)
(input/scratch) dimension(LWORK)
On entry for the first call to ZCNVCOR2, WORKIN(1) must contain CMPLX(0.0,0.0). After the first call, WORKIN(1) must be set to CMPLX(0.0,0.0) iff WORKIN has been altered since the last call to this subroutine or if the sizes of the arrays have changed.

LWORK (input)
Length of the work vector. If the FFT is to be used then for best performance LWORK should be at least 30 words longer than the amount of memory needed to hold the trig tables. If the FFT is not used, the value of LWORK is unimportant.

Contents

NAME

zcoomm - coordinate matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE ZCOOMM( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, JNDX, NNZ,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, K, DESCRA(5), NNZ
*           LDB, LDC, LWORK
 INTEGER    INDX(NNZ), JNDX(NNZ)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE ZCOOMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, JNDX, NNZ,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, K, DESCRA(5), NNZ
*           LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), JNDX(NNZ)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE COOMM( TRANSA, M, [N], K, ALPHA, DESCRA,
*           VAL, INDX, JNDX, NNZ, B, [LDB], BETA, C, [LDC],
*           [WORK], [LWORK] )
 INTEGER TRANSA, M, K,  NNZ
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, JNDX
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

```
       SUBROUTINE COOMM_64( TRANSA, M, [N], K, ALPHA, DESCRA,
      *           VAL, INDX, JNDX, NNZ, B, [LDB], BETA, C, [LDC],
      *           [WORK], [LWORK] )
       INTEGER*8 TRANSA, M, K,  NNZ
       INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, JNDX
       DOUBLE COMPLEX    ALPHA, BETA
       DOUBLE COMPLEX, DIMENSION(:) :: VAL
       DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
            C <- alpha op(A) B + beta C

      where ALPHA and BETA are scalar, C and B are dense matrices,
      A is a matrix represented in coordinate format and
      op( A )  is one  of
      op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                            ( ' indicates matrix transpose)
```

ARGUMENTS

```
      TRANSA        Indicates how to operate with the sparse matrix
                      0 : operate with matrix
                      1 : operate with transpose matrix
                      2 : operate with the conjugate transpose of matrix.
                          2 is equivalent to 1 if matrix is real.

      M             Number of rows in matrix A

      N             Number of columns in matrix C

      K             Number of columns in matrix A

      ALPHA         Scalar parameter

      DESCRA()      Descriptor argument.  Five element integer array
                    DESCRA(1) matrix structure
                      0 : general
                      1 : symmetric (A=A')
                      2 : Hermitian (A= CONJG(A'))
                      3 : Triangular
                      4 : Skew(Anti)-Symmetric (A=-A')
                      5 : Diagonal
                      6 : Skew-Hermitian (A= -CONJG(A'))
                    DESCRA(2) upper/lower triangular indicator
                      1 : lower
                      2 : upper
```

```
               DESCRA(3) main diagonal type
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices

 VAL()         scalar array of length NNZ consisting of the
               non-zero entries of A, in any order.

 INDX()        integer array of length NNZ consisting of the
               corresponding row indices of the entries of A.

 JNDX()        integer array of length NNZ consisting of the
               corresponding column indices of the entries of A.

 NNZ           number of non-zero elements in A.
 B()           rectangular array with first dimension LDB.

 LDB           leading dimension of B

 BETA          Scalar parameter

 C()           rectangular array with first dimension LDC.

 LDC           leading dimension of C

 WORK()        scratch array of length LWORK. WORK is not
               referenced in the current version.

 LWORK         length of WORK array. LWORK is not referenced
               in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

```
zcopy - Copy x to y
```

SYNOPSIS

```
SUBROUTINE ZCOPY(N, X, INCX, Y, INCY)

DOUBLE COMPLEX X(*), Y(*)
INTEGER N, INCX, INCY

SUBROUTINE ZCOPY_64(N, X, INCX, Y, INCY)

DOUBLE COMPLEX X(*), Y(*)
INTEGER*8 N, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE COPY([N], X, [INCX], Y, [INCY])

COMPLEX(8), DIMENSION(:) :: X, Y
INTEGER :: N, INCX, INCY

SUBROUTINE COPY_64([N], X, [INCX], Y, [INCY])

COMPLEX(8), DIMENSION(:) :: X, Y
INTEGER(8) :: N, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>

void zcopy(int n, doublecomplex *x, int incx,  doublecomplex
          *y, int incy);

void zcopy_64(long n, doublecomplex  *x,  long  incx,  doub-
```

```
          lecomplex *y, long incy);
```

PURPOSE

zcopy Copy x to y where x and y are n-vectors.

ARGUMENTS

```
N (input)
          On entry, N specifies the number  of  elements  in
          the  vector.   N must be at least one for the sub-
          routine to have any visible effect.  Unchanged  on
          exit.
X (input)
          of DIMENSION at least ( 1 + ( n - 1 )*abs( INCX  )
          ).   Before  entry,  the  incremented array X must
          contain the vector x.  Unchanged on exit.

INCX (input)
          On entry, INCX specifies  the  increment  for  the
          elements  of X.  INCX must not be zero.  Unchanged
          on exit.

Y (output)
          of DIMENSION at least ( 1 + ( m - 1 )*abs( INCY  )
          ).  On entry, the incremented array Y must contain
          the vector y. On exit, Y  is  overwritten  by  the
          vector x.

INCY (input)
          On entry, INCY specifies  the  increment  for  the
          elements  of Y.  INCY must not be zero.  Unchanged
          on exit.
```

Contents

NAME

zcscmm - compressed sparse column format matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE ZCSCMM( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, K, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTRB(K), PNTRE(K)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE ZCSCMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, K, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTRB(K), PNTRE(K)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTRE(K)-PNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE CSCMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER TRANSA, M, K
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL
```

```
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C

 SUBROUTINE CSCMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER*8 TRANSA, M, K
 INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in compressed sparse column format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
```

```
                    2 : upper
                  DESCRA(3) main diagonal type
                    0 : non-unit
                    1 : unit
                  DESCRA(4) Array base  (NOT IMPLEMENTED)
                    0 : C/C++ compatible
                    1 : Fortran compatible
                  DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                    0 : unknown
                    1 : no repeated indices

 VAL()            scalar array of length NNZ consisting of nonzero
                  entries of A.

 INDX()           integer array of length NNZ consisting of the row
                  indices of nonzero entries of A.

 PNTRB()          integer array of length K such that PNTRB(J)-PNTRB(1)+1
                  points to location in VAL of the first nonzero element
                  in column J.
 PNTRE()          integer array of length K such that PNTRE(J)-PNTRB(1)
                  points to location in VAL of the last nonzero element
                  in column J.

 B()              rectangular array with first dimension LDB.

 LDB              leading dimension of B

 BETA             Scalar parameter

 C()              rectangular array with first dimension LDC.

 LDC              leading dimension of C

 WORK()           scratch array of length LWORK. WORK is not
                  referenced in the current version.

 LWORK            length of WORK array. LWORK is not referenced
                  in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS)
Standard", University of Tennessee, Knoxville, Tennessee,

1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

It is known that there exists another representation of the compressed sparse column format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each column in the arrays VAL and INDX is used instead of two arrays PNTRB and PNTRE. To use the routine with this kind of sparse column format the following calling sequence should be used

```
  SUBROUTINE SCSCMM( TRANSA, M, N, K, ALPHA, DESCRA,
 *           VAL, INDX, IA, IA(2), B, LDB, BETA,
 *           C, LDC, WORK, LWORK )
```

Contents

NAME

```
zcscsm - compressed sparse column format triangular solve
```

SYNOPSIS

```
 SUBROUTINE ZCSCSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, INDX, PNTRB, PNTRE,
*            B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5),
*            LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTRB(M), PNTRE(M)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE ZCSCSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, INDX, PNTRB, PNTRE,
*            B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5),
*            LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTRB(M), PNTRE(M)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTRE(M)-PNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE CSCSM( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER TRANSA, M, UNITD
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL, DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE CSCSM_64(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER*8 TRANSA, M, UNITD
 INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL, DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C
```

where ALPHA and BETA are scalar, C and B are m by n dense matrices, D is a diagonal scaling matrix, A is a unit, or non-unit, upper or lower triangular matrix represented in compressed sparse column format and op(A) is one of
op(A) = inv(A) or op(A) = inv(A') or op(A) =inv(conjg(A'))
(inv denotes matrix inverse, ' indicates matrix transpose)

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic column scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                  0 : general
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))

                Note: For the routine, DESCRA(1)=3 is only supported.

                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices
VAL()           scalar array of length NNZ consisting of nonzero entries
                of A.

INDX()          integer array of length NNZ consisting of the row indices
                of nonzero entries of A. (Row indices MUST be sorted in
                increasing order for each column).

PNTRB()         integer array of length M such that PNTRB(J)-PNTRB(1)+1
                points to location in VAL of the first nonzero element
                in column J.

PNTRE()         integer array of length M such that PNTRE(J)-PNTRB(1)
                points to location in VAL of the last nonzero element
                in column J.

B()             rectangular array with first dimension LDB.

LDB             leading dimension of B

BETA            Scalar parameter

C()             rectangular array with first dimension LDC.

LDC             leading dimension of C

WORK()          scratch array of length LWORK.

                On exit,  if LWORK = -1, WORK(1) returns the optimum  LWORK.
```

LWORK length of WORK array. LWORK should be at least M.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the columns of A such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the columns have been scaled. UNITD=3 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the column number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the CSC representation

of a sparse matrix. They are not used anyway in these cases. But if UNITD=4, the unit diagonal elements MUST be referenced in the CSC representation.

4. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

5. It is known that there exists another representation of the compressed sparse column format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each column in the arrays VAL and INDX is used instead of two arrays PNTRB and PNTRE. To use the routine with this kind of sparse column format the following calling sequence should be used

```
  SUBROUTINE SCSCSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, INDX, IA, IA(2), B, LDB, BETA,
 *           C, LDC, WORK, LWORK )
```

Contents

NAME

zcsrmm - compressed sparse row format matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE ZCSRMM( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, K, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTRB(M), PNTRE(M)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE ZCSRMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, K, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTRB(M), PNTRE(M)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTRE(M)-PNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE CSRMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER TRANSA, M, K
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE CSRMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER*8 TRANSA, M, K
 INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in compressed sparse row format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
```

```
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

 VAL()          scalar array of length NNZ consisting of nonzero
                entries of A.

 INDX()         integer array of length NNZ consisting of the
                column indices of nonzero entries of A.

 PNTRB()        integer array of length M such that PNTRB(J)-PNTRB(1)+1
                points to location in VAL of the first nonzero element
                in row J.
 PNTRE()        integer array of length M such that PNTRE(J)-PNTRB(1)
                points to location in VAL of the last nonzero element
                in row J.

 B()            rectangular array with first dimension LDB.

 LDB            leading dimension of B

 BETA           Scalar parameter

 C()            rectangular array with first dimension LDC.

 LDC            leading dimension of C

 WORK()         scratch array of length LWORK. WORK is not
                referenced in the current version.

 LWORK          length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

It is known that there exists another representation of the compressed sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each row in the arrays VAL and INDX is used instead of two arrays PNTRB and PNTRE. To use the routine with this kind of compressed sparse row format the following calling sequence should be used

```
  SUBROUTINE SCSRMM( TRANSA, M, N, K, ALPHA, DESCRA,
 *           VAL, INDX, IA, IA(2), B, LDB, BETA,
 *           C, LDC, WORK, LWORK )
```

Contents

NAME

```
zcsrsm - compressed sparse row format triangular solve
```

SYNOPSIS

```
 SUBROUTINE ZCSRSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTRB(M), PNTRE(M)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE ZCSRSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, PNTRB, PNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTRB(M), PNTRE(M)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTRE(M)-PNTRB(1)
```

F95 INTERFACE

```
 SUBROUTINE CSRSM( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER TRANSA, M, UNITD
 INTEGER, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL, DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE CSRSM_64(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*   PNTRB, PNTRE, B, [LDB], BETA, C, [LDC], [WORK], [LWORK] )
 INTEGER*8 TRANSA, M, UNITD
 INTEGER*8, DIMENSION(:) ::   DESCRA, INDX, PNTRB, PNTRE
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL, DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a diagonal scaling matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in compressed sparse row
format and op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of
              the scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                  0 : general
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))

                Note: For the routine, only DESCRA(1)=3 is supported.

                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices
VAL()           scalar array of length NNZ consisting of nonzero entries
                of A.

INDX()          integer array of length NNZ consisting of the column
                indices of nonzero entries of A (column indices MUST be
                sorted in increasing order for each row)

PNTRB()         integer array of length M such that PNTRB(J)-PNTRB(1)+1
                points to location in VAL of the first nonzero element
                in row J.

PNTRE()         integer array of length M such that PNTRE(J)-PNTRB(1)
                points to location in VAL of the last nonzero element
                in row J.

B()             rectangular array with first dimension LDB.

LDB             leading dimension of B

BETA            Scalar parameter

C()             rectangular array with first dimension LDC.

LDC             leading dimension of C

WORK()          scratch array of length LWORK.

                On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.
```

LWORK length of WORK array. LWORK should be at least M.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the rows of A such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows have been scaled. UNITD=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the CSR representation of a sparse matrix. They are not used anyway in these cases.

But if UNITD=4, the unit diagonal elements MUST be referenced in the CSR representation.

4. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

5. It is known that there exists another representation of the compressed sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of three array instead of the four used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each row in the arrays VAL and INDX is used instead of two arrays PNTRB and PNTRE. To use the routine with this kind of compressed sparse row format the following calling sequence should be used

```
  SUBROUTINE SCSRSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
 *           VAL, INDX, IA, IA(2), B, LDB, BETA, C,
 *            LDC, WORK, LWORK )
```

Contents

NAME

zdiamm - diagonal format matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE ZDIAMM( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, LDA, IDIAG, NDIAG,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, K, DESCRA(5), LDA, NDIAG,
*           LDB, LDC, LWORK
 INTEGER    IDIAG(NDIAG)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(LDA,NDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE ZDIAMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, LDA, IDIAG, NDIAG,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, K, DESCRA(5), LDA, NDIAG,
*           LDB, LDC, LWORK
 INTEGER*8  IDIAG(NDIAG)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(LDA,NDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
  SUBROUTINE DIAMM(TRANSA, M, [N], K, ALPHA, DESCRA, VAL, [LDA],
*    IDIAG, NDIAG, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, K,  NDIAG
 INTEGER, DIMENSION(:) ::    DESCRA, IDIAG
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:, :) ::  VAL, B, C

  SUBROUTINE DIAMM_64(TRANSA, M, [N], K, ALPHA, DESCRA, VAL, [LDA],
*    IDIAG, NDIAG, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
```

```
 INTEGER*8    TRANSA, M, K,  NDIAG
 INTEGER*8, DIMENSION(:) ::    DESCRA, IDIAG
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:, :) ::  VAL, B, C
```

DESCRIPTION

```
         C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in diagonal format and op( A ) is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)

TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
                0 : non-unit
                1 : unit
              DESCRA(4) Array base  (NOT IMPLEMENTED)
                0 : C/C++ compatible
                1 : Fortran compatible
              DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                0 : unknown
                1 : no repeated indices
```

VAL()	two-dimensional LDA-by-NDIAG array such that VAL(:,I) consists of non-zero elements on diagonal IDIAG(I) of A. Diagonals in the lower triangular part of A are padded from the top, and those in the upper triangular part are padded from the bottom.
LDA	leading dimension of VAL, must be .GE. MIN(M,K)
IDIAG()	integer array of length NDIAG consisting of the corresponding diagonal offsets of the non-zero diagonals of A in VAL. Lower triangular diagonals have negative offsets, the main diagonal has offset 0, and upper triangular diagonals have positive offset.
NDIAG	number of non-zero diagonals in A.
B()	rectangular array with first dimension LDB.
LDB	leading dimension of B
BETA	Scalar parameter
C()	rectangular array with first dimension LDC.
LDC	leading dimension of C
WORK()	scratch array of length LWORK. WORK is not referenced in the current version.
LWORK	length of WORK array. LWORK is not referenced in the current version.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

zdiasm - diagonal format triangular solve

SYNOPSIS

```
 SUBROUTINE ZDIASM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, LDA, IDIAG, NDIAG,
*            B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER     TRANSA, M, N, UNITD, DESCRA(5), LDA, NDIAG,
*            LDB, LDC, LWORK
 INTEGER     IDIAG(NDIAG)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX DV(M), VAL(LDA,NDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE ZDIASM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, LDA, IDIAG, NDIAG,
*            B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8   TRANSA, M, N, UNITD, DESCRA(5), LDA, NDIAG,
*            LDB, LDC, LWORK
 INTEGER*8   IDIAG(NDIAG)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX DV(M), VAL(LDA,NDIAG), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
  SUBROUTINE DIASM(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
*   [LDA], IDIAG, NDIAG, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER     TRANSA, M, NDIAG
 INTEGER, DIMENSION(:) ::     DESCRA, IDIAG
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) ::   DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::  VAL, B, C

  SUBROUTINE DIASM_64(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
```

```
*   [LDA], IDIAG, NDIAG, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, NDIAG
 INTEGER*8, DIMENSION(:) ::    DESCRA, IDIAG
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) ::  DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::  VAL, B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C
```

where ALPHA and BETA are scalar, C and B are m by n dense matrices, D is a diagonal scaling matrix, A is a unit, or non-unit, upper or lower triangular matrix represented in diagonal format and op(A) is one of

```
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
```

```
                 1 : symmetric (A=A')
                 2 : Hermitian (A= CONJG(A'))
                 3 : Triangular
                 4 : Skew(Anti)-Symmetric (A=-A')
                 5 : Diagonal
                 6 : Skew-Hermitian (A= -CONJG(A'))
               Note: For the routine, only DESCRA(1)=3 is supported.

               DESCRA(2) upper/lower triangular indicator
                 1 : lower
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices
VAL()          two-dimensional LDA-by-NDIAG array such that VAL(:,I)
               consists of non-zero elements on diagonal IDIAG(I)
               of A.  Diagonals in the lower triangular part of A
               are padded from the top, and those in the upper
               triangular part are padded from the bottom.

LDA            leading dimension of VAL, must be .GE. MIN(M,K)

IDIAG()        integer array of length NDIAG consisting of the
               corresponding diagonal offsets of the non-zero
               diagonals of A in VAL.  Lower triangular diagonals
               have negative offsets, the main diagonal has offset
               0, and upper triangular diagonals have positive offset.
               Elements of IDIAG of MUST be sorted in  increasing order.

NDIAG          number of non-zero diagonals in A.

B()            rectangular array with first dimension LDB.

LDB            leading dimension of B

BETA           Scalar parameter

C()            rectangular array with first dimension LDC.

LDC            leading dimension of C

WORK()         scratch array of length LWORK.
               On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.
```

```
LWORK         length of WORK array.  LWORK should be at least M.

              For good performance, LWORK should generally be larger.
              For optimum performance on multiple processors, LWORK
              >=M*N_CPUS where N_CPUS is the maximum number of
              processors available to the program.

              If LWORK=0, the routine is to allocate workspace needed.

              If LWORK = -1, then a workspace query is assumed; the
              routine only calculates the optimum size of the WORK
              array, returns this value as the first entry of the WORK
              array, and no error message related to LWORK is issued
              by XERBLA.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the rows of A such that their 2-norm are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows have been scaled. UNITD=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the DIA representation of a sparse matrix. They are not used anyway in these cases. But if UNITD=4, the unit diagonal elements MUST be referenced in the DIA representation.

4. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

```
zdotc - compute the dot product of two vectors conjg(x)  and
y.
```

SYNOPSIS

```
DOUBLE COMPLEX FUNCTION ZDOTC(N, X, INCX, Y, INCY)

DOUBLE COMPLEX X(*), Y(*)
INTEGER N, INCX, INCY

DOUBLE COMPLEX FUNCTION ZDOTC_64(N, X, INCX, Y, INCY)

DOUBLE COMPLEX X(*), Y(*)
INTEGER*8 N, INCX, INCY
```

F95 INTERFACE

```
COMPLEX(8) FUNCTION DOTC([N], X, [INCX], Y, [INCY])

COMPLEX(8), DIMENSION(:) :: X, Y
INTEGER :: N, INCX, INCY

COMPLEX(8) FUNCTION DOTC_64([N], X, [INCX], Y, [INCY])

COMPLEX(8), DIMENSION(:) :: X, Y
INTEGER(8) :: N, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>

doublecomplex zdotc(int n, doublecomplex *x, int incx, doub-
          lecomplex *y, int incy);
```

```
doublecomplex zdotc_64(long n, doublecomplex *x, long  incx,
          doublecomplex *y, long incy);
```

PURPOSE

zdotc compute the dot product of conjg(x) and y where x and y are n-vectors.

ARGUMENTS

N (input)
: On entry, N specifies the number of elements in the vector. If N is not positive then the function returns the value 0.0. Unchanged on exit.

X (input)
: of DIMENSION at least (1 + (n - 1)*abs(INCX)). On entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
: On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input)
: of DIMENSION at least (1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. Unchanged on exit.

INCY (input)
: On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
zdotci - Compute the complex conjugated indexed dot product.
```

SYNOPSIS

```
 DOUBLE COMPLEX FUNCTION ZDOTCI(NZ, X, INDX, Y)

 DOUBLE COMPLEX X(*), Y(*)
 INTEGER NZ
 INTEGER INDX(*)

 DOUBLE COMPLEX FUNCTION ZDOTCI_64(NZ, X, INDX, Y)

 DOUBLE COMPLEX X(*), Y(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 DOUBLE COMPLEX FUNCTION DOTCI([NZ], X, INDX, Y)

 COMPLEX(8), DIMENSION(:) :: X, Y
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 DOUBLE COMPLEX FUNCTION DOTCI_64([NZ], X, INDX, Y)

 COMPLEX(8), DIMENSION(:) :: X, Y
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
ZDOTCI Compute the complex conjugated indexed dot product of
a complex sparse vector x stored in compressed form with a
```

complex vector y in full storage form.

```
 dot = 0
 do i = 1, n
   dot = dot + conjg(x(i)) * y(indx(i))
 enddo
```

ARGUMENTS

NZ (input)
Number of elements in the compressed form.
Unchanged on exit.

X (input)
Vector in compressed form. Unchanged on exit.

INDX (input)
Vector containing the indices of the compressed form. It is assumed that the elements in INDX are distinct and greater than zero. Unchanged on exit.

Y (input)
Vector in full storage form. Only the elements corresponding to the indices in INDX will be accessed.

Contents

NAME

```
zdotu - compute the dot product of two vectors x and y.
```

SYNOPSIS

```
DOUBLE COMPLEX FUNCTION ZDOTU(N, X, INCX, Y, INCY)

DOUBLE COMPLEX X(*), Y(*)
INTEGER N, INCX, INCY

DOUBLE COMPLEX FUNCTION ZDOTU_64(N, X, INCX, Y, INCY)

DOUBLE COMPLEX X(*), Y(*)
INTEGER*8 N, INCX, INCY
```

F95 INTERFACE

```
COMPLEX(8) FUNCTION DOT([N], X, [INCX], Y, [INCY])

COMPLEX(8), DIMENSION(:) :: X, Y
INTEGER :: N, INCX, INCY

COMPLEX(8) FUNCTION DOT_64([N], X, [INCX], Y, [INCY])

COMPLEX(8), DIMENSION(:) :: X, Y
INTEGER(8) :: N, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>

doublecomplex zdotu(int n, doublecomplex *x, int incx, doub-
          lecomplex *y, int incy);

doublecomplex zdotu_64(long n, doublecomplex *x, long  incx,
```

```
          doublecomplex *y, long incy);
```

PURPOSE

```
zdotu compute the dot product of x and y where x and  y  are
n-vectors.
```

ARGUMENTS

```
N (input)
          On entry, N specifies the number  of  elements  in
          the  vector.   If N is not positive then the func-
          tion returns the value 0.0.  Unchanged on exit.
X (input)
          of DIMENSION at least ( 1 + ( n - 1 )*abs( INCX  )
          ).  On entry, the incremented array X must contain
          the vector x.  Unchanged on exit.

INCX (input)
          On entry, INCX specifies  the  increment  for  the
          elements  of  X. INCX must not be zero.  Unchanged
          on exit.

Y (input)
          of DIMENSION at least ( 1 + ( n - 1 )*abs( INCY  )
          ).  On entry, the incremented array Y must contain
          the vector y.  Unchanged on exit.

INCY (input)
          On entry, INCY specifies  the  increment  for  the
          elements  of  Y. INCY must not be zero.  Unchanged
          on exit.
```

Contents

NAME

```
zdotui - Compute the complex unconjugated indexed dot
product.
```

SYNOPSIS

```
 DOUBLE COMPLEX FUNCTION CDOTCI(NZ, X, INDX, Y)

 DOUBLE COMPLEX X(*), Y(*)
 INTEGER NZ
 INTEGER INDX(*)

 DOUBLE COMPLEX FUNCTION CDOTCI_64(NZ, X, INDX, Y)

 DOUBLE COMPLEX X(*), Y(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 DOUBLE COMPLEX FUNCTION DOTCI([NZ], X, INDX, Y)

 COMPLEX(8), DIMENSION(:) :: X, Y
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 DOUBLE COMPLEX FUNCTION DOTCI_64([NZ], X, INDX, Y)

 COMPLEX(8), DIMENSION(:) :: X, Y
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
ZDOTUI Compute the complex unconjugated indexed dot product
```

of a complex sparse vector x stored in compressed form with
a complex vector y in full storage form.

```
 dot = 0
 do i = 1, n
   dot = dot + x(i) * y(indx(i))
 enddo
```

ARGUMENTS

NZ (input)
Number of elements in the compressed form.
Unchanged on exit.

X (input)
Vector in compressed form. Unchanged on exit.

INDX (input)
Vector containing the indices of the compressed
form. It is assumed that the elements in INDX are
distinct and greater than zero. Unchanged on exit.

Y (input)
Vector in full storage form. Only the elements
corresponding to the indices in INDX will be
accessed.

Contents

NAME

```
zdrot - Apply a plane rotation.
```

SYNOPSIS

```
SUBROUTINE ZDROT(N, CX, INCX, CY, INCY, C, S)

DOUBLE PRECISION C, S
DOUBLE COMPLEX CX(*), CY(*)
INTEGER N, INCX, INCY

SUBROUTINE ZDROT_64(N, CX, INCX, CY, INCY, C, S)

DOUBLE PRECISION C, S
DOUBLE COMPLEX CX(*), CY(*)
INTEGER*8 N, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE ROT([N], CX, [INCX], CY, [INCY], C, S)

REAL(8) :: C, S
COMPLEX(8), DIMENSION(:) :: CX, CY
INTEGER :: N, INCX, INCY

SUBROUTINE ROT_64([N], CX, [INCX], CY, [INCY], C, S)

REAL(8) :: C, S
COMPLEX(8), DIMENSION(:) :: CX, CY
INTEGER(8) :: N, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zdrot(int n, doublecomplex *cx, int incx, doublecomplex
          *cy, int incy, double c, double s);

void zdrot_64(long n, doublecomplex *cx,  long  incx,  doub-
          lecomplex *cy, long incy, double c, double s);
```

PURPOSE

zdrot Apply a plane rotation, where the cos and sin (c and s) are real and the vectors x and y are complex.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

CX (input)
Before entry, the incremented array CX must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of CX. INCX must not be zero. Unchanged on exit.

CY (output)
On entry, the incremented array CY must contain the vector y. On exit, CY is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of CY. INCY must not be zero. Unchanged on exit.

C (input)
On entry, the cosine. Unchanged on exit.

S (input)
On entry, the sin. Unchanged on exit.

Contents

NAME

```
zdscal - Compute y := alpha * y
```

SYNOPSIS

```
SUBROUTINE ZDSCAL(N, ALPHA, Y, INCY)

DOUBLE COMPLEX Y(*)
INTEGER N, INCY
DOUBLE PRECISION ALPHA

SUBROUTINE ZDSCAL_64(N, ALPHA, Y, INCY)

DOUBLE COMPLEX Y(*)
INTEGER*8 N, INCY
DOUBLE PRECISION ALPHA
```

F95 INTERFACE

```
SUBROUTINE SCAL([N], ALPHA, Y, [INCY])

COMPLEX(8), DIMENSION(:) :: Y
INTEGER :: N, INCY
REAL(8) :: ALPHA

SUBROUTINE SCAL_64([N], ALPHA, Y, [INCY])

COMPLEX(8), DIMENSION(:) :: Y
INTEGER(8) :: N, INCY
REAL(8) :: ALPHA
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zdscal(int  n,  double  alpha,  doublecomplex  *y,  int
          incy);

void zdscal_64(long n, double alpha, doublecomplex *y,  long
          incy);
```

PURPOSE

zdscal Compute y := alpha * y where alpha is a scalar and y is an n-vector.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

zellmm - Ellpack format matrix-matrix multiply

SYNOPSIS

```
 SUBROUTINE ZELLMM( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, LDA, MAXNZ,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, K, DESCRA(5), LDA, MAXNZ,
*           LDB, LDC, LWORK
 INTEGER    INDX(LDA,MAXNZ)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(LDA,MAXNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE ZELLMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, INDX, LDA, MAXNZ,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, K, DESCRA(5), LDA, MAXNZ,
*           LDB, LDC, LWORK
 INTEGER*8  INDX(LDA,MAXNZ)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(LDA,MAXNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE ELLMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*        [LDA], MAXNZ, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, K, MAXNZ
 INTEGER, DIMENSION(:) ::  DESCRA
 INTEGER, DIMENSION(:, :) ::    INDX
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:, :) ::  VAL, B, C

 SUBROUTINE ELLMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
```

```
*        [LDA], MAXNZ, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, K, MAXNZ
 INTEGER*8, DIMENSION(:) ::  DESCRA
 INTEGER*8, DIMENSION(:, :) ::    INDX
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:, :) ::  VAL, B, C
```

DESCRIPTION

```
where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in  Ellpack format format and
op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
                2 : upper
              DESCRA(3) main diagonal type
                0 : non-unit
                1 : unit
```

```
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

 VAL()          two-dimensional LDA-by-MAXNZ array such that VAL(I,:)
                consists of non-zero elements in row I of A, padded by
                zero values if the row contains less than MAXNZ.

 INDX()         two-dimensional integer LDA-by-MAXNZ array such
                INDX(I,:) consists of the column indices of the
                nonzero elements in row I, padded by the integer
                value I if the number of nonzeros is less than MAXNZ.

 LDA            leading dimension of VAL and INDX.

 MAXNZ          max number of nonzeros elements per row.

 B()            rectangular array with first dimension LDB.
 LDB            leading dimension of B

 BETA           Scalar parameter

 C()            rectangular array with first dimension LDC.

 LDC            leading dimension of C

 WORK()         scratch array of length LWORK. WORK is not
                referenced in the current version.

 LWORK          length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

```
zellsm - Ellpack format triangular solve
```

SYNOPSIS

```
 SUBROUTINE ZELLSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, INDX, LDA, MAXNZ,
*            B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5), LDA, MAXNZ,
*            LDB, LDC, LWORK
 INTEGER    INDX(LDA,MAXNZ)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX DV(M), VAL(LDA,MAXNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE ZELLSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*            VAL, INDX, LDA, MAXNZ,
*            B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5), LDA, MAXNZ,
*            LDB, LDC, LWORK
 INTEGER*8  INDX(LDA,MAXNZ)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX DV(M), VAL(LDA,MAXNZ), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE ELLSM( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
*    INDX, [LDA], MAXNZ, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M,  MAXNZ
 INTEGER, DIMENSION(:) ::  DESCRA
 INTEGER, DIMENSION(:, :) ::    INDX
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) ::  DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::  VAL, B, C
```

```
 SUBROUTINE ELLSM_64( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
*    INDX, [LDA], MAXNZ, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M,  MAXNZ
 INTEGER*8, DIMENSION(:) ::  DESCRA
 INTEGER*8, DIMENSION(:, :) ::    INDX
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) ::  DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::  VAL, B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a diagonal scaling matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in Ellpack format and
op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                  0 : general
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))

                Note: For the routine, only DESCRA(1)=3 is supported.

                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices
VAL()           two-dimensional LDA-by-MAXNZ array such that VAL(I,:)
                consists of non-zero elements in row I of A, padded by
                zero values if the row contains less than MAXNZ.

INDX()          two-dimensional integer LDA-by-MAXNZ array such
                INDX(I,:) consists of the column indices of the
                nonzero elements in row I, padded by the integer
                value I if the number of nonzeros is less than MAXNZ.
                The column indices MUST be sorted in increasing order
                for each row.

LDA             leading dimension of VAL and INDX.

MAXNZ           max number of nonzeros elements per row.

B()             rectangular array with first dimension LDB.

LDB             leading dimension of B

BETA            Scalar parameter

C()             rectangular array with first dimension LDC.

LDC             leading dimension of C

WORK()          scratch array of length LWORK.

                On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.
```

LWORK length of WORK array. LWORK should be at least M.

For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the rows of A such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows have been scaled. UNITD=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the ELL representation of a sparse matrix. They are not used anyway in these cases.

But if UNITD=4, the unit diagonal elements MUST be referenced in the ELL representation.

4. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

```
zfft2b - compute a periodic sequence from its Fourier  coef-
ficients.  The FFT operations are unnormalized, so a call of
ZFFT2F followed by a call of ZFFT2B will multiply the  input
sequence by M*N.
```

SYNOPSIS

```
SUBROUTINE ZFFT2B(M, N, A, LDA, WORK, LWORK)

DOUBLE COMPLEX A(LDA,*)
INTEGER M, N, LDA, LWORK
DOUBLE PRECISION WORK(*)

SUBROUTINE ZFFT2B_64(M, N, A, LDA, WORK, LWORK)

DOUBLE COMPLEX A(LDA,*)
INTEGER*8 M, N, LDA, LWORK
DOUBLE PRECISION WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT2B([M], [N], A, [LDA], WORK, LWORK)

COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, LWORK
REAL(8), DIMENSION(:) :: WORK

SUBROUTINE FFT2B_64([M], [N], A, [LDA], WORK, LWORK)

COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, LWORK
REAL(8), DIMENSION(:) :: WORK
```

C INTERFACE

```
#include <sunperf.h>

void zfft2b(int m, int n, doublecomplex *a, int lda,  double
          *work, int lwork);

void zfft2b_64(long m, long n, doublecomplex *a,  long  lda,
          double *work, long lwork);
```

ARGUMENTS

M (input) Number of rows to be transformed. These subroutines are most efficient when M is a product of small primes. M >= 0.

N (input) Number of columns to be transformed. These subroutines are most efficient when N is a product of small primes. N >= 0.

A (input/output)
On entry, a two-dimensional array A(M,N) that contains the sequences to be transformed.

LDA (input)
Leading dimension of the array containing the data to be transformed. LDA >= M.

WORK (input)
On entry, an array with dimension of at least LWORK. WORK must have been initialized by ZFFT2I.

LWORK (input)
The dimension of the array WORK. LWORK >= (4 * (M + N) + 30)

Contents

NAME

```
zfft2f - compute the  Fourier  coefficients  of  a  periodic
sequence.  The FFT operations are unnormalized, so a call of
ZFFT2F followed by a call of ZFFT2B will multiply the  input
sequence by M*N.
```

SYNOPSIS

```
SUBROUTINE ZFFT2F(M, N, A, LDA, WORK, LWORK)

DOUBLE COMPLEX A(LDA,*)
INTEGER M, N, LDA, LWORK
DOUBLE PRECISION WORK(*)

SUBROUTINE ZFFT2F_64(M, N, A, LDA, WORK, LWORK)

DOUBLE COMPLEX A(LDA,*)
INTEGER*8 M, N, LDA, LWORK
DOUBLE PRECISION WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT2F([M], [N], A, [LDA], WORK, LWORK)

COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, LWORK
REAL(8), DIMENSION(:) :: WORK

SUBROUTINE FFT2F_64([M], [N], A, [LDA], WORK, LWORK)

COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, LWORK
REAL(8), DIMENSION(:) :: WORK
```

C INTERFACE

```
#include <sunperf.h>

void zfft2f(int m, int n, doublecomplex *a, int lda,  double
          *work, int lwork);

void zfft2f_64(long m, long n, doublecomplex *a,  long  lda,
          double *work, long lwork);
```

ARGUMENTS

```
M (input) Number of rows to be transformed.   These  subrou-
          tines  are  most  efficient when M is a product of
          small primes.  M >= 0.
N (input) Number of columns to be transformed.   These  sub-
          routines are most efficient when N is a product of
          small primes.  N >= 0.

A (input/output)
          On entry, a two-dimensional array A(M,N) that con-
          tains the sequences to be transformed.

LDA (input)
          Leading dimension of the array containing the data
          to be transformed.  LDA >= M.

WORK (input)
          On input, workspace WORK must have  been  initial-
          ized by ZFFT2I.

LWORK (input)
          The dimension of the array WORK.  LWORK >= (4 * (M
          + N) + 30)
```

Contents

NAME

```
zfft2i - initialize the array WSAVE, which is used  in  both
the forward and backward transforms.
```

SYNOPSIS

```
SUBROUTINE ZFFT2I(M, N, WORK)

INTEGER M, N
DOUBLE PRECISION WORK(*)

SUBROUTINE ZFFT2I_64(M, N, WORK)

INTEGER*8 M, N
DOUBLE PRECISION WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ZFFT2I(M, N, WORK)

INTEGER :: M, N
REAL(8), DIMENSION(:) :: WORK

SUBROUTINE ZFFT2I_64(M, N, WORK)

INTEGER(8) :: M, N
REAL(8), DIMENSION(:) :: WORK
```

C INTERFACE

```
#include <sunperf.h>

void zfft2i(int m, int n, double *work);

void zfft2i_64(long m, long n, double *work);
```

ARGUMENTS

M (input) Number of rows to be transformed. M >= 0.

N (input) Number of columns to be transformed. N >= 0.

WORK (input/output)
On entry, an array of dimension (4 * (M + N) + 30) or greater. ZFFT2I needs to be called only once to initialize array WORK before calling ZFFT2F and/or ZFFT2B if M, N and WORK remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
zfft3b - compute a periodic sequence from its Fourier  coef-
ficients.  The FFT operations are unnormalized, so a call of
ZFFT3F followed by a call of ZFFT3B will multiply the  input
sequence by M*N*K.
```

SYNOPSIS

```
SUBROUTINE ZFFT3B(M, N, K, A, LDA, LD2A, WORK, LWORK)

DOUBLE COMPLEX A(LDA,LD2A,*)
INTEGER M, N, K, LDA, LD2A, LWORK
DOUBLE PRECISION WORK(*)

SUBROUTINE ZFFT3B_64(M, N, K, A, LDA, LD2A, WORK, LWORK)

DOUBLE COMPLEX A(LDA,LD2A,*)
INTEGER*8 M, N, K, LDA, LD2A, LWORK
DOUBLE PRECISION WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT3B([M], [N], [K], A, [LDA], LD2A, WORK, LWORK)

COMPLEX(8), DIMENSION(:,:,:) :: A
INTEGER :: M, N, K, LDA, LD2A, LWORK
REAL(8), DIMENSION(:) :: WORK

SUBROUTINE FFT3B_64([M], [N], [K], A, [LDA], LD2A, WORK, LWORK)

COMPLEX(8), DIMENSION(:,:,:) :: A
INTEGER(8) :: M, N, K, LDA, LD2A, LWORK
REAL(8), DIMENSION(:) :: WORK
```

C INTERFACE

```
#include <sunperf.h>

void zfft3b(int m, int n, int k, doublecomplex *a, int  lda,
          int ld2a, double *work, int lwork);

void zfft3b_64(long m, long n,  long  k,  doublecomplex  *a,
          long lda, long ld2a, double *work, long lwork);
```

ARGUMENTS

```
M (input) Number of rows to be transformed.   These  subrou-
          tines  are  most  efficient when M is a product of
          small primes.  M >= 0.
N (input) Number of columns to be transformed.   These  sub-
          routines are most efficient when N is a product of
          small primes.  N >= 0.

K (input) Number of planes to be transformed.  These subrou-
          tines  are  most  efficient when K is a product of
          small primes.  K >= 0.

A (input/output)
          On entry, a three-dimensional array  A(LDA,LD2A,K)
          that contains the sequences to be transformed.

LDA (input)
          Leading dimension of the array containing the data
          to be transformed.  LDA >= M.

LD2A (input)
          Second dimension of the array containing the  data
          to be transformed.  LD2A >= N.

WORK (input)
          On input, workspace WORK must have  been  initial-
          ized by ZFFT3I.

LWORK (input)
          The dimension of the array WORK.  LWORK >= (4*(M +
          N + K) + 45).
```

Contents

NAME

```
zfft3f - compute the  Fourier  coefficients  of  a  periodic
sequence.  The FFT operations are unnormalized, so a call of
ZFFT3F followed by a call of ZFFT3B will multiply the  input
sequence by M*N*K.
```

SYNOPSIS

```
SUBROUTINE ZFFT3F(M, N, K, A, LDA, LD2A, WORK, LWORK)

DOUBLE COMPLEX A(LDA,LD2A,*)
INTEGER M, N, K, LDA, LD2A, LWORK
DOUBLE PRECISION WORK(*)

SUBROUTINE ZFFT3F_64(M, N, K, A, LDA, LD2A, WORK, LWORK)

DOUBLE COMPLEX A(LDA,LD2A,*)
INTEGER*8 M, N, K, LDA, LD2A, LWORK
DOUBLE PRECISION WORK(*)
```

F95 INTERFACE

```
SUBROUTINE FFT3F([M], [N], [K], A, [LDA], LD2A, WORK, LWORK)

COMPLEX(8), DIMENSION(:,:,:) :: A
INTEGER :: M, N, K, LDA, LD2A, LWORK
REAL(8), DIMENSION(:) :: WORK

SUBROUTINE FFT3F_64([M], [N], [K], A, [LDA], LD2A, WORK, LWORK)

COMPLEX(8), DIMENSION(:,:,:) :: A
INTEGER(8) :: M, N, K, LDA, LD2A, LWORK
REAL(8), DIMENSION(:) :: WORK
```

C INTERFACE

```
#include <sunperf.h>

void zfft3f(int m, int n, int k, doublecomplex *a, int  lda,
          int ld2a, double *work, int lwork);

void zfft3f_64(long m, long n,  long  k,  doublecomplex  *a,
          long lda, long ld2a, double *work, long lwork);
```

ARGUMENTS

M (input) Number of rows to be transformed. These subroutines are most efficient when M is a product of small primes. M >= 0.

N (input) Number of columns to be transformed. These subroutines are most efficient when N is a product of small primes. N >= 0.

K (input) Number of planes to be transformed. These subroutines are most efficient when K is a product of small primes. K >= 0.

A (input/output)
On entry, a three-dimensional array A(M,N,K) that contains the sequences to be transformed.

LDA (input)
Leading dimension of the array containing the data to be transformed. LDA >= M.

LD2A (input)
Second dimension of the array containing the data to be transformed. LD2A >= N.

WORK (input)
On input, workspace WORK must have been initialized by ZFFT3I.

LWORK (input)
The dimension of the array WORK. LWORK >= (4*(M + N + K) + 45).

Contents

NAME

```
zfft3i - initialize the array WSAVE, which is used  in  both
ZFFT3F and ZFFT3B.
```

SYNOPSIS

```
SUBROUTINE ZFFT3I(M, N, K, WORK)

INTEGER M, N, K
DOUBLE PRECISION WORK(*)

SUBROUTINE ZFFT3I_64(M, N, K, WORK)

INTEGER*8 M, N, K
DOUBLE PRECISION WORK(*)
```

F95 INTERFACE

```
SUBROUTINE ZFFT3I(M, N, K, WORK)

INTEGER :: M, N, K
REAL(8), DIMENSION(:) :: WORK

SUBROUTINE ZFFT3I_64(M, N, K, WORK)

INTEGER(8) :: M, N, K
REAL(8), DIMENSION(:) :: WORK
```

C INTERFACE

```
#include <sunperf.h>

void zfft3i(int m, int n, int k, double *work);

void zfft3i_64(long m, long n, long k, double *work);
```

ARGUMENTS

M (input) Number of rows to be transformed. M >= 0.

N (input) Number of columns to be transformed. N >= 0.

K (input) Number of planes to be transformed. K >= 0.

WORK (input/output)
On entry, an array of dimension (4*(M + N + K) + 45) or greater. ZFFT3I needs to be called only once to initialize array WORK before calling ZFFT3F and/or ZFFT3B if M, N, K and WORK remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
zfftb - compute a periodic sequence from its Fourier coeffi-
cients.   The  FFT operations are unnormalized, so a call of
ZFFTF followed by a call of ZFFTB will  multiply  the  input
sequence by N.
```

SYNOPSIS

```
SUBROUTINE ZFFTB(N, X, WSAVE)

DOUBLE COMPLEX X(*)
INTEGER N
DOUBLE PRECISION WSAVE(*)

SUBROUTINE ZFFTB_64(N, X, WSAVE)

DOUBLE COMPLEX X(*)
INTEGER*8 N
DOUBLE PRECISION WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE FFTB([N], X, WSAVE)

COMPLEX(8), DIMENSION(:) :: X
INTEGER :: N
REAL(8), DIMENSION(:) :: WSAVE

SUBROUTINE FFTB_64([N], X, WSAVE)

COMPLEX(8), DIMENSION(:) :: X
INTEGER(8) :: N
REAL(8), DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void zfftb(int n, doublecomplex *x, double *wsave);

void zfftb_64(long n, doublecomplex *x, double *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence to be  transformed.   These
          subroutines are most efficient when N is a product
          of small primes.  N >= 0.

X (input) On entry, an array  of  length  N  containing  the
          sequence to be transformed.

WSAVE (input/output)
          On entry, WSAVE must be an array of dimension (4 *
          N  + 15) or greater and must have been initialized
          by ZFFTI.
```

Contents

NAME

```
zfftd -  initialize  the  trigonometric  weight  and  factor
tables  or  compute  the inverse Fast Fourier Transform of a
double complex sequence.
```

SYNOPSIS

```
SUBROUTINE ZFFTD(IOPT, N, SCALE, X, Y, TRIGS, IFAC, WORK, LWORK, IERR)

INTEGER IOPT, N, IFAC(*), LWORK, IERR
DOUBLE COMPLEX X(*)
DOUBLE PRECISION SCALE, Y(*), TRIGS(*), WORK(*)

SUBROUTINE ZFFTD_64(IOPT, N, SCALE, X, Y, TRIGS, IFAC, WORK, LWORK, IERR)

INTEGER*8 IOPT, N, IFAC(*), LWORK, IERR
DOUBLE PRECISION SCALE, Y(*), TRIGS(*), WORK(*)
DOUBLE COMPLEX X(*)
```

F95 INTERFACE

```
SUBROUTINE FFT(IOPT, N, [SCALE], X, Y, TRIGS, IFAC, WORK, [LWORK], IERR)

INTEGER, INTENT(IN) :: IOPT, N
INTEGER, INTENT(IN), OPTIONAL :: LWORK
REAL(8), INTENT(IN), OPTIONAL :: SCALE
COMPLEX(8), INTENT(IN), DIMENSION(:) :: X
REAL(8), INTENT(OUT), DIMENSION(:) :: Y
REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
INTEGER, INTENT(INOUT), DIMENSION(:) :: IFAC
REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
INTEGER, INTENT(OUT) :: IERR

SUBROUTINE FFT_64(IOPT, N, [SCALE], X, Y, TRIGS, IFAC, WORK, [LWORK], IERR)
```

```
INTEGER(8), INTENT(IN) :: IOPT, N
INTEGER(8), INTENT(IN), OPTIONAL :: LWORK
REAL(8), INTENT(IN), OPTIONAL :: SCALE
COMPLEX(8), INTENT(IN), DIMENSION(:) :: X
REAL(8), INTENT(OUT), DIMENSION(:) :: Y
REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
#include <sunperf.h>

void zfftd_ (int *iopt, int *n, double *scale, doublecomplex
          *x,  double  *y,  double *trigs, int *ifac, double
          *work, int *lwork, int *ierr);

void zfftd_64_ (long *iopt, long *n,  double  *scale,  doub-
          lecomplex  *x,  double  *y,  double  *trigs,  long
          *ifac, double *work, long *lwork, long *ierr);
```

PURPOSE

```
zfftd initializes the trigonometric weight and factor tables
or  computes  the inverse Fast Fourier Transform of a double
complex sequence as follows:

               N-1
Y(k) = scale * SUM  W*X(j)
               j=0

where
k ranges from 0 to N-1
i = sqrt(-1)
isign = 1 for inverse transform or -1 for forward transform
W = exp(isign*i*j*k*2*pi/N)
In complex-to-real transform of length N, the  (N/2+1)  com-
plex  input  data  points  stored are the positive-frequency
half of the spectrum of the Discrete Fourier Transform.  The
other  half  can be obtained through complex conjugation and
therefore is not stored.  Furthermore, due to symmetries the
imaginary  of the component of X(0) and X(N/2) (if N is even
in the latter) is assumed to be zero and is not referenced.
```

ARGUMENTS

```
IOPT (input)
          Integer specifying the operation to be performed:
          IOPT = 0 computes the trigonometric  weight  table
          and factor table
          IOPT = 1 computes inverse FFT

N (input)
          Integer specifying length of the input sequence X.
          N  is most efficient when it is a product of small
          primes.  N >= 0.  Unchanged on exit.

SCALE (input)
          Double precision scalar by which transform results
          are   scaled.   Unchanged   on  exit.   SCALE  is
          defaulted to 1.0D0 for F95 INTERFACE.

X (input) On entry, X is a double complex array whose  first
          (N/2+1)  elements  are  the  input  sequence to be
          transformed.
Y (output)
          Double precision array of  dimension  at  least  N
          that  contains the transform results.  X and Y may
          be the same array  starting  at  the  same  memory
          location.   Otherwise, it is assumed that there is
          no overlap between X and Y in memory.

TRIGS (input/output)
          Double precision array of length 2*N that contains
          the  trigonometric  weights.  The weights are com-
          puted when the routine is called with IOPT = 0 and
          they  are  used in subsequent calls when IOPT = 1.
          Unchanged on exit.

IFAC (input/output)
          Integer array of dimension at least 128 that  con-
          tains  the factors of N.  The factors are computed
          when the routine is called with IOPT = 0 and  they
          are  used  in  subsequent  calls  where  IOPT = 1.
          Unchanged on exit.

WORK (workspace)
          Double precision array of dimension  at  least  N.
          The user can also choose to have the routine allo-
          cate its own workspace (see LWORK).

LWORK (input)
          Integer specifying workspace size.  If LWORK =  0,
          the routine will allocate its own workspace.

IERR (output)
```

On exit, integer IERR has one of the following values:
0 = normal return
-1 = IOPT is not 0 or 1
-2 = N < 0
-3 = (LWORK is not 0) and (LWORK is less than N)
-4 = memory allocation for workspace failed

SEE ALSO

fft

Contents

NAME

zfftd2 - initialize the trigonometric weight and factor tables or compute the two-dimensional inverse Fast Fourier Transform of a two-dimensional double complex array.

SYNOPSIS

```
     SUBROUTINE ZFFTD2(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     DOUBLE COMPLEX X(LDX, *)
     DOUBLE PRECISION SCALE, Y(LDY, *), TRIGS(*), WORK(*)

     SUBROUTINE ZFFTD2_64(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER*8 IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     DOUBLE COMPLEX X(LDX, *)
     DOUBLE PRECISION SCALE, Y(LDY, *), TRIGS(*), WORK(*)
```

F95 INTERFACE

```
     SUBROUTINE FFT2(IOPT, N1, [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
               IFAC, WORK, [LWORK], IERR)

     INTEGER, INTENT(IN) :: IOPT, N1
     INTEGER, INTENT(IN), OPTIONAL :: N2, LDX, LDY, LWORK
     REAL(8), INTENT(IN), OPTIONAL :: SCALE
     COMPLEX(8), INTENT(IN), DIMENSION(:,:) :: X
     REAL(8), INTENT(OUT), DIMENSION(:,:) :: Y
     REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER, INTENT(INOUT), DIMENSION(:) :: IFAC
```

```
     REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER, INTENT(OUT) :: IERR

     SUBROUTINE FFT2_64(IOPT, N1, [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS, IFAC,
WORK, [LWORK], IERR)

     INTEGER(8), INTENT(IN) :: IOPT, N1
     INTEGER(8), INTENT(IN), OPTIONAL :: N2, LDX, LDY, LWORK
     REAL(8), INTENT(IN), OPTIONAL :: SCALE
     COMPLEX(8), INTENT(IN), DIMENSION(:,:) :: X
     REAL(8), INTENT(OUT), DIMENSION(:,:) :: Y
     REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
     #include <sunperf.h>
     void zfftd2_ (int *iopt, int *n1, int  *n2,  double  *scale,
               doublecomplex  *x,  int *ldx, double *y, int *ldy,
               double  *trigs,  int  *ifac,  double  *work,   int
               *lwork, int *ierr);

     void zfftd2_64_ (long *iopt,  long  *n1,  long  *n2,  double
               *scale,  doublecomplex  *x,  long *ldx, double *y,
               long  *ldy,  double  *trigs,  long  *ifac,  double
               *work, long *lwork, long *ierr);
```

PURPOSE

```
     zfftd2  initializes  the  trigonometric  weight  and  factor
     tables  or computes the two-dimensional inverse Fast Fourier
     Transform of a two-dimensional  double  complex  array.   In
     computing  the two-dimensional FFT, one-dimensional FFTs are
     computed along the rows of the input array.  One-dimensional
     FFTs are then computed along the columns of the intermediate
     results.

                        N1-1  N2-1
     Y(k1,k2) = scale * SUM   SUM   W2*W1*X(j1,j2)
                        j1=0  j2=0

     where
     k1 ranges from 0 to N1-1 and k2 ranges from 0 to N2-1
     i = sqrt(-1)
     isign = 1 for inverse transform
     W1 = exp(isign*i*j1*k1*2*pi/N1)
     W2 = exp(isign*i*j2*k2*2*pi/N2)
     In complex-to-real transform of length N1, the (N1/2+1) com-
```

plex input data points stored are the positive-frequency half of the spectrum of the Discrete Fourier Transform. The other half can be obtained through complex conjugation and therefore is not stored.

ARGUMENTS

IOPT (input)
Integer specifying the operation to be performed:
IOPT = 0 computes the trigonometric weight table and factor table
IOPT = 1 computes inverse FFT

N1 (input)
Integer specifying length of the transform in the first dimension. N1 is most efficient when it is a product of small primes. N1 >= 0. Unchanged on exit.

N2 (input)
Integer specifying length of the transform in the second dimension. N2 is most efficient when it is a product of small primes. N2 >= 0. Unchanged on exit.

SCALE (input)
Double precision scalar by which transform results are scaled. Unchanged on exit. SCALE is defaulted to 1.0D0 for F95 INTERFACE.

X (input) X is a double complex array of dimensions (LDX, N2) that contains input data to be transformed.

LDX (input)
Leading dimension of X. LDX >= (N1/2 + 1) Unchanged on exit.

Y (output)
Y is a double precision array of dimensions (LDY, N2) that contains the transform results. X and Y can be the same array starting at the same memory location, in which case the input data are overwritten by their transform results. Otherwise, it is assumed that there is no overlap between X and Y in memory.

LDY (input)
Leading dimension of Y. If X and Y are the same

array, LDY = 2*LDX Else LDY >= 2*LDX and LDY must be even. Unchanged on exit.

TRIGS (input/output)
: Double precision array of length 2*(N1+N2) that contains the trigonometric weights. The weights are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1. Unchanged on exit.

IFAC (input/output)
: Integer array of dimension at least 2*128 that contains the factors of N1 and N2. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1. Unchanged on exit.

WORK (workspace)
: Double precision array of dimension at least MAX(N1,2*N2) where NCPUS is the number of threads used to execute the routine. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)
: Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)
: On exit, integer IERR has one of the following values:
0 = normal return
-1 = IOPT is not 0, 1
-2 = N1 < 0
-3 = N2 < 0
-4 = (LDX < N1/2+1)
-5 = LDY not equal 2*LDX when X and Y are same array
-6 = (LDY < 2*LDX or LDY odd) when X and Y are same array
-7 = (LWORK not equal 0) and (LWORK < MAX(N1,2*N2))
-8 = memory allocation failed

SEE ALSO

fft

CAUTIONS

On exit, output subarray Y(1:LDY, 1:N2) is overwritten.

Contents

NAME

```
     zfftd3 - initialize  the  trigonometric  weight  and  factor
     tables or compute the three-dimensional inverse Fast Fourier
     Transform of a three-dimensional double complex array.
```

SYNOPSIS

```
     SUBROUTINE ZFFTD3(IOPT, N1, N2, N3, SCALE, X, LDX1, LDX2, Y, LDY1, LDY2,
TRIGS, IFAC, WORK, LWORK, IERR)

     INTEGER IOPT, N1, N2, N3, LDX1, LDX2, LDY1,  LDY2,  IFAC(*),
     LWORK, IERR
     DOUBLE COMPLEX X(LDX1, LDX2, *)
     DOUBLE PRECISION SCALE, TRIGS(*), WORK(*), Y(LDY1, LDY2, *)

     SUBROUTINE ZFFTD3_64(IOPT, N1, N2, N3, SCALE, X, LDX1, LDX2, Y, LDY1, LDY2,
TRIGS, IFAC, WORK, LWORK, IERR)

     INTEGER*8 IOPT, N1, N2, N3, LDX1, LDX2, LDY1, LDY2, IFAC(*),
     LWORK, IERR
     DOUBLE COMPLEX X(LDX1, LDX2, *)
     DOUBLE PRECISION SCALE, TRIGS(*), WORK(*), Y(LDY1, LDY2, *)
```

F95 INTERFACE

```
     SUBROUTINE FFT3(IOPT, N1, [N2], [N3], [SCALE], X, [LDX1], LDX2, Y, [LDY1],
LDY2, TRIGS, IFAC, WORK, [LWORK], IERR)

     INTEGER, INTENT(IN) :: IOPT, N1, LDX2, LDY2
     INTEGER, INTENT(IN), OPTIONAL :: N2, N3, LDX1, LDY1, LWORK
     REAL(8), INTENT(IN), OPTIONAL :: SCALE
     COMPLEX(8), INTENT(IN), DIMENSION(:,:) :: X
     REAL(8), INTENT(OUT), DIMENSION(:,:) :: Y
```

```
     REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER, INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER, INTENT(OUT) :: IERR

     SUBROUTINE FFT3_64(IOPT, N1, [N2], [N3], [SCALE], X, [LDX1], LDX2, Y,
[LDY1], LDY2, TRIGS, IFAC, WORK, [LWORK], IERR)

     INTEGER(8), INTENT(IN) :: IOPT, N1, LDX2, LDY2
     INTEGER(8), INTENT(IN), OPTIONAL  ::  N2,  N3,  LDX1,  LDY1,
     LWORK
     REAL(8), INTENT(IN), OPTIONAL :: SCALE
     COMPLEX(8), INTENT(IN), DIMENSION(:,:) :: X
     REAL(8), INTENT(OUT), DIMENSION(:,:) :: Y
     REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
     #include <sunperf.h>
     void zfftd3_ (int *iopt, int *n1, int *n2, int  *n3,  double
               *scale,  doublecomplex  *x,  int *ldx1, int *ldx2,
               double *y, int *ldy1, int  *ldy2,  double  *trigs,
               int *ifac, double *work, int *lwork, int *ierr);

     void zfftd3_64_ (long *iopt, long *n1, long *n2,  long  *n3,
               double  *scale, doublecomplex *x, long *ldx1, long
               *ldx2, double *y, long *ldy1, long  *ldy2,  double
               *trigs,  long  *ifac,  double  *work, long *lwork,
               long *ierr);
```

PURPOSE

```
     zfftd3  initializes  the  trigonometric  weight  and  factor
     tables   or  computes  the  three-dimensional  inverse  Fast
     Fourier Transform  of  a  three-dimensional  double  complex
     array.

                           N3-1  N2-1  N1-1
     Y(k1,k2,k3) = scale * SUM   SUM   SUM   W3*W2*W1*X(j1,j2,j3)
                           j3=0  j2=0  j1=0

     where
     k1 ranges from 0 to N1-1; k2 ranges from 0 to  N2-1  and  k3
     ranges from 0 to N3-1
     i = sqrt(-1)
     isign = 1 for inverse transform
     W1 = exp(isign*i*j1*k1*2*pi/N1)
```

```
W2 = exp(isign*i*j2*k2*2*pi/N2)
W3 = exp(isign*i*j3*k3*2*pi/N3)
```

ARGUMENTS

```
IOPT (input)
          Integer specifying the operation to be performed:
          IOPT = 0 computes the trigonometric  weight  table
          and factor table
          IOPT = +1 computes inverse FFT

N1 (input)
          Integer specifying length of the transform in  the
          first  dimension.  N1 is most efficient when it is
          a product of small primes.  N1 >= 0.  Unchanged on
          exit.

N2 (input)
          Integer specifying length of the transform in  the
          second dimension.  N2 is most efficient when it is
          a product of small primes.  N2 >= 0.  Unchanged on
          exit.

N3 (input)
          Integer specifying length of the transform in  the
          third  dimension.  N3 is most efficient when it is
          a product of small primes.  N3 >= 0.  Unchanged on
          exit.

SCALE (input)
          Double precision scalar by which transform results
          are   scaled.   Unchanged   on  exit.   SCALE  is
          defaulted to 1.0D0 for F95 INTERFACE.

X (input) X is a double complex array of  dimensions  (LDX1,
          LDX2,   N3)   that   contains  input  data  to  be
          transformed.

LDX1 (input)
          first dimension of X.  LDX1 >= N1/2+1 Unchanged on
          exit.

LDX2 (input)
          second dimension of X.  LDX2 >=  N2  Unchanged  on
          exit.

Y (output)
          Y is a double complex array of  dimensions  (LDY1,
```

LDY2, N3) that contains the transform results. X and Y can be the same array starting at the same memory location, in which case the input data are overwritten by their transform results. Otherwise, it is assumed that there is no overlap between X and Y in memory.

LDY1 (input)
first dimension of Y. If X and Y are the same array, LDY1 = 2*LDX1 Else LDY1 >= 2*LDX1 and LDY1 is even Unchanged on exit.

LDY2 (input)
second dimension of Y. If X and Y are the same array, LDY2 = LDX2 Else LDY2 >= N2 Unchanged on exit.

TRIGS (input/output)
Double precision array of length 2*(N1+N2+N3) that contains the trigonometric weights. The weights are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1. Unchanged on exit.

IFAC (input/output)
Integer array of dimension at least 3*128 that contains the factors of N1, N2 and N3. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1. Unchanged on exit.

WORK (workspace)
Double precision array of dimension at least (MAX(N,2*N2,2*N3) + 16*N3) * NCPUS where NCPUS is the number of threads used to execute the routine. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)
Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)
On exit, integer IERR has one of the following values:
0 = normal return
-1 = IOPT is not 0 or 1
-2 = N1 < 0
-3 = N2 < 0
-4 = N3 < 0

-5 = (LDX1 < N1/2+1)
-6 = (LDX2 < N2)
-7 = LDY1 not equal 2*LDX1 when X and Y are same array
-8 = (LDY1 < 2*LDX1) or (LDY1 is odd) when X and Y are not same array
-9 = (LDY2 < N2) or (LDY2 not equal LDX2) when X and Y are same array
-10 = (LWORK not equal 0) and ((LWORK < MAX(N,2*N2,2*N3) + 16*N3)*NCPUS)
-11 = memory allocation failed

SEE ALSO

fft

CAUTIONS

On exit, output subarray Y(1:LDY1, 1:N2, 1:N3) is overwritten.

Contents

NAME

```
zfftdm - initialize  the  trigonometric  weight  and  factor
tables  or  compute the one-dimensional inverse Fast Fourier
Transform of a set of double complex data  sequences  stored
in a two-dimensional array.
```

SYNOPSIS

```
     SUBROUTINE ZFFTDM(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     DOUBLE COMPLEX X(LDX, *)
     DOUBLE PRECISION SCALE, Y(LDY, *), TRIGS(*), WORK(*)

     SUBROUTINE ZFFTDM_64(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER*8 IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     DOUBLE COMPLEX X(LDX, *)
     DOUBLE PRECISION SCALE, Y(LDY,*), TRIGS(*), WORK(*)
```

F95 INTERFACE

```
     SUBROUTINE FFTM(IOPT, N1, [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
               IFAC, WORK, [LWORK], IERR)

     INTEGER, INTENT(IN) :: IOPT, N1
     INTEGER, INTENT(IN), OPTIONAL :: N2, LDX, LDY, LWORK
     REAL(8), INTENT(IN), OPTIONAL :: SCALE
     COMPLEX(8), INTENT(IN), DIMENSION(:,:) :: X
     REAL(8), INTENT(OUT), DIMENSION(:,:) :: Y
     REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER, INTENT(INOUT), DIMENSION(:) :: IFAC
```

```
     REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER, INTENT(OUT) :: IERR

     SUBROUTINE FFTM_64(IOPT, N1, [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS, IFAC,
WORK, [LWORK], IERR)

     INTEGER(8), INTENT(IN) :: IOPT, N1
     INTEGER(8), INTENT(IN), OPTIONAL :: N2, LDX, LDY, LWORK
     REAL(8), INTENT(IN), OPTIONAL :: SCALE
     COMPLEX(8), INTENT(IN), DIMENSION(:,:) :: X
     REAL(8), INTENT(OUT), DIMENSION(:,:) :: Y
     REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
     #include <sunperf.h>
     void zfftdm_ (int *iopt, int  *m,  int  *n,  double  *scale,
               doublecomplex  *x,  int *ldx, double *y, int *ldy,
               double  *trigs,  int  *ifac,  double  *work,   int
               *lwork, int *ierr);

     void zfftdm_64_  (long  *iopt,  long  *m,  long  *n,  double
               *scale,  doublecomplex  *x,  long *ldx, double *y,
               long  *ldy,  double  *trigs,  long  *ifac,  double
               *work, long *lwork, long *ierr);
```

PURPOSE

```
     zfftdm  initializes  the  trigonometric  weight  and  factor
     tables  or computes the one-dimensional inverse Fast Fourier
     Transform of a set of double complex data  sequences  stored
     in a two-dimensional array:

                      N1-1
     Y(k,l) = scale * SUM  W*X(j,l)
                      j=0

     where
     k ranges from 0 to N1-1 and l ranges from 0 to N2-1
     i = sqrt(-1)
     isign = 1 for inverse transform
     W = exp(isign*i*j*k*2*pi/N1)
     In complex-to-real transform of length N1, the (N1/2+1) com-
     plex  input  data  points  stored are the positive-frequency
     half of the spectrum of the Discrete Fourier Transform.  The
     other  half  can be obtained through complex conjugation and
     therefore is not stored.  Furthermore, due to symmetries the
```

imaginary of the component of X(0,0:N2-1) and X(N1/2,0:N2-1) (if N1 is even in the latter) is assumed to be zero and is not referenced.

ARGUMENTS

IOPT (input)
Integer specifying the operation to be performed:
IOPT = 0 computes the trigonometric weight table and factor table
IOPT = 1 computes inverse FFT

N1 (input)
Integer specifying length of the input sequences. N1 is most efficient when it is a product of small primes. N1 >= 0. Unchanged on exit.

N2 (input)
Integer specifying number of input sequences. N2 >= 0. Unchanged on exit.

SCALE (input)
Double precision scalar by which transform results are scaled. Unchanged on exit. SCALE is defaulted to 1.0D0 for F95 INTERFACE.

X (input) X is a double complex array of dimensions (LDX, N2) that contains the sequences to be transformed stored in its columns in X(0:N1/2, 0:N2-1).

LDX (input)
Leading dimension of X. LDX >= (N1/2+1) Unchanged on exit.

Y (output)
Y is a double precision array of dimensions (LDY, N2) that contains the transform results of the input sequences in Y(0:N1-1,0:N2-1). X and Y can be the same array starting at the same memory location, in which case the input sequences are overwritten by their transform results. Otherwise, it is assumed that there is no overlap between X and Y in memory.

LDY (input)
Leading dimension of Y. If X and Y are the same array, LDY = 2*LDX Else LDY >= N1 Unchanged on exit.

TRIGS (input/output)
double precision array of length 2*N1 that contains the trigonometric weights. The weights are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1. Unchanged on exit.

IFAC (input/output)
Integer array of dimension at least 128 that contains the factors of N1. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1. Unchanged on exit.

WORK (workspace)
double precision array of dimension at least N1. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)
Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)
On exit, integer IERR has one of the following values:
0 = normal return
-1 = IOPT is not 0 or 1
-2 = N1 < 0
-3 = N2 < 0
-4 = (LDX < N1/2+1)
-5 = (LDY < N1) or (LDY not equal 2*LDX when X and Y are same array)
-6 = (LWORK not equal 0) and (LWORK < N1)
-7 = memory allocation failed

SEE ALSO

fft

Contents

NAME

```
zfftf - compute  the  Fourier  coefficients  of  a  periodic
sequence.  The FFT operations are unnormalized, so a call of
ZFFTF followed by a call of ZFFTB will  multiply  the  input
sequence by N.
```

SYNOPSIS

```
SUBROUTINE ZFFTF(N, X, WSAVE)

DOUBLE COMPLEX X(*)
INTEGER N
DOUBLE PRECISION WSAVE(*)

SUBROUTINE ZFFTF_64(N, X, WSAVE)

DOUBLE COMPLEX X(*)
INTEGER*8 N
DOUBLE PRECISION WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE FFTF([N], X, WSAVE)

COMPLEX(8), DIMENSION(:) :: X
INTEGER :: N
REAL(8), DIMENSION(:) :: WSAVE

SUBROUTINE FFTF_64([N], X, WSAVE)

COMPLEX(8), DIMENSION(:) :: X
INTEGER(8) :: N
REAL(8), DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void zfftf(int n, doublecomplex *x, double *wsave);

void zfftf_64(long n, doublecomplex *x, double *wsave);
```

ARGUMENTS

```
N (input) Length of the sequence to be  transformed.   These
          subroutines are most efficient when N is a product
          of small primes.  N >= 0.

X (input) On entry, an array  of  length  N  containing  the
          sequence to be transformed.

WSAVE (input)
          On entry, WSAVE must be an array of dimension (4 *
          N  + 15) or greater and must have been initialized
          by ZFFTI.
```

Contents

NAME

```
zffti - initialize the array WSAVE, which is  used  in  both
ZFFTF and ZFFTB.
```

SYNOPSIS

```
SUBROUTINE ZFFTI(N, WSAVE)

INTEGER N
DOUBLE PRECISION WSAVE(*)

SUBROUTINE ZFFTI_64(N, WSAVE)

INTEGER*8 N
DOUBLE PRECISION WSAVE(*)
```

F95 INTERFACE

```
SUBROUTINE ZFFTI(N, WSAVE)

INTEGER :: N
REAL(8), DIMENSION(:) :: WSAVE

SUBROUTINE ZFFTI_64(N, WSAVE)

INTEGER(8) :: N
REAL(8), DIMENSION(:) :: WSAVE
```

C INTERFACE

```
#include <sunperf.h>

void zffti(int n, double *wsave);

void zffti_64(long n, double *wsave);
```

ARGUMENTS

N (input) Length of the sequence to be transformed. N >= 0.

WSAVE (input/output)
On entry, an array of dimension (4 * N + 15) or greater. ZFFTI needs to be called only once to initialize array WORK before calling ZFFTF and/or ZFFTB if N and WSAVE remain unchanged between these calls. Thus, subsequent transforms or inverse transforms of same size can be obtained faster than the first since they do not require initialization of the workspace.

Contents

NAME

```
zfftopt - compute the length of the closest fast FFT
```

SYNOPSIS

```
INTEGER FUNCTION ZFFTOPT(LEN)

INTEGER LEN

INTEGER*8 FUNCTION ZFFTOPT_64(LEN)

INTEGER*8 LEN
```

F95 INTERFACE

```
INTEGER FUNCTION ZFFTOPT(LEN)

INTEGER :: LEN

INTEGER(8) FUNCTION ZFFTOPT_64(LEN)

INTEGER(8) :: LEN
```

C INTERFACE

```
#include <sunperf.h>

int zfftopt(int len);

long zfftopt_64(long len);
```

PURPOSE

```
zfftopt computes the length of the closest fast  FFT.   Fast
```

Fourier transform algorithms, including those used in Performance Library, work best with vector lengths that are products of small primes. For example, an FFT of length 32=2**5 will run faster than an FFT of prime length 31 because 32 is a product of small primes and 31 is not. If your application is such that you can taper or zero pad your vector to a larger length then this function may help you select a better length and run your FFT faster.

ZFFTOPT will return an integer no smaller than the input argument N that is the closest number that is the product of small primes. ZFFTOPT will return 16 for an input of N=16 and return 18=2*3*3 for an input of N=17.

Note that the length computed here is not guaranteed to be optimal, only to be a
product of small primes. Also, the value returned may change as the underlying
FFTs become capable of handling larger primes. For example, passing in N=51 to day will return 52=2*2*13 rather than 51=3*17 because the FFTs in Performance Li brary do not have fast radix 17 code. In the future, radix 17 code may be added
and then N=51 will return 51.

Contents

NAME

zfftz - initialize the trigonometric weight and factor tables or compute the Fast Fourier transform (forward or inverse) of a double complex sequence.

SYNOPSIS

```
SUBROUTINE ZFFTZ(IOPT, N, SCALE, X, Y, TRIGS, IFAC, WORK, LWORK, IERR)

INTEGER IOPT, N, IFAC(*), LWORK, IERR
DOUBLE COMPLEX X(*), Y(*)
DOUBLE PRECISION SCALE, TRIGS(*), WORK(*)

SUBROUTINE ZFFTZ_64(IOPT, N, SCALE, X, Y, TRIGS, IFAC, WORK, LWORK, IERR)

INTEGER*8 IOPT, N, IFAC(*), LWORK, IERR
DOUBLE PRECISION SCALE, TRIGS(*), WORK(*)
DOUBLE COMPLEX X(*), Y(*)
```

F95 INTERFACE

```
SUBROUTINE FFT(IOPT, [N], [SCALE], X, Y, TRIGS, IFAC, WORK, [LWORK], IERR)

INTEGER, INTENT(IN) :: IOPT
INTEGER, INTENT(IN), OPTIONAL :: N, LWORK
REAL(8), INTENT(IN), OPTIONAL :: SCALE
COMPLEX(8), INTENT(IN), DIMENSION(:) :: X
COMPLEX(8), INTENT(OUT), DIMENSION(:) :: Y
REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
INTEGER, INTENT(INOUT), DIMENSION(:) :: IFAC
REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
INTEGER, INTENT(OUT) :: IERR

SUBROUTINE FFT_64(IOPT, [N], [SCALE], X, Y, TRIGS, IFAC, WORK, [LWORK], IERR)
```

```
INTEGER(8), INTENT(IN) :: IOPT
INTEGER(8), INTENT(IN), OPTIONAL :: N, LWORK
REAL(8), INTENT(IN), OPTIONAL :: SCALE
COMPLEX(8), INTENT(IN), DIMENSION(:) :: X
COMPLEX(8), INTENT(OUT), DIMENSION(:) :: Y
REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
#include <sunperf.h>

void zfftz_ (int *iopt, int *n, double *scale, doublecomplex
          *x,  doublecomplex  *y,  double *trigs, int *ifac,
          double *work, int *lwork, int *ierr);

void zfftz_64_ (long *iopt, long *n,  double  *scale,  doub-
          lecomplex  *x,  doublecomplex  *y,  double *trigs,
          long  *ifac,  double  *work,  long  *lwork,   long
          *ierr);
```

PURPOSE

zfftz initializes the trigonometric weight and factor tables or computes the Fast Fourier transform (forward or inverse) of a double complex sequence as follows:

```
               N-1
Y(k) = scale * SUM  W*X(j)
               j=0
```

where
k ranges from 0 to N-1
i = sqrt(-1)
isign = 1 for inverse transform or -1 for forward transform
W = exp(isign*i*j*k*2*pi/N)

ARGUMENTS

IOPT (input)
Integer specifying the operation to be performed:
IOPT = 0 computes the trigonometric weight table and factor table
IOPT = -1 computes forward FFT
IOPT = +1 computes inverse FFT

N (input)
Integer specifying length of the input sequence X. N is most efficient when it is a product of small primes. N >= 0. Unchanged on exit.

SCALE (input)
Double precision scalar by which transform results are scaled. Unchanged on exit. SCALE is defaulted to 1.0D0 for F95 INTERFACE.

X (input) On entry, X is a double complex array of dimension at least N that contains the sequence to be transformed.

Y (output)
Double complex array of dimension at least N that contains the transform results. X and Y may be the same array starting at the same memory location. Otherwise, it is assumed that there is no overlap between X and Y in memory.

TRIGS (input/output)
Double precision array of length 2*N that contains the trigonometric weights. The weights are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1 or IOPT = -1. Unchanged on exit.

IFAC (input/output)
Integer array of dimension at least 128 that contains the factors of N. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls where IOPT = 1 or IOPT = -1. Unchanged on exit.

WORK (workspace)
Double precision array of dimension at least 2*N. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)
Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)
On exit, integer IERR has one of the following values:
0 = normal return
-1 = IOPT is not 0, 1 or -1

```
-2 = N < 0
-3 = (LWORK is not 0) and (LWORK is less than 2*N)
-4 = memory allocation for workspace failed
```

SEE ALSO

fft

Contents

NAME

```
     zfftz2 - initialize  the  trigonometric  weight  and  factor
     tables or compute the two-dimensional Fast Fourier Transform
     (forward or inverse) of  a  two-dimensional  double  complex
     array.
```

SYNOPSIS

```
     SUBROUTINE ZFFTZ2(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     DOUBLE COMPLEX X(LDX, *), Y(LDY, *)
     DOUBLE PRECISION SCALE, TRIGS(*), WORK(*)

     SUBROUTINE ZFFTZ2_64(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER*8 IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     DOUBLE PRECISION SCALE, TRIGS(*), WORK(*)
     DOUBLE COMPLEX X(LDX, *), Y(LDY, *)
```

F95 INTERFACE

```
     SUBROUTINE FFT2(IOPT, [N1], [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
               IFAC, WORK, [LWORK], IERR)

     INTEGER, INTENT(IN) :: IOPT
     INTEGER, INTENT(IN), OPTIONAL :: N1, N2, LDX, LDY, LWORK
     REAL(8), INTENT(IN), OPTIONAL :: SCALE
     COMPLEX(8), INTENT(IN), DIMENSION(:,:) :: X
     COMPLEX(8), INTENT(OUT), DIMENSION(:,:) :: Y
     REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
```

```
      INTEGER, INTENT(INOUT), DIMENSION(:) :: IFAC
      REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
      INTEGER, INTENT(OUT) :: IERR

      SUBROUTINE FFT2_64(IOPT, [N1], [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
IFAC, WORK, [LWORK], IERR)

      INTEGER(8), INTENT(IN) :: IOPT
      INTEGER(8), INTENT(IN), OPTIONAL :: N1, N2, LDX, LDY, LWORK
      REAL(8), INTENT(IN), OPTIONAL :: SCALE
      COMPLEX(8), INTENT(IN), DIMENSION(:,:) :: X
      COMPLEX(8), INTENT(OUT), DIMENSION(:,:) :: Y
      REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
      INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
      REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
      INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
      #include <sunperf.h>
      void zfftz2_ (int *iopt, int *n1, int  *n2,  double  *scale,
                doublecomplex  *x, int *ldx, doublecomplex *y, int
                *ldy, double *trigs, int *ifac, double *work,  int
                *lwork, int *ierr);

      void zfftz2_64_ (long *iopt,  long  *n1,  long  *n2,  double
                *scale, doublecomplex *x, long *ldx, doublecomplex
                *y, long *ldy, double *trigs, long  *ifac,  double
                *work, long *lwork, long *ierr);
```

PURPOSE

zfftz2 initializes the trigonometric weight and factor tables or computes the two-dimensional Fast Fourier Transform (forward or inverse) of a two-dimensional double complex array. In computing the two-dimensional FFT, one-dimensional FFTs are computed along the columns of the input array. One-dimensional FFTs are then computed along the rows of the intermediate results.

```
                         N2-1  N1-1
      Y(k1,k2) = scale * SUM   SUM   W2*W1*X(j1,j2)
                         j2=0  j1=0
```

where
k1 ranges from 0 to N1-1 and k2 ranges from 0 to N2-1
i = sqrt(-1)
isign = 1 for inverse transform or -1 for forward transform
W1 = exp(isign*i*j1*k1*2*pi/N1)
W2 = exp(isign*i*j2*k2*2*pi/N2)

ARGUMENTS

```
IOPT (input)
          Integer specifying the operation to be performed:
          IOPT = 0 computes the trigonometric  weight  table
          and factor table
          IOPT = -1 computes forward FFT
          IOPT = +1 computes inverse FFT

N1 (input)
          Integer specifying length of the transform in  the
          first  dimension.  N1 is most efficient when it is
          a product of small primes.  N1 >= 0.  Unchanged on
          exit.

N2 (input)
          Integer specifying length of the transform in  the
          second dimension.  N2 is most efficient when it is
          a product of small primes.  N2 >= 0.  Unchanged on
          exit.
SCALE (input)
          Double precision scalar by which transform results
          are   scaled.   Unchanged   on  exit.   SCALE  is
          defaulted to 1.0D0 for F95 INTERFACE.

X (input) X is a double complex array  of  dimensions  (LDX,
          N2) that contains input data to be transformed.

LDX (input)
          Leading dimension of X.  LDX >=  N1  Unchanged  on
          exit.

Y (output)
          Y is a double complex array  of  dimensions  (LDY,
          N2)  that contains the transform results.  X and Y
          can be the same array starting at the same  memory
          location,   in  which  case  the  input  data  are
          overwritten by their  transform  results.   Other-
          wise,  it  is  assumed  that  there  is no overlap
          between X and Y in memory.

LDY (input)
          Leading dimension of Y.  If X and Y are  the  same
          array, LDY = LDX Else LDY >= N1 Unchanged on exit.

TRIGS (input/output)
          Double precision array of  length  2*(N1+N2)  that
```

contains the trigonometric weights. The weights are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1 or IOPT = -1. Unchanged on exit.

IFAC (input/output)
Integer array of dimension at least 2*128 that contains the factors of N1 and N2. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1 or IOPT = -1. Unchanged on exit.

WORK (workspace)
Double precision array of dimension at least 2*MAX(N1,N2)*NCPUS where NCPUS is the number of threads used to execute the routine. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)
Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)
On exit, integer IERR has one of the following values:
0 = normal return
-1 = IOPT is not 0, 1 or -1
-2 = N1 < 0
-3 = N2 < 0
-4 = (LDX < N1)
-5 = (LDY < N1) or (LDY not equal LDX when X and Y are same array)
-6 = (LWORK not equal 0) and (LWORK < 2*MAX(N1,N2)*NCPUS)
-7 = memory allocation failed

SEE ALSO

fft

CAUTIONS

On exit, entire output array Y(1:LDY, 1:N2) is overwritten.

Contents

NAME

```
     zfftz3 - initialize  the  trigonometric  weight  and  factor
     tables   or   compute  the  three-dimensional  Fast  Fourier
     Transform (forward or inverse) of a three-dimensional double
     complex array.
```

SYNOPSIS

```
     SUBROUTINE ZFFTZ3(IOPT, N1, N2, N3, SCALE, X, LDX1, LDX2, Y, LDY1, LDY2,
TRIGS, IFAC, WORK, LWORK, IERR)

     INTEGER IOPT, N1, N2, N3, LDX1, LDX2, LDY1,  LDY2,  IFAC(*),
     LWORK, IERR
     DOUBLE COMPLEX X(LDX1, LDX2, *), Y(LDY1, LDY2, *)
     DOUBLE PRECISION SCALE, TRIGS(*), WORK(*)

     SUBROUTINE ZFFTZ3_64(IOPT, N1, N2, N3, SCALE, X, LDX1, LDX2, Y, LDY1, LDY2,
TRIGS, IFAC, WORK, LWORK, IERR)

     INTEGER*8 IOPT, N1, N2, N3, LDX1, LDX2, LDY1, LDY2, IFAC(*),
     LWORK, IERR
     DOUBLE COMPLEX X(LDX1, LDX2, *), Y(LDY1, LDY2, *)
     DOUBLE PRECISION SCALE, TRIGS(*), WORK(*)
```

F95 INTERFACE

```
     SUBROUTINE FFT3(IOPT, [N1], [N2], [N3], [SCALE], X, [LDX1], LDX2, Y, [LDY1],
LDY2, TRIGS, IFAC, WORK, [LWORK], IERR)

     INTEGER, INTENT(IN) :: IOPT, LDX2, LDY2
     INTEGER, INTENT(IN), OPTIONAL :: N1,  N2,  N3,  LDX1,  LDY1,
     LWORK
     REAL(8), INTENT(IN), OPTIONAL :: SCALE
```

```
     COMPLEX(8), INTENT(IN), DIMENSION(:,:) :: X
     COMPLEX(8), INTENT(OUT), DIMENSION(:,:) :: Y
     REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER, INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER, INTENT(OUT) :: IERR

     SUBROUTINE FFT3_64(IOPT, [N1], [N2], [N3], [SCALE], X, [LDX1], LDX2, Y,
[LDY1], LDY2, TRIGS, IFAC, WORK, [LWORK], IERR)

     INTEGER(8), INTENT(IN) :: IOPT, LDX2, LDY2
     INTEGER(8), INTENT(IN), OPTIONAL :: N1, N2, N3, LDX1,  LDY1,
     LWORK
     REAL(8), INTENT(IN), OPTIONAL :: SCALE
     COMPLEX(8), INTENT(IN), DIMENSION(:,:) :: X
     COMPLEX(8), INTENT(OUT), DIMENSION(:,:) :: Y
     REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
     #include <sunperf.h>

     void zfftz3_ (int *iopt, int *n1, int *n2, int  *n3,  double
               *scale,  doublecomplex  *x,  int *ldx1, int *ldx2,
               doublecomplex *y, int  *ldy1,  int  *ldy2,  double
               *trigs,  int  *ifac, double *work, int *lwork, int
               *ierr);

     void zfftz3_64_ (long *iopt, long *n1, long *n2,  long  *n3,
               double  *scale, doublecomplex *x, long *ldx1, long
               *ldx2, doublecomplex *y, long *ldy1,  long  *ldy2,
               double  *trigs,  long  *ifac,  double  *work, long
               *lwork, long *ierr);
```

PURPOSE

zfftz3 initializes the trigonometric weight and factor tables or computes the three-dimensional Fast Fourier Transform (forward or inverse) of a three-dimensional double complex array.

```
                          N3-1  N2-1  N1-1
     Y(k1,k2,k3) = scale * SUM   SUM   SUM   W3*W2*W1*X(j1,j2,j3)
                          j3=0  j2=0  j1=0
```

where
k1 ranges from 0 to N1-1; k2 ranges from 0 to N2-1 and k3 ranges from 0 to N3-1

```
i = sqrt(-1)
isign = 1 for inverse transform or -1 for forward transform
W1 = exp(isign*i*j1*k1*2*pi/N1)
W2 = exp(isign*i*j2*k2*2*pi/N2)
W3 = exp(isign*i*j3*k3*2*pi/N3)
```

ARGUMENTS

```
IOPT (input)
          Integer specifying the operation to be performed:
          IOPT = 0 computes the trigonometric  weight  table
          and factor table
          IOPT = -1 computes forward FFT
          IOPT = +1 computes inverse FFT

N1 (input)
          Integer specifying length of the transform in  the
          first  dimension.  N1 is most efficient when it is
          a product of small primes.  N1 >= 0.  Unchanged on
          exit.

N2 (input)
          Integer specifying length of the transform in  the
          second dimension.  N2 is most efficient when it is
          a product of small primes.  N2 >= 0.  Unchanged on
          exit.

N3 (input)
          Integer specifying length of the transform in  the
          third  dimension.  N3 is most efficient when it is
          a product of small primes.  N3 >= 0.  Unchanged on
          exit.

SCALE (input)
          Double precision scalar by which transform results
          are   scaled.   Unchanged   on  exit.   SCALE  is
          defaulted to 1.0D0 for F95 INTERFACE.

X (input) X is a double complex array of  dimensions  (LDX1,
          LDX2,   N3)   that   contains  input  data  to  be
          transformed.

LDX1 (input)
          first dimension of X.  LDX1  >=  N1  Unchanged  on
          exit.

LDX2 (input)
          second dimension of X.  LDX2 >=  N2  Unchanged  on
```

exit.

Y (output)

Y is a double complex array of dimensions (LDY1, LDY2, N3) that contains the transform results. X and Y can be the same array starting at the same memory location, in which case the input data are overwritten by their transform results. Otherwise, it is assumed that there is no overlap between X and Y in memory.

LDY1 (input)

first dimension of Y. If X and Y are the same array, LDY1 = LDX1 Else LDY1 >= N1 Unchanged on exit.

LDY2 (input)

second dimension of Y. If X and Y are the same array, LDY2 = LDX2 Else LDY2 >= N2 Unchanged on exit.

TRIGS (input/output)

Double precision array of length 2*(N1+N2+N3) that contains the trigonometric weights. The weights are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1 or IOPT = -1. Unchanged on exit.

IFAC (input/output)

Integer array of dimension at least 3*128 that contains the factors of N1, N2 and N3. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1 or IOPT = -1. Unchanged on exit.

WORK (workspace)

Double precision array of dimension at least (2*MAX(N,N2,N3) + 32*N3) * NCPUS where NCPUS is the number of threads used to execute the routine. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)

Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)

On exit, integer IERR has one of the following values:

0 = normal return

-1 = IOPT is not 0, 1 or -1

-2 = N1 < 0
-3 = N2 < 0
-4 = N3 < 0
-5 = (LDX1 < N1)
-6 = (LDX2 < N2)
-7 = (LDY1 < N1) or (LDY1 not equal LDX1 when X and Y are same array)
-8 = (LDY2 < N2) or (LDY2 not equal LDX2 when X and Y are same array)
-9 = (LWORK not equal 0) and (LWORK < (2*MAX(N,N2,N3) + 16*N3) * NCPUS)
-10 = memory allocation failed

SEE ALSO

fft

CAUTIONS

On exit, output subarray Y(1:LDY1, 1:N2, 1:N3) is overwritten.

Contents

NAME

zfftzm - initialize the trigonometric weight and factor tables or compute the one-dimensional Fast Fourier Transform (forward or inverse) of a set of data sequences stored in a two-dimensional double complex array.

SYNOPSIS

```
     SUBROUTINE ZFFTZM(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     DOUBLE COMPLEX X(LDX, *), Y(LDY, *)
     DOUBLE PRECISION SCALE, TRIGS(*), WORK(*)

     SUBROUTINE ZFFTZM_64(IOPT, N1, N2, SCALE, X, LDX, Y, LDY, TRIGS, IFAC, WORK,
LWORK, IERR)

     INTEGER*8 IOPT, N1, N2, LDX, LDY, IFAC(*), LWORK, IERR
     DOUBLE PRECISION SCALE, TRIGS(*), WORK(*)
     DOUBLE COMPLEX X(LDX, *), Y(LDY, *)
```

F95 INTERFACE

```
     SUBROUTINE FFTM(IOPT, [N1], [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
               IFAC, WORK, [LWORK], IERR)

     INTEGER, INTENT(IN) :: IOPT
     INTEGER, INTENT(IN), OPTIONAL :: N1, N2, LDX, LDY, LWORK
     REAL(8), INTENT(IN), OPTIONAL :: SCALE
     COMPLEX(8), INTENT(IN), DIMENSION(:,:) :: X
     COMPLEX(8), INTENT(OUT), DIMENSION(:,:) :: Y
     REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER, INTENT(INOUT), DIMENSION(:) :: IFAC
```

```
     REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER, INTENT(OUT) :: IERR

     SUBROUTINE FFTM_64(IOPT, [N1], [N2], [SCALE], X, [LDX], Y, [LDY], TRIGS,
IFAC, WORK, [LWORK], IERR)

     INTEGER(8), INTENT(IN) :: IOPT
     INTEGER(8), INTENT(IN), OPTIONAL :: N1, N2, LDX, LDY, LWORK
     REAL(8), INTENT(IN), OPTIONAL :: SCALE
     COMPLEX(8), INTENT(IN), DIMENSION(:,:) :: X
     COMPLEX(8), INTENT(OUT), DIMENSION(:,:) :: Y
     REAL(8), INTENT(INOUT), DIMENSION(:) :: TRIGS
     INTEGER(8), INTENT(INOUT), DIMENSION(:) :: IFAC
     REAL(8), INTENT(OUT), DIMENSION(:) :: WORK
     INTEGER(8), INTENT(OUT) :: IERR
```

C INTERFACE

```
     #include <sunperf.h>
     void zfftzm_ (int *iopt, int  *m,  int  *n,  double  *scale,
               doublecomplex  *x, int *ldx, doublecomplex *y, int
               *ldy, double *trigs, int *ifac, double *work,  int
               *lwork, int *ierr);

     void zfftzm_64_  (long  *iopt,  long  *m,  long  *n,  double
               *scale, doublecomplex *x, long *ldx, doublecomplex
               *y, long *ldy, double *trigs, long  *ifac,  double
               *work, long *lwork, long *ierr);
```

PURPOSE

zfftzm initializes the trigonometric weight and factor tables or computes the one-dimensional Fast Fourier Transform (forward or inverse) of a set of data sequences stored in a two-dimensional double complex array:

```
            N1-1
     Y(k,l) = SUM  W*X(j,l)
            j=0

     where
     k ranges from 0 to N1-1 and l ranges from 0 to N2-1
     i = sqrt(-1)
     isign = 1 for inverse transform or -1 for forward transform
     W = exp(isign*i*j*k*2*pi/N1)
```

ARGUMENTS

IOPT (input)
Integer specifying the operation to be performed:
IOPT = 0 computes the trigonometric weight table and factor table
IOPT = -1 computes forward FFT
IOPT = +1 computes inverse FFT

N1 (input)
Integer specifying length of the input sequences. N1 is most efficient when it is a product of small primes. N1 >= 0. Unchanged on exit.

N2 (input)
Integer specifying number of input sequences. N2 >= 0. Unchanged on exit.

SCALE (input)
Double precision scalar by which transform results are scaled. Unchanged on exit. SCALE is defaulted to 1.0D0 for F95 INTERFACE.

X (input) X is a double complex array of dimensions (LDX, N2) that contains the sequences to be transformed stored in its columns.

LDX (input)
Leading dimension of X. LDX >= N1 Unchanged on exit.

Y (output)
Y is a double complex array of dimensions (LDY, N2) that contains the transform results of the input sequences. X and Y can be the same array starting at the same memory location, in which case the input sequences are overwritten by their transform results. Otherwise, it is assumed that there is no overlap between X and Y in memory.

LDY (input)
Leading dimension of Y. If X and Y are the same array, LDY = LDX Else LDY >= N1 Unchanged on exit.

TRIGS (input/output)
Double precision array of length 2*N1 that contains the trigonometric weights. The weights are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1 or IOPT = -1. Unchanged on exit.

IFAC (input/output)

Integer array of dimension at least 128 that contains the factors of N1. The factors are computed when the routine is called with IOPT = 0 and they are used in subsequent calls when IOPT = 1 or IOPT = -1. Unchanged on exit.

WORK (workspace)

Double precision array of dimension at least 2*N1*NCPUS where NCPUS is the number of threads used to execute the routine. The user can also choose to have the routine allocate its own workspace (see LWORK).

LWORK (input)

Integer specifying workspace size. If LWORK = 0, the routine will allocate its own workspace.

IERR (output)

On exit, integer IERR has one of the following values:

0 = normal return
-1 = IOPT is not 0, 1 or -1
-2 = N1 < 0
-3 = N2 < 0
-4 = (LDX < N1)
-5 = (LDY < N1) or (LDY not equal LDX when X and Y are same array)
-6 = (LWORK not equal 0) and (LWORK < 2*N1*NCPUS)
-7 = memory allocation failed

SEE ALSO

fft

Contents

NAME

```
zgbbrd - reduce a complex general m-by-n band  matrix  A  to
real upper bidiagonal form B by a unitary transformation
```

SYNOPSIS

```
SUBROUTINE ZGBBRD(VECT, M, N, NCC, KL, KU, AB, LDAB, D, E, Q, LDQ,
      PT, LDPT, C, LDC, WORK, RWORK, INFO)

CHARACTER * 1 VECT
DOUBLE COMPLEX AB(LDAB,*), Q(LDQ,*),  PT(LDPT,*),  C(LDC,*),
WORK(*)
INTEGER M, N, NCC, KL, KU, LDAB, LDQ, LDPT, LDC, INFO
DOUBLE PRECISION D(*), E(*), RWORK(*)

SUBROUTINE ZGBBRD_64(VECT, M, N, NCC, KL, KU, AB, LDAB, D, E, Q, LDQ,
      PT, LDPT, C, LDC, WORK, RWORK, INFO)

CHARACTER * 1 VECT
DOUBLE COMPLEX AB(LDAB,*), Q(LDQ,*),  PT(LDPT,*),  C(LDC,*),
WORK(*)
INTEGER*8 M, N, NCC, KL, KU, LDAB, LDQ, LDPT, LDC, INFO
DOUBLE PRECISION D(*), E(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GBBRD(VECT, M, [N], [NCC], KL, KU, AB, [LDAB], D, E, Q,
       [LDQ], PT, [LDPT], C, [LDC], [WORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: VECT
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: AB, Q, PT, C
INTEGER :: M, N, NCC, KL, KU, LDAB, LDQ, LDPT, LDC, INFO
REAL(8), DIMENSION(:) :: D, E, RWORK
```

```
SUBROUTINE GBBRD_64(VECT, M, [N], [NCC], KL, KU, AB, [LDAB], D, E,
       Q, [LDQ], PT, [LDPT], C, [LDC], [WORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: VECT
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: AB, Q, PT, C
INTEGER(8) :: M, N, NCC, KL, KU, LDAB, LDQ, LDPT, LDC, INFO
REAL(8), DIMENSION(:) :: D, E, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void zgbbrd(char vect, int m, int n, int ncc,  int  kl,  int
          ku, doublecomplex *ab, int ldab, double *d, double
          *e, doublecomplex *q, int ldq, doublecomplex  *pt,
          int ldpt, doublecomplex *c, int ldc, int *info);

void zgbbrd_64(char vect, long m, long n, long ncc, long kl,
          long  ku, doublecomplex *ab, long ldab, double *d,
          double *e, doublecomplex *q, long ldq,  doublecom-
          plex  *pt,  long ldpt, doublecomplex *c, long ldc,
          long *info);
```

PURPOSE

zgbbrd reduces a complex general m-by-n band matrix A to real upper bidiagonal form B by a unitary transformation: Q' * A * P = B.

The routine computes B, and optionally forms Q or P', or computes Q'*C for a given matrix C.

ARGUMENTS

VECT (input)
Specifies whether or not the matrices Q and P' are to be formed. = 'N': do not form Q or P';
= 'Q': form Q only;
= 'P': form P' only;
= 'B': form both.

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

NCC (input)

The number of columns of the matrix C. NCC >= 0.

KL (input)
The number of subdiagonals of the matrix A. KL >= 0.

KU (input)
The number of superdiagonals of the matrix A. KU >= 0.

AB (input/output)
On entry, the m-by-n band matrix A, stored in rows 1 to KL+KU+1. The j-th column of A is stored in the j-th column of the array AB as follows: AB(ku+1+i-j,j) = A(i,j) for max(1,j-ku)<=i<=min(m,j+kl). On exit, A is overwritten by values generated during the reduction.

LDAB (input)
The leading dimension of the array A. LDAB >= KL+KU+1.

D (output)
The diagonal elements of the bidiagonal matrix B.

E (output)
The superdiagonal elements of the bidiagonal matrix B.

Q (output)
If VECT = 'Q' or 'B', the m-by-m unitary matrix Q. If VECT = 'N' or 'P', the array Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= max(1,M) if VECT = 'Q' or 'B'; LDQ >= 1 otherwise.

PT (output)
If VECT = 'P' or 'B', the n-by-n unitary matrix P'. If VECT = 'N' or 'Q', the array PT is not referenced.

LDPT (input)
The leading dimension of the array PT. LDPT >= max(1,N) if VECT = 'P' or 'B'; LDPT >= 1 otherwise.

C (input/output)
On entry, an m-by-ncc matrix C. On exit, C is

```
          overwritten by Q'*C.  C is not referenced if NCC =
          0.

LDC (input)
          The leading dimension of  the  array  C.   LDC  >=
          max(1,M) if NCC > 0; LDC >= 1 if NCC = 0.
WORK (workspace)
          dimension(MAX(M,N))

RWORK (workspace)
          dimension(MAX(M,N))

INFO (output)
          = 0:  successful exit.
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value.
```

Contents

NAME

```
zgbcon - estimate the reciprocal of the condition number  of
a complex general band matrix A, in either the 1-norm or the
infinity-norm,
```

SYNOPSIS

```
SUBROUTINE ZGBCON(NORM, N, NSUB, NSUPER, A, LDA, IPIVOT, ANORM,
      RCOND, WORK, WORK2, INFO)

CHARACTER * 1 NORM
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER N, NSUB, NSUPER, LDA, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION WORK2(*)

SUBROUTINE ZGBCON_64(NORM, N, NSUB, NSUPER, A, LDA, IPIVOT, ANORM,
      RCOND, WORK, WORK2, INFO)

CHARACTER * 1 NORM
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, NSUB, NSUPER, LDA, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GBCON(NORM, [N], NSUB, NSUPER, A, [LDA], IPIVOT, ANORM,
       RCOND, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM
COMPLEX(8), DIMENSION(:) :: WORK
```

```
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, NSUB, NSUPER, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: WORK2

SUBROUTINE GBCON_64(NORM, [N], NSUB, NSUPER, A, [LDA], IPIVOT, ANORM,
       RCOND, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, NSUB, NSUPER, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zgbcon(char norm, int n, int nsub,  int  nsuper,  doub-
          lecomplex  *a, int lda, int *ipivot, double anorm,
          double *rcond, int *info);

void zgbcon_64(char norm, long n, long  nsub,  long  nsuper,
          doublecomplex  *a,  long lda, long *ipivot, double
          anorm, double *rcond, long *info);
```

PURPOSE

zgbcon estimates the reciprocal of the condition number of a complex general band matrix A, in either the 1-norm or the infinity-norm, using the LU factorization computed by CGBTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as

RCOND = 1 / (norm(A) * norm(inv(A))).

ARGUMENTS

NORM (input)
Specifies whether the 1-norm condition number or the infinity-norm condition number is required:
= '1' or 'O': 1-norm;
= 'I':   Infinity-norm.

N (input) The order of the matrix A. N >= 0.

NSUB (input)
The number of subdiagonals within the band of A. NSUB >= 0.

NSUPER (input)
The number of superdiagonals within the band of A. NSUPER >= 0.

A (input) Details of the LU factorization of the band matrix A, as computed by CGBTRF. U is stored as an upper triangular band matrix with NSUB+NSUPER superdiagonals in rows 1 to NSUB+NSUPER+1, and the multipliers used during the factorization are stored in rows NSUB+NSUPER+2 to 2*NSUB+NSUPER+1.

LDA (input)
The leading dimension of the array A. LDA >= 2*NSUB+NSUPER+1.

IPIVOT (input)
The pivot indices; for 1 <= i <= N, row i of the matrix was interchanged with row IPIVOT(i).

ANORM (input)
If NORM = '1' or 'O', the 1-norm of the original matrix A. If NORM = 'I', the infinity-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(norm(A) * norm(inv(A))).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension (N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zgbequ - compute row and column scalings intended to equili-
brate  an  M-by-N  band  matrix  A  and reduce its condition
number
```

SYNOPSIS

```
SUBROUTINE ZGBEQU(M, N, KL, KU, A, LDA, R, C, ROWCN,
      COLCN, AMAX, INFO)

DOUBLE COMPLEX A(LDA,*)
INTEGER M, N, KL, KU, LDA, INFO
DOUBLE PRECISION ROWCN, COLCN, AMAX
DOUBLE PRECISION R(*), C(*)

SUBROUTINE ZGBEQU_64(M, N, KL, KU, A, LDA, R, C, ROWCN,
      COLCN, AMAX, INFO)

DOUBLE COMPLEX A(LDA,*)
INTEGER*8 M, N, KL, KU, LDA, INFO
DOUBLE PRECISION ROWCN, COLCN, AMAX
DOUBLE PRECISION R(*), C(*)
```

F95 INTERFACE

```
SUBROUTINE GBEQU([M], [N], KL, KU, A, [LDA], R, C,
       ROWCN, COLCN, AMAX, [INFO])

COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, KL, KU, LDA, INFO
REAL(8) :: ROWCN, COLCN, AMAX
REAL(8), DIMENSION(:) :: R, C

SUBROUTINE GBEQU_64([M], [N], KL, KU, A, [LDA], R, C,
```

```
          ROWCN, COLCN, AMAX, [INFO])

   COMPLEX(8), DIMENSION(:,:) :: A
   INTEGER(8) :: M, N, KL, KU, LDA, INFO
   REAL(8) :: ROWCN, COLCN, AMAX
   REAL(8), DIMENSION(:) :: R, C
```

C INTERFACE

```
#include <sunperf.h>

void zgbequ(int m, int n, int kl, int ku, doublecomplex  *a,
          int lda, double *r, double *c, double *rowcn, dou-
          ble *colcn, double *amax, int *info);
void zgbequ_64(long m, long n, long kl, long ku,  doublecom-
          plex  *a,  long  lda, double *r, double *c, double
          *rowcn, double *colcn, double *amax, long *info);
```

PURPOSE

zgbequ computes row and column scalings intended to equilibrate an M-by-N band matrix A and reduce its condition number. R returns the row scale factors and C the column scale factors, chosen to try to make the largest element in each row and column of the matrix B with elements B(i,j)=R(i)*A(i,j)*C(j) have absolute value 1.

R(i) and C(j) are restricted to be between SMLNUM = smallest safe number and BIGNUM = largest safe number. Use of these scaling factors is not guaranteed to reduce the condition number of A but works well in practice.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

KL (input)
The number of subdiagonals within the band of A. KL >= 0.

KU (input)
The number of superdiagonals within the band of A. KU >= 0.

A (input) The band matrix A, stored in rows 1 to KL+KU+1. The j-th column of A is stored in the j-th column of the array A as follows: A(ku+1+i-j,j) = A(i,j) for max(1,j-ku)<=i<=min(m,j+kl).

LDA (input)
The leading dimension of the array A. LDA >= KL+KU+1.

R (output)
If INFO = 0, or INFO > M, R contains the row scale factors for A.

C (output)
If INFO = 0, C contains the column scale factors for A.

ROWCN (output)
If INFO = 0 or INFO > M, ROWCN contains the ratio of the smallest R(i) to the largest R(i). If ROWCN >= 0.1 and AMAX is neither too large nor too small, it is not worth scaling by R.

COLCN (output)
If INFO = 0, COLCN contains the ratio of the smallest C(i) to the largest C(i). If COLCN >= 0.1, it is not worth scaling by C.

AMAX (output)
Absolute value of largest matrix element. If AMAX is very close to overflow or very close to underflow, the matrix should be scaled.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= M: the i-th row of A is exactly zero
> M: the (i-M)-th column of A is exactly zero

Contents

NAME

zgbmv - perform one of the matrix-vector operations y := alpha*A*x + beta*y, or y := alpha*A'*x + beta*y, or y := alpha*conjg(A')*x + beta*y

SYNOPSIS

```
SUBROUTINE ZGBMV(TRANSA, M, N, NSUB, NSUPER, ALPHA, A, LDA, X, INCX,
      BETA, Y, INCY)

CHARACTER * 1 TRANSA
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(LDA,*), X(*), Y(*)
INTEGER M, N, NSUB, NSUPER, LDA, INCX, INCY

SUBROUTINE ZGBMV_64(TRANSA, M, N, NSUB, NSUPER, ALPHA, A, LDA, X,
      INCX, BETA, Y, INCY)

CHARACTER * 1 TRANSA
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(LDA,*), X(*), Y(*)
INTEGER*8 M, N, NSUB, NSUPER, LDA, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE GBMV([TRANSA], [M], [N], NSUB, NSUPER, ALPHA, A, [LDA], X,
       [INCX], BETA, Y, [INCY])

CHARACTER(LEN=1) :: TRANSA
COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:) :: X, Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, NSUB, NSUPER, LDA, INCX, INCY
```

```
SUBROUTINE GBMV_64([TRANSA], [M], [N], NSUB, NSUPER, ALPHA, A, [LDA],
       X, [INCX], BETA, Y, [INCY])

CHARACTER(LEN=1) :: TRANSA
COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:) :: X, Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, NSUB, NSUPER, LDA, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>

void zgbmv(char transa, int m, int n, int nsub, int  nsuper,
          doublecomplex  *alpha,  doublecomplex *a, int lda,
          doublecomplex *x, int incx,  doublecomplex  *beta,
          doublecomplex *y, int incy);

void zgbmv_64(char transa, long m, long n, long  nsub,  long
          nsuper,  doublecomplex  *alpha,  doublecomplex *a,
          long lda, doublecomplex *x, long incx,  doublecom-
          plex *beta, doublecomplex *y, long incy);
```

PURPOSE

zgbmv performs one of the matrix-vector operations y := alpha*A*x + beta*y, or y := alpha*A'*x + beta*y, or y := alpha*conjg(A')*x + beta*y where alpha and beta are scalars, x and y are vectors and A is an m by n band matrix, with nsub sub-diagonals and nsuper super-diagonals.

ARGUMENTS

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:
TRANSA = 'N' or 'n'   y := alpha*A*x + beta*y.
TRANSA = 'T' or 't'   y := alpha*A'*x + beta*y.
TRANSA = 'C' or 'c'   y := alpha*conjg(A')*x + beta*y.
Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

M (input)
On entry, M specifies the number of rows of the matrix A. M must be at least zero. Unchanged on exit.

N (input)
On entry, N specifies the number of columns of the matrix A. N must be at least zero. Unchanged on exit.

NSUB (input)
On entry, NSUB specifies the number of sub-diagonals of the matrix A. NSUB must satisfy 0 .le. NSUB. Unchanged on exit.

NSUPER (input)
On entry, NSUPER specifies the number of super-diagonals of the matrix A. NSUPER must satisfy 0 .le. NSUPER. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
Before entry, the leading (nsub + nsuper + 1) by n part of the array A must contain the matrix of coefficients, supplied column by column, with the leading diagonal of the matrix in row (nsuper + 1) of the array, the first super-diagonal starting at position 2 in row nsuper, the first sub-diagonal starting at position 1 in row (nsuper + 2), and so on. Elements in the array A that do not correspond to elements in the band matrix (such as the top left nsuper by nsuper triangle) are not referenced. The following program segment will transfer a band matrix from conventional full matrix storage to band storage:

```
    DO 20, J = 1, N
      K = NSUPER + 1 - J
      DO 10, I = MAX( 1, J - NSUPER ), MIN( M, J +
NSUB )
        A( K + I, J ) = matrix( I, J )
10    CONTINUE
20 CONTINUE
```

Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA must be at least (nsub + nsuper + 1). Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)) when TRANSA = 'N' or 'n' and at least (1 + (m - 1)*abs(INCX)) otherwise. Before entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. When BETA is supplied as zero then Y need not be set on input. Unchanged on exit.

Y (input/output)
(1 + (m - 1)*abs(INCY)) when TRANSA = 'N' or 'n' and at least (1 + (n - 1)*abs(INCY)) otherwise. Before entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
zgbrfs - improve the computed solution to a system of linear
equations  when  the  coefficient matrix is banded, and pro-
vides error bounds and  backward  error  estimates  for  the
solution
```

SYNOPSIS

```
SUBROUTINE ZGBRFS(TRANSA, N, KL, KU, NRHS, A, LDA, AF, LDAF,
      IPIVOT, B, LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZGBRFS_64(TRANSA, N, KL, KU, NRHS, A, LDA, AF, LDAF,
      IPIVOT, B, LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER*8 N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GBRFS([TRANSA], [N], KL, KU, [NRHS], A, [LDA], AF,
       [LDAF], IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2],
       [INFO])
```

```
CHARACTER(LEN=1) :: TRANSA
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER :: N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE GBRFS_64([TRANSA], [N], KL, KU, [NRHS], A, [LDA],
       AF, [LDAF], IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: TRANSA
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zgbrfs(char transa, int n, int kl, int  ku,  int  nrhs,
          doublecomplex  *a, int lda, doublecomplex *af, int
          ldaf, int  *ipivot,  doublecomplex  *b,  int  ldb,
          doublecomplex  *x,  int  ldx, double *ferr, double
          *berr, int *info);

void zgbrfs_64(char transa, long n, long kl, long  ku,  long
          nrhs,  doublecomplex  *a,  long lda, doublecomplex
          *af, long ldaf, long  *ipivot,  doublecomplex  *b,
          long  ldb,  doublecomplex  *x,  long  ldx,  double
          *ferr, double *berr, long *info);
```

PURPOSE

zgbrfs improves the computed solution to a system of linear equations when the coefficient matrix is banded, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

```
TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)
```

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A. N >= 0.

KL (input)
The number of subdiagonals within the band of A. KL >= 0.

KU (input)
The number of superdiagonals within the band of A. KU >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The original band matrix A, stored in rows 1 to KL+KU+1. The j-th column of A is stored in the j-th column of the array A as follows: A(ku+1+i-j,j) = A(i,j) for max(1,j-ku)<=i<=min(n,j+kl).

LDA (input)
The leading dimension of the array A. LDA >= KL+KU+1.

AF (input)
Details of the LU factorization of the band matrix A, as computed by CGBTRF. U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1.

LDAF (input)
The leading dimension of the array AF. LDAF >= 2*KL*KU+1.

IPIVOT (input)
The pivot indices from CGBTRF; for 1<=i<=N, row i of the matrix was interchanged with row IPIVOT(i).

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by CGBTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zgbsv - compute the solution to a complex system  of  linear
equations  A  *  X  = B, where A is a band matrix of order N
with KL subdiagonals and KU superdiagonals, and X and B  are
N-by-NRHS matrices
```

SYNOPSIS

```
SUBROUTINE ZGBSV(N, KL, KU, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, KL, KU, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZGBSV_64(N, KL, KU, NRHS, A, LDA, IPIVOT, B, LDB,
      INFO)

DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, KL, KU, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE GBSV([N], KL, KU, [NRHS], A, [LDA], IPIVOT, B, [LDB],
       [INFO])

COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: N, KL, KU, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE GBSV_64([N], KL, KU, [NRHS], A, [LDA], IPIVOT, B,
       [LDB], [INFO])
```

```
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: N, KL, KU, NRHS, LDA, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void zgbsv(int n, int kl, int ku,  int  nrhs,  doublecomplex
          *a,  int  lda,  int *ipivot, doublecomplex *b, int
          ldb, int *info);

void zgbsv_64(long n, long kl, long  ku,  long  nrhs,  doub-
          lecomplex  *a,  long lda, long *ipivot, doublecom-
          plex *b, long ldb, long *info);
```

PURPOSE

zgbsv computes the solution to a complex system of linear equations A * X = B, where A is a band matrix of order N with KL subdiagonals and KU superdiagonals, and X and B are N-by-NRHS matrices.

The LU decomposition with partial pivoting and row interchanges is used to factor A as A = L * U, where L is a product of permutation and unit lower triangular matrices with KL subdiagonals, and U is upper triangular with KL+KU superdiagonals. The factored form of A is then used to solve the system of equations A * X = B.

ARGUMENTS

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

KL (input)
The number of subdiagonals within the band of A. KL >= 0.

KU (input)
The number of superdiagonals within the band of A. KU >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input/output)

On entry, the matrix A in band storage, in rows KL+1 to 2*KL+KU+1; rows 1 to KL of the array need not be set. The j-th column of A is stored in the j-th column of the array A as follows: A(KL+KU+1+i-j,j) = A(i,j) for max(1,j-KU)<=i<=min(N,j+KL) On exit, details of the factorization: U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1. See below for further details.

LDA (input)
The leading dimension of the array A. LDA >= 2*KL+KU+1.

IPIVOT (output)
The pivot indices that define the permutation matrix P; row i of the matrix was interchanged with row IPIVOT(i).

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly singular, and the solution has not been computed.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when M = N = 6, KL = 2, KU = 1:

```
On entry:                       On exit:

    *    *    *    +    +    +       *    *    *   u14   u25
u36
    *    *    +    +    +    +       *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
```

```
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
   a21  a32  a43  a54  a65   *      m21  m32  m43  m54   m65
*
   a31  a42  a53  a64   *    *      m31  m42  m53   m64    *
*
```

Array elements marked * are not used by the routine; elements marked + need not be set on entry, but are required by the routine to store elements of U because of fill-in resulting from the row interchanges.

Contents

NAME

```
zgbsvx - use the LU factorization to compute the solution to
a  complex  system of linear equations A * X = B, A**T * X =
B, or A**H * X = B,
```

SYNOPSIS

```
SUBROUTINE ZGBSVX(FACT, TRANSA, N, KL, KU, NRHS, A, LDA, AF,
      LDAF, IPIVOT, EQUED, R, C, B, LDB, X, LDX, RCOND, FERR,
      BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, TRANSA, EQUED
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION R(*), C(*), FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZGBSVX_64(FACT, TRANSA, N, KL, KU, NRHS, A, LDA, AF,
      LDAF, IPIVOT, EQUED, R, C, B, LDB, X, LDX, RCOND, FERR,
      BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, TRANSA, EQUED
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER*8 N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION R(*), C(*), FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GBSVX(FACT, [TRANSA], [N], KL, KU, [NRHS], A, [LDA],
```

```
       AF, [LDAF], IPIVOT, EQUED, R, C, B, [LDB], X, [LDX],
       RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, TRANSA, EQUED
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER :: N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: R, C, FERR, BERR, WORK2

SUBROUTINE GBSVX_64(FACT, [TRANSA], [N], KL, KU, [NRHS], A,
       [LDA], AF, [LDAF], IPIVOT, EQUED, R, C, B, [LDB], X, [LDX],
       RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, TRANSA, EQUED
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, KL, KU, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: R, C, FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zgbsvx(char fact, char transa, int n, int kl,  int  ku,
          int nrhs, doublecomplex *a, int lda, doublecomplex
          *af, int ldaf, int *ipivot, char equed, double *r,
          double  *c,  doublecomplex *b, int ldb, doublecom-
          plex *x, int ldx,  double  *rcond,  double  *ferr,
          double *berr, int *info);

void zgbsvx_64(char fact, char transa, long n, long kl, long
          ku,  long  nrhs, doublecomplex *a, long lda, doub-
          lecomplex  *af,  long  ldaf,  long  *ipivot,  char
          equed,  double  *r,  double  *c, doublecomplex *b,
          long  ldb,  doublecomplex  *x,  long  ldx,  double
          *rcond, double *ferr, double *berr, long *info);
```

PURPOSE

zgbsvx uses the LU factorization to compute the solution to a complex system of linear equations A * X = B, A**T * X = B, or A**H * X = B, where A is a band matrix of order N with KL subdiagonals and KU superdiagonals, and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are

```
also provided.

The following steps are performed by this subroutine:

1. If FACT = 'E',  real  scaling  factors  are  computed  to
equilibrate
   the system:
      TRANS = 'N':  diag(R)*A*diag(C)     *inv(diag(C))*X  =
diag(R)*B
      TRANS = 'T': (diag(R)*A*diag(C))**T *inv(diag(R))*X  =
diag(C)*B
      TRANS = 'C': (diag(R)*A*diag(C))**H *inv(diag(R))*X  =
diag(C)*B
   Whether or not the system will be equilibrated depends on
the
   scaling of the matrix A, but if equilibration is used,  A
is
   overwritten by diag(R)*A*diag(C) and B by  diag(R)*B  (if
TRANS='N')
   or diag(C)*B (if TRANS = 'T' or 'C').

2. If FACT = 'N' or 'E', the LU  decomposition  is  used  to
factor the
   matrix A (after equilibration if FACT = 'E') as
      A = L * U,
   where L is a product of permutation and unit  lower  tri-
angular
   matrices with KL subdiagonals, and U is upper  triangular
with
   KL+KU superdiagonals.

3. If some U(i,i)=0, so that U is exactly singular, then the
routine
   returns with INFO = i. Otherwise, the factored form of  A
is used
   to estimate the condition number of the matrix A.  If the
   reciprocal of the condition number is less  than  machine
precision,
   INFO = N+1 is returned as  a  warning,  but  the  routine
still goes on
   to solve for X and  compute  error  bounds  as  described
below.

4. The system of equations is solved for X  using  the  fac-
tored form
   of A.

5. Iterative refinement is applied to improve  the  computed
solution
   matrix and calculate  error  bounds  and  backward  error
```

estimates
for it.

6. If equilibration was used, the matrix X is premultiplied by
diag(C) (if TRANS = 'N') or diag(R) (if TRANS = 'T' or 'C') so
that it solves the original system before equilibration.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored. = 'F': On entry, AF and IPIVOT contain the factored form of A. If EQUED is not 'N', the matrix A has been equilibrated with scaling factors given by R and C. A, AF, and IPIVOT are not modified. = 'N': The matrix A will be copied to AF and factored.
= 'E': The matrix A will be equilibrated if necessary, then copied to AF and factored.

TRANSA (input)
Specifies the form of the system of equations. = 'N': A * X = B (No transpose)
= 'T': A**T * X = B (Transpose)
= 'C': A**H * X = B (Conjugate transpose)

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

KL (input)
The number of subdiagonals within the band of A. KL >= 0.

KU (input)
The number of superdiagonals within the band of A. KU >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input/output)
On entry, the matrix A in band storage, in rows 1

to KL+KU+1. The j-th column of A is stored in the j-th column of the array A as follows: A(KU+1+i-j,j) = A(i,j) for max(1,j-KU)<=i<=min(N,j+kl)

If FACT = 'F' and EQUED is not 'N', then A must have been equilibrated by the scaling factors in R and/or C. A is not modified if FACT = 'F' or 'N', or if FACT = 'E' and EQUED = 'N' on exit.

On exit, if EQUED .ne. 'N', A is scaled as follows: EQUED = 'R': A := diag(R) * A
EQUED = 'C': A := A * diag(C)
EQUED = 'B': A := diag(R) * A * diag(C).

LDA (input)
The leading dimension of the array A. LDA >= KL+KU+1.

AF (input/output)
If FACT = 'F', then AF is an input argument and on entry contains details of the LU factorization of the band matrix A, as computed by CGBTRF. U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1. If EQUED .ne. 'N', then AF is the factored form of the equilibrated matrix A.

If FACT = 'N', then AF is an output argument and on exit returns details of the LU factorization of A.

If FACT = 'E', then AF is an output argument and on exit returns details of the LU factorization of the equilibrated matrix A (see the description of A for the form of the equilibrated matrix).

LDAF (input)
The leading dimension of the array AF. LDAF >= 2*KL+KU+1.

IPIVOT (input)
If FACT = 'F', then IPIVOT is an input argument and on entry contains the pivot indices from the factorization A = L*U as computed by CGBTRF; row i of the matrix was interchanged with row IPIVOT(i).

If FACT = 'N', then IPIVOT is an output argument and on exit contains the pivot indices from the factorization A = L*U of the original matrix A.

If FACT = 'E', then IPIVOT is an output argument and on exit contains the pivot indices from the factorization A = L*U of the equilibrated matrix A.

EQUED (input)

Specifies the form of equilibration that was done. = 'N': No equilibration (always true if FACT = 'N').
= 'R': Row equilibration, i.e., A has been premultiplied by diag(R). = 'C': Column equilibration, i.e., A has been postmultiplied by diag(C). = 'B': Both row and column equilibration, i.e., A has been replaced by diag(R) * A * diag(C). EQUED is an input argument if FACT = 'F'; otherwise, it is an output argument.

R (input/output)

The row scale factors for A. If EQUED = 'R' or 'B', A is multiplied on the left by diag(R); if EQUED = 'N' or 'C', R is not accessed. R is an input argument if FACT = 'F'; otherwise, R is an output argument. If FACT = 'F' and EQUED = 'R' or 'B', each element of R must be positive.

C (input/output)

The column scale factors for A. If EQUED = 'C' or 'B', A is multiplied on the right by diag(C); if EQUED = 'N' or 'R', C is not accessed. C is an input argument if FACT = 'F'; otherwise, C is an output argument. If FACT = 'F' and EQUED = 'C' or 'B', each element of C must be positive.

B (input/output)

On entry, the right hand side matrix B. On exit, if EQUED = 'N', B is not modified; if TRANSA = 'N' and EQUED = 'R' or 'B', B is overwritten by diag(R)*B; if TRANSA = 'T' or 'C' and EQUED = 'C' or 'B', B is overwritten by diag(C)*B.

LDB (input)

The leading dimension of the array B. LDB >= max(1,N).

X (output)

If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X to the original system of equations. Note that A and B are modified on exit if EQUED

.ne. 'N', and the solution to the equilibrated system is inv(diag(C))*X if TRANSA = 'N' and EQUED = 'C' or 'B', or inv(diag(R))*X if TRANSA = 'T' or 'C' and EQUED = 'R' or 'B'.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A after equilibration (if done). If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N) On exit, WORK2(1) contains the reciprocal pivot growth factor norm(A)/norm(U). The "max absolute element" norm is used. If WORK2(1) is much less than 1, then the stability of the LU factorization of the (equilibrated) matrix A could be poor. This also means that the solution X, condition estimator RCOND, and forward error bound FERR could be unreliable. If factorization fails with 0<INFO<=N, then WORK2(1) contains the reciprocal pivot growth factor for the leading INFO columns of A.

INFO (output)

= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

zgbtf2 - compute an LU factorization of a complex m-by-n band matrix A using partial pivoting with row interchanges

SYNOPSIS

```
SUBROUTINE ZGBTF2(M, N, KL, KU, AB, LDAB, IPIV, INFO)

DOUBLE COMPLEX AB(LDAB,*)
INTEGER M, N, KL, KU, LDAB, INFO
INTEGER IPIV(*)

SUBROUTINE ZGBTF2_64(M, N, KL, KU, AB, LDAB, IPIV, INFO)

DOUBLE COMPLEX AB(LDAB,*)
INTEGER*8 M, N, KL, KU, LDAB, INFO
INTEGER*8 IPIV(*)
```

F95 INTERFACE

```
SUBROUTINE GBTF2([M], [N], KL, KU, AB, [LDAB], IPIV, [INFO])

COMPLEX(8), DIMENSION(:,:) :: AB
INTEGER :: M, N, KL, KU, LDAB, INFO
INTEGER, DIMENSION(:) :: IPIV

SUBROUTINE GBTF2_64([M], [N], KL, KU, AB, [LDAB], IPIV, [INFO])

COMPLEX(8), DIMENSION(:,:) :: AB
INTEGER(8) :: M, N, KL, KU, LDAB, INFO
INTEGER(8), DIMENSION(:) :: IPIV
```

C INTERFACE

```
#include <sunperf.h>

void zgbtf2(int m, int n, int kl, int ku, doublecomplex *ab,
          int ldab, int *ipiv, int *info);

void zgbtf2_64(long m, long n, long kl, long ku,  doublecom-
          plex *ab, long ldab, long *ipiv, long *info);
```

PURPOSE

zgbtf2 computes an LU factorization of a complex m-by-n band matrix A using partial pivoting with row interchanges.

This is the unblocked version of the algorithm, calling Level 2 BLAS.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

KL (input)
The number of subdiagonals within the band of A. KL >= 0.

KU (input)
The number of superdiagonals within the band of A. KU >= 0.

AB (input/output)
On entry, the matrix A in band storage, in rows KL+1 to 2*KL+KU+1; rows 1 to KL of the array need not be set. The j-th column of A is stored in the j-th column of the array AB as follows: AB(kl+ku+1+i-j,j) = A(i,j) for max(1,j-ku)<=i<=min(m,j+kl)

On exit, details of the factorization: U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1. See below for further details.

LDAB (input)
The leading dimension of the array AB. LDAB >=

2*KL+KU+1.

IPIV (output)
The pivot indices; for 1 <= i <= min(M,N), row i of the matrix was interchanged with row IPIV(i).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = +i, U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when M = N = 6, KL = 2, KU = 1:

```
On entry:                       On exit:

    *    *    *    +    +    +       *    *    *   u14   u25
u36
    *    *    +    +    +    +       *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
   a21  a32  a43  a54  a65   *      m21  m32  m43  m54   m65
*
   a31  a42  a53  a64   *    *      m31  m42  m53   m64    *
*
```

Array elements marked * are not used by the routine; elements marked + need not be set on entry, but are required by the routine to store elements of U, because of fill-in resulting from the row interchanges.

Contents

NAME

```
zgbtrf - compute an LU factorization  of  a  complex  m-by-n
band matrix A using partial pivoting with row interchanges
```

SYNOPSIS

```
SUBROUTINE ZGBTRF(M, N, KL, KU, AB, LDAB, IPIVOT, INFO)

DOUBLE COMPLEX AB(LDAB,N)
INTEGER M, N, KL, KU, LDAB, INFO
INTEGER IPIVOT(MIN(M,N))

SUBROUTINE ZGBTRF_64(M, N, KL, KU, AB, LDAB, IPIVOT, INFO)

DOUBLE COMPLEX AB(LDAB,N)
INTEGER*8 M, N, KL, KU, LDAB, INFO
INTEGER*8 IPIVOT(MIN(M,N))
```

F95 INTERFACE

```
SUBROUTINE GBTRF(M, [N], KL, KU, AB, [LDAB], IPIVOT, [INFO])

COMPLEX(8), DIMENSION(:,:) :: AB
INTEGER :: M, N, KL, KU, LDAB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE GBTRF_64(M, [N], KL, KU, AB, [LDAB], IPIVOT, [INFO])

COMPLEX(8), DIMENSION(:,:) :: AB
INTEGER(8) :: M, N, KL, KU, LDAB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void zgbtrf(int m, int n, int kl, int ku, doublecomplex *ab,
          int ldab, int *ipivot, int *info);

void zgbtrf_64(long m, long n, long kl, long ku,  doublecom-
          plex *ab, long ldab, long *ipivot, long *info);
```

PURPOSE

zgbtrf computes an LU factorization of a complex m-by-n band matrix A using partial pivoting with row interchanges.

This is the blocked version of the algorithm, calling Level 3 BLAS.

ARGUMENTS

M (input) Integer
The number of rows of the matrix A. M >= 0.

N (input) Integer
The number of columns of the matrix A. N >= 0.

KL (input) Integer
The number of subdiagonals within the band of A. KL >= 0.

KU (input) Integer
The number of superdiagonals within the band of A. KU >= 0.

AB (input/output) Double complex array of dimension (LDAB,N).
On entry, the matrix A in band storage, in rows KL+1 to 2*KL+KU+1; rows 1 to KL of the array need not be set. The j-th column of A is stored in the j-th column of the array AB as follows: AB(KL+KU+1+I-J,J) = A(I,J) for MAX(1,J-KU)<=I<=MIN(M,J+KL)

On exit, details of the factorization: U is stored as an upper triangular band matrix with KL+KU superdiagonals in rows 1 to KL+KU+1, and the multipliers used during the factorization are stored in rows KL+KU+2 to 2*KL+KU+1. See below for further details.

LDAB (input) Integer
The leading dimension of the array A. LDA >= 2*KL+KU+1.

IPIVOT (output) Integer array of dimension MIN(M,N)
The pivot indices; for 1 <= I <= min(M,N), row I of the matrix was interchanged with row IPIVOT(I).

INFO (output) Integer
= 0: successful exit
< 0: if INFO = -I, the I-th argument had an illegal value
> 0: if INFO = +I, U(I,I) is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when M = N = 6, KL = 2, KU = 1:

```
On entry:                       On exit:

    *    *    *    +    +    +       *    *    *   u14   u25
u36
    *    *    +    +    +    +       *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
   a21  a32  a43  a54  a65   *      m21  m32  m43  m54   m65
*
   a31  a42  a53  a64   *    *      m31  m42  m53   m64    *
*
```

Array elements marked * are not used by the routine; elements marked + need not be set on entry, but are required by the routine to store elements of U because of fill-in resulting from the row interchanges.

Contents

NAME

```
zgbtrs - solve a system of linear equations  A * X = B, A**T
*  X = B, or A**H * X = B with a general band matrix A using
the LU factorization computed by CGBTRF
```

SYNOPSIS

```
SUBROUTINE ZGBTRS(TRANSA, N, NSUB, NSUPER, NRHS, A, LDA, IPIVOT, B,
      LDB, INFO)

CHARACTER * 1 TRANSA
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, NSUB, NSUPER, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZGBTRS_64(TRANSA, N, NSUB, NSUPER, NRHS, A, LDA, IPIVOT,
      B, LDB, INFO)

CHARACTER * 1 TRANSA
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, NSUB, NSUPER, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE GBTRS([TRANSA], [N], NSUB, NSUPER, [NRHS], A, [LDA],
       IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: TRANSA
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: N, NSUB, NSUPER, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE GBTRS_64([TRANSA], [N], NSUB, NSUPER, [NRHS], A, [LDA],
```

```
          IPIVOT, B, [LDB], [INFO])

   CHARACTER(LEN=1) :: TRANSA
   COMPLEX(8), DIMENSION(:,:) :: A, B
   INTEGER(8) :: N, NSUB, NSUPER, NRHS, LDA, LDB, INFO
   INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
   #include <sunperf.h>

   void zgbtrs(char transa, int n, int nsub,  int  nsuper,  int
             nrhs,  doublecomplex  *a,  int  lda,  int *ipivot,
             doublecomplex *b, int ldb, int *info);
   void zgbtrs_64(char transa, long n, long nsub, long  nsuper,
             long   nrhs,  doublecomplex  *a,  long  lda,  long
             *ipivot, doublecomplex *b, long ldb, long *info);
```

PURPOSE

```
   zgbtrs solves a system of linear equations
      A * X = B,  A**T * X = B,  or  A**H * X = B with  a  gen-
   eral  band  matrix  A using the LU factorization computed by
   CGBTRF.
```

ARGUMENTS

```
   TRANSA (input)
             Specifies the form of the system of equations.   =
             'N':  A * X = B     (No transpose)
             = 'T':  A**T * X = B  (Transpose)
             = 'C':  A**H * X = B  (Conjugate transpose)

             TRANSA is defaulted to 'N' for F95 INTERFACE.

   N (input) The order of the matrix A.  N >= 0.

   NSUB (input)
             The number of subdiagonals within the band  of  A.
             NSUB >= 0.

   NSUPER (input)
             The number of superdiagonals within the band of A.
             NSUPER >= 0.

   NRHS (input)
             The number of right hand sides, i.e.,  the  number
             of columns of the matrix B.  NRHS >= 0.
```

A (input) Details of the LU factorization of the band matrix A, as computed by CGBTRF. U is stored as an upper triangular band matrix with NSUB+NSUPER superdiagonals in rows 1 to NSUB+NSUPER+1, and the multipliers used during the factorization are stored in rows NSUB+NSUPER+2 to 2*NSUB+NSUPER+1.

LDA (input)
The leading dimension of the array A. LDA >= 2*NSUB+NSUPER+1.

IPIVOT (input)
The pivot indices; for 1 <= i <= N, row i of the matrix was interchanged with row IPIVOT(i).

B (input/output)
On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zgebak - form the right or left eigenvectors  of  a  complex
general  matrix  by  backward transformation on the computed
eigenvectors of the balanced matrix output by CGEBAL
```

SYNOPSIS

```
SUBROUTINE ZGEBAK(JOB, SIDE, N, ILO, IHI, SCALE, M, V, LDV, INFO)

CHARACTER * 1 JOB, SIDE
DOUBLE COMPLEX V(LDV,*)
INTEGER N, ILO, IHI, M, LDV, INFO
DOUBLE PRECISION SCALE(*)

SUBROUTINE ZGEBAK_64(JOB, SIDE, N, ILO, IHI, SCALE, M, V, LDV, INFO)

CHARACTER * 1 JOB, SIDE
DOUBLE COMPLEX V(LDV,*)
INTEGER*8 N, ILO, IHI, M, LDV, INFO
DOUBLE PRECISION SCALE(*)
```

F95 INTERFACE

```
SUBROUTINE GEBAK(JOB, SIDE, [N], ILO, IHI, SCALE, [M], V, [LDV],
       [INFO])

CHARACTER(LEN=1) :: JOB, SIDE
COMPLEX(8), DIMENSION(:,:) :: V
INTEGER :: N, ILO, IHI, M, LDV, INFO
REAL(8), DIMENSION(:) :: SCALE

SUBROUTINE GEBAK_64(JOB, SIDE, [N], ILO, IHI, SCALE, [M], V, [LDV],
       [INFO])
```

```
CHARACTER(LEN=1) :: JOB, SIDE
COMPLEX(8), DIMENSION(:,:) :: V
INTEGER(8) :: N, ILO, IHI, M, LDV, INFO
REAL(8), DIMENSION(:) :: SCALE
```

C INTERFACE

```
#include <sunperf.h>

void zgebak(char job, char side, int n, int  ilo,  int  ihi,
          double  *scale,  int m, doublecomplex *v, int ldv,
          int *info);

void zgebak_64(char job, char side, long n, long  ilo,  long
          ihi, double *scale, long m, doublecomplex *v, long
          ldv, long *info);
```

PURPOSE

zgebak forms the right or left eigenvectors of a complex general matrix by backward transformation on the computed eigenvectors of the balanced matrix output by CGEBAL.

ARGUMENTS

JOB (input)
Specifies the type of backward transformation required: = 'N', do nothing, return immediately; = 'P', do backward transformation for permutation only; = 'S', do backward transformation for scaling only; = 'B', do backward transformations for both permutation and scaling. JOB must be the same as the argument JOB supplied to CGEBAL.

SIDE (input)
= 'R': V contains right eigenvectors;
= 'L': V contains left eigenvectors.

N (input) The number of rows of the matrix V. N >= 0.

ILO (input)
The integer ILO determined by CGEBAL. 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

IHI (input)
The integer IHI determined by CGEBAL. 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

SCALE (input)
Details of the permutation and scaling factors, as returned by CGEBAL.

M (input) The number of columns of the matrix V. M >= 0.

V (input/output)
On entry, the matrix of right or left eigenvectors to be transformed, as returned by CHSEIN or CTREVC. On exit, V is overwritten by the transformed eigenvectors.

LDV (input)
The leading dimension of the array V. LDV >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

```
zgebal - balance a general complex matrix A
```

SYNOPSIS

```
SUBROUTINE ZGEBAL(JOB, N, A, LDA, ILO, IHI, SCALE, INFO)

CHARACTER * 1 JOB
DOUBLE COMPLEX A(LDA,*)
INTEGER N, LDA, ILO, IHI, INFO
DOUBLE PRECISION SCALE(*)

SUBROUTINE ZGEBAL_64(JOB, N, A, LDA, ILO, IHI, SCALE, INFO)

CHARACTER * 1 JOB
DOUBLE COMPLEX A(LDA,*)
INTEGER*8 N, LDA, ILO, IHI, INFO
DOUBLE PRECISION SCALE(*)
```

F95 INTERFACE

```
SUBROUTINE GEBAL(JOB, [N], A, [LDA], ILO, IHI, SCALE, [INFO])

CHARACTER(LEN=1) :: JOB
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, ILO, IHI, INFO
REAL(8), DIMENSION(:) :: SCALE

SUBROUTINE GEBAL_64(JOB, [N], A, [LDA], ILO, IHI, SCALE, [INFO])

CHARACTER(LEN=1) :: JOB
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, ILO, IHI, INFO
```

```
REAL(8), DIMENSION(:) :: SCALE
```

C INTERFACE

```
#include <sunperf.h>

void zgebal(char job, int n, doublecomplex *a, int lda,  int
          *ilo, int *ihi, double *scale, int *info);

void zgebal_64(char job, long n, doublecomplex *a, long lda,
          long *ilo, long *ihi, double *scale, long *info);
```

PURPOSE

zgebal balances a general complex matrix A. This involves, first, permuting A by a similarity transformation to isolate eigenvalues in the first 1 to ILO-1 and last IHI+1 to N elements on the diagonal; and second, applying a diagonal similarity transformation to rows and columns ILO to IHI to make the rows and columns as close in norm as possible. Both steps are optional.

Balancing may reduce the 1-norm of the matrix, and improve the accuracy of the computed eigenvalues and/or eigenvectors.

ARGUMENTS

JOB (input)
Specifies the operations to be performed on A:
= 'N': none: simply set ILO = 1, IHI = N, SCALE(I) = 1.0 for i = 1,...,N; = 'P': permute only;
= 'S': scale only;
= 'B': both permute and scale.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the input matrix A. On exit, A is overwritten by the balanced matrix. If JOB = 'N', A is not referenced. See Further Details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

ILO (output)

ILO and IHI are set to integers such that on exit A(i,j) = 0 if i > j and j = 1,...,ILO-1 or I = IHI+1,...,N. If JOB = 'N' or 'S', ILO = 1 and IHI = N.

IHI (output)

ILO and IHI are set to integers such that on exit A(i,j) = 0 if i > j and j = 1,...,ILO-1 or I = IHI+1,...,N. If JOB = 'N' or 'S', ILO = 1 and IHI = N.

SCALE (output)

Details of the permutations and scaling factors applied to A. If P(j) is the index of the row and column interchanged with row and column j and D(j) is the scaling factor applied to row and column j, then SCALE(j) = P(j) for j = 1,...,ILO-1 = D(j) for j = ILO,...,IHI = P(j) for j = IHI+1,...,N. The order in which the interchanges are made is N to IHI+1, then 1 to ILO-1.

INFO (output)

= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

The permutations consist of row and column interchanges which put the matrix in the form

```
          ( T1   X   Y  )
  P A P = (  0   B   Z  )
          (  0   0   T2 )
```

where T1 and T2 are upper triangular matrices whose eigenvalues lie along the diagonal. The column indices ILO and IHI mark the starting and ending columns of the submatrix B. Balancing consists of applying a diagonal similarity transformation inv(D) * B * D to make the 1-norms of each row of B and its corresponding column nearly equal. The output matrix is

```
  ( T1     X*D          Y    )
  (  0  inv(D)*B*D  inv(D)*Z ).
  (  0      0           T2   )
```

Information about the permutations P and the diagonal matrix D is returned in the vector SCALE.

This subroutine is based on the EISPACK routine CBAL.

Modified by Tzu-Yi Chen, Computer Science Division, University of
California at Berkeley, USA

Contents

NAME

```
zgebrd - reduce a general complex M-by-N matrix A  to  upper
or lower bidiagonal form B by a unitary transformation
```

SYNOPSIS

```
SUBROUTINE ZGEBRD(M, N, A, LDA, D, E, TAUQ, TAUP, WORK, LWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAUQ(*), TAUP(*), WORK(*)
INTEGER M, N, LDA, LWORK, INFO
DOUBLE PRECISION D(*), E(*)

SUBROUTINE ZGEBRD_64(M, N, A, LDA, D, E, TAUQ, TAUP, WORK, LWORK,
      INFO)

DOUBLE COMPLEX A(LDA,*), TAUQ(*), TAUP(*), WORK(*)
INTEGER*8 M, N, LDA, LWORK, INFO
DOUBLE PRECISION D(*), E(*)
```

F95 INTERFACE

```
SUBROUTINE GEBRD([M], [N], A, [LDA], D, E, TAUQ, TAUP, [WORK], [LWORK],
       [INFO])

COMPLEX(8), DIMENSION(:) :: TAUQ, TAUP, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, LWORK, INFO
REAL(8), DIMENSION(:) :: D, E

SUBROUTINE GEBRD_64([M], [N], A, [LDA], D, E, TAUQ, TAUP, [WORK],
       [LWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAUQ, TAUP, WORK
```

```
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, LWORK, INFO
REAL(8), DIMENSION(:) :: D, E
```

C INTERFACE

```
#include <sunperf.h>

void zgebrd(int m, int n, doublecomplex *a, int lda,  double
          *d,  double *e, doublecomplex *tauq, doublecomplex
          *taup, int *info);

void zgebrd_64(long m, long n, doublecomplex *a,  long  lda,
          double  *d,  double *e, doublecomplex *tauq, doub-
          lecomplex *taup, long *info);
```

PURPOSE

zgebrd reduces a general complex M-by-N matrix A to upper or lower bidiagonal form B by a unitary transformation: Q**H * A * P = B.

If m >= n, B is upper bidiagonal; if m < n, B is lower bidiagonal.

ARGUMENTS

M (input) The number of rows in the matrix A. M >= 0.

N (input) The number of columns in the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N general matrix to be reduced. On exit, if m >= n, the diagonal and the first superdiagonal are overwritten with the upper bidiagonal matrix B; the elements below the diagonal, with the array TAUQ, represent the unitary matrix Q as a product of elementary reflectors, and the elements above the first superdiagonal, with the array TAUP, represent the unitary matrix P as a product of elementary reflectors; if m < n, the diagonal and the first subdiagonal are overwritten with the lower bidiagonal matrix B; the elements below the first subdiagonal, with the array TAUQ, represent the unitary matrix Q as a product of elementary reflectors, and the elements above the diagonal, with the array TAUP, represent the unitary matrix P as a product of elementary reflec-

tors. See Further Details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

D (output)
The diagonal elements of the bidiagonal matrix B: D(i) = A(i,i).

E (output)
The off-diagonal elements of the bidiagonal matrix B: if m >= n, E(i) = A(i,i+1) for i = 1,2,...,n-1; if m < n, E(i) = A(i+1,i) for i = 1,2,...,m-1.

TAUQ (output)
The scalar factors of the elementary reflectors which represent the unitary matrix Q. See Further Details.

TAUP (output)
The scalar factors of the elementary reflectors which represent the unitary matrix P. See Further Details.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The length of the array WORK. LWORK >= max(1,M,N). For optimum performance LWORK >= (M+N)*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

The matrices Q and P are represented as products of elementary reflectors:

If m >= n,

```
   Q = H(1) H(2) . . . H(n)  and  P = G(1) G(2) . . . G(n-1)

Each H(i) and G(i) has the form:

   H(i) = I - tauq * v * v'  and G(i) = I - taup * u * u'

where tauq and taup are complex scalars, and  v  and  u  are
complex  vectors;  v(1:i-1)  =  0, v(i) = 1, and v(i+1:m) is
stored on exit in A(i+1:m,i); u(1:i) = 0, u(i+1)  =  1,  and
u(i+2:n)  is stored on exit in A(i,i+2:n); tauq is stored in
TAUQ(i) and taup in TAUP(i).
If m < n,

   Q = H(1) H(2) . . . H(m-1)  and  P = G(1) G(2) . . . G(m)

Each H(i) and G(i) has the form:

   H(i) = I - tauq * v * v'  and G(i) = I - taup * u * u'

where tauq and taup are complex scalars, and  v  and  u  are
complex  vectors;  v(1:i)  =  0, v(i+1) = 1, and v(i+2:m) is
stored on exit in A(i+2:m,i); u(1:i-1) = 0, u(i)  =  1,  and
u(i+1:n)  is stored on exit in A(i,i+1:n); tauq is stored in
TAUQ(i) and taup in TAUP(i).

The contents of A on exit are illustrated by  the  following
examples:

m = 6 and n = 5 (m > n):          m = 5 and n = 6 (m < n):

  (  d   e   u1  u1  u1 )           (  d   u1   u1   u1   u1
u1 )
  (  v1  d   e   u2  u2 )           (  e   d    u2   u2   u2
u2 )
  (  v1  v2  d   e   u3 )           (  v1  e    d    u3   u3
u3 )
  (  v1  v2  v3  d   e  )           (  v1  v2   e    d    u4
u4 )
  (  v1  v2  v3  v4  d  )           (  v1   v2   v3   e    d
u5 )
  (  v1  v2  v3  v4  v5 )

where d and e denote diagonal and off-diagonal  elements  of
B, vi denotes an element of the vector defining H(i), and ui
an element of the vector defining G(i).
```

Contents

NAME

zgecon - estimate the reciprocal of the condition number of a general complex matrix A, in either the 1-norm or the infinity-norm, using the LU factorization computed by CGETRF

SYNOPSIS

```
SUBROUTINE ZGECON(NORM, N, A, LDA, ANORM, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 NORM
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, INFO
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION WORK2(*)

SUBROUTINE ZGECON_64(NORM, N, A, LDA, ANORM, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 NORM
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, INFO
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GECON(NORM, [N], A, [LDA], ANORM, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: NORM
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
REAL(8) :: ANORM, RCOND
```

```
    REAL(8), DIMENSION(:) :: WORK2

    SUBROUTINE GECON_64(NORM, [N], A, [LDA], ANORM, RCOND, [WORK], [WORK2],
           [INFO])

    CHARACTER(LEN=1) :: NORM
    COMPLEX(8), DIMENSION(:) :: WORK
    COMPLEX(8), DIMENSION(:,:) :: A
    INTEGER(8) :: N, LDA, INFO
    REAL(8) :: ANORM, RCOND
    REAL(8), DIMENSION(:) :: WORK2
```

C INTERFACE

```
    #include <sunperf.h>
    void zgecon(char norm, int n,  doublecomplex  *a,  int  lda,
              double anorm, double *rcond, int *info);

    void zgecon_64(char norm, long  n,  doublecomplex  *a,  long
              lda, double anorm, double *rcond, long *info);
```

PURPOSE

zgecon estimates the reciprocal of the condition number of a general complex matrix A, in either the 1-norm or the infinity-norm, using the LU factorization computed by CGETRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as
   RCOND = 1 / (norm(A) * norm(inv(A))).

ARGUMENTS

```
    NORM (input)
              Specifies whether the 1-norm condition  number  or
              the infinity-norm condition number is required:
              = '1' or 'O':  1-norm;
              = 'I':         Infinity-norm.

    N (input) The order of the matrix A.  N >= 0.

    A (input) The factors L and U from  the  factorization  A  =
              P*L*U as computed by CGETRF.

    LDA (input)
              The leading dimension of  the  array  A.   LDA  >=
              max(1,N).
```

ANORM (input)
If NORM = '1' or 'O', the 1-norm of the original matrix A. If NORM = 'I', the infinity-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(norm(A) * norm(inv(A))).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

zgeequ - compute row and column scalings intended to equili-
brate an M-by-N matrix A and reduce its condition number

SYNOPSIS

```
SUBROUTINE ZGEEQU(M, N, A, LDA, R, C, ROWCN, COLCN, AMAX,
      INFO)

DOUBLE COMPLEX A(LDA,*)
INTEGER M, N, LDA, INFO
DOUBLE PRECISION ROWCN, COLCN, AMAX
DOUBLE PRECISION R(*), C(*)

SUBROUTINE ZGEEQU_64(M, N, A, LDA, R, C, ROWCN, COLCN, AMAX,
      INFO)

DOUBLE COMPLEX A(LDA,*)
INTEGER*8 M, N, LDA, INFO
DOUBLE PRECISION ROWCN, COLCN, AMAX
DOUBLE PRECISION R(*), C(*)
```

F95 INTERFACE

```
SUBROUTINE GEEQU([M], [N], A, [LDA], R, C, ROWCN, COLCN,
       AMAX, [INFO])

COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, INFO
REAL(8) :: ROWCN, COLCN, AMAX
REAL(8), DIMENSION(:) :: R, C

SUBROUTINE GEEQU_64([M], [N], A, [LDA], R, C, ROWCN, COLCN,
       AMAX, [INFO])
```

```
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, INFO
REAL(8) :: ROWCN, COLCN, AMAX
REAL(8), DIMENSION(:) :: R, C
```

C INTERFACE

```
#include <sunperf.h>

void zgeequ(int m, int n, doublecomplex *a, int lda,  double
          *r,  double *c, double *rowcn, double *colcn, dou-
          ble *amax, int *info);

void zgeequ_64(long m, long n, doublecomplex *a,  long  lda,
          double   *r,  double  *c,  double  *rowcn,  double
          *colcn, double *amax, long *info);
```

PURPOSE

zgeequ computes row and column scalings intended to equilibrate an M-by-N matrix A and reduce its condition number. R returns the row scale factors and C the column scale factors, chosen to try to make the largest element in each row and column of the matrix B with elements B(i,j)=R(i)*A(i,j)*C(j) have absolute value 1.

R(i) and C(j) are restricted to be between SMLNUM = smallest safe number and BIGNUM = largest safe number. Use of these scaling factors is not guaranteed to reduce the condition number of A but works well in practice.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input) The M-by-N matrix whose equilibration factors are to be computed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

R (output)
If INFO = 0 or INFO > M, R contains the row scale factors for A.

C (output)
If INFO = 0, C contains the column scale factors for A.

ROWCN (output)
If INFO = 0 or INFO > M, ROWCN contains the ratio of the smallest R(i) to the largest R(i). If ROWCN >= 0.1 and AMAX is neither too large nor too small, it is not worth scaling by R.

COLCN (output)
If INFO = 0, COLCN contains the ratio of the smallest C(i) to the largest C(i). If COLCN >= 0.1, it is not worth scaling by C.

AMAX (output)
Absolute value of largest matrix element. If AMAX is very close to overflow or very close to underflow, the matrix should be scaled.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= M: the i-th row of A is exactly zero
> M: the (i-M)-th column of A is exactly zero

Contents

NAME

```
zgees - compute for an N-by-N complex nonsymmetric matrix A,
the  eigenvalues,  the  Schur  form  T, and, optionally, the
matrix of Schur vectors Z
```

SYNOPSIS

```
SUBROUTINE ZGEES(JOBZ, SORTEV, SELECT, N, A, LDA, NOUT, W, Z, LDZ,
      WORK, LDWORK, WORK2, WORK3, INFO)

CHARACTER * 1 JOBZ, SORTEV
DOUBLE COMPLEX A(LDA,*), W(*), Z(LDZ,*), WORK(*)
INTEGER N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL SELECT
LOGICAL WORK3(*)
DOUBLE PRECISION WORK2(*)

SUBROUTINE ZGEES_64(JOBZ, SORTEV, SELECT, N, A, LDA, NOUT, W, Z, LDZ,
      WORK, LDWORK, WORK2, WORK3, INFO)

CHARACTER * 1 JOBZ, SORTEV
DOUBLE COMPLEX A(LDA,*), W(*), Z(LDZ,*), WORK(*)
INTEGER*8 N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL*8 SELECT
LOGICAL*8 WORK3(*)
DOUBLE PRECISION WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GEES(JOBZ, SORTEV, [SELECT], [N], A, [LDA], [NOUT], W, [Z], [LDZ],
       [WORK], [LDWORK], [WORK2], [WORK3], [INFO])

CHARACTER(LEN=1) :: JOBZ, SORTEV
COMPLEX(8), DIMENSION(:) :: W, WORK
```

```
COMPLEX(8), DIMENSION(:,:) :: A, Z
INTEGER :: N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL :: SELECT
LOGICAL, DIMENSION(:) :: WORK3
REAL(8), DIMENSION(:) :: WORK2

SUBROUTINE GEES_64(JOBZ, SORTEV, [SELECT], [N], A, [LDA], [NOUT], W, [Z],
       [LDZ], [WORK], [LDWORK], [WORK2], [WORK3], [INFO])

CHARACTER(LEN=1) :: JOBZ, SORTEV
COMPLEX(8), DIMENSION(:) :: W, WORK
COMPLEX(8), DIMENSION(:,:) :: A, Z
INTEGER(8) :: N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL(8) :: SELECT
LOGICAL(8), DIMENSION(:) :: WORK3
REAL(8), DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>

void       zgees(char       jobz,        char        sortev,
          int(*select)(doublecomplex),  int n, doublecomplex
          *a, int lda, int *nout,  doublecomplex  *w,  doub-
          lecomplex *z, int ldz, int *info);

void      zgees_64(char       jobz,       char       sortev,
          long(*select)(doublecomplex),  long  n, doublecom-
          plex *a, long lda, long *nout,  doublecomplex  *w,
          doublecomplex *z, long ldz, long *info);
```

PURPOSE

zgees computes for an N-by-N complex nonsymmetric matrix A, the eigenvalues, the Schur form T, and, optionally, the matrix of Schur vectors Z. This gives the Schur factorization A = Z*T*(Z**H).

Optionally, it also orders the eigenvalues on the diagonal of the Schur form so that selected eigenvalues are at the top left. The leading columns of Z then form an orthonormal basis for the invariant subspace corresponding to the selected eigenvalues.

A complex matrix is in Schur form if it is upper triangular.

ARGUMENTS

JOBZ (input)

```
          = 'N': Schur vectors are not computed;
          = 'V': Schur vectors are computed.

SORTEV (input)
          Specifies whether or not to order the  eigenvalues
          on  the diagonal of the Schur form.  = 'N': Eigen-
          values are not ordered:
          = 'S': Eigenvalues are ordered (see SELECT).

SELECT (input)
          SELECT must be declared EXTERNAL  in  the  calling
          subroutine.   If  SORTEV  = 'S', SELECT is used to
          select eigenvalues to order to the top left of the
          Schur form.  If SORTEV = 'N', SELECT is not refer-
          enced.   The  eigenvalue  W(j)  is   selected   if
          SELECT(W(j)) is true.
N (input) The order of the matrix A. N >= 0.

A (input/output)
          On entry, the N-by-N matrix A.   On  exit,  A  has
          been overwritten by its Schur form T.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

NOUT (output)
          If SORTEV = 'N', NOUT = 0.  If SORTEV = 'S',  NOUT
          = number of eigenvalues for which SELECT is true.

W (output)
          W contains the computed eigenvalues, in  the  same
          order that they appear on the diagonal of the out-
          put Schur form T.

Z (output)
          If JOBZ = 'V', Z contains the unitary matrix Z  of
          Schur  vectors.   If  JOBZ  = 'N', Z is not refer-
          enced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1;
          if JOBZ = 'V', LDZ >= N.

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LDWORK.

LDWORK (input)
          The  dimension  of  the  array  WORK.   LDWORK  >=
```

max(1,2*N).   For good performance, LDWORK must generally be larger.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
dimension(N)

WORK3 (workspace)
dimension(N) Not referenced if SORTEV = 'N'.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, and i is
<= N: the QR algorithm failed to compute all the eigenvalues; elements 1:ILO-1 and i+1:N of W contain those eigenvalues which have converged; if JOBZ = 'V', Z contains the matrix which reduces A to its partially converged Schur form. = N+1: the eigenvalues could not be reordered because some eigenvalues were too close to separate (the problem is very ill-conditioned); = N+2: after reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the Schur form no longer satisfy SELECT = .TRUE.. This could also be caused by underflow due to scaling.

Contents

NAME

```
zgeesx - compute for an N-by-N complex  nonsymmetric  matrix
A,  the  eigenvalues, the Schur form T, and, optionally, the
matrix of Schur vectors Z
```

SYNOPSIS

```
SUBROUTINE ZGEESX(JOBZ, SORTEV, SELECT, SENSE, N, A, LDA, NOUT, W, Z,
      LDZ, RCONE, RCONV, WORK, LDWORK, WORK2, BWORK3, INFO)

CHARACTER * 1 JOBZ, SORTEV, SENSE
DOUBLE COMPLEX A(LDA,*), W(*), Z(LDZ,*), WORK(*)
INTEGER N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL SELECT
LOGICAL BWORK3(*)
DOUBLE PRECISION RCONE, RCONV
DOUBLE PRECISION WORK2(*)

SUBROUTINE ZGEESX_64(JOBZ, SORTEV, SELECT, SENSE, N, A, LDA, NOUT, W,
      Z, LDZ, RCONE, RCONV, WORK, LDWORK, WORK2, BWORK3, INFO)

CHARACTER * 1 JOBZ, SORTEV, SENSE
DOUBLE COMPLEX A(LDA,*), W(*), Z(LDZ,*), WORK(*)
INTEGER*8 N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL*8 SELECT
LOGICAL*8 BWORK3(*)
DOUBLE PRECISION RCONE, RCONV
DOUBLE PRECISION WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GEESX(JOBZ, SORTEV, [SELECT], SENSE, [N], A, [LDA], NOUT, W,
       [Z], [LDZ], RCONE, RCONV, [WORK], [LDWORK], [WORK2], [BWORK3],
       [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, SORTEV, SENSE
COMPLEX(8), DIMENSION(:) :: W, WORK
COMPLEX(8), DIMENSION(:,:) :: A, Z
INTEGER :: N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL :: SELECT
LOGICAL, DIMENSION(:) :: BWORK3
REAL(8) :: RCONE, RCONV
REAL(8), DIMENSION(:) :: WORK2

SUBROUTINE GEESX_64(JOBZ, SORTEV, [SELECT], SENSE, [N], A, [LDA], NOUT,
       W, [Z], [LDZ], RCONE, RCONV, [WORK], [LDWORK], [WORK2], [BWORK3],
       [INFO])

CHARACTER(LEN=1) :: JOBZ, SORTEV, SENSE
COMPLEX(8), DIMENSION(:) :: W, WORK
COMPLEX(8), DIMENSION(:,:) :: A, Z
INTEGER(8) :: N, LDA, NOUT, LDZ, LDWORK, INFO
LOGICAL(8) :: SELECT
LOGICAL(8), DIMENSION(:) :: BWORK3
REAL(8) :: RCONE, RCONV
REAL(8), DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>

void       zgeesx(char       jobz,       char        sortev,
           int(*select)(doublecomplex),  char  sense,  int n,
           doublecomplex *a, int lda, int  *nout,  doublecom-
           plex *w, doublecomplex *z, int ldz, double *rcone,
           double *rconv, int *info);

void       zgeesx_64(char      jobz,       char       sortev,
           long(*select)(doublecomplex),  char sense, long n,
           doublecomplex *a, long lda, long *nout, doublecom-
           plex   *w,  doublecomplex  *z,  long  ldz,  double
           *rcone, double *rconv, long *info);
```

PURPOSE

zgeesx computes for an N-by-N complex nonsymmetric matrix A, the eigenvalues, the Schur form T, and, optionally, the matrix of Schur vectors Z. This gives the Schur factorization A = Z*T*(Z**H).

Optionally, it also orders the eigenvalues on the diagonal of the Schur form so that selected eigenvalues are at the top left; computes a reciprocal condition number for the average of the selected eigenvalues (RCONDE); and computes a reciprocal condition number for the right invariant subspace corresponding to the selected eigenvalues (RCONDV). The leading columns of Z form an orthonormal basis for this invariant subspace.

For further explanation of the reciprocal condition numbers RCONDE and RCONDV, see Section 4.10 of the LAPACK Users' Guide (where these quantities are called s and sep respectively).

A complex matrix is in Schur form if it is upper triangular.

ARGUMENTS

JOBZ (input)
= 'N': Schur vectors are not computed;
= 'V': Schur vectors are computed.

SORTEV (input)
Specifies whether or not to order the eigenvalues on the diagonal of the Schur form. = 'N': Eigenvalues are not ordered;
= 'S': Eigenvalues are ordered (see SELECT).

SELECT (input)
SELECT must be declared EXTERNAL in the calling subroutine. If SORTEV = 'S', SELECT is used to select eigenvalues to order to the top left of the Schur form. If SORTEV = 'N', SELECT is not referenced. An eigenvalue W(j) is selected if SELECT(W(j)) is true.

SENSE (input)
Determines which reciprocal condition numbers are computed. = 'N': None are computed;
= 'E': Computed for average of selected eigenvalues only;
= 'V': Computed for selected right invariant subspace only;
= 'B': Computed for both. If SENSE = 'E', 'V' or 'B', SORTEV must equal 'S'.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the N-by-N matrix A. On exit, A is overwritten by its Schur form T.

LDA (input)

The leading dimension of the array A. LDA >= max(1,N).

NOUT (output)
If SORTEV = 'N', NOUT = 0. If SORTEV = 'S', NOUT = number of eigenvalues for which SELECT is true.

W (output)
W contains the computed eigenvalues, in the same order that they appear on the diagonal of the output Schur form T.

Z (output)
If JOBZ = 'V', Z contains the unitary matrix Z of Schur vectors. If JOBZ = 'N', Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= N.

RCONE (output)
If SENSE = 'E' or 'B', RCONE contains the reciprocal condition number for the average of the selected eigenvalues. Not referenced if SENSE = 'N' or 'V'.

RCONV (output)
If SENSE = 'V' or 'B', RCONV contains the reciprocal condition number for the selected right invariant subspace. Not referenced if SENSE = 'N' or 'E'.

WORK (workspace)
dimension(LDWORK) On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,2*N). Also, if SENSE = 'E' or 'V' or 'B', LDWORK >= 2*NOUT*(N-NOUT), where NOUT is the number of selected eigenvalues computed by this routine. Note that 2*NOUT*(N-NOUT) <= N*N/2. For good performance, LDWORK must generally be larger.

WORK2 (workspace)
dimension(N)

BWORK3 (workspace)
dimension(N) Not referenced if SORTEV = 'N'.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, and i is
<= N: the QR algorithm failed to compute all the eigenvalues; elements 1:ILO-1 and i+1:N of W contain those eigenvalues which have converged; if JOBZ = 'V', Z contains the transformation which reduces A to its partially converged Schur form.
= N+1: the eigenvalues could not be reordered because some eigenvalues were too close to separate (the problem is very ill-conditioned); = N+2: after reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the Schur form no longer satisfy SELECT=.TRUE. This could also be caused by underflow due to scaling.

Contents

NAME

```
zgeev - compute for an N-by-N complex nonsymmetric matrix A,
the  eigenvalues  and,  optionally,  the  left  and/or right
eigenvectors
```

SYNOPSIS

```
SUBROUTINE ZGEEV(JOBVL, JOBVR, N, A, LDA, W, VL, LDVL, VR, LDVR,
      WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 JOBVL, JOBVR
DOUBLE  COMPLEX  A(LDA,*),  W(*),  VL(LDVL,*),   VR(LDVR,*),
WORK(*)
INTEGER N, LDA, LDVL, LDVR, LDWORK, INFO
DOUBLE PRECISION WORK2(*)

SUBROUTINE ZGEEV_64(JOBVL, JOBVR, N, A, LDA, W, VL, LDVL, VR, LDVR,
      WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 JOBVL, JOBVR
DOUBLE  COMPLEX  A(LDA,*),  W(*),  VL(LDVL,*),   VR(LDVR,*),
WORK(*)
INTEGER*8 N, LDA, LDVL, LDVR, LDWORK, INFO
DOUBLE PRECISION WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GEEV(JOBVL, JOBVR, [N], A, [LDA], W, VL, [LDVL], VR, [LDVR],
       [WORK], [LDWORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBVL, JOBVR
COMPLEX(8), DIMENSION(:) :: W, WORK
COMPLEX(8), DIMENSION(:,:) :: A, VL, VR
INTEGER :: N, LDA, LDVL, LDVR, LDWORK, INFO
```

```
REAL(8), DIMENSION(:) :: WORK2

SUBROUTINE GEEV_64(JOBVL, JOBVR, [N], A, [LDA], W, VL, [LDVL], VR,
       [LDVR], [WORK], [LDWORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBVL, JOBVR
COMPLEX(8), DIMENSION(:) :: W, WORK
COMPLEX(8), DIMENSION(:,:) :: A, VL, VR
INTEGER(8) :: N, LDA, LDVL, LDVR, LDWORK, INFO
REAL(8), DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>
void zgeev(char jobvl, char jobvr, int n, doublecomplex  *a,
          int  lda, doublecomplex *w, doublecomplex *vl, int
          ldvl, doublecomplex *vr, int ldvr, int *info);

void zgeev_64(char jobvl, char jobvr, long n,  doublecomplex
          *a, long lda, doublecomplex *w, doublecomplex *vl,
          long ldvl,  doublecomplex  *vr,  long  ldvr,  long
          *info);
```

PURPOSE

```
zgeev computes for an N-by-N complex nonsymmetric matrix  A,
the  eigenvalues  and,  optionally,  the  left  and/or right
eigenvectors.

The right eigenvector v(j) of A satisfies
                 A * v(j) = lambda(j) * v(j)
where lambda(j) is its eigenvalue.
The left eigenvector u(j) of A satisfies
              u(j)**H * A = lambda(j) * u(j)**H
where u(j)**H denotes the conjugate transpose of u(j).

The computed eigenvectors are normalized to  have  Euclidean
norm equal to 1 and largest component real.
```

ARGUMENTS

```
JOBVL (input)
          = 'N': left eigenvectors of A are not computed;
          = 'V': left eigenvectors of are computed.

JOBVR (input)
          = 'N': right eigenvectors of A are not computed;
          = 'V': right eigenvectors of A are computed.
```

```
N (input) The order of the matrix A. N >= 0.

A (input/output)
          On entry, the N-by-N matrix A.   On  exit,  A  has
          been overwritten.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

W (output)
          W contains the computed eigenvalues.

VL (output)
          If JOBVL = 'V', the  left  eigenvectors  u(j)  are
          stored  one after another in the columns of VL, in
          the same order as their eigenvalues.  If  JOBVL  =
          'N',  VL  is  not referenced.  u(j) = VL(:,j), the
          j-th column of VL.

LDVL (input)
          The leading dimension of the array VL.  LDVL >= 1;
          if JOBVL = 'V', LDVL >= N.

VR (input)
          If JOBVR = 'V', the right  eigenvectors  v(j)  are
          stored  one after another in the columns of VR, in
          the same order as their eigenvalues.  If  JOBVR  =
          'N',  VR  is  not referenced.  v(j) = VR(:,j), the
          j-th column of VR.

LDVR (input)
          The leading dimension of the array VR.  LDVR >= 1;
          if JOBVR = 'V', LDVR >= N.

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LDWORK.

LDWORK (input)
          The  dimension  of  the  array  WORK.   LDWORK  >=
          max(1,2*N).   For  good  performance,  LDWORK must
          generally be larger.

          If LDWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LDWORK is issued by XERBLA.
```

WORK2 (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, the QR algorithm failed to compute all the eigenvalues, and no eigenvectors have been computed; elements and i+1:N of W contain eigenvalues which have converged.

Contents

NAME

```
zgeevx - compute for an N-by-N complex  nonsymmetric  matrix
A,  the  eigenvalues  and, optionally, the left and/or right
eigenvectors
```

SYNOPSIS

```
SUBROUTINE ZGEEVX(BALANC, JOBVL, JOBVR, SENSE, N, A, LDA, W, VL,
      LDVL, VR, LDVR, ILO, IHI, SCALE, ABNRM, RCONE, RCONV, WORK,
      LDWORK, WORK2, INFO)

CHARACTER * 1 BALANC, JOBVL, JOBVR, SENSE
DOUBLE  COMPLEX  A(LDA,*),  W(*),  VL(LDVL,*),   VR(LDVR,*),
WORK(*)
INTEGER N, LDA, LDVL, LDVR, ILO, IHI, LDWORK, INFO
DOUBLE PRECISION ABNRM
DOUBLE PRECISION SCALE(*), RCONE(*), RCONV(*), WORK2(*)

SUBROUTINE ZGEEVX_64(BALANC, JOBVL, JOBVR, SENSE, N, A, LDA, W, VL,
      LDVL, VR, LDVR, ILO, IHI, SCALE, ABNRM, RCONE, RCONV, WORK,
      LDWORK, WORK2, INFO)

CHARACTER * 1 BALANC, JOBVL, JOBVR, SENSE
DOUBLE  COMPLEX  A(LDA,*),  W(*),  VL(LDVL,*),   VR(LDVR,*),
WORK(*)
INTEGER*8 N, LDA, LDVL, LDVR, ILO, IHI, LDWORK, INFO
DOUBLE PRECISION ABNRM
DOUBLE PRECISION SCALE(*), RCONE(*), RCONV(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GEEVX(BALANC, JOBVL, JOBVR, SENSE, [N], A, [LDA], W, VL,
       [LDVL], VR, [LDVR], ILO, IHI, SCALE, ABNRM, RCONE, RCONV, [WORK],
       LDWORK, [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: BALANC, JOBVL, JOBVR, SENSE
COMPLEX(8), DIMENSION(:) :: W, WORK
COMPLEX(8), DIMENSION(:,:) :: A, VL, VR
INTEGER :: N, LDA, LDVL, LDVR, ILO, IHI, LDWORK, INFO
REAL(8) :: ABNRM
REAL(8), DIMENSION(:) :: SCALE, RCONE, RCONV, WORK2

SUBROUTINE GEEVX_64(BALANC, JOBVL, JOBVR, SENSE, [N], A, [LDA], W,
       VL, [LDVL], VR, [LDVR], ILO, IHI, SCALE, ABNRM, RCONE, RCONV,
       [WORK], LDWORK, [WORK2], [INFO])

CHARACTER(LEN=1) :: BALANC, JOBVL, JOBVR, SENSE
COMPLEX(8), DIMENSION(:) :: W, WORK
COMPLEX(8), DIMENSION(:,:) :: A, VL, VR
INTEGER(8) :: N, LDA, LDVL, LDVR, ILO, IHI, LDWORK, INFO
REAL(8) :: ABNRM
REAL(8), DIMENSION(:) :: SCALE, RCONE, RCONV, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zgeevx (char, char, char,  char,  int,  doublecomplex*,
          int,  doublecomplex*,  doublecomplex*,  int, doub-
          lecomplex*, int,  int*,  int*,  double*,  double*,
          double*, double*, int*);

void zgeevx_64 (char, char,  char,  char,  long,  doublecom-
          plex*, long, doublecomplex*, doublecomplex*, long,
          doublecomplex*, long, long*, long*, double*,  dou-
          ble*, double*, double*, long*);
```

PURPOSE

zgeevx computes for an N-by-N complex nonsymmetric matrix A, the eigenvalues and, optionally, the left and/or right eigenvectors.

Optionally also, it computes a balancing transformation to improve the conditioning of the eigenvalues and eigenvectors (ILO, IHI, SCALE, and ABNRM), reciprocal condition numbers for the eigenvalues (RCONDE), and reciprocal condition numbers for the right
eigenvectors (RCONDV).

The right eigenvector v(j) of A satisfies

```
                 A * v(j) = lambda(j) * v(j)
```

where lambda(j) is its eigenvalue.
The left eigenvector u(j) of A satisfies

```
               u(j)**H * A = lambda(j) * u(j)**H
```

where u(j)**H denotes the conjugate transpose of u(j).

The computed eigenvectors are normalized to have Euclidean norm equal to 1 and largest component real.

Balancing a matrix means permuting the rows and columns to make it more nearly upper triangular, and applying a diagonal similarity transformation D * A * D**(-1), where D is a diagonal matrix, to make its rows and columns closer in norm and the condition numbers of its eigenvalues and eigenvectors smaller. The computed reciprocal condition numbers correspond to the balanced matrix. Permuting rows and columns will not change the condition numbers (in exact arithmetic) but diagonal scaling will. For further explanation of balancing, see section 4.10.2 of the LAPACK Users' Guide.

ARGUMENTS

BALANC (input)
Indicates how the input matrix should be diagonally scaled and/or permuted to improve the conditioning of its eigenvalues. = 'N': Do not diagonally scale or permute;
= 'P': Perform permutations to make the matrix more nearly upper triangular. Do not diagonally scale; = 'S': Diagonally scale the matrix, ie. replace A by D*A*D**(-1), where D is a diagonal matrix chosen to make the rows and columns of A more equal in norm. Do not permute; = 'B': Both diagonally scale and permute A.

Computed reciprocal condition numbers will be for the matrix after balancing and/or permuting. Permuting does not change condition numbers (in exact arithmetic), but balancing does.

JOBVL (input)
= 'N': left eigenvectors of A are not computed;
= 'V': left eigenvectors of A are computed. If SENSE = 'E' or 'B', JOBVL must = 'V'.

JOBVR (input)
= 'N': right eigenvectors of A are not computed;
= 'V': right eigenvectors of A are computed. If SENSE = 'E' or 'B', JOBVR must = 'V'.

```
SENSE (input)
          Determines which reciprocal condition numbers  are
          computed.  = 'N': None are computed;
          = 'E': Computed for eigenvalues only;
          = 'V': Computed for right eigenvectors only;
          = 'B': Computed for eigenvalues and  right  eigen-
          vectors.

          If SENSE = 'E' or 'B', both left and right  eigen-
          vectors  must  also  be  computed (JOBVL = 'V' and
          JOBVR = 'V').

N (input) The order of the matrix A. N >= 0.

A (input/output)
          On entry, the N-by-N matrix A.   On  exit,  A  has
          been  overwritten.  If JOBVL = 'V' or JOBVR = 'V',
          A contains the Schur form of the balanced  version
          of the matrix A.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

W (output)
          W contains the computed eigenvalues.

VL (output)
          If JOBVL = 'V', the  left  eigenvectors  u(j)  are
          stored  one after another in the columns of VL, in
          the same order as their eigenvalues.  If  JOBVL  =
          'N',  VL  is  not referenced.  u(j) = VL(:,j), the
          j-th column of VL.

LDVL (input)
          The leading dimension of the array VL.  LDVL >= 1;
          if JOBVL = 'V', LDVL >= N.

VR (input)
          If JOBVR = 'V', the right  eigenvectors  v(j)  are
          stored  one after another in the columns of VR, in
          the same order as their eigenvalues.  If  JOBVR  =
          'N',  VR  is  not referenced.  v(j) = VR(:,j), the
          j-th column of VR.

LDVR (input)
          The leading dimension of the array VR.  LDVR >= 1;
          if JOBVR = 'V', LDVR >= N.

ILO (output)
```

ILO and IHI are integer values determined when A was balanced. The balanced A(i,j) = 0 if I > J and J = 1,...,ILO-1 or I = IHI+1,...,N.

IHI (output)

ILO and IHI are integer values determined when A was balanced. The balanced A(i,j) = 0 if I > J and J = 1,...,ILO-1 or I = IHI+1,...,N.

SCALE (output)

Details of the permutations and scaling factors applied when balancing A. If P(j) is the index of the row and column interchanged with row and column j, and D(j) is the scaling factor applied to row and column j, then SCALE(J) = P(J), for J = 1,...,ILO-1 = D(J), for J = ILO,...,IHI = P(J) for J = IHI+1,...,N. The order in which the interchanges are made is N to IHI+1, then 1 to ILO-1.

ABNRM (output)

The one-norm of the balanced matrix (the maximum of the sum of absolute values of elements of any column).

RCONE (output)

RCONE(j) is the reciprocal condition number of the j-th eigenvalue.

RCONV (output)

RCONV(j) is the reciprocal condition number of the j-th right eigenvector.

WORK (workspace)

On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (output)

The dimension of the array WORK. If SENSE = 'N' or 'E', LDWORK >= max(1,2*N), and if SENSE = 'V' or 'B', LDWORK >= N*N+2*N. For good performance, LDWORK must generally be larger.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)

dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, the QR algorithm failed to compute all the eigenvalues, and no eigenvectors or condition numbers have been computed; elements 1:ILO-1 and i+1:N of W contain eigenvalues which have converged.

Contents

NAME

```
zgegs - routine is deprecated and has been replaced by  rou-
tine CGGES
```

SYNOPSIS

```
SUBROUTINE ZGEGS(JOBVSL, JOBVSR, N, A, LDA, B, LDB, ALPHA, BETA, VSL,
      LDVSL, VSR, LDVSR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 JOBVSL, JOBVSR
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),   ALPHA(*),   BETA(*),
VSL(LDVSL,*), VSR(LDVSR,*), WORK(*)
INTEGER N, LDA, LDB, LDVSL, LDVSR, LDWORK, INFO
DOUBLE PRECISION WORK2(*)

SUBROUTINE ZGEGS_64(JOBVSL, JOBVSR, N, A, LDA, B, LDB, ALPHA, BETA,
      VSL, LDVSL, VSR, LDVSR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 JOBVSL, JOBVSR
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),   ALPHA(*),   BETA(*),
VSL(LDVSL,*), VSR(LDVSR,*), WORK(*)
INTEGER*8 N, LDA, LDB, LDVSL, LDVSR, LDWORK, INFO
DOUBLE PRECISION WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GEGS(JOBVSL, JOBVSR, [N], A, [LDA], B, [LDB], ALPHA, BETA,
       VSL, [LDVSL], VSR, [LDVSR], [WORK], [LDWORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR
COMPLEX(8), DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, VSL, VSR
INTEGER :: N, LDA, LDB, LDVSL, LDVSR, LDWORK, INFO
REAL(8), DIMENSION(:) :: WORK2
```

```
SUBROUTINE GEGS_64(JOBVSL, JOBVSR, [N], A, [LDA], B, [LDB], ALPHA,
       BETA, VSL, [LDVSL], VSR, [LDVSR], [WORK], [LDWORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR
COMPLEX(8), DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, VSL, VSR
INTEGER(8) :: N, LDA, LDB, LDVSL, LDVSR, LDWORK, INFO
REAL(8), DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>
void zgegs(char jobvsl, char jobvsr,  int  n,  doublecomplex
          *a, int lda, doublecomplex *b, int ldb, doublecom-
          plex *alpha,  doublecomplex  *beta,  doublecomplex
          *vsl,  int  ldvsl,  doublecomplex *vsr, int ldvsr,
          int *info);

void zgegs_64(char jobvsl, char jobvsr, long  n,  doublecom-
          plex  *a,  long  lda,  doublecomplex *b, long ldb,
          doublecomplex *alpha, doublecomplex  *beta,  doub-
          lecomplex  *vsl,  long  ldvsl, doublecomplex *vsr,
          long ldvsr, long *info);
```

PURPOSE

zgegs routine is deprecated and has been replaced by routine CGGES.

CGEGS computes for a pair of N-by-N complex nonsymmetric matrices A, B: the generalized eigenvalues (alpha, beta), the complex Schur form (A, B), and optionally left and/or right Schur vectors (VSL and VSR).

(If only the generalized eigenvalues are needed, use the driver CGEGV instead.)

A generalized eigenvalue for a pair of matrices (A,B) is, roughly speaking, a scalar w or a ratio alpha/beta = w, such that A - w*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0, and even for both being zero. A good beginning reference is the book, "Matrix Computations", by G. Golub & C. van Loan (Johns Hopkins U. Press)

The (generalized) Schur form of a pair of matrices is the result of multiplying both matrices on the left by one unitary matrix and both on the right by another unitary matrix, these two unitary matrices being chosen so as to bring the pair of matrices into upper triangular form with the diagonal elements of B being non-negative real numbers (this is also called complex Schur form.)

The left and right Schur vectors are the columns of VSL and VSR, respectively, where VSL and VSR are the unitary matrices
which reduce A and B to Schur form:

Schur form of (A,B) = ((VSL)**H A (VSR), (VSL)**H B (VSR))

ARGUMENTS

JOBVSL (input)
= 'N': do not compute the left Schur vectors;
= 'V': compute the left Schur vectors.

JOBVSR (input)
= 'N': do not compute the right Schur vectors;
= 'V': compute the right Schur vectors.

N (input) The order of the matrices A, B, VSL, and VSR. N >= 0.

A (input/output)
On entry, the first of the pair of matrices whose generalized eigenvalues and (optionally) Schur vectors are to be computed. On exit, the generalized Schur form of A.

LDA (input)
The leading dimension of A. LDA >= max(1,N).

B (input/output)
On entry, the second of the pair of matrices whose generalized eigenvalues and (optionally) Schur vectors are to be computed. On exit, the generalized Schur form of B.

LDB (input)
The leading dimension of B. LDB >= max(1,N).

ALPHA (output)
On exit, ALPHA(j)/BETA(j), j=1,...,N, will be the generalized eigenvalues. ALPHA(j), j=1,...,N and BETA(j), j=1,...,N are the diagonals of the complex Schur form (A,B) output by CGEGS. The

BETA(j) will be non-negative real.

Note: the quotients ALPHA(j)/BETA(j) may easily over- or underflow, and BETA(j) may even be zero. Thus, the user should avoid naively computing the ratio alpha/beta. However, ALPHA will be always less than and usually comparable with norm(A) in magnitude, and BETA always less than and usually comparable with norm(B).

BETA (output)
See the description of ALPHA.

VSL (input)
If JOBVSL = 'V', VSL will contain the left Schur vectors. (See "Purpose", above.) Not referenced if JOBVSL = 'N'.

LDVSL (input)
The leading dimension of the matrix VSL. LDVSL >= 1, and if JOBVSL = 'V', LDVSL >= N.

VSR (input)
If JOBVSR = 'V', VSR will contain the right Schur vectors. (See "Purpose", above.) Not referenced if JOBVSR = 'N'.

LDVSR (input)
The leading dimension of the matrix VSR. LDVSR >= 1, and if JOBVSR = 'V', LDVSR >= N.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,2*N). For good performance, LDWORK must generally be larger. To compute the optimal value of LDWORK, call ILAENV to get blocksizes (for CGEQRF, CUNMQR, and CUNGQR.) Then compute: NB as the MAX of the blocksizes for CGEQRF, CUNMQR, and CUNGQR; the optimal LDWORK is N*(NB+1).

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
dimension(3*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
=1,...,N: The QZ iteration failed. (A,B) are not in Schur form, but ALPHA(j) and BETA(j) should be correct for j=INFO+1,...,N. > N: errors that usually indicate LAPACK problems:
=N+1: error return from CGGBAL
=N+2: error return from CGEQRF
=N+3: error return from CUNMQR
=N+4: error return from CUNGQR
=N+5: error return from CGGHRD
=N+6: error return from CHGEQZ (other than failed iteration) =N+7: error return from CGGBAK (computing VSL)
=N+8: error return from CGGBAK (computing VSR)
=N+9: error return from CLASCL (various places)

Contents

NAME

zgegv - routine is deprecated and has been replaced by rou-
tine CGGEV

SYNOPSIS

```
SUBROUTINE ZGEGV(JOBVL, JOBVR, N, A, LDA, B, LDB, ALPHA, BETA, VL,
      LDVL, VR, LDVR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 JOBVL, JOBVR
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),   ALPHA(*),   BETA(*),
VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER N, LDA, LDB, LDVL, LDVR, LDWORK, INFO
DOUBLE PRECISION WORK2(*)

SUBROUTINE ZGEGV_64(JOBVL, JOBVR, N, A, LDA, B, LDB, ALPHA, BETA, VL,
      LDVL, VR, LDVR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 JOBVL, JOBVR
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),   ALPHA(*),   BETA(*),
VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER*8 N, LDA, LDB, LDVL, LDVR, LDWORK, INFO
DOUBLE PRECISION WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GEGV(JOBVL, JOBVR, [N], A, [LDA], B, [LDB], ALPHA, BETA,
       VL, [LDVL], VR, [LDVR], [WORK], [LDWORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBVL, JOBVR
COMPLEX(8), DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, VL, VR
INTEGER :: N, LDA, LDB, LDVL, LDVR, LDWORK, INFO
```

```
REAL(8), DIMENSION(:) :: WORK2

SUBROUTINE GEGV_64(JOBVL, JOBVR, [N], A, [LDA], B, [LDB], ALPHA,
       BETA, VL, [LDVL], VR, [LDVR], [WORK], [LDWORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBVL, JOBVR
COMPLEX(8), DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, VL, VR
INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, LDWORK, INFO
REAL(8), DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zgegv(char jobvl, char jobvr, int n, doublecomplex  *a,
          int  lda, doublecomplex *b, int ldb, doublecomplex
          *alpha, doublecomplex  *beta,  doublecomplex  *vl,
          int ldvl, doublecomplex *vr, int ldvr, int *info);

void zgegv_64(char jobvl, char jobvr, long n,  doublecomplex
          *a,  long  lda,  doublecomplex *b, long ldb, doub-
          lecomplex *alpha, doublecomplex *beta,  doublecom-
          plex *vl, long ldvl, doublecomplex *vr, long ldvr,
          long *info);
```

PURPOSE

zgegv routine is deprecated and has been replaced by routine CGGEV.

CGEGV computes for a pair of N-by-N complex nonsymmetric matrices A and B, the generalized eigenvalues (alpha, beta), and optionally, the left and/or right generalized eigenvectors (VL and VR).

A generalized eigenvalue for a pair of matrices (A,B) is, roughly speaking, a scalar w or a ratio alpha/beta = w, such that A - w*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0, and even for both being zero. A good beginning reference is the book, "Matrix Computations", by G. Golub & C. van Loan (Johns Hopkins U. Press)

A right generalized eigenvector corresponding to a generalized eigenvalue w for a pair of matrices (A,B) is a vector r such that (A - w B) r = 0 . A left generalized eigenvector is a vector l such that l**H * (A - w B) = 0, where l**H is the
conjugate-transpose of l.

Note: this routine performs "full balancing" on A and B. See "Further Details", below.

ARGUMENTS

JOBVL (input)
= 'N': do not compute the left generalized eigenvectors;
= 'V': compute the left generalized eigenvectors.

JOBVR (input)
= 'N': do not compute the right generalized eigenvectors;
= 'V': compute the right generalized eigenvectors.

N (input) The order of the matrices A, B, VL, and VR. N >= 0.

A (input/output)
On entry, the first of the pair of matrices whose generalized eigenvalues and (optionally) generalized eigenvectors are to be computed. On exit, the contents will have been destroyed. (For a description of the contents of A on exit, see "Further Details", below.)

LDA (input)
The leading dimension of A. LDA >= max(1,N).

B (input/output)
On entry, the second of the pair of matrices whose generalized eigenvalues and (optionally) generalized eigenvectors are to be computed. On exit, the contents will have been destroyed. (For a description of the contents of B on exit, see "Further Details", below.)

LDB (input)
The leading dimension of B. LDB >= max(1,N).

ALPHA (output)
On exit, ALPHA(j)/VL(j), j=1,...,N, will be the generalized eigenvalues.

Note: the quotients ALPHA(j)/VL(j) may easily over- or underflow, and VL(j) may even be zero. Thus, the user should avoid naively computing the ratio alpha/beta. However, ALPHA will be always less than and usually comparable with norm(A) in magnitude, and VL always less than and usually comparable with norm(B).

BETA (output)
If JOBVL = 'V', the left generalized eigenvectors. (See "Purpose", above.) Each eigenvector will be scaled so the largest component will have abs(real part) + abs(imag. part) = 1, *except* that for eigenvalues with alpha=beta=0, a zero vector will be returned as the corresponding eigenvector. Not referenced if JOBVL = 'N'.

VL (output)
If JOBVL = 'V', the left generalized eigenvectors. (See "Purpose", above.) Each eigenvector will be scaled so the largest component will have abs(real part) + abs(imag. part) = 1, *except* that for eigenvalues with alpha=beta=0, a zero vector will be returned as the corresponding eigenvector. Not referenced if JOBVL = 'N'.

LDVL (input)
The leading dimension of the matrix VL. LDVL >= 1, and if JOBVL = 'V', LDVL >= N.

VR (output)
If JOBVR = 'V', the right generalized eigenvectors. (See "Purpose", above.) Each eigenvector will be scaled so the largest component will have abs(real part) + abs(imag. part) = 1, *except* that for eigenvalues with alpha=beta=0, a zero vector will be returned as the corresponding eigenvector. Not referenced if JOBVR = 'N'.

LDVR (input)
The leading dimension of the matrix VR. LDVR >= 1, and if JOBVR = 'V', LDVR >= N.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,2*N). For good performance, LDWORK must generally be larger. To compute the optimal value of LDWORK, call ILAENV to get blocksizes (for CGEQRF, CUNMQR, and CUNGQR.) Then compute: NB as the MAX of the blocksizes for CGEQRF, CUNMQR, and

CUNGQR; The optimal LDWORK is MAX(2*N, N*(NB+1)).

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
dimension(8*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
=1,...,N: The QZ iteration failed. No eigenvectors have been calculated, but ALPHA(j) and VL(j) should be correct for j=INFO+1,...,N. > N: errors that usually indicate LAPACK problems:
=N+1: error return from CGGBAL
=N+2: error return from CGEQRF
=N+3: error return from CUNMQR
=N+4: error return from CUNGQR
=N+5: error return from CGGHRD
=N+6: error return from CHGEQZ (other than failed iteration) =N+7: error return from CTGEVC
=N+8: error return from CGGBAK (computing VL)
=N+9: error return from CGGBAK (computing VR)
=N+10: error return from CLASCL (various calls)

FURTHER DETAILS

Balancing

This driver calls CGGBAL to both permute and scale rows and columns of A and B. The permutations PL and PR are chosen so that PL*A*PR and PL*B*R will be upper triangular except for the diagonal blocks A(i:j,i:j) and B(i:j,i:j), with i and j as close together as possible. The diagonal scaling matrices DL and DR are chosen so that the pair DL*PL*A*PR*DR, DL*PL*B*PR*DR have elements close to one (except for the elements that start out zero.)

After the eigenvalues and eigenvectors of the balanced matrices have been computed, CGGBAK transforms the eigenvectors back to what they would have been (in perfect arithmetic) if they had not been balanced.

Contents of A and B on Exit

-------- -- - --- - -- ----

If any eigenvectors are computed (either JOBVL='V' or JOBVR='V' or both), then on exit the arrays A and B will contain the complex Schur form[*] of the "balanced" versions of A and B. If no eigenvectors are computed, then only the diagonal blocks will be correct.

[*] In other words, upper triangular form.

Contents

NAME

```
zgehrd - reduce a complex general matrix A to upper  Hessen-
berg form H by a unitary similarity transformation
```

SYNOPSIS

```
SUBROUTINE ZGEHRD(N, ILO, IHI, A, LDA, TAU, WORKIN, LWORKIN, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORKIN(*)
INTEGER N, ILO, IHI, LDA, LWORKIN, INFO

SUBROUTINE ZGEHRD_64(N, ILO, IHI, A, LDA, TAU, WORKIN, LWORKIN, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORKIN(*)
INTEGER*8 N, ILO, IHI, LDA, LWORKIN, INFO
```

F95 INTERFACE

```
SUBROUTINE GEHRD([N], ILO, IHI, A, [LDA], TAU, [WORKIN], [LWORKIN],
       [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORKIN
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, ILO, IHI, LDA, LWORKIN, INFO

SUBROUTINE GEHRD_64([N], ILO, IHI, A, [LDA], TAU, [WORKIN], [LWORKIN],
       [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORKIN
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, ILO, IHI, LDA, LWORKIN, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zgehrd(int n, int ilo, int ihi, doublecomplex  *a,  int
          lda, doublecomplex *tau, int *info);

void zgehrd_64(long n, long ilo, long ihi, doublecomplex *a,
          long lda, doublecomplex *tau, long *info);
```

PURPOSE

zgehrd reduces a complex general matrix A to upper Hessenberg form H by a unitary similarity transformation: Q' * A * Q = H .

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

ILO (input)
It is assumed that A is already upper triangular in rows and columns 1:ILO-1 and IHI+1:N. ILO and IHI are normally set by a previous call to CGEBAL; otherwise they should be set to 1 and N respectively. See Further Details.

IHI (input)
See the description of ILO.

A (input/output)
On entry, the N-by-N general matrix to be reduced. On exit, the upper triangle and the first subdiagonal of A are overwritten with the upper Hessenberg matrix H, and the elements below the first subdiagonal, with the array TAU, represent the unitary matrix Q as a product of elementary reflectors. See Further Details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

TAU (output)
The scalar factors of the elementary reflectors (see Further Details). Elements 1:ILO-1 and IHI:N-1 of TAU are set to zero.

WORKIN (workspace)

On exit, if INFO = 0, WORKIN(1) returns the optimal LWORKIN.

LWORKIN (input)
The length of the array WORKIN. LWORKIN >= max(1,N). For optimum performance LWORKIN >= N*NB, where NB is the optimal blocksize.

If LWORKIN = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORKIN array, returns this value as the first entry of the WORKIN array, and no error message related to LWORKIN is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

The matrix Q is represented as a product of (ihi-ilo) elementary reflectors

```
   Q = H(ilo) H(ilo+1) . . . H(ihi-1).
```

Each H(i) has the form

```
   H(i) = I - tau * v * v'
```

where tau is a complex scalar, and v is a complex vector with v(1:i) = 0, v(i+1) = 1 and v(ihi+1:n) = 0; v(i+2:ihi) is stored on exit in A(i+2:ihi,i), and tau in TAU(i).

The contents of A are illustrated by the following example, with n = 7, ilo = 2 and ihi = 6:

```
on entry,                        on exit,

( a   a   a   a   a   a   a )    (  a   a   h    h    h    h
a  )  (      a   a   a   a   a   a )    (      a   h   h   h
h   a ) (     a   a   a   a   a   a )    (       h    h    h
h    h    h  ) (     a   a   a   a   a   a )    (      v2  h
h   h   h   h ) (     a   a   a   a   a   a )     (       v2
v3   h    h    h    h  )  (     a   a   a   a   a   a )    (
v2  v3  v4  h    h   h ) (                          a  )     (
a )
```

where a denotes an element of the original matrix A, h denotes a modified element of the upper Hessenberg matrix H, and vi denotes an element of the vector defining H(i).

Contents

NAME

```
zgelqf - compute an LQ factorization  of  a  complex  M-by-N
matrix A
```

SYNOPSIS

```
SUBROUTINE ZGELQF(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, LDA, LDWORK, INFO

SUBROUTINE ZGELQF_64(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, LDA, LDWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE GELQF([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, LDWORK, INFO

SUBROUTINE GELQF_64([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, LDWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zgelqf(int m, int n, doublecomplex *a, int  lda,  doub-
          lecomplex *tau, int *info);

void zgelqf_64(long m, long n, doublecomplex *a,  long  lda,
          doublecomplex *tau, long *info);
```

PURPOSE

zgelqf computes an LQ factorization of a complex M-by-N matrix A: A = L * Q.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, the elements on and below the diagonal of the array contain the m-by-min(m,n) lower trapezoidal matrix L (L is lower triangular if m <= n); the elements above the diagonal, with the array TAU, represent the unitary matrix Q as a product of elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,M). For optimum performance LDWORK >= M*NB, where NB is the optimal blocksize.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

```
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

FURTHER DETAILS

```
The matrix Q is  represented  as  a  product  of  elementary
reflectors

   Q = H(k)' . . . H(2)' H(1)', where k = min(m,n).

Each H(i) has the form
   H(i) = I - tau * v * v'

where tau is a complex scalar, and v  is  a  complex  vector
with v(1:i-1) = 0 and v(i) = 1; conjg(v(i+1:n)) is stored on
exit in A(i,i+1:n), and tau in TAU(i).
```

Contents

NAME

zgels - solve overdetermined or underdetermined complex linear systems involving an M-by-N matrix A, or its conjugate-transpose, using a QR or LQ factorization of A

SYNOPSIS

```
SUBROUTINE ZGELS(TRANSA, M, N, NRHS, A, LDA, B, LDB, WORK, LDWORK,
      INFO)

CHARACTER * 1 TRANSA
DOUBLE COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER M, N, NRHS, LDA, LDB, LDWORK, INFO

SUBROUTINE ZGELS_64(TRANSA, M, N, NRHS, A, LDA, B, LDB, WORK, LDWORK,
      INFO)

CHARACTER * 1 TRANSA
DOUBLE COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER*8 M, N, NRHS, LDA, LDB, LDWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE GELS([TRANSA], [M], [N], [NRHS], A, [LDA], B, [LDB], [WORK],
       LDWORK, [INFO])

CHARACTER(LEN=1) :: TRANSA
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: M, N, NRHS, LDA, LDB, LDWORK, INFO

SUBROUTINE GELS_64([TRANSA], [M], [N], [NRHS], A, [LDA], B, [LDB],
       [WORK], LDWORK, [INFO])
```

```
CHARACTER(LEN=1) :: TRANSA
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: M, N, NRHS, LDA, LDB, LDWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zgels (char, int, int, int, doublecomplex*, int,  doub-
          lecomplex*, int, int*);

void zgels_64 (char, long, long, long, doublecomplex*, long,
          doublecomplex*, long, long*);
```

PURPOSE

zgels solves overdetermined or underdetermined complex linear systems involving an M-by-N matrix A, or its conjugate-transpose, using a QR or LQ factorization of A. It is assumed that A has full rank.

The following options are provided:

1. If TRANS = 'N' and m >= n: find the least squares solution of
   an overdetermined system, i.e., solve the least squares problem
                minimize || B - A*X ||.

2. If TRANS = 'N' and m < n: find the minimum norm solution of
   an underdetermined system A * X = B.

3. If TRANS = 'C' and m >= n: find the minimum norm solution of
   an undetermined system A**H * X = B.

4. If TRANS = 'C' and m < n: find the least squares solution of
   an overdetermined system, i.e., solve the least squares problem
                minimize || B - A**H * X ||.

Several right hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the M-by-NRHS right hand side matrix B and the N-by-NRHS solution matrix X.

ARGUMENTS

```
TRANSA (input)
          = 'N': the linear system involves A;
          = 'C': the linear system involves A**H.

          TRANSA is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix A.  M >= 0.

N (input) The number of columns of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X. NRHS >= 0.
A (input/output)
          On entry, the M-by-N matrix A.  if M >=  N,  A  is
          overwritten  by details of its QR factorization as
          returned by CGEQRF; if M <  N, A is overwritten by
          details  of  its  LQ  factorization as returned by
          CGELQF.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,M).

B (input/output)
          On entry, the matrix B of right hand side vectors,
          stored columnwise; B is M-by-NRHS if TRANSA = 'N',
          or N-by-NRHS if TRANSA  =  'C'.   On  exit,  B  is
          overwritten   by   the  solution  vectors,  stored
          columnwise:  if TRANSA = 'N' and m >= n, rows 1 to
          n of B contain the least squares solution vectors;
          the residual sum of squares for  the  solution  in
          each column is given by the sum of squares of ele-
          ments N+1 to M in that column; if TRANSA = 'N' and
          m  <  n, rows 1 to N of B contain the minimum norm
          solution vectors; if TRANSA = 'C' and m >= n, rows
          1 to M of B contain the minimum norm solution vec-
          tors; if TRANSA = 'C' and m < n, rows 1 to M of  B
          contain  the  least  squares solution vectors; the
          residual sum of squares for the solution  in  each
          column  is given by the sum of squares of elements
          M+1 to N in that column.

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          MAX(1,M,N).
```

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (output)
The dimension of the array WORK. LDWORK >= max(1, MN + max(MN, NRHS)). For optimal performance, LDWORK >= max(1, MN + max(MN, NRHS)*NB). where MN = min(M,N) and NB is the optimum block size.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zgelsd - compute the minimum-norm solution to a real  linear
least squares problem
```

SYNOPSIS

```
SUBROUTINE ZGELSD(M, N, NRHS, A, LDA, B, LDB, S, RCOND, RANK, WORK,
      LWORK, RWORK, IWORK, INFO)

DOUBLE COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER IWORK(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION S(*), RWORK(*)

SUBROUTINE ZGELSD_64(M, N, NRHS, A, LDA, B, LDB, S, RCOND, RANK,
      WORK, LWORK, RWORK, IWORK, INFO)

DOUBLE COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER*8 M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION S(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GELSD([M], [N], [NRHS], A, [LDA], B, [LDB], S, RCOND,
       RANK, [WORK], [LWORK], [RWORK], [IWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
```

```
    REAL(8) :: RCOND
    REAL(8), DIMENSION(:) :: S, RWORK

    SUBROUTINE GELSD_64([M], [N], [NRHS], A, [LDA], B, [LDB], S, RCOND,
           RANK, [WORK], [LWORK], [RWORK], [IWORK], [INFO])

    COMPLEX(8), DIMENSION(:) :: WORK
    COMPLEX(8), DIMENSION(:,:) :: A, B
    INTEGER(8) :: M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
    INTEGER(8), DIMENSION(:) :: IWORK
    REAL(8) :: RCOND
    REAL(8), DIMENSION(:) :: S, RWORK
```

C INTERFACE

```
    #include <sunperf.h>
    void zgelsd(int m, int n, int nrhs,  doublecomplex  *a,  int
              lda,  doublecomplex *b, int ldb, double *s, double
              rcond, int *rank, int *info);

    void zgelsd_64(long m, long n, long nrhs, doublecomplex  *a,
              long  lda,  doublecomplex *b, long ldb, double *s,
              double rcond, long *rank, long *info);
```

PURPOSE

```
    zgelsd computes the minimum-norm solution to a  real  linear
    least squares problem:
        minimize 2-norm(| b - A*x |)
    using the singular value decomposition (SVD) of A. A  is  an
    M-by-N matrix which may be rank-deficient.

    Several right hand side vectors b and solution vectors x can
    be  handled in a single call; they are stored as the columns
    of the M-by-NRHS right hand side matrix B and the  N-by-NRHS
    solution matrix X.

    The problem is solved in three steps:
    (1) Reduce the coefficient matrix A to bidiagonal form with
        Householder tranformations, reducing the original  prob-
    lem
        into a "bidiagonal least squares problem" (BLS)
    (2) Solve the BLS using a divide and conquer approach.
    (3) Apply back all the Householder tranformations to solve
        the original least squares problem.

    The effective rank of A is determined by  treating  as  zero
    those  singular  values  which are less than RCOND times the
    largest singular value.
```

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, A has been destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the M-by-NRHS right hand side matrix B. On exit, B is overwritten by the N-by-NRHS solution matrix X. If m >= n and RANK = n, the residual sum-of-squares for the solution in the i-th column is given by the sum of squares of elements n+1:m in that column.

LDB (input)
The leading dimension of the array B. LDB >= max(1,M,N).

S (output)
The singular values of A in decreasing order. The condition number of A in the 2-norm = S(1)/S(min(m,n)).

RCOND (input)
RCOND is used to determine the effective rank of A. Singular values S(i) <= RCOND*S(1) are treated as zero. If RCOND < 0, machine precision is used instead.

RANK (output)
The effective rank of A, i.e., the number of singular values which are greater than RCOND*S(1).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 1. The exact minimum amount of workspace needed depends on M, N and NRHS. If M >= N, LWORK >= 2*N + N*NRHS. If M < N, LWORK >= 2*M + M*NRHS. For good performance, LWORK should generally be larger.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
If M >= N, LRWORK >= 8*N + 2*N*SMLSIZ + 8*N*NLVL + N*NRHS. If M < N, LRWORK >= 8*M + 2*M*SMLSIZ + 8*M*NLVL + M*NRHS. SMLSIZ is returned by ILAENV and is equal to the maximum size of the subproblems at the bottom of the computation tree (usually about 25), and NLVL = INT(LOG_2(MIN(M,N)/(SMLSIZ+1))) + 1

IWORK (workspace)
LIWORK >= 3 * MINMN * NLVL + 11 * MINMN, where MINMN = MIN(M,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: the algorithm for computing the SVD failed to converge; if INFO = i, i off-diagonal elements of an intermediate bidiagonal form did not converge to zero.

FURTHER DETAILS

Based on contributions by
Ming Gu and Ren-Cang Li, Computer Science Division, University of California at Berkeley, USA

Osni Marques, LBNL/NERSC, USA

Contents

NAME

```
zgelss - compute the minimum  norm  solution  to  a  complex
linear least squares problem
```

SYNOPSIS

```
SUBROUTINE ZGELSS(M, N, NRHS, A, LDA, B, LDB, SING, RCOND, IRANK,
      WORK, LDWORK, WORK2, INFO)

DOUBLE COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER M, N, NRHS, LDA, LDB, IRANK, LDWORK, INFO
DOUBLE PRECISION RCOND
DOUBLE PRECISION SING(*), WORK2(*)

SUBROUTINE ZGELSS_64(M, N, NRHS, A, LDA, B, LDB, SING, RCOND, IRANK,
      WORK, LDWORK, WORK2, INFO)

DOUBLE COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER*8 M, N, NRHS, LDA, LDB, IRANK, LDWORK, INFO
DOUBLE PRECISION RCOND
DOUBLE PRECISION SING(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GELSS([M], [N], [NRHS], A, [LDA], B, [LDB], SING, RCOND,
       IRANK, [WORK], [LDWORK], [WORK2], [INFO])

COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: M, N, NRHS, LDA, LDB, IRANK, LDWORK, INFO
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: SING, WORK2

SUBROUTINE GELSS_64([M], [N], [NRHS], A, [LDA], B, [LDB], SING,
```

```
       RCOND, IRANK, [WORK], [LDWORK], [WORK2], [INFO])

COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: M, N, NRHS, LDA, LDB, IRANK, LDWORK, INFO
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: SING, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zgelss(int m, int n, int nrhs,  doublecomplex  *a,  int
          lda, doublecomplex *b, int ldb, double *sing, dou-
          ble rcond, int *irank, int *info);
void zgelss_64(long m, long n, long nrhs, doublecomplex  *a,
          long  lda,  doublecomplex  *b,  long  ldb,  double
          *sing, double rcond, long *irank, long *info);
```

PURPOSE

zgelss computes the minimum norm solution to a complex linear least squares problem:

Minimize 2-norm(| b - A*x |).

using the singular value decomposition (SVD) of A. A is an M-by-N matrix which may be rank-deficient.

Several right hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the M-by-NRHS right hand side matrix B and the N-by-NRHS solution matrix X.

The effective rank of A is determined by treating as zero those singular values which are less than RCOND times the largest singular value.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input/output)

On entry, the M-by-N matrix A. On exit, the first min(m,n) rows of A are overwritten with its right singular vectors, stored rowwise.

LDA (input)

The leading dimension of the array A. LDA >= max(1,M).

B (input/output)

On entry, the M-by-NRHS right hand side matrix B. On exit, B is overwritten by the N-by-NRHS solution matrix X. If m >= n and IRANK = n, the residual sum-of-squares for the solution in the i-th column is given by the sum of squares of elements n+1:m in that column.

LDB (input)

The leading dimension of the array B. LDB >= max(1,M,N).

SING (output)

The singular values of A in decreasing order. The condition number of A in the 2-norm = SING(1)/SING(min(m,n)).

RCOND (input)

RCOND is used to determine the effective rank of A. Singular values SING(i) <= RCOND*SING(1) are treated as zero. If RCOND < 0, machine precision is used instead.

IRANK (output)

The effective rank of A, i.e., the number of singular values which are greater than RCOND*SING(1).

WORK (workspace)

On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)

The dimension of the array WORK. LDWORK >= 1, and also: LDWORK >= 2*min(M,N) + max(M,N,NRHS) For good performance, LDWORK should generally be larger.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first

entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
dimension(5*min(M,N))

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: the algorithm for computing the SVD failed to converge; if INFO = i, i off-diagonal elements of an intermediate bidiagonal form did not converge to zero.

Contents

NAME

```
zgelsx - routine is deprecated and has been replaced by rou-
tine CGELSY
```

SYNOPSIS

```
SUBROUTINE ZGELSX(M, N, NRHS, A, LDA, B, LDB, JPIVOT, RCOND, IRANK,
      WORK, WORK2, INFO)

DOUBLE COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER M, N, NRHS, LDA, LDB, IRANK, INFO
INTEGER JPIVOT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION WORK2(*)

SUBROUTINE ZGELSX_64(M, N, NRHS, A, LDA, B, LDB, JPIVOT, RCOND,
      IRANK, WORK, WORK2, INFO)

DOUBLE COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER*8 M, N, NRHS, LDA, LDB, IRANK, INFO
INTEGER*8 JPIVOT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GELSX([M], [N], [NRHS], A, [LDA], B, [LDB], JPIVOT, RCOND,
       IRANK, [WORK], [WORK2], [INFO])

COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: M, N, NRHS, LDA, LDB, IRANK, INFO
INTEGER, DIMENSION(:) :: JPIVOT
REAL(8) :: RCOND
```

```
REAL(8), DIMENSION(:) :: WORK2

SUBROUTINE GELSX_64([M], [N], [NRHS], A, [LDA], B, [LDB], JPIVOT,
       RCOND, IRANK, [WORK], [WORK2], [INFO])

COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: M, N, NRHS, LDA, LDB, IRANK, INFO
INTEGER(8), DIMENSION(:) :: JPIVOT
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>
void zgelsx(int m, int n, int nrhs,  doublecomplex  *a,  int
          lda,  doublecomplex *b, int ldb, int *jpivot, dou-
          ble rcond, int *irank, int *info);

void zgelsx_64(long m, long n, long nrhs, doublecomplex  *a,
          long   lda,   doublecomplex  *b,  long  ldb,  long
          *jpivot, double rcond, long *irank, long *info);
```

PURPOSE

zgelsx routine is deprecated and has been replaced by routine CGELSY.

CGELSX computes the minimum-norm solution to a complex linear least squares problem:

```
    minimize || A * X - B ||
```

using a complete orthogonal factorization of A. A is an M-by-N matrix which may be rank-deficient.

Several right hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the M-by-NRHS right hand side matrix B and the N-by-NRHS solution matrix X.

The routine first computes a QR factorization with column pivoting:

```
    A * P = Q * [ R11 R12 ]
                [  0  R22 ]
```

with R11 defined as the largest leading submatrix whose estimated condition number is less than 1/RCOND. The order of R11, RANK, is the effective rank of A.

Then, R22 is considered to be negligible, and R12 is annihilated by unitary transformations from the right, arriving at the complete orthogonal factorization:

```
   A * P = Q * [ T11 0 ] * Z
               [  0  0 ]
The minimum-norm solution is then
   X = P * Z' [ inv(T11)*Q1'*B ]
              [        0       ]
where Q1 consists of the first RANK columns of Q.
```

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of matrices B and X. NRHS >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, A has been overwritten by details of its complete orthogonal factorization.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the M-by-NRHS right hand side matrix B. On exit, the N-by-NRHS solution matrix X. If m >= n and IRANK = n, the residual sum-of-squares for the solution in the i-th column is given by the sum of squares of elements N+1:M in that column.

LDB (input)
The leading dimension of the array B. LDB >= max(1,M,N).

JPIVOT (input/output)
On entry, if JPIVOT(i) .ne. 0, the i-th column of A is an initial column, otherwise it is a free column. Before the QR factorization of A, all initial columns are permuted to the leading positions; only the remaining free columns are moved as a result of column pivoting during the factorization. On exit, if JPIVOT(i) = k, then the i-th column of A*P was the k-th column of A.

RCOND (input)

RCOND is used to determine the effective rank of A, which is defined as the order of the largest leading triangular submatrix R11 in the QR factorization with pivoting of A, whose estimated condition number < 1/RCOND.

IRANK (output)
The effective rank of A, i.e., the order of the submatrix R11. This is the same as the order of the submatrix T11 in the complete orthogonal factorization of A.

WORK (workspace)
(min(M,N) + max(N, 2*min(M,N)+NRHS)),

WORK2 (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zgelsy - compute the  minimum-norm  solution  to  a  complex
linear least squares problem
```

SYNOPSIS

```
SUBROUTINE ZGELSY(M, N, NRHS, A, LDA, B, LDB, JPVT, RCOND, RANK,
      WORK, LWORK, RWORK, INFO)

DOUBLE COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER JPVT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION RWORK(*)

SUBROUTINE ZGELSY_64(M, N, NRHS, A, LDA, B, LDB, JPVT, RCOND, RANK,
      WORK, LWORK, RWORK, INFO)

DOUBLE COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER*8 M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER*8 JPVT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GELSY([M], [N], [NRHS], A, [LDA], B, [LDB], JPVT, RCOND,
       RANK, [WORK], [LWORK], [RWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER, DIMENSION(:) :: JPVT
```

```
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: RWORK

SUBROUTINE GELSY_64([M], [N], [NRHS], A, [LDA], B, [LDB], JPVT,
       RCOND, RANK, [WORK], [LWORK], [RWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: M, N, NRHS, LDA, LDB, RANK, LWORK, INFO
INTEGER(8), DIMENSION(:) :: JPVT
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>
void zgelsy(int m, int n, int nrhs,  doublecomplex  *a,  int
          lda,  doublecomplex *b, int ldb, int *jpvt, double
          rcond, int *rank, int *info);

void zgelsy_64(long m, long n, long nrhs, doublecomplex  *a,
          long  lda, doublecomplex *b, long ldb, long *jpvt,
          double rcond, long *rank, long *info);
```

PURPOSE

```
zgelsy computes  the  minimum-norm  solution  to  a  complex
linear least squares problem:
    minimize || A * X - B ||
using a complete orthogonal factorization of A.  A is an  M-
by-N matrix which may be rank-deficient.

Several right hand side vectors b and solution vectors x can
be  handled in a single call; they are stored as the columns
of the M-by-NRHS right hand side matrix B and the  N-by-NRHS
solution matrix X.

The routine first computes a QR  factorization  with  column
pivoting:
    A * P = Q * [ R11 R12 ]
                [  0  R22 ]
with R11 defined as  the  largest  leading  submatrix  whose
estimated  condition number is less than 1/RCOND.  The order
of R11, RANK, is the effective rank of A.

Then, R22 is considered to be negligible, and R12 is annihi-
lated by unitary transformations from the right, arriving at
the complete orthogonal factorization:
   A * P = Q * [ T11 0 ] * Z
               [  0  0 ]
```

```
The minimum-norm solution is then
   X = P * Z' [ inv(T11)*Q1'*B ]
              [        0       ]
where Q1 consists of the first RANK columns of Q.

This routine is basically identical to the  original  xGELSX
except three differences:
  o The permutation of matrix B (the  right  hand  side)  is
faster and
    more simple.
  o The call to the subroutine xGEQPF has  been  substituted
by the
    the call to the subroutine xGEQP3. This subroutine is  a
Blas-3
    version of the QR factorization with column pivoting.
  o Matrix B (the right hand side) is updated with Blas-3.
```

ARGUMENTS

```
M (input) The number of rows of the matrix A.  M >= 0.

N (input) The number of columns of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of matrices B and X. NRHS >= 0.

A (input/output)
          On entry, the M-by-N matrix A.   On  exit,  A  has
          been   overwritten  by  details  of  its  complete
          orthogonal factorization.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,M).

B (input/output)
          On entry, the M-by-NRHS right hand side matrix  B.
          On exit, the N-by-NRHS solution matrix X.

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          max(1,M,N).

JPVT (input/output)
          On entry, if JPVT(i) .ne. 0, the i-th column of  A
          is permuted to the front of AP, otherwise column i
          is a free column.  On exit, if JPVT(i) =  k,  then
```

the i-th column of A*P was the k-th column of A.

RCOND (input)
RCOND is used to determine the effective rank of A, which is defined as the order of the largest leading triangular submatrix R11 in the QR factorization with pivoting of A, whose estimated condition number < 1/RCOND.

RANK (output)
The effective rank of A, i.e., the order of the submatrix R11. This is the same as the order of the submatrix T11 in the complete orthogonal factorization of A.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. The unblocked strategy requires that: LWORK >= MN + MAX(2*MN, N+1, MN+NRHS) where MN = min(M,N). The block algorithm requires that: LWORK >= MN + MAX(2*MN, NB*(N+1), MN+MN*NB, MN+NB*NRHS) where NB is an upper bound on the blocksize returned by ILAENV for the routines CGEQP3, CTZRZF, CTZRQF, CUNMQR, and CUNMRZ.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

Based on contributions by
A. Petitet, Computer Science Dept., Univ. of Tenn., Knoxville, USA
E. Quintana-Orti, Depto. de Informatica, Universidad Jaime I, Spain

G. Quintana-Orti, Depto. de Informatica, Universidad Jaime
I, Spain

Contents

NAME

zgemm - perform one of the matrix-matrix operations C := alpha*op(A)*op(B) + beta*C

SYNOPSIS

```
SUBROUTINE ZGEMM(TRANSA, TRANSB, M, N, K, ALPHA, A, LDA, B, LDB,
      BETA, C, LDC)

CHARACTER * 1 TRANSA, TRANSB
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER M, N, K, LDA, LDB, LDC

SUBROUTINE ZGEMM_64(TRANSA, TRANSB, M, N, K, ALPHA, A, LDA, B, LDB,
      BETA, C, LDC)

CHARACTER * 1 TRANSA, TRANSB
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER*8 M, N, K, LDA, LDB, LDC
```

F95 INTERFACE

```
SUBROUTINE GEMM([TRANSA], [TRANSB], [M], [N], [K], ALPHA, A, [LDA],
       B, [LDB], BETA, C, [LDC])

CHARACTER(LEN=1) :: TRANSA, TRANSB
COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:,:) :: A, B, C
INTEGER :: M, N, K, LDA, LDB, LDC

SUBROUTINE GEMM_64([TRANSA], [TRANSB], [M], [N], [K], ALPHA, A, [LDA],
       B, [LDB], BETA, C, [LDC])
```

```
CHARACTER(LEN=1) :: TRANSA, TRANSB
COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:,:) :: A, B, C
INTEGER(8) :: M, N, K, LDA, LDB, LDC
```

C INTERFACE

```
#include <sunperf.h>

void zgemm(char transa, char transb, int m, int  n,  int  k,
          doublecomplex  *alpha,  doublecomplex *a, int lda,
          doublecomplex *b, int  ldb,  doublecomplex  *beta,
          doublecomplex *c, int ldc);
void zgemm_64(char transa, char transb, long m, long n, long
          k,  doublecomplex  *alpha,  doublecomplex *a, long
          lda, doublecomplex  *b,  long  ldb,  doublecomplex
          *beta, doublecomplex *c, long ldc);
```

PURPOSE

zgemm performs one of the matrix-matrix operations

C := alpha*op(A)*op(B) + beta*C

where op(X) is one of

op(X) = X   or   op(X) = X'   or   op(X) = conjg(X'), alpha and beta are scalars, and A, B and C are matrices, with op(A) an m by k matrix, op(B) a k by n matrix and C an m by n matrix.

ARGUMENTS

TRANSA (input)

On entry, TRANSA specifies the form of op(A) to be used in the matrix multiplication as follows:

TRANSA = 'N' or 'n', op(A) = A.

TRANSA = 'T' or 't', op(A) = A'.

TRANSA = 'C' or 'c', op(A) = conjg(A').

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

TRANSB (input)

On entry, TRANSB specifies the form of op(B) to be used in the matrix multiplication as follows:

TRANSB = 'N' or 'n', op(B) = B.

TRANSB = 'T' or 't', op(B) = B'.

TRANSB = 'C' or 'c', op(B) = conjg(B').

Unchanged on exit.

TRANSB is defaulted to 'N' for F95 INTERFACE.

M (input)

On entry, M specifies the number of rows of the matrix op(A) and of the matrix C. M >= 0. Unchanged on exit.

N (input)

On entry, N specifies the number of columns of the matrix op(B) and the number of columns of the matrix C. N >= 0. Unchanged on exit.

K (input)

On entry, K specifies the number of columns of the matrix op(A) and the number of rows of the matrix op(B). K >= 0. Unchanged on exit.

ALPHA (input)

On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)

COMPLEX*16 array of DIMENSION (LDA, ka), where ka is K when TRANSA = 'N' or 'n', and is M otherwise. Before entry with TRANSA = 'N' or 'n', the leading M by K part of the array A must contain the matrix A, otherwise the leading K by M part of the array A must contain the matrix A. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDA >= max(1, M), otherwise LDA >= max(1, K). Unchanged on exit.

B (input)

COMPLEX*16 array of DIMENSION (LDB, kb), where kb is n when TRANSB = 'N' or 'n', and is k otherwise. Before entry with TRANSB = 'N' or 'n', the leading k by n part of the array B must contain the matrix B, otherwise the leading n by k part of the array B must contain the matrix B. Unchanged on exit.

LDB (input)

On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. When TRANSB = 'N' or 'n' then LDB >= max(1, k), otherwise LDB >= max(1, n). Unchanged on exit.

BETA (input)

On entry, BETA specifies the scalar beta. When BETA is supplied as zero then C need not be set on input. Unchanged on exit.

C (input/output)

COMPLEX*16 array of DIMENSION (LDC, n). Before entry, the leading m by n part of the array C must contain the matrix C, except when beta is zero, in which case C need not be set on entry. On exit, the array C is overwritten by the m by n matrix (alpha*op(A)*op(B) + beta*C).

LDC (input)

On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC >= max(1, m). Unchanged on exit.

Contents

NAME

zgemv - perform one of the matrix-vector operations y := alpha*A*x + beta*y, or y := alpha*A'*x + beta*y, or y := alpha*conjg(A')*x + beta*y

SYNOPSIS

```
SUBROUTINE ZGEMV(TRANSA, M, N, ALPHA, A, LDA, X, INCX, BETA, Y, INCY)

CHARACTER * 1 TRANSA
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(LDA,*), X(*), Y(*)
INTEGER M, N, LDA, INCX, INCY

SUBROUTINE ZGEMV_64(TRANSA, M, N, ALPHA, A, LDA, X, INCX, BETA, Y,
      INCY)

CHARACTER * 1 TRANSA
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(LDA,*), X(*), Y(*)
INTEGER*8 M, N, LDA, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE GEMV([TRANSA], [M], [N], ALPHA, A, [LDA], X, [INCX], BETA,
       Y, [INCY])

CHARACTER(LEN=1) :: TRANSA
COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:) :: X, Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, INCX, INCY

SUBROUTINE GEMV_64([TRANSA], [M], [N], ALPHA, A, [LDA], X, [INCX],
```

```
          BETA, Y, [INCY])

   CHARACTER(LEN=1) :: TRANSA
   COMPLEX(8) :: ALPHA, BETA
   COMPLEX(8), DIMENSION(:) :: X, Y
   COMPLEX(8), DIMENSION(:,:) :: A
   INTEGER(8) :: M, N, LDA, INCX, INCY
```

C INTERFACE

```
   #include <sunperf.h>

   void zgemv(char transa, int m, int n, doublecomplex  *alpha,
             doublecomplex  *a,  int lda, doublecomplex *x, int
             incx, doublecomplex *beta, doublecomplex  *y,  int
             incy);
   void zgemv_64(char transa, long  m,  long  n,  doublecomplex
             *alpha,  doublecomplex *a, long lda, doublecomplex
             *x, long incx, doublecomplex *beta,  doublecomplex
             *y, long incy);
```

PURPOSE

zgemv performs one of the matrix-vector operations y := alpha*A*x + beta*y, or y := alpha*A'*x + beta*y, or y := alpha*conjg(A')*x + beta*y where alpha and beta are scalars, x and y are vectors and A is an m by n matrix.

ARGUMENTS

TRANSA (input)

On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   y := alpha*A*x + beta*y.

TRANSA = 'T' or 't'   y := alpha*A'*x + beta*y.

TRANSA = 'C' or 'c'   y := alpha*conjg(A')*x + beta*y.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

M (input)

On entry, M specifies the number of rows of the matrix A. M >= 0. Unchanged on exit.

N (input)
On entry, N specifies the number of columns of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
Before entry, the leading m by n part of the array A must contain the matrix of coefficients. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, m). Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)) when TRANSA = 'N' or 'n' and at least (1 + (m - 1)*abs(INCX)) otherwise. Before entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. When BETA is supplied as zero then Y need not be set on input. Unchanged on exit.

Y (input/output)
(1 + (m - 1)*abs(INCY)) when TRANSA = 'N' or 'n' and at least (1 + (n - 1)*abs(INCY)) otherwise. Before entry with BETA non-zero, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
zgeqlf - compute a QL  factorization  of  a  complex  M-by-N
matrix A
```

SYNOPSIS

```
SUBROUTINE ZGEQLF(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, LDA, LDWORK, INFO

SUBROUTINE ZGEQLF_64(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, LDA, LDWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE GEQLF([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, LDWORK, INFO

SUBROUTINE GEQLF_64([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, LDWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zgeqlf(int m, int n, doublecomplex *a, int  lda,  doub-
          lecomplex *tau, int *info);

void zgeqlf_64(long m, long n, doublecomplex *a,  long  lda,
          doublecomplex *tau, long *info);
```

PURPOSE

zgeqlf computes a QL factorization of a complex M-by-N matrix A: A = Q * L.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, if m >= n, the lower triangle of the subarray A(m-n+1:m,1:n) contains the N-by-N lower triangular matrix L; if m <= n, the elements on and below the (n-m)-th superdiagonal contain the M-by-N lower trapezoidal matrix L; the remaining elements, with the array TAU, represent the unitary matrix Q as a product of elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,N). For optimum performance LDWORK >= N*NB, where NB is the optimal blocksize.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first

entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The matrix Q is represented as a product of elementary reflectors

```
   Q = H(k) . . . H(2) H(1), where k = min(m,n).
```

Each H(i) has the form

```
   H(i) = I - tau * v * v'
```

where tau is a complex scalar, and v is a complex vector with v(m-k+i+1:m) = 0 and v(m-k+i) = 1; v(1:m-k+i-1) is stored on exit in A(1:m-k+i-1,n-k+i), and tau in TAU(i).

Contents

NAME

```
zgeqp3 - compute a QR factorization with column pivoting  of
a matrix A
```

SYNOPSIS

```
SUBROUTINE ZGEQP3(M, N, A, LDA, JPVT, TAU, WORK, LWORK, RWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, LDA, LWORK, INFO
INTEGER JPVT(*)
DOUBLE PRECISION RWORK(*)

SUBROUTINE ZGEQP3_64(M, N, A, LDA, JPVT, TAU, WORK, LWORK, RWORK,
      INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, LDA, LWORK, INFO
INTEGER*8 JPVT(*)
DOUBLE PRECISION RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GEQP3([M], [N], A, [LDA], JPVT, TAU, [WORK], [LWORK],
       [RWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, LWORK, INFO
INTEGER, DIMENSION(:) :: JPVT
REAL(8), DIMENSION(:) :: RWORK

SUBROUTINE GEQP3_64([M], [N], A, [LDA], JPVT, TAU, [WORK], [LWORK],
```

```
        [RWORK], [INFO])

    COMPLEX(8), DIMENSION(:) :: TAU, WORK
    COMPLEX(8), DIMENSION(:,:) :: A
    INTEGER(8) :: M, N, LDA, LWORK, INFO
    INTEGER(8), DIMENSION(:) :: JPVT
    REAL(8), DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>

void zgeqp3(int m, int n, doublecomplex  *a,  int  lda,  int
          *jpvt, doublecomplex *tau, int *info);

void zgeqp3_64(long m, long n, doublecomplex *a,  long  lda,
          long *jpvt, doublecomplex *tau, long *info);
```

PURPOSE

zgeqp3 computes a QR factorization with column pivoting of a matrix A: A*P = Q*R using Level 3 BLAS.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, the upper triangle of the array contains the min(M,N)-by-N upper trapezoidal matrix R; the elements below the diagonal, together with the array TAU, represent the unitary matrix Q as a product of min(M,N) elementary reflectors.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

JPVT (input/output)
On entry, if JPVT(J).ne.0, the J-th column of A is permuted to the front of A*P (a leading column); if JPVT(J)=0, the J-th column of A is a free column. On exit, if JPVT(J)=K, then the J-th column of A*P was the the K-th column of A.

TAU (output)
The scalar factors of the elementary reflectors.

WORK (workspace)
On exit, if INFO=0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= N+1. For optimal performance LWORK >= (N+1)*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

The matrix Q is represented as a product of elementary reflectors

```
   Q = H(1) H(2) . . . H(k), where k = min(m,n).
```

Each H(i) has the form

```
   H(i) = I - tau * v * v'
```

where tau is a real/complex scalar, and v is a real/complex vector with v(1:i-1) = 0 and v(i) = 1; v(i+1:m) is stored on exit in A(i+1:m,i), and tau in TAU(i).

Based on contributions by
G. Quintana-Orti, Depto. de Informatica, Universidad Jaime I, Spain
X. Sun, Computer Science Dept., Duke University, USA

Contents

NAME

```
zgeqpf - routine is deprecated and has been replaced by rou-
tine CGEQP3
```

SYNOPSIS

```
SUBROUTINE ZGEQPF(M, N, A, LDA, JPIVOT, TAU, WORK, WORK2, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, LDA, INFO
INTEGER JPIVOT(*)
DOUBLE PRECISION WORK2(*)

SUBROUTINE ZGEQPF_64(M, N, A, LDA, JPIVOT, TAU, WORK, WORK2, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, LDA, INFO
INTEGER*8 JPIVOT(*)
DOUBLE PRECISION WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GEQPF([M], [N], A, [LDA], JPIVOT, TAU, [WORK], [WORK2],
       [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, INFO
INTEGER, DIMENSION(:) :: JPIVOT
REAL(8), DIMENSION(:) :: WORK2

SUBROUTINE GEQPF_64([M], [N], A, [LDA], JPIVOT, TAU, [WORK], [WORK2],
       [INFO])
```

```
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, INFO
INTEGER(8), DIMENSION(:) :: JPIVOT
REAL(8), DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zgeqpf(int m, int n, doublecomplex  *a,  int  lda,  int
          *jpivot, doublecomplex *tau, int *info);

void zgeqpf_64(long m, long n, doublecomplex *a,  long  lda,
          long *jpivot, doublecomplex *tau, long *info);
```

PURPOSE

zgeqpf routine is deprecated and has been replaced by routine CGEQP3.

CGEQPF computes a QR factorization with column pivoting of a complex M-by-N matrix A: A*P = Q*R.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0

A (input/output)
On entry, the M-by-N matrix A. On exit, the upper triangle of the array contains the min(M,N)-by-N upper triangular matrix R; the elements below the diagonal, together with the array TAU, represent the unitary matrix Q as a product of min(m,n) elementary reflectors.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

JPIVOT (input/output)
On entry, if JPIVOT(i) .ne. 0, the i-th column of A is permuted to the front of A*P (a leading column); if JPIVOT(i) = 0, the i-th column of A is a free column. On exit, if JPIVOT(i) = k, then

```
          the i-th column of A*P was the k-th column of A.

TAU (output)
          The scalar factors of the elementary reflectors.

WORK (workspace)
          dimension(N)

WORK2 (workspace)
          dimension(2*N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO =  -i,  the  i-th  argument  had  an
          illegal value
```

FURTHER DETAILS

```
The matrix Q is  represented  as  a  product  of  elementary
reflectors

   Q = H(1) H(2) . . . H(n)

Each H(i) has the form

   H = I - tau * v * v'

where tau is a complex scalar, and v  is  a  complex  vector
with  v(1:i-1)  = 0 and v(i) = 1; v(i+1:m) is stored on exit
in A(i+1:m,i).

The matrix P is represented in jpvt as follows: If
   jpvt(j) = i
then the jth column of P is the ith canonical unit vector.
```

Contents

NAME

```
zgeqrf - compute a QR  factorization  of  a  complex  M-by-N
matrix A
```

SYNOPSIS

```
SUBROUTINE ZGEQRF(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, LDA, LDWORK, INFO

SUBROUTINE ZGEQRF_64(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, LDA, LDWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE GEQRF([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, LDWORK, INFO

SUBROUTINE GEQRF_64([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, LDWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zgeqrf(int m, int n, doublecomplex *a, int  lda,  doub-
          lecomplex *tau, int *info);

void zgeqrf_64(long m, long n, doublecomplex *a,  long  lda,
          doublecomplex *tau, long *info);
```

PURPOSE

zgeqrf computes a QR factorization of a complex M-by-N matrix A: A = Q * R.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.
N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, the elements on and above the diagonal of the array contain the min(M,N)-by-N upper trapezoidal matrix R (R is upper triangular if m >= n); the elements below the diagonal, with the array TAU, represent the unitary matrix Q as a product of min(m,n) elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,N). For optimum performance LDWORK >= N*NB, where NB is the optimal blocksize.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

```
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

FURTHER DETAILS

```
The matrix Q is  represented  as  a  product  of  elementary
reflectors

   Q = H(1) H(2) . . . H(k), where k = min(m,n).

Each H(i) has the form
   H(i) = I - tau * v * v'

where tau is a complex scalar, and v  is  a  complex  vector
with  v(1:i-1)  = 0 and v(i) = 1; v(i+1:m) is stored on exit
in A(i+1:m,i), and tau in TAU(i).
```

Contents

NAME

```
zgerc - perform the rank 1 operation   A  :=  alpha*x*conjg(
y' ) + A
```

SYNOPSIS

```
SUBROUTINE ZGERC(M, N, ALPHA, X, INCX, Y, INCY, A, LDA)

DOUBLE COMPLEX ALPHA
DOUBLE COMPLEX X(*), Y(*), A(LDA,*)
INTEGER M, N, INCX, INCY, LDA

SUBROUTINE ZGERC_64(M, N, ALPHA, X, INCX, Y, INCY, A, LDA)

DOUBLE COMPLEX ALPHA
DOUBLE COMPLEX X(*), Y(*), A(LDA,*)
INTEGER*8 M, N, INCX, INCY, LDA
```

F95 INTERFACE

```
SUBROUTINE GERC([M], [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

COMPLEX(8) :: ALPHA
COMPLEX(8), DIMENSION(:) :: X, Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, INCX, INCY, LDA

SUBROUTINE GERC_64([M], [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

COMPLEX(8) :: ALPHA
COMPLEX(8), DIMENSION(:) :: X, Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, INCX, INCY, LDA
```

C INTERFACE

```
#include <sunperf.h>

void zgerc(int m, int n, doublecomplex *alpha, doublecomplex
          *x,  int  incx,  doublecomplex *y, int incy, doub-
          lecomplex *a, int lda);

void zgerc_64(long m, long n,  doublecomplex  *alpha,  doub-
          lecomplex  *x,  long  incx, doublecomplex *y, long
          incy, doublecomplex *a, long lda);
```

PURPOSE

zgerc performs the rank 1 operation A := alpha*x*conjg(y') + A where alpha is a scalar, x is an m element vector, y is an n element vector and A is an m by n matrix.

ARGUMENTS

M (input)
On entry, M specifies the number of rows of the matrix A. M >= 0. Unchanged on exit.

N (input)
On entry, N specifies the number of columns of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (m - 1)*abs(INCX)). Before entry, the incremented array X must contain the m element vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. Unchanged on exit.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

A (input/output)

Before entry, the leading m by n part of the array A must contain the matrix of coefficients. On exit, A is overwritten by the updated matrix.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, m). Unchanged on exit.

Contents

NAME

```
zgerfs - improve the computed solution to a system of linear
equations and provides error bounds and backward error esti-
mates for the solution
```

SYNOPSIS

```
SUBROUTINE ZGERFS(TRANSA, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZGERFS_64(TRANSA, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B,
      LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GERFS([TRANSA], [N], [NRHS], A, [LDA], AF, [LDAF], IPIVOT,
       B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: TRANSA
COMPLEX(8), DIMENSION(:) :: WORK
```

```
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE GERFS_64([TRANSA], [N], [NRHS], A, [LDA], AF, [LDAF],
       IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: TRANSA
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zgerfs(char transa, int n, int nrhs, doublecomplex  *a,
          int lda, doublecomplex *af, int ldaf, int *ipivot,
          doublecomplex *b, int ldb, doublecomplex  *x,  int
          ldx, double *ferr, double *berr, int *info);

void zgerfs_64(char transa, long n, long nrhs, doublecomplex
          *a,  long  lda, doublecomplex *af, long ldaf, long
          *ipivot, doublecomplex *b, long ldb, doublecomplex
          *x,  long  ldx,  double  *ferr, double *berr, long
          *info);
```

PURPOSE

zgerfs improves the computed solution to a system of linear equations and provides error bounds and backward error estimates for the solution.

ARGUMENTS

```
TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
```

The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The original N-by-N matrix A.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input)
The factors L and U from the factorization A = P*L*U as computed by CGETRF.

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

IPIVOT (input)
The pivot indices from CGETRF; for 1<=i<=N, row i of the matrix was interchanged with row IPIVOT(i).

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by CGETRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

```
WORK (workspace)
          dimension(2*N)

WORK2 (workspace)
          dimension(N)
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

zgerqf - compute an RQ factorization of a complex M-by-N matrix A

SYNOPSIS

```
SUBROUTINE ZGERQF(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, LDA, LDWORK, INFO

SUBROUTINE ZGERQF_64(M, N, A, LDA, TAU, WORK, LDWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, LDA, LDWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE GERQF([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, LDWORK, INFO

SUBROUTINE GERQF_64([M], [N], A, [LDA], TAU, [WORK], [LDWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, LDWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zgerqf(int m, int n, doublecomplex *a, int  lda,  doub-
          lecomplex *tau, int *info);

void zgerqf_64(long m, long n, doublecomplex *a,  long  lda,
          doublecomplex *tau, long *info);
```

PURPOSE

zgerqf computes an RQ factorization of a complex M-by-N matrix A: A = R * Q.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, if m <= n, the upper triangle of the subarray A(1:m,n-m+1:n) contains the M-by-M upper triangular matrix R; if m >= n, the elements on and above the (m-n)-th subdiagonal contain the M-by-N upper trapezoidal matrix R; the remaining elements, with the array TAU, represent the unitary matrix Q as a product of min(m,n) elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A.   LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK.   LDWORK >= max(1,M). For optimum performance LDWORK >= M*NB, where NB is the optimal blocksize.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first

entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The matrix Q is represented as a product of elementary reflectors

```
   Q = H(1)' H(2)' . . . H(k)', where k = min(m,n).
```

Each H(i) has the form

```
   H(i) = I - tau * v * v'
```

where tau is a complex scalar, and v is a complex vector with v(n-k+i+1:n) = 0 and v(n-k+i) = 1; conjg(v(1:n-k+i-1)) is stored on exit in A(m-k+i,1:n-k+i-1), and tau in TAU(i).

Contents

NAME

```
zgeru - perform the rank 1 operation   A := alpha*x*y' + A
```

SYNOPSIS

```
SUBROUTINE ZGERU(M, N, ALPHA, X, INCX, Y, INCY, A, LDA)

DOUBLE COMPLEX ALPHA
DOUBLE COMPLEX X(*), Y(*), A(LDA,*)
INTEGER M, N, INCX, INCY, LDA

SUBROUTINE ZGERU_64(M, N, ALPHA, X, INCX, Y, INCY, A, LDA)

DOUBLE COMPLEX ALPHA
DOUBLE COMPLEX X(*), Y(*), A(LDA,*)
INTEGER*8 M, N, INCX, INCY, LDA
```

F95 INTERFACE

```
SUBROUTINE GER([M], [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

COMPLEX(8) :: ALPHA
COMPLEX(8), DIMENSION(:) :: X, Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, INCX, INCY, LDA

SUBROUTINE GER_64([M], [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

COMPLEX(8) :: ALPHA
COMPLEX(8), DIMENSION(:) :: X, Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, INCX, INCY, LDA
```

C INTERFACE

```
#include <sunperf.h>

void zgeru(int m, int n, doublecomplex *alpha, doublecomplex
          *x,  int  incx,  doublecomplex *y, int incy, doub-
          lecomplex *a, int lda);

void zgeru_64(long m, long n,  doublecomplex  *alpha,  doub-
          lecomplex  *x,  long  incx, doublecomplex *y, long
          incy, doublecomplex *a, long lda);
```

PURPOSE

zgeru performs the rank 1 operation A := alpha*x*y' + A where alpha is a scalar, x is an m element vector, y is an n element vector and A is an m by n matrix.

ARGUMENTS

M (input)
On entry, M specifies the number of rows of the matrix A. M >= 0. Unchanged on exit.

N (input)
On entry, N specifies the number of columns of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (m - 1)*abs(INCX)). Before entry, the incremented array X must contain the m element vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

Y (input)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. Unchanged on exit.

INCY (input)
On entry, INCY specifies the increment for the

elements of Y. INCY must not be zero. Unchanged on exit.

A (input/output)

Before entry, the leading m by n part of the array A must contain the matrix of coefficients. On exit, A is overwritten by the updated matrix.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, m). Unchanged on exit.

Contents

NAME

```
zgesdd - compute the singular value decomposition (SVD) of a
complex  M-by-N  matrix  A,  optionally  computing  the left
and/or right singular vectors, by  using  divide-and-conquer
method
```

SYNOPSIS

```
SUBROUTINE ZGESDD(JOBZ, M, N, A, LDA, S, U, LDU, VT, LDVT, WORK,
      LWORK, RWORK, IWORK, INFO)

CHARACTER * 1 JOBZ
DOUBLE COMPLEX A(LDA,*), U(LDU,*), VT(LDVT,*), WORK(*)
INTEGER M, N, LDA, LDU, LDVT, LWORK, INFO
INTEGER IWORK(*)
DOUBLE PRECISION S(*), RWORK(*)

SUBROUTINE ZGESDD_64(JOBZ, M, N, A, LDA, S, U, LDU, VT, LDVT, WORK,
      LWORK, RWORK, IWORK, INFO)

CHARACTER * 1 JOBZ
DOUBLE COMPLEX A(LDA,*), U(LDU,*), VT(LDVT,*), WORK(*)
INTEGER*8 M, N, LDA, LDU, LDVT, LWORK, INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION S(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GESDD(JOBZ, [M], [N], A, [LDA], S, U, [LDU], VT, [LDVT],
       [WORK], [LWORK], [RWORK], [IWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ
COMPLEX(8), DIMENSION(:) :: WORK
```

```
COMPLEX(8), DIMENSION(:,:) :: A, U, VT
INTEGER :: M, N, LDA, LDU, LDVT, LWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: S, RWORK

SUBROUTINE GESDD_64(JOBZ, [M], [N], A, [LDA], S, U, [LDU], VT, [LDVT],
       [WORK], [LWORK], [RWORK], [IWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, U, VT
INTEGER(8) :: M, N, LDA, LDU, LDVT, LWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: S, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void zgesdd(char jobz, int m, int n, doublecomplex  *a,  int
          lda,  double  *s, doublecomplex *u, int ldu, doub-
          lecomplex *vt, int ldvt, int *info);

void zgesdd_64(char jobz, long m, long n, doublecomplex  *a,
          long  lda,  double *s, doublecomplex *u, long ldu,
          doublecomplex *vt, long ldvt, long *info);
```

PURPOSE

zgesdd computes the singular value decomposition (SVD) of a complex M-by-N matrix A, optionally computing the left and/or right singular vectors, by using divide-and-conquer method. The SVD is written
 = U * SIGMA * conjugate-transpose(V)

where SIGMA is an M-by-N matrix which is zero except for its min(m,n) diagonal elements, U is an M-by-M unitary matrix, and V is an N-by-N unitary matrix. The diagonal elements of SIGMA are the singular values of A; they are real and non-negative, and are returned in descending order. The first min(m,n) columns of U and V are the left and right singular vectors of A.

Note that the routine returns VT = V**H, not V.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

JOBZ (input)
Specifies options for computing all or part of the matrix U:
= 'A': all M columns of U and all N rows of V**H are returned in the arrays U and VT; = 'S': the first min(M,N) columns of U and the first min(M,N) rows of V**H are returned in the arrays U and VT; = 'O': If M >= N, the first N columns of U are overwritten on the array A and all rows of V**H are returned in the array VT; otherwise, all columns of U are returned in the array U and the first M rows of V**H are overwritten in the array VT; = 'N': no columns of U or rows of V**H are computed.

M (input) The number of rows of the input matrix A. M >= 0.

N (input) The number of columns of the input matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, if JOBZ = 'O', A is overwritten with the first N columns of U (the left singular vectors, stored columnwise) if M >= N; A is overwritten with the first M rows of V**H (the right singular vectors, stored row-wise) otherwise. if JOBZ .ne. 'O', the contents of A are destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

S (output)
The singular values of A, sorted so that S(i) >= S(i+1).

U (output)
UCOL = M if JOBZ = 'A' or JOBZ = 'O' and M < N; UCOL = min(M,N) if JOBZ = 'S'. If JOBZ = 'A' or JOBZ = 'O' and M < N, U contains the M-by-M unitary matrix U; if JOBZ = 'S', U contains the first min(M,N) columns of U (the left singular vectors, stored columnwise); if JOBZ = 'O' and M >= N, or

```
          JOBZ = 'N', U is not referenced.

LDU (input)
          The leading dimension of the array U.  LDU  >=  1;
          if  JOBZ = 'S' or 'A' or JOBZ = 'O' and M < N, LDU
          >= M.

VT (output)
          If JOBZ = 'A' or JOBZ = 'O' and M >=  N,  VT  con-
          tains  the  N-by-N  unitary matrix V**H; if JOBZ =
          'S', VT contains the first min(M,N) rows  of  V**H
          (the  right  singular vectors, stored rowwise); if
          JOBZ = 'O' and M < N, or JOBZ =  'N',  VT  is  not
          referenced.

LDVT (input)
          The leading dimension of the array VT.  LDVT >= 1;
          if JOBZ = 'A' or JOBZ = 'O' and M >= N, LDVT >= N;
          if JOBZ = 'S', LDVT >= min(M,N).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK. LWORK >=  1.   if
          JOBZ = 'N', LWORK >= 2*min(M,N)+max(M,N).  if JOBZ
          =            'O',              LWORK             >=
          2*min(M,N)*min(M,N)+2*min(M,N)+max(M,N).   if JOBZ
          =      'S'      or       'A',       LWORK       >=
          min(M,N)*min(M,N)+2*min(M,N)+max(M,N).   For  good
          performance, LWORK should generally be larger.  If
          LWORK  <  0  but  other input arguments are legal,
          WORK(1) returns optimal LWORK.

RWORK (workspace)
          If JOBZ = 'N', LRWORK >=  7*min(M,N).   Otherwise,
          LRWORK >= 5*min(M,N)*min(M,N) + 5*min(M,N)

IWORK (workspace)
          dimension(8*MIN(M,N))

INFO (output)
          = 0:  successful exit.
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value.
          > 0:  The updating process of SBDSDC did not  con-
          verge.
```

FURTHER DETAILS

Based on contributions by
Ming Gu and Huan Ren, Computer Science Division, University of
California at Berkeley, USA

Contents

NAME

```
zgesv - compute the solution to a complex system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE ZGESV(N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZGESV_64(N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE GESV([N], [NRHS], A, [LDA], IPIVOT, B, [LDB], [INFO])

COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: N, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE GESV_64([N], [NRHS], A, [LDA], IPIVOT, B, [LDB], [INFO])

COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zgesv(int n, int nrhs, doublecomplex *a, int  lda,  int
          *ipivot, doublecomplex *b, int ldb, int *info);

void zgesv_64(long n, long nrhs, doublecomplex *a, long lda,
          long  *ipivot,  doublecomplex  *b,  long ldb, long
          *info);
```

PURPOSE

```
zgesv computes the solution to a complex  system  of  linear
equations
   A * X = B, where A is an N-by-N matrix and X  and  B  are
N-by-NRHS matrices.
The LU decomposition with partial pivoting  and  row  inter-
changes is used to factor A as
   A = P * L * U,
where P is a permutation matrix, L is unit lower triangular,
and  U  is upper triangular.  The factored form of A is then
used to solve the system of equations A * X = B.
```

ARGUMENTS

```
N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

A (input/output)
          On entry, the N-by-N  coefficient  matrix  A.   On
          exit, the factors L and U from the factorization A
          = P*L*U; the unit diagonal elements of L  are  not
          stored.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

IPIVOT (output)
          The pivot  indices  that  define  the  permutation
          matrix  P;  row  i  of the matrix was interchanged
          with row IPIVOT(i).

B (input/output)
          On entry, the N-by-NRHS matrix of right hand  side
```

matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution could not be computed.

Contents

NAME

```
zgesvd - compute the singular value decomposition (SVD) of a
complex  M-by-N  matrix  A,  optionally  computing  the left
and/or right singular vectors
```

SYNOPSIS

```
SUBROUTINE ZGESVD(JOBU, JOBVT, M, N, A, LDA, SING, U, LDU, VT, LDVT,
      WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 JOBU, JOBVT
DOUBLE COMPLEX A(LDA,*), U(LDU,*), VT(LDVT,*), WORK(*)
INTEGER M, N, LDA, LDU, LDVT, LDWORK, INFO
DOUBLE PRECISION SING(*), WORK2(*)

SUBROUTINE ZGESVD_64(JOBU, JOBVT, M, N, A, LDA, SING, U, LDU, VT,
      LDVT, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 JOBU, JOBVT
DOUBLE COMPLEX A(LDA,*), U(LDU,*), VT(LDVT,*), WORK(*)
INTEGER*8 M, N, LDA, LDU, LDVT, LDWORK, INFO
DOUBLE PRECISION SING(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GESVD(JOBU, JOBVT, [M], [N], A, [LDA], SING, U, [LDU], VT,
       [LDVT], [WORK], [LDWORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBU, JOBVT
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, U, VT
INTEGER :: M, N, LDA, LDU, LDVT, LDWORK, INFO
REAL(8), DIMENSION(:) :: SING, WORK2
```

```
SUBROUTINE GESVD_64(JOBU, JOBVT, [M], [N], A, [LDA], SING, U, [LDU],
       VT, [LDVT], [WORK], [LDWORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBU, JOBVT
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, U, VT
INTEGER(8) :: M, N, LDA, LDU, LDVT, LDWORK, INFO
REAL(8), DIMENSION(:) :: SING
```

C INTERFACE

```
#include <sunperf.h>

void zgesvd(char jobu, char jobvt, int m, int n,  doublecom-
          plex  *a, int lda, double *sing, doublecomplex *u,
          int ldu, doublecomplex *vt, int ldvt, int *info);
void zgesvd_64(char jobu, char jobvt, long m, long n,  doub-
          lecomplex  *a,  long lda, double *sing, doublecom-
          plex *u, long ldu, doublecomplex *vt,  long  ldvt,
          long *info);
```

PURPOSE

zgesvd computes the singular value decomposition (SVD) of a complex M-by-N matrix A, optionally computing the left and/or right singular vectors. The SVD is written

= U * SIGMA * conjugate-transpose(V)

where SIGMA is an M-by-N matrix which is zero except for its min(m,n) diagonal elements, U is an M-by-M unitary matrix, and V is an N-by-N unitary matrix. The diagonal elements of SIGMA are the singular values of A; they are real and non-negative, and are returned in descending order. The first min(m,n) columns of U and V are the left and right singular vectors of A.

Note that the routine returns V**H, not V.

ARGUMENTS

JOBU (input)
Specifies options for computing all or part of the matrix U:
= 'A': all M columns of U are returned in array U:
= 'S': the first min(m,n) columns of U (the left singular vectors) are returned in the array U; = 'O': the first min(m,n) columns of U (the left

singular vectors) are overwritten on the array A;
= 'N': no columns of U (no left singular vectors) are computed.

JOBVT (input)
Specifies options for computing all or part of the matrix V**H:
= 'A': all N rows of V**H are returned in the array VT;
= 'S': the first min(m,n) rows of V**H (the right singular vectors) are returned in the array VT; = 'O': the first min(m,n) rows of V**H (the right singular vectors) are overwritten on the array A;
= 'N': no rows of V**H (no right singular vectors) are computed.

JOBVT and JOBU cannot both be 'O'.

M (input) The number of rows of the input matrix A. M >= 0.

N (input) The number of columns of the input matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, if JOBU = 'O', A is overwritten with the first min(m,n) columns of U (the left singular vectors, stored columnwise); if JOBVT = 'O', A is overwritten with the first min(m,n) rows of V**H (the right singular vectors, stored rowwise); if JOBU .ne. 'O' and JOBVT .ne. 'O', the contents of A are destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

SING (output)
The singular values of A, sorted so that SING(i) >= SING(i+1).

U (input) (LDU,M) if JOBU = 'A' or (LDU,min(M,N)) if JOBU = 'S'. If JOBU = 'A', U contains the M-by-M unitary matrix U; if JOBU = 'S', U contains the first min(m,n) columns of U (the left singular vectors, stored columnwise); if JOBU = 'N' or 'O', U is not referenced.

LDU (input)
The leading dimension of the array U. LDU >= 1; if JOBU = 'S' or 'A', LDU >= M.

VT (input)
If JOBVT = 'A', VT contains the N-by-N unitary matrix V**H; if JOBVT = 'S', VT contains the first min(m,n) rows of V**H (the right singular vectors, stored rowwise); if JOBVT = 'N' or 'O', VT is not referenced.

LDVT (input)
The leading dimension of the array VT. LDVT >= 1; if JOBVT = 'A', LDVT >= N; if JOBVT = 'S', LDVT >= min(M,N).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= 1. LDWORK >= 2*MIN(M,N)+MAX(M,N) For good performance, LDWORK should generally be larger.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
DIMENSION(5*MIN(M,N)). On exit, if INFO > 0, WORK2(1:MIN(M,N)-1) contains the unconverged superdiagonal elements of an upper bidiagonal matrix B whose diagonal is in SING (not necessarily sorted). B satisfies A = U * B * VT, so it has the same singular values as A, and singular vectors related by U and VT.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if CBDSQR did not converge, INFO specifies how many superdiagonals of an intermediate bidiagonal form B did not converge to zero. See the description of WORK2 above for details.

Contents

NAME

```
zgesvx - use the LU factorization to compute the solution to
a complex system of linear equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE ZGESVX(FACT, TRANSA, N, NRHS, A, LDA, AF, LDAF, IPIVOT,
      EQUED, R, C, B, LDB, X, LDX, RCOND, FERR, BERR, WORK,
      WORK2, INFO)

CHARACTER * 1 FACT, TRANSA, EQUED
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION R(*), C(*), FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZGESVX_64(FACT, TRANSA, N, NRHS, A, LDA, AF, LDAF, IPIVOT,
      EQUED, R, C, B, LDB, X, LDX, RCOND, FERR, BERR, WORK,
      WORK2, INFO)

CHARACTER * 1 FACT, TRANSA, EQUED
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION R(*), C(*), FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GESVX(FACT, [TRANSA], [N], [NRHS], A, [LDA], AF, [LDAF],
       IPIVOT, EQUED, R, C, B, [LDB], X, [LDX], RCOND, FERR,
```

```
       BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, TRANSA, EQUED
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: R, C, FERR, BERR, WORK2

SUBROUTINE GESVX_64(FACT, [TRANSA], [N], [NRHS], A, [LDA], AF, [LDAF],
       IPIVOT, EQUED, R, C, B, [LDB], X, [LDX], RCOND, FERR,
       BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, TRANSA, EQUED
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: R, C, FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zgesvx(char fact, char transa, int n, int  nrhs,  doub-
          lecomplex  *a,  int  lda,  doublecomplex  *af, int
          ldaf, int *ipivot, char equed, double  *r,  double
          *c,  doublecomplex  *b, int ldb, doublecomplex *x,
          int  ldx,  double  *rcond,  double  *ferr,  double
          *berr, int *info);

void zgesvx_64(char fact, char transa, long  n,  long  nrhs,
          doublecomplex  *a,  long  lda,  doublecomplex *af,
          long ldaf, long *ipivot, char  equed,  double  *r,
          double  *c, doublecomplex *b, long ldb, doublecom-
          plex *x, long ldx, double  *rcond,  double  *ferr,
          double *berr, long *info);
```

PURPOSE

zgesvx uses the LU factorization to compute the solution to a complex system of linear equations
   A * X = B, where A is an N-by-N matrix and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'E', real scaling factors are computed to equilibrate
   the system:
      TRANS = 'N': diag(R)*A*diag(C)   *inv(diag(C))*X = diag(R)*B
      TRANS = 'T': (diag(R)*A*diag(C))**T *inv(diag(R))*X = diag(C)*B
      TRANS = 'C': (diag(R)*A*diag(C))**H *inv(diag(R))*X = diag(C)*B
   Whether or not the system will be equilibrated depends on the
   scaling of the matrix A, but if equilibration is used, A is
   overwritten by diag(R)*A*diag(C) and B by diag(R)*B (if TRANS='N')
   or diag(C)*B (if TRANS = 'T' or 'C').
2. If FACT = 'N' or 'E', the LU decomposition is used to factor the
   matrix A (after equilibration if FACT = 'E') as
      A = P * L * U,
   where P is a permutation matrix, L is a unit lower tri- angular
   matrix, and U is upper triangular.

3. If some U(i,i)=0, so that U is exactly singular, then the routine
   returns with INFO = i. Otherwise, the factored form of A is used
   to estimate the condition number of the matrix A. If the
   reciprocal of the condition number is less than machine precision,
   INFO = N+1 is returned as a warning, but the routine still goes on
   to solve for X and compute error bounds as described below.

4. The system of equations is solved for X using the fac- tored form
   of A.

5. Iterative refinement is applied to improve the computed solution
   matrix and calculate error bounds and backward error estimates
   for it.

6. If equilibration was used, the matrix X is premultiplied by

```
   diag(C) (if TRANS = 'N') or diag(R) (if TRANS  =  'T'  or
'C') so
   that it solves the original system before equilibration.
```

ARGUMENTS

```
FACT (input)
          Specifies whether or not the factored form of  the
          matrix A is supplied on entry, and if not, whether
          the matrix A should be equilibrated before  it  is
          factored.  = 'F':  On entry, AF and IPIVOT contain
          the factored form of A.  If EQUED is not 'N',  the
          matrix  A  has been equilibrated with scaling fac-
          tors given by R and C.  A, AF, and IPIVOT are  not
          modified.   = 'N':  The matrix A will be copied to
          AF and factored.
          = 'E':  The  matrix  A  will  be  equilibrated  if
          necessary, then copied to AF and factored.
TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input/output)
          On entry, the N-by-N matrix A.  If FACT = 'F'  and
          EQUED  is  not  'N', then A must have been equili-
          brated by the scaling factors in R and/or C.  A is
          not  modified  if  FACT = 'F' or 'N', or if FACT =
          'E' and EQUED = 'N' on exit.

          On exit, if EQUED .ne. 'N', A is  scaled  as  fol-
          lows:  EQUED = 'R':  A := diag(R) * A
          EQUED = 'C':  A := A * diag(C)
          EQUED = 'B':  A := diag(R) * A * diag(C).

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

AF (input/output)
          If FACT = 'F', then AF is an input argument and on
```

entry contains the factors L and U from the factorization A = P*L*U as computed by CGETRF. If EQUED .ne. 'N', then AF is the factored form of the equilibrated matrix A.

If FACT = 'N', then AF is an output argument and on exit returns the factors L and U from the factorization A = P*L*U of the original matrix A.

If FACT = 'E', then AF is an output argument and on exit returns the factors L and U from the factorization A = P*L*U of the equilibrated matrix A (see the description of A for the form of the equilibrated matrix).

LDAF (input)

The leading dimension of the array AF. LDAF >= max(1,N).

IPIVOT (input or output)

If FACT = 'F', then IPIVOT is an input argument and on entry contains the pivot indices from the factorization A = P*L*U as computed by CGETRF; row i of the matrix was interchanged with row IPIVOT(i).

If FACT = 'N', then IPIVOT is an output argument and on exit contains the pivot indices from the factorization A = P*L*U of the original matrix A.

If FACT = 'E', then IPIVOT is an output argument and on exit contains the pivot indices from the factorization A = P*L*U of the equilibrated matrix A.

EQUED (input/output)

Specifies the form of equilibration that was done. = 'N': No equilibration (always true if FACT = 'N').
= 'R': Row equilibration, i.e., A has been premultiplied by diag(R). = 'C': Column equilibration, i.e., A has been postmultiplied by diag(C). = 'B': Both row and column equilibration, i.e., A has been replaced by diag(R) * A * diag(C). EQUED is an input argument if FACT = 'F'; otherwise, it is an output argument.

R (input/output)

The row scale factors for A. If EQUED = 'R' or 'B', A is multiplied on the left by diag(R); if EQUED = 'N' or 'C', R is not accessed. R is an input argument if FACT = 'F'; otherwise, R is an output argument. If FACT = 'F' and EQUED = 'R' or 'B', each element of R must be positive.

C (input/output)
The column scale factors for A. If EQUED = 'C' or 'B', A is multiplied on the right by diag(C); if EQUED = 'N' or 'R', C is not accessed. C is an input argument if FACT = 'F'; otherwise, C is an output argument. If FACT = 'F' and EQUED = 'C' or 'B', each element of C must be positive.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if EQUED = 'N', B is not modified; if TRANSA = 'N' and EQUED = 'R' or 'B', B is overwritten by diag(R)*B; if TRANSA = 'T' or 'C' and EQUED = 'C' or 'B', B is overwritten by diag(C)*B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X to the original system of equations. Note that A and B are modified on exit if EQUED .ne. 'N', and the solution to the equilibrated system is inv(diag(C))*X if TRANSA = 'N' and EQUED = 'C' or 'B', or inv(diag(R))*X if TRANSA = 'T' or 'C' and EQUED = 'R' or 'B'.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A after equilibration (if done). If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest ele-

ment in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(2*N) On exit, WORK2(1) contains the reciprocal pivot growth factor norm(A)/norm(U). The "max absolute element" norm is used. If WORK2(1) is much less than 1, then the stability of the LU factorization of the (equilibrated) matrix A could be poor. This also means that the solution X, condition estimator RCOND, and forward error bound FERR could be unreliable. If factorization fails with 0<INFO<=N, then WORK2(1) contains the reciprocal pivot growth factor for the leading INFO columns of A.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
zgetf2 - compute an LU factorization  of  a  general  m-by-n
matrix A using partial pivoting with row interchanges
```

SYNOPSIS

```
SUBROUTINE ZGETF2(M, N, A, LDA, IPIV, INFO)

DOUBLE COMPLEX A(LDA,*)
INTEGER M, N, LDA, INFO
INTEGER IPIV(*)

SUBROUTINE ZGETF2_64(M, N, A, LDA, IPIV, INFO)

DOUBLE COMPLEX A(LDA,*)
INTEGER*8 M, N, LDA, INFO
INTEGER*8 IPIV(*)
```

F95 INTERFACE

```
SUBROUTINE GETF2([M], [N], A, [LDA], IPIV, [INFO])

COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIV

SUBROUTINE GETF2_64([M], [N], A, [LDA], IPIV, [INFO])

COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIV
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zgetf2(int m, int n, doublecomplex  *a,  int  lda,  int
          *ipiv, int *info);

void zgetf2_64(long m, long n, doublecomplex *a,  long  lda,
          long *ipiv, long *info);
```

PURPOSE

zgetf2 computes an LU factorization of a general m-by-n matrix A using partial pivoting with row interchanges.

The factorization has the form
   A = P * L * U
where P is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if m > n), and U is upper triangular (upper trapezoidal if m < n).

This is the right-looking Level 2 BLAS version of the algorithm.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the m by n matrix to be factored. On exit, the factors L and U from the factorization A = P*L*U; the unit diagonal elements of L are not stored.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

IPIV (output)
The pivot indices; for 1 <= i <= min(M,N), row i of the matrix was interchanged with row IPIV(i).

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, U(k,k) is exactly zero. The factorization has been completed, but the factor U is

exactly singular, and division by zero will occur if it is used to solve a system of equations.

Contents

NAME

```
zgetrf - compute an LU factorization  of  a  general  M-by-N
matrix A using partial pivoting with row interchanges
```

SYNOPSIS

```
SUBROUTINE ZGETRF(M, N, A, LDA, IPIVOT, INFO)

DOUBLE COMPLEX A(LDA,*)
INTEGER M, N, LDA, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZGETRF_64(M, N, A, LDA, IPIVOT, INFO)

DOUBLE COMPLEX A(LDA,*)
INTEGER*8 M, N, LDA, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE GETRF([M], [N], A, [LDA], IPIVOT, [INFO])

COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE GETRF_64([M], [N], A, [LDA], IPIVOT, [INFO])

COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zgetrf(int m, int n, doublecomplex  *a,  int  lda,  int
          *ipivot, int *info);

void zgetrf_64(long m, long n, doublecomplex *a,  long  lda,
          long *ipivot, long *info);
```

PURPOSE

zgetrf computes an LU factorization of a general M-by-N matrix A using partial pivoting with row interchanges.

The factorization has the form
   A = P * L * U
where P is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if m > n), and U is upper triangular (upper trapezoidal if m < n).

This is the right-looking Level 3 BLAS version of the algorithm.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the M-by-N matrix to be factored. On exit, the factors L and U from the factorization A = P*L*U; the unit diagonal elements of L are not stored.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

IPIVOT (output)
The pivot indices; for 1 <= i <= min(M,N), row i of the matrix was interchanged with row IPIVOT(i).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, U(i,i) is exactly zero. The factorization has been completed, but the factor U

is exactly singular, and division by zero will occur if it is used to solve a system of equations.

Contents

NAME

```
zgetri - compute the inverse of a matrix using the  LU  fac-
torization computed by CGETRF
```

SYNOPSIS

```
SUBROUTINE ZGETRI(N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, LDWORK, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZGETRI_64(N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, LDWORK, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE GETRI([N], A, [LDA], IPIVOT, [WORK], [LDWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, LDWORK, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE GETRI_64([N], A, [LDA], IPIVOT, [WORK], [LDWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void zgetri(int n, doublecomplex *a, int lda,  int  *ipivot,
          int *info);

void zgetri_64(long n,  doublecomplex  *a,  long  lda,  long
          *ipivot, long *info);
```

PURPOSE

zgetri computes the inverse of a matrix using the LU factorization computed by CGETRF.

This method inverts U and then computes inv(A) by solving the system inv(A)*L = inv(U) for inv(A).

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the factors L and U from the factorization A = P*L*U as computed by CGETRF. On exit, if INFO = 0, the inverse of the original matrix A.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (input)
The pivot indices from CGETRF; for 1<=i<=N, row i of the matrix was interchanged with row IPIVOT(i).

WORK (workspace)
On exit, if INFO=0, then WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,N). For optimal performance LDWORK >= N*NB, where NB is the optimal blocksize returned by ILAENV.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first

entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, U(i,i) is exactly zero; the matrix is singular and its inverse could not be computed.

Contents

NAME

```
zgetrs - solve a system of linear equations  A * X = B, A**T
*  X  =  B,  or  A**H * X = B with a general N-by-N matrix A
using the LU factorization computed by CGETRF
```

SYNOPSIS

```
SUBROUTINE ZGETRS(TRANSA, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 TRANSA
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZGETRS_64(TRANSA, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 TRANSA
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE GETRS([TRANSA], [N], [NRHS], A, [LDA], IPIVOT, B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: TRANSA
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: N, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE GETRS_64([TRANSA], [N], [NRHS], A, [LDA], IPIVOT, B, [LDB],
       [INFO])
```

```
CHARACTER(LEN=1) :: TRANSA
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void zgetrs(char transa, int n, int nrhs, doublecomplex  *a,
          int  lda,  int *ipivot, doublecomplex *b, int ldb,
          int *info);

void zgetrs_64(char transa, long n, long nrhs, doublecomplex
          *a, long lda, long *ipivot, doublecomplex *b, long
          ldb, long *info);
```

PURPOSE

zgetrs solves a system of linear equations
   A * X = B, A**T * X = B, or A**H * X = B with a general N-by-N matrix A using the LU factorization computed by CGETRF.

ARGUMENTS

TRANSA (input)
Specifies the form of the system of equations:
= 'N': A * X = B   (No transpose)
= 'T': A**T * X = B (Transpose)
= 'C': A**H * X = B (Conjugate transpose)

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The factors L and U from the factorization A = P*L*U as computed by CGETRF.

LDA (input)
The leading dimension of the array A.   LDA >= max(1,N).

IPIVOT (input)

The pivot indices from CGETRF; for 1<=i<=N, row i of the matrix was interchanged with row IPIVOT(i).

B (input/output)
On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zggbak - form the right or left eigenvectors  of  a  complex
generalized eigenvalue problem A*x = lambda*B*x, by backward
transformation on the computed eigenvectors of the  balanced
pair of matrices output by CGGBAL
```

SYNOPSIS

```
SUBROUTINE ZGGBAK(JOB, SIDE, N, ILO, IHI, LSCALE, RSCALE, M, V, LDV,
      INFO)

CHARACTER * 1 JOB, SIDE
DOUBLE COMPLEX V(LDV,*)
INTEGER N, ILO, IHI, M, LDV, INFO
DOUBLE PRECISION LSCALE(*), RSCALE(*)

SUBROUTINE ZGGBAK_64(JOB, SIDE, N, ILO, IHI, LSCALE, RSCALE, M, V,
      LDV, INFO)

CHARACTER * 1 JOB, SIDE
DOUBLE COMPLEX V(LDV,*)
INTEGER*8 N, ILO, IHI, M, LDV, INFO
DOUBLE PRECISION LSCALE(*), RSCALE(*)
```

F95 INTERFACE

```
SUBROUTINE GGBAK(JOB, SIDE, [N], ILO, IHI, LSCALE, RSCALE, [M], V,
       [LDV], [INFO])

CHARACTER(LEN=1) :: JOB, SIDE
COMPLEX(8), DIMENSION(:,:) :: V
INTEGER :: N, ILO, IHI, M, LDV, INFO
REAL(8), DIMENSION(:) :: LSCALE, RSCALE
```

```
SUBROUTINE GGBAK_64(JOB, SIDE, [N], ILO, IHI, LSCALE, RSCALE, [M], V,
       [LDV], [INFO])

CHARACTER(LEN=1) :: JOB, SIDE
COMPLEX(8), DIMENSION(:,:) :: V
INTEGER(8) :: N, ILO, IHI, M, LDV, INFO
REAL(8), DIMENSION(:) :: LSCALE, RSCALE
```

C INTERFACE

```
#include <sunperf.h>

void zggbak(char job, char side, int n, int  ilo,  int  ihi,
          double  *lscale, double *rscale, int m, doublecom-
          plex *v, int ldv, int *info);
void zggbak_64(char job, char side, long n, long  ilo,  long
          ihi, double *lscale, double *rscale, long m, doub-
          lecomplex *v, long ldv, long *info);
```

PURPOSE

zggbak forms the right or left eigenvectors of a complex generalized eigenvalue problem A*x = lambda*B*x, by backward transformation on the computed eigenvectors of the balanced pair of matrices output by CGGBAL.

ARGUMENTS

JOB (input)
Specifies the type of backward transformation required:
= 'N': do nothing, return immediately;
= 'P': do backward transformation for permutation only;
= 'S': do backward transformation for scaling only;
= 'B': do backward transformations for both permutation and scaling. JOB must be the same as the argument JOB supplied to CGGBAL.

SIDE (input)
= 'R': V contains right eigenvectors;
= 'L': V contains left eigenvectors.

N (input) The number of rows of the matrix V. N >= 0.

ILO (input)

The integers ILO and IHI determined by CGGBAL. 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

IHI (input)
The integers ILO and IHI determined by CGGBAL. 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

LSCALE (input)
Details of the permutations and/or scaling factors applied to the left side of A and B, as returned by CGGBAL.

RSCALE (input)
Details of the permutations and/or scaling factors applied to the right side of A and B, as returned by CGGBAL.

M (input) The number of columns of the matrix V. M >= 0.

V (input/output)
On entry, the matrix of right or left eigenvectors to be transformed, as returned by CTGEVC. On exit, V is overwritten by the transformed eigenvectors.

LDV (input)
The leading dimension of the matrix V. LDV >= max(1,N).

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

See R.C. Ward, Balancing the generalized eigenvalue problem, SIAM J. Sci. Stat. Comp. 2 (1981), 141-152.

Contents

NAME

```
zggbal - balance a pair of general complex matrices (A,B)
```

SYNOPSIS

```
SUBROUTINE ZGGBAL(JOB, N, A, LDA, B, LDB, ILO, IHI, LSCALE, RSCALE,
      WORK, INFO)

CHARACTER * 1 JOB
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, LDA, LDB, ILO, IHI, INFO
DOUBLE PRECISION LSCALE(*), RSCALE(*), WORK(*)

SUBROUTINE ZGGBAL_64(JOB, N, A, LDA, B, LDB, ILO, IHI, LSCALE,
      RSCALE, WORK, INFO)

CHARACTER * 1 JOB
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, LDA, LDB, ILO, IHI, INFO
DOUBLE PRECISION LSCALE(*), RSCALE(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGBAL(JOB, [N], A, [LDA], B, [LDB], ILO, IHI, LSCALE,
       RSCALE, [WORK], [INFO])

CHARACTER(LEN=1) :: JOB
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: N, LDA, LDB, ILO, IHI, INFO
REAL(8), DIMENSION(:) :: LSCALE, RSCALE, WORK

SUBROUTINE GGBAL_64(JOB, [N], A, [LDA], B, [LDB], ILO, IHI, LSCALE,
       RSCALE, [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOB
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: N, LDA, LDB, ILO, IHI, INFO
REAL(8), DIMENSION(:) :: LSCALE, RSCALE, WORK
```

C INTERFACE

```
#include <sunperf.h>

void zggbal(char job, int  n,  doublecomplex  *a,  int  lda,
          doublecomplex  *b,  int  ldb,  int *ilo, int *ihi,
          double *lscale, double *rscale, int *info);

void zggbal_64(char job, long n, doublecomplex *a, long lda,
          doublecomplex  *b, long ldb, long *ilo, long *ihi,
          double *lscale, double *rscale, long *info);
```

PURPOSE

zggbal balances a pair of general complex matrices (A,B). This involves, first, permuting A and B by similarity transformations to isolate eigenvalues in the first 1 to ILO$-$1 and last IHI+1 to N elements on the diagonal; and second, applying a diagonal similarity transformation to rows and columns ILO to IHI to make the rows and columns as close in norm as possible. Both steps are optional.

Balancing may reduce the 1-norm of the matrices, and improve the accuracy of the computed eigenvalues and/or eigenvectors in the generalized eigenvalue problem A*x = lambda*B*x.

ARGUMENTS

JOB (input)
Specifies the operations to be performed on A and B:
= 'N': none: simply set ILO = 1, IHI = N, LSCALE(I) = 1.0 and RSCALE(I) = 1.0 for i=1,...,N;
= 'P': permute only;
= 'S': scale only;
= 'B': both permute and scale.

N (input) The order of the matrices A and B. N >= 0.

A (input/output)
On entry, the input matrix A. On exit, A is overwritten by the balanced matrix. If JOB = 'N',

A is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input) On entry, the input matrix B. On exit, B is overwritten by the balanced matrix. If JOB = 'N', B is not referenced.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

ILO (output)
ILO and IHI are set to integers such that on exit A(i,j) = 0 and B(i,j) = 0 if i > j and j = 1,...,ILO-1 or i = IHI+1,...,N. If JOB = 'N' or 'S', ILO = 1 and IHI = N.

IHI (output)
ILO and IHI are set to integers such that on exit A(i,j) = 0 and B(i,j) = 0 if i > j and j = 1,...,ILO-1 or i = IHI+1,...,N.

LSCALE (input)
Details of the permutations and scaling factors applied to the left side of A and B. If P(j) is the index of the row interchanged with row j, and D(j) is the scaling factor applied to row j, then LSCALE(j) = P(j) for J = 1,...,ILO-1 = D(j) for J = ILO,...,IHI = P(j) for J = IHI+1,...,N. The order in which the interchanges are made is N to IHI+1, then 1 to ILO-1.

RSCALE (input)
Details of the permutations and scaling factors applied to the right side of A and B. If P(j) is the index of the column interchanged with column j, and D(j) is the scaling factor applied to column j, then RSCALE(j) = P(j) for J = 1,...,ILO-1 = D(j) for J = ILO,...,IHI = P(j) for J = IHI+1,...,N. The order in which the interchanges are made is N to IHI+1, then 1 to ILO-1.

WORK (workspace)
dimension(6*N)

INFO (output)

```
              = 0:  successful exit
              < 0:  if INFO = -i, the i-th argument had an ille-
              gal value.
```

FURTHER DETAILS

```
     See R.C. WARD, Balancing the generalized eigenvalue problem,
                    SIAM J. Sci. Stat. Comp. 2 (1981), 141-152.
```

Contents

NAME

```
zgges - compute for a pair of  N-by-N  complex  nonsymmetric
matrices (A,B), the generalized eigenvalues, the generalized
complex Schur form (S, T), and optionally left and/or  right
Schur vectors (VSL and VSR)
```

SYNOPSIS

```
SUBROUTINE ZGGES(JOBVSL, JOBVSR, SORT, DELZTG, N, A, LDA, B, LDB,
      SDIM, ALPHA, BETA, VSL, LDVSL, VSR, LDVSR, WORK, LWORK, RWORK,
      BWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR, SORT
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),   ALPHA(*),   BETA(*),
VSL(LDVSL,*), VSR(LDVSR,*), WORK(*)
INTEGER N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, INFO
LOGICAL DELZTG
LOGICAL BWORK(*)
DOUBLE PRECISION RWORK(*)

SUBROUTINE ZGGES_64(JOBVSL, JOBVSR, SORT, DELZTG, N, A, LDA, B, LDB,
      SDIM, ALPHA, BETA, VSL, LDVSL, VSR, LDVSR, WORK, LWORK, RWORK,
      BWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR, SORT
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),   ALPHA(*),   BETA(*),
VSL(LDVSL,*), VSR(LDVSR,*), WORK(*)
INTEGER*8 N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, INFO
LOGICAL*8 DELZTG
LOGICAL*8 BWORK(*)
DOUBLE PRECISION RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGES(JOBVSL, JOBVSR, SORT, [DELZTG], [N], A, [LDA], B, [LDB],
       SDIM, ALPHA, BETA, VSL, [LDVSL], VSR, [LDVSR], [WORK], [LWORK],
       [RWORK], [BWORK], [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR, SORT
COMPLEX(8), DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, VSL, VSR
INTEGER :: N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, INFO
LOGICAL :: DELZTG
LOGICAL, DIMENSION(:) :: BWORK
REAL(8), DIMENSION(:) :: RWORK

SUBROUTINE GGES_64(JOBVSL, JOBVSR, SORT, [DELZTG], [N], A, [LDA], B,
       [LDB], SDIM, ALPHA, BETA, VSL, [LDVSL], VSR, [LDVSR], [WORK],
       [LWORK], [RWORK], [BWORK], [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR, SORT
COMPLEX(8), DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, VSL, VSR
INTEGER(8) :: N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, INFO
LOGICAL(8) :: DELZTG
LOGICAL(8), DIMENSION(:) :: BWORK
REAL(8), DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>

void   zgges(char   jobvsl,   char   jobvsr,   char    sort,
          int(*delztg)(doublecomplex,doublecomplex),  int n,
          doublecomplex *a, int lda, doublecomplex  *b,  int
          ldb,  int  *sdim, doublecomplex *alpha, doublecom-
          plex *beta, doublecomplex *vsl, int  ldvsl,  doub-
          lecomplex *vsr, int ldvsr, int *info);

void  zgges_64(char  jobvsl,   char   jobvsr,   char   sort,
          long(*delztg)(doublecomplex,doublecomplex),   long
          n, doublecomplex *a, long lda,  doublecomplex  *b,
          long  ldb, long *sdim, doublecomplex *alpha, doub-
          lecomplex *beta, doublecomplex *vsl,  long  ldvsl,
          doublecomplex *vsr, long ldvsr, long *info);
```

PURPOSE

zgges computes for a pair of N-by-N complex nonsymmetric matrices (A,B), the generalized eigenvalues, the generalized complex Schur form (S, T), and optionally left and/or right Schur vectors (VSL and VSR). This gives the generalized Schur factorization

```
        (A,B) = ( (VSL)*S*(VSR)**H, (VSL)*T*(VSR)**H )
```

where (VSR)**H is the conjugate-transpose of VSR.

Optionally, it also orders the eigenvalues so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper triangular matrix S and the upper triangular matrix T. The leading columns of VSL and VSR then form an unitary basis for the corresponding left and right eigenspaces (deflating subspaces).

(If only the generalized eigenvalues are needed, use the driver CGGEV instead, which is faster.)

A generalized eigenvalue for a pair of matrices (A,B) is a scalar w or a ratio alpha/beta = w, such that A - w*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0, and even for both being zero.

A pair of matrices (S,T) is in generalized complex Schur form if S and T are upper triangular and, in addition, the diagonal elements of T are non-negative real numbers.

ARGUMENTS

JOBVSL (input)
= 'N': do not compute the left Schur vectors;
= 'V': compute the left Schur vectors.

JOBVSR (input)
= 'N': do not compute the right Schur vectors;
= 'V': compute the right Schur vectors.

SORT (input)
Specifies whether or not to order the eigenvalues on the diagonal of the generalized Schur form. = 'N': Eigenvalues are not ordered;
= 'S': Eigenvalues are ordered (see DELZTG).

DELZTG (input)
DELZTG must be declared EXTERNAL in the calling subroutine. If SORT = 'N', DELZTG is not referenced. If SORT = 'S', DELZTG is used to select eigenvalues to sort to the top left of the Schur form. An eigenvalue ALPHA(j)/BETA(j) is selected if DELZTG(ALPHA(j),BETA(j)) is true.

Note that a selected complex eigenvalue may no longer satisfy DELZTG(ALPHA(j),BETA(j)) = .TRUE. after ordering, since ordering may change the value of complex eigenvalues (especially if the eigenvalue is ill-conditioned), in this case INFO is set to N+2 (See INFO below).

N (input) The order of the matrices A, B, VSL, and VSR. N >= 0.

A (input/output)
On entry, the first of the pair of matrices. On exit, A has been overwritten by its generalized Schur form S.

LDA (input)
The leading dimension of A. LDA >= max(1,N).

B (input/output)
On entry, the second of the pair of matrices. On exit, B has been overwritten by its generalized Schur form T.

LDB (input)
The leading dimension of B. LDB >= max(1,N).

SDIM (output)
If SORT = 'N', SDIM = 0. If SORT = 'S', SDIM = number of eigenvalues (after sorting) for which DELZTG is true.

ALPHA (output)
On exit, ALPHA(j)/BETA(j), j=1,...,N, will be the generalized eigenvalues. ALPHA(j), j=1,...,N and BETA(j), j=1,...,N are the diagonals of the complex Schur form (A,B) output by CGGES. The BETA(j) will be non-negative real.

Note: the quotients ALPHA(j)/BETA(j) may easily over- or underflow, and BETA(j) may even be zero. Thus, the user should avoid naively computing the ratio alpha/beta. However, ALPHA will be always less than and usually comparable with norm(A) in magnitude, and BETA always less than and usually comparable with norm(B).

BETA (output)
See description of ALPHA.

VSL (input)
If JOBVSL = 'V', VSL will contain the left Schur

```
            vectors.  Not referenced if JOBVSL = 'N'.

LDVSL (input)
            The leading dimension of the matrix VSL. LDVSL  >=
            1, and if JOBVSL = 'V', LDVSL >= N.

VSR (input)
            If JOBVSR = 'V', VSR will contain the right  Schur
            vectors.  Not referenced if JOBVSR = 'N'.

LDVSR (input)
            The leading dimension of the matrix VSR. LDVSR  >=
            1, and if JOBVSR = 'V', LDVSR >= N.

WORK (workspace)
            On exit, if INFO = 0, WORK(1) returns the  optimal
            LWORK.

LWORK (input)
            The  dimension  of  the  array  WORK.   LWORK   >=
            max(1,2*N).  For good performance, LWORK must gen-
            erally be larger.

            If LWORK = -1, then a workspace query is  assumed;
            the  routine  only  calculates the optimal size of
            the WORK array, returns this value  as  the  first
            entry  of  the  WORK  array,  and no error message
            related to LWORK is issued by XERBLA.

RWORK (workspace)
            dimension(8*N)

BWORK (workspace)
            dimension(N) Not referenced if SORT = 'N'.

INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value.
            =1,...,N:  The QZ iteration failed.  (A,B) are not
            in  Schur form, but ALPHA(j) and BETA(j) should be
            correct for j=INFO+1,...,N.   >  N:   =N+1:  other
            than QZ iteration failed in CHGEQZ
            =N+2: after reordering, roundoff changed values of
            some  complex  eigenvalues  so that leading eigen-
            values in the Generalized  Schur  form  no  longer
            satisfy  DELZTG=.TRUE.   This could also be caused
            due  to  scaling.   =N+3:  reordering  falied   in
            CTGSEN.
```

Contents

NAME

```
zggesx - compute for a pair of N-by-N  complex  nonsymmetric
matrices  (A,B),  the  generalized  eigenvalues, the complex
Schur form (S,T),
```

SYNOPSIS

```
SUBROUTINE ZGGESX(JOBVSL, JOBVSR, SORT, DELCTG, SENSE, N, A, LDA, B,
      LDB, SDIM, ALPHA, BETA, VSL, LDVSL, VSR, LDVSR, RCONDE, RCONDV,
      WORK, LWORK, RWORK, IWORK, LIWORK, BWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR, SORT, SENSE
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),   ALPHA(*),   BETA(*),
VSL(LDVSL,*), VSR(LDVSR,*), WORK(*)
INTEGER N, LDA, LDB, SDIM, LDVSL, LDVSR, LWORK, LIWORK, INFO
INTEGER IWORK(*)
LOGICAL DELCTG
LOGICAL BWORK(*)
DOUBLE PRECISION RCONDE(*), RCONDV(*), RWORK(*)

SUBROUTINE ZGGESX_64(JOBVSL, JOBVSR, SORT, DELCTG, SENSE, N, A, LDA,
      B, LDB, SDIM, ALPHA, BETA, VSL, LDVSL, VSR, LDVSR, RCONDE,
      RCONDV, WORK, LWORK, RWORK, IWORK, LIWORK, BWORK, INFO)

CHARACTER * 1 JOBVSL, JOBVSR, SORT, SENSE
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),   ALPHA(*),   BETA(*),
VSL(LDVSL,*), VSR(LDVSR,*), WORK(*)
INTEGER*8 N, LDA, LDB, SDIM, LDVSL,  LDVSR,  LWORK,  LIWORK,
INFO
INTEGER*8 IWORK(*)
LOGICAL*8 DELCTG
LOGICAL*8 BWORK(*)
DOUBLE PRECISION RCONDE(*), RCONDV(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGESX(JOBVSL, JOBVSR, SORT, [DELCTG], SENSE, [N], A, [LDA],
       B, [LDB], SDIM, ALPHA, BETA, VSL, [LDVSL], VSR, [LDVSR], RCONDE,
       RCONDV, [WORK], [LWORK], [RWORK], [IWORK], [LIWORK], [BWORK],
       [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR, SORT, SENSE
COMPLEX(8), DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, VSL, VSR
INTEGER :: N, LDA, LDB, SDIM, LDVSL, LDVSR,  LWORK,  LIWORK,
INFO
INTEGER, DIMENSION(:) :: IWORK
LOGICAL :: DELCTG
LOGICAL, DIMENSION(:) :: BWORK
REAL(8), DIMENSION(:) :: RCONDE, RCONDV, RWORK
SUBROUTINE GGESX_64(JOBVSL, JOBVSR, SORT, [DELCTG], SENSE, [N], A, [LDA],
       B, [LDB], SDIM, ALPHA, BETA, VSL, [LDVSL], VSR, [LDVSR], RCONDE,
       RCONDV, [WORK], [LWORK], [RWORK], [IWORK], [LIWORK], [BWORK],
       [INFO])

CHARACTER(LEN=1) :: JOBVSL, JOBVSR, SORT, SENSE
COMPLEX(8), DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, VSL, VSR
INTEGER(8) ::  N,  LDA,  LDB,  SDIM,  LDVSL,  LDVSR,  LWORK,
LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
LOGICAL(8) :: DELCTG
LOGICAL(8), DIMENSION(:) :: BWORK
REAL(8), DIMENSION(:) :: RCONDE, RCONDV, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void   zggesx(char   jobvsl,   char   jobvsr,   char   sort,
          int(*delctg)(doublecomplex,doublecomplex),    char
          sense, int n, doublecomplex  *a,  int  lda,  doub-
          lecomplex  *b,  int  ldb, int *sdim, doublecomplex
          *alpha, doublecomplex *beta,  doublecomplex  *vsl,
          int  ldvsl,  doublecomplex *vsr, int ldvsr, double
          *rconde, double *rcondv, int *info);

void  zggesx_64(char  jobvsl,  char   jobvsr,   char   sort,
          long(*delctg)(doublecomplex,doublecomplex),   char
          sense, long n, doublecomplex *a, long  lda,  doub-
          lecomplex  *b, long ldb, long *sdim, doublecomplex
          *alpha, doublecomplex *beta,  doublecomplex  *vsl,
          long ldvsl, doublecomplex *vsr, long ldvsr, double
          *rconde, double *rcondv, long *info);
```

PURPOSE

zggesx computes for a pair of N-by-N complex nonsymmetric matrices (A,B), the generalized eigenvalues, the complex Schur form (S,T), and, optionally, the left and/or right matrices of Schur vectors (VSL and VSR). This gives the generalized Schur factorization A,B) = ((VSL) S (VSR)**H, (VSL) T (VSR)**H)

where (VSR)**H is the conjugate-transpose of VSR.

Optionally, it also orders the eigenvalues so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper triangular matrix S and the upper triangular matrix T; computes a reciprocal condition number for the average of the selected eigenvalues (RCONDE); and computes a reciprocal condition number for the right and left deflating subspaces corresponding to the selected eigenvalues (RCONDV). The leading columns of VSL and VSR then form an orthonormal basis for the corresponding left and right eigenspaces (deflating subspaces).

A generalized eigenvalue for a pair of matrices (A,B) is a scalar w or a ratio alpha/beta = w, such that A - w*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0 or for both being zero.

A pair of matrices (S,T) is in generalized complex Schur form if T is upper triangular with non-negative diagonal and S is upper triangular.

ARGUMENTS

JOBVSL (input)
  = 'N': do not compute the left Schur vectors;
  = 'V': compute the left Schur vectors.

JOBVSR (input)
  = 'N': do not compute the right Schur vectors;
  = 'V': compute the right Schur vectors.

SORT (input)
  Specifies whether or not to order the eigenvalues on the diagonal of the generalized Schur form. = 'N': Eigenvalues are not ordered;
  = 'S': Eigenvalues are ordered (see DELCTG).

DELCTG (input)
DELCTG must be declared EXTERNAL in the calling subroutine. If SORT = 'N', DELCTG is not referenced. If SORT = 'S', DELCTG is used to select eigenvalues to sort to the top left of the Schur form. Note that a selected complex eigenvalue may no longer satisfy DELCTG(ALPHA(j),BETA(j)) = .TRUE. after ordering, since ordering may change the value of complex eigenvalues (especially if the eigenvalue is ill-conditioned), in this case INFO is set to N+3 see INFO below).

SENSE (input)
Determines which reciprocal condition numbers are computed. = 'N' : None are computed;
= 'E' : Computed for average of selected eigenvalues only;
= 'V' : Computed for selected deflating subspaces only;
= 'B' : Computed for both. If SENSE = 'E', 'V', or 'B', SORT must equal 'S'.

N (input) The order of the matrices A, B, VSL, and VSR. N >= 0.

A (input/output)
On entry, the first of the pair of matrices. On exit, A has been overwritten by its generalized Schur form S.

LDA (input)
The leading dimension of A. LDA >= max(1,N).

B (input/output)
On entry, the second of the pair of matrices. On exit, B has been overwritten by its generalized Schur form T.

LDB (input)
The leading dimension of B. LDB >= max(1,N).

SDIM (output)
If SORT = 'N', SDIM = 0. If SORT = 'S', SDIM = number of eigenvalues (after sorting) for which DELCTG is true.

ALPHA (output)
On exit, ALPHA(j)/BETA(j), j=1,...,N, will be the generalized eigenvalues. ALPHA(j) and

BETA(j),j=1,...,N are the diagonals of the complex Schur form (S,T). BETA(j) will be non-negative real.

Note: the quotients ALPHA(j)/BETA(j) may easily over- or underflow, and BETA(j) may even be zero. Thus, the user should avoid naively computing the ratio alpha/beta. However, ALPHA will be always less than and usually comparable with norm(A) in magnitude, and BETA always less than and usually comparable with norm(B).

BETA (output)

See description of ALPHA.

VSL (input)

If JOBVSL = 'V', VSL will contain the left Schur vectors. Not referenced if JOBVSL = 'N'.

LDVSL (input)

The leading dimension of the matrix VSL. LDVSL >=1, and if JOBVSL = 'V', LDVSL >= N.

VSR (input)

If JOBVSR = 'V', VSR will contain the right Schur vectors. Not referenced if JOBVSR = 'N'.

LDVSR (input)

The leading dimension of the matrix VSR. LDVSR >= 1, and if JOBVSR = 'V', LDVSR >= N.

RCONDE (output)

If SENSE = 'E' or 'B', RCONDE(1) and RCONDE(2) contain the reciprocal condition numbers for the average of the selected eigenvalues. Not referenced if SENSE = 'N' or 'V'.

RCONDV (output)

If SENSE = 'V' or 'B', RCONDV(1) and RCONDV(2) contain the reciprocal condition number for the selected deflating subspaces. Not referenced if SENSE = 'N' or 'E'.

WORK (workspace)

On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)

The dimension of the array WORK. LWORK >= 2*N. If SENSE = 'E', 'V', or 'B', LWORK >= MAX(2*N, 2*SDIM*(N-SDIM)).

RWORK (workspace)
dimension(8*N) Real workspace.

IWORK (workspace/output)
Not referenced if SENSE = 'N'. On exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of the array WORK. LIWORK >= N+2.

BWORK (workspace)
dimension(N) Not referenced if SORT = 'N'.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
= 1,...,N: The QZ iteration failed. (A,B) are not in Schur form, but ALPHA(j) and BETA(j) should be correct for j=INFO+1,...,N. > N: =N+1: other than QZ iteration failed in CHGEQZ
=N+2: after reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the Generalized Schur form no longer satisfy DELCTG=.TRUE. This could also be caused due to scaling. =N+3: reordering failed in CTGSEN.

Contents

NAME

```
zggev - compute for a pair of  N-by-N  complex  nonsymmetric
matrices (A,B), the generalized eigenvalues, and optionally,
the left and/or right generalized eigenvectors
```

SYNOPSIS

```
SUBROUTINE ZGGEV(JOBVL, JOBVR, N, A, LDA, B, LDB, ALPHA, BETA, VL,
      LDVL, VR, LDVR, WORK, LWORK, RWORK, INFO)

CHARACTER * 1 JOBVL, JOBVR
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),   ALPHA(*),   BETA(*),
VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER N, LDA, LDB, LDVL, LDVR, LWORK, INFO
DOUBLE PRECISION RWORK(*)

SUBROUTINE ZGGEV_64(JOBVL, JOBVR, N, A, LDA, B, LDB, ALPHA, BETA, VL,
      LDVL, VR, LDVR, WORK, LWORK, RWORK, INFO)

CHARACTER * 1 JOBVL, JOBVR
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),   ALPHA(*),   BETA(*),
VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER*8 N, LDA, LDB, LDVL, LDVR, LWORK, INFO
DOUBLE PRECISION RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGEV(JOBVL, JOBVR, [N], A, [LDA], B, [LDB], ALPHA, BETA,
       VL, [LDVL], VR, [LDVR], [WORK], [LWORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: JOBVL, JOBVR
COMPLEX(8), DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, VL, VR
INTEGER :: N, LDA, LDB, LDVL, LDVR, LWORK, INFO
```

```
REAL(8), DIMENSION(:) :: RWORK

SUBROUTINE GGEV_64(JOBVL, JOBVR, [N], A, [LDA], B, [LDB], ALPHA,
       BETA, VL, [LDVL], VR, [LDVR], [WORK], [LWORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: JOBVL, JOBVR
COMPLEX(8), DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, VL, VR
INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, LWORK, INFO
REAL(8), DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>
void zggev(char jobvl, char jobvr, int n, doublecomplex  *a,
          int  lda, doublecomplex *b, int ldb, doublecomplex
          *alpha, doublecomplex  *beta,  doublecomplex  *vl,
          int ldvl, doublecomplex *vr, int ldvr, int *info);

void zggev_64(char jobvl, char jobvr, long n,  doublecomplex
          *a,  long  lda,  doublecomplex *b, long ldb, doub-
          lecomplex *alpha, doublecomplex *beta,  doublecom-
          plex *vl, long ldvl, doublecomplex *vr, long ldvr,
          long *info);
```

PURPOSE

zggev computes for a pair of N-by-N complex nonsymmetric matrices (A,B), the generalized eigenvalues, and optionally, the left and/or right generalized eigenvectors.

A generalized eigenvalue for a pair of matrices (A,B) is a scalar lambda or a ratio alpha/beta = lambda, such that A - lambda*B is singular. It is usually represented as the pair (alpha,beta), as there is a reasonable interpretation for beta=0, and even for both being zero.

The right generalized eigenvector v(j) corresponding to the generalized eigenvalue lambda(j) of (A,B) satisfies

A * v(j) = lambda(j) * B * v(j).

The left generalized eigenvector u(j) corresponding to the generalized eigenvalues lambda(j) of (A,B) satisfies

u(j)**H * A = lambda(j) * u(j)**H * B

where u(j)**H is the conjugate-transpose of u(j).

ARGUMENTS

```
     JOBVL (input)
               = 'N':  do not compute the left generalized eigen-
               vectors;
               = 'V':  compute the left generalized eigenvectors.

     JOBVR (input)
               = 'N':   do  not  compute  the  right  generalized
               eigenvectors;
               = 'V':  compute the  right  generalized  eigenvec-
               tors.

     N (input) The order of the matrices A, B, VL, and VR.  N  >=
               0.

     A (input/output)
               On entry, the matrix A  in  the  pair  (A,B).   On
               exit, A has been overwritten.

     LDA (input)
               The leading dimension of A.  LDA >= max(1,N).

     B (input/output)
               On entry, the matrix B  in  the  pair  (A,B).   On
               exit, B has been overwritten.

     LDB (input)
               The leading dimension of B.  LDB >= max(1,N).

     ALPHA (output)
               On exit, ALPHA(j)/BETA(j), j=1,...,N, will be  the
               generalized eigenvalues.

               Note: the quotients  ALPHA(j)/BETA(j)  may  easily
               over-  or underflow, and BETA(j) may even be zero.
               Thus, the user should avoid naively computing  the
               ratio  alpha/beta.   However, ALPHA will be always
               less than and usually comparable with  norm(A)  in
               magnitude,  and  BETA always less than and usually
               comparable with norm(B).

     BETA (output)
               See description of ALPHA.

     VL (output)
               If JOBVL = 'V', the left generalized  eigenvectors
               u(j)  are  stored one after another in the columns
               of VL, in the same  order  as  their  eigenvalues.
```

```
          Each  eigenvector  will  be  scaled so the largest
          component will have  abs(real  part)  +  abs(imag.
          part) = 1.  Not referenced if JOBVL = 'N'.

LDVL (input)
          The leading dimension of the matrix VL. LDVL >= 1,
          and if JOBVL = 'V', LDVL >= N.
VR (output)
          If JOBVR = 'V', the right generalized eigenvectors
          v(j)  are  stored one after another in the columns
          of VR, in the same  order  as  their  eigenvalues.
          Each  eigenvector  will  be  scaled so the largest
          component will have  abs(real  part)  +  abs(imag.
          part) = 1.  Not referenced if JOBVR = 'N'.

LDVR (input)
          The leading dimension of the matrix VR. LDVR >= 1,
          and if JOBVR = 'V', LDVR >= N.

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The  dimension  of  the  array  WORK.   LWORK   >=
          max(1,2*N).  For good performance, LWORK must gen-
          erally be larger.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

RWORK (workspace)
          dimension(8*N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value.
          =1,...,N:  The QZ iteration failed.  No  eigenvec-
          tors   have  been  calculated,  but  ALPHA(j)  and
          BETA(j) should be correct for  j=INFO+1,...,N.   >
          N:   =N+1:  other  then  QZ  iteration  failed  in
          SHGEQZ,
          =N+2: error return from STGEVC.
```

Contents

NAME

```
zggevx - compute for a pair of N-by-N  complex  nonsymmetric
matrices  (A,B) the generalized eigenvalues, and optionally,
the left and/or right generalized eigenvectors
```

SYNOPSIS

```
SUBROUTINE ZGGEVX(BALANC, JOBVL, JOBVR, SENSE, N, A, LDA, B, LDB,
      ALPHA, BETA, VL, LDVL, VR, LDVR, ILO, IHI, LSCALE, RSCALE, ABNRM,
      BBNRM, RCONDE, RCONDV, WORK, LWORK, RWORK, IWORK, BWORK, INFO)

CHARACTER * 1 BALANC, JOBVL, JOBVR, SENSE
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),   ALPHA(*),   BETA(*),
VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER N, LDA, LDB, LDVL, LDVR, ILO, IHI, LWORK, INFO
INTEGER IWORK(*)
LOGICAL BWORK(*)
DOUBLE PRECISION ABNRM, BBNRM
DOUBLE PRECISION LSCALE(*), RSCALE(*), RCONDE(*), RCONDV(*),
RWORK(*)

SUBROUTINE ZGGEVX_64(BALANC, JOBVL, JOBVR, SENSE, N, A, LDA, B, LDB,
      ALPHA, BETA, VL, LDVL, VR, LDVR, ILO, IHI, LSCALE, RSCALE, ABNRM,
      BBNRM, RCONDE, RCONDV, WORK, LWORK, RWORK, IWORK, BWORK, INFO)

CHARACTER * 1 BALANC, JOBVL, JOBVR, SENSE
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),   ALPHA(*),   BETA(*),
VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER*8 N, LDA, LDB, LDVL, LDVR, ILO, IHI, LWORK, INFO
INTEGER*8 IWORK(*)
LOGICAL*8 BWORK(*)
DOUBLE PRECISION ABNRM, BBNRM
```

```
DOUBLE PRECISION LSCALE(*), RSCALE(*), RCONDE(*), RCONDV(*),
RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGEVX(BALANC, JOBVL, JOBVR, SENSE, [N], A, [LDA], B, [LDB],
       ALPHA, BETA, VL, [LDVL], VR, [LDVR], ILO, IHI, LSCALE, RSCALE,
       ABNRM, BBNRM, RCONDE, RCONDV, [WORK], [LWORK], [RWORK], [IWORK],
       [BWORK], [INFO])

CHARACTER(LEN=1) :: BALANC, JOBVL, JOBVR, SENSE
COMPLEX(8), DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, VL, VR
INTEGER :: N, LDA, LDB, LDVL, LDVR, ILO, IHI, LWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
LOGICAL, DIMENSION(:) :: BWORK
REAL(8) :: ABNRM, BBNRM
REAL(8), DIMENSION(:) ::  LSCALE,  RSCALE,  RCONDE,  RCONDV,
RWORK
SUBROUTINE GGEVX_64(BALANC, JOBVL, JOBVR, SENSE, [N], A, [LDA], B,
       [LDB], ALPHA, BETA, VL, [LDVL], VR, [LDVR], ILO, IHI, LSCALE,
       RSCALE, ABNRM, BBNRM, RCONDE, RCONDV, [WORK], [LWORK], [RWORK],
       [IWORK], [BWORK], [INFO])

CHARACTER(LEN=1) :: BALANC, JOBVL, JOBVR, SENSE
COMPLEX(8), DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, VL, VR
INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, ILO, IHI, LWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
LOGICAL(8), DIMENSION(:) :: BWORK
REAL(8) :: ABNRM, BBNRM
REAL(8), DIMENSION(:) ::  LSCALE,  RSCALE,  RCONDE,  RCONDV,
RWORK
```

C INTERFACE

```
#include <sunperf.h>

void zggevx(char balanc, char jobvl, char jobvr, char sense,
          int  n,  doublecomplex  *a, int lda, doublecomplex
          *b, int ldb, doublecomplex  *alpha,  doublecomplex
          *beta,  doublecomplex *vl, int ldvl, doublecomplex
          *vr, int ldvr, int *ilo, int *ihi, double *lscale,
          double *rscale, double *abnrm, double *bbnrm, dou-
          ble *rconde, double *rcondv, int *info);

void zggevx_64(char balanc, char  jobvl,  char  jobvr,  char
          sense,  long  n, doublecomplex *a, long lda, doub-
          lecomplex  *b,  long  ldb,  doublecomplex  *alpha,
          doublecomplex *beta, doublecomplex *vl, long ldvl,
          doublecomplex *vr,  long  ldvr,  long  *ilo,  long
          *ihi,   double  *lscale,  double  *rscale,  double
```

```
          *abnrm,  double  *bbnrm,  double  *rconde,  double
          *rcondv, long *info);
```

PURPOSE

```
zggevx computes for a pair of  N-by-N  complex  nonsymmetric
matrices  (A,B) the generalized eigenvalues, and optionally,
the left and/or right generalized eigenvectors.

Optionally, it also computes a balancing  transformation  to
improve the conditioning of the eigenvalues and eigenvectors
(ILO, IHI, LSCALE, RSCALE,  ABNRM,  and  BBNRM),  reciprocal
condition numbers for the eigenvalues (RCONDE), and recipro-
cal condition numbers for the right eigenvectors (RCONDV).

A generalized eigenvalue for a pair of matrices (A,B)  is  a
scalar  lambda or a ratio alpha/beta = lambda, such that A -
lambda*B is singular. It is usually represented as the  pair
(alpha,beta),  as  there  is a reasonable interpretation for
beta=0, and even for both being zero.

The right eigenvector v(j) corresponding to  the  eigenvalue
lambda(j) of (A,B) satisfies
                 A * v(j) = lambda(j) * B * v(j) .
The left eigenvector u(j) corresponding  to  the  eigenvalue
lambda(j) of (A,B) satisfies
                 u(j)**H * A  = lambda(j) * u(j)**H * B.
where u(j)**H is the conjugate-transpose of u(j).
```

ARGUMENTS

```
BALANC (input)
          Specifies the balance option to be performed:
          = 'N':  do not diagonally scale or permute;
          = 'P':  permute only;
          = 'S':  scale only;
          = 'B':  both permute and scale.  Computed recipro-
          cal  condition  numbers  will  be for the matrices
          after permuting and/or balancing.  Permuting  does
          not  change  condition  numbers  (in  exact arith-
          metic), but balancing does.

JOBVL (input)
          = 'N':  do not compute the left generalized eigen-
          vectors;
          = 'V':  compute the left generalized eigenvectors.
```

```
JOBVR (input)
          = 'N':   do  not  compute  the  right  generalized
          eigenvectors;
          = 'V':  compute the  right  generalized  eigenvec-
          tors.

SENSE (input)
          Determines which reciprocal condition numbers  are
          computed.  = 'N': none are computed;
          = 'E': computed for eigenvalues only;
          = 'V': computed for eigenvectors only;
          = 'B': computed for eigenvalues and eigenvectors.

N (input) The order of the matrices A, B, VL, and VR.  N  >=
          0.
A (input/output)
          On entry, the matrix A  in  the  pair  (A,B).   On
          exit,  A  has  been  overwritten.  If JOBVL='V' or
          JOBVR='V' or both, then A contains the first  part
          of  the  complex Schur form of the "balanced" ver-
          sions of the input A and B.

LDA (input)
          The leading dimension of A.  LDA >= max(1,N).

B (input/output)
          On entry, the matrix B  in  the  pair  (A,B).   On
          exit,  B  has  been  overwritten.  If JOBVL='V' or
          JOBVR='V' or both, then B contains the second part
          of  the  complex Schur form of the "balanced" ver-
          sions of the input A and B.

LDB (input)
          The leading dimension of B.  LDB >= max(1,N).

ALPHA (output)
          On exit, ALPHA(j)/BETA(j), j=1,...,N, will be  the
          generalized eigenvalues.

          Note: the quotient ALPHA(j)/BETA(j) )  may  easily
          over-  or underflow, and BETA(j) may even be zero.
          Thus, the user should avoid naively computing  the
          ratio  ALPHA/BETA.   However, ALPHA will be always
          less than and usually comparable with  norm(A)  in
          magnitude,  and  BETA always less than and usually
          comparable with norm(B).

BETA (output)
          See description of ALPHA.
```

VL (output)
If JOBVL = 'V', the left generalized eigenvectors u(j) are stored one after another in the columns of VL, in the same order as their eigenvalues. Each eigenvector will be scaled so the largest component will have abs(real part) + abs(imag. part) = 1. Not referenced if JOBVL = 'N'.

LDVL (input)
The leading dimension of the matrix VL. LDVL >= 1, and if JOBVL = 'V', LDVL >= N.

VR (output)
If JOBVR = 'V', the right generalized eigenvectors v(j) are stored one after another in the columns of VR, in the same order as their eigenvalues. Each eigenvector will be scaled so the largest component will have abs(real part) + abs(imag. part) = 1. Not referenced if JOBVR = 'N'.

LDVR (input)
The leading dimension of the matrix VR. LDVR >= 1, and if JOBVR = 'V', LDVR >= N.

ILO (output)
ILO is an integer value such that on exit A(i,j) = 0 and B(i,j) = 0 if i > j and j = 1,...,ILO-1 or i = IHI+1,...,N. If BALANC = 'N' or 'S', ILO = 1 and IHI = N.

IHI (output)
IHI is an integer value such that on exit A(i,j) = 0 and B(i,j) = 0 if i > j and j = 1,...,ILO-1 or i = IHI+1,...,N. If BALANC = 'N' or 'S', ILO = 1 and IHI = N.

LSCALE (output)
Details of the permutations and scaling factors applied to the left side of A and B. If PL(j) is the index of the row interchanged with row j, and DL(j) is the scaling factor applied to row j, then LSCALE(j) = PL(j) for j = 1,...,ILO-1 = DL(j) for j = ILO,...,IHI = PL(j) for j = IHI+1,...,N. The order in which the interchanges are made is N to IHI+1, then 1 to ILO-1.

RSCALE (output)
Details of the permutations and scaling factors applied to the right side of A and B. If PR(j) is the index of the column interchanged with column

j, and DR(j) is the scaling factor applied to column j, then RSCALE(j) = PR(j) for j = 1,...,ILO-1 = DR(j) for j = ILO,...,IHI = PR(j) for j = IHI+1,...,N The order in which the interchanges are made is N to IHI+1, then 1 to ILO-1.

ABNRM (output)
The one-norm of the balanced matrix A.

BBNRM (output)
The one-norm of the balanced matrix B.

RCONDE (output)
If SENSE = 'E' or 'B', the reciprocal condition numbers of the selected eigenvalues, stored in consecutive elements of the array. If SENSE = 'V', RCONDE is not referenced.

RCONDV (output)
If JOB = 'V' or 'B', the estimated reciprocal condition numbers of the selected eigenvectors, stored in consecutive elements of the array. If the eigenvalues cannot be reordered to compute RCONDV(j), RCONDV(j) is set to 0; this can only occur when the true value would be very small anyway. If SENSE = 'E', RCONDV is not referenced. Not referenced if JOB = 'E'.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,2*N). If SENSE = 'N' or 'E', LWORK >= 2*N. If SENSE = 'V' or 'B', LWORK >= 2*N*N+2*N.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
dimension(6*N) Real workspace.

IWORK (workspace)
dimension(N+2) If SENSE = 'E', IWORK is not referenced.

BWORK (workspace)
dimension(N) If SENSE = 'N', BWORK is not refer-

enced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
= 1,...,N: The QZ iteration failed. No eigenvectors have been calculated, but ALPHA(j) and BETA(j) should be correct for j=INFO+1,...,N. > N: =N+1: other than QZ iteration failed in CHGEQZ.
=N+2: error return from CTGEVC.

FURTHER DETAILS

Balancing a matrix pair (A,B) includes, first, permuting rows and columns to isolate eigenvalues, second, applying diagonal similarity transformation to the rows and columns to make the rows and columns as close in norm as possible. The computed reciprocal condition numbers correspond to the balanced matrix. Permuting rows and columns will not change the condition numbers (in exact arithmetic) but diagonal scaling will. For further explanation of balancing, see section 4.11.1.2 of LAPACK Users' Guide.

An approximate error bound on the chordal distance between the i-th computed generalized eigenvalue w and the corresponding exact eigenvalue lambda is
hord(w, lambda) <= EPS * norm(ABNRM, BBNRM) / RCONDE(I)

An approximate error bound for the angle between the i-th computed eigenvector VL(i) or VR(i) is given by
PS * norm(ABNRM, BBNRM) / DIF(i).

For further explanation of the reciprocal condition numbers RCONDE and RCONDV, see section 4.11 of LAPACK User's Guide.

Contents

NAME

```
zggglm - solve a general  Gauss-Markov  linear  model  (GLM)
problem
```

SYNOPSIS

```
SUBROUTINE ZGGGLM(N, M, P, A, LDA, B, LDB, D, X, Y, WORK, LDWORK,
      INFO)

DOUBLE COMPLEX A(LDA,*), B(LDB,*), D(*), X(*), Y(*), WORK(*)
INTEGER N, M, P, LDA, LDB, LDWORK, INFO

SUBROUTINE ZGGGLM_64(N, M, P, A, LDA, B, LDB, D, X, Y, WORK, LDWORK,
      INFO)

DOUBLE COMPLEX A(LDA,*), B(LDB,*), D(*), X(*), Y(*), WORK(*)
INTEGER*8 N, M, P, LDA, LDB, LDWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE GGGLM([N], [M], [P], A, [LDA], B, [LDB], D, X, Y, [WORK],
       [LDWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: D, X, Y, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: N, M, P, LDA, LDB, LDWORK, INFO

SUBROUTINE GGGLM_64([N], [M], [P], A, [LDA], B, [LDB], D, X, Y, [WORK],
       [LDWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: D, X, Y, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: N, M, P, LDA, LDB, LDWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zggglm(int n, int m, int p, doublecomplex *a, int  lda,
          doublecomplex *b, int ldb, doublecomplex *d, doub-
          lecomplex *x, doublecomplex *y, int *info);

void zggglm_64(long n, long m,  long  p,  doublecomplex  *a,
          long  lda,  doublecomplex *b, long ldb, doublecom-
          plex *d, doublecomplex *x, doublecomplex *y,  long
          *info);
```

PURPOSE

zggglm solves a general Gauss-Markov linear model (GLM) problem:

```
        minimize || y ||_2   subject to   d = A*x + B*y
            x
```

where A is an N-by-M matrix, B is an N-by-P matrix, and d is a given N-vector. It is assumed that M <= N <= M+P, and

```
           rank(A) = M    and    rank( A B ) = N.
```

Under these assumptions, the constrained equation is always consistent, and there is a unique solution x and a minimal 2-norm solution y, which is obtained using a generalized QR factorization of A and B.

In particular, if matrix B is square nonsingular, then the problem GLM is equivalent to the following weighted linear least squares problem

```
             minimize || inv(B)*(d-A*x) ||_2
                 x
```

where inv(B) denotes the inverse of B.

ARGUMENTS

N (input) The number of rows of the matrices A and B. N >= 0.

M (input) The number of columns of the matrix A. 0 <= M <= N.

```
P (input) The number of columns of the matrix B.  P >= N-M.

A (input/output)
          On entry, the N-by-M matrix A.  On exit, A is des-
          troyed.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,N).

B (input/output)
          On entry, the N-by-P matrix B.  On exit, B is des-
          troyed.
LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          max(1,N).

D (input/output)
          On entry, D is the left hand side of the GLM equa-
          tion.  On exit, D is destroyed.

X (output)
          On exit, X and Y are  the  solutions  of  the  GLM
          problem.

Y (output)
          On exit, X and Y are  the  solutions  of  the  GLM
          problem.

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LDWORK.

LDWORK (input)
          The  dimension  of  the  array  WORK.  LDWORK   >=
          max(1,N+M+P).   For optimum performance, LDWORK >=
          M+min(N,P)+max(N,P)*NB, where NB is an upper bound
          for  the  optimal  blocksizes  for CGEQRF, CGERQF,
          CUNMQR and CUNMRQ.

          If LDWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LDWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit.
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value.
```

Contents

NAME

```
zgghrd - reduce a pair of complex matrices (A,B) to general-
ized  upper  Hessenberg  form using unitary transformations,
where A is a general matrix and B is upper triangular
```

SYNOPSIS

```
SUBROUTINE ZGGHRD(COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB, Q, LDQ,
      Z, LDZ, INFO)

CHARACTER * 1 COMPQ, COMPZ
DOUBLE COMPLEX A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*)
INTEGER N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO

SUBROUTINE ZGGHRD_64(COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB, Q,
      LDQ, Z, LDZ, INFO)

CHARACTER * 1 COMPQ, COMPZ
DOUBLE COMPLEX A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*)
INTEGER*8 N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO
```

F95 INTERFACE

```
SUBROUTINE GGHRD(COMPQ, COMPZ, [N], ILO, IHI, A, [LDA], B, [LDB], Q,
       [LDQ], Z, [LDZ], [INFO])

CHARACTER(LEN=1) :: COMPQ, COMPZ
COMPLEX(8), DIMENSION(:,:) :: A, B, Q, Z
INTEGER :: N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO

SUBROUTINE GGHRD_64(COMPQ, COMPZ, [N], ILO, IHI, A, [LDA], B, [LDB],
       Q, [LDQ], Z, [LDZ], [INFO])
```

```
CHARACTER(LEN=1) :: COMPQ, COMPZ
COMPLEX(8), DIMENSION(:,:) :: A, B, Q, Z
INTEGER(8) :: N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zgghrd(char compq, char compz, int n, int ilo, int ihi,
          doublecomplex  *a,  int lda, doublecomplex *b, int
          ldb, doublecomplex *q, int ldq, doublecomplex  *z,
          int ldz, int *info);

void zgghrd_64(char compq, char compz,  long  n,  long  ilo,
          long  ihi,  doublecomplex *a, long lda, doublecom-
          plex *b, long ldb,  doublecomplex  *q,  long  ldq,
          doublecomplex *z, long ldz, long *info);
```

PURPOSE

zgghrd reduces a pair of complex matrices (A,B) to general- ized upper Hessenberg form using unitary transformations, where A is a general matrix and B is upper triangular: Q' * A * Z = H and Q' * B * Z = T, where H is upper Hessenberg, T is upper triangular, and Q and Z are unitary, and ' means conjugate transpose.

The unitary matrices Q and Z are determined as products of Givens rotations. They may either be formed explicitly, or they may be postmultiplied into input matrices Q1 and Z1, so that
1 * A * Z1' = (Q1*Q) * H * (Z1*Z)'

ARGUMENTS

COMPQ (input)
= 'N': do not compute Q;
= 'I': Q is initialized to the unit matrix, and the unitary matrix Q is returned; = 'V': Q must contain a unitary matrix Q1 on entry, and the pro- duct Q1*Q is returned.

COMPZ (input)
= 'N': do not compute Q;
= 'I': Q is initialized to the unit matrix, and the unitary matrix Q is returned; = 'V': Q must contain a unitary matrix Q1 on entry, and the pro- duct Q1*Q is returned.

N (input) The order of the matrices A and B. N >= 0.

ILO (input)
It is assumed that A is already upper triangular in rows and columns 1:ILO-1 and IHI+1:N. ILO and IHI are normally set by a previous call to CGGBAL; otherwise they should be set to 1 and N respectively. 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

IHI (input)
See description of ILO.

A (input/output)
On entry, the N-by-N general matrix to be reduced. On exit, the upper triangle and the first subdiagonal of A are overwritten with the upper Hessenberg matrix H, and the rest is set to zero.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the N-by-N upper triangular matrix B. On exit, the upper triangular matrix T = Q' B Z. The elements below the diagonal are set to zero.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

Q (input/output)
If COMPQ='N': Q is not referenced.
If COMPQ='I': on entry, Q need not be set, and on exit it contains the unitary matrix Q, where Q' is the product of the Givens transformations which are applied to A and B on the left. If COMPQ='V': on entry, Q must contain a unitary matrix Q1, and on exit this is overwritten by Q1*Q.

LDQ (input)
The leading dimension of the array Q. LDQ >= N if COMPQ='V' or 'I'; LDQ >= 1 otherwise.

Z (input/output)
If COMPZ='N': Z is not referenced.
If COMPZ='I': on entry, Z need not be set, and on exit it contains the unitary matrix Z, which is

the product of the Givens transformations which are applied to A and B on the right. If COMPZ='V': on entry, Z must contain a unitary matrix Z1, and on exit this is overwritten by Z1*Z.

LDZ (input)
The leading dimension of the array Z. LDZ >= N if COMPZ='V' or 'I'; LDZ >= 1 otherwise.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

This routine reduces A to Hessenberg and B to triangular form by an unblocked reduction, as described in _Matrix_Computations_, by Golub and van Loan (Johns Hopkins Press).

Contents

NAME

```
zgglse - solve the linear equality-constrained least squares
(LSE) problem
```

SYNOPSIS

```
SUBROUTINE ZGGLSE(M, N, P, A, LDA, B, LDB, C, D, X, WORK, LDWORK,
      INFO)

DOUBLE COMPLEX A(LDA,*), B(LDB,*), C(*), D(*), X(*), WORK(*)
INTEGER M, N, P, LDA, LDB, LDWORK, INFO

SUBROUTINE ZGGLSE_64(M, N, P, A, LDA, B, LDB, C, D, X, WORK, LDWORK,
      INFO)

DOUBLE COMPLEX A(LDA,*), B(LDB,*), C(*), D(*), X(*), WORK(*)
INTEGER*8 M, N, P, LDA, LDB, LDWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE GGLSE([M], [N], [P], A, [LDA], B, [LDB], C, D, X, [WORK],
       [LDWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: C, D, X, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: M, N, P, LDA, LDB, LDWORK, INFO

SUBROUTINE GGLSE_64([M], [N], [P], A, [LDA], B, [LDB], C, D, X, [WORK],
       [LDWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: C, D, X, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: M, N, P, LDA, LDB, LDWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zgglse(int m, int n, int p, doublecomplex *a, int  lda,
          doublecomplex *b, int ldb, doublecomplex *c, doub-
          lecomplex *d, doublecomplex *x, int *info);

void zgglse_64(long m, long n,  long  p,  doublecomplex  *a,
          long  lda,  doublecomplex *b, long ldb, doublecom-
          plex *c, doublecomplex *d, doublecomplex *x,  long
          *info);
```

PURPOSE

zgglse solves the linear equality-constrained least squares (LSE) problem:

```
        minimize || c - A*x ||_2   subject to   B*x = d
```

where A is an M-by-N matrix, B is a P-by-N matrix, c is a given M-vector, and d is a given P-vector. It is assumed that
P <= N <= M+P, and

```
         rank(B) = P and  rank( ( A ) ) = N.
                              ( ( B ) )
```

These conditions ensure that the LSE problem has a unique solution, which is obtained using a GRQ factorization of the matrices B and A.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrices A and B. N >= 0.

P (input) The number of rows of the matrix B. 0 <= P <= N <= M+P.

A (input/output)
On entry, the M-by-N matrix A. On exit, A is destroyed.

LDA (input)
The leading dimension of the array A. LDA >=

max(1,M).

B (input/output)
On entry, the P-by-N matrix B. On exit, B is destroyed.

LDB (input)
The leading dimension of the array B. LDB >= max(1,P).

C (input/output)
On entry, C contains the right hand side vector for the least squares part of the LSE problem. On exit, the residual sum of squares for the solution is given by the sum of squares of elements N-P+1 to M of vector C.

D (input/output)
On entry, D contains the right hand side vector for the constrained equation. On exit, D is destroyed.

X (output)
On exit, X is the solution of the LSE problem.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The dimension of the array WORK. LDWORK >= max(1,M+N+P). For optimum performance LDWORK >= P+min(M,N)+max(M,N)*NB, where NB is an upper bound for the optimal blocksizes for CGEQRF, CGERQF, CUNMQR and CUNMRQ.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

```
zggqrf - compute a generalized QR factorization of an N-by-M
matrix A and an N-by-P matrix B.
```

SYNOPSIS

```
SUBROUTINE ZGGQRF(N, M, P, A, LDA, TAUA, B, LDB, TAUB, WORK, LWORK,
      INFO)

DOUBLE COMPLEX A(LDA,*), TAUA(*), B(LDB,*), TAUB(*), WORK(*)
INTEGER N, M, P, LDA, LDB, LWORK, INFO

SUBROUTINE ZGGQRF_64(N, M, P, A, LDA, TAUA, B, LDB, TAUB, WORK,
      LWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAUA(*), B(LDB,*), TAUB(*), WORK(*)
INTEGER*8 N, M, P, LDA, LDB, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE GGQRF([N], [M], [P], A, [LDA], TAUA, B, [LDB], TAUB, [WORK],
       [LWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAUA, TAUB, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: N, M, P, LDA, LDB, LWORK, INFO

SUBROUTINE GGQRF_64([N], [M], [P], A, [LDA], TAUA, B, [LDB], TAUB,
       [WORK], [LWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAUA, TAUB, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: N, M, P, LDA, LDB, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zggqrf(int n, int m, int p, doublecomplex *a, int  lda,
          doublecomplex  *taua,  doublecomplex  *b, int ldb,
          doublecomplex *taub, int *info);

void zggqrf_64(long n, long m,  long  p,  doublecomplex  *a,
          long  lda,  doublecomplex *taua, doublecomplex *b,
          long ldb, doublecomplex *taub, long *info);
```

PURPOSE

```
zggqrf computes a generalized QR factorization of an  N-by-M
matrix A and an N-by-P matrix B:
            A = Q*R,        B = Q*T*Z,

where Q is an N-by-N unitary matrix, Z is a  P-by-P  unitary
matrix, and R and T assume one of the forms:

if N >= M,  R = ( R11 ) M  ,   or if N < M,  R = ( R11   R12
) N,
                (  0  ) N-M                         N   M-N
                   M

where R11 is upper triangular, and

if N <= P,  T = ( 0  T12 ) N,   or if N > P,  T =  (  T11  )
N-P,
                 P-N  N                           ( T21 ) P
                                                     P

where T12 or T21 is upper triangular.

In particular, if B is square and nonsingular, the GQR  fac-
torization  of A and B implicitly gives the QR factorization
of inv(B)*A:

             inv(B)*A = Z'*(inv(T)*R)

where inv(B) denotes the inverse of the  matrix  B,  and  Z'
denotes the conjugate transpose of matrix Z.
```

ARGUMENTS

N (input) The number of rows of the matrices A and B. N >=

0.

M (input) The number of columns of the matrix A. M >= 0.

P (input) The number of columns of the matrix B. P >= 0.

A (input/output)
On entry, the N-by-M matrix A. On exit, the elements on and above the diagonal of the array contain the min(N,M)-by-M upper trapezoidal matrix R (R is upper triangular if N >= M); the elements below the diagonal, with the array TAUA, represent the unitary matrix Q as a product of min(N,M) elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

TAUA (output)
The scalar factors of the elementary reflectors which represent the unitary matrix Q (see Further Details).

B (input/output)
On entry, the N-by-P matrix B. On exit, if N <= P, the upper triangle of the subarray B(1:N,P-N+1:P) contains the N-by-N upper triangular matrix T; if N > P, the elements on and above the (N-P)-th subdiagonal contain the N-by-P upper trapezoidal matrix T; the remaining elements, with the array TAUB, represent the unitary matrix Z as a product of elementary reflectors (see Further Details).

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

TAUB (output)
The scalar factors of the elementary reflectors which represent the unitary matrix Z (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >=

max(1,N,M,P). For optimum performance LWORK >= max(N,M,P)*max(NB1,NB2,NB3), where NB1 is the optimal blocksize for the QR factorization of an N-by-M matrix, NB2 is the optimal blocksize for the RQ factorization of an N-by-P matrix, and NB3 is the optimal blocksize for a call of CUNMQR.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

The matrix Q is represented as a product of elementary reflectors

```
   Q = H(1) H(2) . . . H(k), where k = min(n,m).
```

Each H(i) has the form

```
   H(i) = I - taua * v * v'
```

where taua is a complex scalar, and v is a complex vector with v(1:i-1) = 0 and v(i) = 1; v(i+1:n) is stored on exit in A(i+1:n,i), and taua in TAUA(i).
To form Q explicitly, use LAPACK subroutine CUNGQR.
To use Q to update another matrix, use LAPACK subroutine CUNMQR.

The matrix Z is represented as a product of elementary reflectors

```
   Z = H(1) H(2) . . . H(k), where k = min(n,p).
```

Each H(i) has the form

```
   H(i) = I - taub * v * v'
```

where taub is a complex scalar, and v is a complex vector with v(p-k+i+1:p) = 0 and v(p-k+i) = 1; v(1:p-k+i-1) is stored on exit in B(n-k+i,1:p-k+i-1), and taub in TAUB(i).
To form Z explicitly, use LAPACK subroutine CUNGRQ.
To use Z to update another matrix, use LAPACK subroutine CUNMRQ.

Contents

NAME

```
zggrqf - compute a generalized RQ factorization of an M-by-N
matrix A and a P-by-N matrix B
```

SYNOPSIS

```
SUBROUTINE ZGGRQF(M, P, N, A, LDA, TAUA, B, LDB, TAUB, WORK, LWORK,
      INFO)

DOUBLE COMPLEX A(LDA,*), TAUA(*), B(LDB,*), TAUB(*), WORK(*)
INTEGER M, P, N, LDA, LDB, LWORK, INFO

SUBROUTINE ZGGRQF_64(M, P, N, A, LDA, TAUA, B, LDB, TAUB, WORK,
      LWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAUA(*), B(LDB,*), TAUB(*), WORK(*)
INTEGER*8 M, P, N, LDA, LDB, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE GGRQF([M], [P], [N], A, [LDA], TAUA, B, [LDB], TAUB, [WORK],
       [LWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAUA, TAUB, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: M, P, N, LDA, LDB, LWORK, INFO

SUBROUTINE GGRQF_64([M], [P], [N], A, [LDA], TAUA, B, [LDB], TAUB,
       [WORK], [LWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAUA, TAUB, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: M, P, N, LDA, LDB, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zggrqf(int m, int p, int n, doublecomplex *a, int  lda,
          doublecomplex  *taua,  doublecomplex  *b, int ldb,
          doublecomplex *taub, int *info);

void zggrqf_64(long m, long p,  long  n,  doublecomplex  *a,
          long  lda,  doublecomplex *taua, doublecomplex *b,
          long ldb, doublecomplex *taub, long *info);
```

PURPOSE

```
zggrqf computes a generalized RQ factorization of an  M-by-N
matrix A and a P-by-N matrix B:
            A = R*Q,        B = Z*T*Q,

where Q is an N-by-N unitary matrix, Z is a  P-by-P  unitary
matrix, and R and T assume one of the forms:

if M <= N,  R = ( 0  R12 ) M,   or if M > N,  R =  (  R11  )
M-N,
                 N-M  M                           ( R21 ) N
                                                     N

where R12 or R21 is upper triangular, and

if P >= N,  T = ( T11 ) N  ,   or if P < N,  T = ( T11   T12
) P,
                (  0  ) P-N                        P   N-P
                   N

where T11 is upper triangular.

In particular, if B is square and nonsingular, the GRQ  fac-
torization  of A and B implicitly gives the RQ factorization
of A*inv(B):

             A*inv(B) = (R*inv(T))*Z'

where inv(B) denotes the inverse of the  matrix  B,  and  Z'
denotes the conjugate transpose of the matrix Z.
```

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

P (input) The number of rows of the matrix B. P >= 0.

N (input) The number of columns of the matrices A and B. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, if M <= N, the upper triangle of the subarray A(1:M,N-M+1:N) contains the M-by-M upper triangular matrix R; if M > N, the elements on and above the (M-N)-th subdiagonal contain the M-by-N upper trapezoidal matrix R; the remaining elements, with the array TAUA, represent the unitary matrix Q as a product of elementary reflectors (see Further Details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

TAUA (output)
The scalar factors of the elementary reflectors which represent the unitary matrix Q (see Further Details).

B (input/output)
On entry, the P-by-N matrix B. On exit, the elements on and above the diagonal of the array contain the min(P,N)-by-N upper trapezoidal matrix T (T is upper triangular if P >= N); the elements below the diagonal, with the array TAUB, represent the unitary matrix Z as a product of elementary reflectors (see Further Details).

LDB (input)
The leading dimension of the array B. LDB >= max(1,P).

TAUB (output)
The scalar factors of the elementary reflectors which represent the unitary matrix Z (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,N,M,P). For optimum performance LWORK >=

```
          max(N,M,P)*max(NB1,NB2,NB3),  where  NB1  is   the
          optimal  blocksize  for the RQ factorization of an
          M-by-N matrix, NB2 is the  optimal  blocksize  for
          the  QR  factorization of a P-by-N matrix, and NB3
          is the optimal blocksize for a call of CUNMRQ.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.
INFO (output)
          = 0:  successful exit
          < 0:  if INFO=-i, the i-th argument had an illegal
          value.
```

FURTHER DETAILS

```
The matrix Q is  represented  as  a  product  of  elementary
reflectors

   Q = H(1) H(2) . . . H(k), where k = min(m,n).

Each H(i) has the form

   H(i) = I - taua * v * v'

where taua is a complex scalar, and v is  a  complex  vector
with  v(n-k+i+1:n)  =  0  and  v(n-k+i) = 1; v(1:n-k+i-1) is
stored on exit in A(m-k+i,1:n-k+i-1), and taua in TAUA(i).
To form Q explicitly, use LAPACK subroutine CUNGRQ.
To use Q to update another  matrix,  use  LAPACK  subroutine
CUNMRQ.

The matrix Z is  represented  as  a  product  of  elementary
reflectors

   Z = H(1) H(2) . . . H(k), where k = min(p,n).

Each H(i) has the form

   H(i) = I - taub * v * v'

where taub is a complex scalar, and v is  a  complex  vector
with  v(1:i-1)  = 0 and v(i) = 1; v(i+1:p) is stored on exit
in B(i+1:p,i), and taub in TAUB(i).
To form Z explicitly, use LAPACK subroutine CUNGQR.
To use Z to update another  matrix,  use  LAPACK  subroutine
CUNMQR.
```

Contents

NAME

```
zggsvd - compute the generalized singular  value  decomposi-
tion (GSVD) of an M-by-N complex matrix A and P-by-N complex
matrix B
```

SYNOPSIS

```
SUBROUTINE ZGGSVD(JOBU, JOBV, JOBQ, M, N, P, K, L, A, LDA, B, LDB,
      ALPHA, BETA, U, LDU, V, LDV, Q, LDQ, WORK, WORK2, IWORK3, INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),  U(LDU,*),   V(LDV,*),
Q(LDQ,*), WORK(*)
INTEGER M, N, P, K, L, LDA, LDB, LDU, LDV, LDQ, INFO
INTEGER IWORK3(*)
DOUBLE PRECISION ALPHA(*), BETA(*), WORK2(*)

SUBROUTINE ZGGSVD_64(JOBU, JOBV, JOBQ, M, N, P, K, L, A, LDA, B, LDB,
      ALPHA, BETA, U, LDU, V, LDV, Q, LDQ, WORK, WORK2, IWORK3, INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),  U(LDU,*),   V(LDV,*),
Q(LDQ,*), WORK(*)
INTEGER*8 M, N, P, K, L, LDA, LDB, LDU, LDV, LDQ, INFO
INTEGER*8 IWORK3(*)
DOUBLE PRECISION ALPHA(*), BETA(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GGSVD(JOBU, JOBV, JOBQ, [M], [N], [P], K, L, A, [LDA], B,
       [LDB], ALPHA, BETA, U, [LDU], V, [LDV], Q, [LDQ], [WORK], [WORK2],
       IWORK3, [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
```

```
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, U, V, Q
INTEGER :: M, N, P, K, L, LDA, LDB, LDU, LDV, LDQ, INFO
INTEGER, DIMENSION(:) :: IWORK3
REAL(8), DIMENSION(:) :: ALPHA, BETA, WORK2

SUBROUTINE GGSVD_64(JOBU, JOBV, JOBQ, [M], [N], [P], K, L, A, [LDA],
       B, [LDB], ALPHA, BETA, U, [LDU], V, [LDV], Q, [LDQ], [WORK],
       [WORK2], IWORK3, [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, U, V, Q
INTEGER(8) :: M, N, P, K, L, LDA, LDB, LDU, LDV, LDQ, INFO
INTEGER(8), DIMENSION(:) :: IWORK3
REAL(8), DIMENSION(:) :: ALPHA, BETA, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zggsvd(char jobu, char jobv, char jobq, int m,  int  n,
          int  p, int *k, int *l, doublecomplex *a, int lda,
          doublecomplex *b, int ldb, double  *alpha,  double
          *beta,  doublecomplex  *u,  int ldu, doublecomplex
          *v,  int  ldv,  doublecomplex  *q,  int  ldq,  int
          *iwork3, int *info);

void zggsvd_64(char jobu, char jobv, char jobq, long m, long
          n,  long  p,  long  *k, long *l, doublecomplex *a,
          long  lda,  doublecomplex  *b,  long  ldb,  double
          *alpha,  double *beta, doublecomplex *u, long ldu,
          doublecomplex *v, long ldv, doublecomplex *q, long
          ldq, long *iwork3, long *info);
```

PURPOSE

zggsvd computes the generalized singular value decomposition (GSVD) of an M-by-N complex matrix A and P-by-N complex matrix B:

```
      U'*A*Q = D1*( 0 R ),    V'*B*Q = D2*( 0 R )
```

where U, V and Q are unitary matrices, and Z' means the conjugate transpose of Z. Let K+L = the effective numerical rank of the matrix (A',B')', then R is a (K+L)-by-(K+L) nonsingular upper triangular matrix, D1 and D2 are M-by-(K+L) and P-by-(K+L) "diagonal" matrices and of the following structures, respectively:

If M-K-L >= 0,

```
                   K  L
       D1 =     K ( I  0 )
                L ( 0  C )
            M-K-L ( 0  0 )

                 K  L
       D2 =   L ( 0  S )
            P-L ( 0  0 )

                N-K-L  K    L
  ( 0 R ) = K (  0   R11  R12 )
            L (  0    0   R22 )
```

where

```
  C = diag( ALPHA(K+1), ... , ALPHA(K+L) ),
  S = diag( BETA(K+1),  ... , BETA(K+L) ),
  C**2 + S**2 = I.
  R is stored in A(1:K+L,N-K-L+1:N) on exit.
```

If M-K-L < 0,

```
                 K M-K K+L-M
       D1 =   K ( I  0    0   )
            M-K ( 0  C    0   )

                   K M-K K+L-M
       D2 =   M-K ( 0  S    0  )
            K+L-M ( 0  0    I  )
              P-L ( 0  0    0  )

                  N-K-L  K   M-K  K+L-M
  ( 0 R ) =     K ( 0    R11  R12  R13  )
              M-K ( 0     0   R22  R23  )
            K+L-M ( 0     0    0   R33  )
```

where

```
  C = diag( ALPHA(K+1), ... , ALPHA(M) ),
  S = diag( BETA(K+1),  ... , BETA(M) ),
  C**2 + S**2 = I.

  (R11 R12 R13 ) is stored in A(1:M, N-K-L+1:N), and R33  is
stored
  ( 0  R22 R23 )
  in B(M-K+1:L,N+M-K-L+1:N) on exit.
```

The routine computes C, S, R, and optionally the unitary transformation matrices U, V and Q.

In particular, if B is an N-by-N nonsingular matrix, then the GSVD of A and B implicitly gives the SVD of A*inv(B):

```
                     A*inv(B) = U*(D1*inv(D2))*V'.
```

If (A',B')' has orthnormal columns, then the GSVD of A and B is also equal to the CS decomposition of A and B. Furthermore, the GSVD can be used to derive the solution of the eigenvalue problem:

```
                     A'*A x = lambda* B'*B x.
```

In some literature, the GSVD of A and B is presented in the form

```
                 U'*A*X = ( 0 D1 ),   V'*B*X = ( 0 D2 )
```

where U and V are orthogonal and X is nonsingular, and D1 and D2 are ``diagonal''. The former GSVD form can be converted to the latter form by taking the nonsingular matrix X as

```
                      X = Q*(  I   0    )
                            (  0 inv(R) )
```

ARGUMENTS

JOBU (input)
= 'U': Unitary matrix U is computed;
= 'N': U is not computed.

JOBV (input)
= 'V': Unitary matrix V is computed;
= 'N': V is not computed.

JOBQ (input)
= 'Q': Unitary matrix Q is computed;
= 'N': Q is not computed.

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrices A and B. N >= 0.

P (input) The number of rows of the matrix B. P >= 0.

K (output)
On exit, K and L specify the dimension of the subblocks described in Purpose. K + L = effective numerical rank of (A',B')'.

L (output)
On exit, K and L specify the dimension of the subblocks described in Purpose. K + L = effective numerical rank of (A',B')'.

A (input/output)
On entry, the M-by-N matrix A. On exit, A contains the triangular matrix R, or part of R. See Purpose for details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the P-by-N matrix B. On exit, B contains part of the triangular matrix R if M-K-L < 0. See Purpose for details.

LDB (input)
The leading dimension of the array B. LDB >= max(1,P).

ALPHA (output)
On exit, ALPHA and BETA contain the generalized singular value pairs of A and B; ALPHA(1:K) = 1,
ALPHA(1:K) = 1,
BETA(1:K) = 0, and if M-K-L >= 0, ALPHA(K+1:K+L) = C,
BETA(K+1:K+L) = S, or if M-K-L < 0, ALPHA(K+1:M)= C, ALPHA(M+1:K+L)= 0
BETA(K+1:M) = S, BETA(M+1:K+L) = 1 and
ALPHA(K+L+1:N) = 0
BETA(K+L+1:N) = 0

BETA (output)
See description of ALPHA.

U (output)
If JOBU = 'U', U contains the M-by-M unitary matrix U. If JOBU = 'N', U is not referenced.

LDU (input)
The leading dimension of the array U. LDU >= max(1,M) if JOBU = 'U'; LDU >= 1 otherwise.

V (output)
If JOBV = 'V', V contains the P-by-P unitary matrix V. If JOBV = 'N', V is not referenced.

LDV (input)
The leading dimension of the array V. LDV >= max(1,P) if JOBV = 'V'; LDV >= 1 otherwise.

Q (output)
If JOBQ = 'Q', Q contains the N-by-N unitary matrix Q. If JOBQ = 'N', Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= max(1,N) if JOBQ = 'Q'; LDQ >= 1 otherwise.

WORK (workspace)
dimension(MAX(3*N,M,P)+N)

WORK2 (workspace)
dimension(2*N)

IWORK3 (output)
dimension(N) On exit, IWORK3 stores the sorting information. More precisely, the following loop will sort ALPHA for I = K+1, min(M,K+L) swap ALPHA(I) and ALPHA(IWORK3(I)) endfor such that ALPHA(1) >= ALPHA(2) >= ... >= ALPHA(N).

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = 1, the Jacobi-type procedure failed to converge. For further details, see subroutine CTGSJA.

Contents

NAME

```
zggsvp - compute unitary matrices U, V and Q such that    N-
K-L K L  U'*A*Q = K ( 0 A12 A13 ) if M-K-L >= 0
```

SYNOPSIS

```
SUBROUTINE ZGGSVP(JOBU, JOBV, JOBQ, M, P, N, A, LDA, B, LDB, TOLA,
      TOLB, K, L, U, LDU, V, LDV, Q, LDQ, IWORK, RWORK, TAU, WORK,
      INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),  U(LDU,*),   V(LDV,*),
Q(LDQ,*), TAU(*), WORK(*)
INTEGER M, P, N, LDA, LDB, K, L, LDU, LDV, LDQ, INFO
INTEGER IWORK(*)
DOUBLE PRECISION TOLA, TOLB
DOUBLE PRECISION RWORK(*)

SUBROUTINE ZGGSVP_64(JOBU, JOBV, JOBQ, M, P, N, A, LDA, B, LDB, TOLA,
      TOLB, K, L, U, LDU, V, LDV, Q, LDQ, IWORK, RWORK, TAU, WORK,
      INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),  U(LDU,*),   V(LDV,*),
Q(LDQ,*), TAU(*), WORK(*)
INTEGER*8 M, P, N, LDA, LDB, K, L, LDU, LDV, LDQ, INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION TOLA, TOLB
DOUBLE PRECISION RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE GGSVP(JOBU, JOBV, JOBQ, [M], [P], [N], A, [LDA], B, [LDB],
```

```
       TOLA, TOLB, K, L, U, [LDU], V, [LDV], Q, [LDQ], [IWORK], [RWORK],
       [TAU], [WORK], [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, U, V, Q
INTEGER :: M, P, N, LDA, LDB, K, L, LDU, LDV, LDQ, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL(8) :: TOLA, TOLB
REAL(8), DIMENSION(:) :: RWORK

SUBROUTINE GGSVP_64(JOBU, JOBV, JOBQ, [M], [P], [N], A, [LDA], B,
       [LDB], TOLA, TOLB, K, L, U, [LDU], V, [LDV], Q, [LDQ], [IWORK],
       [RWORK], [TAU], [WORK], [INFO])

CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, U, V, Q
INTEGER(8) :: M, P, N, LDA, LDB, K, L, LDU, LDV, LDQ, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8) :: TOLA, TOLB
REAL(8), DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>

void zggsvp(char jobu, char jobv, char jobq, int m,  int  p,
          int  n,  doublecomplex  *a, int lda, doublecomplex
          *b, int ldb, double tola, double tolb, int *k, int
          *l,  doublecomplex  *u, int ldu, doublecomplex *v,
          int ldv, doublecomplex *q, int ldq, int *info);

void zggsvp_64(char jobu, char jobv, char jobq, long m, long
          p,  long n, doublecomplex *a, long lda, doublecom-
          plex *b, long ldb, double tola, double tolb,  long
          *k,  long  *l,  doublecomplex  *u, long ldu, doub-
          lecomplex *v, long  ldv,  doublecomplex  *q,  long
          ldq, long *info);
```

PURPOSE

```
zggsvp computes unitary matrices U, V and Q such that
              L ( 0     0   A23 )
          M-K-L ( 0     0    0  )

                   N-K-L  K    L
        =     K ( 0    A12  A13 )  if M-K-L < 0;
            M-K ( 0     0   A23 )
```

```
                N-K-L  K    L
 V'*B*Q =   L ( 0     0   B13 )
          P-L ( 0     0    0  )
```

where the K-by-K matrix A12 and L-by-L matrix B13 are non-singular upper triangular; A23 is L-by-L upper triangular if M-K-L >= 0, otherwise A23 is (M-K)-by-L upper trapezoidal. K+L = the effective numerical rank of the (M+P)-by-N matrix (A',B')'. Z' denotes the conjugate transpose of Z.

This decomposition is the preprocessing step for computing the Generalized Singular Value Decomposition (GSVD), see subroutine CGGSVD.

ARGUMENTS

JOBU (input)
= 'U': Unitary matrix U is computed;
= 'N': U is not computed.

JOBV (input)
= 'V': Unitary matrix V is computed;
= 'N': V is not computed.

JOBQ (input)
= 'Q': Unitary matrix Q is computed;
= 'N': Q is not computed.

M (input) The number of rows of the matrix A. M >= 0.

P (input) The number of rows of the matrix B. P >= 0.

N (input) The number of columns of the matrices A and B. N >= 0.

A (input/output)
On entry, the M-by-N matrix A. On exit, A contains the triangular (or trapezoidal) matrix described in the Purpose section.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the P-by-N matrix B. On exit, B contains the triangular matrix described in the Purpose section.

LDB (input)
The leading dimension of the array B. LDB >= max(1,P).

TOLA (input)
TOLA and TOLB are the thresholds to determine the effective numerical rank of matrix B and a sub-block of A. Generally, they are set to TOLA = MAX(M,N)*norm(A)*MACHEPS, TOLB = MAX(P,N)*norm(B)*MACHEPS. The size of TOLA and TOLB may affect the size of backward errors of the decomposition.

TOLB (input)
See description of TOLA.

K (output)
On exit, K and L specify the dimension of the sub-blocks described in Purpose section. K + L = effective numerical rank of (A',B')'.

L (output)
See the description of K.

U (input) If JOBU = 'U', U contains the unitary matrix U. If JOBU = 'N', U is not referenced.

LDU (input)
The leading dimension of the array U. LDU >= max(1,M) if JOBU = 'U'; LDU >= 1 otherwise.

V (input) If JOBV = 'V', V contains the unitary matrix V. If JOBV = 'N', V is not referenced.

LDV (input)
The leading dimension of the array V. LDV >= max(1,P) if JOBV = 'V'; LDV >= 1 otherwise.

Q (input) If JOBQ = 'Q', Q contains the unitary matrix Q. If JOBQ = 'N', Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= max(1,N) if JOBQ = 'Q'; LDQ >= 1 otherwise.

IWORK (workspace)
dimension(N)

RWORK (workspace)

dimension(2*N)

TAU (workspace)
dimension(N)

WORK (workspace)
dimension(MAX(3*N,M,P))

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.

FURTHER DETAILS

The subroutine uses LAPACK subroutine CGEQPF for the QR factorization with column pivoting to detect the effective numerical rank of the a matrix. It may be replaced by a better rank determination strategy.

Contents

NAME

zgssco - General sparse solver condition number estimate.

SYNOPSIS

```
SUBROUTINE ZGSSCO ( COND, HANDLE, IER )

INTEGER             IER
DOUBLE PRECISION    COND
DOUBLE PRECISION    HANDLE(150)
```

PURPOSE

ZGSSCO - Condition number estimate.

PARAMETERS

```
COND       - DOUBLE PRECISION
        On exit, an estimate of the condition number of the
        factored matrix.  Must be called after the numerical
        factorization subroutine, ZGSSFA().

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER        - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -700 : Invalid calling sequence - need to call ZGSSFA first.
```

```
-710 : Condition number estimate not available (not implemented
       for this HANDLE's matix type).
```

Contents

NAME

```
zgssda - Deallocate working storage for the general sparse
solver.
```

SYNOPSIS

```
 SUBROUTINE ZGSSDA ( HANDLE, IER )

 INTEGER            IER
 DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

ZGSSDA - Deallocate dynamically allocated working storage.

PARAMETERS

```
HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER        - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         none
```

Contents

NAME

```
zgssfa - General sparse solver numeric factorization.
```

SYNOPSIS

```
 SUBROUTINE ZGSSFA ( NEQNS, COLSTR, ROWIND, VALUES, HANDLE, IER )

 INTEGER            NEQNS, COLSTR(*), ROWIND(*), IER
 DOUBLE COMPLEX     VALUES(*)
 DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

ZGSSFA - Numeric factorization of a sparse matrix.

PARAMETERS

```
NEQNS     - INTEGER
        On entry, NEQNS specifies the number of equations in
        coefficient matrix.  Unchanged on exit.

COLSTR(*) - INTEGER array
        On entry, COLSTR(*) is an array of size (NEQNS+1),
        containing the pointers of the matrix structure.
        Unchanged on exit.

ROWIND(*) - INTEGER array
        On entry, ROWIND(*) is an array of size
        COLSTR(NEQNS+1)-1, containing the indices of the
        matrix structure.  Unchanged on exit.

VALUES(*) - DOUBLE COMPLEX array
        On entry, VALUES(*) is an array of size
        COLSTR(NEQNS+1)-1, containing the numeric values of
```

```
        the sparse matrix to be factored.  Unchanged on
        exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -300 : Invalid calling sequence - need to call ZGSSOR first.
         -301 : Failure to dynamically allocate memory.
         -666 : Internal error.
```

Contents

NAME

```
zgssfs - General sparse solver one call interface.
```

SYNOPSIS

```
 SUBROUTINE ZGSSFS ( MTXTYP, PIVOT , NEQNS, COLSTR, ROWIND,
                     VALUES, NRHS  , RHS   , LDRHS , ORDMTHD,
                     OUTUNT, MSGLVL, HANDLE, IER )

 CHARACTER*2        MTXTYP
 CHARACTER*1        PIVOT
 INTEGER            NEQNS, COLSTR(*), ROWIND(*), NRHS, LDRHS,
                    OUTUNT, MSGLVL, IER
 CHARACTER*3        ORDMTHD
 DOUBLE COMPLEX     VALUES(*), RHS(*)
 DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

ZGSSFS - General sparse solver one call interface.

PARAMETERS

```
MTXTYP    - CHARACTER*2
        On entry, MTXTYP specifies the coefficient matrix
        type.  Specifically, the valid options are:

         'sp' or 'SP' - symmetric structure, Hermitian positive definite
values
         'ss' or 'SS' - symmetric structure, symmetric values
         'su' or 'SU' - symmetric structure, unsymmetric values
         'uu' or 'UU' - unsymmetric structure, unsymmetric values

        Unchanged on exit.
```

```
PIVOT     - CHARACTER*1
        On entry, pivot specifies whether or not pivoting is
        used in the course of the numeric factorization.
        The valid options are:

         'n' or 'N' - no pivoting is used
         (Pivoting is not supported for this release).

        Unchanged on exit.

NEQNS     - INTEGER
        On entry, NEQNS specifies the number of equations in
        the coefficient matrix.  NEQNS must be at least one.
        Unchanged on exit.

COLSTR(*) - INTEGER array
        On entry, COLSTR(*) is an array of size (NEQNS+1),
        containing the pointers of the matrix structure.
        Unchanged on exit.
ROWIND(*) - INTEGER array
        On entry, ROWIND(*) is an array of size
        COLSTR(NEQNS+1)-1, containing the indices of the
        matrix structure.  Unchanged on exit.

VALUES(*) - DOUBLE COMPLEX array
        On entry, VALUES(*) is an array of size
        COLSTR(NEQNS+1)-1, containing the non-zero numeric
        values of the sparse matrix to be factored.
        Unchanged on exit.

NRHS      - INTEGER
        On entry, NRHS specifies the number of right hand
        sides to solve for.  Unchanged on exit.

RHS(*)    - DOUBLE COMPLEX array
        On entry, RHS(LDRHS,NRHS) contains the NRHS right
        hand sides.  On exit, it contains the solutions.

LDRHS     - INTEGER
        On entry, LDRHS specifies the leading dimension of
        the RHS array. Unchanged on exit.

ORDMTHD   - CHARACTER*3
        On entry, ORDMTHD specifies the fill-reducing
        ordering to be used by the sparse solver.
        Specifically, the valid options are:

         'nat' or 'NAT' - natural ordering (no ordering)
         'mmd' or 'MMD' - multiple minimum degree
```

```
          'gnd' or 'GND' - general nested dissection
          'uso' or 'USO' - user specified ordering (see ZGSSUO)

         Unchanged on exit.

 OUTUNT    - INTEGER
         Output unit.  Unchanged on exit.

 MSGLVL    - INTEGER
         Message level.

          0 - no output from solver.
          (No messages supported for this release.)

         Unchanged on exit.

 HANDLE(150)  - DOUBLE PRECISION array
         On entry, HANDLE(*) is an array of containing
         information needed by the solver, and must be passed
         unchanged to each sparse solver subroutine.
         Modified on exit.
 IER       - INTEGER
         Error number.  If no error encountered, unchanged on
         exit.  If error encountered, it is set to a non-zero
         integer.  Error numbers set by this subroutine:

          -101 : Failure to dynamically allocate memory.
          -102 : Invalid matrix type.
          -103 : Invalid pivot option.
          -104 : Number of nonzeros is less than NEQNS.
          -105 : NEQNS < 1
          -201 : Failure to dynamically allocate memory.
          -301 : Failure to dynamically allocate memory.
          -401 : Failure to dynamically allocate memory.
          -402 : NRHS < 1
          -403 : NEQNS > LDRHS
          -666 : Internal error.
```

Contents

NAME

```
zgssin - Initialize the general sparse solver.
```

SYNOPSIS

```
SUBROUTINE ZGSSIN ( MTXTYP, PIVOT, NEQNS, COLSTR, ROWIND, OUTUNT,
                    MSGLVL, HANDLE, IER )

CHARACTER*2       MTXTYP
CHARACTER*1       PIVOT
INTEGER           NEQNS, COLSTR(*), ROWIND(*), OUTUNT, MSGLVL, IER
DOUBLE PRECISION  HANDLE(150)
```

PURPOSE

ZGSSIN - Initialize the sparse solver and input the matrix structure.

PARAMETERS

```
MTXTYP    - CHARACTER*2
        On entry, MTXTYP specifies the coefficient matrix
        type.  Specifically, the valid options are:

         'sp' or 'SP' - symmetric structure, Hermitian positive definite
values
         'ss' or 'SS' - symmetric structure, symmetric values
         'su' or 'SU' - symmetric structure, unsymmetric values
         'uu' or 'UU' - unsymmetric structure, unsymmetric values

        Unchanged on exit.

PIVOT     - CHARACTER*1
        On entry, PIVOT specifies whether or not pivoting is
```

```
        used in the course of the numeric factorization.
        The valid options are:

         'n' or 'N' - no pivoting is used
         (Pivoting is not supported for this release).

        Unchanged on exit.

NEQNS     - INTEGER
        On entry, NEQNS specifies the number of equations in
        the coefficient matrix.  NEQNS must be at least one.
        Unchanged on exit.

COLSTR(*) - INTEGER array
        On entry, COLSTR(*) is an array of size (NEQNS+1),
        containing the pointers of the matrix structure.
        Unchanged on exit.

ROWIND(*) - INTEGER array
        On entry, ROWIND(*) is an array of size
        COLSTR(NEQNS+1)-1, containing the indices of the
        matrix structure.  Unchanged on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

OUTUNT    - INTEGER
        Output unit.  Unchanged on exit.

MSGLVL    - INTEGER
        Message level.

         0 - no output from solver.
         (No messages supported for this release.)

        Unchanged on exit.

IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -101 : Failure to dynamically allocate memory.
         -102 : Invalid matrix type.
         -103 : Invalid pivot option.
         -104 : Number of nonzeros less than NEQNS.
         -105 : NEQNS < 1
```

Contents

NAME

```
zgssor - General sparse solver ordering and symbolic
factorization.
```

SYNOPSIS

```
 SUBROUTINE ZGSSOR ( ORDMTHD, HANDLE, IER )

 CHARACTER*3       ORDMTHD
 INTEGER           IER
 DOUBLE PRECISION  HANDLE(150)
```

PURPOSE

ZGSSOR - Orders and symbolically factors a sparse matrix.

PARAMETERS

```
ORDMTHD   - CHARACTER*3
        On entry, ORDMTHD specifies the fill-reducing
        ordering to be used by the sparse solver.
        Specifically, the valid options are:

         'nat' or 'NAT' - natural ordering (no ordering)
         'mmd' or 'MMD' - multiple minimum degree
         'gnd' or 'GND' - general nested dissection
         'uso' or 'USO' - user specified ordering (see ZGSSUO)

        Unchanged on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
```

Modified on exit.

IER - INTEGER
Error number. If no error encountered, unchanged on exit. If error encountered, it is set to a non-zero integer. Error numbers set by this subroutine:

-200 : Invalid calling sequence - need to call ZGSSIN first.
-201 : Failure to dynamically allocate memory.
-666 : Internal error.

Contents

NAME

```
zgssps - Print general sparse solver statics.
```

SYNOPSIS

```
 SUBROUTINE ZGSSPS ( HANDLE, IER )

 INTEGER            IER
 DOUBLE PRECISION  HANDLE(150)
```

PURPOSE

ZGSSPS - Print solver statistics.

PARAMETERS

```
HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -800 : Invalid calling sequence - need to call ZGSSSL first.
         -899 : Printed solver statistics not supported this release.
```

Contents

NAME

zgssrp - Return permutation used by the general sparse solver.

SYNOPSIS

```
 SUBROUTINE ZGSSRP ( PERM, HANDLE, IER )

 INTEGER           PERM(*), IER
 DOUBLE PRECISION  HANDLE(150)
```

PURPOSE

ZGSSRP - Returns the permutation used by the solver for the fill-reducing ordering.

PARAMETERS

```
PERM(NEQNS)  - INTEGER array
        Undefined on entry.  PERM(NEQNS) is the permutation
        array used by the sparse solver for the fill-
        reducing ordering.  Modified on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER      - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:
```

-600 : Invalid calling sequence - need to call ZGSSOR first.

Contents

NAME

zgsssl - Solve routine for the general sparse solver.

SYNOPSIS

```
 SUBROUTINE ZGSSSL ( NRHS, RHS, LDRHS, HANDLE, IER )

 INTEGER            NRHS, LDRHS, IER
 DOUBLE COMPLEX     RHS(LDRHS,NRHS)
 DOUBLE PRECISION   HANDLE(150)
```

PURPOSE

ZGSSSL - Triangular solve of a factored sparse matrix.

PARAMETERS

```
NRHS      - INTEGER
        On entry, NRHS specifies the number of right hand
        sides to solve for.  Unchanged on exit.

RHS(LDRHS,*) - DOUBLE COMPLEX array
        On entry, RHS(LDRHS,NRHS) contains the NRHS right
        hand sides.  On exit, it contains the solutions.

LDRHS     - INTEGER
        On entry, LDRHS specifies the leading dimension of
        the RHS array. Unchanged on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.
```

```
IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:

         -400 : Invalid calling sequence - need to call ZGSSFA first.
         -401 : Failure to dynamically allocate memory.
         -402 : NRHS < 1
         -403 : NEQNS > LDRHS
```

Contents

NAME

```
zgssuo - User supplied permutation for ordering used in the
general sparse solver.
```

SYNOPSIS

```
 SUBROUTINE ZGSSUO ( PERM, HANDLE, IER )

 INTEGER           PERM(*), IER
 DOUBLE PRECISION  HANDLE(150)
```

PURPOSE

ZGSSUO - User supplied permutation for ordering. Must be called after *ZGSSIN*() (sparse solver initialization) and before *ZGSSOR*() (sparse solver ordering).

PARAMETERS

```
PERM(NEQNS)  - INTEGER array
        On entry, PERM(NEQNS) is a permutation array
        supplied by the user for the fill-reducing ordering.
        Unchanged on exit.

HANDLE(150)  - DOUBLE PRECISION array
        On entry, HANDLE(*) is an array containing
        information needed by the solver, and must be passed
        unchanged to each sparse solver subroutine.
        Modified on exit.

IER       - INTEGER
        Error number.  If no error encountered, unchanged on
        exit.  If error encountered, it is set to a non-zero
        integer.  Error numbers set by this subroutine:
```

-500 : Invalid calling sequence - need to call ZGSSIN first.

Contents

NAME

```
zgtcon - estimate the reciprocal of the condition number  of
a complex tridiagonal matrix A using the LU factorization as
computed by CGTTRF
```

SYNOPSIS

```
SUBROUTINE ZGTCON(NORM, N, LOW, DIAG, UP1, UP2, IPIVOT, ANORM, RCOND,
      WORK, INFO)

CHARACTER * 1 NORM
DOUBLE COMPLEX LOW(*), DIAG(*), UP1(*), UP2(*), WORK(*)
INTEGER N, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION ANORM, RCOND

SUBROUTINE ZGTCON_64(NORM, N, LOW, DIAG, UP1, UP2, IPIVOT, ANORM,
      RCOND, WORK, INFO)

CHARACTER * 1 NORM
DOUBLE COMPLEX LOW(*), DIAG(*), UP1(*), UP2(*), WORK(*)
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION ANORM, RCOND
```

F95 INTERFACE

```
SUBROUTINE GTCON(NORM, [N], LOW, DIAG, UP1, UP2, IPIVOT, ANORM,
       RCOND, [WORK], [INFO])

CHARACTER(LEN=1) :: NORM
COMPLEX(8), DIMENSION(:) :: LOW, DIAG, UP1, UP2, WORK
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT
```

```
REAL(8) :: ANORM, RCOND

SUBROUTINE GTCON_64(NORM, [N], LOW, DIAG, UP1, UP2, IPIVOT, ANORM,
       RCOND, [WORK], [INFO])

CHARACTER(LEN=1) :: NORM
COMPLEX(8), DIMENSION(:) :: LOW, DIAG, UP1, UP2, WORK
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8) :: ANORM, RCOND
```

C INTERFACE

```
#include <sunperf.h>
void zgtcon(char norm, int n, doublecomplex *low, doublecom-
          plex   *diag,  doublecomplex  *up1,  doublecomplex
          *up2, int *ipivot, double  anorm,  double  *rcond,
          int *info);

void zgtcon_64(char norm, long n, doublecomplex *low,  doub-
          lecomplex *diag, doublecomplex *up1, doublecomplex
          *up2, long *ipivot, double anorm,  double  *rcond,
          long *info);
```

PURPOSE

zgtcon estimates the reciprocal of the condition number of a complex tridiagonal matrix A using the LU factorization as computed by CGTTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

NORM (input)
Specifies whether the 1-norm condition number or the infinity-norm condition number is required:
= '1' or 'O': 1-norm;
= 'I':   Infinity-norm.

N (input) The order of the matrix A. N >= 0.

LOW (input)
The (n-1) multipliers that define the matrix L from the LU factorization of A as computed by CGTTRF.

DIAG (input)
The n diagonal elements of the upper triangular matrix U from the LU factorization of A.

UP1 (input)
The (n-1) elements of the first superdiagonal of U.

UP2 (input)
The (n-2) elements of the second superdiagonal of U.

IPIVOT (input)
The pivot indices; for 1 <= i <= n, row i of the matrix was interchanged with row IPIVOT(i). IPIVOT(i) will always be either i or i+1; IPIVOT(i) = i indicates a row interchange was not required.

ANORM (input)
If NORM = '1' or 'O', the 1-norm of the original matrix A. If NORM = 'I', the infinity-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zgthr - Gathers specified elements from y into x.
```

SYNOPSIS

```
 SUBROUTINE ZGTHR(NZ, Y, X, INDX)

 DOUBLE COMPLEX Y(*), X(*)
 INTEGER NZ
 INTEGER INDX(*)

 SUBROUTINE ZGTHR_64(NZ, Y, X, INDX)

 DOUBLE COMPLEX Y(*), X(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 SUBROUTINE GTHR([NZ], Y, X, INDX)

 COMPLEX(8), DIMENSION(:) :: Y, X
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 SUBROUTINE GTHR_64([NZ], Y, X, INDX)

 COMPLEX(8), DIMENSION(:) :: Y, X
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
ZGTHR - Gathers the specified elements from a vector y in
full storage form into a vector x in compressed form.  Only
```

the elements of y whose indices are listed in indx are referenced.

```
do i = 1, n
  x(i) = y(indx(i))
enddo
```

ARGUMENTS

NZ (input) - INTEGER
Number of elements in the compressed form. Unchanged on exit.

Y (input)
Vector in full storage form. Unchanged on exit.

X (output)
Vector in compressed form. Contains elements of y whose indices are listed in indx on exit.

INDX (input) - INTEGER
Vector containing the indices of the compressed form. It is assumed that the elements in INDX are distinct and greater than zero. Unchanged on exit.

Contents

NAME

```
zgthrz - Gather and zero.
```

SYNOPSIS

```
 SUBROUTINE ZGTHRZ(NZ, Y, X, INDX)

 DOUBLE COMPLEX Y(*), X(*)
 INTEGER NZ
 INTEGER INDX(*)

 SUBROUTINE ZGTHRZ_64(NZ, Y, X, INDX)

 DOUBLE COMPLEX Y(*), X(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 SUBROUTINE GTHRZ([NZ], Y, X, INDX)

 COMPLEX(8), DIMENSION(:) :: Y, X
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 SUBROUTINE GTHRZ_64([NZ], Y, X, INDX)

 COMPLEX(8), DIMENSION(:) :: Y, X
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
ZGTHRZ - Gathers the specified elements from a vector y in
full storage form into a vector x in compressed form.  The
```

gathered elements of y are set to zero. Only the elements of y whose indices are listed in indx are referenced.

```
 do i = 1, n
   x(i) = y(indx(i))
   y(indx(i)) = 0
 enddo
```

ARGUMENTS

NZ (input) - INTEGER
Number of elements in the compressed form. Unchanged on exit.

Y (input/output)
Vector in full storage form. Gathered elements are set to zero.

X (output)
Vector in compressed form. Contains elements of y whose indices are listed in indx on exit.

INDX (input) - INTEGER
Vector containing the indices of the compressed form. It is assumed that the elements in INDX are distinct and greater than zero. Unchanged on exit.

Contents

NAME

```
zgtrfs - improve the computed solution to a system of linear
equations  when  the  coefficient matrix is tridiagonal, and
provides error bounds and backward error estimates  for  the
solution
```

SYNOPSIS

```
SUBROUTINE ZGTRFS(TRANSA, N, NRHS, LOW, DIAG, UP, LOWF, DIAGF, UPF1,
      UPF2, IPIVOT, B, LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 TRANSA
DOUBLE COMPLEX LOW(*), DIAG(*),  UP(*),  LOWF(*),  DIAGF(*),
UPF1(*), UPF2(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZGTRFS_64(TRANSA, N, NRHS, LOW, DIAG, UP, LOWF, DIAGF,
      UPF1, UPF2, IPIVOT, B, LDB, X, LDX, FERR, BERR, WORK, WORK2,
      INFO)

CHARACTER * 1 TRANSA
DOUBLE COMPLEX LOW(*), DIAG(*),  UP(*),  LOWF(*),  DIAGF(*),
UPF1(*), UPF2(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GTRFS([TRANSA], [N], [NRHS], LOW, DIAG, UP, LOWF, DIAGF,
       UPF1, UPF2, IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK],
       [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: TRANSA
COMPLEX(8), DIMENSION(:) ::  LOW,  DIAG,  UP,  LOWF,  DIAGF,
UPF1, UPF2, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE GTRFS_64([TRANSA], [N], [NRHS], LOW, DIAG, UP, LOWF,
       DIAGF, UPF1, UPF2, IPIVOT, B, [LDB], X, [LDX], FERR, BERR, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: TRANSA
COMPLEX(8), DIMENSION(:) ::  LOW,  DIAG,  UP,  LOWF,  DIAGF,
UPF1, UPF2, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zgtrfs(char transa,  int  n,  int  nrhs,  doublecomplex
          *low,   doublecomplex  *diag,  doublecomplex  *up,
          doublecomplex *lowf, doublecomplex  *diagf,  doub-
          lecomplex *upf1, doublecomplex *upf2, int *ipivot,
          doublecomplex *b, int ldb, doublecomplex  *x,  int
          ldx, double *ferr, double *berr, int *info);

void zgtrfs_64(char transa, long n, long nrhs, doublecomplex
          *low,   doublecomplex  *diag,  doublecomplex  *up,
          doublecomplex *lowf, doublecomplex  *diagf,  doub-
          lecomplex   *upf1,   doublecomplex   *upf2,   long
          *ipivot, doublecomplex *b, long ldb, doublecomplex
          *x,  long  ldx,  double  *ferr, double *berr, long
          *info);
```

PURPOSE

zgtrfs improves the computed solution to a system of linear equations when the coefficient matrix is tridiagonal, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

TRANSA (input)
Specifies the form of the system of equations:
= 'N': A * X = B (No transpose)
= 'T': A**T * X = B (Transpose)
= 'C': A**H * X = B (Conjugate transpose)

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

LOW (input)
The (n-1) subdiagonal elements of A.

DIAG (input)
The diagonal elements of A.

UP (input)
The (n-1) superdiagonal elements of A.

LOWF (input)
The (n-1) multipliers that define the matrix L from the LU factorization of A as computed by CGTTRF.

DIAGF (input)
The n diagonal elements of the upper triangular matrix U from the LU factorization of A.

UPF1 (input)
The (n-1) elements of the first superdiagonal of U.

UPF2 (input)
The (n-2) elements of the second superdiagonal of U.

IPIVOT (input)
The pivot indices; for 1 <= i <= n, row i of the matrix was interchanged with row IPIVOT(i). IPIVOT(i) will always be either i or i+1; IPIVOT(i) = i indicates a row interchange was not required.

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by CGTTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zgtsv - solve the equation   A*X = B,
```

SYNOPSIS

```
SUBROUTINE ZGTSV(N, NRHS, LOW, DIAG, UP, B, LDB, INFO)

DOUBLE COMPLEX LOW(*), DIAG(*), UP(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO

SUBROUTINE ZGTSV_64(N, NRHS, LOW, DIAG, UP, B, LDB, INFO)

DOUBLE COMPLEX LOW(*), DIAG(*), UP(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE GTSV([N], [NRHS], LOW, DIAG, UP, B, [LDB], [INFO])

COMPLEX(8), DIMENSION(:) :: LOW, DIAG, UP
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO

SUBROUTINE GTSV_64([N], [NRHS], LOW, DIAG, UP, B, [LDB], [INFO])

COMPLEX(8), DIMENSION(:) :: LOW, DIAG, UP
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zgtsv(int n, int nrhs, doublecomplex  *low,  doublecom-
          plex  *diag,  doublecomplex *up, doublecomplex *b,
```

```
          int ldb, int *info);

void zgtsv_64(long n, long nrhs, doublecomplex  *low,  doub-
          lecomplex  *diag, doublecomplex *up, doublecomplex
          *b, long ldb, long *info);
```

PURPOSE

zgtsv solves the equation

where A is an N-by-N tridiagonal matrix, by Gaussian elimination with partial pivoting.

Note that the equation A'*X = B may be solved by interchanging the order of the arguments DU and DL.

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

LOW (input/output)
On entry, LOW must contain the (n-1) subdiagonal elements of A. On exit, LOW is overwritten by the (n-2) elements of the second superdiagonal of the upper triangular matrix U from the LU factorization of A, in LOW(1), ..., LOW(n-2).

DIAG (input/output)
On entry, DIAG must contain the diagonal elements of A. On exit, DIAG is overwritten by the n diagonal elements of U.

UP (input/output)
On entry, UP must contain the (n-1) superdiagonal elements of A. On exit, UP is overwritten by the (n-1) elements of the first superdiagonal of U.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)

The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, U(i,i) is exactly zero, and the solution has not been computed. The factorization has not been completed unless i = N.

Contents

NAME

```
zgtsvx - use the LU factorization to compute the solution to
a  complex  system of linear equations A * X = B, A**T * X =
B, or A**H * X = B,
```

SYNOPSIS

```
SUBROUTINE ZGTSVX(FACT, TRANSA, N, NRHS, LOW, DIAG, UP, LOWF, DIAGF,
      UPF1, UPF2, IPIVOT, B, LDB, X, LDX, RCOND, FERR, BERR, WORK,
      WORK2, INFO)

CHARACTER * 1 FACT, TRANSA
DOUBLE COMPLEX LOW(*), DIAG(*),  UP(*),  LOWF(*),  DIAGF(*),
UPF1(*), UPF2(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZGTSVX_64(FACT, TRANSA, N, NRHS, LOW, DIAG, UP, LOWF,
      DIAGF, UPF1, UPF2, IPIVOT, B, LDB, X, LDX, RCOND, FERR, BERR,
      WORK, WORK2, INFO)

CHARACTER * 1 FACT, TRANSA
DOUBLE COMPLEX LOW(*), DIAG(*),  UP(*),  LOWF(*),  DIAGF(*),
UPF1(*), UPF2(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE GTSVX(FACT, [TRANSA], [N], [NRHS], LOW, DIAG, UP, LOWF,
```

```
       DIAGF, UPF1, UPF2, IPIVOT, B, [LDB], X, [LDX], RCOND, FERR, BERR,
       [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, TRANSA
COMPLEX(8), DIMENSION(:) ::  LOW,  DIAG,  UP,  LOWF,  DIAGF,
UPF1, UPF2, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE GTSVX_64(FACT, [TRANSA], [N], [NRHS], LOW, DIAG, UP, LOWF,
       DIAGF, UPF1, UPF2, IPIVOT, B, [LDB], X, [LDX], RCOND, FERR, BERR,
       [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, TRANSA
COMPLEX(8), DIMENSION(:) ::  LOW,  DIAG,  UP,  LOWF,  DIAGF,
UPF1, UPF2, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zgtsvx(char fact, char transa, int n, int  nrhs,  doub-
          lecomplex *low, doublecomplex *diag, doublecomplex
          *up, doublecomplex  *lowf,  doublecomplex  *diagf,
          doublecomplex   *upf1,  doublecomplex  *upf2,  int
          *ipivot, doublecomplex *b, int ldb,  doublecomplex
          *x,  int  ldx, double *rcond, double *ferr, double
          *berr, int *info);

void zgtsvx_64(char fact, char transa, long  n,  long  nrhs,
          doublecomplex  *low,  doublecomplex  *diag,  doub-
          lecomplex *up, doublecomplex *lowf,  doublecomplex
          *diagf,  doublecomplex *upf1, doublecomplex *upf2,
          long *ipivot, doublecomplex *b,  long  ldb,  doub-
          lecomplex  *x,  long  ldx,  double  *rcond, double
          *ferr, double *berr, long *info);
```

PURPOSE

zgtsvx uses the LU factorization to compute the solution to a complex system of linear equations A * X = B, A**T * X = B, or A**H * X = B, where A is a tridiagonal matrix of order

N and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'N', the LU decomposition is used to factor the matrix A
   as A = L * U, where L is a product of permutation and unit lower
   bidiagonal matrices and U is upper triangular with nonzeros in
   only the main diagonal and first two superdiagonals.

2. If some U(i,i)=0, so that U is exactly singular, then the routine
   returns with INFO = i. Otherwise, the factored form of A is used
   to estimate the condition number of the matrix A. If the
   reciprocal of the condition number is less than machine precision,
   INFO = N+1 is returned as a warning, but the routine still goes on
   to solve for X and compute error bounds as described below.

3. The system of equations is solved for X using the factored form
   of A.

4. Iterative refinement is applied to improve the computed solution
   matrix and calculate error bounds and backward error estimates
   for it.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of A has been supplied on entry. = 'F': LOWF, DIAGF, UPF1, UPF2, and IPIVOT contain the factored form of A; LOW, DIAG, UP, LOWF, DIAGF, UPF1, UPF2 and IPIVOT will not be modified. = 'N': The matrix will be copied to LOWF, DIAGF, and UPF1 and factored.

```
TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

LOW (input)
          The (n-1) subdiagonal elements of A.
DIAG (input)
          The n diagonal elements of A.

UP (input/output)
          The (n-1) superdiagonal elements of A.

LOWF (input/output)
          If FACT = 'F', then LOWF is an input argument  and
          on  entry  contains  the  (n-1)  multipliers  that
          define the matrix L from the LU factorization of A
          as computed by CGTTRF.

          If FACT = 'N', then LOWF is an output argument and
          on exit contains the (n-1) multipliers that define
          the matrix L from the LU factorization of A.

DIAGF (input/output)
          If FACT = 'F', then DIAGF is an input argument and
          on  entry  contains the n diagonal elements of the
          upper triangular matrix U from the  LU  factoriza-
          tion of A.

          If FACT = 'N', then DIAGF is  an  output  argument
          and  on  exit  contains the n diagonal elements of
          the upper triangular matrix U from the LU factori-
          zation of A.

UPF1 (input/output)
          If FACT = 'F', then UPF1 is an input argument  and
          on  entry contains the (n-1) elements of the first
          superdiagonal of U.

          If FACT = 'N', then UPF1 is an output argument and
          on  exit  contains the (n-1) elements of the first
```

superdiagonal of U.

UPF2 (input/output)
If FACT = 'F', then UPF2 is an input argument and on entry contains the (n-2) elements of the second superdiagonal of U.

If FACT = 'N', then UPF2 is an output argument and on exit contains the (n-2) elements of the second superdiagonal of U.

IPIVOT (input/output)
If FACT = 'F', then IPIVOT is an input argument and on entry contains the pivot indices from the LU factorization of A as computed by CGTTRF.

If FACT = 'N', then IPIVOT is an output argument and on exit contains the pivot indices from the LU factorization of A; row i of the matrix was interchanged with row IPIVOT(i). IPIVOT(i) will always be either i or i+1; IPIVOT(i) = i indicates a row interchange was not required.

B (input) The N-by-NRHS right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A. If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated

upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: U(i,i) is exactly zero. The factorization has not been completed unless i = N, but the factor U is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
zgttrf - compute an LU factorization of a complex  tridiago-
nal matrix A using elimination with partial pivoting and row
interchanges
```

SYNOPSIS

```
SUBROUTINE ZGTTRF(N, LOW, DIAG, UP1, UP2, IPIVOT, INFO)

DOUBLE COMPLEX LOW(*), DIAG(*), UP1(*), UP2(*)
INTEGER N, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZGTTRF_64(N, LOW, DIAG, UP1, UP2, IPIVOT, INFO)

DOUBLE COMPLEX LOW(*), DIAG(*), UP1(*), UP2(*)
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE GTTRF([N], LOW, DIAG, UP1, UP2, IPIVOT, [INFO])

COMPLEX(8), DIMENSION(:) :: LOW, DIAG, UP1, UP2
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE GTTRF_64([N], LOW, DIAG, UP1, UP2, IPIVOT, [INFO])

COMPLEX(8), DIMENSION(:) :: LOW, DIAG, UP1, UP2
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void zgttrf(int n, doublecomplex *low, doublecomplex  *diag,
          doublecomplex   *up1,   doublecomplex   *up2,  int
          *ipivot, int *info);

void zgttrf_64(long  n,  doublecomplex  *low,  doublecomplex
          *diag,  doublecomplex  *up1,  doublecomplex  *up2,
          long *ipivot, long *info);
```

PURPOSE

zgttrf computes an LU factorization of a complex tridiagonal matrix A using elimination with partial pivoting and row interchanges.
The factorization has the form
   A = L * U
where L is a product of permutation and unit lower bidiagonal matrices and U is upper triangular with nonzeros in only the main diagonal and first two superdiagonals.

ARGUMENTS

N (input) The order of the matrix A.

LOW (input/output)
On entry, LOW must contain the (n-1) sub-diagonal elements of A.

On exit, LOW is overwritten by the (n-1) multipliers that define the matrix L from the LU factorization of A.

DIAG (input/output)
On entry, DIAG must contain the diagonal elements of A.

On exit, DIAG is overwritten by the n diagonal elements of the upper triangular matrix U from the LU factorization of A.

UP1 (input/output)
On entry, UP1 must contain the (n-1) super-diagonal elements of A.

On exit, UP1 is overwritten by the (n-1) elements of the first super-diagonal of U.

UP2 (output)

On exit, UP2 is overwritten by the (n-2) elements of the second super-diagonal of U.

IPIVOT (output)

The pivot indices; for 1 <= i <= n, row i of the matrix was interchanged with row IPIVOT(i). IPIVOT(i) will always be either i or i+1; IPIVOT(i) = i indicates a row interchange was not required.

INFO (output)

= 0: successful exit

< 0: if INFO = -k, the k-th argument had an illegal value

> 0: if INFO = k, U(k,k) is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

Contents

NAME

```
zgttrs - solve one of the systems of equations  A * X  =  B,
A**T * X = B, or A**H * X = B,
```

SYNOPSIS

```
SUBROUTINE ZGTTRS(TRANSA, N, NRHS, LOW, DIAG, UP1, UP2, IPIVOT, B,
      LDB, INFO)

CHARACTER * 1 TRANSA
DOUBLE COMPLEX LOW(*), DIAG(*), UP1(*), UP2(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZGTTRS_64(TRANSA, N, NRHS, LOW, DIAG, UP1, UP2, IPIVOT, B,
      LDB, INFO)

CHARACTER * 1 TRANSA
DOUBLE COMPLEX LOW(*), DIAG(*), UP1(*), UP2(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE GTTRS([TRANSA], [N], [NRHS], LOW, DIAG, UP1, UP2, IPIVOT,
       B, [LDB], [INFO])

CHARACTER(LEN=1) :: TRANSA
COMPLEX(8), DIMENSION(:) :: LOW, DIAG, UP1, UP2
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE GTTRS_64([TRANSA], [N], [NRHS], LOW, DIAG, UP1, UP2,
```

```
          IPIVOT, B, [LDB], [INFO])

   CHARACTER(LEN=1) :: TRANSA
   COMPLEX(8), DIMENSION(:) :: LOW, DIAG, UP1, UP2
   COMPLEX(8), DIMENSION(:,:) :: B
   INTEGER(8) :: N, NRHS, LDB, INFO
   INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
   #include <sunperf.h>

   void zgttrs(char transa,  int  n,  int  nrhs,  doublecomplex
             *low,  doublecomplex  *diag,  doublecomplex  *up1,
             doublecomplex *up2, int *ipivot, doublecomplex *b,
             int ldb, int *info);
   void zgttrs_64(char transa, long n, long nrhs, doublecomplex
             *low,  doublecomplex  *diag,  doublecomplex  *up1,
             doublecomplex *up2,  long  *ipivot,  doublecomplex
             *b, long ldb, long *info);
```

PURPOSE

```
   zgttrs solves one of the systems of equations
      A * X = B,  A**T * X = B,  or  A**H * X = B, with a  tri-
   diagonal  matrix  A  using  the LU factorization computed by
   CGTTRF.
```

ARGUMENTS

```
   TRANSA (input)
             Specifies the form of the system of equations.   =
             'N':  A * X = B     (No transpose)
             = 'T':  A**T * X = B  (Transpose)
             = 'C':  A**H * X = B  (Conjugate transpose)

             TRANSA is defaulted to 'N' for F95 INTERFACE.

   N (input) The order of the matrix A.

   NRHS (input)
             The number of right hand sides, i.e.,  the  number
             of columns of the matrix B.  NRHS >= 0.

   LOW (input)
             The (n-1) multipliers that  define  the  matrix  L
             from the LU factorization of A.
```

DIAG (input)

The n diagonal elements of the upper triangular matrix U from the LU factorization of A.

UP1 (input)

The (n-1) elements of the first super-diagonal of U.

UP2 (input)

The (n-2) elements of the second super-diagonal of U.

IPIVOT (input)

The pivot indices; for 1 <= i <= n, row i of the matrix was interchanged with row IPIVOT(i). IPIVOT(i) will always be either i or i+1; IPIVOT(i) = i indicates a row interchange was not required.

B (input/output)

On entry, the matrix of right hand side vectors B. On exit, B is overwritten by the solution vectors X.

LDB (input)

The leading dimension of the array B. LDB >= max(1,N).

INFO (output)

= 0: successful exit

< 0: if INFO = -k, the k-th argument had an illegal value

Contents

NAME

zhbev - compute all the eigenvalues and, optionally, eigen-
vectors of a complex Hermitian band matrix A

SYNOPSIS

```
SUBROUTINE ZHBEV(JOBZ, UPLO, N, KD, A, LDA, W, Z, LDZ, WORK,
      WORK2, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX A(LDA,*), Z(LDZ,*), WORK(*)
INTEGER N, KD, LDA, LDZ, INFO
DOUBLE PRECISION W(*), WORK2(*)

SUBROUTINE ZHBEV_64(JOBZ, UPLO, N, KD, A, LDA, W, Z, LDZ, WORK,
      WORK2, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX A(LDA,*), Z(LDZ,*), WORK(*)
INTEGER*8 N, KD, LDA, LDZ, INFO
DOUBLE PRECISION W(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HBEV(JOBZ, UPLO, [N], KD, A, [LDA], W, Z, [LDZ], [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, Z
INTEGER :: N, KD, LDA, LDZ, INFO
REAL(8), DIMENSION(:) :: W, WORK2

SUBROUTINE HBEV_64(JOBZ, UPLO, [N], KD, A, [LDA], W, Z, [LDZ],
```

```
          [WORK], [WORK2], [INFO])

   CHARACTER(LEN=1) :: JOBZ, UPLO
   COMPLEX(8), DIMENSION(:) :: WORK
   COMPLEX(8), DIMENSION(:,:) :: A, Z
   INTEGER(8) :: N, KD, LDA, LDZ, INFO
   REAL(8), DIMENSION(:) :: W, WORK2
```

C INTERFACE

```
   #include <sunperf.h>

   void zhbev(char jobz, char uplo, int n, int  kd,  doublecom-
             plex *a, int lda, double *w, doublecomplex *z, int
             ldz, int *info);
   void zhbev_64(char jobz, char uplo, long n, long  kd,  doub-
             lecomplex  *a,  long lda, double *w, doublecomplex
             *z, long ldz, long *info);
```

PURPOSE

zhbev computes all the eigenvalues and, optionally, eigenvectors of a complex Hermitian band matrix A.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  KD >= 0.

A (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  band  matrix  A,  stored in the first KD+1
          rows of the array.  The j-th column of A is stored
          in  the j-th column of the array A as follows:  if
          UPLO = 'U', A(kd+1+i-j,j) =  A(i,j)  for  max(1,j-
          kd)<=i<=j;  if  UPLO = 'L', A(1+i-j,j)    = A(i,j)
```

```
          for j<=i<=min(n,j+kd).

          On exit, A is overwritten by values generated dur-
          ing  the reduction to tridiagonal form.  If UPLO =
          'U', the first superdiagonal and the  diagonal  of
          the  tridiagonal  matrix T are returned in rows KD
          and KD+1 of A, and if UPLO = 'L', the diagonal and
          first  subdiagonal  of T are returned in the first
          two rows of A.

LDA (input)
          The leading dimension of the array A.  LDA >= KD +
          1.
W (output)
          If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0,  Z  contains  the
          orthonormal eigenvectors of the matrix A, with the
          i-th column of Z holding the  eigenvector  associ-
          ated  with  W(i).   If  JOBZ  = 'N', then Z is not
          referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
          dimension(N)

WORK2 (workspace)
          dimension(max(1,3*N-2))

INFO (output)
          = 0:  successful exit.
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value.
          > 0:  if INFO = i, the algorithm  failed  to  con-
          verge;  i off-diagonal elements of an intermediate
          tridiagonal form did not converge to zero.
```

Contents

NAME

```
zhbevd - compute all the eigenvalues and, optionally, eigen-
vectors of a complex Hermitian band matrix A
```

SYNOPSIS

```
SUBROUTINE ZHBEVD(JOBZ, UPLO, N, KD, AB, LDAB, W, Z, LDZ, WORK,
      LWORK, RWORK, LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX AB(LDAB,*), Z(LDZ,*), WORK(*)
INTEGER N, KD, LDAB, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER IWORK(*)
DOUBLE PRECISION W(*), RWORK(*)

SUBROUTINE ZHBEVD_64(JOBZ, UPLO, N, KD, AB, LDAB, W, Z, LDZ, WORK,
      LWORK, RWORK, LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX AB(LDAB,*), Z(LDZ,*), WORK(*)
INTEGER*8 N, KD, LDAB, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HBEVD(JOBZ, UPLO, [N], KD, AB, [LDAB], W, Z, [LDZ], [WORK],
       [LWORK], [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: AB, Z
INTEGER :: N, KD, LDAB, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
```

```
REAL(8), DIMENSION(:) :: W, RWORK

SUBROUTINE HBEVD_64(JOBZ, UPLO, [N], KD, AB, [LDAB], W, Z, [LDZ],
       [WORK], [LWORK], [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: AB, Z
INTEGER(8) :: N, KD, LDAB, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>
void zhbevd(char jobz, char uplo, int n, int kd,  doublecom-
          plex  *ab,  int ldab, double *w, doublecomplex *z,
          int ldz, int *info);

void zhbevd_64(char jobz, char uplo, long n, long kd,  doub-
          lecomplex *ab, long ldab, double *w, doublecomplex
          *z, long ldz, long *info);
```

PURPOSE

zhbevd computes all the eigenvalues and, optionally, eigenvectors of a complex Hermitian band matrix A. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

JOBZ (input)
- = 'N': Compute eigenvalues only;
- = 'V': Compute eigenvalues and eigenvectors.

UPLO (input)
- = 'U': Upper triangle of A is stored;
- = 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

AB (input/output)
On entry, the upper or lower triangle of the Hermitian band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).
On exit, AB is overwritten by values generated during the reduction to tridiagonal form. If UPLO = 'U', the first superdiagonal and the diagonal of the tridiagonal matrix T are returned in rows KD and KD+1 of AB, and if UPLO = 'L', the diagonal and first subdiagonal of T are returned in the first two rows of AB.

LDAB (input)
The leading dimension of the array AB. LDAB >= KD + 1.

W (output)
If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, Z contains the orthonormal eigenvectors of the matrix A, with the i-th column of Z holding the eigenvector associated with W(i). If JOBZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If N <= 1, LWORK must be at least 1. If JOBZ = 'N' and N > 1, LWORK must be at least N. If JOBZ = 'V' and N > 1, LWORK must be at least 2*N**2.

If LWORK = -1, then a workspace query is assumed;

the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
dimension (LRWORK) On exit, if INFO = 0, RWORK(1) returns the optimal LRWORK.

LRWORK (input)
The dimension of array RWORK. If N <= 1, LRWORK must be at least 1. If JOBZ = 'N' and N > 1, LRWORK must be at least N. If JOBZ = 'V' and N > 1, LRWORK must be at least 1 + 5*N + 2*N**2.

If LRWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the RWORK array, returns this value as the first entry of the RWORK array, and no error message related to LRWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of array IWORK. If JOBZ = 'N' or N <= 1, LIWORK must be at least 1. If JOBZ = 'V' and N > 1, LIWORK must be at least 3 + 5*N .

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

Contents

NAME

zhbevx - compute selected eigenvalues and, optionally, eigenvectors of a complex Hermitian band matrix A

SYNOPSIS

```
SUBROUTINE ZHBEVX(JOBZ, RANGE, UPLO, N, KD, A, LDA, Q, LDQ, VL,
      VU, IL, IU, ABTOL, NFOUND, W, Z, LDZ, WORK, WORK2, IWORK3, IFAIL,
      INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
DOUBLE COMPLEX A(LDA,*), Q(LDQ,*), Z(LDZ,*), WORK(*)
INTEGER N, KD, LDA, LDQ, IL, IU, NFOUND, LDZ, INFO
INTEGER IWORK3(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABTOL
DOUBLE PRECISION W(*), WORK2(*)

SUBROUTINE ZHBEVX_64(JOBZ, RANGE, UPLO, N, KD, A, LDA, Q, LDQ, VL,
      VU, IL, IU, ABTOL, NFOUND, W, Z, LDZ, WORK, WORK2, IWORK3, IFAIL,
      INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
DOUBLE COMPLEX A(LDA,*), Q(LDQ,*), Z(LDZ,*), WORK(*)
INTEGER*8 N, KD, LDA, LDQ, IL, IU, NFOUND, LDZ, INFO
INTEGER*8 IWORK3(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABTOL
DOUBLE PRECISION W(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HBEVX(JOBZ, RANGE, UPLO, [N], KD, A, [LDA], Q, [LDQ],
       VL, VU, IL, IU, ABTOL, [NFOUND], W, Z, [LDZ], [WORK], [WORK2],
       [IWORK3], IFAIL, [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, Q, Z
INTEGER :: N, KD, LDA, LDQ, IL, IU, NFOUND, LDZ, INFO
INTEGER, DIMENSION(:) :: IWORK3, IFAIL
REAL(8) :: VL, VU, ABTOL
REAL(8), DIMENSION(:) :: W, WORK2

SUBROUTINE HBEVX_64(JOBZ, RANGE, UPLO, [N], KD, A, [LDA], Q, [LDQ],
       VL, VU, IL, IU, ABTOL, [NFOUND], W, Z, [LDZ], [WORK], [WORK2],
       [IWORK3], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, Q, Z
INTEGER(8) :: N, KD, LDA, LDQ, IL, IU, NFOUND, LDZ, INFO
INTEGER(8), DIMENSION(:) :: IWORK3, IFAIL
REAL(8) :: VL, VU, ABTOL
REAL(8), DIMENSION(:) :: W, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zhbevx(char jobz, char range, char uplo, int n, int kd,
          doublecomplex  *a,  int lda, doublecomplex *q, int
          ldq, double vl, double vu, int il, int iu,  double
          abtol,  int  *nfound, double *w, doublecomplex *z,
          int ldz, int *ifail, int *info);

void zhbevx_64(char jobz, char range,  char  uplo,  long  n,
          long kd, doublecomplex *a, long lda, doublecomplex
          *q, long ldq, double vl, double vu, long il,  long
          iu,  double  abtol, long *nfound, double *w, doub-
          lecomplex *z, long ldz, long *ifail, long *info);
```

PURPOSE

zhbevx computes selected eigenvalues and, optionally, eigenvectors of a complex Hermitian band matrix A. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;

= 'V': Compute eigenvalues and eigenvectors.

RANGE (input)
= 'A': all eigenvalues will be found;
= 'V': all eigenvalues in the half-open interval (VL,VU] will be found; = 'I': the IL-th through IU-th eigenvalues will be found.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

A (input/output)
On entry, the upper or lower triangle of the Hermitian band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).

On exit, A is overwritten by values generated during the reduction to tridiagonal form.

LDA (input)
The leading dimension of the array A. LDA >= KD + 1.

Q (output)
If JOBZ = 'V', the N-by-N unitary matrix used in the reduction to tridiagonal form. If JOBZ = 'N', the array Q is not referenced.

LDQ (input)
The leading dimension of the array Q. If JOBZ = 'V', then LDQ >= max(1,N).

VL (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)

If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

IL (input)

If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)

If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

ABTOL (input)

The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing A to tri- diagonal form.

Eigenvalues will be computed most accurately when ABTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABTOL to 2*SLAMCH('S').

See "Computing Small Singular Values of Bidiagonal Matrices with Guaranteed High Relative Accuracy," by Demmel and Kahan, LAPACK Working Note #3.

NFOUND (output)

The total number of eigenvalues found. 0 <= NFOUND <= N. If RANGE = 'A', NFOUND = N, and if RANGE = 'I', NFOUND = IU-IL+1.

W (output)

The first NFOUND elements contain the selected eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, the first NFOUND columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). If an eigenvector fails to converge, then that column of Z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in IFAIL. If JOBZ = 'N', then Z is not referenced. Note: the user must ensure that at least max(1,NFOUND) columns are supplied in the array Z; if RANGE = 'V', the exact value of NFOUND is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
dimension(N)

WORK2 (workspace)
dimension(7*N)

IWORK3 (workspace)
dimension(5*N)

IFAIL (output)
If JOBZ = 'V', then if INFO = 0, the first NFOUND elements of IFAIL are zero. If INFO > 0, then IFAIL contains the indices of the eigenvectors that failed to converge. If JOBZ = 'N', then IFAIL is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, then i eigenvectors failed to converge. Their indices are stored in array IFAIL.

Contents

NAME

```
zhbgst - reduce a complex Hermitian-definite banded general-
ized  eigenproblem  A*x  = lambda*B*x to standard form C*y =
lambda*y,
```

SYNOPSIS

```
SUBROUTINE ZHBGST(VECT, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, X, LDX,
      WORK, RWORK, INFO)

CHARACTER * 1 VECT, UPLO
DOUBLE COMPLEX AB(LDAB,*), BB(LDBB,*), X(LDX,*), WORK(*)
INTEGER N, KA, KB, LDAB, LDBB, LDX, INFO
DOUBLE PRECISION RWORK(*)

SUBROUTINE ZHBGST_64(VECT, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, X,
      LDX, WORK, RWORK, INFO)

CHARACTER * 1 VECT, UPLO
DOUBLE COMPLEX AB(LDAB,*), BB(LDBB,*), X(LDX,*), WORK(*)
INTEGER*8 N, KA, KB, LDAB, LDBB, LDX, INFO
DOUBLE PRECISION RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HBGST(VECT, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB], X,
       [LDX], [WORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: VECT, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: AB, BB, X
INTEGER :: N, KA, KB, LDAB, LDBB, LDX, INFO
REAL(8), DIMENSION(:) :: RWORK
```

```
SUBROUTINE HBGST_64(VECT, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB],
       X, [LDX], [WORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: VECT, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: AB, BB, X
INTEGER(8) :: N, KA, KB, LDAB, LDBB, LDX, INFO
REAL(8), DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>

void zhbgst(char vect, char uplo, int n,  int  ka,  int  kb,
          doublecomplex  *ab,  int  ldab, doublecomplex *bb,
          int ldbb, doublecomplex *x, int ldx, int *info);
void zhbgst_64(char vect, char uplo, long n, long  ka,  long
          kb,  doublecomplex  *ab,  long ldab, doublecomplex
          *bb, long ldbb, doublecomplex *x, long  ldx,  long
          *info);
```

PURPOSE

zhbgst reduces a complex Hermitian-definite banded generalized eigenproblem A*x = lambda*B*x to standard form C*y = lambda*y, such that C has the same bandwidth as A.

B must have been previously factorized as S**H*S by CPBSTF, using a split Cholesky factorization. A is overwritten by C = X**H*A*X, where X = S**(-1)*Q and Q is a unitary matrix chosen to preserve the bandwidth of A.

ARGUMENTS

```
VECT (input)
          = 'N':  do not form the transformation matrix X;
          = 'V':  form X.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrices A and B.  N >= 0.

KA (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  KA >= 0.
```

KB (input)
The number of superdiagonals of the matrix B if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KA >= KB >= 0.

AB (input/output)
On entry, the upper or lower triangle of the Hermitian band matrix A, stored in the first ka+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(ka+1+i-j,j) = A(i,j) for max(1,j-ka)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+ka).
On exit, the transformed matrix X**H*A*X, stored in the same format as A.

LDAB (input)
The leading dimension of the array AB. LDAB >= KA+1.

BB (input)
The banded factor S from the split Cholesky factorization of B, as returned by CPBSTF, stored in the first kb+1 rows of the array.

LDBB (input)
The leading dimension of the array BB. LDBB >= KB+1.

X (output)
If VECT = 'V', the n-by-n matrix X. If VECT = 'N', the array X is not referenced.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N) if VECT = 'V'; LDX >= 1 otherwise.

WORK (workspace)
dimension(N)

RWORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

```
zhbgv - compute all the  eigenvalues,  and  optionally,  the
eigenvectors  of  a  complex  generalized Hermitian-definite
banded eigenproblem, of the form A*x=(lambda)*B*x
```

SYNOPSIS

```
SUBROUTINE ZHBGV(JOBZ, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, W, Z,
      LDZ, WORK, RWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX AB(LDAB,*), BB(LDBB,*), Z(LDZ,*), WORK(*)
INTEGER N, KA, KB, LDAB, LDBB, LDZ, INFO
DOUBLE PRECISION W(*), RWORK(*)

SUBROUTINE ZHBGV_64(JOBZ, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, W, Z,
      LDZ, WORK, RWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX AB(LDAB,*), BB(LDBB,*), Z(LDZ,*), WORK(*)
INTEGER*8 N, KA, KB, LDAB, LDBB, LDZ, INFO
DOUBLE PRECISION W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HBGV(JOBZ, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB], W,
       Z, [LDZ], [WORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: AB, BB, Z
INTEGER :: N, KA, KB, LDAB, LDBB, LDZ, INFO
REAL(8), DIMENSION(:) :: W, RWORK
```

```
SUBROUTINE HBGV_64(JOBZ, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB],
       W, Z, [LDZ], [WORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: AB, BB, Z
INTEGER(8) :: N, KA, KB, LDAB, LDBB, LDZ, INFO
REAL(8), DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void zhbgv(char jobz, char uplo, int  n,  int  ka,  int  kb,
          doublecomplex  *ab,  int  ldab, doublecomplex *bb,
          int ldbb, double *w, doublecomplex  *z,  int  ldz,
          int *info);

void zhbgv_64(char jobz, char uplo, long n,  long  ka,  long
          kb,  doublecomplex  *ab,  long ldab, doublecomplex
          *bb, long ldbb, double *w, doublecomplex *z,  long
          ldz, long *info);
```

PURPOSE

zhbgv computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite banded eigenproblem, of the form A*x=(lambda)*B*x. Here A and B are assumed to be Hermitian and banded, and B is also positive definite.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

UPLO (input)
= 'U': Upper triangles of A and B are stored;
= 'L': Lower triangles of A and B are stored.

N (input) The order of the matrices A and B. N >= 0.

KA (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KA >= 0.

```
KB (input)
          The number of superdiagonals of the  matrix  B  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'. KB >= 0.

AB (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  band  matrix  A,  stored in the first ka+1
          rows of the array.  The j-th column of A is stored
          in the j-th column of the array AB as follows:  if
          UPLO = 'U', AB(ka+1+i-j,j) = A(i,j)  for  max(1,j-
          ka)<=i<=j;  if UPLO = 'L', AB(1+i-j,j)    = A(i,j)
          for j<=i<=min(n,j+ka).
          On exit, the contents of AB are destroyed.

LDAB (input)
          The leading dimension of the array  AB.   LDAB  >=
          KA+1.

BB (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  band  matrix  B,  stored in the first kb+1
          rows of the array.  The j-th column of B is stored
          in the j-th column of the array BB as follows:  if
          UPLO = 'U', BB(kb+1+i-j,j) = B(i,j)  for  max(1,j-
          kb)<=i<=j;  if UPLO = 'L', BB(1+i-j,j)    = B(i,j)
          for j<=i<=min(n,j+kb).

          On exit, the factor S from the split Cholesky fac-
          torization B = S**H*S, as returned by CPBSTF.

LDBB (input)
          The leading dimension of the array  BB.   LDBB  >=
          KB+1.

W (output)
          If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0,  Z  contains  the
          matrix  Z of eigenvectors, with the i-th column of
          Z holding the eigenvector  associated  with  W(i).
          The eigenvectors are normalized so that Z**H*B*Z =
          I.  If JOBZ = 'N', then Z is not referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= N.

WORK (workspace)
          dimension(N)
```

RWORK (workspace)
dimension(3*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-
gal value
> 0: if INFO = i, and i is:
<= N: the algorithm failed to converge: i off-
diagonal elements of an intermediate tridiagonal
form did not converge to zero; > N: if INFO = N
+ i, for 1 <= i <= N, then CPBSTF
returned INFO = i: B is not positive definite.
The factorization of B could not be completed and
no eigenvalues or eigenvectors were computed.

Contents

NAME

```
zhbgvd - compute all the eigenvalues,  and  optionally,  the
eigenvectors  of  a  complex  generalized Hermitian-definite
banded eigenproblem, of the form A*x=(lambda)*B*x
```

SYNOPSIS

```
SUBROUTINE ZHBGVD(JOBZ, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, W, Z,
      LDZ, WORK, LWORK, RWORK, LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX AB(LDAB,*), BB(LDBB,*), Z(LDZ,*), WORK(*)
INTEGER N, KA, KB, LDAB, LDBB, LDZ, LWORK,  LRWORK,  LIWORK,
INFO
INTEGER IWORK(*)
DOUBLE PRECISION W(*), RWORK(*)

SUBROUTINE ZHBGVD_64(JOBZ, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, W, Z,
      LDZ, WORK, LWORK, RWORK, LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX AB(LDAB,*), BB(LDBB,*), Z(LDZ,*), WORK(*)
INTEGER*8 N, KA, KB, LDAB, LDBB, LDZ, LWORK, LRWORK, LIWORK,
INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HBGVD(JOBZ, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB], W,
       Z, [LDZ], [WORK], [LWORK], [RWORK], [LRWORK], [IWORK], [LIWORK],
       [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: AB, BB, Z
INTEGER :: N,  KA,  KB,  LDAB,  LDBB,  LDZ,  LWORK,  LRWORK,
LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: W, RWORK

SUBROUTINE HBGVD_64(JOBZ, UPLO, [N], KA, KB, AB, [LDAB], BB, [LDBB],
       W, Z, [LDZ], [WORK], [LWORK], [RWORK], [LRWORK], [IWORK], [LIWORK],
       [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: AB, BB, Z
INTEGER(8) :: N, KA, KB, LDAB,  LDBB,  LDZ,  LWORK,  LRWORK,
LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void zhbgvd(char jobz, char uplo, int n,  int  ka,  int  kb,
          doublecomplex  *ab,  int  ldab, doublecomplex *bb,
          int ldbb, double *w, doublecomplex  *z,  int  ldz,
          int *info);

void zhbgvd_64(char jobz, char uplo, long n, long  ka,  long
          kb,  doublecomplex  *ab,  long ldab, doublecomplex
          *bb, long ldbb, double *w, doublecomplex *z,  long
          ldz, long *info);
```

PURPOSE

zhbgvd computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite banded eigenproblem, of the form A*x=(lambda)*B*x. Here A and B are assumed to be Hermitian and banded, and B is also positive definite. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

UPLO (input)
= 'U': Upper triangles of A and B are stored;
= 'L': Lower triangles of A and B are stored.

N (input) The order of the matrices A and B. N >= 0.

KA (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KA >= 0.

KB (input)
The number of superdiagonals of the matrix B if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KB >= 0.

AB (input/output)
On entry, the upper or lower triangle of the Hermitian band matrix A, stored in the first ka+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(ka+1+i-j,j) = A(i,j) for max(1,j-ka)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+ka).

On exit, the contents of AB are destroyed.

LDAB (input)
The leading dimension of the array AB. LDAB >= KA+1.

BB (input/output)
On entry, the upper or lower triangle of the Hermitian band matrix B, stored in the first kb+1 rows of the array. The j-th column of B is stored in the j-th column of the array BB as follows: if UPLO = 'U', BB(kb+1+i-j,j) = B(i,j) for max(1,j-kb)<=i<=j; if UPLO = 'L', BB(1+i-j,j) = B(i,j) for j<=i<=min(n,j+kb).

On exit, the factor S from the split Cholesky factorization B = S**H*S, as returned by CPBSTF.

LDBB (input)
The leading dimension of the array BB. LDBB >= KB+1.

W (output)
If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, Z contains the matrix Z of eigenvectors, with the i-th column of Z holding the eigenvector associated with W(i). The eigenvectors are normalized so that Z**H*B*Z = I. If JOBZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= N.

WORK (workspace)
On exit, if INFO=0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If N <= 1, LWORK >= 1. If JOBZ = 'N' and N > 1, LWORK >= N. If JOBZ = 'V' and N > 1, LWORK >= 2*N**2.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
On exit, if INFO=0, RWORK(1) returns the optimal LRWORK.

LRWORK (input)
The dimension of array RWORK. If N <= 1, LRWORK >= 1. If JOBZ = 'N' and N > 1, LRWORK >= N. If JOBZ = 'V' and N > 1, LRWORK >= 1 + 5*N + 2*N**2.

If LRWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the RWORK array, returns this value as the first entry of the RWORK array, and no error message related to LRWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if INFO=0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of array IWORK. If JOBZ = 'N' or N <= 1, LIWORK >= 1. If JOBZ = 'V' and N > 1, LIWORK >= 3 + 5*N.

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is:
<= N: the algorithm failed to converge: i off-diagonal elements of an intermediate tridiagonal form did not converge to zero; > N: if INFO = N + i, for 1 <= i <= N, then CPBSTF
returned INFO = i: B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
Mark Fahey, Department of Mathematics, Univ. of Kentucky, USA

Contents

NAME

```
zhbgvx - compute all the eigenvalues,  and  optionally,  the
eigenvectors  of  a  complex  generalized Hermitian-definite
banded eigenproblem, of the form A*x=(lambda)*B*x
```

SYNOPSIS

```
SUBROUTINE ZHBGVX(JOBZ, RANGE, UPLO, N, KA, KB, AB, LDAB, BB, LDBB,
      Q, LDQ, VL, VU, IL, IU, ABSTOL, M, W, Z, LDZ, WORK, RWORK, IWORK,
      IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
DOUBLE COMPLEX AB(LDAB,*), BB(LDBB,*),  Q(LDQ,*),  Z(LDZ,*),
WORK(*)
INTEGER N, KA, KB, LDAB, LDBB, LDQ, IL, IU, M, LDZ, INFO
INTEGER IWORK(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION W(*), RWORK(*)

SUBROUTINE ZHBGVX_64(JOBZ, RANGE, UPLO, N, KA, KB, AB, LDAB, BB,
      LDBB, Q, LDQ, VL, VU, IL, IU, ABSTOL, M, W, Z, LDZ, WORK, RWORK,
      IWORK, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
DOUBLE COMPLEX AB(LDAB,*), BB(LDBB,*),  Q(LDQ,*),  Z(LDZ,*),
WORK(*)
INTEGER*8 N, KA, KB, LDAB, LDBB, LDQ, IL, IU, M, LDZ, INFO
INTEGER*8 IWORK(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HBGVX(JOBZ, RANGE, UPLO, [N], KA, KB, AB, [LDAB], BB,
       [LDBB], Q, [LDQ], VL, VU, IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK],
       [RWORK], [IWORK], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: AB, BB, Q, Z
INTEGER :: N, KA, KB, LDAB, LDBB, LDQ, IL, IU, M, LDZ, INFO
INTEGER, DIMENSION(:) :: IWORK, IFAIL
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: W, RWORK

SUBROUTINE HBGVX_64(JOBZ, RANGE, UPLO, [N], KA, KB, AB, [LDAB], BB,
       [LDBB], Q, [LDQ], VL, VU, IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK],
       [RWORK], [IWORK], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: AB, BB, Q, Z
INTEGER(8) :: N, KA, KB, LDAB, LDBB, LDQ, IL,  IU,  M,  LDZ,
INFO
INTEGER(8), DIMENSION(:) :: IWORK, IFAIL
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void zhbgvx(char jobz, char range, char uplo, int n, int ka,
          int kb, doublecomplex *ab, int ldab, doublecomplex
          *bb, int ldbb, doublecomplex *q, int  ldq,  double
          vl,  double vu, int il, int iu, double abstol, int
          *m, double *w,  doublecomplex  *z,  int  ldz,  int
          *ifail, int *info);

void zhbgvx_64(char jobz, char range,  char  uplo,  long  n,
          long  ka,  long  kb, doublecomplex *ab, long ldab,
          doublecomplex *bb, long  ldbb,  doublecomplex  *q,
          long  ldq, double vl, double vu, long il, long iu,
          double abstol, long *m, double  *w,  doublecomplex
          *z, long ldz, long *ifail, long *info);
```

PURPOSE

zhbgvx computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite banded eigenproblem, of the form A*x=(lambda)*B*x. Here A and B are assumed to be Hermitian and banded, and B is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either all eigenvalues, a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

RANGE (input)
= 'A': all eigenvalues will be found;
= 'V': all eigenvalues in the half-open interval (VL,VU] will be found; = 'I': the IL-th through IU-th eigenvalues will be found.

UPLO (input)
= 'U': Upper triangles of A and B are stored;
= 'L': Lower triangles of A and B are stored.

N (input) The order of the matrices A and B. N >= 0.

KA (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KA >= 0.

KB (input)
The number of superdiagonals of the matrix B if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KB >= 0.

AB (input/output)
On entry, the upper or lower triangle of the Hermitian band matrix A, stored in the first ka+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(ka+1+i-j,j) = A(i,j) for max(1,j-ka)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+ka).

On exit, the contents of AB are destroyed.

LDAB (input)
The leading dimension of the array AB. LDAB >= KA+1.

BB (input/output)
On entry, the upper or lower triangle of the Her-

mitian band matrix B, stored in the first kb+1 rows of the array. The j-th column of B is stored in the j-th column of the array BB as follows: if UPLO = 'U', BB(kb+1+i-j,j) = B(i,j) for max(1,j-kb)<=i<=j; if UPLO = 'L', BB(1+i-j,j) = B(i,j) for j<=i<=min(n,j+kb).

On exit, the factor S from the split Cholesky factorization B = S**H*S, as returned by CPBSTF.

LDBB (input)

The leading dimension of the array BB. LDBB >= KB+1.

Q (output)

If JOBZ = 'V', the n-by-n matrix used in the reduction of A*x = (lambda)*B*x to standard form, i.e. C*x = (lambda)*x, and consequently C to tri-diagonal form. If JOBZ = 'N', the array Q is not referenced.

LDQ (input)

The leading dimension of the array Q. If JOBZ = 'N', LDQ >= 1. If JOBZ = 'V', LDQ >= max(1,N).

VL (input)

If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)

If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

IL (input)

If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)

If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

ABSTOL (input)

The absolute error tolerance for the eigenvalues.

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABSTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABSTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing AP to tridiagonal form.

Eigenvalues will be computed most accurately when ABSTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABSTOL to 2*SLAMCH('S').

M (output)
The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)
If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, Z contains the matrix Z of eigenvectors, with the i-th column of Z holding the eigenvector associated with W(i). The eigenvectors are normalized so that Z**H*B*Z = I. If JOBZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= N.

WORK (workspace)
dimension(N)

RWORK (workspace)
dimension(7*N)

IWORK (workspace)
dimension(5*N)

IFAIL (output)
If JOBZ = 'V', then if INFO = 0, the first M elements of IFAIL are zero. If INFO > 0, then IFAIL contains the indices of the eigenvectors that failed to converge. If JOBZ = 'N', then IFAIL is

not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is:
<= N: then i eigenvectors failed to converge. Their indices are stored in array IFAIL. > N: if INFO = N + i, for 1 <= i <= N, then CPBSTF returned INFO = i: B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
Mark Fahey, Department of Mathematics, Univ. of Kentucky, USA

Contents

NAME

```
zhbmv - perform the matrix-vector operation   y := alpha*A*x
+ beta*y
```

SYNOPSIS

```
SUBROUTINE ZHBMV(UPLO, N, K, ALPHA, A, LDA, X, INCX, BETA, Y,
      INCY)

CHARACTER * 1 UPLO
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(LDA,*), X(*), Y(*)
INTEGER N, K, LDA, INCX, INCY

SUBROUTINE ZHBMV_64(UPLO, N, K, ALPHA, A, LDA, X, INCX, BETA, Y,
      INCY)

CHARACTER * 1 UPLO
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(LDA,*), X(*), Y(*)
INTEGER*8 N, K, LDA, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE HBMV(UPLO, [N], K, ALPHA, A, [LDA], X, [INCX], BETA,
       Y, [INCY])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:) :: X, Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, K, LDA, INCX, INCY

SUBROUTINE HBMV_64(UPLO, [N], K, ALPHA, A, [LDA], X, [INCX],
```

```
          BETA, Y, [INCY])

   CHARACTER(LEN=1) :: UPLO
   COMPLEX(8) :: ALPHA, BETA
   COMPLEX(8), DIMENSION(:) :: X, Y
   COMPLEX(8), DIMENSION(:,:) :: A
   INTEGER(8) :: N, K, LDA, INCX, INCY
```

C INTERFACE

```
   #include <sunperf.h>

   void zhbmv(char uplo, int n, int  k,  doublecomplex  *alpha,
             doublecomplex  *a,  int lda, doublecomplex *x, int
             incx, doublecomplex *beta, doublecomplex  *y,  int
             incy);
   void zhbmv_64(char  uplo,  long  n,  long  k,  doublecomplex
             *alpha,  doublecomplex *a, long lda, doublecomplex
             *x, long incx, doublecomplex *beta,  doublecomplex
             *y, long incy);
```

PURPOSE

zhbmv performs the matrix-vector operation y := alpha*A*x + beta*y where alpha and beta are scalars, x and y are n element vectors and A is an n by n hermitian band matrix, with k super-diagonals.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the band matrix A is being supplied as follows:

UPLO = 'U' or 'u'   The upper triangular part of A is being supplied.

UPLO = 'L' or 'l'   The lower triangular part of A is being supplied.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

K (input)

On entry, K specifies the number of super-diagonals of the matrix A. K must satisfy 0 .le. K. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
Before entry with UPLO = 'U' or 'u', the leading (k + 1) by n part of the array A must contain the upper triangular band part of the hermitian matrix, supplied column by column, with the leading diagonal of the matrix in row (k + 1) of the array, the first super-diagonal starting at position 2 in row k, and so on. The top left k by k triangle of the array A is not referenced. The following program segment will transfer the upper triangular part of a hermitian band matrix from conventional full matrix storage to band storage:

```
    DO 20, J = 1, N
      M = K + 1 - J
      DO 10, I = MAX( 1, J - K ), J
        A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Before entry with UPLO = 'L' or 'l', the leading (k + 1) by n part of the array A must contain the lower triangular band part of the hermitian matrix, supplied column by column, with the leading diagonal of the matrix in row 1 of the array, the first sub-diagonal starting at position 1 in row 2, and so on. The bottom right k by k triangle of the array A is not referenced. The following program segment will transfer the lower triangular part of a hermitian band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = 1 - J
     DO 10, I = J, MIN( N, J + K )
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Note that the imaginary parts of the diagonal elements need not be set and are assumed to be zero. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= (k + 1). Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX must not be zero. Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

zhbtrd - reduce a complex Hermitian band matrix A to real symmetric tridiagonal form T by a unitary similarity transformation

SYNOPSIS

```
SUBROUTINE ZHBTRD(VECT, UPLO, N, KD, AB, LDAB, D, E, Q, LDQ, WORK,
      INFO)

CHARACTER * 1 VECT, UPLO
DOUBLE COMPLEX AB(LDAB,*), Q(LDQ,*), WORK(*)
INTEGER N, KD, LDAB, LDQ, INFO
DOUBLE PRECISION D(*), E(*)

SUBROUTINE ZHBTRD_64(VECT, UPLO, N, KD, AB, LDAB, D, E, Q, LDQ, WORK,
      INFO)

CHARACTER * 1 VECT, UPLO
DOUBLE COMPLEX AB(LDAB,*), Q(LDQ,*), WORK(*)
INTEGER*8 N, KD, LDAB, LDQ, INFO
DOUBLE PRECISION D(*), E(*)
```

F95 INTERFACE

```
SUBROUTINE HBTRD(VECT, UPLO, [N], KD, AB, [LDAB], D, E, Q, [LDQ],
       [WORK], [INFO])

CHARACTER(LEN=1) :: VECT, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: AB, Q
INTEGER :: N, KD, LDAB, LDQ, INFO
REAL(8), DIMENSION(:) :: D, E
```

```
SUBROUTINE HBTRD_64(VECT, UPLO, [N], KD, AB, [LDAB], D, E, Q, [LDQ],
       [WORK], [INFO])

CHARACTER(LEN=1) :: VECT, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: AB, Q
INTEGER(8) :: N, KD, LDAB, LDQ, INFO
REAL(8), DIMENSION(:) :: D, E
```

C INTERFACE

```
#include <sunperf.h>

void zhbtrd(char vect, char uplo, int n, int kd,  doublecom-
          plex  *ab,  int  ldab, double *d, double *e, doub-
          lecomplex *q, int ldq, int *info);
void zhbtrd_64(char vect, char uplo, long n, long kd,  doub-
          lecomplex  *ab,  long  ldab, double *d, double *e,
          doublecomplex *q, long ldq, long *info);
```

PURPOSE

zhbtrd reduces a complex Hermitian band matrix A to real symmetric tridiagonal form T by a unitary similarity transformation: Q**H * A * Q = T.

ARGUMENTS

VECT (input)
- = 'N': do not form Q;
- = 'V': form Q;
- = 'U': update a matrix X, by forming X*Q.

UPLO (input)
- = 'U': Upper triangle of A is stored;
- = 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

AB (input/output)
On entry, the upper or lower triangle of the Hermitian band matrix A, stored in the first KD+1

rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd). On exit, the diagonal elements of AB are overwritten by the diagonal elements of the tridiagonal matrix T; if KD > 0, the elements on the first superdiagonal (if UPLO = 'U') or the first subdiagonal (if UPLO = 'L') are overwritten by the off-diagonal elements of T; the rest of AB is overwritten by values generated during the reduction.

LDAB (input)
The leading dimension of the array AB. LDAB >= KD+1.

D (output)
The diagonal elements of the tridiagonal matrix T.

E (output)
The off-diagonal elements of the tridiagonal matrix T: E(i) = T(i,i+1) if UPLO = 'U'; E(i) = T(i+1,i) if UPLO = 'L'.

Q (input/output)
On entry, if VECT = 'U', then Q must contain an N-by-N matrix X; if VECT = 'N' or 'V', then Q need not be set.

On exit: if VECT = 'V', Q contains the N-by-N unitary matrix Q; if VECT = 'U', Q contains the product X*Q; if VECT = 'N', the array Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= 1, and LDQ >= N if VECT = 'V' or 'U'.

WORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

Modified by Linda Kaufman, Bell Labs.

Contents

NAME

zhecon - estimate the reciprocal of the condition number of a complex Hermitian matrix A using the factorization A = U*D*U**H or A = L*D*L**H computed by CHETRF

SYNOPSIS

```
SUBROUTINE ZHECON(UPLO, N, A, LDA, IPIVOT, ANORM, RCOND, WORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION ANORM, RCOND

SUBROUTINE ZHECON_64(UPLO, N, A, LDA, IPIVOT, ANORM, RCOND, WORK,
      INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION ANORM, RCOND
```

F95 INTERFACE

```
SUBROUTINE HECON(UPLO, [N], A, [LDA], IPIVOT, ANORM, RCOND, [WORK],
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT
```

```
REAL(8) :: ANORM, RCOND

SUBROUTINE HECON_64(UPLO, [N], A, [LDA], IPIVOT, ANORM, RCOND, [WORK],
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8) :: ANORM, RCOND
```

C INTERFACE

```
#include <sunperf.h>
void zhecon(char uplo, int n, doublecomplex *a, int lda, int
          *ipivot, double anorm, double *rcond, int *info);

void zhecon_64(char uplo, long  n,  doublecomplex  *a,  long
          lda,  long  *ipivot,  double anorm, double *rcond,
          long *info);
```

PURPOSE

zhecon estimates the reciprocal of the condition number of a complex Hermitian matrix A using the factorization A = U*D*U**H or A = L*D*L**H computed by CHETRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**H;
= 'L': Lower triangular, form is A = L*D*L**H.

N (input) The order of the matrix A. N >= 0.

A (input) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CHETRF.

LDA (input)
The leading dimension of the array A. LDA >=

max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CHETRF.

ANORM (input)
The 1-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

zheev - compute all eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A

SYNOPSIS

```
SUBROUTINE ZHEEV(JOBZ, UPLO, N, A, LDA, W, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, LDWORK, INFO
DOUBLE PRECISION W(*), WORK2(*)

SUBROUTINE ZHEEV_64(JOBZ, UPLO, N, A, LDA, W, WORK, LDWORK, WORK2,
      INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, LDWORK, INFO
DOUBLE PRECISION W(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HEEV(JOBZ, UPLO, [N], A, [LDA], W, [WORK], [LDWORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: W, WORK2

SUBROUTINE HEEV_64(JOBZ, UPLO, [N], A, [LDA], W, [WORK], [LDWORK],
       [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, LDWORK, INFO
REAL(8), DIMENSION(:) :: W, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zheev(char jobz, char uplo, int  n,  doublecomplex  *a,
          int lda, double *w, int *info);

void zheev_64(char jobz, char uplo,  long  n,  doublecomplex
          *a, long lda, double *w, long *info);
```

PURPOSE

zheev computes all eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A. On exit, if JOBZ = 'V', then if INFO = 0, A contains the orthonormal eigenvectors of the matrix A. If JOBZ = 'N', then on exit the lower triangle (if UPLO='L') or the upper triangle (if UPLO='U') of A, including the diagonal, is destroyed.

LDA (input)

The leading dimension of the array A. LDA >= max(1,N).

W (output)
If INFO = 0, the eigenvalues in ascending order.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of the array WORK. LDWORK >= max(1,2*N-1). For optimal efficiency, LDWORK >= (NB+1)*N, where NB is the blocksize for CHETRD returned by ILAENV.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
dimension(max(1,3*N-2))

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

Contents

NAME

zheevd - compute all eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A

SYNOPSIS

```
SUBROUTINE ZHEEVD(JOBZ, UPLO, N, A, LDA, W, WORK, LWORK, RWORK,
      LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, LWORK, LRWORK, LIWORK, INFO
INTEGER IWORK(*)
DOUBLE PRECISION W(*), RWORK(*)

SUBROUTINE ZHEEVD_64(JOBZ, UPLO, N, A, LDA, W, WORK, LWORK, RWORK,
      LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, LWORK, LRWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HEEVD(JOBZ, UPLO, [N], A, [LDA], W, [WORK], [LWORK],
       [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, LWORK, LRWORK, LIWORK, INFO
```

```
INTEGER, DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: W, RWORK

SUBROUTINE HEEVD_64(JOBZ, UPLO, [N], A, [LDA], W, [WORK], [LWORK],
       [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, LWORK, LRWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>
void zheevd(char jobz, char uplo, int n,  doublecomplex  *a,
          int lda, double *w, int *info);

void zheevd_64(char jobz, char uplo, long  n,  doublecomplex
          *a, long lda, double *w, long *info);
```

PURPOSE

zheevd computes all eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

JOBZ (input)
- = 'N': Compute eigenvalues only;
- = 'V': Compute eigenvalues and eigenvectors.

UPLO (input)
- = 'U': Upper triangle of A is stored;
- = 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A. On exit, if JOBZ = 'V', then if INFO = 0, A contains the orthonormal eigenvectors of the matrix A. If JOBZ = 'N', then on exit the lower triangle (if UPLO='L') or the upper triangle (if UPLO='U') of A, including the diagonal, is destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

W (output)
If INFO = 0, the eigenvalues in ascending order.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The length of the array WORK. If N <= 1, LWORK must be at least 1. If JOBZ = 'N' and N > 1, LWORK must be at least N + 1. If JOBZ = 'V' and N > 1, LWORK must be at least 2*N + N**2.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
dimension (LRWORK) On exit, if INFO = 0, RWORK(1) returns the optimal LRWORK.

LRWORK (input)
The dimension of the array RWORK. If N <= 1, LRWORK must be at least 1. If JOBZ = 'N' and N > 1, LRWORK must be at least N. If JOBZ = 'V' and N > 1, LRWORK must be at least 1 + 5*N + 2*N**2.

If LRWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the RWORK array, returns this value as the first entry of the RWORK array, and no error message

```
          related to LRWORK is issued by XERBLA.

IWORK (workspace/output)
          On exit, if INFO = 0, IWORK(1) returns the optimal
          LIWORK.

LIWORK (input)
          The dimension of the array  IWORK.   If  N  <=  1,
          LIWORK must be at least 1.  If JOBZ  = 'N' and N >
          1, LIWORK must be at least 1.  If JOBZ  = 'V'  and
          N > 1, LIWORK must be at least 3 + 5*N.

          If LIWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the IWORK array, returns this value as  the  first
          entry  of  the  IWORK  array, and no error message
          related to LIWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, the algorithm  failed  to  con-
          verge;  i off-diagonal elements of an intermediate
          tridiagonal form did not converge to zero.
```

FURTHER DETAILS

```
Based on contributions by
   Jeff Rutter, Computer  Science  Division,  University  of
California
   at Berkeley, USA
```

Contents

NAME

zheevr - compute selected eigenvalues and, optionally, eigenvectors of a complex Hermitian tridiagonal matrix T

SYNOPSIS

```
SUBROUTINE ZHEEVR(JOBZ, RANGE, UPLO, N, A, LDA, VL, VU, IL, IU,
      ABSTOL, M, W, Z, LDZ, ISUPPZ, WORK, LWORK, RWORK, LRWORK, IWORK,
      LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
DOUBLE COMPLEX A(LDA,*), Z(LDZ,*), WORK(*)
INTEGER N, LDA, IL, IU, M, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER ISUPPZ(*), IWORK(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION W(*), RWORK(*)

SUBROUTINE ZHEEVR_64(JOBZ, RANGE, UPLO, N, A, LDA, VL, VU, IL, IU,
      ABSTOL, M, W, Z, LDZ, ISUPPZ, WORK, LWORK, RWORK, LRWORK, IWORK,
      LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
DOUBLE COMPLEX A(LDA,*), Z(LDZ,*), WORK(*)
INTEGER*8 N, LDA, IL, IU, M,  LDZ,  LWORK,  LRWORK,  LIWORK,
INFO
INTEGER*8 ISUPPZ(*), IWORK(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HEEVR(JOBZ, RANGE, UPLO, [N], A, [LDA], VL, VU, IL, IU,
       ABSTOL, M, W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [RWORK], [LRWORK],
```

```
       [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, Z
INTEGER :: N, LDA, IL, IU, M, LDZ,  LWORK,  LRWORK,  LIWORK,
INFO
INTEGER, DIMENSION(:) :: ISUPPZ, IWORK
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: W, RWORK

SUBROUTINE HEEVR_64(JOBZ, RANGE, UPLO, [N], A, [LDA], VL, VU, IL, IU,
       ABSTOL, M, W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [RWORK], [LRWORK],
       [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, Z
INTEGER(8) :: N, LDA, IL, IU, M, LDZ, LWORK, LRWORK, LIWORK,
INFO
INTEGER(8), DIMENSION(:) :: ISUPPZ, IWORK
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void zheevr(char jobz, char range, char uplo, int  n,  doub-
          lecomplex  *a,  int lda, double vl, double vu, int
          il, int iu, double  abstol,  int  *m,  double  *w,
          doublecomplex   *z,  int  ldz,  int  *isuppz,  int
          *info);

void zheevr_64(char jobz, char range,  char  uplo,  long  n,
          doublecomplex  *a, long lda, double vl, double vu,
          long il, long iu, double abstol, long  *m,  double
          *w, doublecomplex *z, long ldz, long *isuppz, long
          *info);
```

PURPOSE

zheevr computes selected eigenvalues and, optionally, eigenvectors of a complex Hermitian tridiagonal matrix T. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Whenever possible, CHEEVR calls CSTEGR to compute the eigenspectrum using Relatively Robust Representations.

CSTEGR computes eigenvalues by the dqds algorithm, while orthogonal eigenvectors are computed from various "good" L D L^T representations (also known as Relatively Robust Representations). Gram-Schmidt orthogonalization is avoided as far as possible. More specifically, the various steps of the algorithm are as follows. For the i-th unreduced block of T,

(a) Compute T - sigma_i = L_i D_i L_i^T, such that L_i D_i L_i^T is a relatively robust representation,

(b) Compute the eigenvalues, lambda_j, of L_i D_i L_i^T to high relative accuracy by the dqds algorithm,

(c) If there is a cluster of close eigenvalues, "choose" sigma_i close to the cluster, and go to step (a),

(d) Given the approximate eigenvalue lambda_j of L_i D_i L_i^T, compute the corresponding eigenvector by forming a rank-revealing twisted factorization.

The desired accuracy of the output can be specified by the input parameter ABSTOL.

For more details, see "A new O(n^2) algorithm for the symmetric tridiagonal eigenvalue/eigenvector problem", by Inderjit Dhillon, Computer Science Division Technical Report No. UCB//CSD-97-971, UC Berkeley, May 1997.

Note 1 : CHEEVR calls CSTEGR when the full spectrum is requested on machines which conform to the ieee-754 floating point standard. CHEEVR calls SSTEBZ and CSTEIN on non-ieee machines and
when partial spectrum requests are made.

Normal execution of CSTEGR may create NaNs and infinities and hence may abort due to a floating point exception in environments which do not handle NaNs and infinities in the ieee standard default manner.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

RANGE (input)
= 'A': all eigenvalues will be found.
= 'V': all eigenvalues in the half-open interval

```
          (VL,VU]  will  be found.  = 'I': the IL-th through
          IU-th eigenvalues will be found.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the Hermitian matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of A contains the lower  triangular  part  of
          the  matrix  A.   On  exit, the lower triangle (if
          UPLO='L') or the upper triangle (if  UPLO='U')  of
          A, including the diagonal, is destroyed.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

VL (input)
          If RANGE='V', the lower and upper  bounds  of  the
          interval  to be searched for eigenvalues. VL < VU.
          Not referenced if RANGE = 'A' or 'I'.

VU (input)
          If RANGE='V', the lower and upper  bounds  of  the
          interval  to be searched for eigenvalues. VL < VU.
          Not referenced if RANGE = 'A' or 'I'.

IL (input)
          If RANGE='I', the indices (in ascending order)  of
          the   smallest   and  largest  eigenvalues  to  be
          returned.  1 <= IL <= IU <= N, if N > 0;  IL  =  1
          and  IU  =  0 if N = 0.  Not referenced if RANGE =
          'A' or 'V'.

IU (input)
          If RANGE='I', the indices (in ascending order)  of
          the   smallest   and  largest  eigenvalues  to  be
          returned.  1 <= IL <= IU <= N, if N > 0;  IL  =  1
          and  IU  =  0 if N = 0.  Not referenced if RANGE =
          'A' or 'V'.

ABSTOL (input)
          The absolute error tolerance for the  eigenvalues.
          An approximate eigenvalue is accepted as converged
```

when it is determined to lie in an interval [a,b] of width less than or equal to

ABSTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABSTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing A to tridiagonal form.

See "Computing Small Singular Values of Bidiagonal Matrices with Guaranteed High Relative Accuracy," by Demmel and Kahan, LAPACK Working Note #3.

If high relative accuracy is important, set ABSTOL to SLAMCH('Safe minimum'). Doing so will guarantee that eigenvalues are computed to high relative accuracy when possible in future releases. The current code does not make any guarantees about high relative accuracy, but furutre releases will. See J. Barlow and J. Demmel, "Computing Accurate Eigensystems of Scaled Diagonally Dominant Matrices", LAPACK Working Note #7, for a discussion of which matrices define their eigenvalues to high relative accuracy.

M (output)

The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)

The first M elements contain the selected eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, the first M columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). If JOBZ = 'N', then Z is not referenced. Note: the user must ensure that at least max(1,M) columns are supplied in the array Z; if RANGE = 'V', the exact value of M is not known in advance and an upper bound must be used.

LDZ (input)

The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

```
ISUPPZ (output)
          The support of the eigenvectors in  Z,  i.e.,  the
          indices  indicating the nonzero elements in Z. The
          i-th  eigenvector  is  nonzero  only  in  elements
          ISUPPZ( 2*i-1 ) through ISUPPZ( 2*i ).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The  length  of  the   array   WORK.    LWORK   >=
          max(1,2*N).   For  optimal  efficiency,  LWORK  >=
          (NB+1)*N, where NB is the max of the blocksize for
          CHETRD and for CUNMTR as returned by ILAENV.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

RWORK (workspace)
          On exit, if INFO = 0, RWORK(1) returns the optimal
          (and minimal) LRWORK.

LRWORK (input)
          The  length  of  the  array  RWORK.    LRWORK   >=
          max(1,24*N).

          If LRWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the RWORK array, returns this value as  the  first
          entry  of  the  RWORK  array, and no error message
          related to LRWORK is issued by XERBLA.

IWORK (workspace/output)
          On exit, if INFO = 0, IWORK(1) returns the optimal
          (and minimal) LIWORK.

LIWORK (input)
          The dimension  of  the  array  IWORK.   LIWORK  >=
          max(1,10*N).

          If LIWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the IWORK array, returns this value as  the  first
          entry  of  the  IWORK  array, and no error message
          related to LIWORK is issued by XERBLA.
```

```
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  Internal error
```

FURTHER DETAILS

```
Based on contributions by
   Inderjit Dhillon, IBM Almaden, USA
   Osni Marques, LBNL/NERSC, USA
   Ken Stanley, Computer Science Division, University of
     California at Berkeley, USA
```

Contents

NAME

zheevx - compute selected eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A

SYNOPSIS

```
SUBROUTINE ZHEEVX(JOBZ, RANGE, UPLO, N, A, LDA, VL, VU, IL, IU,
      ABTOL, NFOUND, W, Z, LDZ, WORK, LDWORK, WORK2, IWORK3, IFAIL,
      INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
DOUBLE COMPLEX A(LDA,*), Z(LDZ,*), WORK(*)
INTEGER N, LDA, IL, IU, NFOUND, LDZ, LDWORK, INFO
INTEGER IWORK3(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABTOL
DOUBLE PRECISION W(*), WORK2(*)

SUBROUTINE ZHEEVX_64(JOBZ, RANGE, UPLO, N, A, LDA, VL, VU, IL, IU,
      ABTOL, NFOUND, W, Z, LDZ, WORK, LDWORK, WORK2, IWORK3, IFAIL,
      INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
DOUBLE COMPLEX A(LDA,*), Z(LDZ,*), WORK(*)
INTEGER*8 N, LDA, IL, IU, NFOUND, LDZ, LDWORK, INFO
INTEGER*8 IWORK3(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABTOL
DOUBLE PRECISION W(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HEEVX(JOBZ, RANGE, UPLO, [N], A, [LDA], VL, VU, IL, IU,
       ABTOL, [NFOUND], W, Z, [LDZ], [WORK], [LDWORK], [WORK2], [IWORK3],
       IFAIL, [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, Z
INTEGER :: N, LDA, IL, IU, NFOUND, LDZ, LDWORK, INFO
INTEGER, DIMENSION(:) :: IWORK3, IFAIL
REAL(8) :: VL, VU, ABTOL
REAL(8), DIMENSION(:) :: W, WORK2

SUBROUTINE HEEVX_64(JOBZ, RANGE, UPLO, [N], A, [LDA], VL, VU, IL, IU,
       ABTOL, [NFOUND], W, Z, [LDZ], [WORK], [LDWORK], [WORK2], [IWORK3],
       IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, Z
INTEGER(8) :: N, LDA, IL, IU, NFOUND, LDZ, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK3, IFAIL
REAL(8) :: VL, VU, ABTOL
REAL(8), DIMENSION(:) :: W, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zheevx(char jobz, char range, char uplo, int  n,  doub-
          lecomplex  *a,  int lda, double vl, double vu, int
          il, int iu, double abtol, int *nfound, double  *w,
          doublecomplex *z, int ldz, int *ifail, int *info);

void zheevx_64(char jobz, char range,  char  uplo,  long  n,
          doublecomplex  *a, long lda, double vl, double vu,
          long il, long iu, double abtol, long *nfound, dou-
          ble  *w,  doublecomplex *z, long ldz, long *ifail,
          long *info);
```

PURPOSE

zheevx computes selected eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

RANGE (input)
= 'A': all eigenvalues will be found.
= 'V': all eigenvalues in the half-open interval (VL,VU] will be found. = 'I': the IL-th through IU-th eigenvalues will be found.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A. On exit, the lower triangle (if UPLO='L') or the upper triangle (if UPLO='U') of A, including the diagonal, is destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

VL (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

ABTOL (input)
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing A to tridiagonal form.

Eigenvalues will be computed most accurately when ABTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABTOL to 2*SLAMCH('S').

See "Computing Small Singular Values of Bidiagonal Matrices with Guaranteed High Relative Accuracy," by Demmel and Kahan, LAPACK Working Note #3.

NFOUND (output)
The total number of eigenvalues found. 0 <= NFOUND <= N. If RANGE = 'A', NFOUND = N, and if RANGE = 'I', NFOUND = IU-IL+1.

W (output)
On normal exit, the first NFOUND elements contain the selected eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, the first NFOUND columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). If an eigenvector fails to converge, then that column of Z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in IFAIL. If JOBZ = 'N', then Z is not referenced. Note: the user must ensure that at least max(1,NFOUND) columns are supplied in the array Z; if RANGE = 'V', the exact value of NFOUND is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of the array WORK. LDWORK >= max(1,2*N). For optimal efficiency, LDWORK >= (NB+1)*N, where NB is the max of the blocksize for CHETRD and for CUNMTR as returned by ILAENV.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
dimension(7*N)

IWORK3 (workspace)
dimension(5*N)

IFAIL (output)
If JOBZ = 'V', then if INFO = 0, the first NFOUND elements of IFAIL are zero. If INFO > 0, then IFAIL contains the indices of the eigenvectors that failed to converge. If JOBZ = 'N', then IFAIL is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, then i eigenvectors failed to converge. Their indices are stored in array IFAIL.

Contents

NAME

```
zhegs2 - reduce  a  complex  Hermitian-definite  generalized
eigenproblem to standard form
```

SYNOPSIS

```
SUBROUTINE ZHEGS2(ITYPE, UPLO, N, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER ITYPE, N, LDA, LDB, INFO

SUBROUTINE ZHEGS2_64(ITYPE, UPLO, N, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 ITYPE, N, LDA, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE HEGS2(ITYPE, UPLO, N, A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: ITYPE, N, LDA, LDB, INFO

SUBROUTINE HEGS2_64(ITYPE, UPLO, N, A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: ITYPE, N, LDA, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zhegs2(int itype, char uplo, int n,  doublecomplex  *a,
          int lda, doublecomplex *b, int ldb, int *info);

void zhegs2_64(long itype, char uplo, long n,  doublecomplex
          *a,  long  lda,  doublecomplex  *b, long ldb, long
          *info);
```

PURPOSE

zhegs2 reduces a complex Hermitian-definite generalized eigenproblem to standard form.

If ITYPE = 1, the problem is A*x = lambda*B*x,
and A is overwritten by inv(U')*A*inv(U) or inv(L)*A*inv(L')
If ITYPE = 2 or 3, the problem is A*B*x = lambda*x or
B*A*x = lambda*x, and A is overwritten by U*A*U` or L'*A*L.

B must have been previously factorized as U'*U or L*L' by CPOTRF.

ARGUMENTS

ITYPE (input)
= 1: compute inv(U')*A*inv(U) or inv(L)*A*inv(L');
= 2 or 3: compute U*A*U' or L'*A*L.

UPLO (input)
Specifies whether the upper or lower triangular part of the Hermitian matrix A is stored, and how B has been factorized. = 'U': Upper triangular
= 'L': Lower triangular

N (input) The order of the matrices A and B. N >= 0.

A (input/output)
On entry, the Hermitian matrix A. If UPLO = 'U', the leading n by n upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading n by n lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, if INFO = 0, the transformed matrix, stored in the same format as A.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input) The triangular factor from the Cholesky factorization of B, as returned by CPOTRF.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

zhegst - reduce a complex Hermitian-definite generalized eigenproblem to standard form

SYNOPSIS

```
SUBROUTINE ZHEGST(ITYPE, UPLO, N, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER ITYPE, N, LDA, LDB, INFO

SUBROUTINE ZHEGST_64(ITYPE, UPLO, N, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 ITYPE, N, LDA, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE HEGST(ITYPE, UPLO, N, A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: ITYPE, N, LDA, LDB, INFO

SUBROUTINE HEGST_64(ITYPE, UPLO, N, A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: ITYPE, N, LDA, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zhegst(int itype, char uplo, int n,  doublecomplex  *a,
          int lda, doublecomplex *b, int ldb, int *info);

void zhegst_64(long itype, char uplo, long n,  doublecomplex
          *a,  long  lda,  doublecomplex  *b, long ldb, long
          *info);
```

PURPOSE

zhegst reduces a complex Hermitian-definite generalized eigenproblem to standard form.

If ITYPE = 1, the problem is A*x = lambda*B*x,
and A is overwritten by inv(U**H)*A*inv(U) or inv(L)*A*inv(L**H)

If ITYPE = 2 or 3, the problem is A*B*x = lambda*x or
B*A*x = lambda*x, and A is overwritten by U*A*U**H or L**H*A*L.

B must have been previously factorized as U**H*U or L*L**H by CPOTRF.

ARGUMENTS

ITYPE (input)
= 1: compute inv(U**H)*A*inv(U) or inv(L)*A*inv(L**H);
= 2 or 3: compute U*A*U**H or L**H*A*L.

UPLO (input)
= 'U': Upper triangle of A is stored and B is factored as U**H*U; = 'L': Lower triangle of A is stored and B is factored as L*L**H.

N (input) The order of the matrices A and B. N >= 0.

A (input/output)
On entry, the Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, if INFO = 0, the transformed matrix, stored in the same format as A.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input) The triangular factor from the Cholesky factorization of B, as returned by CPOTRF.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zhegv - compute all the  eigenvalues,  and  optionally,  the
eigenvectors  of  a  complex  generalized Hermitian-definite
eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x,
or B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE ZHEGV(ITYPE, JOBZ, UPLO, N, A, LDA, B, LDB, W, WORK,
      LDWORK, WORK2, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER ITYPE, N, LDA, LDB, LDWORK, INFO
DOUBLE PRECISION W(*), WORK2(*)

SUBROUTINE ZHEGV_64(ITYPE, JOBZ, UPLO, N, A, LDA, B, LDB, W, WORK,
      LDWORK, WORK2, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER*8 ITYPE, N, LDA, LDB, LDWORK, INFO
DOUBLE PRECISION W(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HEGV(ITYPE, JOBZ, UPLO, N, A, [LDA], B, [LDB], W, [WORK],
       [LDWORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: ITYPE, N, LDA, LDB, LDWORK, INFO
REAL(8), DIMENSION(:) :: W, WORK2
```

```
SUBROUTINE HEGV_64(ITYPE, JOBZ, UPLO, N, A, [LDA], B, [LDB], W, [WORK],
       [LDWORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: ITYPE, N, LDA, LDB, LDWORK, INFO
REAL(8), DIMENSION(:) :: W, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zhegv(int itype, char jobz, char  uplo,  int  n,  doub-
          lecomplex  *a, int lda, doublecomplex *b, int ldb,
          double *w, int *info);

void zhegv_64(long itype, char  jobz,  char  uplo,  long  n,
          doublecomplex *a, long lda, doublecomplex *b, long
          ldb, double *w, long *info);
```

PURPOSE

zhegv computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be Hermitian and B is also
positive definite.

ARGUMENTS

```
ITYPE (input)
          Specifies the problem type to be solved:
          = 1:  A*x = (lambda)*B*x
          = 2:  A*B*x = (lambda)*x
          = 3:  B*A*x = (lambda)*x

JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.
```

A (input/output)
On entry, the Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A.

On exit, if JOBZ = 'V', then if INFO = 0, A contains the matrix Z of eigenvectors. The eigenvectors are normalized as follows: if ITYPE = 1 or 2, Z**H*B*Z = I; if ITYPE = 3, Z**H*inv(B)*Z = I. If JOBZ = 'N', then on exit the upper triangle (if UPLO='U') or the lower triangle (if UPLO='L') of A, including the diagonal, is destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the Hermitian positive definite matrix B. If UPLO = 'U', the leading N-by-N upper triangular part of B contains the upper triangular part of the matrix B. If UPLO = 'L', the leading N-by-N lower triangular part of B contains the lower triangular part of the matrix B.

On exit, if INFO <= N, the part of B containing the matrix is overwritten by the triangular factor U or L from the Cholesky factorization B = U**H*U or B = L*L**H.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

W (output)
If INFO = 0, the eigenvalues in ascending order.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of the array WORK. LDWORK >= max(1,2*N-1). For optimal efficiency, LDWORK >= (NB+1)*N, where NB is the blocksize for CHETRD returned by ILAENV.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
dimension(max(1,3*N-2))

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: CPOTRF or CHEEV returned an error code:
<= N: if INFO = i, CHEEV failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero; > N: if INFO = N + i, for 1 <= i <= N, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

Contents

NAME

zhegvd - compute all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x

SYNOPSIS

```
SUBROUTINE ZHEGVD(ITYPE, JOBZ, UPLO, N, A, LDA, B, LDB, W, WORK,
      LWORK, RWORK, LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER ITYPE, N, LDA, LDB, LWORK, LRWORK, LIWORK, INFO
INTEGER IWORK(*)
DOUBLE PRECISION W(*), RWORK(*)

SUBROUTINE ZHEGVD_64(ITYPE, JOBZ, UPLO, N, A, LDA, B, LDB, W, WORK,
      LWORK, RWORK, LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER*8 ITYPE, N, LDA, LDB, LWORK, LRWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HEGVD(ITYPE, JOBZ, UPLO, [N], A, [LDA], B, [LDB], W, [WORK],
       [LWORK], [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
```

```
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: ITYPE, N, LDA, LDB, LWORK, LRWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: W, RWORK

SUBROUTINE HEGVD_64(ITYPE, JOBZ, UPLO, [N], A, [LDA], B, [LDB], W,
       [WORK], [LWORK], [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: ITYPE, N, LDA,  LDB,  LWORK,  LRWORK,  LIWORK,
INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void zhegvd(int itype, char jobz, char uplo,  int  n,  doub-
          lecomplex  *a, int lda, doublecomplex *b, int ldb,
          double *w, int *info);

void zhegvd_64(long itype, char jobz,  char  uplo,  long  n,
          doublecomplex *a, long lda, doublecomplex *b, long
          ldb, double *w, long *info);
```

PURPOSE

zhegvd computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be Hermitian and B is also positive definite. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

ITYPE (input)

```
          Specifies the problem type to be solved:
          = 1:  A*x = (lambda)*B*x
          = 2:  A*B*x = (lambda)*x
          = 3:  B*A*x = (lambda)*x

JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.
A (input/output)
          On entry, the Hermitian matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of A contains the lower  triangular  part  of
          the matrix A.

          On exit, if JOBZ = 'V', then if INFO = 0,  A  con-
          tains the matrix Z of eigenvectors.  The eigenvec-
          tors are normalized as follows:  if ITYPE =  1  or
          2,  Z**H*B*Z = I; if ITYPE = 3, Z**H*inv(B)*Z = I.
          If JOBZ = 'N', then on exit the upper triangle (if
          UPLO='U')  or  the lower triangle (if UPLO='L') of
          A, including the diagonal, is destroyed.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

B (input/output)
          On entry, the Hermitian matrix B.  If UPLO =  'U',
          the leading N-by-N upper triangular part of B con-
          tains the upper triangular part of the  matrix  B.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of B contains the lower  triangular  part  of
          the matrix B.

          On exit, if INFO <= N, the part  of  B  containing
          the matrix is overwritten by the triangular factor
          U or L from the Cholesky factorization B =  U**H*U
          or B = L*L**H.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).
```

```
W (output)
          If INFO = 0, the eigenvalues in ascending order.

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The  length  of  the  array  WORK.   If  N  <=  1,
          LWORK  >= 1.  If JOBZ  = 'N' and N > 1, LWORK >= N
          + 1.  If JOBZ  = 'V' and N > 1,  LWORK  >=  2*N  +
          N**2.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

RWORK (workspace)
          On exit, if INFO = 0, RWORK(1) returns the optimal
          LRWORK.

LRWORK (input)
          The dimension of the array  RWORK.   If  N  <=  1,
          LRWORK  >= 1.  If JOBZ  = 'N' and N > 1, LRWORK >=
          N.  If JOBZ  = 'V' and N > 1, LRWORK >= 1 + 5*N  +
          2*N**2.

          If LRWORK = -1, then a workspace query is assumed;
          the  routine  only  calculates the optimal size of
          the RWORK array, returns this value as  the  first
          entry  of  the  RWORK  array, and no error message
          related to LRWORK is issued by XERBLA.

IWORK (workspace/output)
          On exit, if INFO = 0, IWORK(1) returns the optimal
          LIWORK.

LIWORK (input)
          The dimension of the array  IWORK.   If  N  <=  1,
          LIWORK  >= 1.  If JOBZ  = 'N' and N > 1, LIWORK >=
          1.  If JOBZ  = 'V' and N > 1, LIWORK >= 3 + 5*N.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  CPOTRF or CHEEVD returned an error code:
```

```
<= N:  if INFO = i, CHEEVD failed to  converge;  i
off-diagonal elements of an intermediate tridiago-
nal form did not converge to zero; > N:   if  INFO
=  N  + i, for 1 <= i <= N, then the leading minor
of order i of B is  not  positive  definite.   The
factorization  of  B could not be completed and no
eigenvalues or eigenvectors were computed.
```

FURTHER DETAILS

```
Based on contributions by
   Mark Fahey, Department of Mathematics, Univ. of Kentucky,
USA
```

Contents

NAME

```
zhegvx  -  compute  selected  eigenvalues,  and  optionally,
eigenvectors  of  a  complex  generalized Hermitian-definite
eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x,
or B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE ZHEGVX(ITYPE, JOBZ, RANGE, UPLO, N, A, LDA, B, LDB, VL,
      VU, IL, IU, ABSTOL, M, W, Z, LDZ, WORK, LWORK, RWORK, IWORK,
      IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*), Z(LDZ,*), WORK(*)
INTEGER ITYPE, N, LDA, LDB, IL, IU, M, LDZ, LWORK, INFO
INTEGER IWORK(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION W(*), RWORK(*)

SUBROUTINE ZHEGVX_64(ITYPE, JOBZ, RANGE, UPLO, N, A, LDA, B, LDB, VL,
      VU, IL, IU, ABSTOL, M, W, Z, LDZ, WORK, LWORK, RWORK, IWORK,
      IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*), Z(LDZ,*), WORK(*)
INTEGER*8 ITYPE, N, LDA, LDB, IL, IU, M, LDZ, LWORK, INFO
INTEGER*8 IWORK(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HEGVX(ITYPE, JOBZ, RANGE, UPLO, [N], A, [LDA], B, [LDB],
```

```
       VL, VU, IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK], [LWORK], [RWORK],
       [IWORK], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, Z
INTEGER :: ITYPE, N, LDA, LDB, IL, IU, M, LDZ, LWORK, INFO
INTEGER, DIMENSION(:) :: IWORK, IFAIL
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: W, RWORK

SUBROUTINE HEGVX_64(ITYPE, JOBZ, RANGE, UPLO, [N], A, [LDA], B, [LDB],
       VL, VU, IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK], [LWORK], [RWORK],
       [IWORK], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, Z
INTEGER(8) :: ITYPE, N, LDA, LDB, IL,  IU,  M,  LDZ,  LWORK,
INFO
INTEGER(8), DIMENSION(:) :: IWORK, IFAIL
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void zhegvx(int itype, char jobz, char range, char uplo, int
          n,  doublecomplex  *a,  int lda, doublecomplex *b,
          int ldb, double vl, double vu,  int  il,  int  iu,
          double  abstol,  int  *m, double *w, doublecomplex
          *z, int ldz, int *ifail, int *info);

void zhegvx_64(long itype, char jobz, char range, char uplo,
          long  n, doublecomplex *a, long lda, doublecomplex
          *b, long ldb, double vl, double vu, long il,  long
          iu,  double abstol, long *m, double *w, doublecom-
          plex *z, long ldz, long *ifail, long *info);
```

PURPOSE

zhegvx computes selected eigenvalues, and optionally, eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be Hermitian and B is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

```
ITYPE (input)
          Specifies the problem type to be solved:
          = 1:  A*x = (lambda)*B*x
          = 2:  A*B*x = (lambda)*x
          = 3:  B*A*x = (lambda)*x

JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

RANGE (input)
          = 'A': all eigenvalues will be found.
          = 'V': all eigenvalues in the  half-open  interval
          (VL,VU]  will  be found.  = 'I': the IL-th through
          IU-th eigenvalues will be found.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.

A (input/output)
          On entry, the Hermitian matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of A contains the lower  triangular  part  of
          the matrix A.

          On exit,  the lower triangle (if UPLO='L') or  the
          upper  triangle  (if UPLO='U') of A, including the
          diagonal, is destroyed.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

B (input/output)
          On entry, the Hermitian matrix B.  If UPLO =  'U',
          the leading N-by-N upper triangular part of B con-
          tains the upper triangular part of the  matrix  B.
          If UPLO = 'L', the leading N-by-N lower triangular
          part of B contains the lower  triangular  part  of
          the matrix B.
```

```
            On exit, if INFO <= N, the part  of  B  containing
            the matrix is overwritten by the triangular factor
            U or L from the Cholesky factorization B =  U**H*U
            or B = L*L**H.

LDB (input)
            The leading dimension of  the  array  B.   LDB  >=
            max(1,N).

VL (input)
            If RANGE='V', the lower and upper  bounds  of  the
            interval  to be searched for eigenvalues. VL < VU.
            Not referenced if RANGE = 'A' or 'I'.
VU (input)
            If RANGE='V', the lower and upper  bounds  of  the
            interval  to be searched for eigenvalues. VL < VU.
            Not referenced if RANGE = 'A' or 'I'.

IL (input)
            If RANGE='I', the indices (in ascending order)  of
            the   smallest   and  largest  eigenvalues  to  be
            returned.  1 <= IL <= IU <= N, if N > 0;  IL  =  1
            and  IU  =  0 if N = 0.  Not referenced if RANGE =
            'A' or 'V'.

IU (input)
            If RANGE='I', the indices (in ascending order)  of
            the   smallest   and  largest  eigenvalues  to  be
            returned.  1 <= IL <= IU <= N, if N > 0;  IL  =  1
            and  IU  =  0 if N = 0.  Not referenced if RANGE =
            'A' or 'V'.

ABSTOL (input)
            The absolute error tolerance for the  eigenvalues.
            An approximate eigenvalue is accepted as converged
            when it is determined to lie in an interval  [a,b]
            of width less than or equal to

            ABSTOL + EPS *   max( |a|,|b| ) ,

            where EPS is the machine precision.  If ABSTOL  is
            less than or equal to zero, then  EPS*|T|  will be
            used in its place, where |T| is the 1-norm of  the
            tridiagonal  matrix obtained by reducing A to tri-
            diagonal form.

            Eigenvalues will be computed most accurately  when
            ABSTOL  is  set  to  twice the underflow threshold
            2*SLAMCH('S'), not zero.  If this routine  returns
            with INFO>0, indicating that some eigenvectors did
```

not converge, try setting ABSTOL to 2*SLAMCH('S').

M (output)
The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)
The first M elements contain the selected eigenvalues in ascending order.

Z (output)
If JOBZ = 'N', then Z is not referenced. If JOBZ = 'V', then if INFO = 0, the first M columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). The eigenvectors are normalized as follows: if ITYPE = 1 or 2, Z**T*B*Z = I; if ITYPE = 3, Z**T*inv(B)*Z = I.

If an eigenvector fails to converge, then that column of Z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in IFAIL. Note: the user must ensure that at least max(1,M) columns are supplied in the array Z; if RANGE = 'V', the exact value of M is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace/output)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The length of the array WORK. LWORK >= max(1,2*N-1). For optimal efficiency, LWORK >= (NB+1)*N, where NB is the blocksize for CHETRD returned by ILAENV.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)

dimension(7*N)

IWORK (workspace)
dimension(5*N)

IFAIL (output)
If JOBZ = 'V', then if INFO = 0, the first M elements of IFAIL are zero. If INFO > 0, then IFAIL contains the indices of the eigenvectors that failed to converge. If JOBZ = 'N', then IFAIL is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: CPOTRF or CHEEVX returned an error code:
<= N: if INFO = i, CHEEVX failed to converge; i eigenvectors failed to converge. Their indices are stored in array IFAIL. > N: if INFO = N + i, for 1 <= i <= N, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
Mark Fahey, Department of Mathematics, Univ. of Kentucky, USA

Contents

NAME

zhemm - perform one of the matrix-matrix operations C := alpha*A*B + beta*C or C := alpha*B*A + beta*C

SYNOPSIS

```
SUBROUTINE ZHEMM(SIDE, UPLO, M, N, ALPHA, A, LDA, B, LDB, BETA, C,
      LDC)

CHARACTER * 1 SIDE, UPLO
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER M, N, LDA, LDB, LDC

SUBROUTINE ZHEMM_64(SIDE, UPLO, M, N, ALPHA, A, LDA, B, LDB, BETA, C,
      LDC)

CHARACTER * 1 SIDE, UPLO
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER*8 M, N, LDA, LDB, LDC
```

F95 INTERFACE

```
SUBROUTINE HEMM(SIDE, UPLO, [M], [N], ALPHA, A, [LDA], B, [LDB],
       BETA, C, [LDC])

CHARACTER(LEN=1) :: SIDE, UPLO
COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:,:) :: A, B, C
INTEGER :: M, N, LDA, LDB, LDC

SUBROUTINE HEMM_64(SIDE, UPLO, [M], [N], ALPHA, A, [LDA], B, [LDB],
       BETA, C, [LDC])
```

```
CHARACTER(LEN=1) :: SIDE, UPLO
COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:,:) :: A, B, C
INTEGER(8) :: M, N, LDA, LDB, LDC
```

C INTERFACE

```
#include <sunperf.h>

void zhemm(char side, char uplo, int m, int n, doublecomplex
          *alpha,  doublecomplex  *a, int lda, doublecomplex
          *b, int ldb,  doublecomplex  *beta,  doublecomplex
          *c, int ldc);
void zhemm_64(char side, char uplo, long m,  long  n,  doub-
          lecomplex  *alpha,  doublecomplex  *a,  long  lda,
          doublecomplex *b, long ldb,  doublecomplex  *beta,
          doublecomplex *c, long ldc);
```

PURPOSE

zhemm performs one of the matrix-matrix operations C := alpha*A*B + beta*C or C := alpha*B*A + beta*C where alpha and beta are scalars, A is an hermitian matrix and B and C are m by n matrices.

ARGUMENTS

SIDE (input)
On entry, SIDE specifies whether the hermitian matrix A appears on the left or right in the operation as follows:

SIDE = 'L' or 'l'   C := alpha*A*B + beta*C,

SIDE = 'R' or 'r'   C := alpha*B*A + beta*C,

Unchanged on exit.

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the hermitian matrix A is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of the hermitian matrix is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of the hermitian matrix is to be referenced.

Unchanged on exit.

M (input)
On entry, M specifies the number of rows of the matrix C. M >= 0. Unchanged on exit.

N (input)
On entry, N specifies the number of columns of the matrix C. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
COMPLEX*16 array of DIMENSION (LDA, ka), where ka is m when SIDE = 'L' or 'l' and is n otherwise.

Before entry with SIDE = 'L' or 'l', the m by m part of the array A must contain the hermitian matrix, such that when UPLO = 'U' or 'u', the leading m by m upper triangular part of the array A must contain the upper triangular part of the hermitian matrix and the strictly lower triangular part of A is not referenced, and when UPLO = 'L' or 'l', the leading m by m lower triangular part of the array A must contain the lower triangular part of the hermitian matrix and the strictly upper triangular part of A is not referenced.

Before entry with SIDE = 'R' or 'r', the n by n part of the array A must contain the hermitian matrix, such that when UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular part of the hermitian matrix and the strictly lower triangular part of A is not referenced, and when UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular part of the hermitian matrix and the strictly upper triangular part of A is not referenced.

Note that the imaginary parts of the diagonal elements need not be set, they are assumed to be zero. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When SIDE = 'L' or 'l' then LDA >= max(1, m), other- wise LDA >= max(1, n). Unchanged on exit.

B (input)
COMPLEX*16 array of DIMENSION (LDB, n). Before entry, the leading m by n part of the array B must contain the matrix B. Unchanged on exit.

LDB (input)
On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. LDB must be at least max(1, m). Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. When BETA is supplied as zero then C need not be set on input. Unchanged on exit.

C (input/output)
COMPLEX*16 array of DIMENSION (LDC, n).

Before entry, the leading m by n part of the array C must contain the matrix C, except when beta is zero, in which case C need not be set on entry.

On exit, the array C is overwritten by the m by n updated matrix.

LDC (input)
On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC must be at least max(1, m). Unchanged on exit.

Contents

NAME

zhemv - perform the matrix-vector operation   y := alpha*A*x + beta*y

SYNOPSIS

```
SUBROUTINE ZHEMV(UPLO, N, ALPHA, A, LDA, X, INCX, BETA, Y, INCY)

CHARACTER * 1 UPLO
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(LDA,*), X(*), Y(*)
INTEGER N, LDA, INCX, INCY

SUBROUTINE ZHEMV_64(UPLO, N, ALPHA, A, LDA, X, INCX, BETA, Y, INCY)

CHARACTER * 1 UPLO
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(LDA,*), X(*), Y(*)
INTEGER*8 N, LDA, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE HEMV(UPLO, [N], ALPHA, A, [LDA], X, [INCX], BETA, Y, [INCY])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:) :: X, Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, INCX, INCY

SUBROUTINE HEMV_64(UPLO, [N], ALPHA, A, [LDA], X, [INCX], BETA, Y,
       [INCY])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:) :: X, Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>

void zhemv(char uplo, int  n,  doublecomplex  *alpha,  doub-
          lecomplex *a, int lda, doublecomplex *x, int incx,
          doublecomplex *beta, doublecomplex *y, int incy);

void zhemv_64(char uplo, long n, doublecomplex *alpha, doub-
          lecomplex  *a,  long  lda,  doublecomplex *x, long
          incx, doublecomplex *beta, doublecomplex *y,  long
          incy);
```

PURPOSE

zhemv performs the matrix-vector operation y := alpha*A*x + beta*y where alpha and beta are scalars, x and y are n element vectors and A is an n by n hermitian matrix.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array A is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of A is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of A is to be referenced.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular part of the hermitian matrix and the strictly lower triangular part of A is not referenced. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular part of the hermitian matrix and the strictly upper triangular part of A is not referenced. Note that the imaginary parts of the diagonal elements need not be set and are assumed to be zero. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

X (input)

(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)

On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

BETA (input)

On entry, BETA specifies the scalar beta. When BETA is supplied as zero then Y need not be set on input. Unchanged on exit.

Y (input/output)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
zher - perform the hermitian rank 1 operation   A :=
alpha*x*conjg( x' ) + A
```

SYNOPSIS

```
SUBROUTINE ZHER(UPLO, N, ALPHA, X, INCX, A, LDA)

CHARACTER * 1 UPLO
DOUBLE COMPLEX X(*), A(LDA,*)
INTEGER N, INCX, LDA
DOUBLE PRECISION ALPHA

SUBROUTINE ZHER_64(UPLO, N, ALPHA, X, INCX, A, LDA)

CHARACTER * 1 UPLO
DOUBLE COMPLEX X(*), A(LDA,*)
INTEGER*8 N, INCX, LDA
DOUBLE PRECISION ALPHA
```

F95 INTERFACE

```
SUBROUTINE HER(UPLO, [N], ALPHA, X, [INCX], A, [LDA])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: X
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, INCX, LDA
REAL(8) :: ALPHA

SUBROUTINE HER_64(UPLO, [N], ALPHA, X, [INCX], A, [LDA])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: X
```

```
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, INCX, LDA
REAL(8) :: ALPHA
```

C INTERFACE

```
#include <sunperf.h>

void zher(char uplo, int n, double alpha, doublecomplex  *x,
          int incx, doublecomplex *a, int lda);

void zher_64(char uplo, long n, double alpha,  doublecomplex
          *x, long incx, doublecomplex *a, long lda);
```

PURPOSE

zher performs the hermitian rank 1 operation A := alpha*x*conjg(x') + A where alpha is a real scalar, x is an n element vector and A is an n by n hermitian matrix.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array A is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of A is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of A is to be referenced.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

A (input/output)
Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular part of the hermitian matrix and the strictly lower triangular part of A is not referenced. On exit, the upper triangular part of the array A is overwritten by the upper triangular part of the updated matrix. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular part of the hermitian matrix and the strictly upper triangular part of A is not referenced. On exit, the lower triangular part of the array A is overwritten by the lower triangular part of the updated matrix. Note that the imaginary parts of the diagonal elements need not be set, they are assumed to be zero, and on exit they are set to zero.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

Contents

NAME

zher2 - perform the hermitian rank 2 operation A := alpha*x*conjg(y') + conjg(alpha)*y*conjg(x') + A

SYNOPSIS

```
SUBROUTINE ZHER2(UPLO, N, ALPHA, X, INCX, Y, INCY, A, LDA)

CHARACTER * 1 UPLO
DOUBLE COMPLEX ALPHA
DOUBLE COMPLEX X(*), Y(*), A(LDA,*)
INTEGER N, INCX, INCY, LDA

SUBROUTINE ZHER2_64(UPLO, N, ALPHA, X, INCX, Y, INCY, A, LDA)

CHARACTER * 1 UPLO
DOUBLE COMPLEX ALPHA
DOUBLE COMPLEX X(*), Y(*), A(LDA,*)
INTEGER*8 N, INCX, INCY, LDA
```

F95 INTERFACE

```
SUBROUTINE HER2(UPLO, [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8) :: ALPHA
COMPLEX(8), DIMENSION(:) :: X, Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, INCX, INCY, LDA

SUBROUTINE HER2_64(UPLO, [N], ALPHA, X, [INCX], Y, [INCY], A, [LDA])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8) :: ALPHA
```

```
COMPLEX(8), DIMENSION(:) :: X, Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, INCX, INCY, LDA
```

C INTERFACE

```
#include <sunperf.h>

void zher2(char uplo, int  n,  doublecomplex  *alpha,  doub-
          lecomplex  *x,  int  incx,  doublecomplex  *y, int
          incy, doublecomplex *a, int lda);

void zher2_64(char uplo, long n, doublecomplex *alpha, doub-
          lecomplex  *x,  long  incx, doublecomplex *y, long
          incy, doublecomplex *a, long lda);
```

PURPOSE

zher2 performs the hermitian rank 2 operation A := alpha*x*conjg(y') + conjg(alpha)*y*conjg(x') + A where alpha is a scalar, x and y are n element vectors and A is an n by n hermitian matrix.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array A is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of A is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of A is to be referenced.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the

incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

Y (input)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. Unchanged on exit.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

A (input/output)
Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular part of the hermitian matrix and the strictly lower triangular part of A is not referenced. On exit, the upper triangular part of the array A is overwritten by the upper triangular part of the updated matrix. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular part of the hermitian matrix and the strictly upper triangular part of A is not referenced. On exit, the lower triangular part of the array A is overwritten by the lower triangular part of the updated matrix. Note that the imaginary parts of the diagonal elements need not be set, they are assumed to be zero, and on exit they are set to zero.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

Contents

NAME

```
zher2k - perform one of the Hermitian rank 2k operations   C
:=  alpha*A*conjg(  B'  )  +  conjg( alpha )*B*conjg( A' ) +
beta*C or C := alpha*conjg( A' )*B + conjg(  alpha  )*conjg(
B' )*A + beta*C
```

SYNOPSIS

```
SUBROUTINE ZHER2K(UPLO, TRANSA, N, K, ALPHA, A, LDA, B, LDB, BETA, C,
      LDC)

CHARACTER * 1 UPLO, TRANSA
DOUBLE COMPLEX ALPHA
DOUBLE COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER N, K, LDA, LDB, LDC
DOUBLE PRECISION BETA

SUBROUTINE ZHER2K_64(UPLO, TRANSA, N, K, ALPHA, A, LDA, B, LDB, BETA,
      C, LDC)

CHARACTER * 1 UPLO, TRANSA
DOUBLE COMPLEX ALPHA
DOUBLE COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER*8 N, K, LDA, LDB, LDC
DOUBLE PRECISION BETA
```

F95 INTERFACE

```
SUBROUTINE HER2K(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], B, [LDB],
       BETA, C, [LDC])

CHARACTER(LEN=1) :: UPLO, TRANSA
COMPLEX(8) :: ALPHA
COMPLEX(8), DIMENSION(:,:) :: A, B, C
```

```
INTEGER :: N, K, LDA, LDB, LDC
REAL(8) :: BETA

SUBROUTINE HER2K_64(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], B,
       [LDB], BETA, C, [LDC])

CHARACTER(LEN=1) :: UPLO, TRANSA
COMPLEX(8) :: ALPHA
COMPLEX(8), DIMENSION(:,:) :: A, B, C
INTEGER(8) :: N, K, LDA, LDB, LDC
REAL(8) :: BETA
```

C INTERFACE

```
#include <sunperf.h>
void zher2k(char uplo, char transa, int n, int k, doublecom-
          plex *alpha, doublecomplex *a, int lda, doublecom-
          plex *b, int ldb, double beta,  doublecomplex  *c,
          int ldc);

void zher2k_64(char uplo, char transa, long n, long k, doub-
          lecomplex  *alpha,  doublecomplex  *a,  long  lda,
          doublecomplex *b, long  ldb,  double  beta,  doub-
          lecomplex *c, long ldc);
```

PURPOSE

zher2k performs one of the Hermitian rank 2k operations C := alpha*A*conjg(B') + conjg(alpha)*B*conjg(A') + beta*C or C := alpha*conjg(A')*B + conjg(alpha)*conjg(B')*A + beta*C where alpha and beta are scalars with beta real, C is an n by n Hermitian matrix and A and B are n by k matrices in the first case and k by n matrices in the second case.

ARGUMENTS

UPLO (input)
On entry,   UPLO specifies whether the   upper or   lower triangular part of the array C is to be referenced as follows:

UPLO = 'U' or 'u'   Only the   upper triangular part of C is to be referenced.

UPLO = 'L' or 'l'   Only the   lower triangular part of C is to be referenced.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n' C := alpha*A*conjg(B') + conjg(alpha)*B*conjg(A') + beta*C.

TRANSA = 'C' or 'c' C := alpha*conjg(A')*B + conjg(alpha)*conjg(B')*A + beta*C.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input)
On entry, N specifies the order of the matrix C. N must be at least zero. Unchanged on exit.

K (input)
On entry with TRANSA = 'N' or 'n', K specifies the number of columns of the matrices A and B, and on entry with TRANSA = 'C' or 'c', K specifies the number of rows of the matrices A and B. K must be at least zero. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
COMPLEX*16 array of DIMENSION (LDA, ka), where ka is k when TRANSA = 'N' or 'n', and is n otherwise. Before entry with TRANSA = 'N' or 'n', the leading n by k part of the array A must contain the matrix A, otherwise the leading k by n part of the array A must contain the matrix A. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDA must be at least max(1, n), otherwise LDA must be at least max(1, k). Unchanged on exit.

B (input)
COMPLEX*16 array of DIMENSION (LDB, kb), where kb is k when TRANSA = 'N' or 'n', and is

n otherwise. Before entry with TRANSA = 'N' or 'n', the leading n by k part of the array B must contain the matrix B, otherwise the leading k by n part of the array B must contain the matrix B. Unchanged on exit.

LDB (input)
On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDB must be at least max(1, n), otherwise LDB must be at least max(1, k). Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. Unchanged on exit.

C (input/output)
COMPLEX*16 array of DIMENSION (LDC, n).

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array C must contain the upper triangular part of the Hermitian matrix and the strictly lower triangular part of C is not referenced. On exit, the upper triangular part of the array C is overwritten by the upper triangular part of the updated matrix.

Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array C must contain the lower triangular part of the Hermitian matrix and the strictly upper triangular part of C is not referenced. On exit, the lower triangular part of the array C is overwritten by the lower triangular part of the updated matrix.

Note that the imaginary parts of the diagonal elements need not be set, they are assumed to be zero, and on exit they are set to zero.

LDC (input)
On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC must be at least max(1, n). Unchanged on exit.

Contents

NAME

```
zherfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is Hermitian indefin-
ite, and provides error bounds and backward error  estimates
for the solution
```

SYNOPSIS

```
SUBROUTINE ZHERFS(UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZHERFS_64(UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HERFS(UPLO, [N], [NRHS], A, [LDA], AF, [LDAF], IPIVOT, B,
       [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
    COMPLEX(8), DIMENSION(:) :: WORK
    COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
    INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
    INTEGER, DIMENSION(:) :: IPIVOT
    REAL(8), DIMENSION(:) :: FERR, BERR, WORK2

    SUBROUTINE HERFS_64(UPLO, [N], [NRHS], A, [LDA], AF, [LDAF], IPIVOT,
           B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

    CHARACTER(LEN=1) :: UPLO
    COMPLEX(8), DIMENSION(:) :: WORK
    COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
    INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
    INTEGER(8), DIMENSION(:) :: IPIVOT
    REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
    #include <sunperf.h>

    void zherfs(char uplo, int n, int  nrhs,  doublecomplex  *a,
              int lda, doublecomplex *af, int ldaf, int *ipivot,
              doublecomplex *b, int ldb, doublecomplex  *x,  int
              ldx, double *ferr, double *berr, int *info);

    void zherfs_64(char uplo, long n, long  nrhs,  doublecomplex
              *a,  long  lda, doublecomplex *af, long ldaf, long
              *ipivot, doublecomplex *b, long ldb, doublecomplex
              *x,  long  ldx,  double  *ferr, double *berr, long
              *info);
```

PURPOSE

zherfs improves the computed solution to a system of linear equations when the coefficient matrix is Hermitian indefinite, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

```
    UPLO (input)
              = 'U':  Upper triangle of A is stored;
              = 'L':  Lower triangle of A is stored.

    N (input) The order of the matrix A.  N >= 0.

    NRHS (input)
              The number of right hand sides, i.e.,  the  number
              of columns of the matrices B and X.  NRHS >= 0.
```

A (input) The Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input)
The factored form of the matrix A. AF contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**H or A = L*D*L**H as computed by CHETRF.

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CHETRF.

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by CHETRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as

reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zherk - perform one of the Hermitian rank k  operations    C
:=  alpha*A*conjg( A' ) + beta*C or C := alpha*conjg( A' )*A
+ beta*C
```

SYNOPSIS

```
SUBROUTINE ZHERK(UPLO, TRANSA, N, K, ALPHA, A, LDA, BETA, C, LDC)

CHARACTER * 1 UPLO, TRANSA
DOUBLE COMPLEX A(LDA,*), C(LDC,*)
INTEGER N, K, LDA, LDC
DOUBLE PRECISION ALPHA, BETA

SUBROUTINE ZHERK_64(UPLO, TRANSA, N, K, ALPHA, A, LDA, BETA, C, LDC)

CHARACTER * 1 UPLO, TRANSA
DOUBLE COMPLEX A(LDA,*), C(LDC,*)
INTEGER*8 N, K, LDA, LDC
DOUBLE PRECISION ALPHA, BETA
```

F95 INTERFACE

```
SUBROUTINE HERK(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], BETA, C,
       [LDC])

CHARACTER(LEN=1) :: UPLO, TRANSA
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER :: N, K, LDA, LDC
REAL(8) :: ALPHA, BETA

SUBROUTINE HERK_64(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], BETA,
       C, [LDC])
```

```
CHARACTER(LEN=1) :: UPLO, TRANSA
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER(8) :: N, K, LDA, LDC
REAL(8) :: ALPHA, BETA
```

C INTERFACE

```
#include <sunperf.h>

void zherk(char uplo, char transa,  int  n,  int  k,  double
          alpha,  doublecomplex  *a,  int  lda, double beta,
          doublecomplex *c, int ldc);

void zherk_64(char uplo, char transa, long n, long k, double
          alpha,  doublecomplex  *a,  long lda, double beta,
          doublecomplex *c, long ldc);
```

PURPOSE

zherk performs one of the Hermitian rank k operations C := alpha*A*conjg(A') + beta*C or C := alpha*conjg(A')*A + beta*C where alpha and beta are real scalars, C is an n by n Hermitian matrix and A is an n by k matrix in the first case and a k by n matrix in the second case.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array C is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of C is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of C is to be referenced.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   C := alpha*A*conjg(A') + beta*C.

TRANSA = 'C' or 'c'   C := alpha*conjg(A')*A +

beta*C.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input)
On entry, N specifies the order of the matrix C. N must be at least zero. Unchanged on exit.

K (input)
On entry with TRANSA = 'N' or 'n', K specifies the number of columns of the matrix A, and on entry with TRANSA = 'C' or 'c', K specifies the number of rows of the matrix A. K must be at least zero. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
COMPLEX*16 array of DIMENSION (LDA, ka), where ka is k when TRANSA = 'N' or 'n', and is n otherwise. Before entry with TRANSA = 'N' or 'n', the leading n by k part of the array A must contain the matrix A, otherwise the leading k by n part of the array A must contain the matrix A. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDA must be at least max(1, n), otherwise LDA must be at least max(1, k). Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. Unchanged on exit.

C (input/output)
COMPLEX*16 array of DIMENSION (LDC, n).

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array C must contain the upper triangular part of the Hermitian matrix and the strictly lower triangular part of C is not referenced. On exit, the upper triangular part of the array C is overwritten by the upper triangular part of the updated

matrix.

Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array C must contain the lower triangular part of the Hermitian matrix and the strictly upper triangular part of C is not referenced. On exit, the lower triangular part of the array C is overwritten by the lower triangular part of the updated matrix.

Note that the imaginary parts of the diagonal elements need not be set, they are assumed to be zero, and on exit they are set to zero.

LDC (input)

On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC must be at least max(1, n). Unchanged on exit.

Contents

NAME

```
zhesv - compute the solution to a complex system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE ZHESV(UPLO, N, NRHS, A, LDA, IPIVOT, B, LDB, WORK, LDWORK,
      INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER N, NRHS, LDA, LDB, LDWORK, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZHESV_64(UPLO, N, NRHS, A, LDA, IPIVOT, B, LDB, WORK,
      LDWORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER*8 N, NRHS, LDA, LDB, LDWORK, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE HESV(UPLO, [N], [NRHS], A, [LDA], IPIVOT, B, [LDB], [WORK],
       [LDWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: N, NRHS, LDA, LDB, LDWORK, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE HESV_64(UPLO, [N], [NRHS], A, [LDA], IPIVOT, B, [LDB],
```

```
          [WORK], [LDWORK], [INFO])

   CHARACTER(LEN=1) :: UPLO
   COMPLEX(8), DIMENSION(:) :: WORK
   COMPLEX(8), DIMENSION(:,:) :: A, B
   INTEGER(8) :: N, NRHS, LDA, LDB, LDWORK, INFO
   INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
   #include <sunperf.h>

   void zhesv(char uplo, int n, int nrhs, doublecomplex *a, int
             lda,  int  *ipivot, doublecomplex *b, int ldb, int
             *info);
   void zhesv_64(char uplo, long n,  long  nrhs,  doublecomplex
             *a, long lda, long *ipivot, doublecomplex *b, long
             ldb, long *info);
```

PURPOSE

zhesv computes the solution to a complex system of linear equations
   A * X = B, where A is an N-by-N Hermitian matrix and X and B are N-by-NRHS matrices.

The diagonal pivoting method is used to factor A as
   A = U * D * U**H, if UPLO = 'U', or
   A = L * D * L**H, if UPLO = 'L',
where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is Hermitian and block diagonal with 1-by-1 and 2-by-2 diagonal blocks. The factored form of A is then used to solve the system of equations A * X = B.

ARGUMENTS

UPLO (input)
  = 'U': Upper triangle of A is stored;
  = 'L': Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NRHS (input)
  The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input/output)
On entry, the Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, if INFO = 0, the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**H or A = L*D*L**H as computed by CHETRF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (output)
Details of the interchanges and the block structure of D, as determined by CHETRF. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged, and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of WORK. LDWORK >= 1, and for best performance LDWORK >= N*NB, where NB is the optimal blocksize for CHETRF.

If LDWORK = -1, then a workspace query is assumed;

the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, so the solution could not be computed.

Contents

NAME

```
zhesvx - use the diagonal pivoting factorization to  compute
the solution to a complex system of linear equations A * X =
B,
```

SYNOPSIS

```
SUBROUTINE ZHESVX(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B,
      LDB, X, LDX, RCOND, FERR, BERR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZHESVX_64(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT,
      B, LDB, X, LDX, RCOND, FERR, BERR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HESVX(FACT, UPLO, [N], [NRHS], A, [LDA], AF, [LDAF],
       IPIVOT, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [LDWORK],
       [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: FACT, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE HESVX_64(FACT, UPLO, [N], [NRHS], A, [LDA], AF, [LDAF],
       IPIVOT, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [LDWORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zhesvx(char fact, char uplo, int  n,  int  nrhs,  doub-
          lecomplex  *a,  int  lda,  doublecomplex  *af, int
          ldaf, int  *ipivot,  doublecomplex  *b,  int  ldb,
          doublecomplex  *x,  int ldx, double *rcond, double
          *ferr, double *berr, int *info);

void zhesvx_64(char fact, char  uplo,  long  n,  long  nrhs,
          doublecomplex  *a,  long  lda,  doublecomplex *af,
          long ldaf, long *ipivot,  doublecomplex  *b,  long
          ldb,  doublecomplex  *x,  long ldx, double *rcond,
          double *ferr, double *berr, long *info);
```

PURPOSE

zhesvx uses the diagonal pivoting factorization to compute the solution to a complex system of linear equations A * X = B, where A is an N-by-N Hermitian matrix and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'N', the diagonal pivoting method is used to

```
factor A.
   The form of the factorization is
      A = U * D * U**H,  if UPLO = 'U', or
      A = L * D * L**H,  if UPLO = 'L',
   where U (or L) is a product of permutation and unit upper
(lower)
   triangular matrices, and D is Hermitian and block  diago-
nal with
   1-by-1 and 2-by-2 diagonal blocks.

2. If some D(i,i)=0, so that D is exactly singular, then the
routine
   returns with INFO = i. Otherwise, the factored form of  A
is used
   to estimate the condition number of the matrix A.  If the
   reciprocal of the condition number is less  than  machine
precision,
   INFO = N+1 is returned as  a  warning,  but  the  routine
still goes on
   to solve for X and  compute  error  bounds  as  described
below.

3. The system of equations is solved for X  using  the  fac-
tored form
   of A.

4. Iterative refinement is applied to improve  the  computed
solution
   matrix and calculate  error  bounds  and  backward  error
estimates
   for it.
```

ARGUMENTS

```
FACT (input)
          Specifies whether or not the factored  form  of  A
          has  been supplied on entry.  = 'F':  On entry, AF
          and IPIVOT contain the factored form of A.  A,  AF
          and  IPIVOT  will  not  be  modified.  = 'N':  The
          matrix A will be copied to AF and factored.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.
```

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input/output)
If FACT = 'F', then AF is an input argument and on entry contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**H or A = L*D*L**H as computed by CHETRF.

If FACT = 'N', then AF is an output argument and on exit returns the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**H or A = L*D*L**H.

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

IPIVOT (input or output)
If FACT = 'F', then IPIVOT is an input argument and on entry contains details of the interchanges and the block structure of D, as determined by CHETRF. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

If FACT = 'N', then IPIVOT is an output argument and on exit contains details of the interchanges and the block structure of D, as determined by

CHETRF.

B (input) The N-by-NRHS right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A. If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of WORK. LDWORK >= 2*N, and for best performance LDWORK >= N*NB, where NB is the optimal blocksize for CHETRF.

If LDWORK = -1, then a workspace query is assumed;

the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: D(i,i) is exactly zero. The factorization has been completed but the factor D is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: D is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

zhetf2 - compute the factorization of a complex Hermitian matrix A using the Bunch-Kaufman diagonal pivoting method

SYNOPSIS

```
SUBROUTINE ZHETF2(UPLO, N, A, LDA, IPIV, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*)
INTEGER N, LDA, INFO
INTEGER IPIV(*)

SUBROUTINE ZHETF2_64(UPLO, N, A, LDA, IPIV, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*)
INTEGER*8 N, LDA, INFO
INTEGER*8 IPIV(*)
```

F95 INTERFACE

```
SUBROUTINE HETF2(UPLO, [N], A, [LDA], IPIV, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIV

SUBROUTINE HETF2_64(UPLO, [N], A, [LDA], IPIV, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A
```

```
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIV
```

C INTERFACE

```
#include <sunperf.h>

void zhetf2(char uplo, int n, doublecomplex *a, int lda, int
          *ipiv, int *info);

void zhetf2_64(char uplo, long  n,  doublecomplex  *a,  long
          lda, long *ipiv, long *info);
```

PURPOSE

zhetf2 computes the factorization of a complex Hermitian matrix A using the Bunch-Kaufman diagonal pivoting method:

   A = U*D*U' or A = L*D*L'

where U (or L) is a product of permutation and unit upper (lower) triangular matrices, U' is the conjugate transpose of U, and D is Hermitian and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

This is the unblocked version of the algorithm, calling Level 2 BLAS.

ARGUMENTS

UPLO (input)
Specifies whether the upper or lower triangular part of the Hermitian matrix A is stored:
= 'U': Upper triangular
= 'L': Lower triangular

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the Hermitian matrix A. If UPLO = 'U', the leading n-by-n upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, the block diagonal matrix D and the mul-

tipliers used to obtain the factor U or L (see below for further details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIV (output)
Details of the interchanges and the block structure of D. If IPIV(k) > 0, then rows and columns k and IPIV(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIV(k) = IPIV(k-1) < 0, then rows and columns k-1 and -IPIV(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIV(k) = IPIV(k+1) < 0, then rows and columns k+1 and -IPIV(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, D(k,k) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

```
1-96 - Based on modifications by
  J. Lewis, Boeing Computer Services Company
  A. Petitet, Computer Science Dept., Univ. of Tenn., Knox-
ville, USA

If UPLO = 'U', then A = U*D*U', where
   U = P(n)*U(n)* ... *P(k)U(k)* ...,
i.e., U is a product of terms P(k)*U(k), where  k  decreases
from  n  to  1 in steps of 1 or 2, and D is a block diagonal
matrix with 1-by-1 and 2-by-2 diagonal blocks D(k).  P(k) is
a  permutation  matrix  as defined by IPIV(k), and U(k) is a
unit upper triangular matrix,  such  that  if  the  diagonal
block D(k) is of order s (s = 1 or 2), then

           (   I    v    0   )   k-s
   U(k) =  (   0    I    0   )   s
           (   0    0    I   )   n-k
              k-s   s   n-k
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(1:k-1,k). If s = 2, the upper triangle of D(k) overwrites A(k-1,k-1), A(k-1,k), and A(k,k), and v overwrites A(1:k-2,k-1:k).

If UPLO = 'L', then A = L*D*L', where

```
   L = P(1)*L(1)* ... *P(k)*L(k)* ...,
```

i.e., L is a product of terms P(k)*L(k), where k increases from 1 to n in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIV(k), and L(k) is a unit lower triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    0     0   )  k-1
   L(k) =  (   0    I     0   )  s
           (   0    v     I   )  n-k-s+1
              k-1   s  n-k-s+1
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(k+1:n,k). If s = 2, the lower triangle of D(k) overwrites A(k,k), A(k+1,k), and A(k+1,k+1), and v overwrites A(k+2:n,k:k+1).

Contents

NAME

```
zhetrd - reduce a complex Hermitian matrix A  to  real  sym-
metric  tridiagonal form T by a unitary similarity transfor-
mation
```

SYNOPSIS

```
SUBROUTINE ZHETRD(UPLO, N, A, LDA, D, E, TAU, WORK, LWORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER N, LDA, LWORK, INFO
DOUBLE PRECISION D(*), E(*)

SUBROUTINE ZHETRD_64(UPLO, N, A, LDA, D, E, TAU, WORK, LWORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 N, LDA, LWORK, INFO
DOUBLE PRECISION D(*), E(*)
```

F95 INTERFACE

```
SUBROUTINE HETRD(UPLO, [N], A, [LDA], D, E, TAU, [WORK], [LWORK],
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, LWORK, INFO
REAL(8), DIMENSION(:) :: D, E

SUBROUTINE HETRD_64(UPLO, [N], A, [LDA], D, E, TAU, [WORK], [LWORK],
```

```
          [INFO])

   CHARACTER(LEN=1) :: UPLO
   COMPLEX(8), DIMENSION(:) :: TAU, WORK
   COMPLEX(8), DIMENSION(:,:) :: A
   INTEGER(8) :: N, LDA, LWORK, INFO
   REAL(8), DIMENSION(:) :: D, E
```

C INTERFACE

```
   #include <sunperf.h>

   void zhetrd(char uplo, int n,  doublecomplex  *a,  int  lda,
             double  *d,  double  *e,  doublecomplex  *tau, int
             *info);
   void zhetrd_64(char uplo, long  n,  doublecomplex  *a,  long
             lda,  double  *d,  double  *e, doublecomplex *tau,
             long *info);
```

PURPOSE

zhetrd reduces a complex Hermitian matrix A to real symmetric tridiagonal form T by a unitary similarity transformation: Q**H * A * Q = T.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input) On entry, the Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced. On exit, if UPLO = 'U', the diagonal and first superdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements above the first superdiagonal, with the array TAU, represent the unitary matrix Q as a product of elementary reflectors; if UPLO = 'L', the diagonal and first subdiagonal of A are overwritten by the corresponding elements of the tri- diagonal matrix T, and the elements below the first subdiagonal, with the array TAU, represent the unitary matrix Q as a product of elementary reflectors. See Further Details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

D (output)
The diagonal elements of the tridiagonal matrix T: D(i) = A(i,i).

E (output)
The off-diagonal elements of the tridiagonal matrix T: E(i) = A(i,i+1) if UPLO = 'U', E(i) = A(i+1,i) if UPLO = 'L'.

TAU (output)
The scalar factors of the elementary reflectors (see Further Details).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 1. For optimum performance LWORK >= N*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille- gal value

FURTHER DETAILS

If UPLO = 'U', the matrix Q is represented as a product of elementary reflectors

Q = H(n-1) . . . H(2) H(1).

Each H(i) has the form

```
   H(i) = I - tau * v * v'

where tau is a complex scalar, and v  is  a  complex  vector
with  v(i+1:n)  = 0 and v(i) = 1; v(1:i-1) is stored on exit
in
A(1:i-1,i+1), and tau in TAU(i).

If UPLO = 'L', the matrix Q is represented as a  product  of
elementary reflectors

   Q = H(1) H(2) . . . H(n-1).
Each H(i) has the form

   H(i) = I - tau * v * v'

where tau is a complex scalar, and v  is  a  complex  vector
with  v(1:i)  = 0 and v(i+1) = 1; v(i+2:n) is stored on exit
in A(i+2:n,i), and tau in TAU(i).

The contents of A on exit are illustrated by  the  following
examples with n = 5:

if UPLO = 'U':                       if UPLO = 'L':

  (    d     e     v2    v3    v4   )                (     d
)
  (       d    e    v3    v4   )                (    e     d
)
  (           d    e    v4  )               (   v1   e     d
)
  (              d   e  )               (   v1   v2   e    d
)
  (                  d  )              (  v1  v2  v3  e    d
)

where d and e denote diagonal and off-diagonal  elements  of
T, and vi denotes an element of the vector defining H(i).
```

Contents

NAME

```
zhetrf - compute the factorization of  a  complex  Hermitian
matrix A using the Bunch-Kaufman diagonal pivoting method
```

SYNOPSIS

```
SUBROUTINE ZHETRF(UPLO, N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, LDWORK, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZHETRF_64(UPLO, N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, LDWORK, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE HETRF(UPLO, [N], A, [LDA], IPIVOT, [WORK], [LDWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, LDWORK, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE HETRF_64(UPLO, [N], A, [LDA], IPIVOT, [WORK], [LDWORK],
       [INFO])
```

```
CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void zhetrf(char uplo, int n, doublecomplex *a, int lda, int
          *ipivot, int *info);

void zhetrf_64(char uplo, long  n,  doublecomplex  *a,  long
          lda, long *ipivot, long *info);
```

PURPOSE

zhetrf computes the factorization of a complex Hermitian matrix A using the Bunch-Kaufman diagonal pivoting method. The form of the factorization is

A = U*D*U**H or A = L*D*L**H

where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is Hermitian and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

This is the blocked version of the algorithm, calling Level 3 BLAS.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, the block diagonal matrix D and the multipliers used to obtain the factor U or L (see below for further details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (output)
Details of the interchanges and the block structure of D. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of WORK. LDWORK >=1. For best performance LDWORK >= N*NB, where NB is the block size returned by ILAENV.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

If UPLO = 'U', then A = U*D*U', where
   U = P(n)*U(n)* ... *P(k)U(k)* ...,
i.e., U is a product of terms P(k)*U(k), where k decreases from n to 1 in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIVOT(k), and U(k) is a unit upper triangular matrix, such that if the diagonal

block D(k) is of order s (s = 1 or 2), then

```
           (   I    v    0   )   k-s
   U(k) =  (   0    I    0   )   s
           (   0    0    I   )   n-k
              k-s   s   n-k
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(1:k-1,k). If s = 2, the upper triangle of D(k) overwrites A(k-1,k-1), A(k-1,k), and A(k,k), and v overwrites A(1:k-2,k-1:k).

If UPLO = 'L', then A = L*D*L', where
   L = P(1)*L(1)* ... *P(k)*L(k)* ...,
i.e., L is a product of terms P(k)*L(k), where k increases from 1 to n in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIVOT(k), and L(k) is a unit lower triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    0     0   )  k-1
   L(k) =  (   0    I     0   )  s
           (   0    v     I   )  n-k-s+1
              k-1   s  n-k-s+1
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(k+1:n,k). If s = 2, the lower triangle of D(k) overwrites A(k,k), A(k+1,k), and A(k+1,k+1), and v overwrites A(k+2:n,k:k+1).

Contents

NAME

zhetri - compute the inverse of a complex Hermitian indefinite matrix A using the factorization A = U*D*U**H or A = L*D*L**H computed by CHETRF

SYNOPSIS

```
SUBROUTINE ZHETRI(UPLO, N, A, LDA, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZHETRI_64(UPLO, N, A, LDA, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE HETRI(UPLO, [N], A, [LDA], IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE HETRI_64(UPLO, [N], A, [LDA], IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void zhetri(char uplo, int n, doublecomplex *a, int lda, int
          *ipivot, int *info);

void zhetri_64(char uplo, long  n,  doublecomplex  *a,  long
          lda, long *ipivot, long *info);
```

PURPOSE

zhetri computes the inverse of a complex Hermitian indefinite matrix A using the factorization A = U*D*U**H or A = L*D*L**H computed by CHETRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**H;
= 'L': Lower triangular, form is A = L*D*L**H.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CHETRF.

On exit, if INFO = 0, the (Hermitian) inverse of the original matrix. If UPLO = 'U', the upper triangular part of the inverse is formed and the part of A below the diagonal is not referenced; if UPLO = 'L' the lower triangular part of the inverse is formed and the part of A above the diagonal is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CHETRF.

WORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) = 0; the matrix is singular and its inverse could not be computed.

Contents

NAME

```
zhetrs - solve a system of linear equations A*X = B  with  a
complex  Hermitian  matrix  A  using  the  factorization A =
U*D*U**H or A = L*D*L**H computed by CHETRF
```

SYNOPSIS

```
SUBROUTINE ZHETRS(UPLO, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZHETRS_64(UPLO, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE HETRS(UPLO, [N], [NRHS], A, [LDA], IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: N, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE HETRS_64(UPLO, [N], [NRHS], A, [LDA], IPIVOT, B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void zhetrs(char uplo, int n, int  nrhs,  doublecomplex  *a,
          int  lda,  int *ipivot, doublecomplex *b, int ldb,
          int *info);

void zhetrs_64(char uplo, long n, long  nrhs,  doublecomplex
          *a, long lda, long *ipivot, doublecomplex *b, long
          ldb, long *info);
```

PURPOSE

zhetrs solves a system of linear equations A*X = B with a complex Hermitian matrix A using the factorization A = U*D*U**H or A = L*D*L**H computed by CHETRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**H;
= 'L': Lower triangular, form is A = L*D*L**H.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CHETRF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CHETRF.

B (input/output)
On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

zhgeqz - implement a single-shift version of the QZ method for finding the generalized eigenvalues w(i)=ALPHA(i)/BETA(i) of the equation det(A-w(i) B) = 0 If JOB='S', then the pair (A,B) is simultaneously reduced to Schur form (i.e., A and B are both upper triangular) by applying one unitary tranformation (usually called Q) on the left and another (usually called Z) on the right

SYNOPSIS

```
SUBROUTINE ZHGEQZ(JOB, COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB,
      ALPHA, BETA, Q, LDQ, Z, LDZ, WORK, LWORK, RWORK, INFO)

CHARACTER * 1 JOB, COMPQ, COMPZ
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),   ALPHA(*),   BETA(*),
Q(LDQ,*), Z(LDZ,*), WORK(*)
INTEGER N, ILO, IHI, LDA, LDB, LDQ, LDZ, LWORK, INFO
DOUBLE PRECISION RWORK(*)

SUBROUTINE ZHGEQZ_64(JOB, COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB,
      ALPHA, BETA, Q, LDQ, Z, LDZ, WORK, LWORK, RWORK, INFO)

CHARACTER * 1 JOB, COMPQ, COMPZ
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),   ALPHA(*),   BETA(*),
Q(LDQ,*), Z(LDZ,*), WORK(*)
INTEGER*8 N, ILO, IHI, LDA, LDB, LDQ, LDZ, LWORK, INFO
DOUBLE PRECISION RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HGEQZ(JOB, COMPQ, COMPZ, [N], ILO, IHI, A, [LDA], B, [LDB],
       ALPHA, BETA, Q, [LDQ], Z, [LDZ], [WORK], [LWORK], [RWORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOB, COMPQ, COMPZ
COMPLEX(8), DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, Q, Z
INTEGER :: N, ILO, IHI, LDA, LDB, LDQ, LDZ, LWORK, INFO
REAL(8), DIMENSION(:) :: RWORK

SUBROUTINE HGEQZ_64(JOB, COMPQ, COMPZ, [N], ILO, IHI, A, [LDA], B,
       [LDB], ALPHA, BETA, Q, [LDQ], Z, [LDZ], [WORK], [LWORK], [RWORK],
       [INFO])

CHARACTER(LEN=1) :: JOB, COMPQ, COMPZ
COMPLEX(8), DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, Q, Z
INTEGER(8) :: N, ILO, IHI, LDA, LDB, LDQ, LDZ, LWORK, INFO
REAL(8), DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>

void zhgeqz(char job, char compq, char  compz,  int  n,  int
          ilo,  int  ihi,  doublecomplex  *a, int lda, doub-
          lecomplex *b, int ldb, doublecomplex *alpha, doub-
          lecomplex  *beta, doublecomplex *q, int ldq, doub-
          lecomplex *z, int ldz, int *info);

void zhgeqz_64(char job, char compq,  char  compz,  long  n,
          long  ilo,  long  ihi, doublecomplex *a, long lda,
          doublecomplex *b, long ldb, doublecomplex  *alpha,
          doublecomplex  *beta,  doublecomplex *q, long ldq,
          doublecomplex *z, long ldz, long *info);
```

PURPOSE

zhgeqz implements a single-shift version of the QZ method for finding the generalized eigenvalues w(i)=ALPHA(i)/BETA(i) of the equation A are then ALPHA(1),...,ALPHA(N), and of B are BETA(1),...,BETA(N).

If JOB='S' and COMPQ and COMPZ are 'V' or 'I', then the unitary transformations used to reduce (A,B) are accumulated into the arrays Q and Z s.t.:
(in) A(in) Z(in)* = Q(out) A(out) Z(out)*

Ref: C.B. Moler & G.W. Stewart, "An Algorithm for Generalized Matrixigenvalue Problems", SIAM J. Numer. Anal., 10(1973),p. 241--256.

ARGUMENTS

JOB (input)
= 'E': compute only ALPHA and BETA. A and B will not necessarily be put into generalized Schur form. = 'S': put A and B into generalized Schur form, as well as computing ALPHA and BETA.

COMPQ (input)
= 'N': do not modify Q.
= 'V': multiply the array Q on the right by the conjugate transpose of the unitary tranformation that is applied to the left side of A and B to reduce them to Schur form. = 'I': like COMPQ='V', except that Q will be initialized to the identity first.

COMPZ (input)
= 'N': do not modify Z.
= 'V': multiply the array Z on the right by the unitary tranformation that is applied to the right side of A and B to reduce them to Schur form. = 'I': like COMPZ='V', except that Z will be initialized to the identity first.

N (input) The order of the matrices A, B, Q, and Z. N >= 0.

ILO (input)
It is assumed that A is already upper triangular in rows and columns 1:ILO-1 and IHI+1:N. 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

IHI (input)
It is assumed that A is already upper triangular in rows and columns 1:ILO-1 and IHI+1:N. 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

A (input) On entry, the N-by-N upper Hessenberg matrix A. Elements below the subdiagonal must be zero. If JOB='S', then on exit A and B will have been simultaneously reduced to upper triangular form. If JOB='E', then on exit A will have been destroyed.

LDA (input)
The leading dimension of the array A. LDA >= max(1, N).

B (input) On entry, the N-by-N upper triangular matrix B. Elements below the diagonal must be zero. If

JOB='S', then on exit A and B will have been simultaneously reduced to upper triangular form. If JOB='E', then on exit B will have been destroyed.

LDB (input)
The leading dimension of the array B. LDB >= max(1, N).

ALPHA (output)
The diagonal elements of A when the pair (A,B) has been reduced to Schur form. ALPHA(i)/BETA(i) i=1,...,N are the generalized eigenvalues.

BETA (output)
The diagonal elements of B when the pair (A,B) has been reduced to Schur form. ALPHA(i)/BETA(i) i=1,...,N are the generalized eigenvalues. A and B are normalized so that BETA(1),...,BETA(N) are non-negative real numbers.

Q (input/output)
If COMPQ='N', then Q will not be referenced. If COMPQ='V' or 'I', then the conjugate transpose of the unitary transformations which are applied to A and B on the left will be applied to the array Q on the right.

LDQ (input)
The leading dimension of the array Q. LDQ >= 1. If COMPQ='V' or 'I', then LDQ >= N.

Z (input/output)
If COMPZ='N', then Z will not be referenced. If COMPZ='V' or 'I', then the unitary transformations which are applied to A and B on the right will be applied to the array Z on the right.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1. If COMPZ='V' or 'I', then LDZ >= N.

WORK (workspace)
On exit, if INFO >= 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,N).

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
= 1,...,N: the QZ iteration did not converge. (A,B) is not in Schur form, but ALPHA(i) and BETA(i), i=INFO+1,...,N should be correct. = N+1,...,2*N: the shift calculation failed. (A,B) is not in Schur form, but ALPHA(i) and BETA(i), i=INFO-N+1,...,N should be correct. > 2*N: various "impossible" errors.

FURTHER DETAILS

We assume that complex ABS works as long as its value is less than overflow.

Contents

NAME

```
zhpcon - estimate the reciprocal of the condition number  of
a  complex Hermitian packed matrix A using the factorization
A = U*D*U**H or A = L*D*L**H computed by CHPTRF
```

SYNOPSIS

```
SUBROUTINE ZHPCON(UPLO, N, A, IPIVOT, ANORM, RCOND, WORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), WORK(*)
INTEGER N, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION ANORM, RCOND

SUBROUTINE ZHPCON_64(UPLO, N, A, IPIVOT, ANORM, RCOND, WORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), WORK(*)
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION ANORM, RCOND
```

F95 INTERFACE

```
SUBROUTINE HPCON(UPLO, N, A, IPIVOT, ANORM, RCOND, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A, WORK
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8) :: ANORM, RCOND

SUBROUTINE HPCON_64(UPLO, N, A, IPIVOT, ANORM, RCOND, [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A, WORK
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8) :: ANORM, RCOND
```

C INTERFACE

```
#include <sunperf.h>

void zhpcon(char uplo, int n, doublecomplex *a, int *ipivot,
          double anorm, double *rcond, int *info);

void zhpcon_64(char uplo, long  n,  doublecomplex  *a,  long
          *ipivot, double anorm, double *rcond, long *info);
```

PURPOSE

zhpcon estimates the reciprocal of the condition number of a complex Hermitian packed matrix A using the factorization A = U*D*U**H or A = L*D*L**H computed by CHPTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**H;
= 'L': Lower triangular, form is A = L*D*L**H.

N (input) The order of the matrix A. N >= 0.

A (input) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CHPTRF, stored as a packed triangular matrix.

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CHPTRF.

ANORM (input)
The 1-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zhpev - compute all the eigenvalues and, optionally,  eigen-
vectors of a complex Hermitian matrix in packed storage
```

SYNOPSIS

```
SUBROUTINE ZHPEV(JOBZ, UPLO, N, A, W, Z, LDZ, WORK, WORK2, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX A(*), Z(LDZ,*), WORK(*)
INTEGER N, LDZ, INFO
DOUBLE PRECISION W(*), WORK2(*)

SUBROUTINE ZHPEV_64(JOBZ, UPLO, N, A, W, Z, LDZ, WORK, WORK2, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX A(*), Z(LDZ,*), WORK(*)
INTEGER*8 N, LDZ, INFO
DOUBLE PRECISION W(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HPEV(JOBZ, UPLO, N, A, W, Z, [LDZ], [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: A, WORK
COMPLEX(8), DIMENSION(:,:) :: Z
INTEGER :: N, LDZ, INFO
REAL(8), DIMENSION(:) :: W, WORK2

SUBROUTINE HPEV_64(JOBZ, UPLO, N, A, W, Z, [LDZ], [WORK], [WORK2],
       [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: A, WORK
COMPLEX(8), DIMENSION(:,:) :: Z
INTEGER(8) :: N, LDZ, INFO
REAL(8), DIMENSION(:) :: W, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zhpev(char jobz, char uplo, int  n,  doublecomplex  *a,
          double *w, doublecomplex *z, int ldz, int *info);

void zhpev_64(char jobz, char uplo,  long  n,  doublecomplex
          *a,  double  *w,  doublecomplex *z, long ldz, long
          *info);
```

PURPOSE

zhpev computes all the eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix in packed storage.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the upper or lower triangle of the Hermitian matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

On exit, A is overwritten by values generated during the reduction to tridiagonal form. If UPLO = 'U', the diagonal and first superdiagonal of the tridiagonal matrix T overwrite the corresponding elements of A, and if UPLO = 'L', the diagonal and first subdiagonal of T overwrite the corresponding

```
          elements of A.

W (output)
          If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0,  Z  contains  the
          orthonormal eigenvectors of the matrix A, with the
          i-th column of Z holding the  eigenvector  associ-
          ated  with  W(i).   If  JOBZ  = 'N', then Z is not
          referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).
WORK (workspace)
          dimension(MAX(1,2*N-1))

WORK2 (workspace)
          dimension(max(1,3*N-2))

INFO (output)
          = 0:  successful exit.
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value.
          > 0:  if INFO = i, the algorithm  failed  to  con-
          verge;  i off-diagonal elements of an intermediate
          tridiagonal form did not converge to zero.
```

Contents

NAME

zhpevd - compute all the eigenvalues and, optionally, eigen-
vectors of a complex Hermitian matrix A in packed storage

SYNOPSIS

```
SUBROUTINE ZHPEVD(JOBZ, UPLO, N, AP, W, Z, LDZ, WORK, LWORK, RWORK,
      LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX AP(*), Z(LDZ,*), WORK(*)
INTEGER N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER IWORK(*)
DOUBLE PRECISION W(*), RWORK(*)

SUBROUTINE ZHPEVD_64(JOBZ, UPLO, N, AP, W, Z, LDZ, WORK, LWORK,
      RWORK, LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX AP(*), Z(LDZ,*), WORK(*)
INTEGER*8 N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HPEVD(JOBZ, UPLO, N, AP, W, Z, [LDZ], [WORK], [LWORK],
       [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: AP, WORK
COMPLEX(8), DIMENSION(:,:) :: Z
INTEGER :: N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
```

```
REAL(8), DIMENSION(:) :: W, RWORK

SUBROUTINE HPEVD_64(JOBZ, UPLO, N, AP, W, Z, [LDZ], [WORK], [LWORK],
       [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: AP, WORK
COMPLEX(8), DIMENSION(:,:) :: Z
INTEGER(8) :: N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>
void zhpevd(char jobz, char uplo, int n, doublecomplex  *ap,
          double *w, doublecomplex *z, int ldz, int *info);

void zhpevd_64(char jobz, char uplo, long  n,  doublecomplex
          *ap,  double  *w, doublecomplex *z, long ldz, long
          *info);
```

PURPOSE

zhpevd computes all the eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A in packed storage. If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

```
AP (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  matrix  A,  packed  columnwise in a linear
          array.  The j-th column of  A  is  stored  in  the
          array  AP  as  follows:  if UPLO = 'U', AP(i + (j-
          1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L',  AP(i
          + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

          On exit, AP is  overwritten  by  values  generated
          during the reduction to tridiagonal form.  If UPLO
          = 'U', the diagonal and first superdiagonal of the
          tridiagonal  matrix  T overwrite the corresponding
          elements of A, and if UPLO = 'L', the diagonal and
          first subdiagonal of T overwrite the corresponding
          elements of A.

W (output)
          If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0,  Z  contains  the
          orthonormal eigenvectors of the matrix A, with the
          i-th column of Z holding the  eigenvector  associ-
          ated  with  W(i).   If  JOBZ  = 'N', then Z is not
          referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The  dimension  of  array  WORK.   If  N   <=   1,
          LWORK  must  be at least 1.  If JOBZ = 'N' and N >
          1, LWORK must be at least N.  If JOBZ = 'V' and  N
          > 1, LWORK must be at least 2*N.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

RWORK (workspace)
          dimension (LRWORK) On exit, if INFO = 0,  RWORK(1)
          returns the optimal LRWORK.
```

LRWORK (input)

The dimension of array RWORK. If N <= 1, LRWORK must be at least 1. If JOBZ = 'N' and N > 1, LRWORK must be at least N. If JOBZ = 'V' and N > 1, LRWORK must be at least 1 + 5*N + 2*N**2.

If LRWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the RWORK array, returns this value as the first entry of the RWORK array, and no error message related to LRWORK is issued by XERBLA.

IWORK (workspace/output)

On exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)

The dimension of array IWORK. If JOBZ = 'N' or N <= 1, LIWORK must be at least 1. If JOBZ = 'V' and N > 1, LIWORK must be at least 3 + 5*N.

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value.

> 0: if INFO = i, the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

Contents

NAME

zhpevx - compute selected eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A in packed storage

SYNOPSIS

```
SUBROUTINE ZHPEVX(JOBZ, RANGE, UPLO, N, A, VL, VU, IL, IU, ABTOL,
      NFOUND, W, Z, LDZ, WORK, WORK2, IWORK3, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
DOUBLE COMPLEX A(*), Z(LDZ,*), WORK(*)
INTEGER N, IL, IU, NFOUND, LDZ, INFO
INTEGER IWORK3(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABTOL
DOUBLE PRECISION W(*), WORK2(*)

SUBROUTINE ZHPEVX_64(JOBZ, RANGE, UPLO, N, A, VL, VU, IL, IU, ABTOL,
      NFOUND, W, Z, LDZ, WORK, WORK2, IWORK3, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
DOUBLE COMPLEX A(*), Z(LDZ,*), WORK(*)
INTEGER*8 N, IL, IU, NFOUND, LDZ, INFO
INTEGER*8 IWORK3(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABTOL
DOUBLE PRECISION W(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HPEVX(JOBZ, RANGE, UPLO, N, A, VL, VU, IL, IU, ABTOL,
       [NFOUND], W, Z, [LDZ], [WORK], [WORK2], [IWORK3], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX(8), DIMENSION(:) :: A, WORK
```

```
COMPLEX(8), DIMENSION(:,:) :: Z
INTEGER :: N, IL, IU, NFOUND, LDZ, INFO
INTEGER, DIMENSION(:) :: IWORK3, IFAIL
REAL(8) :: VL, VU, ABTOL
REAL(8), DIMENSION(:) :: W, WORK2

SUBROUTINE HPEVX_64(JOBZ, RANGE, UPLO, N, A, VL, VU, IL, IU, ABTOL,
       [NFOUND], W, Z, [LDZ], [WORK], [WORK2], [IWORK3], IFAIL, [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX(8), DIMENSION(:) :: A, WORK
COMPLEX(8), DIMENSION(:,:) :: Z
INTEGER(8) :: N, IL, IU, NFOUND, LDZ, INFO
INTEGER(8), DIMENSION(:) :: IWORK3, IFAIL
REAL(8) :: VL, VU, ABTOL
REAL(8), DIMENSION(:) :: W, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zhpevx(char jobz, char range, char uplo, int  n,  doub-
          lecomplex  *a,  double  vl, double vu, int il, int
          iu, double abtol, int *nfound,  double  *w,  doub-
          lecomplex *z, int ldz, int *ifail, int *info);

void zhpevx_64(char jobz, char range,  char  uplo,  long  n,
          doublecomplex  *a,  double vl, double vu, long il,
          long iu, double abtol, long  *nfound,  double  *w,
          doublecomplex  *z,  long  ldz,  long  *ifail, long
          *info);
```

PURPOSE

zhpevx computes selected eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A in packed storage. Eigenvalues/vectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

```
JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

RANGE (input)
          = 'A': all eigenvalues will be found;
          = 'V': all eigenvalues in the  half-open  interval
```

(VL,VU] will be found; = 'I': the IL-th through IU-th eigenvalues will be found.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the upper or lower triangle of the Hermitian matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.
On exit, A is overwritten by values generated during the reduction to tridiagonal form. If UPLO = 'U', the diagonal and first superdiagonal of the tridiagonal matrix T overwrite the corresponding elements of A, and if UPLO = 'L', the diagonal and first subdiagonal of T overwrite the corresponding elements of A.

VL (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

ABTOL (input)
The absolute error tolerance for the eigenvalues.

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing A to tridiagonal form.

Eigenvalues will be computed most accurately when ABTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABTOL to 2*SLAMCH('S').

See "Computing Small Singular Values of Bidiagonal Matrices with Guaranteed High Relative Accuracy," by Demmel and Kahan, LAPACK Working Note #3.

NFOUND (output)
The total number of eigenvalues found. 0 <= NFOUND <= N. If RANGE = 'A', NFOUND = N, and if RANGE = 'I', NFOUND = IU-IL+1.

W (output)
If INFO = 0, the selected eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, the first NFOUND columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). If an eigenvector fails to converge, then that column of Z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in IFAIL. If JOBZ = 'N', then Z is not referenced. Note: the user must ensure that at least max(1,NFOUND) columns are supplied in the array Z; if RANGE = 'V', the exact value of NFOUND is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)

dimension(2*N)

WORK2 (workspace)
dimension(7*N)

IWORK3 (workspace)
dimension(5*N)

IFAIL (output)
If JOBZ = 'V', then if INFO = 0, the first NFOUND elements of IFAIL are zero. If INFO > 0, then IFAIL contains the indices of the eigenvectors that failed to converge. If JOBZ = 'N', then IFAIL is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, then i eigenvectors failed to converge. Their indices are stored in array IFAIL.

Contents

NAME

```
zhpgst - reduce  a  complex  Hermitian-definite  generalized
eigenproblem to standard form, using packed storage
```

SYNOPSIS

```
SUBROUTINE ZHPGST(ITYPE, UPLO, N, AP, BP, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AP(*), BP(*)
INTEGER ITYPE, N, INFO

SUBROUTINE ZHPGST_64(ITYPE, UPLO, N, AP, BP, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AP(*), BP(*)
INTEGER*8 ITYPE, N, INFO
```

F95 INTERFACE

```
SUBROUTINE HPGST(ITYPE, UPLO, N, AP, BP, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: AP, BP
INTEGER :: ITYPE, N, INFO

SUBROUTINE HPGST_64(ITYPE, UPLO, N, AP, BP, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: AP, BP
INTEGER(8) :: ITYPE, N, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zhpgst(int itype, char uplo, int n, doublecomplex  *ap,
          doublecomplex *bp, int *info);

void zhpgst_64(long itype, char uplo, long n,  doublecomplex
          *ap, doublecomplex *bp, long *info);
```

PURPOSE

zhpgst reduces a complex Hermitian-definite generalized eigenproblem to standard form, using packed storage.

If ITYPE = 1, the problem is A*x = lambda*B*x,
and A is overwritten by inv(U**H)*A*inv(U) or inv(L)*A*inv(L**H)
If ITYPE = 2 or 3, the problem is A*B*x = lambda*x or
B*A*x = lambda*x, and A is overwritten by U*A*U**H or L**H*A*L.

B must have been previously factorized as U**H*U or L*L**H by CPPTRF.

ARGUMENTS

ITYPE (input)
= 1: compute inv(U**H)*A*inv(U) or inv(L)*A*inv(L**H);
= 2 or 3: compute U*A*U**H or L**H*A*L.

UPLO (input)
= 'U': Upper triangle of A is stored and B is factored as U**H*U; = 'L': Lower triangle of A is stored and B is factored as L*L**H.

N (input) The order of the matrices A and B. N >= 0.

AP (input/output)
On entry, the upper or lower triangle of the Hermitian matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n.

On exit, if INFO = 0, the transformed matrix, stored in the same format as A.

BP (input)
The triangular factor from the Cholesky factorization of B, stored in the same format as A, as returned by CPPTRF.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zhpgv - compute all the  eigenvalues  and,  optionally,  the
eigenvectors  of  a  complex  generalized Hermitian-definite
eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x,
or B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE ZHPGV(ITYPE, JOBZ, UPLO, N, A, B, W, Z, LDZ, WORK, WORK2,
      INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX A(*), B(*), Z(LDZ,*), WORK(*)
INTEGER ITYPE, N, LDZ, INFO
DOUBLE PRECISION W(*), WORK2(*)

SUBROUTINE ZHPGV_64(ITYPE, JOBZ, UPLO, N, A, B, W, Z, LDZ, WORK,
      WORK2, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX A(*), B(*), Z(LDZ,*), WORK(*)
INTEGER*8 ITYPE, N, LDZ, INFO
DOUBLE PRECISION W(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HPGV(ITYPE, JOBZ, UPLO, N, A, B, W, Z, [LDZ], [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: A, B, WORK
COMPLEX(8), DIMENSION(:,:) :: Z
INTEGER :: ITYPE, N, LDZ, INFO
REAL(8), DIMENSION(:) :: W, WORK2
```

```
SUBROUTINE HPGV_64(ITYPE, JOBZ, UPLO, N, A, B, W, Z, [LDZ], [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: A, B, WORK
COMPLEX(8), DIMENSION(:,:) :: Z
INTEGER(8) :: ITYPE, N, LDZ, INFO
REAL(8), DIMENSION(:) :: W, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zhpgv(int itype, char jobz, char  uplo,  int  n,  doub-
          lecomplex   *a,   doublecomplex   *b,  double  *w,
          doublecomplex *z, int ldz, int *info);

void zhpgv_64(long itype, char  jobz,  char  uplo,  long  n,
          doublecomplex  *a,  doublecomplex  *b,  double *w,
          doublecomplex *z, long ldz, long *info);
```

PURPOSE

zhpgv computes all the eigenvalues and, optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be Hermitian, stored in packed format, and B is also positive definite.

ARGUMENTS

```
ITYPE (input)
          Specifies the problem type to be solved:
          = 1:  A*x = (lambda)*B*x
          = 2:  A*B*x = (lambda)*x
          = 3:  B*A*x = (lambda)*x

JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.
```

A (input/output)
On entry, the upper or lower triangle of the Hermitian matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

On exit, the contents of A are destroyed.

B (input/output)
On entry, the upper or lower triangle of the Hermitian matrix B, packed columnwise in a linear array. The j-th column of B is stored in the array B as follows: if UPLO = 'U', B(i + (j-1)*j/2) = B(i,j) for 1<=i<=j; if UPLO = 'L', B(i + (j-1)*(2*n-j)/2) = B(i,j) for j<=i<=n.

On exit, the triangular factor U or L from the Cholesky factorization B = U**H*U or B = L*L**H, in the same storage format as B.

W (output)
If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, Z contains the matrix Z of eigenvectors. The eigenvectors are normalized as follows: if ITYPE = 1 or 2, Z**H*B*Z = I; if ITYPE = 3, Z**H*inv(B)*Z = I. If JOBZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
dimension(MAX(1,2*N-1))

WORK2 (workspace)
dimension(MAX(1,3*N-2))

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: CPPTRF or CHPEV returned an error code:
<= N: if INFO = i, CHPEV failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not convergeto zero; > N: if INFO =

N + i, for 1 <= i <= n, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

Contents

NAME

```
zhpgvd - compute all the eigenvalues  and,  optionally,  the
eigenvectors  of  a  complex  generalized Hermitian-definite
eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x,
or B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE ZHPGVD(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, LDZ, WORK,
      LWORK, RWORK, LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX AP(*), BP(*), Z(LDZ,*), WORK(*)
INTEGER ITYPE, N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER IWORK(*)
DOUBLE PRECISION W(*), RWORK(*)

SUBROUTINE ZHPGVD_64(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, LDZ, WORK,
      LWORK, RWORK, LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, UPLO
DOUBLE COMPLEX AP(*), BP(*), Z(LDZ,*), WORK(*)
INTEGER*8 ITYPE, N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HPGVD(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, [LDZ], [WORK],
       [LWORK], [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: AP, BP, WORK
```

```
COMPLEX(8), DIMENSION(:,:) :: Z
INTEGER :: ITYPE, N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: W, RWORK

SUBROUTINE HPGVD_64(ITYPE, JOBZ, UPLO, N, AP, BP, W, Z, [LDZ],
       [WORK], [LWORK], [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, UPLO
COMPLEX(8), DIMENSION(:) :: AP, BP, WORK
COMPLEX(8), DIMENSION(:,:) :: Z
INTEGER(8) :: ITYPE, N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void zhpgvd(int itype, char jobz, char uplo,  int  n,  doub-
          lecomplex *ap, doublecomplex *bp, double *w, doub-
          lecomplex *z, int ldz, int *info);

void zhpgvd_64(long itype, char jobz,  char  uplo,  long  n,
          doublecomplex  *ap,  doublecomplex *bp, double *w,
          doublecomplex *z, long ldz, long *info);
```

PURPOSE

zhpgvd computes all the eigenvalues and, optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be Hermitian, stored in packed format, and B is also positive definite.
If eigenvectors are desired, it uses a divide and conquer algorithm.

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

ARGUMENTS

ITYPE (input)

```
          Specifies the problem type to be solved:
          = 1:  A*x = (lambda)*B*x
          = 2:  A*B*x = (lambda)*x
          = 3:  B*A*x = (lambda)*x

JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.
AP (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  matrix  A,  packed  columnwise in a linear
          array.  The j-th column of  A  is  stored  in  the
          array  AP  as  follows:  if UPLO = 'U', AP(i + (j-
          1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L',  AP(i
          + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

          On exit, the contents of AP are destroyed.

BP (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  matrix  B,  packed  columnwise in a linear
          array.  The j-th column of  B  is  stored  in  the
          array  BP  as  follows:  if UPLO = 'U', BP(i + (j-
          1)*j/2) = B(i,j) for 1<=i<=j; if UPLO = 'L',  BP(i
          + (j-1)*(2*n-j)/2) = B(i,j) for j<=i<=n.

          On exit, the triangular factor U  or  L  from  the
          Cholesky  factorization  B = U**H*U or B = L*L**H,
          in the same storage format as B.

W (output)
          If INFO = 0, the eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0,  Z  contains  the
          matrix  Z  of  eigenvectors.  The eigenvectors are
          normalized  as  follows:   if  ITYPE  =  1  or  2,
          Z**H*B*Z = I; if ITYPE = 3, Z**H*inv(B)*Z = I.  If
          JOBZ = 'N', then Z is not referenced.

LDZ (input)
          The leading dimension of the array Z.  LDZ  >=  1,
          and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
```

```
            On exit, if INFO = 0, WORK(1) returns the  optimal
            LWORK.

LWORK (input)
            The  dimension  of  array  WORK.   If  N   <=   1,
            LWORK  >= 1.  If JOBZ = 'N' and N > 1, LWORK >= N.
            If JOBZ = 'V' and N > 1, LWORK >= 2*N.

            If LWORK = -1, then a workspace query is  assumed;
            the  routine  only  calculates the optimal size of
            the WORK array, returns this value  as  the  first
            entry  of  the  WORK  array,  and no error message
            related to LWORK is issued by XERBLA.

RWORK (workspace)
            On exit, if INFO = 0, RWORK(1) returns the optimal
            LRWORK.

LRWORK (input)
            The  dimension  of  array  RWORK.   If  N  <=   1,
            LRWORK  >=  1.  If JOBZ = 'N' and N > 1, LRWORK >=
            N.  If JOBZ = 'V' and N > 1, LRWORK >= 1 +  5*N  +
            2*N**2.

            If LRWORK = -1, then a workspace query is assumed;
            the  routine  only  calculates the optimal size of
            the RWORK array, returns this value as  the  first
            entry  of  the  RWORK  array, and no error message
            related to LRWORK is issued by XERBLA.

IWORK (workspace/output)
            On exit, if INFO = 0, IWORK(1) returns the optimal
            LIWORK.

LIWORK (input)
            The dimension of array IWORK.  If JOBZ  = 'N' or N
            <=  1,  LIWORK  >=  1.   If JOBZ  = 'V' and N > 1,
            LIWORK >= 3 + 5*N.

            If LIWORK = -1, then a workspace query is assumed;
            the  routine  only  calculates the optimal size of
            the IWORK array, returns this value as  the  first
            entry  of  the  IWORK  array, and no error message
            related to LIWORK is issued by XERBLA.

INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value
            > 0:  CPPTRF or CHPEVD returned an error code:
```

<= N: if INFO = i, CHPEVD failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not convergeto zero; > N: if INFO = N + i, for 1 <= i <= n, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
Mark Fahey, Department of Mathematics, Univ. of Kentucky, USA

Contents

NAME

```
zhpgvx  -  compute  selected  eigenvalues  and,  optionally,
eigenvectors  of  a  complex  generalized Hermitian-definite
eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x,
or B*A*x=(lambda)*x
```

SYNOPSIS

```
SUBROUTINE ZHPGVX(ITYPE, JOBZ, RANGE, UPLO, N, AP, BP, VL, VU, IL,
      IU, ABSTOL, M, W, Z, LDZ, WORK, RWORK, IWORK, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
DOUBLE COMPLEX AP(*), BP(*), Z(LDZ,*), WORK(*)
INTEGER ITYPE, N, IL, IU, M, LDZ, INFO
INTEGER IWORK(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION W(*), RWORK(*)

SUBROUTINE ZHPGVX_64(ITYPE, JOBZ, RANGE, UPLO, N, AP, BP, VL, VU, IL,
      IU, ABSTOL, M, W, Z, LDZ, WORK, RWORK, IWORK, IFAIL, INFO)

CHARACTER * 1 JOBZ, RANGE, UPLO
DOUBLE COMPLEX AP(*), BP(*), Z(LDZ,*), WORK(*)
INTEGER*8 ITYPE, N, IL, IU, M, LDZ, INFO
INTEGER*8 IWORK(*), IFAIL(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION W(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HPGVX(ITYPE, JOBZ, RANGE, UPLO, N, AP, BP, VL, VU, IL,
       IU, ABSTOL, M, W, Z, [LDZ], [WORK], [RWORK], [IWORK], IFAIL,
       [INFO])
```

```
CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX(8), DIMENSION(:) :: AP, BP, WORK
COMPLEX(8), DIMENSION(:,:) :: Z
INTEGER :: ITYPE, N, IL, IU, M, LDZ, INFO
INTEGER, DIMENSION(:) :: IWORK, IFAIL
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: W, RWORK

SUBROUTINE HPGVX_64(ITYPE, JOBZ, RANGE, UPLO, N, AP, BP, VL, VU,
       IL, IU, ABSTOL, M, W, Z, [LDZ], [WORK], [RWORK], [IWORK], IFAIL,
       [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE, UPLO
COMPLEX(8), DIMENSION(:) :: AP, BP, WORK
COMPLEX(8), DIMENSION(:,:) :: Z
INTEGER(8) :: ITYPE, N, IL, IU, M, LDZ, INFO
INTEGER(8), DIMENSION(:) :: IWORK, IFAIL
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: W, RWORK
```

C INTERFACE

```
#include <sunperf.h>

void zhpgvx(int itype, char jobz, char range, char uplo, int
          n,  doublecomplex  *ap,  doublecomplex *bp, double
          vl, double vu, int il, int iu, double abstol,  int
          *m,  double  *w,  doublecomplex  *z,  int ldz, int
          *ifail, int *info);

void zhpgvx_64(long itype, char jobz, char range, char uplo,
          long n, doublecomplex *ap, doublecomplex *bp, dou-
          ble vl,  double  vu,  long  il,  long  iu,  double
          abstol, long *m, double *w, doublecomplex *z, long
          ldz, long *ifail, long *info);
```

PURPOSE

zhpgvx computes selected eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form A*x=(lambda)*B*x, A*Bx=(lambda)*x, or B*A*x=(lambda)*x. Here A and B are assumed to be Hermitian, stored in packed format, and B is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

ARGUMENTS

```
ITYPE (input)
          Specifies the problem type to be solved:
          = 1:  A*x = (lambda)*B*x
          = 2:  A*B*x = (lambda)*x
          = 3:  B*A*x = (lambda)*x

JOBZ (input)
          = 'N':  Compute eigenvalues only;
          = 'V':  Compute eigenvalues and eigenvectors.

RANGE (input)
          = 'A': all eigenvalues will be found;
          = 'V': all eigenvalues in the  half-open  interval
          (VL,VU]  will  be  found; = 'I': the IL-th through
          IU-th eigenvalues will be found.
UPLO (input)
          = 'U':  Upper triangles of A and B are stored;
          = 'L':  Lower triangles of A and B are stored.

N (input) The order of the matrices A and B.  N >= 0.

AP (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  matrix  A,  packed  columnwise in a linear
          array.  The j-th column of  A  is  stored  in  the
          array  AP  as  follows:  if UPLO = 'U', AP(i + (j-
          1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L',  AP(i
          + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

          On exit, the contents of AP are destroyed.

BP (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  matrix  B,  packed  columnwise in a linear
          array.  The j-th column of  B  is  stored  in  the
          array  BP  as  follows:  if UPLO = 'U', BP(i + (j-
          1)*j/2) = B(i,j) for 1<=i<=j; if UPLO = 'L',  BP(i
          + (j-1)*(2*n-j)/2) = B(i,j) for j<=i<=n.

          On exit, the triangular factor U  or  L  from  the
          Cholesky  factorization  B = U**H*U or B = L*L**H,
          in the same storage format as B.

VL (input)
          If RANGE='V', the lower and upper  bounds  of  the
          interval  to be searched for eigenvalues. VL < VU.
          Not referenced if RANGE = 'A' or 'I'.
```

VU (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

ABSTOL (input)
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to

ABSTOL + EPS * max(|a|,|b|) ,

where EPS is the machine precision. If ABSTOL is less than or equal to zero, then EPS*|T| will be used in its place, where |T| is the 1-norm of the tridiagonal matrix obtained by reducing AP to tri-diagonal form.

Eigenvalues will be computed most accurately when ABSTOL is set to twice the underflow threshold 2*SLAMCH('S'), not zero. If this routine returns with INFO>0, indicating that some eigenvectors did not converge, try setting ABSTOL to 2*SLAMCH('S').

M (output)
The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)
On normal exit, the first M elements contain the selected eigenvalues in ascending order.

Z (input) If JOBZ = 'N', then Z is not referenced. If JOBZ = 'V', then if INFO = 0, the first M columns of Z contain the orthonormal eigenvectors of the matrix

A corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). The eigenvectors are normalized as follows: if ITYPE = 1 or 2, Z**H*B*Z = I; if ITYPE = 3, Z**H*inv(B)*Z = I.

If an eigenvector fails to converge, then that column of Z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in IFAIL. Note: the user must ensure that at least max(1,M) columns are supplied in the array Z; if RANGE = 'V', the exact value of M is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

WORK (workspace)
dimension(2*N)

RWORK (workspace)
dimension(7*N)

IWORK (workspace)
dimension(5*N)

IFAIL (output)
If JOBZ = 'V', then if INFO = 0, the first M elements of IFAIL are zero. If INFO > 0, then IFAIL contains the indices of the eigenvectors that failed to converge. If JOBZ = 'N', then IFAIL is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: CPPTRF or CHPEVX returned an error code:
<= N: if INFO = i, CHPEVX failed to converge; i eigenvectors failed to converge. Their indices are stored in array IFAIL. > N: if INFO = N + i, for 1 <= i <= n, then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

FURTHER DETAILS

Based on contributions by
   Mark Fahey, Department of Mathematics, Univ. of Kentucky,
USA

Contents

NAME

```
zhpmv - perform the matrix-vector operation   y := alpha*A*x
+ beta*y
```

SYNOPSIS

```
SUBROUTINE ZHPMV(UPLO, N, ALPHA, A, X, INCX, BETA, Y, INCY)

CHARACTER * 1 UPLO
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(*), X(*), Y(*)
INTEGER N, INCX, INCY

SUBROUTINE ZHPMV_64(UPLO, N, ALPHA, A, X, INCX, BETA, Y, INCY)

CHARACTER * 1 UPLO
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(*), X(*), Y(*)
INTEGER*8 N, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE HPMV(UPLO, [N], ALPHA, A, X, [INCX], BETA, Y, [INCY])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:) :: A, X, Y
INTEGER :: N, INCX, INCY

SUBROUTINE HPMV_64(UPLO, [N], ALPHA, A, X, [INCX], BETA, Y, [INCY])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:) :: A, X, Y
```

```
INTEGER(8) :: N, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>

void zhpmv(char uplo, int  n,  doublecomplex  *alpha,  doub-
          lecomplex  *a,  doublecomplex  *x, int incx, doub-
          lecomplex *beta, doublecomplex *y, int incy);

void zhpmv_64(char uplo, long n, doublecomplex *alpha, doub-
          lecomplex  *a,  doublecomplex *x, long incx, doub-
          lecomplex *beta, doublecomplex *y, long incy);
```

PURPOSE

zhpmv performs the matrix-vector operation y := alpha*A*x + beta*y where alpha and beta are scalars, x and y are n element vectors and A is an n by n hermitian matrix, supplied in packed form.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the matrix A is supplied in the packed array A as follows:

UPLO = 'U' or 'u'   The upper triangular part of A is supplied in A.

UPLO = 'L' or 'l'   The lower triangular part of A is supplied in A.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
((n*(n + 1))/2). Before entry with UPLO = 'U' or 'u', the array A must contain the upper triangular part of the hermitian matrix packed

sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(1, 2) and a(2, 2) respectively, and so on. Before entry with UPLO = 'L' or 'l', the array A must contain the lower triangular part of the hermitian matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(2, 1) and a(3, 1) respectively, and so on. Note that the imaginary parts of the diagonal elements need not be set and are assumed to be zero. Unchanged on exit.

X (input)

(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)

On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

BETA (input)

On entry, BETA specifies the scalar beta. When BETA is supplied as zero then Y need not be set on input. Unchanged on exit.

Y (input/output)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

zhpr - perform the hermitian rank 1 operation A := alpha*x*conjg(x') + A

SYNOPSIS

```
SUBROUTINE ZHPR(UPLO, N, ALPHA, X, INCX, A)

CHARACTER * 1 UPLO
DOUBLE COMPLEX X(*), A(*)
INTEGER N, INCX
DOUBLE PRECISION ALPHA

SUBROUTINE ZHPR_64(UPLO, N, ALPHA, X, INCX, A)

CHARACTER * 1 UPLO
DOUBLE COMPLEX X(*), A(*)
INTEGER*8 N, INCX
DOUBLE PRECISION ALPHA
```

F95 INTERFACE

```
SUBROUTINE HPR(UPLO, [N], ALPHA, X, [INCX], A)

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: X, A
INTEGER :: N, INCX
REAL(8) :: ALPHA

SUBROUTINE HPR_64(UPLO, [N], ALPHA, X, [INCX], A)

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: X, A
INTEGER(8) :: N, INCX
```

```
REAL(8) :: ALPHA
```

C INTERFACE

```
#include <sunperf.h>

void zhpr(char uplo, int n, double alpha, doublecomplex  *x,
          int incx, doublecomplex *a);

void zhpr_64(char uplo, long n, double alpha,  doublecomplex
          *x, long incx, doublecomplex *a);
```

PURPOSE

zhpr performs the hermitian rank 1 operation A := alpha*x*conjg(x') + A where alpha is a real scalar, x is an n element vector and A is an n by n hermitian matrix, supplied in packed form.

ARGUMENTS

UPLO (input)

On entry, UPLO specifies whether the upper or lower triangular part of the matrix A is supplied in the packed array A as follows:

UPLO = 'U' or 'u'   The upper triangular part of A is supplied in A.

UPLO = 'L' or 'l'   The lower triangular part of A is supplied in A.

Unchanged on exit.

N (input)

On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)

On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)

(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)

On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

A (input/output)

((n*(n + 1))/2). Before entry with UPLO = 'U' or 'u', the array A must contain the upper triangular part of the hermitian matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(1, 2) and a(2, 2) respectively, and so on. On exit, the array A is overwritten by the upper triangular part of the updated matrix. Before entry with UPLO = 'L' or 'l', the array A must contain the lower triangular part of the hermitian matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(2, 1) and a(3, 1) respectively, and so on. On exit, the array A is overwritten by the lower triangular part of the updated matrix. Note that the imaginary parts of the diagonal elements need not be set, they are assumed to be zero, and on exit they are set to zero.

Contents

NAME

zhpr2 - perform the Hermitian rank 2 operation A := alpha*x*conjg(y') + conjg(alpha)*y*conjg(x') + A

SYNOPSIS

```
SUBROUTINE ZHPR2(UPLO, N, ALPHA, X, INCX, Y, INCY, AP)

CHARACTER * 1 UPLO
DOUBLE COMPLEX ALPHA
DOUBLE COMPLEX X(*), Y(*), AP(*)
INTEGER N, INCX, INCY

SUBROUTINE ZHPR2_64(UPLO, N, ALPHA, X, INCX, Y, INCY, AP)

CHARACTER * 1 UPLO
DOUBLE COMPLEX ALPHA
DOUBLE COMPLEX X(*), Y(*), AP(*)
INTEGER*8 N, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE HPR2(UPLO, [N], ALPHA, X, [INCX], Y, [INCY], AP)

CHARACTER(LEN=1) :: UPLO
COMPLEX(8) :: ALPHA
COMPLEX(8), DIMENSION(:) :: X, Y, AP
INTEGER :: N, INCX, INCY

SUBROUTINE HPR2_64(UPLO, [N], ALPHA, X, [INCX], Y, [INCY], AP)

CHARACTER(LEN=1) :: UPLO
COMPLEX(8) :: ALPHA
COMPLEX(8), DIMENSION(:) :: X, Y, AP
```

```
INTEGER(8) :: N, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>

void zhpr2(char uplo, int  n,  doublecomplex  *alpha,  doub-
          lecomplex  *x,  int  incx,  doublecomplex  *y, int
          incy, doublecomplex *ap);

void zhpr2_64(char uplo, long n, doublecomplex *alpha, doub-
          lecomplex  *x,  long  incx, doublecomplex *y, long
          incy, doublecomplex *ap);
```

PURPOSE

zhpr2 performs the Hermitian rank 2 operation A := alpha*x*conjg(y') + conjg(alpha)*y*conjg(x') + A where alpha is a scalar, x and y are n element vectors and A is an n by n hermitian matrix, supplied in packed form.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the matrix A is supplied in the packed array AP as follows:

UPLO = 'U' or 'u'   The upper triangular part of A is supplied in AP.

UPLO = 'L' or 'l'   The lower triangular part of A is supplied in AP.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

X (input)
(1 + (n - 1)*abs(INCX)). Before entry, the incremented array X must contain the n element vector x. Unchanged on exit.

INCX (input)
On entry, INCX specifies the increment for the elements of X. INCX <> 0. Unchanged on exit.

Y (input)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector y. Unchanged on exit.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

AP (input/output)
((n*(n + 1))/2). Before entry with UPLO = 'U' or 'u', the array AP must contain the upper triangular part of the hermitian matrix packed sequentially, column by column, so that AP(1) contains a(1, 1), AP(2) and AP(3) contain a(1, 2) and a(2, 2) respectively, and so on. On exit, the array AP is overwritten by the upper triangular part of the updated matrix. Before entry with UPLO = 'L' or 'l', the array AP must contain the lower triangular part of the hermitian matrix packed sequentially, column by column, so that AP(1) contains a(1, 1), AP(2) and AP(3) contain a(2, 1) and a(3, 1) respectively, and so on. On exit, the array AP is overwritten by the lower triangular part of the updated matrix. Note that the imaginary parts of the diagonal elements need not be set, they are assumed to be zero, and on exit they are set to zero.

Contents

NAME

```
zhprfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is Hermitian indefin-
ite and packed, and provides error bounds and backward error
estimates for the solution
```

SYNOPSIS

```
SUBROUTINE ZHPRFS(UPLO, N, NRHS, A, AF, IPIVOT, B, LDB, X, LDX, FERR,
      BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZHPRFS_64(UPLO, N, NRHS, A, AF, IPIVOT, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HPRFS(UPLO, N, [NRHS], A, AF, IPIVOT, B, [LDB], X, [LDX],
       FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A, AF, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
```

```
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE HPRFS_64(UPLO, N, [NRHS], A, AF, IPIVOT, B, [LDB], X,
       [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A, AF, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zhprfs(char uplo, int n, int  nrhs,  doublecomplex  *a,
          doublecomplex  *af, int *ipivot, doublecomplex *b,
          int ldb, doublecomplex *x, int ldx, double  *ferr,
          double *berr, int *info);

void zhprfs_64(char uplo, long n, long  nrhs,  doublecomplex
          *a, doublecomplex *af, long *ipivot, doublecomplex
          *b, long ldb, doublecomplex *x, long  ldx,  double
          *ferr, double *berr, long *info);
```

PURPOSE

zhprfs improves the computed solution to a system of linear equations when the coefficient matrix is Hermitian indefinite and packed, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input) The upper  or  lower  triangle  of  the  Hermitian
          matrix  A,  packed  columnwise  in a linear array.
```

The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

AF (input)
The factored form of the matrix A. AF contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**H or A = L*D*L**H as computed by CHPTRF, stored as a packed triangular matrix.

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CHPTRF.

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by CHPTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)

```
          dimension(N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO =  -i,  the  i-th  argument  had  an
          illegal value
```

Contents

NAME

```
zhpsv - compute the solution to a complex system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE ZHPSV(UPLO, N, NRHS, A, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZHPSV_64(UPLO, N, NRHS, A, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE HPSV(UPLO, N, [NRHS], A, IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE HPSV_64(UPLO, N, [NRHS], A, IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX(8), DIMENSION(:) :: A
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void zhpsv(char uplo, int n, int nrhs, doublecomplex *a, int
          *ipivot, doublecomplex *b, int ldb, int *info);

void zhpsv_64(char uplo, long n,  long  nrhs,  doublecomplex
          *a, long *ipivot, doublecomplex *b, long ldb, long
          *info);
```

PURPOSE

```
zhpsv computes the solution to a complex  system  of  linear
equations
   A * X = B, where A is an N-by-N Hermitian  matrix  stored
in packed format and X and B are N-by-NRHS matrices.

The diagonal pivoting method is used to factor A as
   A = U * D * U**H,  if UPLO = 'U', or
   A = L * D * L**H,  if UPLO = 'L',
where U (or L) is a product of permutation  and  unit  upper
(lower) triangular matrices, D is Hermitian and block diago-
nal with 1-by-1 and 2-by-2 diagonal  blocks.   The  factored
form  of A is then used to solve the system of equations A *
X = B.
```

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

A (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  matrix  A,  packed  columnwise in a linear
```

array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n. See below for further details.

On exit, the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**H or A = L*D*L**H as computed by CHPTRF, stored as a packed triangular matrix in the same storage format as A.

IPIVOT (output)
Details of the interchanges and the block structure of D, as determined by CHPTRF. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged, and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, so the solution could not be computed.

FURTHER DETAILS

The packed storage scheme is illustrated by the following example when N = 4, UPLO = 'U':

Two-dimensional storage of the Hermitian matrix A:

```
   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = conjg(aji))
               a44

Packed storage of the upper triangle of A:

A = [ a11, a12, a22, a13, a23, a33, a14, a24, a34, a44 ]
```

Contents

NAME

```
zhpsvx  -  use  the  diagonal  pivoting  factorization  A  =
U*D*U**H  or  A = L*D*L**H to compute the solution to a com-
plex system of linear equations A * X = B, where A is an  N-
by-N  Hermitian  matrix  stored in packed format and X and B
are N-by-NRHS matrices
```

SYNOPSIS

```
SUBROUTINE ZHPSVX(FACT, UPLO, N, NRHS, A, AF, IPIVOT, B, LDB, X, LDX,
      RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
DOUBLE COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZHPSVX_64(FACT, UPLO, N, NRHS, A, AF, IPIVOT, B, LDB, X,
      LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
DOUBLE COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE HPSVX(FACT, UPLO, N, [NRHS], A, AF, IPIVOT, B, [LDB], X,
       [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: FACT, UPLO
COMPLEX(8), DIMENSION(:) :: A, AF, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE HPSVX_64(FACT, UPLO, N, [NRHS], A, AF, IPIVOT, B, [LDB],
       X, [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO
COMPLEX(8), DIMENSION(:) :: A, AF, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zhpsvx(char fact, char uplo, int  n,  int  nrhs,  doub-
          lecomplex  *a,  doublecomplex  *af,  int  *ipivot,
          doublecomplex *b, int ldb, doublecomplex  *x,  int
          ldx,  double  *rcond,  double *ferr, double *berr,
          int *info);

void zhpsvx_64(char fact, char  uplo,  long  n,  long  nrhs,
          doublecomplex *a, doublecomplex *af, long *ipivot,
          doublecomplex *b, long ldb, doublecomplex *x, long
          ldx,  double  *rcond,  double *ferr, double *berr,
          long *info);
```

PURPOSE

zhpsvx uses the diagonal pivoting factorization A = U*D*U**H or A = L*D*L**H to compute the solution to a complex system of linear equations A * X = B, where A is an N-by-N Hermitian matrix stored in packed format and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'N', the diagonal pivoting method is used to

```
factor A as
      A = U * D * U**H,  if UPLO = 'U', or
      A = L * D * L**H,  if UPLO = 'L',
   where U (or L) is a product of permutation and unit upper
(lower)
   triangular matrices and D is Hermitian and block diagonal
with
   1-by-1 and 2-by-2 diagonal blocks.

2. If some D(i,i)=0, so that D is exactly singular, then the
routine
   returns with INFO = i. Otherwise, the factored form of  A
is used
   to estimate the condition number of the matrix A.  If the
   reciprocal of the condition number is less  than  machine
precision,
   INFO = N+1 is returned as  a  warning,  but  the  routine
still goes on
   to solve for X and  compute  error  bounds  as  described
below.

3. The system of equations is solved for X  using  the  fac-
tored form
   of A.

4. Iterative refinement is applied to improve  the  computed
solution
   matrix and calculate  error  bounds  and  backward  error
estimates
   for it.
```

ARGUMENTS

```
FACT (input)
          Specifies whether or not the factored  form  of  A
          has  been supplied on entry.  = 'F':  On entry, AF
          and IPIVOT contain the factored form of A.  AF and
          IPIVOT will not be modified.  = 'N':  The matrix A
          will be copied to AF and factored.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
```

The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The upper or lower triangle of the Hermitian matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n. See below for further details.

AF (input/output)

If FACT = 'F', then AF is an input argument and on entry contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**H or A = L*D*L**H as computed by CHPTRF, stored as a packed triangular matrix in the same storage format as A.

If FACT = 'N', then AF is an output argument and on exit contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**H or A = L*D*L**H as computed by CHPTRF, stored as a packed triangular matrix in the same storage format as A.

IPIVOT (input or output)

If FACT = 'F', then IPIVOT is an input argument and on entry contains details of the interchanges and the block structure of D, as determined by CHPTRF. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

If FACT = 'N', then IPIVOT is an output argument and on exit contains details of the interchanges and the block structure of D, as determined by CHPTRF.

B (input) The N-by-NRHS right hand side matrix B.

LDB (input)

The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A. If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: D(i,i) is exactly zero. The factorization has been completed but the factor D is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: D is nonsingular, but RCOND is less than machine

precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

FURTHER DETAILS

The packed storage scheme is illustrated by the following example when N = 4, UPLO = 'U':
Two-dimensional storage of the Hermitian matrix A:

```
   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = conjg(aji))
               a44
```

Packed storage of the upper triangle of A:

A = [a11, a12, a22, a13, a23, a33, a14, a24, a34, a44]

Contents

NAME

```
zhptrd - reduce a  complex  Hermitian  matrix  A  stored  in
packed  form  to real symmetric tridiagonal form T by a uni-
tary similarity transformation
```

SYNOPSIS

```
SUBROUTINE ZHPTRD(UPLO, N, AP, D, E, TAU, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AP(*), TAU(*)
INTEGER N, INFO
DOUBLE PRECISION D(*), E(*)

SUBROUTINE ZHPTRD_64(UPLO, N, AP, D, E, TAU, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AP(*), TAU(*)
INTEGER*8 N, INFO
DOUBLE PRECISION D(*), E(*)
```

F95 INTERFACE

```
SUBROUTINE HPTRD(UPLO, N, AP, D, E, TAU, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: AP, TAU
INTEGER :: N, INFO
REAL(8), DIMENSION(:) :: D, E

SUBROUTINE HPTRD_64(UPLO, N, AP, D, E, TAU, [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX(8), DIMENSION(:) :: AP, TAU
INTEGER(8) :: N, INFO
REAL(8), DIMENSION(:) :: D, E
```

C INTERFACE

```
#include <sunperf.h>

void zhptrd(char uplo, int n, doublecomplex *ap, double  *d,
          double *e, doublecomplex *tau, int *info);

void zhptrd_64(char uplo, long n, doublecomplex *ap,  double
          *d, double *e, doublecomplex *tau, long *info);
```

PURPOSE

zhptrd reduces a complex Hermitian matrix A stored in packed form to real symmetric tridiagonal form T by a unitary similarity transformation: Q**H * A * Q = T.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

AP (input)
On entry, the upper or lower triangle of the Hermitian matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n. On exit, if UPLO = 'U', the diagonal and first superdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements above the first superdiagonal, with the array TAU, represent the unitary matrix Q as a product of elementary reflectors; if UPLO = 'L', the diagonal and first subdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements below the first subdiagonal, with the array TAU, represent the unitary matrix Q as a product of elementary reflectors. See Further Details.

```
D (output)
          The diagonal elements of the tridiagonal matrix T:
          D(i) = A(i,i).

E (output)
          The  off-diagonal  elements  of  the   tridiagonal
          matrix  T:   E(i) = A(i,i+1) if UPLO = 'U', E(i) =
          A(i+1,i) if UPLO = 'L'.

TAU (output)
          The scalar factors of  the  elementary  reflectors
          (see Further Details).

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

FURTHER DETAILS

```
If UPLO = 'U', the matrix Q is represented as a  product  of
elementary reflectors

   Q = H(n-1) . . . H(2) H(1).

Each H(i) has the form

   H(i) = I - tau * v * v'

where tau is a complex scalar, and v  is  a  complex  vector
with  v(i+1:n)  = 0 and v(i) = 1; v(1:i-1) is stored on exit
in AP,  overwriting  A(1:i-1,i+1),  and  tau  is  stored  in
TAU(i).

If UPLO = 'L', the matrix Q is represented as a  product  of
elementary reflectors

   Q = H(1) H(2) . . . H(n-1).

Each H(i) has the form

   H(i) = I - tau * v * v'

where tau is a complex scalar, and v  is  a  complex  vector
with  v(1:i)  = 0 and v(i+1) = 1; v(i+2:n) is stored on exit
in AP, overwriting A(i+2:n,i), and tau is stored in TAU(i).
```

Contents

NAME

```
zhptrf - compute the factorization of  a  complex  Hermitian
packed  matrix  A  using the Bunch-Kaufman diagonal pivoting
method
```

SYNOPSIS

```
SUBROUTINE ZHPTRF(UPLO, N, A, IPIVOT, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*)
INTEGER N, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZHPTRF_64(UPLO, N, A, IPIVOT, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*)
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE HPTRF(UPLO, N, A, IPIVOT, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE HPTRF_64(UPLO, N, A, IPIVOT, [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX(8), DIMENSION(:) :: A
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void zhptrf(char uplo, int n, doublecomplex *a, int *ipivot,
          int *info);

void zhptrf_64(char uplo, long  n,  doublecomplex  *a,  long
          *ipivot, long *info);
```

PURPOSE

zhptrf computes the factorization of a complex Hermitian packed matrix A using the Bunch-Kaufman diagonal pivoting method:

```
A = U*D*U**H  or  A = L*D*L**H
```

where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is Hermitian and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the upper or lower triangle of the Hermitian matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n.

On exit, the block diagonal matrix D and the multipliers used to obtain the factor U or L, stored as a packed triangular matrix overwriting A (see below for further details).

IPIVOT (output)

Details of the interchanges and the block structure of D. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

```
5-96 - Based on modifications by J. Lewis,  Boeing  Computer
Services
       Company

If UPLO = 'U', then A = U*D*U', where
   U = P(n)*U(n)* ... *P(k)U(k)* ...,
i.e., U is a product of terms P(k)*U(k), where  k  decreases
from  n  to  1 in steps of 1 or 2, and D is a block diagonal
matrix with 1-by-1 and 2-by-2 diagonal blocks D(k).  P(k) is
a  permutation matrix as defined by IPIVOT(k), and U(k) is a
unit upper triangular matrix,  such  that  if  the  diagonal
block D(k) is of order s (s = 1 or 2), then

           (   I    v    0   )   k-s
   U(k) =  (   0    I    0   )   s
           (   0    0    I   )   n-k
              k-s   s   n-k

If s = 1, D(k) overwrites A(k,k), and  v  overwrites  A(1:k-
1,k).   If s = 2, the upper triangle of D(k) overwrites A(k-
1,k-1), A(k-1,k), and A(k,k), and  v  overwrites  A(1:k-2,k-
1:k).

If UPLO = 'L', then A = L*D*L', where
   L = P(1)*L(1)* ... *P(k)*L(k)* ...,
i.e., L is a product of terms P(k)*L(k), where  k  increases
```

from 1 to n in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIVOT(k), and L(k) is a unit lower triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    0     0   )  k-1
   L(k) =  (   0    I     0   )  s
           (   0    v     I   )  n-k-s+1
              k-1   s  n-k-s+1
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(k+1:n,k). If s = 2, the lower triangle of D(k) overwrites A(k,k), A(k+1,k), and A(k+1,k+1), and v overwrites A(k+2:n,k:k+1).

Contents

NAME

```
zhptri - compute the inverse of a complex Hermitian indefin-
ite  matrix  A in packed storage using the factorization A =
U*D*U**H or A = L*D*L**H computed by CHPTRF
```

SYNOPSIS

```
SUBROUTINE ZHPTRI(UPLO, N, A, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), WORK(*)
INTEGER N, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZHPTRI_64(UPLO, N, A, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), WORK(*)
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE HPTRI(UPLO, N, A, IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A, WORK
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE HPTRI_64(UPLO, N, A, IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A, WORK
```

```
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void zhptri(char uplo, int n, doublecomplex *a, int *ipivot,
          int *info);

void zhptri_64(char uplo, long  n,  doublecomplex  *a,  long
          *ipivot, long *info);
```

PURPOSE

zhptri computes the inverse of a complex Hermitian indefinite matrix A in packed storage using the factorization A = U*D*U**H or A = L*D*L**H computed by CHPTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**H;
= 'L': Lower triangular, form is A = L*D*L**H.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CHPTRF, stored as a packed triangular matrix.

On exit, if INFO = 0, the (Hermitian) inverse of the original matrix, stored as a packed triangular matrix. The j-th column of inv(A) is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = inv(A)(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = inv(A)(i,j) for j<=i<=n.

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CHPTRF.

WORK (workspace)
dimension(N)

```
INFO (output)
          = 0: successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
          gal value
          > 0: if INFO = i, D(i,i) = 0; the matrix is singu-
          lar and its inverse could not be computed.
```

Contents

NAME

```
zhptrs - solve a system of linear equations A*X = B  with  a
complex Hermitian matrix A stored in packed format using the
factorization A = U*D*U**H  or  A  =  L*D*L**H  computed  by
CHPTRF
```

SYNOPSIS

```
SUBROUTINE ZHPTRS(UPLO, N, NRHS, A, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZHPTRS_64(UPLO, N, NRHS, A, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE HPTRS(UPLO, N, [NRHS], A, IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE HPTRS_64(UPLO, N, [NRHS], A, IPIVOT, B, [LDB], [INFO])
```

```
CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void zhptrs(char uplo, int n, int  nrhs,  doublecomplex  *a,
          int   *ipivot,  doublecomplex  *b,  int  ldb,  int
          *info);

void zhptrs_64(char uplo, long n, long  nrhs,  doublecomplex
          *a, long *ipivot, doublecomplex *b, long ldb, long
          *info);
```

PURPOSE

zhptrs solves a system of linear equations A*X = B with a complex Hermitian matrix A stored in packed format using the factorization A = U*D*U**H or A = L*D*L**H computed by CHPTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**H;
= 'L': Lower triangular, form is A = L*D*L**H.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CHPTRF, stored as a packed triangular matrix.

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CHPTRF.

B (input/output)

On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zhsein - use  inverse  iteration  to  find  specified  right
and/or  left  eigenvectors  of  a  complex  upper Hessenberg
matrix H
```

SYNOPSIS

```
SUBROUTINE ZHSEIN(SIDE, EIGSRC, INITV, SELECT, N, H, LDH, W, VL,
      LDVL, VR, LDVR, MM, M, WORK, RWORK, IFAILL, IFAILR, INFO)

CHARACTER * 1 SIDE, EIGSRC, INITV
DOUBLE  COMPLEX  H(LDH,*),  W(*),  VL(LDVL,*),   VR(LDVR,*),
WORK(*)
INTEGER N, LDH, LDVL, LDVR, MM, M, INFO
INTEGER IFAILL(*), IFAILR(*)
LOGICAL SELECT(*)
DOUBLE PRECISION RWORK(*)

SUBROUTINE ZHSEIN_64(SIDE, EIGSRC, INITV, SELECT, N, H, LDH, W, VL,
      LDVL, VR, LDVR, MM, M, WORK, RWORK, IFAILL, IFAILR, INFO)

CHARACTER * 1 SIDE, EIGSRC, INITV
DOUBLE  COMPLEX  H(LDH,*),  W(*),  VL(LDVL,*),   VR(LDVR,*),
WORK(*)
INTEGER*8 N, LDH, LDVL, LDVR, MM, M, INFO
INTEGER*8 IFAILL(*), IFAILR(*)
LOGICAL*8 SELECT(*)
DOUBLE PRECISION RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE HSEIN(SIDE, EIGSRC, INITV, SELECT, [N], H, [LDH], W, VL,
       [LDVL], VR, [LDVR], MM, M, [WORK], [RWORK], IFAILL, IFAILR, [INFO])
```

```
CHARACTER(LEN=1) :: SIDE, EIGSRC, INITV
COMPLEX(8), DIMENSION(:) :: W, WORK
COMPLEX(8), DIMENSION(:,:) :: H, VL, VR
INTEGER :: N, LDH, LDVL, LDVR, MM, M, INFO
INTEGER, DIMENSION(:) :: IFAILL, IFAILR
LOGICAL, DIMENSION(:) :: SELECT
REAL(8), DIMENSION(:) :: RWORK

SUBROUTINE HSEIN_64(SIDE, EIGSRC, INITV, SELECT, [N], H, [LDH], W,
       VL, [LDVL], VR, [LDVR], MM, M, [WORK], [RWORK], IFAILL, IFAILR,
       [INFO])

CHARACTER(LEN=1) :: SIDE, EIGSRC, INITV
COMPLEX(8), DIMENSION(:) :: W, WORK
COMPLEX(8), DIMENSION(:,:) :: H, VL, VR
INTEGER(8) :: N, LDH, LDVL, LDVR, MM, M, INFO
INTEGER(8), DIMENSION(:) :: IFAILL, IFAILR
LOGICAL(8), DIMENSION(:) :: SELECT
REAL(8), DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>

void zhsein(char side, char eigsrc, char initv, int *select,
          int  n,  doublecomplex  *h, int ldh, doublecomplex
          *w, doublecomplex  *vl,  int  ldvl,  doublecomplex
          *vr,  int  ldvr,  int mm, int *m, int *ifaill, int
          *ifailr, int *info);

void zhsein_64(char side,  char  eigsrc,  char  initv,  long
          *select, long n, doublecomplex *h, long ldh, doub-
          lecomplex *w, doublecomplex *vl, long ldvl,  doub-
          lecomplex  *vr,  long ldvr, long mm, long *m, long
          *ifaill, long *ifailr, long *info);
```

PURPOSE

zhsein uses inverse iteration to find specified right and/or left eigenvectors of a complex upper Hessenberg matrix H.

The right eigenvector x and the left eigenvector y of the matrix H corresponding to an eigenvalue w are defined by:

```
H * x = w * x,     y**h * H = w * y**h
```

where y**h denotes the conjugate transpose of the vector y.

ARGUMENTS

```
SIDE (input)
          = 'R': compute right eigenvectors only;
          = 'L': compute left eigenvectors only;
          = 'B': compute both right and left eigenvectors.

EIGSRC (input)
          Specifies the source of eigenvalues supplied in W:
          = 'Q': the eigenvalues were  found  using  CHSEQR;
          thus,  if  H has zero subdiagonal elements, and so
          is block-triangular, then the j-th eigenvalue  can
          be  assumed  to be an eigenvalue of the block con-
          taining the j-th row/column.  This property allows
          CHSEIN  to  perform  inverse iteration on just one
          diagonal block.  = 'N': no assumptions are made on
          the  correspondence between eigenvalues and diago-
          nal blocks.  In  this  case,  CHSEIN  must  always
          perform  inverse  iteration using the whole matrix
          H.

INITV (input)
          = 'N': no initial vectors are supplied;
          = 'U': user-supplied initial vectors are stored in
          the arrays VL and/or VR.

SELECT (input)
          Specifies the  eigenvectors  to  be  computed.  To
          select the eigenvector corresponding to the eigen-
          value W(j), SELECT(j) must be set to .TRUE..

N (input) The order of the matrix H.  N >= 0.

H (input) The upper Hessenberg matrix H.

LDH (input)
          The leading dimension of  the  array  H.   LDH  >=
          max(1,N).

W (input/output)
          On entry, the eigenvalues of H.  On exit, the real
          parts  of  W  may  have  been  altered since close
          eigenvalues are perturbed  slightly  in  searching
          for independent eigenvectors.

VL (input/output)
          On entry, if INITV = 'U' and SIDE = 'L' or 'B', VL
          must  contain  starting  vectors  for  the inverse
          iteration for the left eigenvectors; the  starting
```

vector for each eigenvector must be in the same column in which the eigenvector will be stored. On exit, if SIDE = 'L' or 'B', the left eigenvectors specified by SELECT will be stored consecutively in the columns of VL, in the same order as their eigenvalues. If SIDE = 'R', VL is not referenced.

LDVL (input)
The leading dimension of the array VL. LDVL >= max(1,N) if SIDE = 'L' or 'B'; LDVL >= 1 otherwise.

VR (input/output)
On entry, if INITV = 'U' and SIDE = 'R' or 'B', VR must contain starting vectors for the inverse iteration for the right eigenvectors; the starting vector for each eigenvector must be in the same column in which the eigenvector will be stored. On exit, if SIDE = 'R' or 'B', the right eigenvectors specified by SELECT will be stored consecutively in the columns of VR, in the same order as their eigenvalues. If SIDE = 'L', VR is not referenced.

LDVR (input)
The leading dimension of the array VR. LDVR >= max(1,N) if SIDE = 'R' or 'B'; LDVR >= 1 otherwise.

MM (input)
The number of columns in the arrays VL and/or VR. MM >= M.

M (output)
The number of columns in the arrays VL and/or VR required to store the eigenvectors (= the number of .TRUE. elements in SELECT).

WORK (workspace)
dimension(N*N)

RWORK (workspace)
dimension(N)

IFAILL (output)
If SIDE = 'L' or 'B', IFAILL(i) = j > 0 if the left eigenvector in the i-th column of VL (corresponding to the eigenvalue w(j)) failed to converge; IFAILL(i) = 0 if the eigenvector converged satisfactorily. If SIDE = 'R', IFAILL is

not referenced.

IFAILR (output)
If SIDE = 'R' or 'B', IFAILR(i) = j > 0 if the right eigenvector in the i-th column of VR (corresponding to the eigenvalue w(j)) failed to converge; IFAILR(i) = 0 if the eigenvector converged satisfactorily. If SIDE = 'L', IFAILR is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, i is the number of eigenvectors which failed to converge; see IFAILL and IFAILR for further details.

FURTHER DETAILS

Each eigenvector is normalized so that the element of largest magnitude has magnitude 1; here the magnitude of a complex number (x,y) is taken to be |x|+|y|.

Contents

NAME

```
zhseqr - compute the eigenvalues of a complex upper  Hessen-
berg  matrix  H,  and, optionally, the matrices T and Z from
the Schur decomposition H = Z T Z**H, where T  is  an  upper
triangular  matrix  (the  Schur  form), and Z is the unitary
matrix of Schur vectors
```

SYNOPSIS

```
SUBROUTINE ZHSEQR(JOB, COMPZ, N, ILO, IHI, H, LDH, W, Z, LDZ, WORK,
      LWORK, INFO)

CHARACTER * 1 JOB, COMPZ
DOUBLE COMPLEX H(LDH,*), W(*), Z(LDZ,*), WORK(*)
INTEGER N, ILO, IHI, LDH, LDZ, LWORK, INFO

SUBROUTINE ZHSEQR_64(JOB, COMPZ, N, ILO, IHI, H, LDH, W, Z, LDZ,
      WORK, LWORK, INFO)

CHARACTER * 1 JOB, COMPZ
DOUBLE COMPLEX H(LDH,*), W(*), Z(LDZ,*), WORK(*)
INTEGER*8 N, ILO, IHI, LDH, LDZ, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE HSEQR(JOB, COMPZ, N, ILO, IHI, H, [LDH], W, Z, [LDZ],
       [WORK], LWORK, [INFO])

CHARACTER(LEN=1) :: JOB, COMPZ
COMPLEX(8), DIMENSION(:) :: W, WORK
COMPLEX(8), DIMENSION(:,:) :: H, Z
INTEGER :: N, ILO, IHI, LDH, LDZ, LWORK, INFO

SUBROUTINE HSEQR_64(JOB, COMPZ, N, ILO, IHI, H, [LDH], W, Z, [LDZ],
```

```
          [WORK], LWORK, [INFO])

   CHARACTER(LEN=1) :: JOB, COMPZ
   COMPLEX(8), DIMENSION(:) :: W, WORK
   COMPLEX(8), DIMENSION(:,:) :: H, Z
   INTEGER(8) :: N, ILO, IHI, LDH, LDZ, LWORK, INFO
```

C INTERFACE

```
   #include <sunperf.h>

   void zhseqr (char, char, int, int, int, doublecomplex*, int,
             doublecomplex*, doublecomplex*, int, int*);

   void zhseqr_64 (char, char,  long,  long,  long,  doublecom-
             plex*, long, doublecomplex*, doublecomplex*, long,
             long*);
```

PURPOSE

zhseqr computes the eigenvalues of a complex upper Hessenberg matrix H, and, optionally, the matrices T and Z from the Schur decomposition H = Z T Z**H, where T is an upper triangular matrix (the Schur form), and Z is the unitary matrix of Schur vectors.

Optionally Z may be postmultiplied into an input unitary matrix Q, so that this routine can give the Schur factorization of a matrix A which has been reduced to the Hessenberg form H by the unitary matrix Q: A = Q*H*Q**H = (QZ)*T*(QZ)**H.

ARGUMENTS

JOB (input)
= 'E': compute eigenvalues only;
= 'S': compute eigenvalues and the Schur form T.

COMPZ (input)
= 'N': no Schur vectors are computed;
= 'I': Z is initialized to the unit matrix and the matrix Z of Schur vectors of H is returned; = 'V': Z must contain an unitary matrix Q on entry, and the product Q*Z is returned.

N (input) The order of the matrix H. N >= 0.

ILO (input)

```
            It is assumed that H is already  upper  triangular
            in  rows  and columns 1:ILO-1 and IHI+1:N. ILO and
            IHI are normally set by a previous call to CGEBAL,
            and  then  passed to CGEHRD when the matrix output
            by CGEBAL is reduced to Hessenberg form. Otherwise
            ILO and IHI should be set to 1 and N respectively.
            1 <= ILO <= IHI <= N, if N > 0; ILO=1  and  IHI=0,
            if N=0.

IHI (input)
            See the description of ILO.

H (input/output)
            On entry, the upper Hessenberg matrix H.  On exit,
            if  JOB  =  'S',  H  contains the upper triangular
            matrix T from the Schur decomposition  (the  Schur
            form).  If  JOB  =  'E',  the  contents  of  H are
            unspecified on exit.

LDH (input)
            The leading dimension  of  the  array  H.  LDH  >=
            max(1,N).

W (output)
            The computed eigenvalues. If JOB = 'S', the eigen-
            values  are  stored  in  the  same order as on the
            diagonal of the Schur form  returned  in  H,  with
            W(i) = H(i,i).

Z (input)   If COMPZ = 'N': Z is not referenced.
            If COMPZ = 'I': on entry, Z need not be  set,  and
            on  exit,  Z  contains the unitary matrix Z of the
            Schur vectors of H.  If COMPZ = 'V':  on  entry  Z
            must  contain an N-by-N matrix Q, which is assumed
            to be equal to the unit matrix except for the sub-
            matrix Z(ILO:IHI,ILO:IHI); on exit Z contains Q*Z.
            Normally Q is  the  unitary  matrix  generated  by
            CUNGHR  after  the call to CGEHRD which formed the
            Hessenberg matrix H.

LDZ (input)
            The leading dimension of  the  array  Z.   LDZ  >=
            max(1,N)  if  COMPZ  = 'I' or 'V'; LDZ >= 1 other-
            wise.

WORK (workspace)
            On exit, if INFO = 0, WORK(1) returns the  optimal
            LWORK.

LWORK (output)
```

The dimension of the array WORK. LWORK >= max(1,N).

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, CHSEQR failed to compute all the eigenvalues in a total of 30*(IHI-ILO+1) iterations; elements 1:ilo-1 and i+1:n of W contain those eigenvalues which have been successfully computed.

Contents

NAME

```
zjadmm - Jagged diagonal matrix-matrix multiply (modified
Ellpack)
```

SYNOPSIS

```
 SUBROUTINE ZJADMM( TRANSA, M, N, K, ALPHA, DESCRA,
*            VAL, INDX, PNTR, MAXNZ, IPERM,
*            B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, K, DESCRA(5), MAXNZ,
*            LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTR(MAXNZ+1), IPERM(M)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE ZJADMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*            VAL, INDX, PNTR, MAXNZ, IPERM,
*            B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, K, DESCRA(5), MAXNZ,
*            LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTR(MAXNZ+1), IPERM(M)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

where NNZ=PNTR(MAXNZ+1)-PNTR(1)+1 is the number of non-zero elements
```

F95 INTERFACE

```
 SUBROUTINE JADMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
*    PNTR, MAXNZ, IPERM, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, K, MAXNZ
 INTEGER, DIMENSION(:) ::  DESCRA, INDX, PNTR, IPERM
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) ::  VAL
```

```
      DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C

      SUBROUTINE JADMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL, INDX,
     *    PNTR, MAXNZ, IPERM, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
      INTEGER*8     TRANSA, M, K, MAXNZ
      INTEGER*8, DIMENSION(:) ::  DESCRA, INDX, PNTR, IPERM
      DOUBLE COMPLEX    ALPHA, BETA
      DOUBLE COMPLEX, DIMENSION(:) ::  VAL
      DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
             C <- alpha op(A) B + beta C

     where ALPHA and BETA are scalar, C and B are dense matrices,
     A is a matrix represented in jagged-diagonal format and
     op( A )  is one  of
     op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                           ( ' indicates matrix transpose)
```

ARGUMENTS

```
     TRANSA        Indicates how to operate with the sparse matrix
                     0 : operate with matrix
                     1 : operate with transpose matrix
                     2 : operate with the conjugate transpose of matrix.
                         2 is equivalent to 1 if matrix is real.

     M             Number of rows in matrix A

     N             Number of columns in matrix C

     K             Number of columns in matrix A

     ALPHA         Scalar parameter

     DESCRA()      Descriptor argument.  Five element integer array
                   DESCRA(1) matrix structure
                     0 : general
                     1 : symmetric (A=A')
                     2 : Hermitian (A= CONJG(A'))
                     3 : Triangular
                     4 : Skew(Anti)-Symmetric (A=-A')
                     5 : Diagonal
                     6 : Skew-Hermitian (A= -CONJG(A'))
                   DESCRA(2) upper/lower triangular indicator
                     1 : lower
```

```
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-unit
                 1 : unit
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices

VAL()          array of length NNZ consisting of entries of A.
               VAL can be viewed as a column major ordering of a
               row permutation of the Ellpack representation of A,
               where the Ellpack representation is permuted so that
               the rows are non-increasing in the number of nonzero
               entries.  Values added for padding in Ellpack are
               not included in the Jagged-Diagonal format.

INDX()         array of length NNZ consisting of the column indices
               of the corresponding entries in VAL.
PNTR()         array of length MAXNZ+1, where PNTR(I)-PNTR(1)+1
               points to the location in VAL of the first element
               in the row-permuted Ellpack represenation of A.

MAXNZ          max number of nonzeros elements per row.

IPERM()        integer array of length M such that I = IPERM(I'),
               where row I in the original Ellpack representation
               corresponds to row I' in the permuted representation.
               If IPERM(1) = 0, it is assumed by convention that
               IPERM(I) = I. IPERM is used to determine the order
               in which rows of C are updated.

B()            rectangular array with first dimension LDB.

LDB            leading dimension of B

BETA           Scalar parameter

C()            rectangular array with first dimension LDC.

LDC            leading dimension of C

WORK()         scratch array of length LWORK. WORK is not
               referenced in the current version.

LWORK          length of WORK array. LWORK is not referenced
               in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

zjadrp - right permutation of a jagged diagonal matrix

SYNOPSIS

```
 SUBROUTINE ZJADRP( TRANSP, M, K, VAL, INDX, PNTR, MAXNZ,
*                      IPERM,WORK,LWORK)
 INTEGER    TRANSP, M, K, MAXNZ, LWORK
 INTEGER    INDX(*), PNTR(MAXNZ+1), IPERM(K), WORK(LWORK)
 DOUBLE COMPLEX VAL(*)

 SUBROUTINE ZJADRP_64( TRANSP, M, K, VAL, INDX, PNTR, MAXNZ,
*                      IPERM,WORK,LWORK)
 INTEGER*8  TRANSP, M, K, MAXNZ, LWORK
 INTEGER*8  INDX(*), PNTR(MAXNZ+1), IPERM(K), WORK(LWORK)
 DOUBLE COMPLEX VAL(*)
```

F95 INTERFACE

```
 SUBROUTINE JADRP( TRANSP, M, K, VAL, INDX, PNTR, MAXNZ,
*                   IPERM, [WORK], [LWORK] )
 INTEGER TRANSP, M, K,  MAXNZ
 INTEGER, DIMENSION(:) :: INDX, PNTR, IPERM
 DOUBLE COMPLEX, DIMENSION(:) :: VAL

 SUBROUTINE JADRP_64( TRANSP, M, K, VAL, INDX, PNTR, MAXNZ,
*                   IPERM, [WORK], [LWORK] )
 INTEGER*8 TRANSP, M, K,  MAXNZ
 INTEGER*8, DIMENSION(:) :: INDX, PNTR, IPERM
 DOUBLE COMPLEX, DIMENSION(:) :: VAL
```

DESCRIPTION

```
A <- A P
A <- A P'
                                   ( ' indicates matrix transpose)
```

where permutation P is represented by an integer vector IPERM, such that IPERM(I) is equal to the position of the only nonzero element in row I of permutation matrix P.

NOTE: In order to get a symetrically permuted jagged diagonal matrix P A P', one can explicitly permute the columns P A by calling

```
   SJADRP(0, M, M, VAL, INDX, PNTR, MAXNZ, IPERM, WORK, LWORK)
```

where parameters VAL, INDX, PNTR, MAXNZ, IPERM are the representation of A in the jagged diagonal format. The operation makes sense if the original matrix A is square.

ARGUMENTS

TRANSP — Indicates how to operate with the permutation matrix
  0 : operate with matrix
  1 : operate with transpose matrix

M — Number of rows in matrix A

K — Number of columns in matrix A

VAL() — array of length PNTR(MAXNZ+1)-PNTR(1) consisting of entries of A. VAL can be viewed as a column major ordering of a row permutation of the Ellpack representation of A, where the Ellpack representation is permuted so that the rows are non-increasing in the number of nonzero entries. Values added for padding in Ellpack are not included in the Jagged-Diagonal format.

INDX() — array of length PNTR(MAXNZ+1)-PNTR(1) consisting of the column indices of the corresponding entries in VAL.

PNTR() — array of length MAXNZ+1, where PNTR(I)-PNTR(1)+1 points to the location in VAL of the first element in the row-permuted Ellpack represenation of A.

MAXNZ — max number of nonzeros elements per row.

IPERM() — integer array of length K such that I = IPERM(I').

```
                Array IPERM represents a permutation P, such that
                IPERM(I) is equal to the position of the only nonzero
                element in row I of permutation matrix P.
                For example, if
                               | 0 0 1 |
                           P  =| 1 0 0 |
                               | 0 1 0 |
                then IPERM = (3, 1, 2).

WORK()          scratch array of length LWORK.  LWORK should be at
                least K.

LWORK           length of WORK array

                If LWORK = -1, then a workspace query is assumed;
                the routine only calculates the optimal size of the
                WORK array, returns this value as the first entry of
                the WORK array, and no error message related to LWORK
                is issued by XERBLA.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

Contents

NAME

zjadsm - Jagged-diagonal format triangular solve

SYNOPSIS

```
 SUBROUTINE ZJADSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, PNTR, MAXNZ, IPERM,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5), MAXNZ,
*           LDB, LDC, LWORK
 INTEGER    INDX(NNZ), PNTR(MAXNZ+1), IPERM(M)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE ZJADSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, PNTR, MAXNZ, IPERM,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5), MAXNZ,
*           LDB, LDC, LWORK
 INTEGER*8  INDX(NNZ), PNTR(MAXNZ+1), IPERM(M)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

where NNZ=PNTR(MAXNZ+1)-PNTR(1)+1 is the number of non-zero elements
```

F95 INTERFACE

```
 SUBROUTINE JADSM(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*    PNTR, MAXNZ, IPERM, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, MAXNZ
 INTEGER, DIMENSION(:) ::  DESCRA, INDX, PNTR, IPERM
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) ::  VAL, DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE JADSM_64(TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL, INDX,
*    PNTR, MAXNZ, IPERM, B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, MAXNZ
 INTEGER*8, DIMENSION(:) ::  DESCRA, INDX, PNTR, IPERM
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) ::  VAL, DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C

where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a diagonal scaling matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in jagged-diagonal format and
op( A )  is one  of
 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row scaling (see section NOTES for
                    further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
```

```
                  0 : general
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))
                Note: For the routine, DESCRA(1)=3 is only supported.

                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices
VAL()           array of length NNZ consisting of entries of A.
                VAL can be viewed as a column major ordering of a
                row permutation of the Ellpack representation of A,
                where the Ellpack representation is permuted so that
                the rows are non-increasing in the number of nonzero
                entries.  Values added for padding in Ellpack are
                not included in the Jagged-Diagonal format.

INDX()          array of length NNZ consisting of the column indices
                of the corresponding entries in VAL.

PNTR()          array of length MAXNZ+1, where PNTR(I)-PNTR(1)+1
                points to the location in VAL of the first element
                in the row-permuted Ellpack represenation of A.

MAXNZ           max number of nonzeros elements per row.

IPERM()         integer array of length M such that I = IPERM(I'),
                where row I in the original Ellpack representation
                corresponds to row I' in the permuted representation.
                If IPERM(1)=0,   it's assumed by convention that
                IPERM(I)=I. IPERM is used to determine the order
                in which rows of C are updated.

B()             rectangular array with first dimension LDB.

LDB             leading dimension of B

BETA            Scalar parameter
```

```
C()            rectangular array with first dimension LDC.

LDC            leading dimension of C

WORK()         scratch array of length LWORK.
               On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.

LWORK          length of WORK array.  LWORK should be at least 2*M.

               For good performance, LWORK should generally be larger.
               For optimum performance on multiple processors, LWORK
               >=2*M*N_CPUS where N_CPUS is the maximum number of
               processors available to the program.

               If LWORK=0, the routine is to allocate workspace needed.

               If LWORK = -1, then a workspace query is assumed; the
               routine only calculates the optimum size of the WORK
               array, returns this value as the first entry of the WORK
               array, and no error message related to LWORK is issued
               by XERBLA.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If UNITD =4, the routine scales the rows of A such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in the particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows have been scaled. UNITD=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row number which 2-norm is exactly zero.

3. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements might or might not be referenced in the JAD representation of a sparse matrix. They are not used anyway in these cases. But if UNITD=4, the unit diagonal elements MUST be referenced in the JAD representation.

4. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

```
zlarz - applie a complex elementary reflector H to a complex
M-by-N matrix C, from either the left or the right
```

SYNOPSIS

```
SUBROUTINE ZLARZ(SIDE, M, N, L, V, INCV, TAU, C, LDC, WORK)

CHARACTER * 1 SIDE
DOUBLE COMPLEX TAU
DOUBLE COMPLEX V(*), C(LDC,*), WORK(*)
INTEGER M, N, L, INCV, LDC

SUBROUTINE ZLARZ_64(SIDE, M, N, L, V, INCV, TAU, C, LDC, WORK)

CHARACTER * 1 SIDE
DOUBLE COMPLEX TAU
DOUBLE COMPLEX V(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, L, INCV, LDC
```

F95 INTERFACE

```
SUBROUTINE LARZ(SIDE, [M], [N], L, V, [INCV], TAU, C, [LDC], [WORK])

CHARACTER(LEN=1) :: SIDE
COMPLEX(8) :: TAU
COMPLEX(8), DIMENSION(:) :: V, WORK
COMPLEX(8), DIMENSION(:,:) :: C
INTEGER :: M, N, L, INCV, LDC

SUBROUTINE LARZ_64(SIDE, [M], [N], L, V, [INCV], TAU, C, [LDC], [WORK])

CHARACTER(LEN=1) :: SIDE
```

```
COMPLEX(8) :: TAU
COMPLEX(8), DIMENSION(:) :: V, WORK
COMPLEX(8), DIMENSION(:,:) :: C
INTEGER(8) :: M, N, L, INCV, LDC
```

C INTERFACE

```
#include <sunperf.h>

void zlarz(char side, int m, int n, int l, doublecomplex *v,
          int  incv,  doublecomplex  *tau, doublecomplex *c,
          int ldc);

void zlarz_64(char side, long m, long n, long l,  doublecom-
          plex *v, long incv, doublecomplex *tau, doublecom-
          plex *c, long ldc);
```

PURPOSE

zlarz applies a complex elementary reflector H to a complex M-by-N matrix C, from either the left or the right. H is represented in the form

H = I - tau * v * v'

where tau is a complex scalar and v is a complex vector.

If tau = 0, then H is taken to be the unit matrix.

To apply H' (the conjugate transpose of H), supply conjg(tau) instead tau.

H is a product of k elementary reflectors as returned by CTZRZF.

ARGUMENTS

SIDE (input)
= 'L': form H * C
= 'R': form C * H

M (input) The number of rows of the matrix C.

N (input) The number of columns of the matrix C.

L (input) The number of entries of the vector V containing the meaningful part of the Householder vectors. If SIDE = 'L', M >= L >= 0, if SIDE = 'R', N >= L

>= 0.

V (input) The vector v in the representation of H as returned by CTZRZF. V is not used if TAU = 0.

INCV (input)
The increment between elements of v. INCV <> 0.

TAU (input)
The value tau in the representation of H.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by the matrix H * C if SIDE = 'L', or C * H if SIDE = 'R'.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
(N) if SIDE = 'L' or (M) if SIDE = 'R'

FURTHER DETAILS

Based on contributions by
A. Petitet, Computer Science Dept., Univ. of Tenn., Knoxville, USA

Contents

NAME

```
zlarzb - applie a complex block reflector H or its transpose
H**H  to a complex distributed M-by-N C from the left or the
right
```

SYNOPSIS

```
SUBROUTINE ZLARZB(SIDE, TRANS, DIRECT, STOREV, M, N, K, L, V, LDV, T,
      LDT, C, LDC, WORK, LDWORK)

CHARACTER * 1 SIDE, TRANS, DIRECT, STOREV
DOUBLE COMPLEX V(LDV,*), T(LDT,*), C(LDC,*), WORK(LDWORK,*)
INTEGER M, N, K, L, LDV, LDT, LDC, LDWORK

SUBROUTINE ZLARZB_64(SIDE, TRANS, DIRECT, STOREV, M, N, K, L, V, LDV,
      T, LDT, C, LDC, WORK, LDWORK)

CHARACTER * 1 SIDE, TRANS, DIRECT, STOREV
DOUBLE COMPLEX V(LDV,*), T(LDT,*), C(LDC,*), WORK(LDWORK,*)
INTEGER*8 M, N, K, L, LDV, LDT, LDC, LDWORK
```

F95 INTERFACE

```
SUBROUTINE LARZB(SIDE, TRANS, DIRECT, STOREV, [M], [N], K, L, V, [LDV],
       T, [LDT], C, [LDC], [WORK], [LDWORK])

CHARACTER(LEN=1) :: SIDE, TRANS, DIRECT, STOREV
COMPLEX(8), DIMENSION(:,:) :: V, T, C, WORK
INTEGER :: M, N, K, L, LDV, LDT, LDC, LDWORK

SUBROUTINE LARZB_64(SIDE, TRANS, DIRECT, STOREV, [M], [N], K, L, V,
       [LDV], T, [LDT], C, [LDC], [WORK], [LDWORK])
```

```
CHARACTER(LEN=1) :: SIDE, TRANS, DIRECT, STOREV
COMPLEX(8), DIMENSION(:,:) :: V, T, C, WORK
INTEGER(8) :: M, N, K, L, LDV, LDT, LDC, LDWORK
```

C INTERFACE

```
#include <sunperf.h>

void zlarzb(char side, char trans, char direct, char storev,
          int  m, int n, int k, int l, doublecomplex *v, int
          ldv, doublecomplex *t, int ldt, doublecomplex  *c,
          int ldc, int ldwork);

void zlarzb_64(char side,  char  trans,  char  direct,  char
          storev, long m, long n, long k, long l, doublecom-
          plex *v, long ldv,  doublecomplex  *t,  long  ldt,
          doublecomplex *c, long ldc, long ldwork);
```

PURPOSE

zlarzb applies a complex block reflector H or its transpose H**H to a complex distributed M-by-N C from the left or the right.

Currently, only STOREV = 'R' and DIRECT = 'B' are supported.

ARGUMENTS

```
SIDE (input)
          = 'L': apply H or H' from the Left
          = 'R': apply H or H' from the Right

TRANS (input)
          = 'N': apply H (No transpose)
          = 'C': apply H' (Conjugate transpose)

DIRECT (input)
          Indicates how H is formed from a product  of  ele-
          mentary reflectors = 'F': H = H(1) H(2) . . . H(k)
          (Forward, not supported yet)
          = 'B': H = H(k) . . . H(2) H(1) (Backward)

STOREV (input)
          Indicates how the vectors which define the elemen-
          tary reflectors are stored:
          = 'C': Columnwise                        (not sup-
          ported yet)
          = 'R': Rowwise
```

M (input) The number of rows of the matrix C.

N (input) The number of columns of the matrix C.

K (input) The order of the matrix T (= the number of elementary reflectors whose product defines the block reflector).

L (input) The number of columns of the matrix V containing the meaningful part of the Householder reflectors. If SIDE = 'L', M >= L >= 0, if SIDE = 'R', N >= L >= 0.

V (input) If STOREV = 'C', NV = K; if STOREV = 'R', NV = L.

LDV (input)
The leading dimension of the array V. If STOREV = 'C', LDV >= L; if STOREV = 'R', LDV >= K.

T (input) The triangular K-by-K matrix T in the representation of the block reflector.

LDT (input)
The leading dimension of the array T. LDT >= K.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by H*C or H'*C or C*H or C*H'.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
dimension(MAX(M,N),K)

LDWORK (input)
The leading dimension of the array WORK. If SIDE = 'L', LDWORK >= max(1,N); if SIDE = 'R', LDWORK >= max(1,M).

FURTHER DETAILS

Based on contributions by
A. Petitet, Computer Science Dept., Univ. of Tenn., Knoxville, USA

Contents

NAME

```
zlarzt - form the triangular factor T  of  a  complex  block
reflector H of order > n, which is defined as a product of k
elementary reflectors
```

SYNOPSIS

```
SUBROUTINE ZLARZT(DIRECT, STOREV, N, K, V, LDV, TAU, T, LDT)

CHARACTER * 1 DIRECT, STOREV
DOUBLE COMPLEX V(LDV,*), TAU(*), T(LDT,*)
INTEGER N, K, LDV, LDT

SUBROUTINE ZLARZT_64(DIRECT, STOREV, N, K, V, LDV, TAU, T, LDT)

CHARACTER * 1 DIRECT, STOREV
DOUBLE COMPLEX V(LDV,*), TAU(*), T(LDT,*)
INTEGER*8 N, K, LDV, LDT
```

F95 INTERFACE

```
SUBROUTINE LARZT(DIRECT, STOREV, N, K, V, [LDV], TAU, T, [LDT])

CHARACTER(LEN=1) :: DIRECT, STOREV
COMPLEX(8), DIMENSION(:) :: TAU
COMPLEX(8), DIMENSION(:,:) :: V, T
INTEGER :: N, K, LDV, LDT

SUBROUTINE LARZT_64(DIRECT, STOREV, N, K, V, [LDV], TAU, T, [LDT])

CHARACTER(LEN=1) :: DIRECT, STOREV
COMPLEX(8), DIMENSION(:) :: TAU
COMPLEX(8), DIMENSION(:,:) :: V, T
```

```
INTEGER(8) :: N, K, LDV, LDT
```

C INTERFACE

```
#include <sunperf.h>

void zlarzt(char direct, char storev, int n,  int  k,  doub-
          lecomplex  *v,  int ldv, doublecomplex *tau, doub-
          lecomplex *t, int ldt);

void zlarzt_64(char direct, char storev,  long  n,  long  k,
          doublecomplex  *v,  long  ldv, doublecomplex *tau,
          doublecomplex *t, long ldt);
```

PURPOSE

zlarzt forms the triangular factor T of a complex block reflector H of order > n, which is defined as a product of k elementary reflectors.

If DIRECT = 'F', H = H(1) H(2) . . . H(k) and T is upper triangular;

If DIRECT = 'B', H = H(k) . . . H(2) H(1) and T is lower triangular.

If STOREV = 'C', the vector which defines the elementary reflector H(i) is stored in the i-th column of the array V, and

   H = I - V * T * V'

If STOREV = 'R', the vector which defines the elementary reflector H(i) is stored in the i-th row of the array V, and

   H = I - V' * T * V

Currently, only STOREV = 'R' and DIRECT = 'B' are supported.

ARGUMENTS

DIRECT (input)
Specifies the order in which the elementary reflectors are multiplied to form the block reflector:
= 'F': H = H(1) H(2) . . . H(k) (Forward, not supported yet)
= 'B': H = H(k) . . . H(2) H(1) (Backward)

STOREV (input)
Specifies how the vectors which define the elementary reflectors are stored (see also Further Details):
= 'R': rowwise

N (input) The order of the block reflector H. N >= 0.

K (input) The order of the triangular factor T (= the number of elementary reflectors). K >= 1.

V (input) (LDV,K) if STOREV = 'C' (LDV,N) if STOREV = 'R'
The matrix V. See further details.

LDV (input)
The leading dimension of the array V. If STOREV = 'C', LDV >= max(1,N); if STOREV = 'R', LDV >= K.

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i).

T (input) The k by k triangular factor T of the block reflector. If DIRECT = 'F', T is upper triangular; if DIRECT = 'B', T is lower triangular. The rest of the array is not used.

LDT (input)
The leading dimension of the array T. LDT >= K.

FURTHER DETAILS

Based on contributions by
A. Petitet, Computer Science Dept., Univ. of Tenn., Knoxville, USA

The shape of the matrix V and the storage of the vectors which define the H(i) is best illustrated by the following example with n = 5 and k = 3. The elements equal to 1 are not stored; the corresponding array elements are modified but restored on exit. The rest of the array is not used.

DIRECT = 'F' and STOREV = 'C': DIRECT = 'F' and STOREV = 'R':

```
                                                  ______V_____
       (    v1    v2    v3    )                              /
( v1 v2 v3 )                      ( v1 v1 v1 v1 v1 . . . . 1
)
   V = ( v1 v2 v3 )                      ( v2 v2 v2 v2 v2  .
```

```
. . 1   )
       ( v1 v2 v3 )                      ( v3 v3 v3 v3 v3  .
. 1     )
       ( v1 v2 v3 )
          .  .  .
          1  .  .
             1  .
                1

DIRECT = 'B' and STOREV =  'C':          DIRECT  =  'B'  and
STOREV = 'R':

                                                     ______V_____
          1                                                 /
.  1                          ( 1 . . . . v1 v1 v1 v1 v1 )
          .  .  1                        ( . 1 . . .  v2  v2
v2 v2 v2 )
          .  .  .                        ( . . 1 . .  v3  v3
v3 v3 v3 )
          .  .  .
       ( v1 v2 v3 )
    V = ( v1 v2 v3 )
       ( v1 v2 v3 )
```

Contents

NAME

```
zlatzm - routine is deprecated and has been replaced by rou-
tine CUNMRZ
```

SYNOPSIS

```
SUBROUTINE ZLATZM(SIDE, M, N, V, INCV, TAU, C1, C2, LDC, WORK)

CHARACTER * 1 SIDE
DOUBLE COMPLEX TAU
DOUBLE COMPLEX V(*), C1(LDC,*), C2(LDC,*), WORK(*)
INTEGER M, N, INCV, LDC

SUBROUTINE ZLATZM_64(SIDE, M, N, V, INCV, TAU, C1, C2, LDC, WORK)

CHARACTER * 1 SIDE
DOUBLE COMPLEX TAU
DOUBLE COMPLEX V(*), C1(LDC,*), C2(LDC,*), WORK(*)
INTEGER*8 M, N, INCV, LDC
```

F95 INTERFACE

```
SUBROUTINE LATZM(SIDE, [M], [N], V, [INCV], TAU, C1, C2, [LDC], [WORK])

CHARACTER(LEN=1) :: SIDE
COMPLEX(8) :: TAU
COMPLEX(8), DIMENSION(:) :: V, WORK
COMPLEX(8), DIMENSION(:,:) :: C1, C2
INTEGER :: M, N, INCV, LDC

SUBROUTINE LATZM_64(SIDE, [M], [N], V, [INCV], TAU, C1, C2, [LDC],
       [WORK])

CHARACTER(LEN=1) :: SIDE
```

```
COMPLEX(8) :: TAU
COMPLEX(8), DIMENSION(:) :: V, WORK
COMPLEX(8), DIMENSION(:,:) :: C1, C2
INTEGER(8) :: M, N, INCV, LDC
```

C INTERFACE

```
#include <sunperf.h>

void zlatzm(char side, int m, int n, doublecomplex  *v,  int
          incv, doublecomplex *tau, doublecomplex *c1, doub-
          lecomplex *c2, int ldc);

void zlatzm_64(char side, long m, long n, doublecomplex  *v,
          long  incv, doublecomplex *tau, doublecomplex *c1,
          doublecomplex *c2, long ldc);
```

PURPOSE

zlatzm routine is deprecated and has been replaced by routine CUNMRZ.

CLATZM applies a Householder matrix generated by CTZRQF to a matrix.

```
Let P = I - tau*u*u',   u = ( 1 ),
                            ( v )
where v is an (m-1) vector if SIDE = 'L', or a (n-1)  vector
if SIDE = 'R'.

If SIDE equals 'L', let
       C = [ C1 ] 1
           [ C2 ] m-1
             n
Then C is overwritten by P*C.

If SIDE equals 'R', let
       C = [ C1, C2 ] m
              1  n-1
Then C is overwritten by C*P.
```

ARGUMENTS

SIDE (input)
= 'L': form P * C
= 'R': form C * P

M (input) The number of rows of the matrix C.

N (input) The number of columns of the matrix C.

V (input) (1 + (M-1)*abs(INCV)) if SIDE = 'L' (1 + (N-1)*abs(INCV)) if SIDE = 'R' The vector v in the representation of P. V is not used if TAU = 0.

INCV (input)
The increment between elements of v. INCV <> 0

TAU (input)
The value tau in the representation of P.

C1 (input/output)
(LDC,N) if SIDE = 'L' (M,1) if SIDE = 'R' On entry, the n-vector C1 if SIDE = 'L', or the m-vector C1 if SIDE = 'R'.

On exit, the first row of P*C if SIDE = 'L', or the first column of C*P if SIDE = 'R'.

C2 (input/output)
(LDC, N) if SIDE = 'L' (LDC, N-1) if SIDE = 'R' On entry, the (m - 1) x n matrix C2 if SIDE = 'L', or the m x (n - 1) matrix C2 if SIDE = 'R'.

On exit, rows 2:m of P*C if SIDE = 'L', or columns 2:m of C*P if SIDE = 'R'.

LDC (input)
The leading dimension of the arrays C1 and C2. LDC >= max(1,M).

WORK (workspace)
(N) if SIDE = 'L' (M) if SIDE = 'R'

Contents

NAME

zpbcon - estimate the reciprocal of the condition number (in the 1-norm) of a complex Hermitian positive definite band matrix using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPBTRF

SYNOPSIS

```
SUBROUTINE ZPBCON(UPLO, N, KD, A, LDA, ANORM, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER N, KD, LDA, INFO
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION WORK2(*)

SUBROUTINE ZPBCON_64(UPLO, N, KD, A, LDA, ANORM, RCOND, WORK,
      WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, KD, LDA, INFO
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE PBCON(UPLO, [N], KD, A, [LDA], ANORM, RCOND, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
```

```
INTEGER :: N, KD, LDA, INFO
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: WORK2

SUBROUTINE PBCON_64(UPLO, [N], KD, A, [LDA], ANORM, RCOND, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, KD, LDA, INFO
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zpbcon(char uplo, int n, int kd, doublecomplex *a,  int
          lda, double anorm, double *rcond, int *info);

void zpbcon_64(char uplo, long n, long kd, doublecomplex *a,
          long   lda,  double  anorm,  double  *rcond,  long
          *info);
```

PURPOSE

zpbcon estimates the reciprocal of the condition number (in the 1-norm) of a complex Hermitian positive definite band matrix using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPBTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
= 'U': Upper triangular factor stored in A;
= 'L': Lower triangular factor stored in A.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of sub-diagonals if UPLO = 'L'. KD >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H of the band matrix A, stored in the first KD+1 rows of the array. The j-th column of U or L is stored in the j-th column of the array A as follows: if UPLO ='U', A(kd+1+i-j,j) = U(i,j) for max(1,j-kd)<=i<=j; if UPLO ='L', A(1+i-j,j) = L(i,j) for j<=i<=min(n,j+kd).

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

ANORM (input)
The 1-norm (or infinity-norm) of the Hermitian band matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

zpbequ - compute row and column scalings intended to equilibrate a Hermitian positive definite band matrix A and reduce its condition number (with respect to the two-norm)

SYNOPSIS

```
SUBROUTINE ZPBEQU(UPLO, N, KD, A, LDA, SCALE, SCOND, AMAX, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*)
INTEGER N, KD, LDA, INFO
DOUBLE PRECISION SCOND, AMAX
DOUBLE PRECISION SCALE(*)

SUBROUTINE ZPBEQU_64(UPLO, N, KD, A, LDA, SCALE, SCOND, AMAX,
      INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*)
INTEGER*8 N, KD, LDA, INFO
DOUBLE PRECISION SCOND, AMAX
DOUBLE PRECISION SCALE(*)
```

F95 INTERFACE

```
SUBROUTINE PBEQU(UPLO, [N], KD, A, [LDA], SCALE, SCOND, AMAX,
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, KD, LDA, INFO
REAL(8) :: SCOND, AMAX
REAL(8), DIMENSION(:) :: SCALE
```

```
SUBROUTINE PBEQU_64(UPLO, [N], KD, A, [LDA], SCALE, SCOND, AMAX,
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, KD, LDA, INFO
REAL(8) :: SCOND, AMAX
REAL(8), DIMENSION(:) :: SCALE
```

C INTERFACE

```
#include <sunperf.h>

void zpbequ(char uplo, int n, int kd, doublecomplex *a,  int
          lda,  double  *scale, double *scond, double *amax,
          int *info);

void zpbequ_64(char uplo, long n, long kd, doublecomplex *a,
          long  lda,  double  *scale,  double *scond, double
          *amax, long *info);
```

PURPOSE

zpbequ computes row and column scalings intended to equilibrate a Hermitian positive definite band matrix A and reduce its condition number (with respect to the two-norm). S contains the scale factors, S(i) = 1/sqrt(A(i,i)), chosen so that the scaled matrix B with elements B(i,j) = S(i)*A(i,j)*S(j) has ones on the diagonal. This choice of S puts the condition number of B within a factor N of the smallest possible condition number over all possible diagonal scalings.

ARGUMENTS

UPLO (input)
= 'U': Upper triangular of A is stored;
= 'L': Lower triangular of A is stored.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

A (input) The upper or lower triangle of the Hermitian band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

SCALE (output)
If INFO = 0, SCALE contains the scale factors for A.

SCOND (output)
If INFO = 0, SCALE contains the ratio of the smallest SCALE(i) to the largest SCALE(i). If SCOND >= 0.1 and AMAX is neither too large nor too small, it is not worth scaling by SCALE.

AMAX (output)
Absolute value of largest matrix element. If AMAX is very close to overflow or very close to underflow, the matrix should be scaled.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, the i-th diagonal element is nonpositive.

Contents

NAME

```
zpbrfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is Hermitian positive
definite and banded, and provides error bounds and  backward
error estimates for the solution
```

SYNOPSIS

```
SUBROUTINE ZPBRFS(UPLO, N, KD, NRHS, A, LDA, AF, LDAF, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER N, KD, NRHS, LDA, LDAF, LDB, LDX, INFO
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZPBRFS_64(UPLO, N, KD, NRHS, A, LDA, AF, LDAF, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER*8 N, KD, NRHS, LDA, LDAF, LDB, LDX, INFO
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE PBRFS(UPLO, [N], KD, [NRHS], A, [LDA], AF, [LDAF], B,
       [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
```

```
INTEGER :: N, KD, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE PBRFS_64(UPLO, [N], KD, [NRHS], A, [LDA], AF, [LDAF],
       B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, KD, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>
void zpbrfs(char uplo, int n, int kd, int  nrhs,  doublecom-
          plex  *a,  int  lda,  doublecomplex *af, int ldaf,
          doublecomplex *b, int ldb, doublecomplex  *x,  int
          ldx, double *ferr, double *berr, int *info);

void zpbrfs_64(char uplo, long n, long kd, long nrhs,  doub-
          lecomplex  *a,  long  lda, doublecomplex *af, long
          ldaf, doublecomplex *b,  long  ldb,  doublecomplex
          *x,  long  ldx,  double  *ferr, double *berr, long
          *info);
```

PURPOSE

zpbrfs improves the computed solution to a system of linear equations when the coefficient matrix is Hermitian positive definite and banded, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

NRHS (input)
The number of right hand sides, i.e., the number

of columns of the matrices B and X. NRHS >= 0.

A (input) The upper or lower triangle of the Hermitian band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

AF (input)
The triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H of the band matrix A as computed by CPBTRF, in the same storage format as A (see A).

LDAF (input)
The leading dimension of the array AF. LDAF >= KD+1.

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by CPBTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each

solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zpbstf - compute a split Cholesky factorization of a complex
Hermitian positive definite band matrix A
```

SYNOPSIS

```
SUBROUTINE ZPBSTF(UPLO, N, KD, AB, LDAB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AB(LDAB,*)
INTEGER N, KD, LDAB, INFO

SUBROUTINE ZPBSTF_64(UPLO, N, KD, AB, LDAB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AB(LDAB,*)
INTEGER*8 N, KD, LDAB, INFO
```

F95 INTERFACE

```
SUBROUTINE PBSTF(UPLO, [N], KD, AB, [LDAB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: AB
INTEGER :: N, KD, LDAB, INFO

SUBROUTINE PBSTF_64(UPLO, [N], KD, AB, [LDAB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: AB
INTEGER(8) :: N, KD, LDAB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zpbstf(char uplo, int n, int kd, doublecomplex *ab, int
          ldab, int *info);

void zpbstf_64(char uplo, long  n,  long  kd,  doublecomplex
          *ab, long ldab, long *info);
```

PURPOSE

zpbstf computes a split Cholesky factorization of a complex Hermitian positive definite band matrix A.

This routine is designed to be used in conjunction with CHBGST.
The factorization has the form A = S**H*S where S is a band matrix of the same bandwidth as A and the following structure:

```
  S = ( U    )
      ( M  L )
```

where U is upper triangular of order m = (n+kd)/2, and L is lower triangular of order n-m.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

AB (input/output)
On entry, the upper or lower triangle of the Hermitian band matrix A, stored in the first kd+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).

On exit, if INFO = 0, the factor S from the split Cholesky factorization A = S**H*S. See Further Details.

LDAB (input)
The leading dimension of the array AB. LDAB >= KD+1.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the factorization could not be completed, because the updated element a(i,i) was negative; the matrix A is not positive definite.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when N = 7, KD = 2:

```
S = ( s11  s12  s13                     )
    (      s22  s23  s24                )
    (           s33  s34                )
    (                s44                )
    (           s53  s54  s55           )
    (                s64  s65  s66      )
    (                     s75  s76  s77 )
```

If UPLO = 'U', the array AB holds:

on entry: on exit:

```
 *    *   a13  a24  a35  a46  a57   *    *   s13  s24   s53'
s64' s75'
 *   a12  a23  a34  a45  a56  a67   *   s12  s23  s34   s54'
s65'  s76'  a11  a22  a33  a44  a55  a66  a77  s11  s22  s33
s44  s55  s66  s77
```

If UPLO = 'L', the array AB holds:

on entry: on exit:

```
a11  a22  a33  a44  a55  a66  a77  s11  s22  s33   s44   s55
s66   s77  a21  a32  a43  a54  a65  a76   *   s12' s23' s34'
s54  s65  s76   * a31  a42  a53  a64  a64    *     *    s13'
s24' s53  s64  s75   *    *
```

Array elements marked * are not used by the routine; s12' denotes conjg(s12); the diagonal elements of S are real.

Contents

NAME

```
zpbsv - compute the solution to a complex system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE ZPBSV(UPLO, N, NDIAG, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, NDIAG, NRHS, LDA, LDB, INFO

SUBROUTINE ZPBSV_64(UPLO, N, NDIAG, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, NDIAG, NRHS, LDA, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE PBSV(UPLO, [N], NDIAG, [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: N, NDIAG, NRHS, LDA, LDB, INFO

SUBROUTINE PBSV_64(UPLO, [N], NDIAG, [NRHS], A, [LDA], B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: N, NDIAG, NRHS, LDA, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zpbsv(char uplo, int n, int ndiag, int nrhs, doublecom-
          plex  *a,  int lda, doublecomplex *b, int ldb, int
          *info);

void zpbsv_64(char uplo, long  n,  long  ndiag,  long  nrhs,
          doublecomplex *a, long lda, doublecomplex *b, long
          ldb, long *info);
```

PURPOSE

```
zpbsv computes the solution to a complex  system  of  linear
equations
   A * X = B,  where  A  is  an  N-by-N  Hermitian  positive
definite band matrix and X and B are N-by-NRHS matrices.

The Cholesky decomposition is used to factor A as
   A = U**H * U,  if UPLO = 'U', or
   A = L * L**H,  if UPLO = 'L',
where U is an upper triangular band matrix, and L is a lower
triangular  band matrix, with the same number of superdiago-
nals or subdiagonals as A.  The factored form of A  is  then
used to solve the system of equations A * X = B.
```

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NDIAG (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  NDIAG >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

A (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  band matrix A, stored in the first NDIAG+1
```

rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(NDIAG+1+i-j,j) = A(i,j) for max(1,j-NDIAG)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(N,j+NDIAG). See below for further details.

On exit, if INFO = 0, the triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H of the band matrix A, in the same storage format as A.

LDA (input)
The leading dimension of the array A. LDA >= NDIAG+1.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when N = 6, NDIAG = 2, and UPLO = 'U':

```
On entry:                       On exit:

    *    *   a13  a24  a35  a46      *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
```

Similarly, if UPLO = 'L' the format of A is as follows:

```
On entry:                       On exit:
```

```
   a11  a22  a33  a44  a55  a66     l11  l22  l33  l44   l55
l66
   a21  a32  a43  a54  a65   *      l21  l32  l43  l54   l65
*
   a31  a42  a53  a64   *    *      l31  l42  l53   l64    *
*
```

Array elements marked * are not used by the routine.

Contents

NAME

```
zpbsvx - use the Cholesky factorization A = U**H*U  or  A  =
L*L**H to compute the solution to a complex system of linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE ZPBSVX(FACT, UPLO, N, NDIAG, NRHS, A, LDA, AF, LDAF,
      EQUED, S, B, LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2,
      INFO)

CHARACTER * 1 FACT, UPLO, EQUED
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER N, NDIAG, NRHS, LDA, LDAF, LDB, LDX, INFO
DOUBLE PRECISION RCOND
DOUBLE PRECISION S(*), FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZPBSVX_64(FACT, UPLO, N, NDIAG, NRHS, A, LDA, AF, LDAF,
      EQUED, S, B, LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2,
      INFO)

CHARACTER * 1 FACT, UPLO, EQUED
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER*8 N, NDIAG, NRHS, LDA, LDAF, LDB, LDX, INFO
DOUBLE PRECISION RCOND
DOUBLE PRECISION S(*), FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE PBSVX(FACT, UPLO, [N], NDIAG, [NRHS], A, [LDA], AF, [LDAF],
       EQUED, S, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK],
```

```
       [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER :: N, NDIAG, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: S, FERR, BERR, WORK2

SUBROUTINE PBSVX_64(FACT, UPLO, [N], NDIAG, [NRHS], A, [LDA], AF,
       [LDAF], EQUED, S, B, [LDB], X, [LDX], RCOND, FERR, BERR,
       [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, NDIAG, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: S, FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zpbsvx(char fact, char uplo,  int  n,  int  ndiag,  int
          nrhs,  doublecomplex  *a,  int  lda, doublecomplex
          *af, int ldaf, char equed, double  *s,  doublecom-
          plex  *b, int ldb, doublecomplex *x, int ldx, dou-
          ble  *rcond,  double  *ferr,  double  *berr,   int
          *info);

void zpbsvx_64(char fact, char uplo,  long  n,  long  ndiag,
          long  nrhs, doublecomplex *a, long lda, doublecom-
          plex *af, long ldaf, char equed, double *s,  doub-
          lecomplex  *b,  long  ldb,  doublecomplex *x, long
          ldx, double *rcond, double  *ferr,  double  *berr,
          long *info);
```

PURPOSE

zpbsvx uses the Cholesky factorization A = U**H*U or A = L*L**H to compute the solution to a complex system of linear equations
   A * X = B, where A is an N-by-N Hermitian positive definite band matrix and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

```
1. If FACT = 'E',  real  scaling  factors  are  computed  to
equilibrate
   the system:
      diag(S) * A * diag(S) * inv(diag(S)) * X = diag(S) * B
   Whether or not the system will be equilibrated depends on
the
   scaling of the matrix A, but if equilibration is used,  A
is
   overwritten by diag(S)*A*diag(S) and B by diag(S)*B.

2. If FACT = 'N' or 'E', the Cholesky decomposition is  used
to
   factor the matrix A (after equilibration if FACT  =  'E')
as
      A = U**H * U,  if UPLO = 'U', or
      A = L * L**H,  if UPLO = 'L',
   where U is an upper triangular band matrix, and  L  is  a
lower
   triangular band matrix.

3. If the leading i-by-i principal  minor  is  not  positive
definite,
   then the routine returns with INFO =  i.  Otherwise,  the
factored
   form of A is used to estimate the condition number of the
matrix
   A.  If the reciprocal of the  condition  number  is  less
than machine
   precision, INFO = N+1 is returned as a warning,  but  the
routine
   still goes on to solve for X and compute error bounds as
   described below.

4. The system of equations is solved for X  using  the  fac-
tored form
   of A.

5. Iterative refinement is applied to improve  the  computed
solution
   matrix and calculate  error  bounds  and  backward  error
estimates
   for it.

6. If equilibration was used, the matrix X is  premultiplied
by
   diag(S) so that it solves the original system before
   equilibration.
```

ARGUMENTS

```
FACT (input)
          Specifies whether or not the factored form of  the
          matrix A is supplied on entry, and if not, whether
          the matrix A should be equilibrated before  it  is
          factored.   = 'F':  On entry, AF contains the fac-
          tored form of A.  If EQUED = 'Y', the matrix A has
          been equilibrated with scaling factors given by S.
          A and AF will not be modified.  = 'N':  The matrix
          A will be copied to AF and factored.
          = 'E':  The  matrix  A  will  be  equilibrated  if
          necessary, then copied to AF and factored.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NDIAG (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  NDIAG >= 0.

NRHS (input)
          The number of right-hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  band matrix A, stored in the first NDIAG+1
          rows of the array, except if FACT = 'F' and  EQUED
          = 'Y', then A must contain the equilibrated matrix
          diag(S)*A*diag(S).  The j-th column of A is stored
          in  the j-th column of the array A as follows:  if
          UPLO  =  'U',  A(NDIAG+1+i-j,j)   =   A(i,j)   for
          max(1,j-NDIAG)<=i<=j;  if  UPLO  = 'L', A(1+i-j,j)
          = A(i,j) for j<=i<=min(N,j+NDIAG).  See below  for
          further details.

          On exit, if FACT = 'E'  and  EQUED  =  'Y',  A  is
          overwritten by diag(S)*A*diag(S).

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          NDIAG+1.

AF (input/output)
```

If FACT = 'F', then AF is an input argument and on entry contains the triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H of the band matrix A, in the same storage format as A (see A). If EQUED = 'Y', then AF is the factored form of the equilibrated matrix A.

If FACT = 'N', then AF is an output argument and on exit returns the triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H.

If FACT = 'E', then AF is an output argument and on exit returns the triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H of the equilibrated matrix A (see the description of A for the form of the equilibrated matrix).

LDAF (input)
The leading dimension of the array AF. LDAF >= NDIAG+1.

EQUED (input)
Specifies the form of equilibration that was done.
= 'N': No equilibration (always true if FACT = 'N').
= 'Y': Equilibration was done, i.e., A has been replaced by diag(S) * A * diag(S). EQUED is an input argument if FACT = 'F'; otherwise, it is an output argument.

S (input/output)
The scale factors for A; not accessed if EQUED = 'N'. S is an input argument if FACT = 'F'; otherwise, S is an output argument. If FACT = 'F' and EQUED = 'Y', each element of S must be positive.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if EQUED = 'N', B is not modified; if EQUED = 'Y', B is overwritten by diag(S) * B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X to the original system of equations.

Note that if EQUED = 'Y', A and B are modified on exit, and the solution to the equilibrated system is inv(diag(S))*X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A after equilibration (if done). If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to work-

ing precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when N = 6, NDIAG = 2, and UPLO = 'U':

Two-dimensional storage of the Hermitian matrix A:

```
   a11  a12  a13
        a22  a23  a24
             a33  a34  a35
                  a44  a45  a46
                       a55  a56
   (aij=conjg(aji))         a66
```

Band storage of the upper triangle of A:

```
    *    *   a13  a24  a35  a46
    *   a12  a23  a34  a45  a56
   a11  a22  a33  a44  a55  a66
```

Similarly, if UPLO = 'L' the format of A is as follows:

```
   a11  a22  a33  a44  a55  a66
   a21  a32  a43  a54  a65   *
   a31  a42  a53  a64   *    *
```

Array elements marked * are not used by the routine.

Contents

NAME

zpbtf2 - compute the Cholesky factorization of a complex Hermitian positive definite band matrix A

SYNOPSIS

```
SUBROUTINE ZPBTF2(UPLO, N, KD, AB, LDAB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AB(LDAB,*)
INTEGER N, KD, LDAB, INFO

SUBROUTINE ZPBTF2_64(UPLO, N, KD, AB, LDAB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AB(LDAB,*)
INTEGER*8 N, KD, LDAB, INFO
```

F95 INTERFACE

```
SUBROUTINE PBTF2(UPLO, [N], KD, AB, [LDAB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: AB
INTEGER :: N, KD, LDAB, INFO

SUBROUTINE PBTF2_64(UPLO, [N], KD, AB, [LDAB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: AB
INTEGER(8) :: N, KD, LDAB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zpbtf2(char uplo, int n, int kd, doublecomplex *ab, int
          ldab, int *info);

void zpbtf2_64(char uplo, long  n,  long  kd,  doublecomplex
          *ab, long ldab, long *info);
```

PURPOSE

zpbtf2 computes the Cholesky factorization of a complex Hermitian positive definite band matrix A.

The factorization has the form
```
   A = U' * U ,  if UPLO = 'U', or
   A = L  * L',  if UPLO = 'L',
```
where U is an upper triangular matrix, U' is the conjugate transpose of U, and L is lower triangular.

This is the unblocked version of the algorithm, calling Level 2 BLAS.

ARGUMENTS

UPLO (input)
Specifies whether the upper or lower triangular part of the Hermitian matrix A is stored:
= 'U': Upper triangular
= 'L': Lower triangular

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of super-diagonals of the matrix A if UPLO = 'U', or the number of sub-diagonals if UPLO = 'L'. KD >= 0.

AB (input/output)
On entry, the upper or lower triangle of the Hermitian band matrix A, stored in the first KD+1 rows of the array. The j-th column of A is stored in the j-th column of the array AB as follows: if UPLO = 'U', AB(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', AB(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd).

On exit, if INFO = 0, the triangular factor U or L

from the Cholesky factorization A = U'*U or A = L*L' of the band matrix A, in the same storage format as A.

LDAB (input)
The leading dimension of the array AB. LDAB >= KD+1.

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, the leading minor of order k is not positive definite, and the factorization could not be completed.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when N = 6, KD = 2, and UPLO = 'U':

```
On entry:                       On exit:

    *    *   a13  a24  a35  a46      *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
```

Similarly, if UPLO = 'L' the format of A is as follows:

```
On entry:                       On exit:

   a11  a22  a33  a44  a55  a66     l11  l22  l33  l44   l55
l66
   a21  a32  a43  a54  a65   *      l21  l32  l43  l54   l65
*
   a31  a42  a53  a64   *    *      l31  l42  l53   l64    *
*
```

Array elements marked * are not used by the routine.

Contents

NAME

```
zpbtrf - compute the Cholesky  factorization  of  a  complex
Hermitian positive definite band matrix A
```

SYNOPSIS

```
SUBROUTINE ZPBTRF(UPLO, N, KD, A, LDA, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*)
INTEGER N, KD, LDA, INFO

SUBROUTINE ZPBTRF_64(UPLO, N, KD, A, LDA, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*)
INTEGER*8 N, KD, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE PBTRF(UPLO, [N], KD, A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, KD, LDA, INFO

SUBROUTINE PBTRF_64(UPLO, [N], KD, A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, KD, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zpbtrf(char uplo, int n, int kd, doublecomplex *a,  int
          lda, int *info);

void zpbtrf_64(char uplo, long n, long kd, doublecomplex *a,
          long lda, long *info);
```

PURPOSE

```
zpbtrf computes the Cholesky factorization of a complex Her-
mitian positive definite band matrix A.

The factorization has the form
   A = U**H * U,  if UPLO = 'U', or
   A = L  * L**H,  if UPLO = 'L',
where U is an upper triangular matrix and L  is  lower  tri-
angular.
```

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals of the  matrix  A  if
          UPLO  = 'U', or the number of subdiagonals if UPLO
          = 'L'.  KD >= 0.

A (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  band  matrix  A,  stored in the first KD+1
          rows of the array.  The j-th column of A is stored
          in  the j-th column of the array A as follows:  if
          UPLO = 'U', A(kd+1+i-j,j) =  A(i,j)  for  max(1,j-
          kd)<=i<=j;  if  UPLO = 'L', A(1+i-j,j)    = A(i,j)
          for j<=i<=min(n,j+kd).

          On exit, if INFO = 0, the triangular factor U or L
          from  the Cholesky factorization A = U**H*U or A =
          L*L**H of the band matrix A, in the  same  storage
          format as A.

LDA (input)
```

The leading dimension of the array A. LDA >= KD+1.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i is not positive definite, and the factorization could not be completed.

FURTHER DETAILS

The band storage scheme is illustrated by the following example, when N = 6, KD = 2, and UPLO = 'U':

```
On entry:                       On exit:

    *    *   a13  a24  a35  a46      *    *   u13  u24   u35
u46
    *   a12  a23  a34  a45  a56      *   u12  u23  u34   u45
u56
   a11  a22  a33  a44  a55  a66     u11  u22  u33  u44   u55
u66
```

Similarly, if UPLO = 'L' the format of A is as follows:

```
On entry:                       On exit:

   a11  a22  a33  a44  a55  a66     l11  l22  l33  l44   l55
l66
   a21  a32  a43  a54  a65   *      l21  l32  l43  l54   l65
*
   a31  a42  a53  a64   *    *      l31  l42  l53   l64    *
*
```

Array elements marked * are not used by the routine.

Contributed by
Peter Mayes and Giuseppe Radicati, IBM ECSEC, Rome, March 23, 1989

Contents

NAME

```
zpbtrs - solve a system of linear equations A*X = B  with  a
Hermitian positive definite band matrix A using the Cholesky
factorization A = U**H*U or A = L*L**H computed by CPBTRF
```

SYNOPSIS

```
SUBROUTINE ZPBTRS(UPLO, N, KD, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, KD, NRHS, LDA, LDB, INFO

SUBROUTINE ZPBTRS_64(UPLO, N, KD, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, KD, NRHS, LDA, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE PBTRS(UPLO, [N], KD, [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: N, KD, NRHS, LDA, LDB, INFO

SUBROUTINE PBTRS_64(UPLO, [N], KD, [NRHS], A, [LDA], B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: N, KD, NRHS, LDA, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zpbtrs(char uplo, int n, int kd, int  nrhs,  doublecom-
          plex  *a,  int lda, doublecomplex *b, int ldb, int
          *info);

void zpbtrs_64(char uplo, long n, long kd, long nrhs,  doub-
          lecomplex  *a,  long  lda,  doublecomplex *b, long
          ldb, long *info);
```

PURPOSE

zpbtrs solves a system of linear equations A*X = B with a Hermitian positive definite band matrix A using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPBTRF.

ARGUMENTS

UPLO (input)
= 'U': Upper triangular factor stored in A;
= 'L': Lower triangular factor stored in A.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'. KD >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H of the band matrix A, stored in the first KD+1 rows of the array. The j-th column of U or L is stored in the j-th column of the array A as follows: if UPLO ='U', A(kd+1+i-j,j) = U(i,j) for max(1,j-kd)<=i<=j; if UPLO ='L', A(1+i-j,j) = L(i,j) for j<=i<=min(n,j+kd).

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

```
B (input/output)
          On entry, the right hand side matrix B.  On  exit,
          the solution matrix X.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

INFO (output)
          = 0:  successful exit
          < 0:  if INFO =  -i,  the  i-th  argument  had  an
          illegal value
```

Contents

NAME

```
zpocon - estimate the reciprocal of the condition number (in
the  1-norm) of a complex Hermitian positive definite matrix
using the Cholesky factorization A = U**H*U or  A  =  L*L**H
computed by CPOTRF
```

SYNOPSIS

```
SUBROUTINE ZPOCON(UPLO, N, A, LDA, ANORM, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, INFO
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION WORK2(*)

SUBROUTINE ZPOCON_64(UPLO, N, A, LDA, ANORM, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, INFO
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE POCON(UPLO, [N], A, [LDA], ANORM, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
```

```
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: WORK2

SUBROUTINE POCON_64(UPLO, [N], A, [LDA], ANORM, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>
void zpocon(char uplo, int n,  doublecomplex  *a,  int  lda,
          double anorm, double *rcond, int *info);

void zpocon_64(char uplo, long  n,  doublecomplex  *a,  long
          lda, double anorm, double *rcond, long *info);
```

PURPOSE

zpocon estimates the reciprocal of the condition number (in the 1-norm) of a complex Hermitian positive definite matrix using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPOTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H, as computed by CPOTRF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

ANORM (input)
The 1-norm (or infinity-norm) of the Hermitian matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

zpoequ - compute row and column scalings intended to equilibrate a Hermitian positive definite matrix A and reduce its condition number (with respect to the two-norm)

SYNOPSIS

```
SUBROUTINE ZPOEQU(N, A, LDA, SCALE, SCOND, AMAX, INFO)

DOUBLE COMPLEX A(LDA,*)
INTEGER N, LDA, INFO
DOUBLE PRECISION SCOND, AMAX
DOUBLE PRECISION SCALE(*)

SUBROUTINE ZPOEQU_64(N, A, LDA, SCALE, SCOND, AMAX, INFO)

DOUBLE COMPLEX A(LDA,*)
INTEGER*8 N, LDA, INFO
DOUBLE PRECISION SCOND, AMAX
DOUBLE PRECISION SCALE(*)
```

F95 INTERFACE

```
SUBROUTINE POEQU([N], A, [LDA], SCALE, SCOND, AMAX, [INFO])

COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
REAL(8) :: SCOND, AMAX
REAL(8), DIMENSION(:) :: SCALE

SUBROUTINE POEQU_64([N], A, [LDA], SCALE, SCOND, AMAX, [INFO])

COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
```

```
REAL(8) :: SCOND, AMAX
REAL(8), DIMENSION(:) :: SCALE
```

C INTERFACE

```
#include <sunperf.h>

void zpoequ(int n, doublecomplex *a, int lda, double *scale,
          double *scond, double *amax, int *info);

void zpoequ_64(long n, doublecomplex *a,  long  lda,  double
          *scale, double *scond, double *amax, long *info);
```

PURPOSE

zpoequ computes row and column scalings intended to equilibrate a Hermitian positive definite matrix A and reduce its condition number (with respect to the two-norm). S contains the scale factors, S(i) = 1/sqrt(A(i,i)), chosen so that the scaled matrix B with elements B(i,j) = S(i)*A(i,j)*S(j) has ones on the diagonal. This choice of S puts the condition number of B within a factor N of the smallest possible condition number over all possible diagonal scalings.

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

A (input) The N-by-N Hermitian positive definite matrix whose scaling factors are to be computed. Only the diagonal elements of A are referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

SCALE (output)
If INFO = 0, SCALE contains the scale factors for A.

SCOND (output)
If INFO = 0, SCALE contains the ratio of the smallest SCALE(i) to the largest SCALE(i). If SCOND >= 0.1 and AMAX is neither too large nor too small, it is not worth scaling by SCALE.

AMAX (output)

Absolute value of largest matrix element. If AMAX is very close to overflow or very close to underflow, the matrix should be scaled.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the i-th diagonal element is nonpositive.

Contents

NAME

```
zporfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is Hermitian positive
definite,
```

SYNOPSIS

```
SUBROUTINE ZPORFS(UPLO, N, NRHS, A, LDA, AF, LDAF, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZPORFS_64(UPLO, N, NRHS, A, LDA, AF, LDAF, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE PORFS(UPLO, [N], [NRHS], A, [LDA], AF, [LDAF], B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
```

```
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE PORFS_64(UPLO, [N], [NRHS], A, [LDA], AF, [LDAF], B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>
void zporfs(char uplo, int n, int  nrhs,  doublecomplex  *a,
          int  lda,  doublecomplex *af, int ldaf, doublecom-
          plex *b, int ldb, doublecomplex *x, int ldx,  dou-
          ble *ferr, double *berr, int *info);

void zporfs_64(char uplo, long n, long  nrhs,  doublecomplex
          *a,  long lda, doublecomplex *af, long ldaf, doub-
          lecomplex *b, long  ldb,  doublecomplex  *x,  long
          ldx, double *ferr, double *berr, long *info);
```

PURPOSE

zporfs improves the computed solution to a system of linear equations when the coefficient matrix is Hermitian positive definite, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The Hermitian matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input)
The triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H, as computed by CPOTRF.

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by CPOTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

```
WORK2 (workspace)
          dimension(N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO =  -i,  the  i-th  argument  had  an
          illegal value
```

Contents

NAME

```
zposv - compute the solution to a complex system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE ZPOSV(UPLO, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, NRHS, LDA, LDB, INFO

SUBROUTINE ZPOSV_64(UPLO, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, NRHS, LDA, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE POSV(UPLO, [N], [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: N, NRHS, LDA, LDB, INFO

SUBROUTINE POSV_64(UPLO, [N], [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zposv(char uplo, int n, int nrhs, doublecomplex *a, int
          lda, doublecomplex *b, int ldb, int *info);

void zposv_64(char uplo, long n,  long  nrhs,  doublecomplex
          *a,  long  lda,  doublecomplex  *b, long ldb, long
          *info);
```

PURPOSE

```
zposv computes the solution to a complex  system  of  linear
equations
   A * X = B, where A is an N-by-N Hermitian positive defin-
ite matrix and X and B are N-by-NRHS matrices.
The Cholesky decomposition is used to factor A as
   A = U**H* U,  if UPLO = 'U', or
   A = L * L**H,  if UPLO = 'L',
where U is an upper triangular matrix and  L is a lower tri-
angular  matrix.   The  factored  form  of A is then used to
solve the system of equations A * X = B.
```

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

A (input/output)
          On entry, the Hermitian matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A,
          and the strictly lower triangular part of A is not
          referenced.  If UPLO =  'L',  the  leading  N-by-N
          lower triangular part of A contains the lower tri-
          angular part of the matrix  A,  and  the  strictly
          upper triangular part of A is not referenced.

          On exit, if INFO = 0, the factor U or L  from  the
          Cholesky factorization A = U**H*U or A = L*L**H.
```

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed.

Contents

NAME

```
zposvx - use the Cholesky factorization A = U**H*U  or  A  =
L*L**H to compute the solution to a complex system of linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE ZPOSVX(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, EQUED,
      S, B, LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO, EQUED
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
DOUBLE PRECISION RCOND
DOUBLE PRECISION S(*), FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZPOSVX_64(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, EQUED,
      S, B, LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO, EQUED
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
DOUBLE PRECISION RCOND
DOUBLE PRECISION S(*), FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE POSVX(FACT, UPLO, [N], [NRHS], A, [LDA], AF, [LDAF],
       EQUED, S, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
```

```
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: S, FERR, BERR, WORK2

SUBROUTINE POSVX_64(FACT, UPLO, [N], [NRHS], A, [LDA], AF, [LDAF],
       EQUED, S, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: S, FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zposvx(char fact, char uplo, int  n,  int  nrhs,  doub-
          lecomplex  *a,  int  lda,  doublecomplex  *af, int
          ldaf, char equed, double *s, doublecomplex *b, int
          ldb,  doublecomplex  *x,  int  ldx, double *rcond,
          double *ferr, double *berr, int *info);

void zposvx_64(char fact, char  uplo,  long  n,  long  nrhs,
          doublecomplex  *a,  long  lda,  doublecomplex *af,
          long ldaf, char equed,  double  *s,  doublecomplex
          *b,  long  ldb, doublecomplex *x, long ldx, double
          *rcond, double *ferr, double *berr, long *info);
```

PURPOSE

```
zposvx uses the Cholesky factorization A =  U**H*U  or  A  =
L*L**H to compute the solution to a complex system of linear
equations
   A * X = B, where A is an N-by-N Hermitian positive defin-
ite matrix and X and B are N-by-NRHS matrices.

Error bounds on the solution and a  condition  estimate  are
also provided.

The following steps are performed:

1. If FACT = 'E',  real  scaling  factors  are  computed  to
equilibrate
   the system:
      diag(S) * A * diag(S) * inv(diag(S)) * X = diag(S) * B
   Whether or not the system will be equilibrated depends on
```

the
scaling of the matrix A, but if equilibration is used, A is
overwritten by diag(S)*A*diag(S) and B by diag(S)*B.

2. If FACT = 'N' or 'E', the Cholesky decomposition is used to
factor the matrix A (after equilibration if FACT = 'E') as
```
A = U**H* U,  if UPLO = 'U', or
A = L * L**H,  if UPLO = 'L',
```
where U is an upper triangular matrix and L is a lower triangular
matrix.

3. If the leading i-by-i principal minor is not positive definite,
then the routine returns with INFO = i. Otherwise, the factored
form of A is used to estimate the condition number of the matrix
A. If the reciprocal of the condition number is less than machine
precision, INFO = N+1 is returned as a warning, but the routine
still goes on to solve for X and compute error bounds as
described below.

4. The system of equations is solved for X using the factored form
of A.

5. Iterative refinement is applied to improve the computed solution
matrix and calculate error bounds and backward error estimates
for it.

6. If equilibration was used, the matrix X is premultiplied by
diag(S) so that it solves the original system before
equilibration.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of the
matrix A is supplied on entry, and if not, whether

the matrix A should be equilibrated before it is factored. = 'F': On entry, AF contains the factored form of A. If EQUED = 'Y', the matrix A has been equilibrated with scaling factors given by S. A and AF will not be modified. = 'N': The matrix A will be copied to AF and factored.
= 'E': The matrix A will be equilibrated if necessary, then copied to AF and factored.

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input/output)
On entry, the Hermitian matrix A, except if FACT = 'F' and EQUED = 'Y', then A must contain the equilibrated matrix diag(S)*A*diag(S). If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced. A is not modified if FACT = 'F' or 'N', or if FACT = 'E' and EQUED = 'N' on exit.

On exit, if FACT = 'E' and EQUED = 'Y', A is overwritten by diag(S)*A*diag(S).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (output)
If FACT = 'F', then AF is an input argument and on entry contains the triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H, in the same storage format as A. If EQUED .ne. 'N', then AF is the factored form of the equilibrated matrix diag(S)*A*diag(S).

If FACT = 'N', then AF is an output argument and on exit returns the triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H of the original matrix A.

If FACT = 'E', then AF is an output argument and on exit returns the triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H of the equilibrated matrix A (see the description of A for the form of the equilibrated matrix).

LDAF (input)

The leading dimension of the array AF. LDAF >= max(1,N).

EQUED (input)

Specifies the form of equilibration that was done.
= 'N': No equilibration (always true if FACT = 'N').
= 'Y': Equilibration was done, i.e., A has been replaced by diag(S) * A * diag(S). EQUED is an input argument if FACT = 'F'; otherwise, it is an output argument.

S (input/output)

The scale factors for A; not accessed if EQUED = 'N'. S is an input argument if FACT = 'F'; otherwise, S is an output argument. If FACT = 'F' and EQUED = 'Y', each element of S must be positive.

B (input/output)

On entry, the N-by-NRHS righthand side matrix B. On exit, if EQUED = 'N', B is not modified; if EQUED = 'Y', B is overwritten by diag(S) * B.

LDB (input)

The leading dimension of the array B. LDB >= max(1,N).

X (output)

If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X to the original system of equations. Note that if EQUED = 'Y', A and B are modified on exit, and the solution to the equilibrated system is inv(diag(S))*X.

LDX (input)

The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)

The estimate of the reciprocal condition number of the matrix A after equilibration (if done). If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

zpotf2 - compute the Cholesky factorization of a complex Hermitian positive definite matrix A

SYNOPSIS

```
SUBROUTINE ZPOTF2(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*)
INTEGER N, LDA, INFO

SUBROUTINE ZPOTF2_64(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*)
INTEGER*8 N, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE POTF2(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO

SUBROUTINE POTF2_64(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zpotf2(char uplo, int n, doublecomplex *a, int lda, int
          *info);

void zpotf2_64(char uplo, long  n,  doublecomplex  *a,  long
          lda, long *info);
```

PURPOSE

```
zpotf2 computes the Cholesky factorization of a complex Her-
mitian positive definite matrix A.

The factorization has the form
   A = U' * U ,  if UPLO = 'U', or
   A = L  * L',  if UPLO = 'L',
where U is an upper triangular matrix and L  is  lower  tri-
angular.

This is the unblocked  version  of  the  algorithm,  calling
Level 2 BLAS.
```

ARGUMENTS

```
UPLO (input)
          Specifies whether the upper  or  lower  triangular
          part  of the Hermitian matrix A is stored.  = 'U':
          Upper triangular
          = 'L':  Lower triangular

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the Hermitian matrix A.  If UPLO =  'U',
          the leading n by n upper triangular part of A con-
          tains the upper triangular part of the  matrix  A,
          and the strictly lower triangular part of A is not
          referenced.  If UPLO = 'L', the  leading  n  by  n
          lower triangular part of A contains the lower tri-
          angular part of the matrix  A,  and  the  strictly
          upper triangular part of A is not referenced.

          On exit, if INFO = 0, the factor U or L  from  the
          Cholesky factorization A = U'*U  or A = L*L'.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).
```

```
INFO (output)
          = 0: successful exit
          < 0: if INFO = -k, the k-th argument had an  ille-
          gal value
          > 0: if INFO = k, the leading minor of order k  is
          not positive definite, and the factorization could
          not be completed.
```

Contents

NAME

```
zpotrf - compute the Cholesky  factorization  of  a  complex
Hermitian positive definite matrix A
```

SYNOPSIS

```
SUBROUTINE ZPOTRF(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*)
INTEGER N, LDA, INFO

SUBROUTINE ZPOTRF_64(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*)
INTEGER*8 N, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE POTRF(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO

SUBROUTINE POTRF_64(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zpotrf(char uplo, int n, doublecomplex *a, int lda, int
          *info);

void zpotrf_64(char uplo, long  n,  doublecomplex  *a,  long
          lda, long *info);
```

PURPOSE

```
zpotrf computes the Cholesky factorization of a complex Her-
mitian positive definite matrix A.

The factorization has the form
   A = U**H * U,  if UPLO = 'U', or
   A = L  * L**H,  if UPLO = 'L',
where U is an upper triangular matrix and L  is  lower  tri-
angular.

This is the block version of the algorithm, calling Level  3
BLAS.
```

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the Hermitian matrix A.  If UPLO =  'U',
          the leading N-by-N upper triangular part of A con-
          tains the upper triangular part of the  matrix  A,
          and the strictly lower triangular part of A is not
          referenced.  If UPLO =  'L',  the  leading  N-by-N
          lower triangular part of A contains the lower tri-
          angular part of the matrix  A,  and  the  strictly
          upper triangular part of A is not referenced.

          On exit, if INFO = 0, the factor U or L  from  the
          Cholesky factorization A = U**H*U or A = L*L**H.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

INFO (output)
```

= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-
gal value
> 0: if INFO = i, the leading minor of order i is
not positive definite, and the factorization could
not be completed.

Contents

NAME

```
zpotri - compute the inverse of a complex Hermitian positive
definite  matrix  A  using  the  Cholesky  factorization A =
U**H*U or A = L*L**H computed by CPOTRF
```

SYNOPSIS

```
SUBROUTINE ZPOTRI(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*)
INTEGER N, LDA, INFO

SUBROUTINE ZPOTRI_64(UPLO, N, A, LDA, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*)
INTEGER*8 N, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE POTRI(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO

SUBROUTINE POTRI_64(UPLO, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zpotri(char uplo, int n, doublecomplex *a, int lda, int
          *info);

void zpotri_64(char uplo, long  n,  doublecomplex  *a,  long
          lda, long *info);
```

PURPOSE

zpotri computes the inverse of a complex Hermitian positive definite matrix A using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPOTRF.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H, as computed by CPOTRF. On exit, the upper or lower triangle of the (Hermitian) inverse of A, overwriting the input factor U or L.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the (i,i) element of the factor U or L is zero, and the inverse could not be computed.

Contents

NAME

zpotrs - solve a system of linear equations A*X = B with a Hermitian positive definite matrix A using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPOTRF

SYNOPSIS

```
SUBROUTINE ZPOTRS(UPLO, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, NRHS, LDA, LDB, INFO

SUBROUTINE ZPOTRS_64(UPLO, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, NRHS, LDA, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE POTRS(UPLO, [N], [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: N, NRHS, LDA, LDB, INFO

SUBROUTINE POTRS_64(UPLO, [N], [NRHS], A, [LDA], B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zpotrs(char uplo, int n, int  nrhs,  doublecomplex  *a,
          int lda, doublecomplex *b, int ldb, int *info);

void zpotrs_64(char uplo, long n, long  nrhs,  doublecomplex
          *a,  long  lda,  doublecomplex  *b, long ldb, long
          *info);
```

PURPOSE

zpotrs solves a system of linear equations A*X = B with a Hermitian positive definite matrix A using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPOTRF.

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

A (input) The triangular factor U or  L  from  the  Cholesky
          factorization  A  =  U**H*U or A = L*L**H, as com-
          puted by CPOTRF.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

B (input/output)
          On entry, the right hand side matrix B.  On  exit,
          the solution matrix X.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
zppcon - estimate the reciprocal of the condition number (in
the  1-norm) of a complex Hermitian positive definite packed
matrix using the Cholesky factorization A = U**H*U  or  A  =
L*L**H computed by CPPTRF
```

SYNOPSIS

```
SUBROUTINE ZPPCON(UPLO, N, A, ANORM, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), WORK(*)
INTEGER N, INFO
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION WORK2(*)

SUBROUTINE ZPPCON_64(UPLO, N, A, ANORM, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), WORK(*)
INTEGER*8 N, INFO
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE PPCON(UPLO, N, A, ANORM, RCOND, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A, WORK
INTEGER :: N, INFO
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: WORK2
```

```
SUBROUTINE PPCON_64(UPLO, N, A, ANORM, RCOND, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A, WORK
INTEGER(8) :: N, INFO
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zppcon(char  uplo,  int  n,  doublecomplex  *a,  double
          anorm, double *rcond, int *info);
void zppcon_64(char uplo, long n, doublecomplex  *a,  double
          anorm, double *rcond, long *info);
```

PURPOSE

zppcon estimates the reciprocal of the condition number (in the 1-norm) of a complex Hermitian positive definite packed matrix using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPPTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

A (input) The triangular factor U or  L  from  the  Cholesky
          factorization  A  =  U**H*U  or A = L*L**H, packed
          columnwise in a linear array.  The j-th column  of
          U  or  L  is stored in the array A as follows:  if
          UPLO = 'U', A(i + (j-1)*j/2) = U(i,j) for 1<=i<=j;
          if  UPLO = 'L', A(i + (j-1)*(2n-j)/2) = L(i,j) for
          j<=i<=n.

ANORM (input)
          The 1-norm (or  infinity-norm)  of  the  Hermitian
          matrix A.
```

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zppequ - compute row and column scalings intended to equili-
brate  a  Hermitian  positive  definite  matrix  A in packed
storage and reduce its condition number (with respect to the
two-norm)
```

SYNOPSIS

```
SUBROUTINE ZPPEQU(UPLO, N, A, SCALE, SCOND, AMAX, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*)
INTEGER N, INFO
DOUBLE PRECISION SCOND, AMAX
DOUBLE PRECISION SCALE(*)

SUBROUTINE ZPPEQU_64(UPLO, N, A, SCALE, SCOND, AMAX, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*)
INTEGER*8 N, INFO
DOUBLE PRECISION SCOND, AMAX
DOUBLE PRECISION SCALE(*)
```

F95 INTERFACE

```
SUBROUTINE PPEQU(UPLO, [N], A, SCALE, SCOND, AMAX, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A
INTEGER :: N, INFO
REAL(8) :: SCOND, AMAX
REAL(8), DIMENSION(:) :: SCALE
```

```
SUBROUTINE PPEQU_64(UPLO, [N], A, SCALE, SCOND, AMAX, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A
INTEGER(8) :: N, INFO
REAL(8) :: SCOND, AMAX
REAL(8), DIMENSION(:) :: SCALE
```

C INTERFACE

```
#include <sunperf.h>

void zppequ(char  uplo,  int  n,  doublecomplex  *a,  double
          *scale, double *scond, double *amax, int *info);
void zppequ_64(char uplo, long n, doublecomplex  *a,  double
          *scale, double *scond, double *amax, long *info);
```

PURPOSE

zppequ computes row and column scalings intended to equilibrate a Hermitian positive definite matrix A in packed storage and reduce its condition number (with respect to the two-norm). S contains the scale factors, S(i)=1/sqrt(A(i,i)), chosen so that the scaled matrix B with elements B(i,j)=S(i)*A(i,j)*S(j) has ones on the diagonal. This choice of S puts the condition number of B within a factor N of the smallest possible condition number over all possible diagonal scalings.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input) The upper or lower triangle of the Hermitian matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n.

SCALE (output)
If INFO = 0, SCALE contains the scale factors for A.

```
SCOND (output)
          If INFO = 0, SCALE contains the ratio of the smal-
          lest  SCALE(i)  to the largest SCALE(i).  If SCOND
          >= 0.1 and AMAX  is  neither  too  large  nor  too
          small, it is not worth scaling by SCALE.

AMAX (output)
          Absolute value of largest matrix element.  If AMAX
          is  very close to overflow or very close to under-
          flow, the matrix should be scaled.
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, the i-th  diagonal  element  is
          nonpositive.
```

Contents

NAME

```
zpprfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is Hermitian positive
definite and packed, and provides error bounds and  backward
error estimates for the solution
```

SYNOPSIS

```
SUBROUTINE ZPPRFS(UPLO, N, NRHS, A, AF, B, LDB, X, LDX, FERR, BERR,
      WORK, WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZPPRFS_64(UPLO, N, NRHS, A, AF, B, LDB, X, LDX, FERR,
      BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE PPRFS(UPLO, N, [NRHS], A, AF, B, [LDB], X, [LDX], FERR,
       BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A, AF, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
INTEGER :: N, NRHS, LDB, LDX, INFO
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

```
SUBROUTINE PPRFS_64(UPLO, N, [NRHS], A, AF, B, [LDB], X, [LDX], FERR,
       BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A, AF, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zpprfs(char uplo, int n, int  nrhs,  doublecomplex  *a,
          doublecomplex  *af,  doublecomplex  *b,  int  ldb,
          doublecomplex *x, int ldx,  double  *ferr,  double
          *berr, int *info);

void zpprfs_64(char uplo, long n, long  nrhs,  doublecomplex
          *a, doublecomplex *af, doublecomplex *b, long ldb,
          doublecomplex *x, long ldx, double  *ferr,  double
          *berr, long *info);
```

PURPOSE

zpprfs improves the computed solution to a system of linear equations when the coefficient matrix is Hermitian positive definite and packed, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input) The upper  or  lower  triangle  of  the  Hermitian
          matrix  A,  packed  columnwise  in a linear array.
          The j-th column of A is stored in the array  A  as
          follows:  if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j)
          for 1<=i<=j; if UPLO = 'L', A(i +  (j-1)*(2n-j)/2)
```

= A(i,j) for j<=i<=n.

AF (input)
The triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H, as computed by SPPTRF/CPPTRF, packed columnwise in a linear array in the same format as A (see A).

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by CPPTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zppsv - compute the solution to a complex system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE ZPPSV(UPLO, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO

SUBROUTINE ZPPSV_64(UPLO, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE PPSV(UPLO, N, [NRHS], A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO

SUBROUTINE PPSV_64(UPLO, N, [NRHS], A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zppsv(char uplo, int n,  int  nrhs,  doublecomplex  *a,
          doublecomplex *b, int ldb, int *info);

void zppsv_64(char uplo, long n,  long  nrhs,  doublecomplex
          *a, doublecomplex *b, long ldb, long *info);
```

PURPOSE

```
zppsv computes the solution to a complex  system  of  linear
equations
   A * X = B, where A is an N-by-N Hermitian positive defin-
ite matrix stored in packed format and X and B are N-by-NRHS
matrices.

The Cholesky decomposition is used to factor A as
   A = U**H* U,  if UPLO = 'U', or
   A = L * L**H,  if UPLO = 'L',
where U is an upper triangular matrix and L is a lower  tri-
angular  matrix.   The  factored  form  of A is then used to
solve the system of equations A * X = B.
```

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

A (input/output)
          On entry, the upper or lower triangle of the  Her-
          mitian  matrix  A,  packed  columnwise in a linear
          array.  The j-th column of  A  is  stored  in  the
          array  A  as  follows:   if  UPLO = 'U', A(i + (j-
          1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i +
          (j-1)*(2n-j)/2)  =  A(i,j) for j<=i<=n.  See below
          for further details.
```

```
          On exit, if INFO = 0, the factor U or L  from  the
          Cholesky  factorization  A = U**H*U or A = L*L**H,
          in the same storage format as A.

B (input/output)
          On entry, the N-by-NRHS right hand side matrix  B.
          On  exit,  if  INFO  =  0,  the N-by-NRHS solution
          matrix X.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, the leading minor of order i of
          A  is  not positive definite, so the factorization
          could not be completed, and the solution  has  not
          been computed.
```

FURTHER DETAILS

```
The packed storage scheme is illustrated  by  the  following
example when N = 4, UPLO = 'U':

Two-dimensional storage of the Hermitian matrix A:

   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = conjg(aji))
               a44

Packed storage of the upper triangle of A:

A = [ a11, a12, a22, a13, a23, a33, a14, a24, a34, a44 ]
```

Contents

NAME

```
zppsvx - use the Cholesky factorization A = U**H*U  or  A  =
L*L**H to compute the solution to a complex system of linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE ZPPSVX(FACT, UPLO, N, NRHS, A, AF, EQUED, S, B, LDB,
      X, LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO, EQUED
DOUBLE COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
DOUBLE PRECISION RCOND
DOUBLE PRECISION S(*), FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZPPSVX_64(FACT, UPLO, N, NRHS, A, AF, EQUED, S, B,
      LDB, X, LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO, EQUED
DOUBLE COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
DOUBLE PRECISION RCOND
DOUBLE PRECISION S(*), FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE PPSVX(FACT, UPLO, [N], [NRHS], A, AF, EQUED, S, B,
       [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
COMPLEX(8), DIMENSION(:) :: A, AF, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
```

```
INTEGER :: N, NRHS, LDB, LDX, INFO
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: S, FERR, BERR, WORK2

SUBROUTINE PPSVX_64(FACT, UPLO, [N], [NRHS], A, AF, EQUED, S, B,
       [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO, EQUED
COMPLEX(8), DIMENSION(:) :: A, AF, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: S, FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>
void zppsvx(char fact, char uplo, int  n,  int  nrhs,  doub-
          lecomplex  *a, doublecomplex *af, char equed, dou-
          ble *s, doublecomplex *b, int  ldb,  doublecomplex
          *x,  int  ldx, double *rcond, double *ferr, double
          *berr, int *info);

void zppsvx_64(char fact, char  uplo,  long  n,  long  nrhs,
          doublecomplex  *a,  doublecomplex *af, char equed,
          double *s, doublecomplex *b, long ldb,  doublecom-
          plex  *x,  long  ldx, double *rcond, double *ferr,
          double *berr, long *info);
```

PURPOSE

```
zppsvx uses the Cholesky factorization A =  U**H*U  or  A  =
L*L**H to compute the solution to a complex system of linear
equations
   A * X = B, where A is an N-by-N Hermitian positive defin-
ite matrix stored in packed format and X and B are N-by-NRHS
matrices.

Error bounds on the solution and a  condition  estimate  are
also provided.

The following steps are performed:

1. If FACT = 'E',  real  scaling  factors  are  computed  to
equilibrate
   the system:
      diag(S) * A * diag(S) * inv(diag(S)) * X = diag(S) * B
   Whether or not the system will be equilibrated depends on
the
   scaling of the matrix A, but if equilibration is used,  A
```

is
   overwritten by diag(S)*A*diag(S) and B by diag(S)*B.

2. If FACT = 'N' or 'E', the Cholesky decomposition is used to
   factor the matrix A (after equilibration if FACT = 'E') as
      A = U'* U , if UPLO = 'U', or
      A = L * L', if UPLO = 'L',
   where U is an upper triangular matrix, L is a lower triangular
   matrix, and ' indicates conjugate transpose.

3. If the leading i-by-i principal minor is not positive definite,
   then the routine returns with INFO = i. Otherwise, the factored
   form of A is used to estimate the condition number of the matrix
   A. If the reciprocal of the condition number is less than machine
   precision, INFO = N+1 is returned as a warning, but the routine
   still goes on to solve for X and compute error bounds as
   described below.

4. The system of equations is solved for X using the factored form
   of A.

5. Iterative refinement is applied to improve the computed solution
   matrix and calculate error bounds and backward error estimates
   for it.

6. If equilibration was used, the matrix X is premultiplied by
   diag(S) so that it solves the original system before
   equilibration.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored. = 'F': On entry, AF contains the fac-

```
            tored form of A.  If EQUED = 'Y', the matrix A has
            been equilibrated with scaling factors given by S.
            A and AF will not be modified.  = 'N':  The matrix
            A will be copied to AF and factored.
            = 'E':  The  matrix  A  will  be  equilibrated  if
            necessary, then copied to AF and factored.

UPLO (input)
            = 'U':  Upper triangle of A is stored;
            = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
            the matrix A.  N >= 0.

NRHS (input)
            The number of right hand sides, i.e.,  the  number
            of columns of the matrices B and X.  NRHS >= 0.
A (input/output)
            On entry, the upper or lower triangle of the  Her-
            mitian  matrix  A,  packed  columnwise in a linear
            array, except if FACT = 'F' and EQUED = 'Y',  then
            A    must    contain    the    equilibrated    matrix
            diag(S)*A*diag(S).  The j-th column of A is stored
            in  the  array A as follows:  if UPLO = 'U', A(i +
            (j-1)*j/2) = A(i,j) for 1<=i<=j; if  UPLO  =  'L',
            A(i  +  (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n.  See
            below for further details.  A is not  modified  if
            FACT  =  'F'  or 'N', or if FACT = 'E' and EQUED =
            'N' on exit.

            On exit, if FACT = 'E'  and  EQUED  =  'Y',  A  is
            overwritten by diag(S)*A*diag(S).

AF (input/output)
            If FACT = 'F', then AF is an input argument and on
            entry  contains  the triangular factor U or L from
            the Cholesky factorization  A  =  U**H*U  or  A  =
            L*L**H, in the same storage format as A.  If EQUED
            .ne. 'N', then AF is  the  factored  form  of  the
            equilibrated matrix A.

            If FACT = 'N', then AF is an output  argument  and
            on  exit returns the triangular factor U or L from
            the Cholesky factorization  A  =  U**H*U  or  A  =
            L*L**H of the original matrix A.

            If FACT = 'E', then AF is an output  argument  and
            on  exit returns the triangular factor U or L from
            the Cholesky factorization  A  =  U**H*U  or  A  =
            L*L**H  of  the  equilibrated  matrix  A  (see the
```

description of A for the form of the equilibrated matrix).

EQUED (input)

Specifies the form of equilibration that was done.
= 'N': No equilibration (always true if FACT = 'N').
= 'Y': Equilibration was done, i.e., A has been replaced by diag(S) * A * diag(S). EQUED is an input argument if FACT = 'F'; otherwise, it is an output argument.

S (input/output)

The scale factors for A; not accessed if EQUED = 'N'. S is an input argument if FACT = 'F'; otherwise, S is an output argument. If FACT = 'F' and EQUED = 'Y', each element of S must be positive.

B (input/output)

On entry, the N-by-NRHS right hand side matrix B. On exit, if EQUED = 'N', B is not modified; if EQUED = 'Y', B is overwritten by diag(S) * B.

LDB (input)

The leading dimension of the array B. LDB >= max(1,N).

X (output)

If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X to the original system of equations. Note that if EQUED = 'Y', A and B are modified on exit, and the solution to the equilibrated system is inv(diag(S))*X.

LDX (input)

The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)

The estimate of the reciprocal condition number of the matrix A after equilibration (if done). If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)

The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

FURTHER DETAILS

The packed storage scheme is illustrated by the following example when N = 4, UPLO = 'U':

Two-dimensional storage of the Hermitian matrix A:

```
   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = conjg(aji))
               a44
```

Packed storage of the upper triangle of A:

```
A = [ a11, a12, a22, a13, a23, a33, a14, a24, a34, a44 ]
```

Contents

NAME

zpptrf - compute the Cholesky factorization of a complex Hermitian positive definite matrix A stored in packed format

SYNOPSIS

```
SUBROUTINE ZPPTRF(UPLO, N, A, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*)
INTEGER N, INFO

SUBROUTINE ZPPTRF_64(UPLO, N, A, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*)
INTEGER*8 N, INFO
```

F95 INTERFACE

```
SUBROUTINE PPTRF(UPLO, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A
INTEGER :: N, INFO

SUBROUTINE PPTRF_64(UPLO, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A
INTEGER(8) :: N, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zpptrf(char uplo, int n, doublecomplex *a, int *info);

void zpptrf_64(char uplo, long  n,  doublecomplex  *a,  long
          *info);
```

PURPOSE

zpptrf computes the Cholesky factorization of a complex Hermitian positive definite matrix A stored in packed format.

The factorization has the form

```
   A = U**H * U,  if UPLO = 'U', or
   A = L  * L**H,  if UPLO = 'L',
```

where U is an upper triangular matrix and L is lower triangular.

ARGUMENTS

UPLO (input)
  = 'U': Upper triangle of A is stored;
  = 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
  On entry, the upper or lower triangle of the Hermitian matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n. See below for further details.

  On exit, if INFO = 0, the triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H, in the same storage format as A.

INFO (output)
  = 0: successful exit
  < 0: if INFO = -i, the i-th argument had an illegal value
  > 0: if INFO = i, the leading minor of order i is not positive definite, and the factorization could not be completed.

FURTHER DETAILS

```
The packed storage scheme is illustrated  by  the  following
example when N = 4, UPLO = 'U':

Two-dimensional storage of the Hermitian matrix A:

   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = conjg(aji))
               a44

Packed storage of the upper triangle of A:

A = [ a11, a12, a22, a13, a23, a33, a14, a24, a34, a44 ]
```

Contents

NAME

```
zpptri - compute the inverse of a complex Hermitian positive
definite  matrix  A  using  the  Cholesky  factorization A =
U**H*U or A = L*L**H computed by CPPTRF
```

SYNOPSIS

```
SUBROUTINE ZPPTRI(UPLO, N, A, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*)
INTEGER N, INFO

SUBROUTINE ZPPTRI_64(UPLO, N, A, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*)
INTEGER*8 N, INFO
```

F95 INTERFACE

```
SUBROUTINE PPTRI(UPLO, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A
INTEGER :: N, INFO

SUBROUTINE PPTRI_64(UPLO, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A
INTEGER(8) :: N, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zpptri(char uplo, int n, doublecomplex *a, int *info);

void zpptri_64(char uplo, long  n,  doublecomplex  *a,  long
          *info);
```

PURPOSE

```
zpptri computes the inverse of a complex Hermitian  positive
definite  matrix  A  using  the  Cholesky  factorization A =
U**H*U or A = L*L**H computed by CPPTRF.
```

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangular factor is stored in A;
          = 'L':  Lower triangular factor is stored in A.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the triangular factor U or  L  from  the
          Cholesky  factorization  A = U**H*U or A = L*L**H,
          packed columnwise as a  linear  array.   The  j-th
          column  of U or L is stored in the array A as fol-
          lows:  if UPLO = 'U', A(i +  (j-1)*j/2)  =  U(i,j)
          for  1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2)
          = L(i,j) for j<=i<=n.

          On exit, the upper or lower triangle of the  (Her-
          mitian) inverse of A, overwriting the input factor
          U or L.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, the (i,i) element of the factor
          U  or L is zero, and the inverse could not be com-
          puted.
```

Contents

NAME

```
zpptrs - solve a system of linear equations A*X = B  with  a
Hermitian positive definite matrix A in packed storage using
the Cholesky factorization A = U**H*U or A = L*L**H computed
by CPPTRF
```

SYNOPSIS

```
SUBROUTINE ZPPTRS(UPLO, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO

SUBROUTINE ZPPTRS_64(UPLO, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE PPTRS(UPLO, N, [NRHS], A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO

SUBROUTINE PPTRS_64(UPLO, N, [NRHS], A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A
COMPLEX(8), DIMENSION(:,:) :: B
```

```
INTEGER(8) :: N, NRHS, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zpptrs(char uplo, int n, int  nrhs,  doublecomplex  *a,
          doublecomplex *b, int ldb, int *info);

void zpptrs_64(char uplo, long n, long  nrhs,  doublecomplex
          *a, doublecomplex *b, long ldb, long *info);
```

PURPOSE

zpptrs solves a system of linear equations A*X = B with a Hermitian positive definite matrix A in packed storage using the Cholesky factorization A = U**H*U or A = L*L**H computed by CPPTRF.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The triangular factor U or L from the Cholesky factorization A = U**H*U or A = L*L**H, packed columnwise in a linear array. The j-th column of U or L is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = U(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = L(i,j) for j<=i<=n.

B (input/output)
On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B.   LDB >= max(1,N).

INFO (output)

```
= 0:  successful exit
< 0:  if INFO = -i, the i-th argument had an ille-
gal value
```

Contents

NAME

```
zptcon - compute the reciprocal of the condition number  (in
the  1-norm) of a complex Hermitian positive definite tridi-
agonal matrix using the factorization A = L*D*L**H  or  A  =
U**H*D*U computed by CPTTRF
```

SYNOPSIS

```
SUBROUTINE ZPTCON(N, DIAG, OFFD, ANORM, RCOND, WORK, INFO)

DOUBLE COMPLEX OFFD(*)
INTEGER N, INFO
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION DIAG(*), WORK(*)

SUBROUTINE ZPTCON_64(N, DIAG, OFFD, ANORM, RCOND, WORK, INFO)

DOUBLE COMPLEX OFFD(*)
INTEGER*8 N, INFO
DOUBLE PRECISION ANORM, RCOND
DOUBLE PRECISION DIAG(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PTCON([N], DIAG, OFFD, ANORM, RCOND, [WORK], [INFO])

COMPLEX(8), DIMENSION(:) :: OFFD
INTEGER :: N, INFO
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: DIAG, WORK

SUBROUTINE PTCON_64([N], DIAG, OFFD, ANORM, RCOND, [WORK], [INFO])
```

```
COMPLEX(8), DIMENSION(:) :: OFFD
INTEGER(8) :: N, INFO
REAL(8) :: ANORM, RCOND
REAL(8), DIMENSION(:) :: DIAG, WORK
```

C INTERFACE

```
#include <sunperf.h>

void zptcon(int n, double *diag, doublecomplex *offd, double
          anorm, double *rcond, int *info);

void zptcon_64(long n, double  *diag,  doublecomplex  *offd,
          double anorm, double *rcond, long *info);
```

PURPOSE

zptcon computes the reciprocal of the condition number (in the 1-norm) of a complex Hermitian positive definite tridiagonal matrix using the factorization A = L*D*L**H or A = U**H*D*U computed by CPTTRF.

Norm(inv(A)) is computed by a direct method, and the reciprocal of the condition number is computed as
RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

DIAG (input)
The n diagonal elements of the diagonal matrix DIAG from the factorization of A, as computed by CPTTRF.

OFFD (input)
The (n-1) off-diagonal elements of the unit bidiagonal factor U or L from the factorization of A, as computed by CPTTRF.

ANORM (input)
The 1-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is the 1-norm of inv(A) computed in this routine.

WORK (workspace)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The method used is described in Nicholas J. Higham, "Efficient Algorithms for Computing the Condition Number of a Tridiagonal Matrix", SIAM J. Sci. Stat. Comput., Vol. 7, No. 1, January 1986.

Contents

NAME

```
zpteqr - compute all eigenvalues and, optionally,  eigenvec-
tors  of a symmetric positive definite tridiagonal matrix by
first factoring the matrix using  SPTTRF  and  then  calling
CBDSQR to compute the singular values of the bidiagonal fac-
tor
```

SYNOPSIS

```
SUBROUTINE ZPTEQR(COMPZ, N, D, E, Z, LDZ, WORK, INFO)

CHARACTER * 1 COMPZ
DOUBLE COMPLEX Z(LDZ,*)
INTEGER N, LDZ, INFO
DOUBLE PRECISION D(*), E(*), WORK(*)

SUBROUTINE ZPTEQR_64(COMPZ, N, D, E, Z, LDZ, WORK, INFO)

CHARACTER * 1 COMPZ
DOUBLE COMPLEX Z(LDZ,*)
INTEGER*8 N, LDZ, INFO
DOUBLE PRECISION D(*), E(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE PTEQR(COMPZ, [N], D, E, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
COMPLEX(8), DIMENSION(:,:) :: Z
INTEGER :: N, LDZ, INFO
REAL(8), DIMENSION(:) :: D, E, WORK

SUBROUTINE PTEQR_64(COMPZ, [N], D, E, Z, [LDZ], [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: COMPZ
COMPLEX(8), DIMENSION(:,:) :: Z
INTEGER(8) :: N, LDZ, INFO
REAL(8), DIMENSION(:) :: D, E, WORK
```

C INTERFACE

```
#include <sunperf.h>

void zpteqr(char compz, int n, double *d, double  *e,  doub-
          lecomplex *z, int ldz, int *info);

void zpteqr_64(char compz, long n,  double  *d,  double  *e,
          doublecomplex *z, long ldz, long *info);
```

PURPOSE

zpteqr computes all eigenvalues and, optionally, eigenvectors of a symmetric positive definite tridiagonal matrix by first factoring the matrix using SPTTRF and then calling CBDSQR to compute the singular values of the bidiagonal factor.

This routine computes the eigenvalues of the positive definite tridiagonal matrix to high relative accuracy. This means that if the eigenvalues range over many orders of magnitude in size, then the small eigenvalues and corresponding eigenvectors will be computed more accurately than, for example, with the standard QR method.

The eigenvectors of a full or band positive definite Hermitian matrix can also be found if CHETRD, CHPTRD, or CHBTRD has been used to reduce this matrix to tridiagonal form. (The reduction to tridiagonal form, however, may preclude the possibility of obtaining high relative accuracy in the small eigenvalues of the original matrix, if these eigenvalues range over many orders of magnitude.)

ARGUMENTS

COMPZ (input)
= 'N': Compute eigenvalues only.
= 'V': Compute eigenvectors of original Hermitian matrix also. Array Z contains the unitary matrix used to reduce the original matrix to tridiagonal form. = 'I': Compute eigenvectors of tridiagonal matrix also.

N (input) The order of the matrix. N >= 0.

D (input/output)
On entry, the n diagonal elements of the tridiagonal matrix. On normal exit, D contains the eigenvalues, in descending order.

E (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix. On exit, E has been destroyed.

Z (input) On entry, if COMPZ = 'V', the unitary matrix used in the reduction to tridiagonal form. On exit, if COMPZ = 'V', the orthonormal eigenvectors of the original Hermitian matrix; if COMPZ = 'I', the orthonormal eigenvectors of the tridiagonal matrix. If INFO > 0 on exit, Z contains the eigenvectors associated with only the stored eigenvalues. If COMPZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if COMPZ = 'V' or 'I', LDZ >= max(1,N).

WORK (workspace)
dimension(4*N)

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = i, and i is: <= N the Cholesky factorization of the matrix could not be performed because the i-th principal minor was not positive definite. > N the SVD algorithm failed to converge; if INFO = N+i, i off-diagonal elements of the bidiagonal factor did not converge to zero.

Contents

NAME

zptrfs - improve the computed solution to a system of linear equations when the coefficient matrix is Hermitian positive definite and tridiagonal, and provides error bounds and backward error estimates for the solution

SYNOPSIS

```
SUBROUTINE ZPTRFS(UPLO, N, NRHS, DIAG, OFFD, DIAGF, OFFDF, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE  COMPLEX  OFFD(*),  OFFDF(*),   B(LDB,*),   X(LDX,*),
WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
DOUBLE  PRECISION  DIAG(*),  DIAGF(*),   FERR(*),   BERR(*),
WORK2(*)

SUBROUTINE ZPTRFS_64(UPLO, N, NRHS, DIAG, OFFD, DIAGF, OFFDF, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE  COMPLEX  OFFD(*),  OFFDF(*),   B(LDB,*),   X(LDX,*),
WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
DOUBLE  PRECISION  DIAG(*),  DIAGF(*),   FERR(*),   BERR(*),
WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE PTRFS(UPLO, [N], [NRHS], DIAG, OFFD, DIAGF, OFFDF, B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX(8), DIMENSION(:) :: OFFD, OFFDF, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
INTEGER :: N, NRHS, LDB, LDX, INFO
REAL(8), DIMENSION(:) :: DIAG, DIAGF, FERR, BERR, WORK2

SUBROUTINE PTRFS_64(UPLO, [N], [NRHS], DIAG, OFFD, DIAGF, OFFDF, B,
       [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: OFFD, OFFDF, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
REAL(8), DIMENSION(:) :: DIAG, DIAGF, FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zptrfs(char uplo, int n, int nrhs, double *diag,  doub-
          lecomplex   *offd,  double  *diagf,  doublecomplex
          *offdf, doublecomplex *b, int  ldb,  doublecomplex
          *x,  int  ldx,  double  *ferr,  double  *berr, int
          *info);

void zptrfs_64(char uplo, long n, long nrhs,  double  *diag,
          doublecomplex  *offd, double *diagf, doublecomplex
          *offdf, doublecomplex *b, long ldb,  doublecomplex
          *x,  long  ldx,  double  *ferr, double *berr, long
          *info);
```

PURPOSE

zptrfs improves the computed solution to a system of linear equations when the coefficient matrix is Hermitian positive definite and tridiagonal, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

UPLO (input)
Specifies whether the superdiagonal or the subdiagonal of the tridiagonal matrix A is stored and the form of the factorization:
= 'U': OFFD is the superdiagonal of A, and A = U**H*DIAG*U;
= 'L': OFFD is the subdiagonal of A, and A = L*DIAG*L**H. (The two forms are equivalent if A is real.)

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

DIAG (input)
The n real diagonal elements of the tridiagonal matrix A.

OFFD (input)
The (n-1) off-diagonal elements of the tridiagonal matrix A (see UPLO).

DIAGF (input)
The n diagonal elements of the diagonal matrix DIAG from the factorization computed by CPTTRF.

OFFDF (input)
The (n-1) off-diagonal elements of the unit bidiagonal factor U or L from the factorization computed by CPTTRF (see UPLO).

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by CPTTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j).

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

```
WORK (workspace)
          dimension(N)

WORK2 (workspace)
          dimension(N)
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
zptsv - compute the solution to a complex system  of  linear
equations  A*X  = B, where A is an N-by-N Hermitian positive
definite tridiagonal matrix,  and  X  and  B  are  N-by-NRHS
matrices.
```

SYNOPSIS

```
SUBROUTINE ZPTSV(N, NRHS, DIAG, SUB, B, LDB, INFO)

DOUBLE COMPLEX SUB(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO
DOUBLE PRECISION DIAG(*)

SUBROUTINE ZPTSV_64(N, NRHS, DIAG, SUB, B, LDB, INFO)

DOUBLE COMPLEX SUB(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
DOUBLE PRECISION DIAG(*)
```

F95 INTERFACE

```
SUBROUTINE PTSV([N], [NRHS], DIAG, SUB, B, [LDB], [INFO])

COMPLEX(8), DIMENSION(:) :: SUB
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO
REAL(8), DIMENSION(:) :: DIAG

SUBROUTINE PTSV_64([N], [NRHS], DIAG, SUB, B, [LDB], [INFO])

COMPLEX(8), DIMENSION(:) :: SUB
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
```

```
REAL(8), DIMENSION(:) :: DIAG
```

C INTERFACE

```
#include <sunperf.h>

void zptsv(int n,  int  nrhs,  double  *diag,  doublecomplex
          *sub, doublecomplex *b, int ldb, int *info);

void zptsv_64(long n, long nrhs, double *diag, doublecomplex
          *sub, doublecomplex *b, long ldb, long *info);
```

PURPOSE

zptsv computes the solution to a complex system of linear equations A*X = B, where A is an N-by-N Hermitian positive definite tridiagonal matrix, and X and B are N-by-NRHS matrices.

A is factored as A = L*D*L**H, and the factored form of A is then used to solve the system of equations.

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

DIAG (input/output)
On entry, the n diagonal elements of the tridiagonal matrix A. On exit, the n diagonal elements of the diagonal matrix DIAG from the factorization A = L*DIAG*L**H.

SUB (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix A. On exit, the (n-1) subdiagonal elements of the unit bidiagonal factor L from the L*DIAG*L**H factorization of A. SUB can also be regarded as the superdiagonal of the unit bidiagonal factor U from the U**H*DIAG*U factorization of A.

B (input/output)
On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution

```
            matrix X.

LDB (input)
            The leading dimension of  the  array  B.   LDB  >=
            max(1,N).

INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument had an ille-
            gal value
            > 0:  if INFO = i, the leading minor of order i is
            not  positive  definite,  and the solution has not
            been computed.  The  factorization  has  not  been
            completed unless i = N.
```

Contents

NAME

```
zptsvx - use the factorization A = L*D*L**H to  compute  the
solution  to  a  complex system of linear equations A*X = B,
where A is an N-by-N Hermitian positive definite tridiagonal
matrix and X and B are N-by-NRHS matrices
```

SYNOPSIS

```
SUBROUTINE ZPTSVX(FACT, N, NRHS, DIAG, SUB, DIAGF, SUBF, B, LDB, X,
      LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT
DOUBLE COMPLEX SUB(*), SUBF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
DOUBLE PRECISION RCOND
DOUBLE  PRECISION  DIAG(*),  DIAGF(*),   FERR(*),   BERR(*),
WORK2(*)

SUBROUTINE ZPTSVX_64(FACT, N, NRHS, DIAG, SUB, DIAGF, SUBF, B, LDB,
      X, LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT
DOUBLE COMPLEX SUB(*), SUBF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
DOUBLE PRECISION RCOND
DOUBLE  PRECISION  DIAG(*),  DIAGF(*),   FERR(*),   BERR(*),
WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE PTSVX(FACT, [N], [NRHS], DIAG, SUB, DIAGF, SUBF, B, [LDB],
       X, [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT
```

```
COMPLEX(8), DIMENSION(:) :: SUB, SUBF, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
INTEGER :: N, NRHS, LDB, LDX, INFO
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: DIAG, DIAGF, FERR, BERR, WORK2

SUBROUTINE PTSVX_64(FACT, [N], [NRHS], DIAG, SUB, DIAGF, SUBF, B,
       [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT
COMPLEX(8), DIMENSION(:) :: SUB, SUBF, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: DIAG, DIAGF, FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zptsvx(char fact, int n, int nrhs, double *diag,  doub-
          lecomplex   *sub,   double  *diagf,  doublecomplex
          *subf, doublecomplex *b,  int  ldb,  doublecomplex
          *x,  int  ldx, double *rcond, double *ferr, double
          *berr, int *info);

void zptsvx_64(char fact, long n, long nrhs,  double  *diag,
          doublecomplex  *sub,  double *diagf, doublecomplex
          *subf, doublecomplex *b, long  ldb,  doublecomplex
          *x,  long ldx, double *rcond, double *ferr, double
          *berr, long *info);
```

PURPOSE

zptsvx uses the factorization A = L*D*L**H to compute the solution to a complex system of linear equations A*X = B, where A is an N-by-N Hermitian positive definite tridiagonal matrix and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'N', the matrix A is factored as A = L*D*L**H, where L
   is a unit lower bidiagonal matrix and D is diagonal. The factorization can also be regarded as having the form
   A = U**H*D*U.

2. If the leading i-by-i principal minor is not positive

definite,
then the routine returns with INFO = i. Otherwise, the factored
form of A is used to estimate the condition number of the matrix
A. If the reciprocal of the condition number is less than machine
precision, INFO = N+1 is returned as a warning, but the routine
still goes on to solve for X and compute error bounds as
described below.

3. The system of equations is solved for X using the factored form
of A.

4. Iterative refinement is applied to improve the computed solution
matrix and calculate error bounds and backward error estimates
for it.

ARGUMENTS

FACT (input)
Specifies whether or not the factored form of the matrix A is supplied on entry. = 'F': On entry, DIAGF and SUBF contain the factored form of A. DIAG, SUB, DIAGF, and SUBF will not be modified. = 'N': The matrix A will be copied to DIAGF and SUBF and factored.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

DIAG (input)
The n diagonal elements of the tridiagonal matrix A.

SUB (input)
The (n-1) subdiagonal elements of the tridiagonal matrix A.

DIAGF (input/output)
If FACT = 'F', then DIAGF is an input argument and

on entry contains the n diagonal elements of the diagonal matrix DIAG from the L*DIAG*L**H factorization of A. If FACT = 'N', then DIAGF is an output argument and on exit contains the n diagonal elements of the diagonal matrix DIAG from the L*DIAG*L**H factorization of A.

SUBF (input/output)
If FACT = 'F', then SUBF is an input argument and on entry contains the (n-1) subdiagonal elements of the unit bidiagonal factor L from the L*DIAG*L**H factorization of A. If FACT = 'N', then SUBF is an output argument and on exit contains the (n-1) subdiagonal elements of the unit bidiagonal factor L from the L*DIAG*L**H factorization of A.

B (input) On entry, the N-by-NRHS right hand side matrix B. Unchanged on exit.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The reciprocal condition number of the matrix A. If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
The forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j).

BERR (output)
The componentwise relative backward error of each

solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: the leading minor of order i of A is not positive definite, so the factorization could not be completed, and the solution has not been computed. RCOND = 0 is returned. = N+1: U is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
zpttrf - compute the L*D*L' factorization of a complex  Her-
mitian positive definite tridiagonal matrix A
```

SYNOPSIS

```
SUBROUTINE ZPTTRF(N, DIAG, OFFD, INFO)

DOUBLE COMPLEX OFFD(*)
INTEGER N, INFO
DOUBLE PRECISION DIAG(*)

SUBROUTINE ZPTTRF_64(N, DIAG, OFFD, INFO)

DOUBLE COMPLEX OFFD(*)
INTEGER*8 N, INFO
DOUBLE PRECISION DIAG(*)
```

F95 INTERFACE

```
SUBROUTINE PTTRF([N], DIAG, OFFD, [INFO])

COMPLEX(8), DIMENSION(:) :: OFFD
INTEGER :: N, INFO
REAL(8), DIMENSION(:) :: DIAG

SUBROUTINE PTTRF_64([N], DIAG, OFFD, [INFO])

COMPLEX(8), DIMENSION(:) :: OFFD
INTEGER(8) :: N, INFO
REAL(8), DIMENSION(:) :: DIAG
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zpttrf(int n, double *diag,  doublecomplex  *offd,  int
          *info);

void zpttrf_64(long n, double  *diag,  doublecomplex  *offd,
          long *info);
```

PURPOSE

zpttrf computes the L*D*L' factorization of a complex Hermitian positive definite tridiagonal matrix A. The factorization may also be regarded as having the form A = U'*D*U.

ARGUMENTS

N (input) The order of the matrix A. N >= 0.

DIAG (input/output)
On entry, the n diagonal elements of the tridiagonal matrix A. On exit, the n diagonal elements of the diagonal matrix DIAG from the L*DIAG*L' factorization of A.

OFFD (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix A. On exit, the (n-1) subdiagonal elements of the unit bidiagonal factor L from the L*DIAG*L' factorization of A. OFFD can also be regarded as the superdiagonal of the unit bidiagonal factor U from the U'*DIAG*U factorization of A.

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, the leading minor of order k is not positive definite; if k < N, the factorization could not be completed, while if k = N, the factorization was completed, but DIAG(N) = 0.

Contents

NAME

```
zpttrs - solve a tridiagonal system of the form  A * X  =  B
using the factorization A = U'*D*U or A = L*D*L' computed by
CPTTRF
```

SYNOPSIS

```
SUBROUTINE ZPTTRS(UPLO, N, NRHS, DIAG, OFFD, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX OFFD(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO
DOUBLE PRECISION DIAG(*)

SUBROUTINE ZPTTRS_64(UPLO, N, NRHS, DIAG, OFFD, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX OFFD(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
DOUBLE PRECISION DIAG(*)
```

F95 INTERFACE

```
SUBROUTINE PTTRS(UPLO, [N], [NRHS], DIAG, OFFD, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: OFFD
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO
REAL(8), DIMENSION(:) :: DIAG

SUBROUTINE PTTRS_64(UPLO, [N], [NRHS], DIAG, OFFD, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX(8), DIMENSION(:) :: OFFD
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
REAL(8), DIMENSION(:) :: DIAG
```

C INTERFACE

```
#include <sunperf.h>

void zpttrs(char uplo, int n, int nrhs, double *diag,  doub-
          lecomplex  *offd,  doublecomplex  *b, int ldb, int
          *info);

void zpttrs_64(char uplo, long n, long nrhs,  double  *diag,
          doublecomplex  *offd,  doublecomplex *b, long ldb,
          long *info);
```

PURPOSE

zpttrs solves a tridiagonal system of the form
A * X = B using the factorization A = U'*D*U or A = L*D*L' computed by CPTTRF. D is a diagonal matrix specified in the vector D, U (or L) is a unit bidiagonal matrix whose superdiagonal (subdiagonal) is specified in the vector E, and X and B are N by NRHS matrices.

ARGUMENTS

UPLO (input)
Specifies the form of the factorization and whether the vector OFFD is the superdiagonal of the upper bidiagonal factor U or the subdiagonal of the lower bidiagonal factor L. = 'U': A = U'*DIAG*U, OFFD is the superdiagonal of U
= 'L': A = L*DIAG*L', OFFD is the subdiagonal of L

N (input) The order of the tridiagonal matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

DIAG (input)
The n diagonal elements of the diagonal matrix DIAG from the factorization A = U'*DIAG*U or A = L*DIAG*L'.

OFFD (input/output)

If UPLO = 'U', the (n-1) superdiagonal elements of the unit bidiagonal factor U from the factorization A = U'*DIAG*U. If UPLO = 'L', the (n-1) subdiagonal elements of the unit bidiagonal factor L from the factorization A = L*DIAG*L'.

B (input/output)

On entry, the right hand side vectors B for the system of linear equations. On exit, the solution vectors, X.

LDB (input)

The leading dimension of the array B. LDB >= max(1,N).

INFO (output)

= 0: successful exit

< 0: if INFO = -k, the k-th argument had an illegal value

Contents

NAME

```
zptts2 - solve a tridiagonal system of the form  A * X  =  B
using the factorization A = U'*D*U or A = L*D*L' computed by
CPTTRF
```

SYNOPSIS

```
SUBROUTINE ZPTTS2(IUPLO, N, NRHS, D, E, B, LDB)

DOUBLE COMPLEX E(*), B(LDB,*)
INTEGER IUPLO, N, NRHS, LDB
DOUBLE PRECISION D(*)

SUBROUTINE ZPTTS2_64(IUPLO, N, NRHS, D, E, B, LDB)

DOUBLE COMPLEX E(*), B(LDB,*)
INTEGER*8 IUPLO, N, NRHS, LDB
DOUBLE PRECISION D(*)
```

F95 INTERFACE

```
SUBROUTINE ZPTTS2(IUPLO, N, NRHS, D, E, B, LDB)

COMPLEX(8), DIMENSION(:) :: E
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER :: IUPLO, N, NRHS, LDB
REAL(8), DIMENSION(:) :: D

SUBROUTINE ZPTTS2_64(IUPLO, N, NRHS, D, E, B, LDB)

COMPLEX(8), DIMENSION(:) :: E
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER(8) :: IUPLO, N, NRHS, LDB
REAL(8), DIMENSION(:) :: D
```

C INTERFACE

```
#include <sunperf.h>

void zptts2(int iuplo, int n, int  nrhs,  double  *d,  doub-
          lecomplex *e, doublecomplex *b, int ldb);

void zptts2_64(long iuplo, long n,  long  nrhs,  double  *d,
          doublecomplex *e, doublecomplex *b, long ldb);
```

PURPOSE

zptts2 solves a tridiagonal system of the form
   A * X = B using the factorization A = U'*D*U or A =
L*D*L' computed by CPTTRF. D is a diagonal matrix specified
in the vector D, U (or L) is a unit bidiagonal matrix whose
superdiagonal (subdiagonal) is specified in the vector E,
and X and B are N by NRHS matrices.

ARGUMENTS

IUPLO (input)
Specifies the form of the factorization   and
whether the vector E is the superdiagonal of the
upper bidiagonal factor U or the subdiagonal of
the lower bidiagonal factor L. = 1: A = U'*D*U,
E is the superdiagonal of U
= 0: A = L*D*L', E is the subdiagonal of L

N (input) The order of the tridiagonal matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number
of columns of the matrix B. NRHS >= 0.

D (input) The n diagonal elements of the diagonal matrix D
from the factorization A = U'*D*U or A = L*D*L'.

E (input) If IUPLO = 1, the (n-1) superdiagonal elements of
the unit bidiagonal factor U from the factoriza-
tion A = U'*D*U. If IUPLO = 0, the (n-1) subdiag-
onal elements of the unit bidiagonal factor L from
the factorization A = L*D*L'.

B (input/output)
On entry, the right hand side vectors B for the
system of linear equations. On exit, the solution

vectors, X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

Contents

NAME

zrot - apply a plane rotation, where the cos (C) is real and the sin (S) is complex, and the vectors X and Y are complex

SYNOPSIS

```
SUBROUTINE ZROT(N, X, INCX, Y, INCY, C, S)

DOUBLE COMPLEX S
DOUBLE COMPLEX X(*), Y(*)
INTEGER N, INCX, INCY
DOUBLE PRECISION C

SUBROUTINE ZROT_64(N, X, INCX, Y, INCY, C, S)

DOUBLE COMPLEX S
DOUBLE COMPLEX X(*), Y(*)
INTEGER*8 N, INCX, INCY
DOUBLE PRECISION C
```

F95 INTERFACE

```
SUBROUTINE ROT([N], X, [INCX], Y, [INCY], C, S)

COMPLEX(8) :: S
COMPLEX(8), DIMENSION(:) :: X, Y
INTEGER :: N, INCX, INCY
REAL(8) :: C

SUBROUTINE ROT_64([N], X, [INCX], Y, [INCY], C, S)

COMPLEX(8) :: S
COMPLEX(8), DIMENSION(:) :: X, Y
INTEGER(8) :: N, INCX, INCY
```

```
REAL(8) :: C
```

C INTERFACE

```
#include <sunperf.h>

void zrot(int n, doublecomplex *x, int  incx,  doublecomplex
          *y, int incy, double c, doublecomplex *s);

void zrot_64(long n, doublecomplex *x, long incx, doublecom-
          plex *y, long incy, double c, doublecomplex *s);
```

PURPOSE

zrot applies a plane rotation, where the cos (C) is real and the sin (S) is complex, and the vectors X and Y are complex.

ARGUMENTS

N (input)
The number of elements in the vectors X and Y.

X (input/output)
On input, the vector X. On output, X is overwritten with C*X + S*Y.

INCX (input)
The increment between successive values of Y. INCX <> 0.

Y (input/output)
On input, the vector Y. On output, Y is overwritten with -CONJG(S)*X + C*Y.

INCY (input)
The increment between successive values of Y. INCY <> 0.

C (input)

S (input)

```
C and S define a rotation
[  C          S  ]
[ -conjg(S)   C  ]

where C*C + S*CONJG(S) = 1.0.
```

Contents

NAME

```
zrotg - Construct a Given's plane rotation
```

SYNOPSIS

```
SUBROUTINE ZROTG(A, B, C, S)

DOUBLE COMPLEX A, B, S
DOUBLE PRECISION C

SUBROUTINE ZROTG_64(A, B, C, S)

DOUBLE COMPLEX A, B, S
DOUBLE PRECISION C
```

F95 INTERFACE

```
SUBROUTINE ROTG(A, B, C, S)

COMPLEX(8) :: A, B, S
REAL(8) :: C

SUBROUTINE ROTG_64(A, B, C, S)

COMPLEX(8) :: A, B, S
REAL(8) :: C
```

C INTERFACE

```
#include <sunperf.h>

void zrotg(doublecomplex *a, doublecomplex  *b,  double  *c,
          doublecomplex *s);

void zrotg_64(doublecomplex *a, doublecomplex *b, double *c,
```

```
              doublecomplex *s);
```

PURPOSE

zrotg Construct a Given's plane rotation that will annihilate an element of a vector.

ARGUMENTS

A (input/output)
On entry, A contains the entry in the first vector that corresponds to the element to be annihilated in the second vector. On exit, contains the nonzero element of the rotated vector.

B (input)
On entry, B contains the entry to be annihilated in the second vector. Unchanged on exit.

C (output)
On exit, C and S are the elements of the rotation matrix that will be applied to annihilate B.

S (output)
On exit, C and S are the elements of the rotation matrix that will be applied to annihilate B.

Contents

NAME

```
zscal - Compute y := alpha * y
```

SYNOPSIS

```
SUBROUTINE ZSCAL(N, ALPHA, Y, INCY)

DOUBLE COMPLEX ALPHA
DOUBLE COMPLEX Y(*)
INTEGER N, INCY

SUBROUTINE ZSCAL_64(N, ALPHA, Y, INCY)

DOUBLE COMPLEX ALPHA
DOUBLE COMPLEX Y(*)
INTEGER*8 N, INCY
```

F95 INTERFACE

```
SUBROUTINE SCAL([N], ALPHA, Y, [INCY])

COMPLEX(8) :: ALPHA
COMPLEX(8), DIMENSION(:) :: Y
INTEGER :: N, INCY

SUBROUTINE SCAL_64([N], ALPHA, Y, [INCY])

COMPLEX(8) :: ALPHA
COMPLEX(8), DIMENSION(:) :: Y
INTEGER(8) :: N, INCY
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zscal(int n, doublecomplex  *alpha,  doublecomplex  *y,
          int incy);

void zscal_64(long n,  doublecomplex  *alpha,  doublecomplex
          *y, long incy);
```

PURPOSE

zscal Compute y := alpha * y where alpha is a scalar and y is an n-vector.

ARGUMENTS

N (input)
On entry, N specifies the number of elements in the vector. N must be at least one for the subroutine to have any visible effect. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). On entry, the incremented array Y must contain the vector y. On exit, Y is overwritten by the updated vector y.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY must not be zero. Unchanged on exit.

Contents

NAME

```
zsctr - Scatters elements from x into y.
```

SYNOPSIS

```
 SUBROUTINE ZSCTR(NZ, X, INDX, Y)

 DOUBLE COMPLEX X(*), Y(*)
 INTEGER NZ
 INTEGER INDX(*)

 SUBROUTINE ZSCTR_64(NZ, X, INDX, Y)

 DOUBLE COMPLEX X(*), Y(*)
 INTEGER*8 NZ
 INTEGER*8 INDX(*)

F95 INTERFACE
 SUBROUTINE SCTR([NZ], X, INDX, Y)

 COMPLEX(8), DIMENSION(:) :: X, Y
 INTEGER :: NZ
 INTEGER, DIMENSION(:) :: INDX

 SUBROUTINE SCTR_64([NZ], X, INDX, Y)

 COMPLEX(8), DIMENSION(:) :: X, Y
 INTEGER(8) :: NZ
 INTEGER(8), DIMENSION(:) :: INDX
```

PURPOSE

```
ZSCTR - Scatters the components of a sparse vector x stored
in compressed form into specified components of a vector y
```

in full storage form.

```
 do i = 1, n
   y(indx(i)) = x(i)
 enddo
```

ARGUMENTS

```
NZ (input) - INTEGER
        Number of elements in the compressed form.
        Unchanged on exit.

X (input)
        Vector containing the values to be scattered from
        compressed form into full storage form.  Unchanged
        on exit.

INDX (input) - INTEGER
        Vector containing the indices of the compressed
        form.  It is assumed that the elements in INDX are
        distinct and greater than zero.  Unchanged on exit.

Y (output)
        Vector whose elements specified by indx have been
        set to the corresponding entries of x.  Only the
        elements corresponding to the indices in indx have
        been modified.
```

Contents

NAME

```
zskymm - Skyline format matrix-matrix multiply
```

SYNOPSIS

```
 SUBROUTINE ZSKYMM( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, PNTR,  B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, K, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER    PNTR(*),
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE ZSKYMM_64( TRANSA, M, N, K, ALPHA, DESCRA,
*           VAL, PNTR,  B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, K, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER*8  PNTR(*),
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTR(K+1)-PNTR(1) (upper triangular)
       NNZ = PNTR(M+1)-PNTR(1) (lower triangular)
       PNTR() size = (K+1) (upper triangular)
       PNTR() size = (M+1) (lower triangular)
```

F95 INTERFACE

```
 SUBROUTINE SKYMM( TRANSA, M, [N], K, ALPHA, DESCRA, VAL,
*    PNTR,  B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, K
 INTEGER, DIMENSION(:) ::  DESCRA,  PNTR
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) ::  VAL
```

```
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C

 SUBROUTINE SKYMM_64( TRANSA, M, [N], K, ALPHA, DESCRA, VAL,
*    PNTR,  B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, K
 INTEGER*8, DIMENSION(:) ::  DESCRA,  PNTR
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) ::  VAL
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C

where ALPHA and BETA are scalar, C and B are dense matrices,
A is a matrix represented in skyline format and
op( A )  is one  of
op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                    ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

K             Number of columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general (NOT SUPPORTED)
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
              DESCRA(2) upper/lower triangular indicator
                1 : lower
```

```
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

 VAL()          array contain the nonzeros of A in skyline profile form.
                Row-oriented if DESCRA(2) = 1 (lower triangular),
                column oriented if DESCRA(2) = 2 (upper triangular).

 PNTR()         integer array of length M+1 (lower triangular) or
                K+1 (upper triangular) such that PNTR(I)-PNTR(1)+1
                points to the location in VAL of the first element of
                the skyline profile in row (column) I.

 B()            rectangular array with first dimension LDB.
 LDB            leading dimension of B

 BETA           Scalar parameter

 C()            rectangular array with first dimension LDC.

 LDC            leading dimension of C

 WORK()         scratch array of length LWORK. WORK is not
                referenced in the current version.

 LWORK          length of WORK array. LWORK is not referenced
                in the current version.
```

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

The SKY data structure is not supported for a general matrix structure (*DESCRA*(1)=0).

Also not supported:

1. lower triangular matrix A of size m by n where m > n
2. upper triangular matrix A of size m by n where m < n

Contents

NAME

```
zskysm - Skyline format triangular solve
```

SYNOPSIS

```
 SUBROUTINE ZSKYSM( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, PNTR,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER    PNTR(*),
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE ZSKYSM_64( TRANSA, M, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, PNTR,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, M, N, UNITD, DESCRA(5),
*           LDB, LDC, LWORK
 INTEGER*8  PNTR(*),
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX DV(M), VAL(NNZ), B(LDB,*), C(LDC,*), WORK(LWORK)

 where NNZ = PNTR(M+1)-PNTR(1) (upper triangular)
       NNZ = PNTR(K+1)-PNTR(1) (lower triangular)
       PNTR() size = (M+1) (upper triangular)
       PNTR() size = (K+1) (lower triangular)
```

F95 INTERFACE

```
 SUBROUTINE SKYSM( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA, VAL,
*    PNTR,  B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER    TRANSA, M, UNITD
 INTEGER, DIMENSION(:) ::  DESCRA,  PNTR
```

```
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) ::  VAL, DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C

 SUBROUTINE SKYSM_64( TRANSA, M, [N], UNITD, DV, ALPHA, DESCRA,
*    VAL, PNTR,  B, [LDB], BETA, C, [LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, M, UNITD
 INTEGER*8, DIMENSION(:) ::  DESCRA,  PNTR
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) ::  VAL, DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C
where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a diagonal scaling matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in skyline format and
op( A )  is one  of

 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' )).
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

M             Number of rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row scaling)
                3 : Scale on right (column scaling)
                4 : Automatic row or column scaling (see section
                    NOTES for further details)

DV()          Array of length M containing the diagonal entries of the
              scaling diagonal matrix D.

ALPHA         Scalar parameter
```

```
DESCRA()        Descriptor argument.  Five element integer array
                DESCRA(1) matrix structure
                  0 : general
                  1 : symmetric (A=A')
                  2 : Hermitian (A= CONJG(A'))
                  3 : Triangular
                  4 : Skew(Anti)-Symmetric (A=-A')
                  5 : Diagonal
                  6 : Skew-Hermitian (A= -CONJG(A'))

                Note: For the routine, DESCRA(1)=3 is only supported.
                DESCRA(2) upper/lower triangular indicator
                  1 : lower
                  2 : upper
                DESCRA(3) main diagonal type
                  0 : non-unit
                  1 : unit
                DESCRA(4) Array base  (NOT IMPLEMENTED)
                  0 : C/C++ compatible
                  1 : Fortran compatible
                DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                  0 : unknown
                  1 : no repeated indices

VAL()           array contain the nonzeros of A in skyline profile form.
                Row-oriented if DESCRA(2) = 1 (lower triangular),
                column oriented if DESCRA(2) = 2 (upper triangular).

PNTR()          integer array of length M+1 (lower triangular) or
                K+1 (upper triangular) such that PNTR(I)-PNTR(1)+1
                points to the location in VAL of the first element of
                the skyline profile in row (column) I.

B()             rectangular array with first dimension LDB.

LDB             leading dimension of B

BETA            Scalar parameter

C()             rectangular array with first dimension LDC.

LDC             leading dimension of C

WORK()          scratch array of length LWORK.
                On exit,  if LWORK = -1, WORK(1) returns the optimum LWORK.

LWORK           length of WORK array.  LWORK should be at least M.

                For good performance, LWORK should generally be larger.
```

For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program.

If LWORK=0, the routine is to allocate workspace needed.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. Also not supported:
   a. lower triangular matrix A of size m by n where m > n
   b. upper triangular matrix A of size m by n where m < n

2. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

3. If UNITD =4, the routine scales the rows of A if *DESCRA*(2)=1 and the columns of A if *DESCRA*(2)=2 such that their 2-norms are one. The scaling may improve the accuracy of the computed solution. Corresponding entries of VAL are changed only in this particular case. On return DV matrix stored as a vector contains the diagonal matrix by which the rows (columns) have been scaled. UNITD=2 if *DESCRA*(2)=1 and UNITD=3 if *DESCRA*(2)=2 should be used for the next calls to the routine with overwritten VAL and DV.

WORK(1)=0 on return if the scaling has been completed successfully, otherwise *WORK*(1) = -i where i is the row (column) number which 2-norm is exactly zero.

4. If *DESCRA*(3)=1 and UNITD < 4, the unit diagonal elements

might or might not be referenced in the SKY representation of a sparse matrix. They are not used anyway in these cases. But if UNITD=4, the unit diagonal elements MUST be referenced in the SKY representation.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. However *DESCRA*(1) must be equal to 3 in this case.

Contents

NAME

zspcon - estimate the reciprocal of the condition number (in the 1-norm) of a complex symmetric packed matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by ZSPTRF

SYNOPSIS

```
SUBROUTINE ZSPCON(UPLO, N, AP, IPIVOT, ANORM, RCOND, WORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AP(*), WORK(*)
INTEGER N, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION ANORM, RCOND

SUBROUTINE ZSPCON_64(UPLO, N, AP, IPIVOT, ANORM, RCOND, WORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AP(*), WORK(*)
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION ANORM, RCOND
```

F95 INTERFACE

```
SUBROUTINE SPCON(UPLO, N, AP, IPIVOT, ANORM, RCOND, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: AP, WORK
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8) :: ANORM, RCOND
```

```
SUBROUTINE SPCON_64(UPLO, N, AP, IPIVOT, ANORM, RCOND, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: AP, WORK
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8) :: ANORM, RCOND
```

C INTERFACE

```
#include <sunperf.h>

void  zspcon(char  uplo,  int  n,  doublecomplex  *ap,   int
          *ipivot, double anorm, double *rcond, int *info);
void zspcon_64(char uplo, long n,  doublecomplex  *ap,  long
          *ipivot, double anorm, double *rcond, long *info);
```

PURPOSE

zspcon estimates the reciprocal of the condition number (in the 1-norm) of a complex symmetric packed matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by ZSPTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

AP (input)
Double complex array, dimension (N*(N+1)/2) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by ZSPTRF, stored as a packed triangular matrix.

IPIVOT (input)
Integer array, dimension (N) Details of the interchanges and the block structure of D as determined by ZSPTRF.

```
ANORM (input)
          The 1-norm of the original matrix A.

RCOND (output)
          The reciprocal of  the  condition  number  of  the
          matrix  A, computed as RCOND = 1/(ANORM * AINVNM),
          where AINVNM is  an  estimate  of  the  1-norm  of
          inv(A) computed in this routine.

WORK (workspace)
          Double complex array, dimension(2*N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
zsprfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is symmetric indefin-
ite and packed, and provides error bounds and backward error
estimates for the solution
```

SYNOPSIS

```
SUBROUTINE ZSPRFS(UPLO, N, NRHS, A, AF, IPIVOT, B, LDB, X, LDX, FERR,
      BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZSPRFS_64(UPLO, N, NRHS, A, AF, IPIVOT, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE SPRFS(UPLO, N, [NRHS], A, AF, IPIVOT, B, [LDB], X, [LDX],
       FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A, AF, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
```

```
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE SPRFS_64(UPLO, N, [NRHS], A, AF, IPIVOT, B, [LDB], X, [LDX],
       FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: A, AF, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zsprfs(char uplo, int n, int  nrhs,  doublecomplex  *a,
          doublecomplex  *af, int *ipivot, doublecomplex *b,
          int ldb, doublecomplex *x, int ldx, double  *ferr,
          double *berr, int *info);

void zsprfs_64(char uplo, long n, long  nrhs,  doublecomplex
          *a, doublecomplex *af, long *ipivot, doublecomplex
          *b, long ldb, doublecomplex *x, long  ldx,  double
          *ferr, double *berr, long *info);
```

PURPOSE

zsprfs improves the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite and packed, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input) Double complex array,  dimension  (N*(N+1)/2)  The
          upper or lower triangle of the symmetric matrix A,
```

packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n.

AF (input)
Double complex array, dimension (N*(N+1)/2) The factored form of the matrix A. AF contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by CSPTRF, stored as a packed triangular matrix.

IPIVOT (input)
Integer array, dimension (N) Details of the interchanges and the block structure of D as determined by CSPTRF.

B (input) Double complex array, dimension (LDB,NRHS) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
Double complex array, dimension (LDX,NRHS) On entry, the solution matrix X, as computed by CSPTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
Double precision array, dimension (NRHS) The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
Double precision array, dimension (NRHS) The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative

change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
Double precision array, dimension(2*N)

WORK2 (workspace)
Integer array, dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zspsv - compute the solution to a complex system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE ZSPSV(UPLO, N, NRHS, AP, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AP(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZSPSV_64(UPLO, N, NRHS, AP, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AP(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE SPSV(UPLO, N, [NRHS], AP, IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: AP
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE SPSV_64(UPLO, N, [NRHS], AP, IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX(8), DIMENSION(:) :: AP
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void zspsv(char uplo, int n, int  nrhs,  doublecomplex  *ap,
          int   *ipivot,  doublecomplex  *b,  int  ldb,  int
          *info);

void zspsv_64(char uplo, long n,  long  nrhs,  doublecomplex
          *ap,  long  *ipivot,  doublecomplex  *b, long ldb,
          long *info);
```

PURPOSE

```
zspsv computes the solution to a complex  system  of  linear
equations
   A * X = B, where A is an N-by-N symmetric  matrix  stored
in packed format and X and B are N-by-NRHS matrices.

The diagonal pivoting method is used to factor A as
   A = U * D * U**T,  if UPLO = 'U', or
   A = L * D * L**T,  if UPLO = 'L',
where U (or L) is a product of permutation  and  unit  upper
(lower) triangular matrices, D is symmetric and block diago-
nal with 1-by-1 and 2-by-2 diagonal  blocks.   The  factored
form  of A is then used to solve the system of equations A *
X = B.
```

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

AP (input/output)
          Double complex  array,  dimension  (N*(N+1)/2)  On
```

entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n. See below for further details.

On exit, the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by CSPTRF, stored as a packed triangular matrix in the same storage format as A.

IPIVOT (output)
Integer array, dimension (N) Details of the interchanges and the block structure of D, as determined by CSPTRF. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged, and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

B (input/output)
Double complex array, dimension (LDB,NRHS) On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, so the solution could not be computed.

FURTHER DETAILS

The packed storage scheme is illustrated by the following

```
example when N = 4, UPLO = 'U':

Two-dimensional storage of the symmetric matrix A:

   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = aji)
               a44

Packed storage of the upper triangle of A:

AP = [ a11, a12, a22, a13, a23, a33, a14, a24, a34, a44 ]
```

Contents

NAME

zspsvx - use the diagonal pivoting factorization A = U*D*U**T or A = L*D*L**T to compute the solution to a complex system of linear equations A * X = B, where A is an N-by-N symmetric matrix stored in packed format and X and B are N-by-NRHS matrices

SYNOPSIS

```
SUBROUTINE ZSPSVX(FACT, UPLO, N, NRHS, A, AF, IPIVOT, B, LDB, X, LDX,
      RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
DOUBLE COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZSPSVX_64(FACT, UPLO, N, NRHS, A, AF, IPIVOT, B, LDB, X,
      LDX, RCOND, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
DOUBLE COMPLEX A(*), AF(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE SPSVX(FACT, UPLO, N, [NRHS], A, AF, IPIVOT, B, [LDB], X,
       [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])
```

```
CHARACTER(LEN=1) :: FACT, UPLO
COMPLEX(8), DIMENSION(:) :: A, AF, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
INTEGER :: N, NRHS, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE SPSVX_64(FACT, UPLO, N, [NRHS], A, AF, IPIVOT, B, [LDB], X,
       [LDX], RCOND, FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO
COMPLEX(8), DIMENSION(:) :: A, AF, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zspsvx(char fact, char uplo, int  n,  int  nrhs,  doub-
          lecomplex  *a,  doublecomplex  *af,  int  *ipivot,
          doublecomplex *b, int ldb, doublecomplex  *x,  int
          ldx,  double  *rcond,  double *ferr, double *berr,
          int *info);

void zspsvx_64(char fact, char  uplo,  long  n,  long  nrhs,
          doublecomplex *a, doublecomplex *af, long *ipivot,
          doublecomplex *b, long ldb, doublecomplex *x, long
          ldx,  double  *rcond,  double *ferr, double *berr,
          long *info);
```

PURPOSE

zspsvx uses the diagonal pivoting factorization A = U*D*U**T or A = L*D*L**T to compute the solution to a complex system of linear equations A * X = B, where A is an N-by-N symmetric matrix stored in packed format and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'N', the diagonal pivoting method is used to

```
factor A as
      A = U * D * U**T,  if UPLO = 'U', or
      A = L * D * L**T,  if UPLO = 'L',
   where U (or L) is a product of permutation and unit upper
(lower)
   triangular matrices and D is symmetric and block diagonal
with
   1-by-1 and 2-by-2 diagonal blocks.

2. If some D(i,i)=0, so that D is exactly singular, then the
routine
   returns with INFO = i. Otherwise, the factored form of  A
is used
   to estimate the condition number of the matrix A.  If the
   reciprocal of the condition number is less  than  machine
precision,
   INFO = N+1 is returned as  a  warning,  but  the  routine
still goes on
   to solve for X and  compute  error  bounds  as  described
below.

3. The system of equations is solved for X  using  the  fac-
tored form
   of A.

4. Iterative refinement is applied to improve  the  computed
solution
   matrix and calculate  error  bounds  and  backward  error
estimates
   for it.
```

ARGUMENTS

```
FACT (input)
          Specifies whether or not the factored  form  of  A
          has  been supplied on entry.  = 'F':  On entry, AF
          and IPIVOT contain the factored form of A.  A,  AF
          and  IPIVOT  will  not  be  modified.  = 'N':  The
          matrix A will be copied to AF and factored.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.

NRHS (input)
```

The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) Double complex array, dimension (N*(N+1)/2) The upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2*n-j)/2) = A(i,j) for j<=i<=n. See below for further details.

AF (input/output)

Double complex array, dimension (N*(N+1)/2) If FACT = 'F', then AF is an input argument and on entry contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by CSPTRF, stored as a packed triangular matrix in the same storage format as A.

If FACT = 'N', then AF is an output argument and on exit contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by CSPTRF, stored as a packed triangular matrix in the same storage format as A.

IPIVOT (input or output)

Integer array, dimension (N) If FACT = 'F', then IPIVOT is an input argument and on entry contains details of the interchanges and the block structure of D, as determined by CSPTRF. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

If FACT = 'N', then IPIVOT is an output argument and on exit contains details of the interchanges and the block structure of D, as determined by CSPTRF.

B (input) Double complex array, dimension (LDB,NRHS) The N-by-NRHS right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (output)
Double complex array, dimension (LDX,NRHS) If INFO = 0 or INFO = N+1, the N-by-NRHS solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

RCOND (output)
The estimate of the reciprocal condition number of the matrix A. If RCOND is less than the machine precision (in particular, if RCOND = 0), the matrix is singular to working precision. This condition is indicated by a return code of INFO > 0.

FERR (output)
Double complex array, dimension (NRHS) The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
Double complex array, dimension (NRHS) The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
Double complex array, dimension(2*N)

WORK2 (workspace)
Integer array, dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, and i is
<= N: D(i,i) is exactly zero. The factorization has been completed but the factor D is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: D is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

FURTHER DETAILS

The packed storage scheme is illustrated by the following example when N = 4, UPLO = 'U':

Two-dimensional storage of the symmetric matrix A:

```
   a11 a12 a13 a14
       a22 a23 a24
           a33 a34     (aij = aji)
               a44
```

Packed storage of the upper triangle of A:

A = [a11, a12, a22, a13, a23, a33, a14, a24, a34, a44]

Contents

NAME

zsptrf - compute the factorization of a complex symmetric matrix A stored in packed format using the Bunch-Kaufman diagonal pivoting method

SYNOPSIS

```
SUBROUTINE ZSPTRF(UPLO, N, AP, IPIVOT, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AP(*)
INTEGER N, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZSPTRF_64(UPLO, N, AP, IPIVOT, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AP(*)
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE SPTRF(UPLO, N, AP, IPIVOT, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: AP
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE SPTRF_64(UPLO, N, AP, IPIVOT, [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX(8), DIMENSION(:) :: AP
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void  zsptrf(char  uplo,  int  n,  doublecomplex  *ap,   int
          *ipivot, int *info);

void zsptrf_64(char uplo, long n,  doublecomplex  *ap,  long
          *ipivot, long *info);
```

PURPOSE

zsptrf computes the factorization of a complex symmetric matrix A stored in packed format using the Bunch-Kaufman diagonal pivoting method:

A = U*D*U**T or A = L*D*L**T

where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

AP (input/output)
Double complex array, dimension (N*(N+1)/2) On entry, the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n.

On exit, the block diagonal matrix D and the multipliers used to obtain the factor U or L, stored as a packed triangular matrix overwriting A (see below for further details).

IPIVOT (output)
Integer array, dimension (N) Details of the interchanges and the block structure of D. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

5-96 - Based on modifications by J. Lewis, Boeing Computer Services
Company

If UPLO = 'U', then A = U*D*U', where
U = P(n)*U(n)* ... *P(k)U(k)* ...,
i.e., U is a product of terms P(k)*U(k), where k decreases from n to 1 in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIVOT(k), and U(k) is a unit upper triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    v    0   )   k-s
   U(k) =  (   0    I    0   )   s
           (   0    0    I   )   n-k
              k-s   s   n-k
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(1:k-1,k). If s = 2, the upper triangle of D(k) overwrites A(k-1,k-1), A(k-1,k), and A(k,k), and v overwrites A(1:k-2,k-1:k).

If UPLO = 'L', then A = L*D*L', where
L = P(1)*L(1)* ... *P(k)*L(k)* ...,

i.e., L is a product of terms P(k)*L(k), where k increases from 1 to n in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIVOT(k), and L(k) is a unit lower triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    0     0   )  k-1
   L(k) =  (   0    I     0   )  s
           (   0    v     I   )  n-k-s+1
              k-1   s  n-k-s+1
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(k+1:n,k). If s = 2, the lower triangle of D(k) overwrites A(k,k), A(k+1,k), and A(k+1,k+1), and v overwrites A(k+2:n,k:k+1).

Contents

NAME

zsptri - compute the inverse of a complex symmetric indefinite matrix A in packed storage using the factorization A = U*D*U**T or A = L*D*L**T computed by CSPTRF

SYNOPSIS

```
SUBROUTINE ZSPTRI(UPLO, N, AP, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AP(*), WORK(*)
INTEGER N, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZSPTRI_64(UPLO, N, AP, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AP(*), WORK(*)
INTEGER*8 N, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE SPTRI(UPLO, N, AP, IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: AP, WORK
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE SPTRI_64(UPLO, N, AP, IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: AP, WORK
```

```
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void  zsptri(char  uplo,  int  n,  doublecomplex  *ap,   int
          *ipivot, int *info);

void zsptri_64(char uplo, long n,  doublecomplex  *ap,  long
          *ipivot, long *info);
```

PURPOSE

zsptri computes the inverse of a complex symmetric indefinite matrix A in packed storage using the factorization A = U*D*U**T or A = L*D*L**T computed by ZSPTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

AP (input/output)
Double complex array, dimension (N*(N+1)/2) On entry, the block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by ZSPTRF, stored as a packed triangular matrix.

On exit, if INFO = 0, the (symmetric) inverse of the original matrix, stored as a packed triangular matrix. The j-th column of inv(A) is stored in the array AP as follows: if UPLO = 'U', AP(i + (j-1)*j/2) = inv(A)(i,j) for 1<=i<=j; if UPLO = 'L', AP(i + (j-1)*(2n-j)/2) = inv(A)(i,j) for j<=i<=n.

IPIVOT (input)
Integer array, dimension (N) Details of the interchanges and the block structure of D as determined by ZSPTRF.

WORK (workspace)
Double complex array, dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) = 0; the matrix is singular and its inverse could not be computed.

Contents

NAME

```
zsptrs - solve a system of linear equations A*X = B  with  a
complex symmetric matrix A stored in packed format using the
factorization A = U*D*U**T  or  A  =  L*D*L**T  computed  by
ZSPTRF
```

SYNOPSIS

```
SUBROUTINE ZSPTRS(UPLO, N, NRHS, AP, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AP(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZSPTRS_64(UPLO, N, NRHS, AP, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AP(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE SPTRS(UPLO, N, [NRHS], AP, IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: AP
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE SPTRS_64(UPLO, N, [NRHS], AP, IPIVOT, B, [LDB], [INFO])
```

```
CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: AP
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void zsptrs(char uplo, int n, int nrhs,  doublecomplex  *ap,
          int   *ipivot,  doublecomplex  *b,  int  ldb,  int
          *info);

void zsptrs_64(char uplo, long n, long  nrhs,  doublecomplex
          *ap,  long  *ipivot,  doublecomplex  *b, long ldb,
          long *info);
```

PURPOSE

zsptrs solves a system of linear equations A*X = B with a complex symmetric matrix A stored in packed format using the factorization A = U*D*U**T or A = L*D*L**T computed by ZSPTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

AP (input)
Double complex array, dimension (N*(N+1)/2) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by ZSPTRF, stored as a packed triangular matrix.

IPIVOT (input)
Integer array, dimension (N) Details of the interchanges and the block structure of D as determined

by ZSPTRF.

B (input/output)
Double complex array, dimension (LDB,NRHS) On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zstedc - compute all eigenvalues and, optionally,  eigenvec-
tors  of a symmetric tridiagonal matrix using the divide and
conquer method
```

SYNOPSIS

```
SUBROUTINE ZSTEDC(COMPZ, N, D, E, Z, LDZ, WORK, LWORK, RWORK, LRWORK,
      IWORK, LIWORK, INFO)

CHARACTER * 1 COMPZ
DOUBLE COMPLEX Z(LDZ,*), WORK(*)
INTEGER N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER IWORK(*)
DOUBLE PRECISION D(*), E(*), RWORK(*)

SUBROUTINE ZSTEDC_64(COMPZ, N, D, E, Z, LDZ, WORK, LWORK, RWORK,
      LRWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 COMPZ
DOUBLE COMPLEX Z(LDZ,*), WORK(*)
INTEGER*8 N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION D(*), E(*), RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEDC(COMPZ, [N], D, E, Z, [LDZ], [WORK], [LWORK], [RWORK],
       [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: Z
```

```
INTEGER :: N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: D, E, RWORK

SUBROUTINE STEDC_64(COMPZ, [N], D, E, Z, [LDZ], [WORK], [LWORK],
       [RWORK], [LRWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: Z
INTEGER(8) :: N, LDZ, LWORK, LRWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8), DIMENSION(:) :: D, E, RWORK
```

C INTERFACE

```
#include <sunperf.h>
void zstedc(char compz, int n, double *d, double  *e,  doub-
          lecomplex *z, int ldz, int *info);

void zstedc_64(char compz, long n,  double  *d,  double  *e,
          doublecomplex *z, long ldz, long *info);
```

PURPOSE

zstedc computes all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the divide and conquer method. The eigenvectors of a full or band complex Hermitian matrix can also be found if CHETRD or CHPTRD or CHBTRD has been used to reduce this matrix to tridiagonal form.

This code makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none. See SLAED3 for details.

ARGUMENTS

COMPZ (input)
= 'N': Compute eigenvalues only.
= 'I': Compute eigenvectors of tridiagonal matrix also.
= 'V': Compute eigenvectors of original Hermitian matrix also. On entry, Z contains the unitary

matrix used to reduce the original matrix to tri-
diagonal form.

N (input) The dimension of the symmetric tridiagonal matrix.
N >= 0.

D (input/output)
On entry, the diagonal elements of the tridiagonal matrix. On exit, if INFO = 0, the eigenvalues in ascending order.

E (input/output)
On entry, the subdiagonal elements of the tridiagonal matrix. On exit, E has been destroyed.

Z (input) On entry, if COMPZ = 'V', then Z contains the unitary matrix used in the reduction to tridiagonal form. On exit, if INFO = 0, then if COMPZ = 'V', Z contains the orthonormal eigenvectors of the original Hermitian matrix, and if COMPZ = 'I', Z contains the orthonormal eigenvectors of the symmetric tridiagonal matrix. If COMPZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1. If eigenvectors are desired, then LDZ >= max(1,N).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If COMPZ = 'N' or 'I', or N <= 1, LWORK must be at least 1. If COMPZ = 'V' and N > 1, LWORK must be at least N*N.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

RWORK (workspace)
dimension (LRWORK) On exit, if INFO = 0, RWORK(1) returns the optimal LRWORK.

LRWORK (input)
The dimension of the array RWORK. If COMPZ = 'N' or N <= 1, LRWORK must be at least 1. If COMPZ =

'V' and N > 1, LRWORK must be at least 1 + 3*N + 2*N*lg N + 3*N**2 , where lg(N) = smallest integer k such that 2**k >= N. If COMPZ = 'I' and N > 1, LRWORK must be at least 1 + 4*N + 2*N**2 .

If LRWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the RWORK array, returns this value as the first entry of the RWORK array, and no error message related to LRWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of the array IWORK. If COMPZ = 'N' or N <= 1, LIWORK must be at least 1. If COMPZ = 'V' or N > 1, LIWORK must be at least 6 + 6*N + 5*N*lg N. If COMPZ = 'I' or N > 1, LIWORK must be at least 3 + 5*N .

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: The algorithm failed to compute an eigenvalue while working on the submatrix lying in rows and columns INFO/(N+1) through mod(INFO,N+1).

FURTHER DETAILS

Based on contributions by
Jeff Rutter, Computer Science Division, University of California
at Berkeley, USA

Contents

NAME

```
zstegr - Compute T-sigma_i = L_i D_i L_i^T,  such  that  L_i
D_i L_i^T  is a relatively robust representation
```

SYNOPSIS

```
SUBROUTINE ZSTEGR(JOBZ, RANGE, N, D, E, VL, VU, IL, IU, ABSTOL, M, W,
      Z, LDZ, ISUPPZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE
DOUBLE COMPLEX Z(LDZ,*)
INTEGER N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER ISUPPZ(*), IWORK(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION D(*), E(*), W(*), WORK(*)

SUBROUTINE ZSTEGR_64(JOBZ, RANGE, N, D, E, VL, VU, IL, IU, ABSTOL, M,
      W, Z, LDZ, ISUPPZ, WORK, LWORK, IWORK, LIWORK, INFO)

CHARACTER * 1 JOBZ, RANGE
DOUBLE COMPLEX Z(LDZ,*)
INTEGER*8 N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER*8 ISUPPZ(*), IWORK(*)
DOUBLE PRECISION VL, VU, ABSTOL
DOUBLE PRECISION D(*), E(*), W(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEGR(JOBZ, RANGE, [N], D, E, VL, VU, IL, IU, ABSTOL, M,
       W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE
COMPLEX(8), DIMENSION(:,:) :: Z
```

```
INTEGER :: N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER, DIMENSION(:) :: ISUPPZ, IWORK
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: D, E, W, WORK

SUBROUTINE STEGR_64(JOBZ, RANGE, [N], D, E, VL, VU, IL, IU, ABSTOL,
       M, W, Z, [LDZ], ISUPPZ, [WORK], [LWORK], [IWORK], [LIWORK], [INFO])

CHARACTER(LEN=1) :: JOBZ, RANGE
COMPLEX(8), DIMENSION(:,:) :: Z
INTEGER(8) :: N, IL, IU, M, LDZ, LWORK, LIWORK, INFO
INTEGER(8), DIMENSION(:) :: ISUPPZ, IWORK
REAL(8) :: VL, VU, ABSTOL
REAL(8), DIMENSION(:) :: D, E, W, WORK
```

C INTERFACE

```
#include <sunperf.h>

void zstegr(char jobz, char range, int n, double *d,  double
          *e,  double  vl, double vu, int il, int iu, double
          abstol, int *m, double *w, doublecomplex  *z,  int
          ldz, int *isuppz, int *info);

void zstegr_64(char jobz, char range,  long  n,  double  *d,
          double *e, double vl, double vu, long il, long iu,
          double abstol, long *m, double  *w,  doublecomplex
          *z, long ldz, long *isuppz, long *info);
```

PURPOSE

```
zstegr b) Compute the  eigenvalues,  lambda_j,  of  L_i  D_i
L_i^T to high
       relative accuracy by the dqds algorithm,
   (c) If there is a cluster of close eigenvalues,  "choose"
sigma_i
       close to the cluster, and go to step (a),
   (d) Given the approximate eigenvalue lambda_j of L_i  D_i
L_i^T,
       compute the corresponding eigenvector by forming a
       rank-revealing twisted factorization.
The desired accuracy of the output can be specified  by  the
input parameter ABSTOL.

For more details, see "A new O(n^2) algorithm for  the  sym-
metric   tridiagonal   eigenvalue/eigenvector  problem",  by
Inderjit Dhillon, Computer Science Division Technical Report
No. UCB/CSD-97-971, UC Berkeley, May 1997.

Note 1 : Currently CSTEGR is only set up to find ALL  the  n
eigenvalues and eigenvectors of T in O(n^2) time
```

Note 2 : Currently the routine CSTEIN is called when an appropriate sigma_i cannot be chosen in step (c) above. CSTEIN invokes modified Gram-Schmidt when eigenvalues are close.
Note 3 : CSTEGR works only on machines which follow ieee-754 floating-point standard in their handling of infinities and NaNs. Normal execution of CSTEGR may create NaNs and infinities and hence may abort due to a floating point exception in environments which do not conform to the ieee standard.

ARGUMENTS

JOBZ (input)
= 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

RANGE (input)
= 'A': all eigenvalues will be found.
= 'V': all eigenvalues in the half-open interval (VL,VU] will be found. = 'I': the IL-th through IU-th eigenvalues will be found.

N (input) The order of the matrix. N >= 0.

D (input/output)
On entry, the n diagonal elements of the tridiagonal matrix T. On exit, D is overwritten.

E (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix T in elements 1 to N-1 of E; E(N) need not be set. On exit, E is overwritten.

VL (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

VU (input)
If RANGE='V', the lower and upper bounds of the interval to be searched for eigenvalues. VL < VU. Not referenced if RANGE = 'A' or 'I'.

IL (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

IU (input)
If RANGE='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0. Not referenced if RANGE = 'A' or 'V'.

ABSTOL (input)
The absolute error tolerance for the eigenvalues/eigenvectors. IF JOBZ = 'V', the eigenvalues and eigenvectors output have residual norms bounded by ABSTOL, and the dot products between different eigenvectors are bounded by ABSTOL. If ABSTOL is less than N*EPS*|T|, then N*EPS*|T| will be used in its place, where EPS is the machine precision and |T| is the 1-norm of the tridiagonal matrix. The eigenvalues are computed to an accuracy of EPS*|T| irrespective of ABSTOL. If high relative accuracy is important, set ABSTOL to DLAMCH('Safe minimum'). See Barlow and Demmel "Computing Accurate Eigensystems of Scaled Diagonally Dominant Matrices", LAPACK Working Note #7 for a discussion of which matrices define their eigenvalues to high relative accuracy.

M (output)
The total number of eigenvalues found. 0 <= M <= N. If RANGE = 'A', M = N, and if RANGE = 'I', M = IU-IL+1.

W (output)
The first M elements contain the selected eigenvalues in ascending order.

Z (input) If JOBZ = 'V', then if INFO = 0, the first M columns of Z contain the orthonormal eigenvectors of the matrix T corresponding to the selected eigenvalues, with the i-th column of Z holding the eigenvector associated with W(i). If JOBZ = 'N', then Z is not referenced. Note: the user must ensure that at least max(1,M) columns are supplied in the array Z; if RANGE = 'V', the exact value of M is not known in advance and an upper bound must be used.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if JOBZ = 'V', LDZ >= max(1,N).

ISUPPZ (output)
The support of the eigenvectors in Z, i.e., the indices indicating the nonzero elements in Z. The i-th eigenvector is nonzero only in elements ISUPPZ(2*i-1) through ISUPPZ(2*i).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal (and minimal) LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,18*N)

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace/output)
On exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of the array IWORK. LIWORK >= max(1,10*N)

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = 1, internal error in SLARRE, if INFO = 2, internal error in CLARRV.

FURTHER DETAILS

Based on contributions by
Inderjit Dhillon, IBM Almaden, USA
Osni Marques, LBNL/NERSC, USA
Ken Stanley, Computer Science Division, University of California at Berkeley, USA

Contents

NAME

```
zstein - compute the eigenvectors of a real symmetric tridi-
agonal  matrix  T  corresponding  to  specified eigenvalues,
using inverse iteration
```

SYNOPSIS

```
SUBROUTINE ZSTEIN(N, D, E, M, W, IBLOCK, ISPLIT, Z, LDZ, WORK, IWORK,
      IFAIL, INFO)

DOUBLE COMPLEX Z(LDZ,*)
INTEGER N, M, LDZ, INFO
INTEGER IBLOCK(*), ISPLIT(*), IWORK(*), IFAIL(*)
DOUBLE PRECISION D(*), E(*), W(*), WORK(*)

SUBROUTINE ZSTEIN_64(N, D, E, M, W, IBLOCK, ISPLIT, Z, LDZ, WORK,
      IWORK, IFAIL, INFO)

DOUBLE COMPLEX Z(LDZ,*)
INTEGER*8 N, M, LDZ, INFO
INTEGER*8 IBLOCK(*), ISPLIT(*), IWORK(*), IFAIL(*)
DOUBLE PRECISION D(*), E(*), W(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEIN([N], D, E, [M], W, IBLOCK, ISPLIT, Z, [LDZ], [WORK],
       [IWORK], IFAIL, [INFO])

COMPLEX(8), DIMENSION(:,:) :: Z
INTEGER :: N, M, LDZ, INFO
INTEGER, DIMENSION(:) :: IBLOCK, ISPLIT, IWORK, IFAIL
REAL(8), DIMENSION(:) :: D, E, W, WORK

SUBROUTINE STEIN_64([N], D, E, [M], W, IBLOCK, ISPLIT, Z, [LDZ],
```

```
        [WORK], [IWORK], IFAIL, [INFO])

   COMPLEX(8), DIMENSION(:,:) :: Z
   INTEGER(8) :: N, M, LDZ, INFO
   INTEGER(8), DIMENSION(:) :: IBLOCK, ISPLIT, IWORK, IFAIL
   REAL(8), DIMENSION(:) :: D, E, W, WORK
```

C INTERFACE

```
   #include <sunperf.h>

   void zstein(int n, double *d, double *e, int m,  double  *w,
             int  *iblock,  int  *isplit, doublecomplex *z, int
             ldz, int *ifail, int *info);
   void zstein_64(long n, double *d, double *e, long m,  double
             *w,  long *iblock, long *isplit, doublecomplex *z,
             long ldz, long *ifail, long *info);
```

PURPOSE

zstein computes the eigenvectors of a real symmetric tridiagonal matrix T corresponding to specified eigenvalues, using inverse iteration.

The maximum number of iterations allowed for each eigenvector is specified by an internal parameter MAXITS (currently set to 5).

Although the eigenvectors are real, they are stored in a complex array, which may be passed to CUNMTR or CUPMTR for back
transformation to the eigenvectors of a complex Hermitian matrix which was reduced to tridiagonal form.

ARGUMENTS

N (input) The order of the matrix. N >= 0.

D (input) The n diagonal elements of the tridiagonal matrix T.

E (input) The (n-1) subdiagonal elements of the tridiagonal matrix T, stored in elements 1 to N-1; E(N) need not be set.

M (input) The number of eigenvectors to be found. 0 <= M <= N.

```
W (input) The first M elements of W contain the  eigenvalues
          for  which  eigenvectors  are to be computed.  The
          eigenvalues should be grouped by  split-off  block
          and  ordered  from  smallest to largest within the
          block.  ( The output  array  W  from  SSTEBZ  with
          ORDER = 'B' is expected here. )

IBLOCK (input)
          The  submatrix   indices   associated   with   the
          corresponding  eigenvalues  in  W;  IBLOCK(i)=1 if
          eigenvalue W(i) belongs  to  the  first  submatrix
          from  the  top,  =2  if W(i) belongs to the second
          submatrix, etc.  ( The output  array  IBLOCK  from
          SSTEBZ is expected here. )

ISPLIT (input)
          The splitting points, at which T  breaks  up  into
          submatrices.   The  first  submatrix  consists  of
          rows/columns 1 to  ISPLIT(  1  ),  the  second  of
          rows/columns  ISPLIT(  1  )+1 through ISPLIT( 2 ),
          etc.  ( The output array  ISPLIT  from  SSTEBZ  is
          expected here. )

Z (output)
          The computed eigenvectors.  The eigenvector  asso-
          ciated  with  the eigenvalue W(i) is stored in the
          i-th column of Z.  Any vector which fails to  con-
          verge  is  set to its current iterate after MAXITS
          iterations.  The imaginary parts of the  eigenvec-
          tors are set to zero.

LDZ (input)
          The leading dimension of  the  array  Z.   LDZ  >=
          max(1,N).

WORK (workspace)
          dimension(5*N)

IWORK (workspace)
          dimension(N)

IFAIL (output)
          On normal exit, all elements of  IFAIL  are  zero.
          If one or more eigenvectors fail to converge after
          MAXITS iterations, then their indices  are  stored
          in array IFAIL.

INFO (output)
          = 0: successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
```

gal value
> 0: if INFO = i, then i eigenvectors failed to converge in MAXITS iterations. Their indices are stored in array IFAIL.

Contents

NAME

zsteqr - compute all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the implicit QL or QR method

SYNOPSIS

```
SUBROUTINE ZSTEQR(COMPZ, N, D, E, Z, LDZ, WORK, INFO)

CHARACTER * 1 COMPZ
DOUBLE COMPLEX Z(LDZ,*)
INTEGER N, LDZ, INFO
DOUBLE PRECISION D(*), E(*), WORK(*)

SUBROUTINE ZSTEQR_64(COMPZ, N, D, E, Z, LDZ, WORK, INFO)

CHARACTER * 1 COMPZ
DOUBLE COMPLEX Z(LDZ,*)
INTEGER*8 N, LDZ, INFO
DOUBLE PRECISION D(*), E(*), WORK(*)
```

F95 INTERFACE

```
SUBROUTINE STEQR(COMPZ, [N], D, E, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
COMPLEX(8), DIMENSION(:,:) :: Z
INTEGER :: N, LDZ, INFO
REAL(8), DIMENSION(:) :: D, E, WORK

SUBROUTINE STEQR_64(COMPZ, [N], D, E, Z, [LDZ], [WORK], [INFO])

CHARACTER(LEN=1) :: COMPZ
COMPLEX(8), DIMENSION(:,:) :: Z
```

```
INTEGER(8) :: N, LDZ, INFO
REAL(8), DIMENSION(:) :: D, E, WORK
```

C INTERFACE

```
#include <sunperf.h>

void zsteqr(char compz, int n, double *d, double  *e,  doub-
          lecomplex *z, int ldz, int *info);

void zsteqr_64(char compz, long n,  double  *d,  double  *e,
          doublecomplex *z, long ldz, long *info);
```

PURPOSE

zsteqr computes all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the implicit QL or QR method. The eigenvectors of a full or band complex Hermitian matrix can also be found if CHETRD or CHPTRD or CHBTRD has been used to reduce this matrix to tridiagonal form.

ARGUMENTS

COMPZ (input)
= 'N': Compute eigenvalues only.
= 'V': Compute eigenvalues and eigenvectors of the original Hermitian matrix. On entry, Z must contain the unitary matrix used to reduce the original matrix to tridiagonal form. = 'I': Compute eigenvalues and eigenvectors of the tridiagonal matrix. Z is initialized to the identity matrix.

N (input) The order of the matrix. N >= 0.

D (input/output)
On entry, the diagonal elements of the tridiagonal matrix. On exit, if INFO = 0, the eigenvalues in ascending order.

E (input/output)
On entry, the (n-1) subdiagonal elements of the tridiagonal matrix. On exit, E has been destroyed.

Z (input) On entry, if COMPZ = 'V', then Z contains the unitary matrix used in the reduction to tridiagonal form. On exit, if INFO = 0, then if COMPZ =

'V', Z contains the orthonormal eigenvectors of the original Hermitian matrix, and if COMPZ = 'I', Z contains the orthonormal eigenvectors of the symmetric tridiagonal matrix. If COMPZ = 'N', then Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1, and if eigenvectors are desired, then LDZ >= max(1,N).

WORK (workspace)
dimension(max(1,2*N-2)) If COMPZ = 'N', then WORK is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: the algorithm has failed to find all the eigenvalues in a total of 30*N iterations; if INFO = i, then i elements of E have not converged to zero; on exit, D and E contain the elements of a symmetric tridiagonal matrix which is unitarily similar to the original matrix.

Contents

NAME

```
zstsv - compute the solution to a complex system  of  linear
equations  A  *  X  =  B  where A is a Hermitian tridiagonal
matrix
```

SYNOPSIS

```
SUBROUTINE ZSTSV(N, NRHS, L, D, SUBL, B, LDB, IPIV, INFO)

DOUBLE COMPLEX L(*), D(*), SUBL(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO
INTEGER IPIV(*)

SUBROUTINE ZSTSV_64(N, NRHS, L, D, SUBL, B, LDB, IPIV, INFO)

DOUBLE COMPLEX L(*), D(*), SUBL(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIV(*)
```

F95 INTERFACE

```
SUBROUTINE STSV(N, NRHS, L, D, SUBL, B, [LDB], IPIV, [INFO])

COMPLEX(8), DIMENSION(:) :: L, D, SUBL
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIV

SUBROUTINE STSV_64(N, NRHS, L, D, SUBL, B, [LDB], IPIV, [INFO])

COMPLEX(8), DIMENSION(:) :: L, D, SUBL
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIV
```

C INTERFACE

```
#include <sunperf.h>

void zstsv(int n, int nrhs, doublecomplex *l,  doublecomplex
          *d,  doublecomplex  *subl,  doublecomplex  *b, int
          ldb, int *ipiv, int *info);

void zstsv_64(long n, long  nrhs,  doublecomplex  *l,  doub-
          lecomplex  *d,  doublecomplex *subl, doublecomplex
          *b, long ldb, long *ipiv, long *info);
```

PURPOSE

zstsv computes the solution to a complex system of linear equations A * X = B where A is a Hermitian tridiagonal matrix.

ARGUMENTS

N (input)
The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides in B.

L (input/output)
COMPLEX array, dimension (N)
On entry, the n-1 subdiagonal elements of the tridiagonal matrix A. On exit, part of the factorization of A.

D (input/output)
REAL array, dimension (N)
On entry, the n diagonal elements of the tridiagonal matrix A. On exit, the n diagonal elements of the diagonal matrix D from the factorization of A.

SUBL (output)
COMPLEX array, dimension (N)
On exit, part of the factorization of A.

B (input/output)
The columns of B contain the right hand sides.

LDB (input)
The leading dimension of B as specified in a type

```
          or DIMENSION statement.

IPIV (output)
           INTEGER array, dimension (N)
          On exit, the pivot indices of the factorization.

INFO (output)
           INTEGER
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, D(k,k) is  exactly  zero.   The
          factorization  has  been  completed, but the block
          diagonal matrix D is exactly singular and division
          by zero will occur if it is used to solve a system
          of equations.
```

Contents

NAME

```
zsttrf - compute the factorization of  a  complex  Hermitian
tridiagonal matrix A
```

SYNOPSIS

```
SUBROUTINE ZSTTRF(N, L, D, SUBL, IPIV, INFO)

DOUBLE COMPLEX L(*), D(*), SUBL(*)
INTEGER N, INFO
INTEGER IPIV(*)

SUBROUTINE ZSTTRF_64(N, L, D, SUBL, IPIV, INFO)

DOUBLE COMPLEX L(*), D(*), SUBL(*)
INTEGER*8 N, INFO
INTEGER*8 IPIV(*)
```

F95 INTERFACE

```
SUBROUTINE STTRF([N], L, D, SUBL, IPIV, [INFO])

COMPLEX(8), DIMENSION(:) :: L, D, SUBL
INTEGER :: N, INFO
INTEGER, DIMENSION(:) :: IPIV

SUBROUTINE STTRF_64([N], L, D, SUBL, IPIV, [INFO])

COMPLEX(8), DIMENSION(:) :: L, D, SUBL
INTEGER(8) :: N, INFO
INTEGER(8), DIMENSION(:) :: IPIV
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zsttrf(int n, doublecomplex *l, doublecomplex *d, doub-
          lecomplex *subl, int *ipiv, int *info);

void zsttrf_64(long n, doublecomplex *l,  doublecomplex  *d,
          doublecomplex *subl, long *ipiv, long *info);
```

PURPOSE

zsttrf computes the L*D*L**H factorization of a complex Hermitian tridiagonal matrix A.

ARGUMENTS

```
N (input)  INTEGER
          The order of the matrix A.  N >= 0.

L (input/output)
           COMPLEX array, dimension (N)
          On entry, the n-1 subdiagonal elements of the tri-
          diagonal  matrix A.  On exit, part of the factori-
          zation of A.

D (input/output)
           REAL array, dimension (N)
          On entry, the n diagonal elements of the tridiago-
          nal matrix A.  On exit, the n diagonal elements of
          the diagonal matrix D from the factorization of A.

SUBL (output)
           COMPLEX array, dimension (N)
          On exit, part of the factorization of A.

IPIV (output)
           INTEGER array, dimension (N)
          On exit, the pivot indices of the factorization.

INFO (output)
           INTEGER
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, D(k,k) is  exactly  zero.   The
          factorization  has  been  completed, but the block
          diagonal matrix D is exactly singular and division
          by zero will occur if it is used to solve a system
          of equations.
```

Contents

NAME

zsttrs - computes the solution to a complex system of linear equations A * X = B

SYNOPSIS

```
SUBROUTINE ZSTTRS(N, NRHS, L, D, SUBL, B, LDB, IPIV, INFO)

DOUBLE COMPLEX L(*), D(*), SUBL(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO
INTEGER IPIV(*)

SUBROUTINE ZSTTRS_64(N, NRHS, L, D, SUBL, B, LDB, IPIV, INFO)

DOUBLE COMPLEX L(*), D(*), SUBL(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
INTEGER*8 IPIV(*)
```

F95 INTERFACE

```
SUBROUTINE STTRS(N, NRHS, L, D, SUBL, B, [LDB], IPIV, [INFO])

COMPLEX(8), DIMENSION(:) :: L, D, SUBL
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO
INTEGER, DIMENSION(:) :: IPIV

SUBROUTINE STTRS_64(N, NRHS, L, D, SUBL, B, [LDB], IPIV, [INFO])

COMPLEX(8), DIMENSION(:) :: L, D, SUBL
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIV
```

C INTERFACE

```
#include <sunperf.h>

void zsttrs(int n, int nrhs, doublecomplex *l, doublecomplex
          *d,  doublecomplex  *subl,  doublecomplex  *b, int
          ldb, int *ipiv, int *info);

void zsttrs_64(long n, long nrhs,  doublecomplex  *l,  doub-
          lecomplex  *d,  doublecomplex *subl, doublecomplex
          *b, long ldb, long *ipiv, long *info);
```

PURPOSE

zsttrs computes the solution to a complex system of linear equations A * X = B, where A is an N-by-N symmetric tridiagonal matrix and X and B are N-by-NRHS matrices.

ARGUMENTS

```
N (input)  INTEGER
          The order of the matrix A.  N >= 0.

NRHS (input)
           INTEGER
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B. NRHS >= 0.

L (input)  COMPLEX array, dimension (N-1)
          On entry, the subdiagonal elements of LL and DD.

D (input)  COMPLEX array, dimension (N)
          On entry, the diagonal elements of DD.

SUBL (input)
           COMPLEX array, dimension (N-2)
          On entry, the second subdiagonal elements of LL.

B (input/output)
           COMPLEX array, dimension (LDB, NRHS)
          On entry, the N-by-NRHS right hand side matrix  B.
          On  exit,  if  INFO  =  0,  the N-by-NRHS solution
          matrix X.

LDB (input)
           INTEGER
          The leading dimension of  the  array  B.   LDB  >=
          max(1, N)
```

```
IPIV (output)
           INTEGER array, dimension (N)
          Details of the interchanges and block  pivot.   If
          IPIV(K)  > 0, 1 by 1 pivot, and if IPIV(K) = K + 1
          an interchange done;  If  IPIV(K)  <  0,  2  by  2
          pivot, no interchange required.

INFO (output)
           INTEGER
          = 0:  successful exit
          < 0:  if INFO = -k, the k-th argument had an ille-
          gal value
```

Contents

NAME

zswap - Exchange vectors x and y.

SYNOPSIS

```
SUBROUTINE ZSWAP(N, X, INCX, Y, INCY)

DOUBLE COMPLEX X(*), Y(*)
INTEGER N, INCX, INCY

SUBROUTINE ZSWAP_64(N, X, INCX, Y, INCY)

DOUBLE COMPLEX X(*), Y(*)
INTEGER*8 N, INCX, INCY
```

F95 INTERFACE

```
SUBROUTINE SWAP([N], X, [INCX], Y, [INCY])

COMPLEX(8), DIMENSION(:) :: X, Y
INTEGER :: N, INCX, INCY

SUBROUTINE SWAP_64([N], X, [INCX], Y, [INCY])

COMPLEX(8), DIMENSION(:) :: X, Y
INTEGER(8) :: N, INCX, INCY
```

C INTERFACE

```
#include <sunperf.h>

void zswap(int n, doublecomplex *x, int incx,  doublecomplex
          *y, int incy);

void zswap_64(long n, doublecomplex  *x,  long  incx,  doub-
```

```
          lecomplex *y, long incy);
```

PURPOSE

zswap Exchange x and y where x and y are n-vectors.

ARGUMENTS

```
N (input)
          On entry, N specifies the number  of  elements  in
          the  vector.   N must be at least one for the sub-
          routine to have any visible effect.  Unchanged  on
          exit.
X (input/output)
          ( 1 + ( n - 1 )*abs( INCX  )  ).   On  entry,  the
          incremented array X must contain the vector x.  On
          exit, the y vector.

INCX (input)
          On entry, INCX specifies  the  increment  for  the
          elements  of  X. INCX must not be zero.  Unchanged
          on exit.

Y (input/output)
          ( 1 + ( n - 1 )*abs( INCY  )  ).   On  entry,  the
          incremented array Y must contain the vector y.  On
          exit, the x vector.

INCY (input)
          On entry, INCY specifies  the  increment  for  the
          elements  of  Y. INCY must not be zero.  Unchanged
          on exit.
```

Contents

NAME

zsycon - estimate the reciprocal of the condition number (in the 1-norm) of a complex symmetric matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by CSYTRF

SYNOPSIS

```
SUBROUTINE ZSYCON(UPLO, N, A, LDA, IPIVOT, ANORM, RCOND, WORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION ANORM, RCOND

SUBROUTINE ZSYCON_64(UPLO, N, A, LDA, IPIVOT, ANORM, RCOND, WORK,
      INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION ANORM, RCOND
```

F95 INTERFACE

```
SUBROUTINE SYCON(UPLO, [N], A, [LDA], IPIVOT, ANORM, RCOND, [WORK],
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT
```

```
REAL(8) :: ANORM, RCOND

SUBROUTINE SYCON_64(UPLO, [N], A, [LDA], IPIVOT, ANORM, RCOND, [WORK],
       [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8) :: ANORM, RCOND
```

C INTERFACE

```
#include <sunperf.h>
void zsycon(char uplo, int n, doublecomplex *a, int lda, int
          *ipivot, double anorm, double *rcond, int *info);

void zsycon_64(char uplo, long  n,  doublecomplex  *a,  long
          lda,  long  *ipivot,  double anorm, double *rcond,
          long *info);
```

PURPOSE

zsycon estimates the reciprocal of the condition number (in the 1-norm) of a complex symmetric matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by CSYTRF.

An estimate is obtained for norm(inv(A)), and the reciprocal of the condition number is computed as RCOND = 1 / (ANORM * norm(inv(A))).

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

A (input) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CSYTRF.

LDA (input)
The leading dimension of the array A.   LDA >=

max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CSYTRF.

ANORM (input)
The 1-norm of the original matrix A.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(ANORM * AINVNM), where AINVNM is an estimate of the 1-norm of inv(A) computed in this routine.

WORK (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

zsymm - perform one of the matrix-matrix operations C := alpha*A*B + beta*C or C := alpha*B*A + beta*C

SYNOPSIS

```
SUBROUTINE ZSYMM(SIDE, UPLO, M, N, ALPHA, A, LDA, B, LDB, BETA, C,
      LDC)

CHARACTER * 1 SIDE, UPLO
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER M, N, LDA, LDB, LDC

SUBROUTINE ZSYMM_64(SIDE, UPLO, M, N, ALPHA, A, LDA, B, LDB, BETA, C,
      LDC)

CHARACTER * 1 SIDE, UPLO
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER*8 M, N, LDA, LDB, LDC
```

F95 INTERFACE

```
SUBROUTINE SYMM(SIDE, UPLO, [M], [N], ALPHA, A, [LDA], B, [LDB],
       BETA, C, [LDC])

CHARACTER(LEN=1) :: SIDE, UPLO
COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:,:) :: A, B, C
INTEGER :: M, N, LDA, LDB, LDC

SUBROUTINE SYMM_64(SIDE, UPLO, [M], [N], ALPHA, A, [LDA], B, [LDB],
       BETA, C, [LDC])
```

```
CHARACTER(LEN=1) :: SIDE, UPLO
COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:,:) :: A, B, C
INTEGER(8) :: M, N, LDA, LDB, LDC
```

C INTERFACE

```
#include <sunperf.h>

void zsymm(char side, char uplo, int m, int n, doublecomplex
          *alpha,  doublecomplex  *a, int lda, doublecomplex
          *b, int ldb,  doublecomplex  *beta,  doublecomplex
          *c, int ldc);
void zsymm_64(char side, char uplo, long m,  long  n,  doub-
          lecomplex  *alpha,  doublecomplex  *a,  long  lda,
          doublecomplex *b, long ldb,  doublecomplex  *beta,
          doublecomplex *c, long ldc);
```

PURPOSE

zsymm performs one of the matrix-matrix operations C := alpha*A*B + beta*C or C := alpha*B*A + beta*C where alpha and beta are scalars, A is a symmetric matrix and B and C are m by n matrices.

ARGUMENTS

SIDE (input)
On entry, SIDE specifies whether the symmetric matrix A appears on the left or right in the operation as follows:

SIDE = 'L' or 'l'   C := alpha*A*B + beta*C,

SIDE = 'R' or 'r'   C := alpha*B*A + beta*C,

Unchanged on exit.

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the symmetric matrix A is to be referenced as follows:

UPLO = 'U' or 'u'   Only the upper triangular part of the symmetric matrix is to be referenced.

UPLO = 'L' or 'l'   Only the lower triangular part of the symmetric matrix is to be referenced.

Unchanged on exit.

M (input)

On entry, M specifies the number of rows of the matrix C. M >= 0. Unchanged on exit.

N (input)

On entry, N specifies the number of columns of the matrix C. N >= 0. Unchanged on exit.

ALPHA (input)

On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)

COMPLEX*16 array of DIMENSION (LDA, ka), where ka is m when SIDE = 'L' or 'l' and is n otherwise.

Before entry with SIDE = 'L' or 'l', the m by m part of the array A must contain the symmetric matrix, such that when UPLO = 'U' or 'u', the leading m by m upper triangular part of the array A must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of A is not referenced, and when UPLO = 'L' or 'l', the leading m by m lower triangular part of the array A must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of A is not referenced.

Before entry with SIDE = 'R' or 'r', the n by n part of the array A must contain the symmetric matrix, such that when UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of A is not referenced, and when UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of A is not referenced.

Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When SIDE = 'L' or 'l' then LDA >= max(1, m), otherwise LDA >= max(1, n). Unchanged on exit.

B (input)

COMPLEX*16 array of DIMENSION (LDB, n). Before entry, the leading m by n part of the array B must contain the matrix B. Unchanged on exit.

LDB (input)

On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. LDB >= max(1, m). Unchanged on exit.

BETA (input)

On entry, BETA specifies the scalar beta. When BETA is supplied as zero then C need not be set on input. Unchanged on exit.

C (input/output)

COMPLEX*16 array of DIMENSION (LDC, n). Before entry, the leading m by n part of the array C must contain the matrix C, except when beta is zero, in which case C need not be set on entry. On exit, the array C is overwritten by the m by n updated matrix.

LDC (input)

On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC >= max(1, m). Unchanged on exit.

Contents

NAME

```
zsyr2k - perform one of the symmetric rank 2k operations   C
:=  alpha*A*B'  +  alpha*B*A'  + beta*C or C := alpha*A'*B +
alpha*B'*A + beta*C
```

SYNOPSIS

```
SUBROUTINE ZSYR2K(UPLO, TRANSA, N, K, ALPHA, A, LDA, B, LDB, BETA, C,
      LDC)

CHARACTER * 1 UPLO, TRANSA
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER N, K, LDA, LDB, LDC

SUBROUTINE ZSYR2K_64(UPLO, TRANSA, N, K, ALPHA, A, LDA, B, LDB, BETA,
      C, LDC)

CHARACTER * 1 UPLO, TRANSA
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER*8 N, K, LDA, LDB, LDC
```

F95 INTERFACE

```
SUBROUTINE SYR2K(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], B, [LDB],
       BETA, C, [LDC])

CHARACTER(LEN=1) :: UPLO, TRANSA
COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:,:) :: A, B, C
INTEGER :: N, K, LDA, LDB, LDC

SUBROUTINE SYR2K_64(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], B,
```

```
       [LDB], BETA, C, [LDC])

CHARACTER(LEN=1) :: UPLO, TRANSA
COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:,:) :: A, B, C
INTEGER(8) :: N, K, LDA, LDB, LDC
```

C INTERFACE

```
#include <sunperf.h>

void zsyr2k(char uplo, char transa, int n, int k, doublecom-
          plex *alpha, doublecomplex *a, int lda, doublecom-
          plex *b, int ldb, doublecomplex *beta,  doublecom-
          plex *c, int ldc);
void zsyr2k_64(char uplo, char transa, long n, long k, doub-
          lecomplex  *alpha,  doublecomplex  *a,  long  lda,
          doublecomplex *b, long ldb,  doublecomplex  *beta,
          doublecomplex *c, long ldc);
```

PURPOSE

zsyr2k performs one of the symmetric rank 2k operations C := alpha*A*B' + alpha*B*A' + beta*C or C := alpha*A'*B + alpha*B'*A + beta*C where alpha and beta are scalars, C is an n by n symmetric matrix and A and B are n by k matrices in the first case and k by n matrices in the second case.

ARGUMENTS

UPLO (input)

On entry, UPLO specifies whether the upper or lower triangular part of the array C is to be referenced as follows:

UPLO = 'U' or 'u' Only the upper triangular part of C is to be referenced.

UPLO = 'L' or 'l' Only the lower triangular part of C is to be referenced.

Unchanged on exit.

TRANSA (input)

On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   C := alpha*A*B' + alpha*B*A' + beta*C.

TRANSA = 'T' or 't'   C := alpha*A'*B + alpha*B'*A + beta*C.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input)
On entry, N specifies the order of the matrix C. N must be at least zero. Unchanged on exit.

K (input)
On entry with TRANSA = 'N' or 'n', K specifies the number of columns of the matrices A and B, and on entry with TRANSA = 'T' or 't', K specifies the number of rows of the matrices A and B. K must be at least zero. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
COMPLEX*16 array of DIMENSION (LDA, ka), where ka is k when TRANSA = 'N' or 'n', and is n otherwise. Before entry with TRANSA = 'N' or 'n', the leading n by k part of the array A must contain the matrix A, otherwise the leading k by n part of the array A must contain the matrix A. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDA must be at least max(1, n), otherwise LDA must be at least max(1, k). Unchanged on exit.

B (input)
COMPLEX*16 array of DIMENSION (LDB, kb), where kb is k when TRANSA = 'N' or 'n', and is n otherwise. Before entry with TRANSA = 'N' or 'n', the leading n by k part of the array B must contain the matrix B, otherwise the leading k by n part of the array B must contain the matrix B. Unchanged on exit.

LDB (input)

On entry, LDB specifies the first dimension of B as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDB must be at least max(1, n), otherwise LDB must be at least max(1, k). Unchanged on exit.

BETA (input)

On entry, BETA specifies the scalar beta. Unchanged on exit.

C (input/output)

COMPLEX*16 array of DIMENSION (LDC, n).

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array C must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of C is not referenced. On exit, the upper triangular part of the array C is overwritten by the upper triangular part of the updated matrix.

Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array C must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of C is not referenced. On exit, the lower triangular part of the array C is overwritten by the lower triangular part of the updated matrix.

LDC (input)

On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC must be at least max(1, n). Unchanged on exit.

Contents

NAME

```
zsyrfs - improve the computed solution to a system of linear
equations  when the coefficient matrix is symmetric indefin-
ite, and provides error bounds and backward error  estimates
for the solution
```

SYNOPSIS

```
SUBROUTINE ZSYRFS(UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZSYRFS_64(UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE SYRFS(UPLO, N, NRHS, A, [LDA], AF, [LDAF], IPIVOT, B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE SYRFS_64(UPLO, N, NRHS, A, [LDA], AF, [LDAF], IPIVOT, B,
       [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zsyrfs(char uplo, int n, int  nrhs,  doublecomplex  *a,
          int lda, doublecomplex *af, int ldaf, int *ipivot,
          doublecomplex *b, int ldb, doublecomplex  *x,  int
          ldx, double *ferr, double *berr, int *info);

void zsyrfs_64(char uplo, long n, long  nrhs,  doublecomplex
          *a,  long  lda, doublecomplex *af, long ldaf, long
          *ipivot, doublecomplex *b, long ldb, doublecomplex
          *x,  long  ldx,  double  *ferr, double *berr, long
          *info);
```

PURPOSE

zsyrfs improves the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite, and provides error bounds and backward error estimates for the solution.

ARGUMENTS

```
UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.
```

A (input) The symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input)
The factored form of the matrix A. AF contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by CSYTRF.

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CSYTRF.

B (input) The right hand side matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

X (input/output)
On entry, the solution matrix X, as computed by CSYTRS. On exit, the improved solution matrix X.

LDX (input)
The leading dimension of the array X. LDX >= max(1,N).

FERR (output)
The estimated forward error bound for each solution vector X(j) (the j-th column of the solution matrix X). If XTRUE is the true solution corresponding to X(j), FERR(j) is an estimated upper bound for the magnitude of the largest element in (X(j) - XTRUE) divided by the magnitude of the largest element in X(j). The estimate is as

reliable as the estimate for RCOND, and is almost always a slight overestimate of the true error.

BERR (output)
The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zsyrk - perform one of the symmetric rank k  operations    C
:= alpha*A*A' + beta*C or C := alpha*A'*A + beta*C
```

SYNOPSIS

```
SUBROUTINE ZSYRK(UPLO, TRANSA, N, K, ALPHA, A, LDA, BETA, C, LDC)

CHARACTER * 1 UPLO, TRANSA
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(LDA,*), C(LDC,*)
INTEGER N, K, LDA, LDC

SUBROUTINE ZSYRK_64(UPLO, TRANSA, N, K, ALPHA, A, LDA, BETA, C, LDC)

CHARACTER * 1 UPLO, TRANSA
DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX A(LDA,*), C(LDC,*)
INTEGER*8 N, K, LDA, LDC
```

F95 INTERFACE

```
SUBROUTINE SYRK(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], BETA, C,
       [LDC])

CHARACTER(LEN=1) :: UPLO, TRANSA
COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER :: N, K, LDA, LDC

SUBROUTINE SYRK_64(UPLO, [TRANSA], [N], [K], ALPHA, A, [LDA], BETA,
       C, [LDC])

CHARACTER(LEN=1) :: UPLO, TRANSA
```

```
COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER(8) :: N, K, LDA, LDC
```

C INTERFACE

```
#include <sunperf.h>

void zsyrk(char uplo, char transa, int n, int k,  doublecom-
          plex *alpha, doublecomplex *a, int lda, doublecom-
          plex *beta, doublecomplex *c, int ldc);

void zsyrk_64(char uplo, char transa, long n, long k,  doub-
          lecomplex  *alpha,  doublecomplex  *a,  long  lda,
          doublecomplex *beta, doublecomplex *c, long ldc);
```

PURPOSE

zsyrk performs one of the symmetric rank k operations C := alpha*A*A' + beta*C or C := alpha*A'*A + beta*C where alpha and beta are scalars, C is an n by n symmetric matrix and A is an n by k matrix in the first case and a k by n matrix in the second case.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the upper or lower triangular part of the array C is to be referenced as follows:

UPLO = 'U' or 'u' Only the upper triangular part of C is to be referenced.

UPLO = 'L' or 'l' Only the lower triangular part of C is to be referenced.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n' C := alpha*A*A' + beta*C.

TRANSA = 'T' or 't' C := alpha*A'*A + beta*C.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

N (input)
On entry, N specifies the order of the matrix C. N must be at least zero. Unchanged on exit.

K (input)
On entry with TRANSA = 'N' or 'n', K specifies the number of columns of the matrix A, and on entry with TRANSA = 'T' or 't', K specifies the number of rows of the matrix A. K must be at least zero. Unchanged on exit.

ALPHA (input)
On entry, ALPHA specifies the scalar alpha. Unchanged on exit.

A (input)
COMPLEX*16 array of DIMENSION (LDA, ka), where ka is k when TRANSA = 'N' or 'n', and is n otherwise. Before entry with TRANSA = 'N' or 'n', the leading n by k part of the array A must contain the matrix A, otherwise the leading k by n part of the array A must contain the matrix A. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When TRANSA = 'N' or 'n' then LDA must be at least max(1, n), otherwise LDA must be at least max(1, k). Unchanged on exit.

BETA (input)
On entry, BETA specifies the scalar beta. Unchanged on exit.

C (input/output)
COMPLEX*16 array of DIMENSION (LDC, n).

Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array C must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of C is not referenced. On exit, the upper triangular part of the array C is overwritten by the upper triangular part of the updated matrix.

Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array C must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of C is not referenced. On exit, the lower triangular part of the array C is overwritten by the lower triangular part of the updated matrix.

LDC (input)

On entry, LDC specifies the first dimension of C as declared in the calling (sub) program. LDC must be at least max(1, n). Unchanged on exit.

Contents

NAME

```
zsysv - compute the solution to a complex system  of  linear
equations  A * X = B,
```

SYNOPSIS

```
SUBROUTINE ZSYSV(UPLO, N, NRHS, A, LDA, IPIV, B, LDB, WORK, LWORK,
      INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER N, NRHS, LDA, LDB, LWORK, INFO
INTEGER IPIV(*)

SUBROUTINE ZSYSV_64(UPLO, N, NRHS, A, LDA, IPIV, B, LDB, WORK,
      LWORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*), WORK(*)
INTEGER*8 N, NRHS, LDA, LDB, LWORK, INFO
INTEGER*8 IPIV(*)
```

F95 INTERFACE

```
SUBROUTINE SYSV(UPLO, [N], [NRHS], A, [LDA], IPIV, B, [LDB], [WORK],
       [LWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: N, NRHS, LDA, LDB, LWORK, INFO
INTEGER, DIMENSION(:) :: IPIV

SUBROUTINE SYSV_64(UPLO, [N], [NRHS], A, [LDA], IPIV, B, [LDB], [WORK],
```

```
          [LWORK], [INFO])

     CHARACTER(LEN=1) :: UPLO
     COMPLEX(8), DIMENSION(:) :: WORK
     COMPLEX(8), DIMENSION(:,:) :: A, B
     INTEGER(8) :: N, NRHS, LDA, LDB, LWORK, INFO
     INTEGER(8), DIMENSION(:) :: IPIV
```

C INTERFACE

```
#include <sunperf.h>

void zsysv(char uplo, int n, int nrhs, doublecomplex *a, int
          lda,  int  *ipivot, doublecomplex *b, int ldb, int
          *info);
void zsysv_64(char uplo, long n,  long  nrhs,  doublecomplex
          *a, long lda, long *ipivot, doublecomplex *b, long
          ldb, long *info);
```

PURPOSE

zsysv computes the solution to a complex system of linear equations
   A * X = B, where A is an N-by-N symmetric matrix and X and B are N-by-NRHS matrices.

The diagonal pivoting method is used to factor A as
   A = U * D * U**T, if UPLO = 'U', or
   A = L * D * L**T, if UPLO = 'L',
where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks. The factored form of A is then used to solve the system of equations A * X = B.

ARGUMENTS

UPLO (input)
          = 'U': Upper triangle of A is stored;
          = 'L': Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of the matrix A. N >= 0.

NRHS (input)
          The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input/output)

On entry, the symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, if INFO = 0, the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by CSYTRF.

LDA (input)

The leading dimension of the array A. LDA >= max(1,N).

IPIV (output)

Details of the interchanges and the block structure of D, as determined by CSYTRF. If IPIV(k) > 0, then rows and columns k and IPIV(k) were interchanged, and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIV(k) = IPIV(k-1) < 0, then rows and columns k-1 and -IPIV(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIV(k) = IPIV(k+1) < 0, then rows and columns k+1 and -IPIV(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

B (input/output)

On entry, the N-by-NRHS right hand side matrix B. On exit, if INFO = 0, the N-by-NRHS solution matrix X.

LDB (input)

The leading dimension of the array B. LDB >= max(1,N).

WORK (workspace)

On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)

The length of WORK. LWORK >= 1, and for best performance LWORK >= N*NB, where NB is the optimal blocksize for CSYTRF.

If LWORK = -1, then a workspace query is assumed;

the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, so the solution could not be computed.

Contents

NAME

```
zsysvx - use the diagonal pivoting factorization to  compute
the solution to a complex system of linear equations A * X =
B,
```

SYNOPSIS

```
SUBROUTINE ZSYSVX(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT, B,
      LDB, X, LDX, RCOND, FERR, BERR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER IPIVOT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZSYSVX_64(FACT, UPLO, N, NRHS, A, LDA, AF, LDAF, IPIVOT,
      B, LDB, X, LDX, RCOND, FERR, BERR, WORK, LDWORK, WORK2, INFO)

CHARACTER * 1 FACT, UPLO
DOUBLE COMPLEX  A(LDA,*),  AF(LDAF,*),  B(LDB,*),  X(LDX,*),
WORK(*)
INTEGER*8 N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER*8 IPIVOT(*)
DOUBLE PRECISION RCOND
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE SYSVX(FACT, UPLO, N, NRHS, A, [LDA], AF, [LDAF], IPIVOT,
       B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [LDWORK], [WORK2],
       [INFO])
```

```
CHARACTER(LEN=1) :: FACT, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER :: N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER, DIMENSION(:) :: IPIVOT
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2

SUBROUTINE SYSVX_64(FACT, UPLO, N, NRHS, A, [LDA], AF, [LDAF],
       IPIVOT, B, [LDB], X, [LDX], RCOND, FERR, BERR, [WORK], [LDWORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: FACT, UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, AF, B, X
INTEGER(8) :: N, NRHS, LDA, LDAF, LDB, LDX, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void zsysvx(char fact, char uplo, int  n,  int  nrhs,  doub-
          lecomplex  *a,  int  lda,  doublecomplex  *af, int
          ldaf, int  *ipivot,  doublecomplex  *b,  int  ldb,
          doublecomplex  *x,  int ldx, double *rcond, double
          *ferr, double *berr, int *info);

void zsysvx_64(char fact, char  uplo,  long  n,  long  nrhs,
          doublecomplex  *a,  long  lda,  doublecomplex *af,
          long ldaf, long *ipivot,  doublecomplex  *b,  long
          ldb,  doublecomplex  *x,  long ldx, double *rcond,
          double *ferr, double *berr, long *info);
```

PURPOSE

zsysvx uses the diagonal pivoting factorization to compute the solution to a complex system of linear equations A * X = B, where A is an N-by-N symmetric matrix and X and B are N-by-NRHS matrices.

Error bounds on the solution and a condition estimate are also provided.

The following steps are performed:

1. If FACT = 'N', the diagonal pivoting method is used to

```
factor A.
   The form of the factorization is
      A = U * D * U**T,  if UPLO = 'U', or
      A = L * D * L**T,  if UPLO = 'L',
   where U (or L) is a product of permutation and unit upper
(lower)
   triangular matrices, and D is symmetric and block  diago-
nal with
   1-by-1 and 2-by-2 diagonal blocks.

2. If some D(i,i)=0, so that D is exactly singular, then the
routine
   returns with INFO = i. Otherwise, the factored form of  A
is used
   to estimate the condition number of the matrix A.  If the
   reciprocal of the condition number is less  than  machine
precision,
   INFO = N+1 is returned as  a  warning,  but  the  routine
still goes on
   to solve for X and  compute  error  bounds  as  described
below.

3. The system of equations is solved for X  using  the  fac-
tored form
   of A.

4. Iterative refinement is applied to improve  the  computed
solution
   matrix and calculate  error  bounds  and  backward  error
estimates
   for it.
```

ARGUMENTS

```
FACT (input)
          Specifies whether or not the factored  form  of  A
          has  been supplied on entry.  = 'F':  On entry, AF
          and IPIVOT contain the factored form of A.  A,  AF
          and  IPIVOT  will  not  be  modified.  = 'N':  The
          matrix A will be copied to AF and factored.

UPLO (input)
          = 'U':  Upper triangle of A is stored;
          = 'L':  Lower triangle of A is stored.

N (input) The number of linear equations, i.e., the order of
          the matrix A.  N >= 0.
```

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrices B and X. NRHS >= 0.

A (input) The symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

AF (input/output)
If FACT = 'F', then AF is an input argument and on entry contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T as computed by CSYTRF.

If FACT = 'N', then AF is an output argument and on exit returns the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization A = U*D*U**T or A = L*D*L**T.

LDAF (input)
The leading dimension of the array AF. LDAF >= max(1,N).

IPIVOT (input or output)
If FACT = 'F', then IPIVOT is an input argument and on entry contains details of the interchanges and the block structure of D, as determined by CSYTRF. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

If FACT = 'N', then IPIVOT is an output argument and on exit contains details of the interchanges and the block structure of D, as determined by

```
          CSYTRF.

B (input) The N-by-NRHS right hand side matrix B.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

X (output)
          If INFO = 0 or INFO = N+1, the N-by-NRHS  solution
          matrix X.
LDX (input)
          The leading dimension of  the  array  X.   LDX  >=
          max(1,N).

RCOND (output)
          The estimate of the reciprocal condition number of
          the  matrix  A.  If RCOND is less than the machine
          precision (in  particular,  if  RCOND  =  0),  the
          matrix  is  singular  to  working precision.  This
          condition is indicated by a return code of INFO  >
          0.

FERR (output)
          The estimated forward error bound for  each  solu-
          tion  vector X(j) (the j-th column of the solution
          matrix  X).   If  XTRUE  is  the   true   solution
          corresponding  to  X(j),  FERR(j)  is an estimated
          upper bound for the magnitude of the largest  ele-
          ment in (X(j) - XTRUE) divided by the magnitude of
          the largest element in X(j).  The estimate  is  as
          reliable  as the estimate for RCOND, and is almost
          always a slight overestimate of the true error.

BERR (output)
          The componentwise relative backward error of  each
          solution  vector X(j) (i.e., the smallest relative
          change in any element of A or B that makes X(j) an
          exact solution).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LDWORK.

LDWORK (input)
          The length of WORK.  LDWORK >= 2*N, and  for  best
          performance  LDWORK  >=  N*NB,  where  NB  is  the
          optimal blocksize for CSYTRF.

          If LDWORK = -1, then a workspace query is assumed;
```

the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, and i is
<= N: D(i,i) is exactly zero. The factorization has been completed but the factor D is exactly singular, so the solution and error bounds could not be computed. RCOND = 0 is returned. = N+1: D is nonsingular, but RCOND is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of RCOND would suggest.

Contents

NAME

```
zsytf2 - compute the factorization of  a  complex  symmetric
matrix A using the Bunch-Kaufman diagonal pivoting method
```

SYNOPSIS

```
SUBROUTINE ZSYTF2(UPLO, N, A, LDA, IPIV, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*)
INTEGER N, LDA, INFO
INTEGER IPIV(*)

SUBROUTINE ZSYTF2_64(UPLO, N, A, LDA, IPIV, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*)
INTEGER*8 N, LDA, INFO
INTEGER*8 IPIV(*)
```

F95 INTERFACE

```
SUBROUTINE SYTF2(UPLO, [N], A, [LDA], IPIV, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIV

SUBROUTINE SYTF2_64(UPLO, [N], A, [LDA], IPIV, [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A
```

```
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIV
```

C INTERFACE

```
#include <sunperf.h>

void zsytf2(char uplo, int n, doublecomplex *a, int lda, int
          *ipiv, int *info);

void zsytf2_64(char uplo, long  n,  doublecomplex  *a,  long
          lda, long *ipiv, long *info);
```

PURPOSE

zsytf2 computes the factorization of a complex symmetric matrix A using the Bunch-Kaufman diagonal pivoting method:

```
   A = U*D*U'  or  A = L*D*L'
```

where U (or L) is a product of permutation and unit upper (lower) triangular matrices, U' is the transpose of U, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

This is the unblocked version of the algorithm, calling Level 2 BLAS.

ARGUMENTS

UPLO (input)
Specifies whether the upper or lower triangular part of the symmetric matrix A is stored:
= 'U': Upper triangular
= 'L': Lower triangular

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the symmetric matrix A. If UPLO = 'U', the leading n-by-n upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, the block diagonal matrix D and the mul-

tipliers used to obtain the factor U or L (see below for further details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIV (output)
Details of the interchanges and the block structure of D. If IPIV(k) > 0, then rows and columns k and IPIV(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIV(k) = IPIV(k-1) < 0, then rows and columns k-1 and -IPIV(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIV(k) = IPIV(k+1) < 0, then rows and columns k+1 and -IPIV(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an illegal value
> 0: if INFO = k, D(k,k) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

1-96 - Based on modifications by J. Lewis, Boeing Computer Services
Company

If UPLO = 'U', then A = U*D*U', where
U = P(n)*U(n)* ... *P(k)U(k)* ...,
i.e., U is a product of terms P(k)*U(k), where k decreases from n to 1 in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIV(k), and U(k) is a unit upper triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    v    0   )   k-s
   U(k) =  (   0    I    0   )   s
           (   0    0    I   )   n-k
              k-s   s   n-k
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(1:k-

1,k).   If s = 2, the upper triangle of D(k) overwrites A(k-1,k-1), A(k-1,k), and A(k,k), and v overwrites A(1:k-2,k-1:k).

If UPLO = 'L', then A = L*D*L', where

```
   L = P(1)*L(1)* ... *P(k)*L(k)* ...,
```

i.e., L is a product of terms P(k)*L(k), where k increases from 1 to n in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k). P(k) is a permutation matrix as defined by IPIV(k), and L(k) is a unit lower triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

```
           (   I    0     0   )  k-1
   L(k) =  (   0    I     0   )  s
           (   0    v     I   )  n-k-s+1
              k-1   s  n-k-s+1
```

If s = 1, D(k) overwrites A(k,k), and v overwrites A(k+1:n,k). If s = 2, the lower triangle of D(k) overwrites A(k,k), A(k+1,k), and A(k+1,k+1), and v overwrites A(k+2:n,k:k+1).

Contents

NAME

zsytrf - compute the factorization of a complex symmetric matrix A using the Bunch-Kaufman diagonal pivoting method

SYNOPSIS

```
SUBROUTINE ZSYTRF(UPLO, N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, LDWORK, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZSYTRF_64(UPLO, N, A, LDA, IPIVOT, WORK, LDWORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, LDWORK, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE SYTRF(UPLO, N, A, [LDA], IPIVOT, [WORK], [LDWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, LDWORK, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE SYTRF_64(UPLO, N, A, [LDA], IPIVOT, [WORK], [LDWORK],
       [INFO])
```

```
CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, LDWORK, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void zsytrf(char uplo, int n, doublecomplex *a, int lda, int
          *ipivot, int *info);

void zsytrf_64(char uplo, long  n,  doublecomplex  *a,  long
          lda, long *ipivot, long *info);
```

PURPOSE

zsytrf computes the factorization of a complex symmetric matrix A using the Bunch-Kaufman diagonal pivoting method. The form of the factorization is

A = U*D*U**T or A = L*D*L**T

where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with with 1-by-1 and 2-by-2 diagonal blocks.

This is the blocked version of the algorithm, calling Level 3 BLAS.

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A is stored;
= 'L': Lower triangle of A is stored.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the symmetric matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit, the block diagonal matrix D and the multipliers used to obtain the factor U or L (see below for further details).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (output)
Details of the interchanges and the block structure of D. If IPIVOT(k) > 0, then rows and columns k and IPIVOT(k) were interchanged and D(k,k) is a 1-by-1 diagonal block. If UPLO = 'U' and IPIVOT(k) = IPIVOT(k-1) < 0, then rows and columns k-1 and -IPIVOT(k) were interchanged and D(k-1:k,k-1:k) is a 2-by-2 diagonal block. If UPLO = 'L' and IPIVOT(k) = IPIVOT(k+1) < 0, then rows and columns k+1 and -IPIVOT(k) were interchanged and D(k:k+1,k:k+1) is a 2-by-2 diagonal block.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LDWORK.

LDWORK (input)
The length of WORK. LDWORK >=1. For best performance LDWORK >= N*NB, where NB is the block size returned by ILAENV.

If LDWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LDWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

FURTHER DETAILS

If UPLO = 'U', then A = U*D*U', where

```
   U = P(n)*U(n)* ... *P(k)U(k)* ...,
i.e., U is a product of terms P(k)*U(k), where  k  decreases
from  n  to  1 in steps of 1 or 2, and D is a block diagonal
matrix with 1-by-1 and 2-by-2 diagonal blocks D(k).  P(k) is
a  permutation matrix as defined by IPIVOT(k), and U(k) is a
unit upper triangular matrix,  such  that  if  the  diagonal
block D(k) is of order s (s = 1 or 2), then

           (   I    v    0   )   k-s
   U(k) =  (   0    I    0   )   s
           (   0    0    I   )   n-k
              k-s   s   n-k

If s = 1, D(k) overwrites A(k,k), and  v  overwrites  A(1:k-
1,k).   If s = 2, the upper triangle of D(k) overwrites A(k-
1,k-1), A(k-1,k), and A(k,k), and  v  overwrites  A(1:k-2,k-
1:k).

If UPLO = 'L', then A = L*D*L', where
   L = P(1)*L(1)* ... *P(k)*L(k)* ...,
i.e., L is a product of terms P(k)*L(k), where  k  increases
from  1  to  n in steps of 1 or 2, and D is a block diagonal
matrix with 1-by-1 and 2-by-2 diagonal blocks D(k).  P(k) is
a  permutation matrix as defined by IPIVOT(k), and L(k) is a
unit lower triangular matrix,  such  that  if  the  diagonal
block D(k) is of order s (s = 1 or 2), then

           (   I    0     0   )  k-1
   L(k) =  (   0    I     0   )  s
           (   0    v     I   )  n-k-s+1
              k-1   s  n-k-s+1

If  s  =  1,  D(k)  overwrites  A(k,k),  and  v   overwrites
A(k+1:n,k).  If s = 2, the lower triangle of D(k) overwrites
A(k,k),  A(k+1,k),  and   A(k+1,k+1),   and   v   overwrites
A(k+2:n,k:k+1).
```

Contents

NAME

zsytri - compute the inverse of a complex symmetric indefinite matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by CSYTRF

SYNOPSIS

```
SUBROUTINE ZSYTRI(UPLO, N, A, LDA, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZSYTRI_64(UPLO, N, A, LDA, IPIVOT, WORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE SYTRI(UPLO, N, A, [LDA], IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE SYTRI_64(UPLO, N, A, [LDA], IPIVOT, [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
```

```
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void zsytri(char uplo, int n, doublecomplex *a, int lda, int
          *ipivot, int *info);

void zsytri_64(char uplo, long  n,  doublecomplex  *a,  long
          lda, long *ipivot, long *info);
```

PURPOSE

zsytri computes the inverse of a complex symmetric indefinite matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by CSYTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

A (input/output)
On entry, the block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CSYTRF.

On exit, if INFO = 0, the (symmetric) inverse of the original matrix. If UPLO = 'U', the upper triangular part of the inverse is formed and the part of A below the diagonal is not referenced; if UPLO = 'L' the lower triangular part of the inverse is formed and the part of A above the diagonal is not referenced.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CSYTRF.

WORK (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, D(i,i) = 0; the matrix is singular and its inverse could not be computed.

Contents

NAME

zsytrs - solve a system of linear equations A*X = B with a complex symmetric matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by CSYTRF

SYNOPSIS

```
SUBROUTINE ZSYTRS(UPLO, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, NRHS, LDA, LDB, INFO
INTEGER IPIVOT(*)

SUBROUTINE ZSYTRS_64(UPLO, N, NRHS, A, LDA, IPIVOT, B, LDB, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, NRHS, LDA, LDB, INFO
INTEGER*8 IPIVOT(*)
```

F95 INTERFACE

```
SUBROUTINE SYTRS(UPLO, N, NRHS, A, [LDA], IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: N, NRHS, LDA, LDB, INFO
INTEGER, DIMENSION(:) :: IPIVOT

SUBROUTINE SYTRS_64(UPLO, N, NRHS, A, [LDA], IPIVOT, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:,:) :: A, B
```

```
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
INTEGER(8), DIMENSION(:) :: IPIVOT
```

C INTERFACE

```
#include <sunperf.h>

void zsytrs(char uplo, int n, int  nrhs,  doublecomplex  *a,
          int  lda,  int *ipivot, doublecomplex *b, int ldb,
          int *info);

void zsytrs_64(char uplo, long n, long  nrhs,  doublecomplex
          *a, long lda, long *ipivot, doublecomplex *b, long
          ldb, long *info);
```

PURPOSE

zsytrs solves a system of linear equations A*X = B with a complex symmetric matrix A using the factorization A = U*D*U**T or A = L*D*L**T computed by CSYTRF.

ARGUMENTS

UPLO (input)
Specifies whether the details of the factorization are stored as an upper or lower triangular matrix.
= 'U': Upper triangular, form is A = U*D*U**T;
= 'L': Lower triangular, form is A = L*D*L**T.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by CSYTRF.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

IPIVOT (input)
Details of the interchanges and the block structure of D as determined by CSYTRF.

B (input/output)

On entry, the right hand side matrix B. On exit, the solution matrix X.

LDB (input)

The leading dimension of the array B. LDB >= max(1,N).

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
ztbcon - estimate the reciprocal of the condition number  of
a  triangular  band  matrix  A,  in either the 1-norm or the
infinity-norm
```

SYNOPSIS

```
SUBROUTINE ZTBCON(NORM, UPLO, DIAG, N, KD, A, LDA, RCOND, WORK,
      WORK2, INFO)

CHARACTER * 1 NORM, UPLO, DIAG
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER N, KD, LDA, INFO
DOUBLE PRECISION RCOND
DOUBLE PRECISION WORK2(*)

SUBROUTINE ZTBCON_64(NORM, UPLO, DIAG, N, KD, A, LDA, RCOND, WORK,
      WORK2, INFO)

CHARACTER * 1 NORM, UPLO, DIAG
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, KD, LDA, INFO
DOUBLE PRECISION RCOND
DOUBLE PRECISION WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE TBCON(NORM, UPLO, DIAG, N, KD, A, [LDA], RCOND, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, KD, LDA, INFO
```

```
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: WORK2

SUBROUTINE TBCON_64(NORM, UPLO, DIAG, N, KD, A, [LDA], RCOND,
       [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, KD, LDA, INFO
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>
void ztbcon(char norm, char uplo, char diag, int n, int  kd,
          doublecomplex  *a,  int  lda,  double  *rcond, int
          *info);

void ztbcon_64(char norm, char uplo, char diag, long n, long
          kd,  doublecomplex  *a,  long  lda, double *rcond,
          long *info);
```

PURPOSE

ztbcon estimates the reciprocal of the condition number of a triangular band matrix A, in either the 1-norm or the infinity-norm.

The norm of A is computed and an estimate is obtained for norm(inv(A)), then the reciprocal of the condition number is computed as

RCOND = 1 / (norm(A) * norm(inv(A))).

ARGUMENTS

```
NORM (input)
          Specifies whether the 1-norm condition  number  or
          the infinity-norm condition number is required:
          = '1' or 'O':  1-norm;
          = 'I':         Infinity-norm.

UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

DIAG (input)
```

= 'N': A is non-unit triangular;
= 'U': A is unit triangular.

N (input) The order of the matrix A. N >= 0.

KD (input)
The number of superdiagonals or subdiagonals of the triangular band matrix A. KD >= 0.

A (input) The upper or lower triangular band matrix A, stored in the first kd+1 rows of the array. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd). If DIAG = 'U', the diagonal elements of A are not referenced and are assumed to be 1.

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(norm(A) * norm(inv(A))).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

ztbmv - perform one of the matrix-vector operations x := A*x, or x := A'*x, or x := conjg(A')*x

SYNOPSIS

```
SUBROUTINE ZTBMV(UPLO, TRANSA, DIAG, N, K, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(LDA,*), Y(*)
INTEGER N, K, LDA, INCY

SUBROUTINE ZTBMV_64(UPLO, TRANSA, DIAG, N, K, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(LDA,*), Y(*)
INTEGER*8 N, K, LDA, INCY
```

F95 INTERFACE

```
SUBROUTINE TBMV(UPLO, [TRANSA], DIAG, [N], K, A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, K, LDA, INCY

SUBROUTINE TBMV_64(UPLO, [TRANSA], DIAG, [N], K, A, [LDA], Y,
       [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, K, LDA, INCY
```

C INTERFACE

```
#include <sunperf.h>

void ztbmv(char uplo, char transa, char diag, int n, int  k,
          doublecomplex  *a,  int lda, doublecomplex *y, int
          incy);

void ztbmv_64(char uplo, char transa,  char  diag,  long  n,
          long  k, doublecomplex *a, long lda, doublecomplex
          *y, long incy);
```

PURPOSE

ztbmv performs one of the matrix-vector operations x := A*x, or x := A'*x, or x := conjg(A')*x where x is an n element vector and A is an n by n unit, or non-unit, upper or lower triangular band matrix, with (k + 1) diagonals.

ARGUMENTS

UPLO (input)

On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)

On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   x := A*x.

TRANSA = 'T' or 't'   x := A'*x.

TRANSA = 'C' or 'c'   x := conjg(A')*x.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit triangular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

K (input)
On entry with UPLO = 'U' or 'u', K specifies the number of super-diagonals of the matrix A. On entry with UPLO = 'L' or 'l', K specifies the number of sub-diagonals of the matrix A. K >= 0. Unchanged on exit.

A (input)
Before entry with UPLO = 'U' or 'u', the leading (k + 1) by n part of the array A must contain the upper triangular band part of the matrix of coefficients, supplied column by column, with the leading diagonal of the matrix in row (k + 1) of the array, the first super-diagonal starting at position 2 in row k, and so on. The top left k by k triangle of the array A is not referenced. The following program segment will transfer an upper triangular band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = K + 1 - J
     DO 10, I = MAX( 1, J - K ), J
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Before entry with UPLO = 'L' or 'l', the leading (k + 1) by n part of the array A must contain the lower triangular band part of the matrix of coefficients, supplied column by column, with the leading diagonal of the matrix in row 1 of the array, the first sub-diagonal starting at position 1 in row 2, and so on. The bottom right k by k triangle of the array A is not referenced. The following program segment will transfer a lower triangular band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = 1 - J
     DO 10, I = J, MIN( N, J + K )
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Note that when DIAG = 'U' or 'u' the elements of the array A corresponding to the diagonal elements of the matrix are not referenced, but are assumed to be unity. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= (k + 1). Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector x. On exit, Y is overwritten with the transformed vector x.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

ztbrfs - provide error bounds and backward error estimates for the solution to a system of linear equations with a triangular band coefficient matrix

SYNOPSIS

```
SUBROUTINE ZTBRFS(UPLO, TRANSA, DIAG, N, KD, NRHS, A, LDA, B, LDB,
      X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(LDA,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, KD, NRHS, LDA, LDB, LDX, INFO
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZTBRFS_64(UPLO, TRANSA, DIAG, N, KD, NRHS, A, LDA, B,
      LDB, X, LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(LDA,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, KD, NRHS, LDA, LDB, LDX, INFO
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE TBRFS(UPLO, [TRANSA], DIAG, N, KD, NRHS, A, [LDA], B,
       [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, X
INTEGER :: N, KD, NRHS, LDA, LDB, LDX, INFO
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

```
SUBROUTINE TBRFS_64(UPLO, [TRANSA], DIAG, N, KD, NRHS, A, [LDA],
       B, [LDB], X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, X
INTEGER(8) :: N, KD, NRHS, LDA, LDB, LDX, INFO
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void ztbrfs(char uplo, char transa, char diag,  int  n,  int
          kd,  int  nrhs,  doublecomplex  *a, int lda, doub-
          lecomplex *b, int ldb, doublecomplex *x, int  ldx,
          double *ferr, double *berr, int *info);

void ztbrfs_64(char uplo, char transa, char  diag,  long  n,
          long  kd,  long  nrhs, doublecomplex *a, long lda,
          doublecomplex *b, long ldb, doublecomplex *x, long
          ldx, double *ferr, double *berr, long *info);
```

PURPOSE

ztbrfs provides error bounds and backward error estimates for the solution to a system of linear equations with a triangular band coefficient matrix.

The solution matrix X must be computed by CTBTRS or some other means before entering this routine. CTBRFS does not do iterative refinement because doing so cannot improve the backward error.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.
```

```
DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals  or  subdiagonals  of
          the triangular band matrix A.  KD >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.
A (input) The upper  or  lower  triangular  band  matrix  A,
          stored  in  the  first kd+1 rows of the array. The
          j-th column of A is stored in the j-th  column  of
          the  array A as follows:  if UPLO = 'U', A(kd+1+i-
          j,j) = A(i,j) for  max(1,j-kd)<=i<=j;  if  UPLO  =
          'L', A(1+i-j,j)    = A(i,j) for j<=i<=min(n,j+kd).
          If DIAG = 'U', the diagonal elements of A are  not
          referenced and are assumed to be 1.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          KD+1.

B (input) The right hand side matrix B.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

X (input) The solution matrix X.

LDX (input)
          The leading dimension of  the  array  X.   LDX  >=
          max(1,N).

FERR (output)
          The estimated forward error bound for  each  solu-
          tion  vector X(j) (the j-th column of the solution
          matrix  X).   If  XTRUE  is  the   true   solution
          corresponding  to  X(j),  FERR(j)  is an estimated
          upper bound for the magnitude of the largest  ele-
          ment in (X(j) - XTRUE) divided by the magnitude of
          the largest element in X(j).  The estimate  is  as
          reliable  as the estimate for RCOND, and is almost
          always a slight overestimate of the true error.

BERR (output)
```

The componentwise relative backward error of each solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
ztbsv - solve one of the systems of equations   A*x = b,  or
A'*x = b, or conjg( A' )*x = b
```

SYNOPSIS

```
SUBROUTINE ZTBSV(UPLO, TRANSA, DIAG, N, K, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(LDA,*), Y(*)
INTEGER N, K, LDA, INCY

SUBROUTINE ZTBSV_64(UPLO, TRANSA, DIAG, N, K, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(LDA,*), Y(*)
INTEGER*8 N, K, LDA, INCY
```

F95 INTERFACE

```
SUBROUTINE TBSV(UPLO, [TRANSA], DIAG, [N], K, A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, K, LDA, INCY

SUBROUTINE TBSV_64(UPLO, [TRANSA], DIAG, [N], K, A, [LDA], Y,
       [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, K, LDA, INCY
```

C INTERFACE

```
#include <sunperf.h>

void ztbsv(char uplo, char transa, char diag, int n, int  k,
          doublecomplex  *a,  int lda, doublecomplex *y, int
          incy);

void ztbsv_64(char uplo, char transa,  char  diag,  long  n,
          long  k, doublecomplex *a, long lda, doublecomplex
          *y, long incy);
```

PURPOSE

ztbsv solves one of the systems of equations A*x = b, or A'*x = b, or conjg(A')*x = b where b and x are n element vectors and A is an n by n unit, or non-unit, upper or lower triangular band matrix, with (k + 1) diagonals.

No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the equations to be solved as follows:

TRANSA = 'N' or 'n'   A*x = b.

TRANSA = 'T' or 't'   A'*x = b.

TRANSA = 'C' or 'c'   conjg(A')*x = b.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit triangular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

K (input)
On entry with UPLO = 'U' or 'u', K specifies the number of super-diagonals of the matrix A. On entry with UPLO = 'L' or 'l', K specifies the number of sub-diagonals of the matrix A. K >= 0. Unchanged on exit.

A (input)
Before entry with UPLO = 'U' or 'u', the leading (k + 1) by n part of the array A must contain the upper triangular band part of the matrix of coefficients, supplied column by column, with the leading diagonal of the matrix in row (k + 1) of the array, the first super-diagonal starting at position 2 in row k, and so on. The top left k by k triangle of the array A is not referenced. The following program segment will transfer an upper triangular band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = K + 1 - J
     DO 10, I = MAX( 1, J - K ), J
       A( M + I, J ) = matrix( I, J )
10   CONTINUE
20 CONTINUE
```

Before entry with UPLO = 'L' or 'l', the leading (k + 1) by n part of the array A must contain the lower triangular band part of the matrix of coefficients, supplied column by column, with the

leading diagonal of the matrix in row 1 of the array, the first sub-diagonal starting at position 1 in row 2, and so on. The bottom right k by k triangle of the array A is not referenced. The following program segment will transfer a lower triangular band matrix from conventional full matrix storage to band storage:

```
   DO 20, J = 1, N
     M = 1 - J
     DO 10, I = J, MIN( N, J + K )
       A( M + I, J ) = matrix( I, J )
10  CONTINUE
20 CONTINUE
```

Note that when DIAG = 'U' or 'u' the elements of the array A corresponding to the diagonal elements of the matrix are not referenced, but are assumed to be unity. Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= (k + 1). Unchanged on exit.

Y (input/output)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element right-hand side vector b. On exit, Y is overwritten with the solution vector x.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
ztbtrs - solve a triangular system of the form   A * X =  B,
A**T * X = B, or A**H * X = B,
```

SYNOPSIS

```
SUBROUTINE ZTBTRS(UPLO, TRANSA, DIAG, N, KD, NRHS, A, LDA, B, LDB,
      INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, KD, NRHS, LDA, LDB, INFO

SUBROUTINE ZTBTRS_64(UPLO, TRANSA, DIAG, N, KD, NRHS, A, LDA, B,
      LDB, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, KD, NRHS, LDA, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE TBTRS(UPLO, TRANSA, DIAG, N, KD, NRHS, A, [LDA], B,
       [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: N, KD, NRHS, LDA, LDB, INFO

SUBROUTINE TBTRS_64(UPLO, TRANSA, DIAG, N, KD, NRHS, A, [LDA], B,
       [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:,:) :: A, B
```

```
INTEGER(8) :: N, KD, NRHS, LDA, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void ztbtrs(char uplo, char transa, char diag,  int  n,  int
          kd,  int  nrhs,  doublecomplex  *a, int lda, doub-
          lecomplex *b, int ldb, int *info);

void ztbtrs_64(char uplo, char transa, char  diag,  long  n,
          long  kd,  long  nrhs, doublecomplex *a, long lda,
          doublecomplex *b, long ldb, long *info);
```

PURPOSE

ztbtrs solves a triangular system of the form

where A is a triangular band matrix of order N, and B is an N-by-NRHS matrix. A check is made to verify that A is non-singular.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

KD (input)
          The number of superdiagonals  or  subdiagonals  of
          the triangular band matrix A.  KD >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.
```

A (input) The upper or lower triangular band matrix A, stored in the first kd+1 rows of A. The j-th column of A is stored in the j-th column of the array A as follows: if UPLO = 'U', A(kd+1+i-j,j) = A(i,j) for max(1,j-kd)<=i<=j; if UPLO = 'L', A(1+i-j,j) = A(i,j) for j<=i<=min(n,j+kd). If DIAG = 'U', the diagonal elements of A are not referenced and are assumed to be 1.

LDA (input)
The leading dimension of the array A. LDA >= KD+1.

B (input/output)
On entry, the right hand side matrix B. On exit, if INFO = 0, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the i-th diagonal element of A is zero, indicating that the matrix is singular and the solutions X have not been computed.

Contents

NAME

ztgevc - compute some or all of the right and/or left generalized eigenvectors of a pair of complex upper triangular matrices (A,B)

SYNOPSIS

```
SUBROUTINE ZTGEVC(SIDE, HOWMNY, SELECT, N, A, LDA, B, LDB, VL, LDVL,
      VR, LDVR, MM, M, WORK, RWORK, INFO)

CHARACTER * 1 SIDE, HOWMNY
DOUBLE COMPLEX A(LDA,*), B(LDB,*),  VL(LDVL,*),  VR(LDVR,*),
WORK(*)
INTEGER N, LDA, LDB, LDVL, LDVR, MM, M, INFO
LOGICAL SELECT(*)
DOUBLE PRECISION RWORK(*)

SUBROUTINE ZTGEVC_64(SIDE, HOWMNY, SELECT, N, A, LDA, B, LDB, VL,
      LDVL, VR, LDVR, MM, M, WORK, RWORK, INFO)

CHARACTER * 1 SIDE, HOWMNY
DOUBLE COMPLEX A(LDA,*), B(LDB,*),  VL(LDVL,*),  VR(LDVR,*),
WORK(*)
INTEGER*8 N, LDA, LDB, LDVL, LDVR, MM, M, INFO
LOGICAL*8 SELECT(*)
DOUBLE PRECISION RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE TGEVC(SIDE, HOWMNY, SELECT, [N], A, [LDA], B, [LDB], VL,
       [LDVL], VR, [LDVR], MM, M, [WORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, HOWMNY
COMPLEX(8), DIMENSION(:) :: WORK
```

```
COMPLEX(8), DIMENSION(:,:) :: A, B, VL, VR
INTEGER :: N, LDA, LDB, LDVL, LDVR, MM, M, INFO
LOGICAL, DIMENSION(:) :: SELECT
REAL(8), DIMENSION(:) :: RWORK

SUBROUTINE TGEVC_64(SIDE, HOWMNY, SELECT, [N], A, [LDA], B, [LDB],
       VL, [LDVL], VR, [LDVR], MM, M, [WORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, HOWMNY
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, VL, VR
INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, MM, M, INFO
LOGICAL(8), DIMENSION(:) :: SELECT
REAL(8), DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>

void ztgevc(char side, char  howmny,  int  *select,  int  n,
          doublecomplex  *a,  int lda, doublecomplex *b, int
          ldb, doublecomplex *vl,  int  ldvl,  doublecomplex
          *vr, int ldvr, int mm, int *m, int *info);

void ztgevc_64(char side, char howmny, long *select, long n,
          doublecomplex *a, long lda, doublecomplex *b, long
          ldb, doublecomplex *vl, long  ldvl,  doublecomplex
          *vr, long ldvr, long mm, long *m, long *info);
```

PURPOSE

ztgevc computes some or all of the right and/or left generalized eigenvectors of a pair of complex upper triangular matrices (A,B).

The right generalized eigenvector x and the left generalized eigenvector y of (A,B) corresponding to a generalized eigenvalue w are defined by:

```
        (A - wB) * x = 0  and  y**H * (A - wB) = 0
```

where y**H denotes the conjugate tranpose of y.

If an eigenvalue w is determined by zero diagonal elements of both A and B, a unit vector is returned as the corresponding eigenvector.

If all eigenvectors are requested, the routine may either return the matrices X and/or Y of right or left eigenvectors of (A,B), or the products Z*X and/or Q*Y, where Z and Q are input unitary matrices. If (A,B) was obtained from the gen-

eralized Schur factorization of an original pair of matrices
   (A0,B0) = (Q*A*Z**H,Q*B*Z**H),
then Z*X and Q*Y are the matrices of right or left eigenvectors of A.

ARGUMENTS

SIDE (input)
= 'R': compute right eigenvectors only;
= 'L': compute left eigenvectors only;
= 'B': compute both right and left eigenvectors.

HOWMNY (input)
= 'A': compute all right and/or left eigenvectors;
= 'B': compute all right and/or left eigenvectors, and backtransform them using the input matrices supplied in VR and/or VL; = 'S': compute selected right and/or left eigenvectors, specified by the logical array SELECT.

SELECT (input)
If HOWMNY='S', SELECT specifies the eigenvectors to be computed. If HOWMNY='A' or 'B', SELECT is not referenced. To select the eigenvector corresponding to the j-th eigenvalue, SELECT(j) must be set to .TRUE..

N (input) The order of the matrices A and B. N >= 0.

A (input) The upper triangular matrix A.

LDA (input)
The leading dimension of array A. LDA >= max(1,N).

B (input) The upper triangular matrix B. B must have real diagonal elements.

LDB (input)
The leading dimension of array B. LDB >= max(1,N).

VL (input/output)
On entry, if SIDE = 'L' or 'B' and HOWMNY = 'B', VL must contain an N-by-N matrix Q (usually the unitary matrix Q of left Schur vectors returned by CHGEQZ). On exit, if SIDE = 'L' or 'B', VL contains: if HOWMNY = 'A', the matrix Y of left

eigenvectors of (A,B); if HOWMNY = 'B', the matrix Q*Y; if HOWMNY = 'S', the left eigenvectors of (A,B) specified by SELECT, stored consecutively in the columns of VL, in the same order as their eigenvalues. If SIDE = 'R', VL is not referenced.

LDVL (input)
The leading dimension of array VL. LDVL >= max(1,N) if SIDE = 'L' or 'B'; LDVL >= 1 otherwise.

VR (input/output)
On entry, if SIDE = 'R' or 'B' and HOWMNY = 'B', VR must contain an N-by-N matrix Q (usually the unitary matrix Z of right Schur vectors returned by CHGEQZ). On exit, if SIDE = 'R' or 'B', VR contains: if HOWMNY = 'A', the matrix X of right eigenvectors of (A,B); if HOWMNY = 'B', the matrix Z*X; if HOWMNY = 'S', the right eigenvectors of (A,B) specified by SELECT, stored consecutively in the columns of VR, in the same order as their eigenvalues. If SIDE = 'L', VR is not referenced.

LDVR (input)
The leading dimension of the array VR. LDVR >= max(1,N) if SIDE = 'R' or 'B'; LDVR >= 1 otherwise.

MM (input)
The number of columns in the arrays VL and/or VR. MM >= M.

M (output)
The number of columns in the arrays VL and/or VR actually used to store the eigenvectors. If HOWMNY = 'A' or 'B', M is set to N. Each selected eigenvector occupies one column.

WORK (workspace)
dimension(2*N)

RWORK (workspace)
dimension(2*N)

INFO (output)
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

Contents

NAME

ztgexc - reorder the generalized Schur decomposition of a complex matrix pair (A,B), using an unitary equivalence transformation (A, B) := Q * (A, B) * Z', so that the diagonal block of (A, B) with row index IFST is moved to row ILST

SYNOPSIS

```
SUBROUTINE ZTGEXC(WANTQ, WANTZ, N, A, LDA, B, LDB, Q, LDQ, Z, LDZ,
      IFST, ILST, INFO)

DOUBLE COMPLEX A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*)
INTEGER N, LDA, LDB, LDQ, LDZ, IFST, ILST, INFO
LOGICAL WANTQ, WANTZ

SUBROUTINE ZTGEXC_64(WANTQ, WANTZ, N, A, LDA, B, LDB, Q, LDQ, Z, LDZ,
      IFST, ILST, INFO)

DOUBLE COMPLEX A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*)
INTEGER*8 N, LDA, LDB, LDQ, LDZ, IFST, ILST, INFO
LOGICAL*8 WANTQ, WANTZ
```

F95 INTERFACE

```
SUBROUTINE TGEXC(WANTQ, WANTZ, [N], A, [LDA], B, [LDB], Q, [LDQ], Z,
       [LDZ], IFST, ILST, [INFO])

COMPLEX(8), DIMENSION(:,:) :: A, B, Q, Z
INTEGER :: N, LDA, LDB, LDQ, LDZ, IFST, ILST, INFO
LOGICAL :: WANTQ, WANTZ

SUBROUTINE TGEXC_64(WANTQ, WANTZ, [N], A, [LDA], B, [LDB], Q, [LDQ],
       Z, [LDZ], IFST, ILST, [INFO])
```

```
COMPLEX(8), DIMENSION(:,:) :: A, B, Q, Z
INTEGER(8) :: N, LDA, LDB, LDQ, LDZ, IFST, ILST, INFO
LOGICAL(8) :: WANTQ, WANTZ
```

C INTERFACE

```
#include <sunperf.h>

void ztgexc(int wantq, int wantz, int n,  doublecomplex  *a,
          int  lda, doublecomplex *b, int ldb, doublecomplex
          *q, int ldq, doublecomplex *z, int ldz, int *ifst,
          int *ilst, int *info);

void ztgexc_64(long wantq, long wantz, long n, doublecomplex
          *a,  long  lda,  doublecomplex *b, long ldb, doub-
          lecomplex *q, long  ldq,  doublecomplex  *z,  long
          ldz, long *ifst, long *ilst, long *info);
```

PURPOSE

ztgexc reorders the generalized Schur decomposition of a complex matrix pair (A,B), using an unitary equivalence transformation (A, B) := Q * (A, B) * Z', so that the diagonal block of (A, B) with row index IFST is moved to row ILST.

(A, B) must be in generalized Schur canonical form, that is, A and B are both upper triangular.

Optionally, the matrices Q and Z of generalized Schur vectors are updated.

```
Q(in) * A(in) * Z(in)' = Q(out) * A(out) * Z(out)'
Q(in) * B(in) * Z(in)' = Q(out) * B(out) * Z(out)'
```

ARGUMENTS

WANTQ (input)

WANTZ (input)

N (input) The order of the matrices A and B. N >= 0.

A (input/output)
On entry, the upper triangular matrix A in the pair (A, B). On exit, the updated matrix A.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the upper triangular matrix B in the pair (A, B). On exit, the updated matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

Q (input/output)
On entry, if WANTQ = .TRUE., the unitary matrix Q. On exit, the updated matrix Q. If WANTQ = .FALSE., Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= 1; If WANTQ = .TRUE., LDQ >= N.

Z (input/output)
On entry, if WANTZ = .TRUE., the unitary matrix Z. On exit, the updated matrix Z. If WANTZ = .FALSE., Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1; If WANTZ = .TRUE., LDZ >= N.

IFST (input/output)
Specify the reordering of the diagonal blocks of (A, B). The block with row index IFST is moved to row ILST, by a sequence of swapping between adjacent blocks.

ILST (input/output)
See the description of IFST.

INFO (output)
=0: Successful exit.
<0: if INFO = -i, the i-th argument had an illegal value.
=1: The transformed matrix pair (A, B) would be too far from generalized Schur form; the problem is ill- conditioned. (A, B) may have been partially reordered, and ILST points to the first row of the current position of the block being moved.

FURTHER DETAILS

Based on contributions by
Bo Kagstrom and Peter Poromaa, Department of Computing Science,
Umea University, S-901 87 Umea, Sweden.

[1] B. Kagstrom; A Direct Method for Reordering Eigenvalues in the Generalized Real Schur Form of a Regular Matrix Pair (A, B), in M.S. Moonen et al (eds), Linear Algebra for Large Scale and Real-Time Applications, Kluwer Academic Publ. 1993, pp 195-218.

[2] B. Kagstrom and P. Poromaa; Computing Eigenspaces with Specified Eigenvalues of a Regular Matrix Pair (A, B) and Condi- tion Estimation: Theory, Algorithms and Software, Report UMINF - 94.04, Department of Computing Science, Umea University, S-901 87 Umea, Sweden, 1994. Also as LAPACK Working Note 87. To appear in Numerical Algorithms, 1996.

[3] B. Kagstrom and P. Poromaa, LAPACK-Style Algorithms and Software for Solving the Generalized Sylvester Equation and Estimating the Separation between Regular Matrix Pairs, Report UMINF - 93.23, Department of Computing Science, Umea University, S-901 87 Umea, Sweden, December 1993, Revised April 1994, Also as LAPACK working Note 75. To appear in ACM Trans. on Math. Software, Vol 22, No 1, 1996.

Contents

NAME

```
ztgsen - reorder the generalized Schur  decomposition  of  a
complex   matrix   pair   (A,   B)  (in  terms  of  an  unitary
equivalence trans- formation Q' * (A, B) *  Z),  so  that  a
selected cluster of eigenvalues appears in the leading diag-
onal blocks of the pair (A,B)
```

SYNOPSIS

```
SUBROUTINE ZTGSEN(IJOB, WANTQ, WANTZ, SELECT, N, A, LDA, B, LDB,
      ALPHA, BETA, Q, LDQ, Z, LDZ, M, PL, PR, DIF, WORK, LWORK, IWORK,
      LIWORK, INFO)

DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),   ALPHA(*),   BETA(*),
Q(LDQ,*), Z(LDZ,*), WORK(*)
INTEGER IJOB, N, LDA, LDB, LDQ, LDZ, M, LWORK, LIWORK, INFO
INTEGER IWORK(*)
LOGICAL WANTQ, WANTZ
LOGICAL SELECT(*)
DOUBLE PRECISION PL, PR
DOUBLE PRECISION DIF(*)

SUBROUTINE ZTGSEN_64(IJOB, WANTQ, WANTZ, SELECT, N, A, LDA, B, LDB,
      ALPHA, BETA, Q, LDQ, Z, LDZ, M, PL, PR, DIF, WORK, LWORK, IWORK,
      LIWORK, INFO)

DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),   ALPHA(*),   BETA(*),
Q(LDQ,*), Z(LDZ,*), WORK(*)
INTEGER*8 IJOB, N, LDA, LDB, LDQ,  LDZ,  M,  LWORK,  LIWORK,
INFO
INTEGER*8 IWORK(*)
LOGICAL*8 WANTQ, WANTZ
```

```
LOGICAL*8 SELECT(*)
DOUBLE PRECISION PL, PR
DOUBLE PRECISION DIF(*)
```

F95 INTERFACE

```
SUBROUTINE TGSEN(IJOB, WANTQ, WANTZ, SELECT, N, A, [LDA], B, [LDB],
       ALPHA, BETA, Q, [LDQ], Z, [LDZ], M, PL, PR, DIF, [WORK], [LWORK],
       [IWORK], [LIWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, Q, Z
INTEGER :: IJOB, N, LDA, LDB, LDQ, LDZ,  M,  LWORK,  LIWORK,
INFO
INTEGER, DIMENSION(:) :: IWORK
LOGICAL :: WANTQ, WANTZ
LOGICAL, DIMENSION(:) :: SELECT
REAL(8) :: PL, PR
REAL(8), DIMENSION(:) :: DIF
SUBROUTINE TGSEN_64(IJOB, WANTQ, WANTZ, SELECT, N, A, [LDA], B, [LDB],
       ALPHA, BETA, Q, [LDQ], Z, [LDZ], M, PL, PR, DIF, [WORK], [LWORK],
       [IWORK], [LIWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: ALPHA, BETA, WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, Q, Z
INTEGER(8) :: IJOB, N, LDA, LDB, LDQ, LDZ, M, LWORK, LIWORK,
INFO
INTEGER(8), DIMENSION(:) :: IWORK
LOGICAL(8) :: WANTQ, WANTZ
LOGICAL(8), DIMENSION(:) :: SELECT
REAL(8) :: PL, PR
REAL(8), DIMENSION(:) :: DIF
```

C INTERFACE

```
#include <sunperf.h>

void ztgsen(int ijob, int wantq, int wantz, int *select, int
          n,  doublecomplex  *a,  int lda, doublecomplex *b,
          int  ldb,  doublecomplex   *alpha,   doublecomplex
          *beta,  doublecomplex  *q,  int ldq, doublecomplex
          *z, int ldz, int *m, double *pl, double *pr,  dou-
          ble *dif, int *info);

void ztgsen_64(long  ijob,  long  wantq,  long  wantz,  long
          *select, long n, doublecomplex *a, long lda, doub-
          lecomplex  *b,  long  ldb,  doublecomplex  *alpha,
          doublecomplex  *beta,  doublecomplex *q, long ldq,
          doublecomplex *z, long ldz, long *m,  double  *pl,
          double *pr, double *dif, long *info);
```

PURPOSE

ztgsen reorders the generalized Schur decomposition of a complex matrix pair (A, B) (in terms of an unitary equivalence trans- formation Q' * (A, B) * Z), so that a selected cluster of eigenvalues appears in the leading diag- onal blocks of the pair (A,B). The leading columns of Q and Z form unitary bases of the corresponding left and right eigenspaces (deflating subspaces). (A, B) must be in gen- eralized Schur canonical form, that is, A and B are both upper triangular.

ZTGSEN also computes the generalized eigenvalues

w(j)= ALPHA(j) / BETA(j)

of the reordered matrix pair (A, B).

Optionally, the routine computes estimates of reciprocal condition numbers for eigenvalues and eigenspaces. These are Difu[(A11,B11), (A22,B22)] and Difl[(A11,B11), (A22,B22)], i.e. the separation(s) between the matrix pairs (A11, B11) and (A22,B22) that correspond to the selected cluster and the eigenvalues outside the cluster, resp., and norms of "projections" onto left and right eigenspaces w.r.t. the selected cluster in the (1,1)-block.

ARGUMENTS

IJOB (input)

Specifies whether condition numbers are required for the cluster of eigenvalues (PL and PR) or the deflating subspaces (Difu and Difl):
=0: Only reorder w.r.t. SELECT. No extras.
=1: Reciprocal of norms of "projections" onto left and right eigenspaces w.r.t. the selected cluster (PL and PR). =2: Upper bounds on Difu and Difl. F-norm-based estimate
(DIF(1:2)).
=3: Estimate of Difu and Difl. 1-norm-based esti- mate
(DIF(1:2)). About 5 times as expensive as IJOB = 2. =4: Compute PL, PR and DIF (i.e. 0, 1 and 2 above): Economic version to get it all. =5: Com- pute PL, PR and DIF (i.e. 0, 1 and 3 above)

WANTQ (input)

```
WANTZ (input)

SELECT (input)
          SELECT specifies the eigenvalues in  the  selected
          cluster.  To  select an eigenvalue w(j), SELECT(j)
          must be set to

N (input) The order of the matrices A and B. N >= 0.

A (input/output)
          On entry, the upper triangular matrix A,  in  gen-
          eralized  Schur  canonical  form.   On  exit, A is
          overwritten by the reordered matrix A.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,N).

B (input/output)
          On entry, the upper triangular matrix B,  in  gen-
          eralized  Schur  canonical  form.   On  exit, B is
          overwritten by the reordered matrix B.

LDB (input)
          The leading dimension  of  the  array  B.  LDB  >=
          max(1,N).

ALPHA (output)
          The diagonal elements of A  and  B,  respectively,
          when  the  pair (A,B) has been reduced to general-
          ized Schur form.  ALPHA(i)/BETA(i)  i=1,...,N  are
          the generalized eigenvalues.

BETA (output)
          See the description of ALPHA.

Q (input/output)
          On entry, if  WANTQ  =  .TRUE.,  Q  is  an  N-by-N
          matrix.  On exit, Q has been postmultiplied by the
          left unitary transformation matrix  which  reorder
          (A,  B); The leading M columns of Q form orthonor-
          mal bases for the specified pair  of  left  eigen-
          spaces (deflating subspaces).  If WANTQ = .FALSE.,
          Q is not referenced.

LDQ (input)
          The leading dimension of the array Q.  LDQ  >=  1.
          If WANTQ = .TRUE., LDQ >= N.

Z (input/output)
```

On entry, if WANTZ = .TRUE., Z is an N-by-N matrix. On exit, Z has been postmultiplied by the left unitary transformation matrix which reorder (A, B); The leading M columns of Z form orthonormal bases for the specified pair of left eigenspaces (deflating subspaces). If WANTZ = .FALSE., Z is not referenced.

LDZ (input)
The leading dimension of the array Z. LDZ >= 1. If WANTZ = .TRUE., LDZ >= N.

M (output)
The dimension of the specified pair of left and right eigenspaces, (deflating subspaces) 0 <= M <= N.

PL (output)
IF IJOB = 1, 4, or 5, PL, PR are lower bounds on the reciprocal of the norm of "projections" onto left and right eigenspace with respect to the selected cluster.
0 < PL, PR <= 1. If M = 0 or M = N, PL = PR = 1.
If IJOB = 0, 2, or 3 PL, PR are not referenced.

PR (output)
See the description of PL.

DIF (output)
If IJOB >= 2, DIF(1:2) store the estimates of Difu and Difl.
If IJOB = 2 or 4, DIF(1:2) are F-norm-based upper bounds on
Difu and Difl. If IJOB = 3 or 5, DIF(1:2) are 1-norm-based estimates of Difu and Difl, computed using reversed communication with CLACON. If M = 0 or N, DIF(1:2) = F-norm([A, B]). If IJOB = 0 or 1, DIF is not referenced.

WORK (workspace)
If IJOB = 0, WORK is not referenced. Otherwise, on exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 1 If IJOB = 1, 2 or 4, LWORK >= 2*M*(N-M) If IJOB = 3 or 5, LWORK >= 4*M*(N-M)

If LWORK = -1, then a workspace query is assumed;

the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace/output)
If IJOB = 0, IWORK is not referenced. Otherwise, on exit, if INFO = 0, IWORK(1) returns the optimal LIWORK.

LIWORK (input)
The dimension of the array IWORK. LIWORK >= 1. If IJOB = 1, 2 or 4, LIWORK >= N+2; If IJOB = 3 or 5, LIWORK >= MAX(N+2, 2*M*(N-M));

If LIWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the IWORK array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by XERBLA.

INFO (output)
=0: Successful exit.
<0: If INFO = -i, the i-th argument had an illegal value.
=1: Reordering of (A, B) failed because the transformed matrix pair (A, B) would be too far from generalized Schur form; the problem is very ill-conditioned. (A, B) may have been partially reordered. If requested, 0 is returned in DIF(*), PL and PR.

FURTHER DETAILS

ZTGSEN first collects the selected eigenvalues by computing unitary U and W that move them to the top left corner of (A, B). In other words, the selected eigenvalues are the eigenvalues of (A11, B11) in

```
U'*(A, B)*W = (A11 A12) (B11 B12) n1
              ( 0  A22),( 0  B22) n2
                n1  n2    n1  n2
```

where N = n1+n2 and U' means the conjugate transpose of U. The first n1 columns of U and W span the specified pair of left and right eigenspaces (deflating subspaces) of (A, B).

If (A, B) has been obtained from the generalized real Schur decomposition of a matrix pair (C, D) = Q*(A, B)*Z', then the reordered generalized Schur form of (C, D) is given by

```
(C, D) = (Q*U)*(U'*(A, B)*W)*(Z*W)',
```

and the first n1 columns of Q*U and Z*W span the corresponding deflating subspaces of (C, D) (Q and Z store Q*U and Z*W, resp.).
Note that if the selected eigenvalue is sufficiently ill-conditioned, then its value may differ significantly from its value before reordering.

The reciprocal condition numbers of the left and right eigenspaces spanned by the first n1 columns of U and W (or Q*U and Z*W) may be returned in DIF(1:2), corresponding to Difu and Difl, resp.

The Difu and Difl are defined as:
ifu[(A11, B11), (A22, B22)] = sigma-min(Zu)
and

where sigma-min(Zu) is the smallest singular value of the (2*n1*n2)-by-(2*n1*n2) matrix

```
u = [ kron(In2, A11)  -kron(A22', In1) ]
          [ kron(In2, B11)  -kron(B22', In1) ].
```

Here, Inx is the identity matrix of size nx and A22' is the transpose of A22. kron(X, Y) is the Kronecker product between the matrices X and Y.

When DIF(2) is small, small changes in (A, B) can cause large changes in the deflating subspace. An approximate (asymptotic) bound on the maximum angular error in the computed deflating subspaces is PS * norm((A, B)) / DIF(2),

where EPS is the machine precision.

The reciprocal norm of the projectors on the left and right eigenspaces associated with (A11, B11) may be returned in PL and PR. They are computed as follows. First we compute L and R so that P*(A, B)*Q is block diagonal, where

```
 = ( I -L ) n1           Q = ( I R ) n1
         ( 0  I ) n2    and        ( 0 I ) n2
           n1 n2                    n1 n2
```

and (L, R) is the solution to the generalized Sylvester equation 11*R - L*A22 = -A12

Then PL = (F-norm(L)**2+1)**(-1/2) and PR = (F-norm(R)**2+1)**(-1/2). An approximate (asymptotic) bound on the average absolute error of the selected eigenvalues is
EPS * norm((A, B)) / PL.

There are also global error bounds which valid for perturbations up to a certain restriction: A lower bound (x) on the smallest F-norm(E,F) for which an eigenvalue of (A11, B11) may move and coalesce with an eigenvalue of (A22, B22) under perturbation (E,F), (i.e. (A + E, B + F), is

x = min(Difu,Difl)/((1/(PL*PL)+1/(PR*PR))**(1/2)+2*max(1/PL,1/PR)).

An approximate bound on x can be computed from DIF(1:2), PL and PR.

If y = (F-norm(E,F) / x) <= 1, the angles between the perturbed (L', R') and unperturbed (L, R) left and right deflating subspaces associated with the selected cluster in the (1,1)-blocks can be bounded as

```
 max-angle(L, L') <= arctan( y * PL / (1 - y *  (1  -  PL  *
PL)**(1/2))
 max-angle(R, R') <= arctan( y * PR / (1 - y *  (1  -  PR  *
PR)**(1/2))
```

See LAPACK User's Guide section 4.11 or the following references for more information.

Note that if the default method for computing the Frobenius-norm- based estimate DIF is not wanted (see CLATDF), then the parameter IDIFJB (see below) should be changed from 3 to 4 (routine CLATDF (IJOB = 2 will be used)). See CTGSYL for more details.

Based on contributions by
Bo Kagstrom and Peter Poromaa, Department of Computing Science,
Umea University, S-901 87 Umea, Sweden.

References
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[1] B. Kagstrom; A Direct Method for Reordering Eigenvalues in the
Generalized Real Schur Form of a Regular Matrix Pair (A, B), in
M.S. Moonen et al (eds), Linear Algebra for Large Scale and
Real-Time Applications, Kluwer Academic Publ. 1993, pp 195-218.

[2] B. Kagstrom and P. Poromaa; Computing Eigenspaces with Specified

Eigenvalues of a Regular Matrix Pair (A, B) and Condition
Estimation: Theory, Algorithms and Software, Report
UMINF - 94.04, Department of Computing Science, Umea University,
S-901 87 Umea, Sweden, 1994. Also as LAPACK Working Note 87.
To appear in Numerical Algorithms, 1996.

[3] B. Kagstrom and P. Poromaa, LAPACK-Style Algorithms and Software
for Solving the Generalized Sylvester Equation and Estimating the
Separation between Regular Matrix Pairs, Report UMINF - 93.23,
Department of Computing Science, Umea University, S-901 87 Umea,
Sweden, December 1993, Revised April 1994, Also as LAPACK working
Note 75. To appear in ACM Trans. on Math. Software, Vol 22, No 1,
1996.

Contents

NAME

```
ztgsja - compute the generalized singular  value  decomposi-
tion (GSVD) of two complex upper triangular (or trapezoidal)
matrices A and B
```

SYNOPSIS

```
SUBROUTINE ZTGSJA(JOBU, JOBV, JOBQ, M, P, N, K, L, A, LDA, B, LDB,
      TOLA, TOLB, ALPHA, BETA, U, LDU, V, LDV, Q, LDQ, WORK, NCYCLE,
      INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),  U(LDU,*),   V(LDV,*),
Q(LDQ,*), WORK(*)
INTEGER M, P, N, K, L, LDA, LDB, LDU, LDV, LDQ, NCYCLE, INFO
DOUBLE PRECISION TOLA, TOLB
DOUBLE PRECISION ALPHA(*), BETA(*)

SUBROUTINE ZTGSJA_64(JOBU, JOBV, JOBQ, M, P, N, K, L, A, LDA, B, LDB,
      TOLA, TOLB, ALPHA, BETA, U, LDU, V, LDV, Q, LDQ, WORK, NCYCLE,
      INFO)

CHARACTER * 1 JOBU, JOBV, JOBQ
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),  U(LDU,*),   V(LDV,*),
Q(LDQ,*), WORK(*)
INTEGER*8 M, P, N, K, L, LDA, LDB, LDU,  LDV,  LDQ,  NCYCLE,
INFO
DOUBLE PRECISION TOLA, TOLB
DOUBLE PRECISION ALPHA(*), BETA(*)
```

F95 INTERFACE

```
SUBROUTINE TGSJA(JOBU, JOBV, JOBQ, [M], [P], [N], K, L, A, [LDA], B,
       [LDB], TOLA, TOLB, ALPHA, BETA, U, [LDU], V, [LDV], Q, [LDQ],
```

```
          [WORK], NCYCLE, [INFO])

   CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
   COMPLEX(8), DIMENSION(:) :: WORK
   COMPLEX(8), DIMENSION(:,:) :: A, B, U, V, Q
   INTEGER :: M, P, N, K, L, LDA, LDB, LDU, LDV,  LDQ,  NCYCLE,
   INFO
   REAL(8) :: TOLA, TOLB
   REAL(8), DIMENSION(:) :: ALPHA, BETA

   SUBROUTINE TGSJA_64(JOBU, JOBV, JOBQ, [M], [P], [N], K, L, A, [LDA],
          B, [LDB], TOLA, TOLB, ALPHA, BETA, U, [LDU], V, [LDV], Q, [LDQ],
          [WORK], NCYCLE, [INFO])

   CHARACTER(LEN=1) :: JOBU, JOBV, JOBQ
   COMPLEX(8), DIMENSION(:) :: WORK
   COMPLEX(8), DIMENSION(:,:) :: A, B, U, V, Q
   INTEGER(8) :: M, P, N, K, L, LDA, LDB, LDU, LDV,  LDQ,  NCY-
   CLE, INFO
   REAL(8) :: TOLA, TOLB
   REAL(8), DIMENSION(:) :: ALPHA, BETA
```

C INTERFACE

```
   #include <sunperf.h>

   void ztgsja(char jobu, char jobv, char jobq, int m,  int  p,
             int  n,  int  k, int l, doublecomplex *a, int lda,
             doublecomplex *b, int  ldb,  double  tola,  double
             tolb,  double  *alpha, double *beta, doublecomplex
             *u, int ldu, doublecomplex *v, int ldv, doublecom-
             plex *q, int ldq, int *ncycle, int *info);

   void ztgsja_64(char jobu, char jobv, char jobq, long m, long
             p,  long n, long k, long l, doublecomplex *a, long
             lda, doublecomplex *b, long ldb, double tola, dou-
             ble  tolb, double *alpha, double *beta, doublecom-
             plex *u, long ldu,  doublecomplex  *v,  long  ldv,
             doublecomplex  *q,  long  ldq,  long *ncycle, long
             *info);
```

PURPOSE

ztgsja computes the generalized singular value decomposition (GSVD) of two complex upper triangular (or trapezoidal) matrices A and B.

On entry, it is assumed that matrices A and B have the following forms, which may be obtained by the preprocessing subroutine CGGSVP from a general M-by-N matrix A and P-by-N matrix B:

```
             N-K-L  K    L
   A =    K ( 0    A12  A13 ) if M-K-L >= 0;
          L ( 0     0   A23 )
      M-K-L ( 0     0    0  )

           N-K-L  K    L
   A =  K ( 0    A12  A13 ) if M-K-L < 0;
      M-K ( 0     0   A23 )

           N-K-L  K    L
   B =  L ( 0     0   B13 )
      P-L ( 0     0    0  )
```

where the K-by-K matrix A12 and L-by-L matrix B13 are non-
singular upper triangular; A23 is L-by-L upper triangular if
M-K-L >= 0, otherwise A23 is (M-K)-by-L upper trapezoidal.
On exit,

```
       U'*A*Q = D1*( 0 R ),    V'*B*Q = D2*( 0 R ),
```

where U, V and Q are unitary matrices, Z' denotes the conju-
gate transpose of Z, R is a nonsingular upper triangular
matrix, and D1 and D2 are ``diagonal'' matrices, which are
of the following structures:

If M-K-L >= 0,

```
                    K  L
       D1 =     K ( I  0 )
                L ( 0  C )
            M-K-L ( 0  0 )

                   K  L
       D2 = L   ( 0  S )
            P-L ( 0  0 )

               N-K-L  K    L
  ( 0 R ) = K (  0   R11  R12 ) K
            L (  0    0   R22 ) L
```

where

```
  C = diag( ALPHA(K+1), ... , ALPHA(K+L) ),
  S = diag( BETA(K+1),  ... , BETA(K+L) ),
  C**2 + S**2 = I.
```

R is stored in A(1:K+L,N-K-L+1:N) on exit.

```
If M-K-L < 0,

               K M-K K+L-M
    D1 =   K ( I  0    0   )
         M-K ( 0  C    0   )

                 K M-K K+L-M
    D2 =   M-K ( 0  S    0   )
         K+L-M ( 0  0    I   )
           P-L ( 0  0    0   )

               N-K-L  K   M-K  K+L-M

          M-K ( 0     0   R22  R23  )
        K+L-M ( 0     0    0   R33  )

where
C = diag( ALPHA(K+1), ... , ALPHA(M) ),
S = diag( BETA(K+1),  ... , BETA(M) ),
C**2 + S**2 = I.
R = ( R11 R12 R13 ) is stored in A(1:M, N-K-L+1:N)  and  R33
is stored
    (  0  R22 R23 )
in B(M-K+1:L,N+M-K-L+1:N) on exit.
```

The computation of the unitary transformation matrices U, V or Q is optional. These matrices may either be formed explicitly, or they may be postmultiplied into input matrices U1, V1, or Q1.

CTGSJA essentially uses a variant of Kogbetliantz algorithm to reduce min(L,M-K)-by-L triangular (or trapezoidal) matrix A23 and L-by-L matrix B13 to the form:

```
   U1'*A13*Q1 = C1*R1; V1'*B13*Q1 = S1*R1,
```

where U1, V1 and Q1 are unitary matrix, and Z' is the conjugate transpose of Z. C1 and S1 are diagonal matrices satisfying

```
   C1**2 + S1**2 = I,
```

and R1 is an L-by-L nonsingular upper triangular matrix.

ARGUMENTS

JOBU (input)
= 'U': U must contain a unitary matrix U1 on entry, and the product U1*U is returned; = 'I': U is initialized to the unit matrix, and the unitary matrix U is returned; = 'N': U is not computed.

JOBV (input)

= 'V': V must contain a unitary matrix V1 on entry, and the product V1*V is returned; = 'I': V is initialized to the unit matrix, and the unitary matrix V is returned; = 'N': V is not computed.

JOBQ (input)
= 'Q': Q must contain a unitary matrix Q1 on entry, and the product Q1*Q is returned; = 'I': Q is initialized to the unit matrix, and the unitary matrix Q is returned; = 'N': Q is not computed.

M (input) The number of rows of the matrix A. M >= 0.

P (input) The number of rows of the matrix B. P >= 0.

N (input) The number of columns of the matrices A and B. N >= 0.

K (input) K and L specify the subblocks in the input matrices A and B:
A23 = A(K+1:MIN(K+L,M),N-L+1:N) and B13 = B(1:L,,N-L+1:N) of A and B, whose GSVD is going to be computed by CTGSJA. See the Further Details section below.

L (input) See the description of K.

A (input/output)
On entry, the M-by-N matrix A. On exit, A(N-K+1:N,1:MIN(K+L,M)) contains the triangular matrix R or part of R. See Purpose for details.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input/output)
On entry, the P-by-N matrix B. On exit, if necessary, B(M-K+1:L,N+M-K-L+1:N) contains a part of R. See Purpose for details.

LDB (input)
The leading dimension of the array B. LDB >= max(1,P).

TOLA (input)
TOLA and TOLB are the convergence criteria for the Jacobi- Kogbetliantz iteration procedure. Generally, they are the same as used in the preprocessing step, say TOLA = MAX(M,N)*norm(A)*MACHEPS, TOLB = MAX(P,N)*norm(B)*MACHEPS.

```
TOLB (input)
          See the description of TOLA.

ALPHA (output)
          On exit, ALPHA and BETA  contain  the  generalized
          singular value pairs of A and B; ALPHA(1:K) = 1,
          BETA(1:K)  = 0, and if M-K-L >= 0,  ALPHA(K+1:K+L)
          = diag(C),
          BETA(K+1:K+L)   =  diag(S),  or  if  M-K-L  <   0,
          ALPHA(K+1:M)= C, ALPHA(M+1:K+L)= 0
          BETA(K+1:M) = S, BETA(M+1:K+L) = 1.   Furthermore,
          if K+L < N, ALPHA(K+L+1:N) = 0
          BETA(K+L+1:N)  = 0.

BETA (output)
          See the description of ALPHA.

U (input) On entry, if JOBU = 'U', U must contain  a  matrix
          U1   (usually   the  unitary  matrix  returned  by
          CGGSVP).  On exit, if JOBU = 'I', U  contains  the
          unitary  matrix  U;  if JOBU = 'U', U contains the
          product U1*U.  If JOBU = 'N', U is not referenced.

LDU (input)
          The leading dimension  of  the  array  U.  LDU  >=
          max(1,M) if JOBU = 'U'; LDU >= 1 otherwise.

V (input) On entry, if JOBV = 'V', V must contain  a  matrix
          V1   (usually   the  unitary  matrix  returned  by
          CGGSVP).  On exit, if JOBV = 'I', V  contains  the
          unitary  matrix  V;  if JOBV = 'V', V contains the
          product V1*V.  If JOBV = 'N', V is not referenced.

LDV (input)
          The leading dimension  of  the  array  V.  LDV  >=
          max(1,P) if JOBV = 'V'; LDV >= 1 otherwise.

Q (input) On entry, if JOBQ = 'Q', Q must contain  a  matrix
          Q1   (usually   the  unitary  matrix  returned  by
          CGGSVP).  On exit, if JOBQ = 'I', Q  contains  the
          unitary  matrix  Q;  if JOBQ = 'Q', Q contains the
          product Q1*Q.  If JOBQ = 'N', Q is not referenced.

LDQ (input)
          The leading dimension  of  the  array  Q.  LDQ  >=
          max(1,N) if JOBQ = 'Q'; LDQ >= 1 otherwise.

WORK (workspace)
          dimension(2*N)
```

NCYCLE (output)
The number of cycles required for convergence.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value.
= 1: the procedure does not converge after MAXIT cycles.

Contents

NAME

```
ztgsna - estimate reciprocal condition numbers for specified
eigenvalues and/or eigenvectors of a matrix pair (A, B)
```

SYNOPSIS

```
SUBROUTINE ZTGSNA(JOB, HOWMNT, SELECT, N, A, LDA, B, LDB, VL, LDVL,
      VR, LDVR, S, DIF, MM, M, WORK, LWORK, IWORK, INFO)

CHARACTER * 1 JOB, HOWMNT
DOUBLE COMPLEX A(LDA,*), B(LDB,*),  VL(LDVL,*),  VR(LDVR,*),
WORK(*)
INTEGER N, LDA, LDB, LDVL, LDVR, MM, M, LWORK, INFO
INTEGER IWORK(*)
LOGICAL SELECT(*)
DOUBLE PRECISION S(*), DIF(*)

SUBROUTINE ZTGSNA_64(JOB, HOWMNT, SELECT, N, A, LDA, B, LDB, VL,
      LDVL, VR, LDVR, S, DIF, MM, M, WORK, LWORK, IWORK, INFO)

CHARACTER * 1 JOB, HOWMNT
DOUBLE COMPLEX A(LDA,*), B(LDB,*),  VL(LDVL,*),  VR(LDVR,*),
WORK(*)
INTEGER*8 N, LDA, LDB, LDVL, LDVR, MM, M, LWORK, INFO
INTEGER*8 IWORK(*)
LOGICAL*8 SELECT(*)
DOUBLE PRECISION S(*), DIF(*)
```

F95 INTERFACE

```
SUBROUTINE TGSNA(JOB, HOWMNT, SELECT, [N], A, [LDA], B, [LDB], VL,
       [LDVL], VR, [LDVR], S, DIF, MM, M, [WORK], [LWORK], [IWORK],
       [INFO])
```

```
CHARACTER(LEN=1) :: JOB, HOWMNT
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, VL, VR
INTEGER :: N, LDA, LDB, LDVL, LDVR, MM, M, LWORK, INFO
INTEGER, DIMENSION(:) :: IWORK
LOGICAL, DIMENSION(:) :: SELECT
REAL(8), DIMENSION(:) :: S, DIF

SUBROUTINE TGSNA_64(JOB, HOWMNT, SELECT, [N], A, [LDA], B, [LDB], VL,
       [LDVL], VR, [LDVR], S, DIF, MM, M, [WORK], [LWORK], [IWORK],
       [INFO])

CHARACTER(LEN=1) :: JOB, HOWMNT
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, VL, VR
INTEGER(8) :: N, LDA, LDB, LDVL, LDVR, MM, M, LWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
LOGICAL(8), DIMENSION(:) :: SELECT
REAL(8), DIMENSION(:) :: S, DIF
```

C INTERFACE

```
#include <sunperf.h>

void ztgsna(char job, char howmnt, int *select, int n, doub-
          lecomplex  *a, int lda, doublecomplex *b, int ldb,
          doublecomplex *vl, int  ldvl,  doublecomplex  *vr,
          int  ldvr, double *s, double *dif, int mm, int *m,
          int *info);

void ztgsna_64(char job, char howmnt, long *select, long  n,
          doublecomplex *a, long lda, doublecomplex *b, long
          ldb, doublecomplex *vl, long  ldvl,  doublecomplex
          *vr,  long  ldvr, double *s, double *dif, long mm,
          long *m, long *info);
```

PURPOSE

ztgsna estimates reciprocal condition numbers for specified eigenvalues and/or eigenvectors of a matrix pair (A, B).

(A, B) must be in generalized Schur canonical form, that is, A and B are both upper triangular.

ARGUMENTS

JOB (input)

Specifies whether condition numbers are required for eigenvalues (S) or eigenvectors (DIF):
= 'E': for eigenvalues only (S);
= 'V': for eigenvectors only (DIF);
= 'B': for both eigenvalues and eigenvectors (S and DIF).

HOWMNT (input)
= 'A': compute condition numbers for all eigenpairs;
= 'S': compute condition numbers for selected eigenpairs specified by the array SELECT.

SELECT (input)
If HOWMNT = 'S', SELECT specifies the eigenpairs for which condition numbers are required. To select condition numbers for the corresponding j-th eigenvalue and/or eigenvector, SELECT(j) must be set to .TRUE.. If HOWMNT = 'A', SELECT is not referenced.

N (input) The order of the square matrix pair (A, B). N >= 0.

A (input) The upper triangular matrix A in the pair (A,B).

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input) The upper triangular matrix B in the pair (A, B).

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

VL (input)
If JOB = 'E' or 'B', VL must contain left eigenvectors of (A, B), corresponding to the eigenpairs specified by HOWMNT and SELECT. The eigenvectors must be stored in consecutive columns of VL, as returned by CTGEVC. If JOB = 'V', VL is not referenced.

LDVL (input)
The leading dimension of the array VL. LDVL >= 1; and If JOB = 'E' or 'B', LDVL >= N.

VR (input)
If JOB = 'E' or 'B', VR must contain right eigen-

vectors of (A, B), corresponding to the eigenpairs specified by HOWMNT and SELECT. The eigenvectors must be stored in consecutive columns of VR, as returned by CTGEVC. If JOB = 'V', VR is not referenced.

LDVR (input)
The leading dimension of the array VR. LDVR >= 1; If JOB = 'E' or 'B', LDVR >= N.

S (output)
If JOB = 'E' or 'B', the reciprocal condition numbers of the selected eigenvalues, stored in consecutive elements of the array. If JOB = 'V', S is not referenced.

DIF (output)
If JOB = 'V' or 'B', the estimated reciprocal condition numbers of the selected eigenvectors, stored in consecutive elements of the array. If the eigenvalues cannot be reordered to compute DIF(j), DIF(j) is set to 0; this can only occur when the true value would be very small anyway. For each eigenvalue/vector specified by SELECT, DIF stores a Frobenius norm-based estimate of Difl. If JOB = 'E', DIF is not referenced.

MM (input)
The number of elements in the arrays S and DIF. MM >= M.

M (output)
The number of elements of the arrays S and DIF used to store the specified condition numbers; for each selected eigenvalue one element is used. If HOWMNT = 'A', M is set to N.

WORK (workspace)
If JOB = 'E', WORK is not referenced. Otherwise, on exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= 1. If JOB = 'V' or 'B', LWORK >= 2*N*N.

IWORK (workspace)
dimension(N+2) If JOB = 'E', IWORK is not referenced.

INFO (output)

```
= 0: Successful exit
< 0: If INFO = -i, the i-th argument had an  ille-
gal value
```

FURTHER DETAILS

```
The reciprocal of the condition number of the i-th  general-
ized eigenvalue w = (a, b) is defined as

        S(I)   =   (|v'Au|**2    +    |v'Bu|**2)**(1/2)    /
(norm(u)*norm(v))

where u and v are the right and left eigenvectors of (A,  B)
corresponding  to  w;  |z| denotes the absolute value of the
complex number, and norm(u) denotes the 2-norm of the vector
u.  The  pair (a, b) corresponds to an eigenvalue w = a/b (=
v'Au/v'Bu) of the matrix pair (A, B). If both a and b  equal
zero, then (A,B) is singular and S(I) = -1 is returned.

An approximate error bound on the chordal  distance  between
the   i-th   computed   generalized  eigenvalue  w  and  the
corresponding exact eigenvalue lambda is

        chord(w, lambda) <=   EPS * norm(A, B) / S(I),

where EPS is the machine precision.

The reciprocal of the condition number of the  right  eigen-
vector  u  and  left eigenvector v corresponding to the gen-
eralized eigenvalue w is defined as follows. Suppose

               (A, B) = ( a   *  ) ( b  *  )  1
                        ( 0  A22 ),( 0 B22 )  n-1
                          1  n-1     1 n-1

Then the reciprocal condition number DIF(I) is

        Difl[(a, b), (A22, B22)]  = sigma-min( Zl )

where sigma-min(Zl) denotes the smallest singular value of

       Zl = [ kron(a, In-1) -kron(1, A22) ]
            [ kron(b, In-1) -kron(1, B22) ].

Here In-1 is the identity matrix of size n-1 and X'  is  the
conjugate  transpose  of X. kron(X, Y) is the Kronecker pro-
duct between the matrices X and Y.

We approximate the smallest singular value  of  Zl  with  an
```

upper bound. This is done by CLATDF.

An approximate error bound for a computed eigenvector VL(i) or VR(i) is given by

EPS * norm(A, B) / DIF(i).

See ref. [2-3] for more details and further references.
Based on contributions by
Bo Kagstrom and Peter Poromaa, Department of Computing Science,
Umea University, S-901 87 Umea, Sweden.

References
==========

[1] B. Kagstrom; A Direct Method for Reordering Eigenvalues in the
Generalized Real Schur Form of a Regular Matrix Pair (A, B), in
M.S. Moonen et al (eds), Linear Algebra for Large Scale and
Real-Time Applications, Kluwer Academic Publ. 1993, pp 195-218.

[2] B. Kagstrom and P. Poromaa; Computing Eigenspaces with Specified
Eigenvalues of a Regular Matrix Pair (A, B) and Condi- tion
Estimation: Theory, Algorithms and Software, Report
UMINF - 94.04, Department of Computing Science, Umea University,
S-901 87 Umea, Sweden, 1994. Also as LAPACK Working Note 87.
To appear in Numerical Algorithms, 1996.

[3] B. Kagstrom and P. Poromaa, LAPACK-Style Algorithms and Software
for Solving the Generalized Sylvester Equation and Estimating the
Separation between Regular Matrix Pairs, Report UMINF - 93.23,
Department of Computing Science, Umea University, S-901 87 Umea,
Sweden, December 1993, Revised April 1994, Also as LAPACK Working
Note 75.
To appear in ACM Trans. on Math. Software, Vol 22, No 1, 1996.

Contents

NAME

```
ztgsyl - solve the generalized Sylvester equation
```

SYNOPSIS

```
SUBROUTINE ZTGSYL(TRANS, IJOB, M, N, A, LDA, B, LDB, C, LDC, D, LDD,
      E, LDE, F, LDF, SCALE, DIF, WORK, LWORK, IWORK, INFO)

CHARACTER * 1 TRANS
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),  C(LDC,*),   D(LDD,*),
E(LDE,*), F(LDF,*), WORK(*)
INTEGER IJOB, M, N, LDA, LDB, LDC,  LDD,  LDE,  LDF,  LWORK,
INFO
INTEGER IWORK(*)
DOUBLE PRECISION SCALE, DIF

SUBROUTINE ZTGSYL_64(TRANS, IJOB, M, N, A, LDA, B, LDB, C, LDC, D,
      LDD, E, LDE, F, LDF, SCALE, DIF, WORK, LWORK, IWORK, INFO)

CHARACTER * 1 TRANS
DOUBLE  COMPLEX  A(LDA,*),  B(LDB,*),  C(LDC,*),   D(LDD,*),
E(LDE,*), F(LDF,*), WORK(*)
INTEGER*8 IJOB, M, N, LDA, LDB, LDC, LDD, LDE,  LDF,  LWORK,
INFO
INTEGER*8 IWORK(*)
DOUBLE PRECISION SCALE, DIF
```

F95 INTERFACE

```
SUBROUTINE TGSYL(TRANS, IJOB, [M], [N], A, [LDA], B, [LDB], C, [LDC],
       D, [LDD], E, [LDE], F, [LDF], SCALE, DIF, [WORK], [LWORK], [IWORK],
       [INFO])
```

```
CHARACTER(LEN=1) :: TRANS
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, C, D, E, F
INTEGER :: IJOB, M, N, LDA, LDB, LDC, LDD, LDE, LDF,  LWORK,
INFO
INTEGER, DIMENSION(:) :: IWORK
REAL(8) :: SCALE, DIF

SUBROUTINE TGSYL_64(TRANS, IJOB, [M], [N], A, [LDA], B, [LDB], C,
       [LDC], D, [LDD], E, [LDE], F, [LDF], SCALE, DIF, [WORK], [LWORK],
       [IWORK], [INFO])

CHARACTER(LEN=1) :: TRANS
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, C, D, E, F
INTEGER(8) :: IJOB, M, N, LDA,  LDB,  LDC,  LDD,  LDE,  LDF,
LWORK, INFO
INTEGER(8), DIMENSION(:) :: IWORK
REAL(8) :: SCALE, DIF
```

C INTERFACE

```
#include <sunperf.h>

void ztgsyl(char trans, int ijob, int m, int  n,  doublecom-
          plex *a, int lda, doublecomplex *b, int ldb, doub-
          lecomplex *c, int ldc, doublecomplex *d, int  ldd,
          doublecomplex  *e,  int lde, doublecomplex *f, int
          ldf, double *scale, double *dif, int *info);

void ztgsyl_64(char trans, long ijob, long m, long n,  doub-
          lecomplex  *a,  long  lda,  doublecomplex *b, long
          ldb, doublecomplex *c, long ldc, doublecomplex *d,
          long  ldd,  doublecomplex *e, long lde, doublecom-
          plex *f, long ldf,  double  *scale,  double  *dif,
          long *info);
```

PURPOSE

ztgsyl solves the generalized Sylvester equation:

```
            A * R - L * B = scale * C            (1)
            D * R - L * E = scale * F
```

where R and L are unknown m-by-n matrices, (A, D), (B, E) and (C, F) are given matrix pairs of size m-by-m, n-by-n and m-by-n, respectively, with complex entries. A, B, D and E are upper triangular (i.e., (A,D) and (B,E) in generalized Schur form).

```
The solution (R, L) overwrites (C, F). 0 <= SCALE <= 1
is an output scaling factor chosen to avoid overflow.

In matrix notation (1) is equivalent to solve Zx =  scale*b,
where Z is defined as

       Z = [ kron(In, A)  -kron(B', Im) ]         (2)
           [ kron(In, D)  -kron(E', Im) ],

Here Ix is the identity matrix of size x and X' is the  con-
jugate  transpose  of X. Kron(X, Y) is the Kronecker product
between the matrices X and Y.

If TRANS = 'C', y in the conjugate transposed system Z'*y  =
scale*b  is  solved  for, which is equivalent to solve for R
and L in

            A' * R + D' * L = scale * C           (3)
            R * B' + L * E' = scale * -F
This case (TRANS = 'C') is used to compute an one-norm-based
estimate  of  Dif[(A,D),  (B,E)], the separation between the
matrix pairs (A,D) and (B,E), using CLACON.

If IJOB >= 1, CTGSYL computes a Frobenius  norm-based  esti-
mate of Dif[(A,D),(B,E)]. That is, the reciprocal of a lower
bound on the reciprocal of the smallest singular value of Z.

This is a level-3 BLAS algorithm.
```

ARGUMENTS

```
TRANS (input)
          = 'N': solve the  generalized  sylvester  equation
          (1).
          = 'C': solve  the  "conjugate  transposed"  system
          (3).

IJOB (input)
          Specifies what kind of functionality  to  be  per-
          formed.  =0: solve (1) only.
          =1: The functionality of 0 and 3.
          =2: The functionality of 0 and 4.
          =3: Only an estimate of Dif[(A,D), (B,E)] is  com-
          puted.   (look  ahead strategy is used).  =4: Only
          an estimate  of  Dif[(A,D),  (B,E)]  is  computed.
          (CGECON  on  sub-systems is used).  Not referenced
          if TRANS = 'C'.
```

M (input) The order of the matrices A and D, and the row dimension of the matrices C, F, R and L.

N (input) The order of the matrices B and E, and the column dimension of the matrices C, F, R and L.

A (input) The upper triangular matrix A.

LDA (input)
The leading dimension of the array A. LDA >= max(1, M).

B (input) The upper triangular matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1, N).

C (input/output)
On entry, C contains the right-hand-side of the first matrix equation in (1) or (3). On exit, if IJOB = 0, 1 or 2, C has been overwritten by the solution R. If IJOB = 3 or 4 and TRANS = 'N', C holds R, the solution achieved during the computation of the Dif-estimate.

LDC (input)
The leading dimension of the array C. LDC >= max(1, M).

D (input) The upper triangular matrix D.

LDD (input)
The leading dimension of the array D. LDD >= max(1, M).

E (input) The upper triangular matrix E.

LDE (input)
The leading dimension of the array E. LDE >= max(1, N).

F (input/output)
On entry, F contains the right-hand-side of the second matrix equation in (1) or (3). On exit, if IJOB = 0, 1 or 2, F has been overwritten by the solution L. If IJOB = 3 or 4 and TRANS = 'N', F holds L, the solution achieved during the computation of the Dif-estimate.

LDF (input)
The leading dimension of the array F. LDF >= max(1, M).

SCALE (output)
On exit SCALE is the reciprocal of a lower bound of the reciprocal of the Dif-function, i.e. SCALE is an upper bound of Dif[(A,D), (B,E)] = sigma-min(Z), where Z as in (2). If IJOB = 0 or TRANS = 'C', SCALE is not referenced.

DIF (output)
On exit SCALE is the reciprocal of a lower bound of the reciprocal of the Dif-function, i.e. SCALE is an upper bound of Dif[(A,D), (B,E)] = sigma-min(Z), where Z as in (2). If IJOB = 0 or TRANS = 'C', SCALE is not referenced.

WORK (workspace)
If IJOB = 0, WORK is not referenced. Otherwise, on exit, if INFO=0 then WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK > = 1. If IJOB = 1 or 2 and TRANS = 'N', LWORK >= 2*M*N.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

IWORK (workspace)
If IJOB = 0, IWORK is not referenced.

INFO (output)
=0: successful exit
<0: If INFO = -i, the i-th argument had an illegal value.
>0: (A, D) and (B, E) have common or very close eigenvalues.

FURTHER DETAILS

Based on contributions by
Bo Kagstrom and Peter Poromaa, Department of Computing Science,
Umea University, S-901 87 Umea, Sweden.

[1] B. Kagstrom and P. Poromaa, LAPACK-Style Algorithms and Software for Solving the Generalized Sylvester Equation and Estimating the Separation between Regular Matrix Pairs, Report UMINF - 93.23, Department of Computing Science, Umea University, S-901 87 Umea, Sweden, December 1993, Revised April 1994, Also as LAPACK Working Note 75. To appear in ACM Trans. on Math. Software, Vol 22, No 1, 1996.

[2] B. Kagstrom, A Perturbation Analysis of the Generalized Sylvester Equation (AR - LB, DR - LE) = (C, F), SIAM J. Matrix Anal. Appl., 15(4):1045-1060, 1994.

[3] B. Kagstrom and L. Westin, Generalized Schur Methods with Condition Estimators for Solving the Generalized Sylvester Equation, IEEE Transactions on Automatic Control, Vol. 34, No. 7, July 1989, pp 745-751.

Contents

NAME

```
ztpcon - estimate the reciprocal of the condition number  of
a  packed  triangular  matrix A, in either the 1-norm or the
infinity-norm
```

SYNOPSIS

```
SUBROUTINE ZTPCON(NORM, UPLO, DIAG, N, A, RCOND, WORK, WORK2, INFO)

CHARACTER * 1 NORM, UPLO, DIAG
DOUBLE COMPLEX A(*), WORK(*)
INTEGER N, INFO
DOUBLE PRECISION RCOND
DOUBLE PRECISION WORK2(*)

SUBROUTINE ZTPCON_64(NORM, UPLO, DIAG, N, A, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 NORM, UPLO, DIAG
DOUBLE COMPLEX A(*), WORK(*)
INTEGER*8 N, INFO
DOUBLE PRECISION RCOND
DOUBLE PRECISION WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE TPCON(NORM, UPLO, DIAG, N, A, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
COMPLEX(8), DIMENSION(:) :: A, WORK
INTEGER :: N, INFO
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: WORK2
```

```
SUBROUTINE TPCON_64(NORM, UPLO, DIAG, N, A, RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
COMPLEX(8), DIMENSION(:) :: A, WORK
INTEGER(8) :: N, INFO
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>

void ztpcon(char norm, char uplo, char diag,  int  n,  doub-
          lecomplex *a, double *rcond, int *info);
void ztpcon_64(char norm, char  uplo,  char  diag,  long  n,
          doublecomplex *a, double *rcond, long *info);
```

PURPOSE

ztpcon estimates the reciprocal of the condition number of a packed triangular matrix A, in either the 1-norm or the infinity-norm.

The norm of A is computed and an estimate is obtained for norm(inv(A)), then the reciprocal of the condition number is computed as

```
   RCOND = 1 / ( norm(A) * norm(inv(A)) ).
```

ARGUMENTS

```
NORM (input)
          Specifies whether the 1-norm condition  number  or
          the infinity-norm condition number is required:
          = '1' or 'O':  1-norm;
          = 'I':         Infinity-norm.

UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.
```

A (input) The upper or lower triangular matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array A as follows: if UPLO = 'U', A(i + (j-1)*j/2) = A(i,j) for 1<=i<=j; if UPLO = 'L', A(i + (j-1)*(2n-j)/2) = A(i,j) for j<=i<=n. If DIAG = 'U', the diagonal elements of A are not referenced and are assumed to be 1.

RCOND (output)
The reciprocal of the condition number of the matrix A, computed as RCOND = 1/(norm(A) * norm(inv(A))).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

ztpmv - perform one of the matrix-vector operations x := A*x, or x := A'*x, or x := conjg(A')*x

SYNOPSIS

```
SUBROUTINE ZTPMV(UPLO, TRANSA, DIAG, N, A, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(*), Y(*)
INTEGER N, INCY

SUBROUTINE ZTPMV_64(UPLO, TRANSA, DIAG, N, A, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(*), Y(*)
INTEGER*8 N, INCY
```

F95 INTERFACE

```
SUBROUTINE TPMV(UPLO, [TRANSA], DIAG, [N], A, Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: A, Y
INTEGER :: N, INCY

SUBROUTINE TPMV_64(UPLO, [TRANSA], DIAG, [N], A, Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: A, Y
INTEGER(8) :: N, INCY
```

C INTERFACE

```
#include <sunperf.h>
```

```
void ztpmv(char uplo, char transa, char diag, int  n,  doub-
          lecomplex *a, doublecomplex *y, int incy);

void ztpmv_64(char uplo, char transa,  char  diag,  long  n,
          doublecomplex *a, doublecomplex *y, long incy);
```

PURPOSE

ztpmv performs one of the matrix-vector operations x := A*x, or x := A'*x, or x := conjg(A')*x where x is an n element vector and A is an n by n unit, or non-unit, upper or lower triangular matrix, supplied in packed form.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   x := A*x.

TRANSA = 'T' or 't'   x := A'*x.

TRANSA = 'C' or 'c'   x := conjg(A')*x.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit tri-

```
          angular.

          DIAG = 'N' or 'n'   A is not assumed  to  be  unit
          triangular.

          Unchanged on exit.

N (input)
          On entry, N specifies the order of the  matrix  A.
          N >= 0.  Unchanged on exit.

A (input)
          ( ( n*( n + 1 ) )/2 ).  Before entry with  UPLO  =
          'U'  or  'u',  the  array A must contain the upper
          triangular matrix packed sequentially,  column  by
          column,  so that A( 1 ) contains a( 1, 1 ), A( 2 )
          and A( 3 ) contain a( 1, 2 ) and a( 2, 2 ) respec-
          tively,  and  so on.  Before entry with UPLO = 'L'
          or 'l', the array A must contain  the  lower  tri-
          angular  matrix  packed  sequentially,  column  by
          column, so that A( 1 ) contains a( 1, 1 ), A( 2  )
          and A( 3 ) contain a( 2, 1 ) and a( 3, 1 ) respec-
          tively, and so on.  Note that when  DIAG = 'U'  or
          'u',  the  diagonal  elements  of A are not refer-
          enced, but are assumed to be unity.  Unchanged  on
          exit.

Y (input/output)
          ( 1 + ( n - 1 )*abs( INCY ) ).  Before entry,  the
          incremented  array  Y  must  contain the n element
          vector x. On exit, Y is overwritten with the tran-
          formed vector x.

INCY (input)
          On entry, INCY specifies  the  increment  for  the
          elements of Y. INCY <> 0.  Unchanged on exit.
```

Contents

NAME

```
ztprfs - provide error bounds and backward  error  estimates
for the solution to a system of linear equations with a tri-
angular packed coefficient matrix
```

SYNOPSIS

```
SUBROUTINE ZTPRFS(UPLO, TRANSA, DIAG, N, NRHS, A, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDB, LDX, INFO
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZTPRFS_64(UPLO, TRANSA, DIAG, N, NRHS, A, B, LDB, X, LDX,
      FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDB, LDX, INFO
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE TPRFS(UPLO, [TRANSA], DIAG, N, NRHS, A, B, [LDB], X, [LDX],
       FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: A, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
INTEGER :: N, NRHS, LDB, LDX, INFO
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

```
SUBROUTINE TPRFS_64(UPLO, [TRANSA], DIAG, N, NRHS, A, B, [LDB], X,
       [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: A, WORK
COMPLEX(8), DIMENSION(:,:) :: B, X
INTEGER(8) :: N, NRHS, LDB, LDX, INFO
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void ztprfs(char uplo, char transa, char diag,  int  n,  int
          nrhs, doublecomplex *a, doublecomplex *b, int ldb,
          doublecomplex *x, int ldx,  double  *ferr,  double
          *berr, int *info);

void ztprfs_64(char uplo, char transa, char  diag,  long  n,
          long  nrhs,  doublecomplex  *a,  doublecomplex *b,
          long  ldb,  doublecomplex  *x,  long  ldx,  double
          *ferr, double *berr, long *info);
```

PURPOSE

ztprfs provides error bounds and backward error estimates for the solution to a system of linear equations with a triangular packed coefficient matrix.

The solution matrix X must be computed by CTPTRS or some other means before entering this routine. CTPRFS does not do iterative refinement because doing so cannot improve the backward error.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.
```

```
DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input) The upper or lower  triangular  matrix  A,  packed
          columnwise  in a linear array.  The j-th column of
          A is stored in the array A as follows:  if UPLO  =
          'U',  A(i  +  (j-1)*j/2)  = A(i,j) for 1<=i<=j; if
          UPLO = 'L', A(i +  (j-1)*(2n-j)/2)  =  A(i,j)  for
          j<=i<=n.   If DIAG = 'U', the diagonal elements of
          A are not referenced and are assumed to be 1.

B (input) The right hand side matrix B.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

X (input) The solution matrix X.

LDX (input)
          The leading dimension of  the  array  X.   LDX  >=
          max(1,N).

FERR (output)
          The estimated forward error bound for  each  solu-
          tion  vector X(j) (the j-th column of the solution
          matrix  X).   If  XTRUE  is  the   true   solution
          corresponding  to  X(j),  FERR(j)  is an estimated
          upper bound for the magnitude of the largest  ele-
          ment in (X(j) - XTRUE) divided by the magnitude of
          the largest element in X(j).  The estimate  is  as
          reliable  as the estimate for RCOND, and is almost
          always a slight overestimate of the true error.

BERR (output)
          The componentwise relative backward error of  each
          solution  vector X(j) (i.e., the smallest relative
          change in any element of A or B that makes X(j) an
          exact solution).

WORK (workspace)
          dimension(2*N)
```

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-
gal value

Contents

NAME

```
ztpsv - solve one of the systems of equations   A*x = b,  or
A'*x = b, or conjg( A' )*x = b
```

SYNOPSIS

```
SUBROUTINE ZTPSV(UPLO, TRANSA, DIAG, N, A, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(*), Y(*)
INTEGER N, INCY

SUBROUTINE ZTPSV_64(UPLO, TRANSA, DIAG, N, A, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(*), Y(*)
INTEGER*8 N, INCY
```

F95 INTERFACE

```
SUBROUTINE TPSV(UPLO, [TRANSA], DIAG, [N], A, Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: A, Y
INTEGER :: N, INCY

SUBROUTINE TPSV_64(UPLO, [TRANSA], DIAG, [N], A, Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: A, Y
INTEGER(8) :: N, INCY
```

C INTERFACE

```
#include <sunperf.h>
```

```
void ztpsv(char uplo, char transa, char diag, int  n,  doub-
          lecomplex *a, doublecomplex *y, int incy);

void ztpsv_64(char uplo, char transa,  char  diag,  long  n,
          doublecomplex *a, doublecomplex *y, long incy);
```

PURPOSE

ztpsv solves one of the systems of equations A*x = b, or A'*x = b, or conjg(A')*x = b where b and x are n element vectors and A is an n by n unit, or non-unit, upper or lower triangular matrix, supplied in packed form.

No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

ARGUMENTS

UPLO (input)

On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)

On entry, TRANSA specifies the equations to be solved as follows:

TRANSA = 'N' or 'n'   A*x = b.

TRANSA = 'T' or 't'   A'*x = b.

TRANSA = 'C' or 'c'   conjg(A')*x = b.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)

On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u' A is assumed to be unit triangular.

DIAG = 'N' or 'n' A is not assumed to be unit triangular.

Unchanged on exit.

N (input)

On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

A (input)

((n*(n + 1))/2). Before entry with UPLO = 'U' or 'u', the array A must contain the upper triangular matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(1, 2) and a(2, 2) respectively, and so on. Before entry with UPLO = 'L' or 'l', the array A must contain the lower triangular matrix packed sequentially, column by column, so that A(1) contains a(1, 1), A(2) and A(3) contain a(2, 1) and a(3, 1) respectively, and so on. Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced, but are assumed to be unity. Unchanged on exit.

Y (input/output)

(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element right-hand side vector b. On exit, Y is overwritten with the solution vector x.

INCY (input)

On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

ztptri - compute the inverse of a complex upper or lower triangular matrix A stored in packed format

SYNOPSIS

```
SUBROUTINE ZTPTRI(UPLO, DIAG, N, A, INFO)

CHARACTER * 1 UPLO, DIAG
DOUBLE COMPLEX A(*)
INTEGER N, INFO

SUBROUTINE ZTPTRI_64(UPLO, DIAG, N, A, INFO)

CHARACTER * 1 UPLO, DIAG
DOUBLE COMPLEX A(*)
INTEGER*8 N, INFO
```

F95 INTERFACE

```
SUBROUTINE TPTRI(UPLO, DIAG, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
COMPLEX(8), DIMENSION(:) :: A
INTEGER :: N, INFO

SUBROUTINE TPTRI_64(UPLO, DIAG, N, A, [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
COMPLEX(8), DIMENSION(:) :: A
INTEGER(8) :: N, INFO
```

C INTERFACE

```
#include <sunperf.h>

void ztptri(char uplo, char diag, int n,  doublecomplex  *a,
          int *info);

void ztptri_64(char uplo, char diag, long  n,  doublecomplex
          *a, long *info);
```

PURPOSE

ztptri computes the inverse of a complex upper or lower triangular matrix A stored in packed format.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the upper or lower triangular matrix  A,
          stored  columnwise  in  a  linear array.  The j-th
          column of A is stored in the array A  as  follows:
          if  UPLO  =  'U',  A(i  +  (j-1)*j/2) = A(i,j) for
          1<=i<=j; if UPLO = 'L', A(i + (j-1)*((2*n-j)/2)  =
          A(i,j)   for   j<=i<=n.   See  below  for  further
          details.  On exit, the (triangular) inverse of the
          original  matrix,  in the same packed storage for-
          mat.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, A(i,i) is  exactly  zero.   The
          triangular  matrix is singular and its inverse can
          not be computed.
```

FURTHER DETAILS

A triangular matrix A can be transferred to packed storage using one of the following program segments:

UPLO = 'U':

```
      JC = 1
      DO 2 J = 1, N
         DO 1 I = 1, J
            A(JC+I-1) = A(I,J)
    1    CONTINUE
         JC = JC + J
    2 CONTINUE
```

UPLO = 'L':

```
      JC = 1
      DO 2 J = 1, N
         DO 1 I = J, N
            A(JC+I-J) = A(I,J)
    1    CONTINUE
         JC = JC + N - J + 1
    2 CONTINUE
```

Contents

NAME

```
ztptrs - solve a triangular system of the form   A * X =  B,
A**T * X = B, or A**H * X = B,
```

SYNOPSIS

```
SUBROUTINE ZTPTRS(UPLO, TRANSA, DIAG, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(*), B(LDB,*)
INTEGER N, NRHS, LDB, INFO

SUBROUTINE ZTPTRS_64(UPLO, TRANSA, DIAG, N, NRHS, A, B, LDB, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(*), B(LDB,*)
INTEGER*8 N, NRHS, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE TPTRS(UPLO, TRANSA, DIAG, N, NRHS, A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: A
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER :: N, NRHS, LDB, INFO

SUBROUTINE TPTRS_64(UPLO, TRANSA, DIAG, N, NRHS, A, B, [LDB], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: A
COMPLEX(8), DIMENSION(:,:) :: B
INTEGER(8) :: N, NRHS, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void ztptrs(char uplo, char transa, char diag,  int  n,  int
          nrhs, doublecomplex *a, doublecomplex *b, int ldb,
          int *info);

void ztptrs_64(char uplo, char transa, char  diag,  long  n,
          long  nrhs,  doublecomplex  *a,  doublecomplex *b,
          long ldb, long *info);
```

PURPOSE

ztptrs solves a triangular system of the form where A is a triangular matrix of order N stored in packed format, and B is an N-by-NRHS matrix. A check is made to verify that A is nonsingular.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrix B.  NRHS >= 0.

A (input) The upper or lower  triangular  matrix  A,  packed
          columnwise  in a linear array.  The j-th column of
          A is stored in the array A as follows:  if UPLO  =
          'U',  A(i  +  (j-1)*j/2)  = A(i,j) for 1<=i<=j; if
          UPLO = 'L', A(i + (j-1)*(2*n-j)/2)  =  A(i,j)  for
          j<=i<=n.
```

```
B (input/output)
          On entry, the right hand side matrix B.  On  exit,
          if INFO = 0, the solution matrix X.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
          > 0:  if INFO = i, the i-th diagonal element of  A
          is  zero,  indicating  that the matrix is singular
          and the solutions X have not been computed.
```

Contents

NAME

```
ztrans - transpose and scale source matrix
```

SYNOPSIS

```
SUBROUTINE ZTRANS(PLACE, SCALE, SOURCE, M, N, DEST)

CHARACTER * 1 PLACE
DOUBLE COMPLEX SCALE
DOUBLE COMPLEX SOURCE(*), DEST(*)
INTEGER M, N

SUBROUTINE ZTRANS_64(PLACE, SCALE, SOURCE, M, N, DEST)

CHARACTER * 1 PLACE
DOUBLE COMPLEX SCALE
DOUBLE COMPLEX SOURCE(*), DEST(*)
INTEGER*8 M, N
```

F95 INTERFACE

```
SUBROUTINE TRANS([PLACE], SCALE, SOURCE, M, N, [DEST])

CHARACTER(LEN=1) :: PLACE
COMPLEX(8) :: SCALE
COMPLEX(8), DIMENSION(:) :: SOURCE, DEST
INTEGER :: M, N

SUBROUTINE TRANS_64([PLACE], SCALE, SOURCE, M, N, [DEST])

CHARACTER(LEN=1) :: PLACE
COMPLEX(8) :: SCALE
COMPLEX(8), DIMENSION(:) :: SOURCE, DEST
INTEGER(8) :: M, N
```

C INTERFACE

```
#include <sunperf.h>

void ztrans(char place, doublecomplex *scale,  doublecomplex
          *source, int m, int n, doublecomplex *dest);

void ztrans_64(char place, doublecomplex *scale,  doublecom-
          plex   *source,  long  m,  long  n,  doublecomplex
          *dest);
```

PURPOSE

ztrans scales and transposes the source matrix. The N2 x N1 result is written into SOURCE when PLACE = 'I' or 'i', and DEST when PLACE = 'O' or 'o'.

PLACE = 'I' or 'i': SOURCE = SCALE * SOURCE'

PLACE = 'O' or 'o': DEST = SCALE * SOURCE'

ARGUMENTS

PLACE (input)
Type of transpose. 'I' or 'i' for in-place, 'O' or 'o' for out-of-place. 'I' is default.

SCALE (input)
Scale factor on the SOURCE matrix.

SOURCE (input/output)
on input. Array of (N, M) on output if in-place transpose.

M (input)
Number of rows in the SOURCE matrix on input.

N (input)
Number of columns in the SOURCE matrix on input.

DEST (output)
Scaled and transposed SOURCE matrix if out-of-place transpose. Not referenced if in-place transpose.

Contents

NAME

ztrcon - estimate the reciprocal of the condition number of a triangular matrix A, in either the 1-norm or the infinity-norm

SYNOPSIS

```
SUBROUTINE ZTRCON(NORM, UPLO, DIAG, N, A, LDA, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 NORM, UPLO, DIAG
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER N, LDA, INFO
DOUBLE PRECISION RCOND
DOUBLE PRECISION WORK2(*)

SUBROUTINE ZTRCON_64(NORM, UPLO, DIAG, N, A, LDA, RCOND, WORK, WORK2,
      INFO)

CHARACTER * 1 NORM, UPLO, DIAG
DOUBLE COMPLEX A(LDA,*), WORK(*)
INTEGER*8 N, LDA, INFO
DOUBLE PRECISION RCOND
DOUBLE PRECISION WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE TRCON(NORM, UPLO, DIAG, N, A, [LDA], RCOND, [WORK], [WORK2],
       [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO
```

```
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: WORK2

SUBROUTINE TRCON_64(NORM, UPLO, DIAG, N, A, [LDA], RCOND, [WORK],
       [WORK2], [INFO])

CHARACTER(LEN=1) :: NORM, UPLO, DIAG
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
REAL(8) :: RCOND
REAL(8), DIMENSION(:) :: WORK2
```

C INTERFACE

```
#include <sunperf.h>
void ztrcon(char norm, char uplo, char diag,  int  n,  doub-
          lecomplex *a, int lda, double *rcond, int *info);

void ztrcon_64(char norm, char  uplo,  char  diag,  long  n,
          doublecomplex  *a,  long  lda, double *rcond, long
          *info);
```

PURPOSE

ztrcon estimates the reciprocal of the condition number of a triangular matrix A, in either the 1-norm or the infinity-norm.

The norm of A is computed and an estimate is obtained for norm(inv(A)), then the reciprocal of the condition number is computed as

RCOND = 1 / (norm(A) * norm(inv(A))).

ARGUMENTS

NORM (input)
Specifies whether the 1-norm condition number or the infinity-norm condition number is required:
= '1' or 'O': 1-norm;
= 'I': Infinity-norm.

UPLO (input)
= 'U': A is upper triangular;
= 'L': A is lower triangular.

DIAG (input)
= 'N': A is non-unit triangular;

```
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

A (input) The triangular matrix A.  If UPLO = 'U', the lead-
          ing  N-by-N  upper  triangular part of the array A
          contains the  upper  triangular  matrix,  and  the
          strictly  lower triangular part of A is not refer-
          enced.  If UPLO = 'L', the  leading  N-by-N  lower
          triangular  part of the array A contains the lower
          triangular matrix, and the strictly upper triangu-
          lar  part  of A is not referenced.  If DIAG = 'U',
          the diagonal elements of A are also not referenced
          and are assumed to be 1.
LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

RCOND (output)
          The reciprocal of  the  condition  number  of  the
          matrix   A,  computed  as  RCOND  =  1/(norm(A)  *
          norm(inv(A))).

WORK (workspace)
          dimension(2*N)

WORK2 (workspace)
          dimension(N)

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

ztrevc - compute some or all of the right and/or left eigen-
vectors of a complex upper triangular matrix T

SYNOPSIS

```
SUBROUTINE ZTREVC(SIDE, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR,
      LDVR, MM, M, WORK, RWORK, INFO)

CHARACTER * 1 SIDE, HOWMNY
DOUBLE COMPLEX T(LDT,*), VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER N, LDT, LDVL, LDVR, MM, M, INFO
LOGICAL SELECT(*)
DOUBLE PRECISION RWORK(*)

SUBROUTINE ZTREVC_64(SIDE, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR,
      LDVR, MM, M, WORK, RWORK, INFO)

CHARACTER * 1 SIDE, HOWMNY
DOUBLE COMPLEX T(LDT,*), VL(LDVL,*), VR(LDVR,*), WORK(*)
INTEGER*8 N, LDT, LDVL, LDVR, MM, M, INFO
LOGICAL*8 SELECT(*)
DOUBLE PRECISION RWORK(*)
```

F95 INTERFACE

```
SUBROUTINE TREVC(SIDE, HOWMNY, SELECT, [N], T, [LDT], VL, [LDVL], VR,
       [LDVR], MM, M, [WORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, HOWMNY
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: T, VL, VR
INTEGER :: N, LDT, LDVL, LDVR, MM, M, INFO
```

```
LOGICAL, DIMENSION(:) :: SELECT
REAL(8), DIMENSION(:) :: RWORK

SUBROUTINE TREVC_64(SIDE, HOWMNY, SELECT, [N], T, [LDT], VL, [LDVL],
       VR, [LDVR], MM, M, [WORK], [RWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, HOWMNY
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: T, VL, VR
INTEGER(8) :: N, LDT, LDVL, LDVR, MM, M, INFO
LOGICAL(8), DIMENSION(:) :: SELECT
REAL(8), DIMENSION(:) :: RWORK
```

C INTERFACE

```
#include <sunperf.h>
void ztrevc(char side, char  howmny,  int  *select,  int  n,
          doublecomplex  *t, int ldt, doublecomplex *vl, int
          ldvl, doublecomplex *vr, int ldvr, int mm, int *m,
          int *info);

void ztrevc_64(char side, char howmny, long *select, long n,
          doublecomplex  *t,  long  ldt,  doublecomplex *vl,
          long ldvl, doublecomplex *vr, long ldvr, long  mm,
          long *m, long *info);
```

PURPOSE

ztrevc computes some or all of the right and/or left eigenvectors of a complex upper triangular matrix T.

The right eigenvector x and the left eigenvector y of T corresponding to an eigenvalue w are defined by:

T*x = w*x,   y'*T = w*y'

where y' denotes the conjugate transpose of the vector y.

If all eigenvectors are requested, the routine may either return the matrices X and/or Y of right or left eigenvectors of T, or the products Q*X and/or Q*Y, where Q is an input unitary
matrix. If T was obtained from the Schur factorization of an original matrix A = Q*T*Q', then Q*X and Q*Y are the matrices of right or left eigenvectors of A.

ARGUMENTS

SIDE (input)
= 'R': compute right eigenvectors only;
= 'L': compute left eigenvectors only;
= 'B': compute both right and left eigenvectors.

HOWMNY (input)
= 'A': compute all right and/or left eigenvectors;
= 'B': compute all right and/or left eigenvectors, and backtransform them using the input matrices supplied in VR and/or VL; = 'S': compute selected right and/or left eigenvectors, specified by the logical array SELECT.

SELECT (input/output)
If HOWMNY = 'S', SELECT specifies the eigenvectors to be computed. If HOWMNY = 'A' or 'B', SELECT is not referenced. To select the eigenvector corresponding to the j-th eigenvalue, SELECT(j) must be set to .TRUE..

N (input) The order of the matrix T. N >= 0.

T (input/output)
The upper triangular matrix T. T is modified, but restored on exit.

LDT (input)
The leading dimension of the array T. LDT >= max(1,N).

VL (input/output)
On entry, if SIDE = 'L' or 'B' and HOWMNY = 'B', VL must contain an N-by-N matrix Q (usually the unitary matrix Q of Schur vectors returned by CHSEQR). On exit, if SIDE = 'L' or 'B', VL contains: if HOWMNY = 'A', the matrix Y of left eigenvectors of T; VL is lower triangular. The i-th column VL(i) of VL is the eigenvector corresponding to T(i,i). if HOWMNY = 'B', the matrix Q*Y; if HOWMNY = 'S', the left eigenvectors of T specified by SELECT, stored consecutively in the columns of VL, in the same order as their eigenvalues. If SIDE = 'R', VL is not referenced.

LDVL (input)
The leading dimension of the array VL. LDVL >= max(1,N) if SIDE = 'L' or 'B'; LDVL >= 1 otherwise.

VR (input/output)
On entry, if SIDE = 'R' or 'B' and HOWMNY = 'B', VR must contain an N-by-N matrix Q (usually the unitary matrix Q of Schur vectors returned by CHSEQR). On exit, if SIDE = 'R' or 'B', VR contains: if HOWMNY = 'A', the matrix X of right eigenvectors of T; VR is upper triangular. The i-th column VR(i) of VR is the eigenvector corresponding to T(i,i). if HOWMNY = 'B', the matrix Q*X; if HOWMNY = 'S', the right eigenvectors of T specified by SELECT, stored consecutively in the columns of VR, in the same order as their eigenvalues. If SIDE = 'L', VR is not referenced.

LDVR (input)
The leading dimension of the array VR. LDVR >= max(1,N) if SIDE = 'R' or 'B'; LDVR >= 1 otherwise.

MM (input)
The number of columns in the arrays VL and/or VR. MM >= M.

M (output)
The number of columns in the arrays VL and/or VR actually used to store the eigenvectors. If HOWMNY = 'A' or 'B', M is set to N. Each selected eigenvector occupies one column.

WORK (workspace)
dimension(2*N)

RWORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The algorithm used in this program is basically backward (forward) substitution, with scaling to make the the code robust against possible overflow.

Each eigenvector is normalized so that the element of largest magnitude has magnitude 1; here the magnitude of a complex number (x,y) is taken to be |x| + |y|.

Contents

NAME

```
ztrexc - reorder the Schur factorization of a complex matrix
A  =  Q*T*Q**H,  so  that the diagonal element of T with row
index IFST is moved to row ILST
```

SYNOPSIS

```
SUBROUTINE ZTREXC(COMPQ, N, T, LDT, Q, LDQ, IFST, ILST, INFO)

CHARACTER * 1 COMPQ
DOUBLE COMPLEX T(LDT,*), Q(LDQ,*)
INTEGER N, LDT, LDQ, IFST, ILST, INFO

SUBROUTINE ZTREXC_64(COMPQ, N, T, LDT, Q, LDQ, IFST, ILST, INFO)

CHARACTER * 1 COMPQ
DOUBLE COMPLEX T(LDT,*), Q(LDQ,*)
INTEGER*8 N, LDT, LDQ, IFST, ILST, INFO
```

F95 INTERFACE

```
SUBROUTINE TREXC(COMPQ, [N], T, [LDT], Q, [LDQ], IFST, ILST, [INFO])

CHARACTER(LEN=1) :: COMPQ
COMPLEX(8), DIMENSION(:,:) :: T, Q
INTEGER :: N, LDT, LDQ, IFST, ILST, INFO

SUBROUTINE TREXC_64(COMPQ, [N], T, [LDT], Q, [LDQ], IFST, ILST, [INFO])

CHARACTER(LEN=1) :: COMPQ
COMPLEX(8), DIMENSION(:,:) :: T, Q
INTEGER(8) :: N, LDT, LDQ, IFST, ILST, INFO
```

C INTERFACE

```
#include <sunperf.h>

void ztrexc(char compq, int n, doublecomplex  *t,  int  ldt,
          doublecomplex *q, int ldq, int ifst, int ilst, int
          *info);

void ztrexc_64(char compq, long n,  doublecomplex  *t,  long
          ldt,  doublecomplex  *q, long ldq, long ifst, long
          ilst, long *info);
```

PURPOSE

ztrexc reorders the Schur factorization of a complex matrix A = Q*T*Q**H, so that the diagonal element of T with row index IFST is moved to row ILST.
The Schur form T is reordered by a unitary similarity transformation Z**H*T*Z, and optionally the matrix Q of Schur vectors is updated by postmultplying it with Z.

ARGUMENTS

COMPQ (input)
= 'V': update the matrix Q of Schur vectors;
= 'N': do not update Q.

N (input) The order of the matrix T. N >= 0.

T (input/output)
On entry, the upper triangular matrix T. On exit, the reordered upper triangular matrix.

LDT (input)
The leading dimension of the array T. LDT >= max(1,N).

Q (input) On entry, if COMPQ = 'V', the matrix Q of Schur vectors. On exit, if COMPQ = 'V', Q has been postmultiplied by the unitary transformation matrix Z which reorders T. If COMPQ = 'N', Q is not referenced.

LDQ (input)
The leading dimension of the array Q. LDQ >= max(1,N).

IFST (input)
Specify the reordering of the diagonal elements of

T: The element with row index IFST is moved to row ILST by a sequence of transpositions between adjacent elements. 1 <= IFST <= N; 1 <= ILST <= N.

ILST (input)
See the description of IFST.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
ztrmm - perform one of the matrix-matrix operations    B  :=
alpha*op(  A  )*B, or B := alpha*B*op( A )  where alpha is a
scalar, B is an m by n matrix, A is  a  unit,  or  non-unit,
upper or lower triangular matrix and op( A ) is one of   op(
A ) = A or op( A ) = A' or op( A ) = conjg( A' )
```

SYNOPSIS

```
SUBROUTINE ZTRMM(SIDE, UPLO, TRANSA, DIAG, M, N, ALPHA, A, LDA, B,
      LDB)

CHARACTER * 1 SIDE, UPLO, TRANSA, DIAG
DOUBLE COMPLEX ALPHA
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER M, N, LDA, LDB

SUBROUTINE ZTRMM_64(SIDE, UPLO, TRANSA, DIAG, M, N, ALPHA, A, LDA, B,
      LDB)

CHARACTER * 1 SIDE, UPLO, TRANSA, DIAG
DOUBLE COMPLEX ALPHA
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 M, N, LDA, LDB
```

F95 INTERFACE

```
SUBROUTINE TRMM(SIDE, UPLO, [TRANSA], DIAG, [M], [N], ALPHA, A, [LDA],
       B, [LDB])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANSA, DIAG
COMPLEX(8) :: ALPHA
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: M, N, LDA, LDB
```

```
SUBROUTINE TRMM_64(SIDE, UPLO, [TRANSA], DIAG, [M], [N], ALPHA, A,
       [LDA], B, [LDB])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANSA, DIAG
COMPLEX(8) :: ALPHA
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: M, N, LDA, LDB
```

C INTERFACE

```
#include <sunperf.h>

void ztrmm(char side, char uplo, char transa, char diag, int
          m,  int n, doublecomplex *alpha, doublecomplex *a,
          int lda, doublecomplex *b, int ldb);
void ztrmm_64(char side, char uplo, char transa, char  diag,
          long  m,  long n, doublecomplex *alpha, doublecom-
          plex *a, long lda, doublecomplex *b, long ldb);
```

PURPOSE

ztrmm performs one of the matrix-matrix operations B := alpha*op(A)*B, or B := alpha*B*op(A) where alpha is a scalar, B is an m by n matrix, A is a unit, or non-unit, upper or lower triangular matrix and op(A) is one of op(A) = A or op(A) = A' or op(A) = conjg(A')

ARGUMENTS

SIDE (input)
On entry, SIDE specifies whether op(A) multiplies B from the left or right as follows:

SIDE = 'L' or 'l'   B := alpha*op(A)*B.

SIDE = 'R' or 'r'   B := alpha*B*op(A).

Unchanged on exit.

UPLO (input)
On entry, UPLO specifies whether the matrix A is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular

```
            matrix.

            Unchanged on exit.

  TRANSA (input)
            On entry, TRANSA specifies the form of op( A )  to
            be used in the matrix multiplication as follows:

            TRANSA = 'N' or 'n'   op( A ) = A.

            TRANSA = 'T' or 't'   op( A ) = A'.

            TRANSA = 'C' or 'c'   op( A ) = conjg( A' ).

            Unchanged on exit.

            TRANSA is defaulted to 'N' for F95 INTERFACE.
  DIAG (input)
            On entry, DIAG specifies whether or not A is  unit
            triangular as follows:

            DIAG = 'U' or 'u'   A is assumed to be  unit  tri-
            angular.

            DIAG = 'N' or 'n'   A is not assumed  to  be  unit
            triangular.

            Unchanged on exit.

  M (input)
            On entry, M specifies the number of rows of  B.  M
            >= 0.  Unchanged on exit.

  N (input)
            On entry, N specifies the number of columns of  B.
            N >= 0.  Unchanged on exit.

  ALPHA (input)
            On entry,  ALPHA specifies the scalar  alpha. When
            alpha  is  zero  then   A is not referenced and  B
            need not be set before entry.  Unchanged on exit.

  A (input)
            COMPLEX*16 array of DIMENSION ( LDA, k ), where  k
            is  m  when   SIDE  =  'L' or 'l'  and is  n  when
            SIDE = 'R' or 'r'.

            Before entry  with  UPLO = 'U' or 'u',  the  lead-
            ing   k by k upper triangular part of the array  A
            must contain the upper triangular matrix  and  the
```

strictly lower triangular part of A is not referenced.

Before entry with UPLO = 'L' or 'l', the leading k by k lower triangular part of the array A must contain the lower triangular matrix and the strictly upper triangular part of A is not referenced.

Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced either, but are assumed to be unity.

Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When SIDE = 'L' or 'l' then LDA >= max(1,M), when SIDE = 'R' or 'r' then LDA >= max(1,N). Unchanged on exit.

B (input/output)

COMPLEX*16 array of DIMENSION (LDB, n). Before entry, the leading M by N part of the array B must contain the matrix B, and on exit is overwritten by the transformed matrix.

LDB (input)

On entry, LDB specifies the first dimension of B as declared in the calling subprogram. LDB must be at least max(1,M). Unchanged on exit.

Contents

NAME

```
ztrmv - perform one of the matrix-vector operations    x  :=
A*x, or x := A'*x, or x := conjg( A' )*x
```

SYNOPSIS

```
SUBROUTINE ZTRMV(UPLO, TRANSA, DIAG, N, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(LDA,*), Y(*)
INTEGER N, LDA, INCY

SUBROUTINE ZTRMV_64(UPLO, TRANSA, DIAG, N, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(LDA,*), Y(*)
INTEGER*8 N, LDA, INCY
```

F95 INTERFACE

```
SUBROUTINE TRMV(UPLO, [TRANSA], DIAG, [N], A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, INCY

SUBROUTINE TRMV_64(UPLO, [TRANSA], DIAG, [N], A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INCY
```

C INTERFACE

```
#include <sunperf.h>

void ztrmv(char uplo, char transa, char diag, int  n,  doub-
          lecomplex  *a,  int  lda,  doublecomplex  *y,  int
          incy);

void ztrmv_64(char uplo, char transa,  char  diag,  long  n,
          doublecomplex *a, long lda, doublecomplex *y, long
          incy);
```

PURPOSE

ztrmv performs one of the matrix-vector operations x := A*x, or x := A'*x, or x := conjg(A')*x where x is an n element vector and A is an n by n unit, or non-unit, upper or lower triangular matrix.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the operation to be performed as follows:

TRANSA = 'N' or 'n'   x := A*x.

TRANSA = 'T' or 't'   x := A'*x.

TRANSA = 'C' or 'c'   x := conjg(A')*x.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)

On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit triangular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

A (input)
Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular matrix and the strictly lower triangular part of A is not referenced. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular matrix and the strictly upper triangular part of A is not referenced. Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced either, but are assumed to be unity. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element vector x. On exit, Y is overwritten with the tranformed vector x.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

```
ztrrfs - provide error bounds and backward  error  estimates
for the solution to a system of linear equations with a tri-
angular coefficient matrix
```

SYNOPSIS

```
SUBROUTINE ZTRRFS(UPLO, TRANSA, DIAG, N, NRHS, A, LDA, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(LDA,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER N, NRHS, LDA, LDB, LDX, INFO
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)

SUBROUTINE ZTRRFS_64(UPLO, TRANSA, DIAG, N, NRHS, A, LDA, B, LDB, X,
      LDX, FERR, BERR, WORK, WORK2, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(LDA,*), B(LDB,*), X(LDX,*), WORK(*)
INTEGER*8 N, NRHS, LDA, LDB, LDX, INFO
DOUBLE PRECISION FERR(*), BERR(*), WORK2(*)
```

F95 INTERFACE

```
SUBROUTINE TRRFS(UPLO, [TRANSA], DIAG, N, NRHS, A, [LDA], B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, X
INTEGER :: N, NRHS, LDA, LDB, LDX, INFO
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

```
SUBROUTINE TRRFS_64(UPLO, [TRANSA], DIAG, N, NRHS, A, [LDA], B, [LDB],
       X, [LDX], FERR, BERR, [WORK], [WORK2], [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: WORK
COMPLEX(8), DIMENSION(:,:) :: A, B, X
INTEGER(8) :: N, NRHS, LDA, LDB, LDX, INFO
REAL(8), DIMENSION(:) :: FERR, BERR, WORK2
```

C INTERFACE

```
#include <sunperf.h>

void ztrrfs(char uplo, char transa, char diag,  int  n,  int
          nrhs, doublecomplex *a, int lda, doublecomplex *b,
          int ldb, doublecomplex *x, int ldx, double  *ferr,
          double *berr, int *info);

void ztrrfs_64(char uplo, char transa, char  diag,  long  n,
          long  nrhs, doublecomplex *a, long lda, doublecom-
          plex *b, long ldb,  doublecomplex  *x,  long  ldx,
          double *ferr, double *berr, long *info);
```

PURPOSE

ztrrfs provides error bounds and backward error estimates for the solution to a system of linear equations with a triangular coefficient matrix.

The solution matrix X must be computed by CTRTRS or some other means before entering this routine. CTRRFS does not do iterative refinement because doing so cannot improve the backward error.

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

TRANSA (input)
          Specifies the form of the system of equations:
          = 'N':  A * X = B     (No transpose)
          = 'T':  A**T * X = B  (Transpose)
          = 'C':  A**H * X = B  (Conjugate transpose)

          TRANSA is defaulted to 'N' for F95 INTERFACE.
```

```
DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

NRHS (input)
          The number of right hand sides, i.e.,  the  number
          of columns of the matrices B and X.  NRHS >= 0.

A (input) The triangular matrix A.  If UPLO = 'U', the lead-
          ing  N-by-N  upper  triangular part of the array A
          contains the  upper  triangular  matrix,  and  the
          strictly   lower  triangular  part  of  A  is  not
          referenced.  If UPLO =  'L',  the  leading  N-by-N
          lower  triangular part of the array A contains the
          lower triangular matrix, and  the  strictly  upper
          triangular part of A is not referenced.  If DIAG =
          'U', the diagonal  elements  of  A  are  also  not
          referenced and are assumed to be 1.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

B (input) The right hand side matrix B.

LDB (input)
          The leading dimension of  the  array  B.   LDB  >=
          max(1,N).

X (input) The solution matrix X.

LDX (input)
          The leading dimension of  the  array  X.   LDX  >=
          max(1,N).

FERR (output)
          The estimated forward error bound for  each  solu-
          tion  vector X(j) (the j-th column of the solution
          matrix  X).   If  XTRUE  is  the   true   solution
          corresponding  to  X(j),  FERR(j)  is an estimated
          upper bound for the magnitude of the largest  ele-
          ment in (X(j) - XTRUE) divided by the magnitude of
          the largest element in X(j).  The estimate  is  as
          reliable  as the estimate for RCOND, and is almost
          always a slight overestimate of the true error.

BERR (output)
          The componentwise relative backward error of  each
```

solution vector X(j) (i.e., the smallest relative change in any element of A or B that makes X(j) an exact solution).

WORK (workspace)
dimension(2*N)

WORK2 (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

ztrsen - reorder the Schur factorization of a complex matrix A = Q*T*Q**H, so that a selected cluster of eigenvalues appears in the leading positions on the diagonal of the upper triangular matrix T, and the leading columns of Q form an orthonormal basis of the corresponding right invariant subspace

SYNOPSIS

```
SUBROUTINE ZTRSEN(JOB, COMPQ, SELECT, N, T, LDT, Q, LDQ, W, M, S,
      SEP, WORK, LWORK, INFO)

CHARACTER * 1 JOB, COMPQ
DOUBLE COMPLEX T(LDT,*), Q(LDQ,*), W(*), WORK(*)
INTEGER N, LDT, LDQ, M, LWORK, INFO
LOGICAL SELECT(*)
DOUBLE PRECISION S, SEP

SUBROUTINE ZTRSEN_64(JOB, COMPQ, SELECT, N, T, LDT, Q, LDQ, W, M, S,
      SEP, WORK, LWORK, INFO)

CHARACTER * 1 JOB, COMPQ
DOUBLE COMPLEX T(LDT,*), Q(LDQ,*), W(*), WORK(*)
INTEGER*8 N, LDT, LDQ, M, LWORK, INFO
LOGICAL*8 SELECT(*)
DOUBLE PRECISION S, SEP
```

F95 INTERFACE

```
SUBROUTINE TRSEN(JOB, COMPQ, SELECT, [N], T, [LDT], Q, [LDQ], W, M,
       S, SEP, [WORK], [LWORK], [INFO])
```

```
CHARACTER(LEN=1) :: JOB, COMPQ
COMPLEX(8), DIMENSION(:) :: W, WORK
COMPLEX(8), DIMENSION(:,:) :: T, Q
INTEGER :: N, LDT, LDQ, M, LWORK, INFO
LOGICAL, DIMENSION(:) :: SELECT
REAL(8) :: S, SEP

SUBROUTINE TRSEN_64(JOB, COMPQ, SELECT, [N], T, [LDT], Q, [LDQ], W,
       M, S, SEP, [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: JOB, COMPQ
COMPLEX(8), DIMENSION(:) :: W, WORK
COMPLEX(8), DIMENSION(:,:) :: T, Q
INTEGER(8) :: N, LDT, LDQ, M, LWORK, INFO
LOGICAL(8), DIMENSION(:) :: SELECT
REAL(8) :: S, SEP
```

C INTERFACE

```
#include <sunperf.h>

void ztrsen(char job, char compq, int *select, int n,  doub-
          lecomplex  *t, int ldt, doublecomplex *q, int ldq,
          doublecomplex *w, int *m, double *s, double  *sep,
          int *info);

void ztrsen_64(char job, char compq, long *select,  long  n,
          doublecomplex *t, long ldt, doublecomplex *q, long
          ldq, doublecomplex *w, long *m, double *s,  double
          *sep, long *info);
```

PURPOSE

ztrsen reorders the Schur factorization of a complex matrix A = Q*T*Q**H, so that a selected cluster of eigenvalues appears in the leading positions on the diagonal of the upper triangular matrix T, and the leading columns of Q form an orthonormal basis of the corresponding right invariant subspace.

Optionally the routine computes the reciprocal condition numbers of the cluster of eigenvalues and/or the invariant subspace.

ARGUMENTS

JOB (input)
Specifies whether condition numbers are required for the cluster of eigenvalues (S) or the invari-

```
          ant subspace (SEP):
          = 'N': none;
          = 'E': for eigenvalues only (S);
          = 'V': for invariant subspace only (SEP);
          = 'B': for both eigenvalues and invariant subspace
          (S and SEP).

COMPQ (input)
          = 'V': update the matrix Q of Schur vectors;
          = 'N': do not update Q.

SELECT (input)
          SELECT specifies the eigenvalues in  the  selected
          cluster.  To select the j-th eigenvalue, SELECT(j)
          must be set to .TRUE..

N (input) The order of the matrix T. N >= 0.
T (input/output)
          On entry, the upper triangular matrix T.  On exit,
          T  is  overwritten by the reordered matrix T, with
          the selected eigenvalues as the  leading  diagonal
          elements.

LDT (input)
          The leading dimension  of  the  array  T.  LDT  >=
          max(1,N).

Q (input) On entry, if COMPQ = 'V', the matrix  Q  of  Schur
          vectors.   On  exit,  if  COMPQ  = 'V', Q has been
          postmultiplied  by  the   unitary   transformation
          matrix  which reorders T; the leading M columns of
          Q form an  orthonormal  basis  for  the  specified
          invariant  subspace.   If  COMPQ  =  'N', Q is not
          referenced.

LDQ (input)
          The leading dimension of the array Q.  LDQ  >=  1;
          and if COMPQ = 'V', LDQ >= N.

W (output)
          The reordered eigenvalues of T, in the same  order
          as they appear on the diagonal of T.

M (output)
          The dimension of the specified invariant subspace.
          0 <= M <= N.

S (output)
          If JOB = 'E' or 'B', S is a  lower  bound  on  the
          reciprocal condition number for the selected clus-
```

```
          ter of eigenvalues.  S  cannot  underestimate  the
          true  reciprocal  condition  number by more than a
          factor of sqrt(N). If M = 0 or N, S = 1.  If JOB =
          'N' or 'V', S is not referenced.

SEP (output)
          If JOB = 'V' or 'B', SEP is the estimated recipro-
          cal  condition  number  of the specified invariant
          subspace. If M = 0 or N, SEP = norm(T).  If JOB  =
          'N' or 'E', SEP is not referenced.
WORK (workspace)
          If JOB = 'N', WORK is not referenced.   Otherwise,
          on  exit, if INFO = 0, WORK(1) returns the optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK.  If  JOB  =  'N',
          LWORK  >= 1; if JOB = 'E', LWORK = M*(N-M); if JOB
          = 'V' or 'B', LWORK >= 2*M*(N-M).

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

FURTHER DETAILS

```
CTRSEN first collects the selected eigenvalues by  computing
a  unitary  transformation  Z  to  move them to the top left
corner of T. In other words, the  selected  eigenvalues  are
the eigenvalues of T11 in:

              Z'*T*Z = ( T11 T12 ) n1
                       (  0  T22 ) n2
                          n1  n2

where N = n1+n2 and Z' means the conjugate transpose  of  Z.
The  first n1 columns of Z span the specified invariant sub-
space of T.

If T has been obtained from the  Schur  factorization  of  a
matrix A = Q*T*Q', then the reordered Schur factorization of
A is given by A = (Q*Z)*(Z'*T*Z)*(Q*Z)', and  the  first  n1
columns  of Q*Z span the corresponding invariant subspace of
```

A.

The reciprocal condition number of the average of the eigenvalues of T11 may be returned in S. S lies between 0 (very badly conditioned) and 1 (very well conditioned). It is computed as follows. First we compute R so that

```
P = ( I  R ) n1
    ( 0  0 ) n2
      n1 n2
```

is the projector on the invariant subspace associated with T11. R is the solution of the Sylvester equation:

```
T11*R - R*T22 = T12.
```

Let F-norm(M) denote the Frobenius-norm of M and 2-norm(M) denote the two-norm of M. Then S is computed as the lower bound

```
(1 + F-norm(R)**2)**(-1/2)
```

on the reciprocal of 2-norm(P), the true reciprocal condition number. S cannot underestimate 1 / 2-norm(P) by more than a factor of sqrt(N).

An approximate error bound for the computed average of the eigenvalues of T11 is

```
EPS * norm(T) / S
```

where EPS is the machine precision.

The reciprocal condition number of the right invariant subspace spanned by the first n1 columns of Z (or of Q*Z) is returned in SEP. SEP is defined as the separation of T11 and T22:

```
sep( T11, T22 ) = sigma-min( C )
```

where sigma-min(C) is the smallest singular value of the n1*n2-by-n1*n2 matrix

```
C  = kprod( I(n2), T11 ) - kprod( transpose(T22), I(n1) )
```

I(m) is an m by m identity matrix, and kprod denotes the Kronecker product. We estimate sigma-min(C) by the reciprocal of an estimate of the 1-norm of inverse(C). The true reciprocal 1-norm of inverse(C) cannot differ from sigma-min(C) by more than a factor of sqrt(n1*n2).

When SEP is small, small changes in T can cause large changes in the invariant subspace. An approximate bound on the maximum angular error in the computed right invariant subspace is

```
EPS * norm(T) / SEP
```

Contents

NAME

```
ztrsm - solve one of  the  matrix  equations  op(  A  )*X  =
alpha*B, or X*op( A ) = alpha*B
```

SYNOPSIS

```
SUBROUTINE ZTRSM(SIDE, UPLO, TRANSA, DIAG, M, N, ALPHA, A, LDA, B,
      LDB)

CHARACTER * 1 SIDE, UPLO, TRANSA, DIAG
DOUBLE COMPLEX ALPHA
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER M, N, LDA, LDB

SUBROUTINE ZTRSM_64(SIDE, UPLO, TRANSA, DIAG, M, N, ALPHA, A, LDA, B,
      LDB)

CHARACTER * 1 SIDE, UPLO, TRANSA, DIAG
DOUBLE COMPLEX ALPHA
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 M, N, LDA, LDB
```

F95 INTERFACE

```
SUBROUTINE TRSM(SIDE, UPLO, [TRANSA], DIAG, [M], [N], ALPHA, A, [LDA],
       B, [LDB])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANSA, DIAG
COMPLEX(8) :: ALPHA
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: M, N, LDA, LDB

SUBROUTINE TRSM_64(SIDE, UPLO, [TRANSA], DIAG, [M], [N], ALPHA, A,
       [LDA], B, [LDB])
```

```
CHARACTER(LEN=1) :: SIDE, UPLO, TRANSA, DIAG
COMPLEX(8) :: ALPHA
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: M, N, LDA, LDB
```

C INTERFACE

```
#include <sunperf.h>

void ztrsm(char side, char uplo, char transa, char diag, int
          m,  int n, doublecomplex *alpha, doublecomplex *a,
          int lda, doublecomplex *b, int ldb);

void ztrsm_64(char side, char uplo, char transa, char  diag,
          long    m,    long    n,   doublecomplex   *alpha,
          doublecomplex *a, long lda, doublecomplex *b, long
          ldb);
```

PURPOSE

ztrsm solves one of the matrix equations op(A)*X = alpha*B, or X*op(A) = alpha*B where alpha is a scalar, X and B are m by n matrices, A is a unit, or non-unit, upper or lower triangular matrix and op(A) is one of

op(A) = A or op(A) = A' or op(A) = conjg(A').

The matrix X is overwritten on B.

ARGUMENTS

SIDE (input)
On entry, SIDE specifies whether op(A) appears on the left or right of X as follows:

SIDE = 'L' or 'l'   op(A)*X = alpha*B.

SIDE = 'R' or 'r'   X*op(A) = alpha*B.

Unchanged on exit.

UPLO (input)
On entry, UPLO specifies whether the matrix A is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular

matrix.

UPLO = 'L' or 'l' A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)

On entry, TRANSA specifies the form of op(A) to be used in the matrix multiplication as follows:

TRANSA = 'N' or 'n' op(A) = A.

TRANSA = 'T' or 't' op(A) = A'.

TRANSA = 'C' or 'c' op(A) = conjg(A').
Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)

On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u' A is assumed to be unit triangular.

DIAG = 'N' or 'n' A is not assumed to be unit triangular.

Unchanged on exit.

M (input)

On entry, M specifies the number of rows of B. M >= 0. Unchanged on exit.

N (input)

On entry, N specifies the number of columns of B. N >= 0. Unchanged on exit.

ALPHA (input)

On entry, ALPHA specifies the scalar alpha. When alpha is zero then A is not referenced and B need not be set before entry. Unchanged on exit.

A (input)

COMPLEX*16 array of DIMENSION (LDA, k), where k is m when SIDE = 'L' or 'l' and is n when SIDE = 'R' or 'r'.

Before entry with UPLO = 'U' or 'u', the leading k by k upper triangular part of the array A must contain the upper triangular matrix and the strictly lower triangular part of A is not referenced.

Before entry with UPLO = 'L' or 'l', the leading k by k lower triangular part of the array A must contain the lower triangular matrix and the strictly upper triangular part of A is not referenced.

Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced either, but are assumed to be unity.

Unchanged on exit.

LDA (input)

On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. When SIDE = 'L' or 'l' then LDA >= max(1,M), when SIDE = 'R' or 'r' then LDA >= max(1,N). Unchanged on exit.

B (input/output)

COMPLEX*16 array of DIMENSION (LDB, n). Before entry, the leading M by N part of the array B must contain the right-hand side matrix B, and on exit is overwritten by the solution matrix X.

LDB (input)

On entry, LDB specifies the first dimension of B as declared in the calling subprogram. LDB >= max(1,M). Unchanged on exit.

Contents

NAME

ztrsna - estimate reciprocal condition numbers for specified eigenvalues and/or right eigenvectors of a complex upper triangular matrix T (or of any matrix Q*T*Q**H with Q unitary)

SYNOPSIS

```
SUBROUTINE ZTRSNA(JOB, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR, LDVR,
      S, SEP, MM, M, WORK, LDWORK, WORK1, INFO)

CHARACTER * 1 JOB, HOWMNY
DOUBLE    COMPLEX    T(LDT,*),    VL(LDVL,*),    VR(LDVR,*),
WORK(LDWORK,*)
INTEGER N, LDT, LDVL, LDVR, MM, M, LDWORK, INFO
LOGICAL SELECT(*)
DOUBLE PRECISION S(*), SEP(*), WORK1(*)

SUBROUTINE ZTRSNA_64(JOB, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR,
      LDVR, S, SEP, MM, M, WORK, LDWORK, WORK1, INFO)

CHARACTER * 1 JOB, HOWMNY
DOUBLE    COMPLEX    T(LDT,*),    VL(LDVL,*),    VR(LDVR,*),
WORK(LDWORK,*)
INTEGER*8 N, LDT, LDVL, LDVR, MM, M, LDWORK, INFO
LOGICAL*8 SELECT(*)
DOUBLE PRECISION S(*), SEP(*), WORK1(*)
```

F95 INTERFACE

```
SUBROUTINE TRSNA(JOB, HOWMNY, SELECT, [N], T, [LDT], VL, [LDVL], VR,
       [LDVR], S, SEP, MM, M, [WORK], [LDWORK], [WORK1], [INFO])
```

```
CHARACTER(LEN=1) :: JOB, HOWMNY
COMPLEX(8), DIMENSION(:,:) :: T, VL, VR, WORK
INTEGER :: N, LDT, LDVL, LDVR, MM, M, LDWORK, INFO
LOGICAL, DIMENSION(:) :: SELECT
REAL(8), DIMENSION(:) :: S, SEP, WORK1

SUBROUTINE TRSNA_64(JOB, HOWMNY, SELECT, [N], T, [LDT], VL, [LDVL],
       VR, [LDVR], S, SEP, MM, M, [WORK], [LDWORK], [WORK1], [INFO])

CHARACTER(LEN=1) :: JOB, HOWMNY
COMPLEX(8), DIMENSION(:,:) :: T, VL, VR, WORK
INTEGER(8) :: N, LDT, LDVL, LDVR, MM, M, LDWORK, INFO
LOGICAL(8), DIMENSION(:) :: SELECT
REAL(8), DIMENSION(:) :: S, SEP, WORK1
```

C INTERFACE

```
#include <sunperf.h>

void ztrsna(char job, char howmny, int *select, int n, doub-
          lecomplex  *t,  int  ldt,  doublecomplex  *vl, int
          ldvl, doublecomplex *vr, int ldvr, double *s, dou-
          ble *sep, int mm, int *m, int ldwork, int *info);

void ztrsna_64(char job, char howmny, long *select, long  n,
          doublecomplex  *t,  long  ldt,  doublecomplex *vl,
          long ldvl, doublecomplex *vr,  long  ldvr,  double
          *s,  double  *sep,  long mm, long *m, long ldwork,
          long *info);
```

PURPOSE

ztrsna estimates reciprocal condition numbers for specified eigenvalues and/or right eigenvectors of a complex upper triangular matrix T (or of any matrix Q*T*Q**H with Q unitary).

ARGUMENTS

JOB (input)
Specifies whether condition numbers are required for eigenvalues (S) or eigenvectors (SEP):
= 'E': for eigenvalues only (S);
= 'V': for eigenvectors only (SEP);
= 'B': for both eigenvalues and eigenvectors (S and SEP).

HOWMNY (input)
= 'A': compute condition numbers for all eigen-

```
          pairs;
          = 'S':  compute  condition  numbers  for  selected
          eigenpairs specified by the array SELECT.

SELECT (input)
          If HOWMNY = 'S', SELECT specifies  the  eigenpairs
          for  which  condition  numbers  are  required.  To
          select condition numbers for the  j-th  eigenpair,
          SELECT(j) must be set to .TRUE..  If HOWMNY = 'A',
          SELECT is not referenced.

N (input) The order of the matrix T. N >= 0.

T (input) The upper triangular matrix T.
LDT (input)
          The leading dimension  of  the  array  T.  LDT  >=
          max(1,N).

VL (input)
          If JOB = 'E' or 'B', VL must contain  left  eigen-
          vectors  of T (or of any Q*T*Q**H with Q unitary),
          corresponding  to  the  eigenpairs  specified   by
          HOWMNY and SELECT. The eigenvectors must be stored
          in consecutive  columns  of  VL,  as  returned  by
          CHSEIN  or CTREVC.  If JOB = 'V', VL is not refer-
          enced.

LDVL (input)
          The leading dimension of the array VL.  LDVL >= 1;
          and if JOB = 'E' or 'B', LDVL >= N.

VR (input)
          If JOB = 'E' or 'B', VR must contain right  eigen-
          vectors  of T (or of any Q*T*Q**H with Q unitary),
          corresponding  to  the  eigenpairs  specified   by
          HOWMNY and SELECT. The eigenvectors must be stored
          in consecutive  columns  of  VR,  as  returned  by
          CHSEIN  or CTREVC.  If JOB = 'V', VR is not refer-
          enced.

LDVR (input)
          The leading dimension of the array VR.  LDVR >= 1;
          and if JOB = 'E' or 'B', LDVR >= N.

S (output)
          If JOB = 'E'  or  'B',  the  reciprocal  condition
          numbers  of  the  selected  eigenvalues, stored in
          consecutive elements  of  the  array.  Thus  S(j),
          SEP(j),  and  the  j-th  columns  of VL and VR all
          correspond to the same eigenpair (but not in  gen-
```

eral the j-th eigenpair, unless all eigenpairs are selected). If JOB = 'V', S is not referenced.

SEP (output)
If JOB = 'V' or 'B', the estimated reciprocal condition numbers of the selected eigenvectors, stored in consecutive elements of the array. If JOB = 'E', SEP is not referenced.

MM (input)
The number of elements in the arrays S (if JOB = 'E' or 'B') and/or SEP (if JOB = 'V' or 'B'). MM >= M.

M (output)
The number of elements of the arrays S and/or SEP actually used to store the estimated condition numbers. If HOWMNY = 'A', M is set to N.

WORK (workspace)
dimension(LDWORK,N+1) If JOB = 'E', WORK is not referenced.

LDWORK (input)
The leading dimension of the array WORK. LDWORK >= 1; and if JOB = 'V' or 'B', LDWORK >= N.

WORK1 (workspace)
dimension(N) If JOB = 'E', WORK1 is not referenced.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

The reciprocal of the condition number of an eigenvalue lambda is defined as

S(lambda) = |v'*u| / (norm(u)*norm(v))

where u and v are the right and left eigenvectors of T corresponding to lambda; v' denotes the conjugate transpose of v, and norm(u) denotes the Euclidean norm. These reciprocal condition numbers always lie between zero (very badly conditioned) and one (very well conditioned). If n = 1, S(lambda) is defined to be 1.

An approximate error bound for a computed eigenvalue W(i) is given by

```
EPS * norm(T) / S(i)
```

where EPS is the machine precision.
The reciprocal of the condition number of the right eigenvector u corresponding to lambda is defined as follows. Suppose

```
T = ( lambda  c  )
    (   0    T22 )
```

Then the reciprocal condition number is

```
SEP( lambda, T22 ) = sigma-min( T22 - lambda*I )
```

where sigma-min denotes the smallest singular value. We approximate the smallest singular value by the reciprocal of an estimate of the one-norm of the inverse of T22 - lambda*I. If n = 1, SEP(1) is defined to be abs(T(1,1)).

An approximate error bound for a computed right eigenvector VR(i) is given by

```
EPS * norm(T) / SEP(i)
```

Contents

NAME

```
ztrsv - solve one of the systems of equations   A*x = b,  or
A'*x = b, or conjg( A' )*x = b
```

SYNOPSIS

```
SUBROUTINE ZTRSV(UPLO, TRANSA, DIAG, N, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(LDA,*), Y(*)
INTEGER N, LDA, INCY

SUBROUTINE ZTRSV_64(UPLO, TRANSA, DIAG, N, A, LDA, Y, INCY)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(LDA,*), Y(*)
INTEGER*8 N, LDA, INCY
```

F95 INTERFACE

```
SUBROUTINE TRSV(UPLO, [TRANSA], DIAG, [N], A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, INCY

SUBROUTINE TRSV_64(UPLO, [TRANSA], DIAG, [N], A, [LDA], Y, [INCY])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:) :: Y
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INCY
```

C INTERFACE

```
#include <sunperf.h>

void ztrsv(char uplo, char transa, char diag, int  n,  doub-
          lecomplex  *a,  int  lda,  doublecomplex  *y,  int
          incy);

void ztrsv_64(char uplo, char transa,  char  diag,  long  n,
          doublecomplex *a, long lda, doublecomplex *y, long
          incy);
```

PURPOSE

ztrsv solves one of the systems of equations A*x = b, or A'*x = b, or conjg(A')*x = b where b and x are n element vectors and A is an n by n unit, or non-unit, upper or lower triangular matrix.

No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

ARGUMENTS

UPLO (input)
On entry, UPLO specifies whether the matrix is an upper or lower triangular matrix as follows:

UPLO = 'U' or 'u'   A is an upper triangular matrix.

UPLO = 'L' or 'l'   A is a lower triangular matrix.

Unchanged on exit.

TRANSA (input)
On entry, TRANSA specifies the equations to be solved as follows:

TRANSA = 'N' or 'n'   A*x = b.

TRANSA = 'T' or 't'   A'*x = b.

TRANSA = 'C' or 'c'   conjg(A')*x = b.

Unchanged on exit.

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
On entry, DIAG specifies whether or not A is unit triangular as follows:

DIAG = 'U' or 'u'   A is assumed to be unit triangular.

DIAG = 'N' or 'n'   A is not assumed to be unit triangular.

Unchanged on exit.

N (input)
On entry, N specifies the order of the matrix A. N >= 0. Unchanged on exit.

A (input)
Before entry with UPLO = 'U' or 'u', the leading n by n upper triangular part of the array A must contain the upper triangular matrix and the strictly lower triangular part of A is not referenced. Before entry with UPLO = 'L' or 'l', the leading n by n lower triangular part of the array A must contain the lower triangular matrix and the strictly upper triangular part of A is not referenced. Note that when DIAG = 'U' or 'u', the diagonal elements of A are not referenced either, but are assumed to be unity. Unchanged on exit.

LDA (input)
On entry, LDA specifies the first dimension of A as declared in the calling (sub) program. LDA >= max(1, n). Unchanged on exit.

Y (input/output)
(1 + (n - 1)*abs(INCY)). Before entry, the incremented array Y must contain the n element right-hand side vector b. On exit, Y is overwritten with the solution vector x.

INCY (input)
On entry, INCY specifies the increment for the elements of Y. INCY <> 0. Unchanged on exit.

Contents

NAME

ztrsyl - solve the complex Sylvester matrix equation

SYNOPSIS

```
SUBROUTINE ZTRSYL(TRANA, TRANB, ISGN, M, N, A, LDA, B, LDB, C, LDC,
      SCALE, INFO)

CHARACTER * 1 TRANA, TRANB
DOUBLE COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER ISGN, M, N, LDA, LDB, LDC, INFO
DOUBLE PRECISION SCALE

SUBROUTINE ZTRSYL_64(TRANA, TRANB, ISGN, M, N, A, LDA, B, LDB, C,
      LDC, SCALE, INFO)

CHARACTER * 1 TRANA, TRANB
DOUBLE COMPLEX A(LDA,*), B(LDB,*), C(LDC,*)
INTEGER*8 ISGN, M, N, LDA, LDB, LDC, INFO
DOUBLE PRECISION SCALE
```

F95 INTERFACE

```
SUBROUTINE TRSYL(TRANA, TRANB, ISGN, [M], [N], A, [LDA], B, [LDB], C,
       [LDC], SCALE, [INFO])

CHARACTER(LEN=1) :: TRANA, TRANB
COMPLEX(8), DIMENSION(:,:) :: A, B, C
INTEGER :: ISGN, M, N, LDA, LDB, LDC, INFO
REAL(8) :: SCALE

SUBROUTINE TRSYL_64(TRANA, TRANB, ISGN, [M], [N], A, [LDA], B, [LDB],
       C, [LDC], SCALE, [INFO])
```

```
CHARACTER(LEN=1) :: TRANA, TRANB
COMPLEX(8), DIMENSION(:,:) :: A, B, C
INTEGER(8) :: ISGN, M, N, LDA, LDB, LDC, INFO
REAL(8) :: SCALE
```

C INTERFACE

```
#include <sunperf.h>

void ztrsyl(char trana, char tranb, int isgn, int m, int  n,
          doublecomplex  *a,  int lda, doublecomplex *b, int
          ldb, doublecomplex *c, int ldc, double *scale, int
          *info);

void ztrsyl_64(char trana, char tranb, long  isgn,  long  m,
          long  n, doublecomplex *a, long lda, doublecomplex
          *b, long ldb, doublecomplex *c, long  ldc,  double
          *scale, long *info);
```

PURPOSE

ztrsyl solves the complex Sylvester matrix equation:

```
   op(A)*X + X*op(B) = scale*C or
   op(A)*X - X*op(B) = scale*C,
```

where op(A) = A or A**H, and A and B are both upper triangular. A is M-by-M and B is N-by-N; the right hand side C and the solution X are M-by-N; and scale is an output scale factor, set <= 1 to avoid overflow in X.

ARGUMENTS

```
TRANA (input)
          Specifies the option op(A):
          = 'N': op(A) = A    (No transpose)
          = 'C': op(A) = A**H (Conjugate transpose)

TRANB (input)
          Specifies the option op(B):
          = 'N': op(B) = B    (No transpose)
          = 'C': op(B) = B**H (Conjugate transpose)

ISGN (input)
          Specifies the sign in the equation:
          = +1: solve op(A)*X + X*op(B) = scale*C
          = -1: solve op(A)*X - X*op(B) = scale*C
```

M (input) The order of the matrix A, and the number of rows in the matrices X and C. M >= 0.

N (input) The order of the matrix B, and the number of columns in the matrices X and C. N >= 0.

A (input) The upper triangular matrix A.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

B (input) The upper triangular matrix B.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

C (input/output)
On entry, the M-by-N right hand side matrix C. On exit, C is overwritten by the solution matrix X.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M)

SCALE (output)
The scale factor, scale, set <= 1 to avoid overflow in X.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
= 1: A and B have common or very close eigenvalues; perturbed values were used to solve the equation (but the matrices A and B are unchanged).

Contents

NAME

ztrti2 - compute the inverse of a complex upper or lower triangular matrix

SYNOPSIS

```
SUBROUTINE ZTRTI2(UPLO, DIAG, N, A, LDA, INFO)

CHARACTER * 1 UPLO, DIAG
DOUBLE COMPLEX A(LDA,*)
INTEGER N, LDA, INFO

SUBROUTINE ZTRTI2_64(UPLO, DIAG, N, A, LDA, INFO)

CHARACTER * 1 UPLO, DIAG
DOUBLE COMPLEX A(LDA,*)
INTEGER*8 N, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE TRTI2(UPLO, DIAG, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO

SUBROUTINE TRTI2_64(UPLO, DIAG, [N], A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void ztrti2(char uplo, char diag, int n,  doublecomplex  *a,
          int lda, int *info);

void ztrti2_64(char uplo, char diag, long  n,  doublecomplex
          *a, long lda, long *info);
```

PURPOSE

ztrti2 computes the inverse of a complex upper or lower triangular matrix.

This is the Level 2 BLAS version of the algorithm.

ARGUMENTS

```
UPLO (input)
          Specifies whether the matrix A is upper  or  lower
          triangular.  = 'U':  Upper triangular
          = 'L':  Lower triangular

DIAG (input)
          Specifies whether or not the matrix A is unit tri-
          angular.  = 'N':  Non-unit triangular
          = 'U':  Unit triangular

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the triangular matrix A.  If UPLO = 'U',
          the  leading  n  by n upper triangular part of the
          array A contains the upper triangular matrix,  and
          the  strictly  lower  triangular  part of A is not
          referenced.  If UPLO = 'L', the  leading  n  by  n
          lower  triangular part of the array A contains the
          lower triangular matrix, and  the  strictly  upper
          triangular part of A is not referenced.  If DIAG =
          'U', the diagonal  elements  of  A  are  also  not
          referenced and are assumed to be 1.

          On exit, the (triangular) inverse of the  original
          matrix, in the same storage format.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).
```

INFO (output)
= 0: successful exit
< 0: if INFO = -k, the k-th argument had an ille-
gal value

Contents

NAME

```
ztrtri - compute the inverse of a  complex  upper  or  lower
triangular matrix A
```

SYNOPSIS

```
SUBROUTINE ZTRTRI(UPLO, DIAG, N, A, LDA, INFO)

CHARACTER * 1 UPLO, DIAG
DOUBLE COMPLEX A(LDA,*)
INTEGER N, LDA, INFO

SUBROUTINE ZTRTRI_64(UPLO, DIAG, N, A, LDA, INFO)

CHARACTER * 1 UPLO, DIAG
DOUBLE COMPLEX A(LDA,*)
INTEGER*8 N, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE TRTRI(UPLO, DIAG, N, A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, INFO

SUBROUTINE TRTRI_64(UPLO, DIAG, N, A, [LDA], [INFO])

CHARACTER(LEN=1) :: UPLO, DIAG
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void ztrtri(char uplo, char diag, int n,  doublecomplex  *a,
          int lda, int *info);

void ztrtri_64(char uplo, char diag, long  n,  doublecomplex
          *a, long lda, long *info);
```

PURPOSE

```
ztrtri computes the inverse of a complex upper or lower tri-
angular matrix A.

This is the Level 3 BLAS version of the algorithm.
```

ARGUMENTS

```
UPLO (input)
          = 'U':  A is upper triangular;
          = 'L':  A is lower triangular.

DIAG (input)
          = 'N':  A is non-unit triangular;
          = 'U':  A is unit triangular.

N (input) The order of the matrix A.  N >= 0.

A (input/output)
          On entry, the triangular matrix A.  If UPLO = 'U',
          the  leading  N-by-N  upper triangular part of the
          array A contains the upper triangular matrix,  and
          the  strictly  lower  triangular  part of A is not
          referenced.  If UPLO =  'L',  the  leading  N-by-N
          lower  triangular part of the array A contains the
          lower triangular matrix, and  the  strictly  upper
          triangular part of A is not referenced.  If DIAG =
          'U', the diagonal  elements  of  A  are  also  not
          referenced  and are assumed to be 1.  On exit, the
          (triangular) inverse of the  original  matrix,  in
          the same storage format.

LDA (input)
          The leading dimension of  the  array  A.   LDA  >=
          max(1,N).

INFO (output)
          = 0: successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
```

gal value
> 0: if INFO = i, A(i,i) is exactly zero. The triangular matrix is singular and its inverse can not be computed.

Contents

NAME

```
ztrtrs - solve a triangular system of the form   A * X =  B,
A**T * X = B, or A**H * X = B,
```

SYNOPSIS

```
SUBROUTINE ZTRTRS(UPLO, TRANSA, DIAG, N, NRHS, A, LDA, B, LDB, INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER N, NRHS, LDA, LDB, INFO

SUBROUTINE ZTRTRS_64(UPLO, TRANSA, DIAG, N, NRHS, A, LDA, B, LDB,
      INFO)

CHARACTER * 1 UPLO, TRANSA, DIAG
DOUBLE COMPLEX A(LDA,*), B(LDB,*)
INTEGER*8 N, NRHS, LDA, LDB, INFO
```

F95 INTERFACE

```
SUBROUTINE TRTRS(UPLO, [TRANSA], DIAG, N, NRHS, A, [LDA], B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER :: N, NRHS, LDA, LDB, INFO

SUBROUTINE TRTRS_64(UPLO, [TRANSA], DIAG, N, NRHS, A, [LDA], B, [LDB],
       [INFO])

CHARACTER(LEN=1) :: UPLO, TRANSA, DIAG
COMPLEX(8), DIMENSION(:,:) :: A, B
INTEGER(8) :: N, NRHS, LDA, LDB, INFO
```

C INTERFACE

```
#include <sunperf.h>

void ztrtrs(char uplo, char transa, char diag,  int  n,  int
          nrhs, doublecomplex *a, int lda, doublecomplex *b,
          int ldb, int *info);

void ztrtrs_64(char uplo, char transa, char  diag,  long  n,
          long  nrhs, doublecomplex *a, long lda, doublecom-
          plex *b, long ldb, long *info);
```

PURPOSE

ztrtrs solves a triangular system of the form where A is a triangular matrix of order N, and B is an N-by-NRHS matrix. A check is made to verify that A is non-singular.

ARGUMENTS

UPLO (input)
= 'U': A is upper triangular;
= 'L': A is lower triangular.

TRANSA (input)
Specifies the form of the system of equations:
= 'N': A * X = B (No transpose)
= 'T': A**T * X = B (Transpose)
= 'C': A**H * X = B (Conjugate transpose)

TRANSA is defaulted to 'N' for F95 INTERFACE.

DIAG (input)
= 'N': A is non-unit triangular;
= 'U': A is unit triangular.

N (input) The order of the matrix A. N >= 0.

NRHS (input)
The number of right hand sides, i.e., the number of columns of the matrix B. NRHS >= 0.

A (input) The triangular matrix A. If UPLO = 'U', the leading N-by-N upper triangular part of the array A contains the upper triangular matrix, and the strictly lower triangular part of A is not referenced. If UPLO = 'L', the leading N-by-N lower triangular part of the array A contains the lower triangular matrix, and the strictly upper triangular part of A is not referenced. If DIAG = 'U', the diagonal elements of A are also not referenced and are assumed to be 1.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

B (input/output)
On entry, the right hand side matrix B. On exit, if INFO = 0, the solution matrix X.

LDB (input)
The leading dimension of the array B. LDB >= max(1,N).

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: if INFO = i, the i-th diagonal element of A is zero, indicating that the matrix is singular and the solutions X have not been computed.

Contents

NAME

```
ztzrqf - routine is deprecated and has been replaced by rou-
tine CTZRZF
```

SYNOPSIS

```
SUBROUTINE ZTZRQF(M, N, A, LDA, TAU, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*)
INTEGER M, N, LDA, INFO

SUBROUTINE ZTZRQF_64(M, N, A, LDA, TAU, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*)
INTEGER*8 M, N, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE TZRQF([M], [N], A, [LDA], TAU, [INFO])

COMPLEX(8), DIMENSION(:) :: TAU
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, INFO

SUBROUTINE TZRQF_64([M], [N], A, [LDA], TAU, [INFO])

COMPLEX(8), DIMENSION(:) :: TAU
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void ztzrqf(int m, int n, doublecomplex *a, int  lda,  doub-
          lecomplex *tau, int *info);

void ztzrqf_64(long m, long n, doublecomplex *a,  long  lda,
          doublecomplex *tau, long *info);
```

PURPOSE

ztzrqf routine is deprecated and has been replaced by routine CTZRZF.

CTZRQF reduces the M-by-N (M<=N) complex upper trapezoidal matrix A to upper triangular form by means of unitary transformations.

The upper trapezoidal matrix A is factored as
   A = (R 0) * Z,

where Z is an N-by-N unitary matrix and R is an M-by-M upper triangular matrix.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= M.

A (input/output)
On entry, the leading M-by-N upper trapezoidal part of the array A must contain the matrix to be factorized. On exit, the leading M-by-M upper triangular part of A contains the upper triangular matrix R, and elements M+1 to N of the first M rows of A, with the array TAU, represent the unitary matrix Z as a product of M elementary reflectors.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an ille-

gal value

FURTHER DETAILS

```
The  factorization is obtained by Householder's method.  The
kth transformation matrix, Z( k ), whose conjugate transpose
is used to introduce zeros into the (m - k + 1)th row of  A,
is given in the form

   Z( k ) = ( I     0   ),
            ( 0  T( k ) )

where

   T( k ) = I - tau*u( k )*u( k )',   u( k ) = (   1    ),
                                               (   0    )
                                               ( z( k ) )

tau is a scalar and z( k ) is an ( n - m )  element  vector.
tau  and z( k ) are chosen to annihilate the elements of the
kth row of X.

The scalar tau is returned in the kth element of TAU and the
vector u( k ) in the kth row of A, such that the elements of
z( k ) are in  a( k, m + 1 ), ..., a( k, n ).  The  elements
of R are returned in the upper triangular part of A.

Z is given by

   Z =  Z( 1 ) * Z( 2 ) * ... * Z( m ).
```

Contents

NAME

```
ztzrzf - reduce the M-by-N  (  M<=N  )  complex  upper  tra-
pezoidal  matrix A to upper triangular form by means of uni-
tary transformations
```

SYNOPSIS

```
SUBROUTINE ZTZRZF(M, N, A, LDA, TAU, WORK, LWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, LDA, LWORK, INFO

SUBROUTINE ZTZRZF_64(M, N, A, LDA, TAU, WORK, LWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, LDA, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE TZRZF([M], [N], A, [LDA], TAU, [WORK], [LWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, LDA, LWORK, INFO

SUBROUTINE TZRZF_64([M], [N], A, [LDA], TAU, [WORK], [LWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, LDA, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void ztzrzf(int m, int n, doublecomplex *a, int  lda,  doub-
          lecomplex *tau, int *info);

void ztzrzf_64(long m, long n, doublecomplex *a,  long  lda,
          doublecomplex *tau, long *info);
```

PURPOSE

ztzrzf reduces the M-by-N (M<=N) complex upper trapezoidal matrix A to upper triangular form by means of unitary transformations.

The upper trapezoidal matrix A is factored as

   A = (R 0) * Z,

where Z is an N-by-N unitary matrix and R is an M-by-M upper triangular matrix.

ARGUMENTS

M (input) The number of rows of the matrix A. M >= 0.

N (input) The number of columns of the matrix A. N >= 0.

A (input/output)
On entry, the leading M-by-N upper trapezoidal part of the array A must contain the matrix to be factorized. On exit, the leading M-by-M upper triangular part of A contains the upper triangular matrix R, and elements M+1 to N of the first M rows of A, with the array TAU, represent the unitary matrix Z as a product of M elementary reflectors.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M).

TAU (output)
The scalar factors of the elementary reflectors.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)

The dimension of the array WORK. LWORK >= max(1,M). For optimum performance LWORK >= M*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

FURTHER DETAILS

Based on contributions by
A. Petitet, Computer Science Dept., Univ. of Tenn., Knoxville, USA

The factorization is obtained by Householder's method. The kth transformation matrix, Z(k), which is used to introduce zeros into the (m - k + 1)th row of A, is given in the form

```
   Z( k ) = ( I     0   ),
            ( 0  T( k ) )
```

where

```
   T( k ) = I - tau*u( k )*u( k )',   u( k ) = (   1    ),
                                               (   0    )
                                               ( z( k ) )
```

tau is a scalar and z(k) is an (n - m) element vector. tau and z(k) are chosen to annihilate the elements of the kth row of X.

The scalar tau is returned in the kth element of TAU and the vector u(k) in the kth row of A, such that the elements of z(k) are in a(k, m + 1), ..., a(k, n). The elements of R are returned in the upper triangular part of A.

Z is given by

```
   Z =  Z( 1 ) * Z( 2 ) * ... * Z( m ).
```

Contents

NAME

```
zung2l - generate an m by n complex matrix Q with  orthonor-
mal columns,
```

SYNOPSIS

```
SUBROUTINE ZUNG2L(M, N, K, A, LDA, TAU, WORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, K, LDA, INFO

SUBROUTINE ZUNG2L_64(M, N, K, A, LDA, TAU, WORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, K, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE UNG2L(M, [N], [K], A, [LDA], TAU, [WORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, K, LDA, INFO

SUBROUTINE UNG2L_64(M, [N], [K], A, [LDA], TAU, [WORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, K, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zung2l(int m, int n, int k, doublecomplex *a, int  lda,
```

```
          doublecomplex *tau, int *info);

void zung2l_64(long m, long n,  long  k,  doublecomplex  *a,
          long lda, doublecomplex *tau, long *info);
```

PURPOSE

zung2l L generates an m by n complex matrix Q with orthonormal columns, which is defined as the last n columns of a product of k elementary reflectors of order m

Q = H(k) . . . H(2) H(1)

as returned by CGEQLF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. M >= N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. N >= K >= 0.

A (input/output)
On entry, the (n-k+i)-th column must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGEQLF in the last k columns of its array argument A. On exit, the m-by-n matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGEQLF.

WORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
zung2r - generate an m by n complex matrix Q with  orthonor-
mal columns,
```

SYNOPSIS

```
SUBROUTINE ZUNG2R(M, N, K, A, LDA, TAU, WORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, K, LDA, INFO

SUBROUTINE ZUNG2R_64(M, N, K, A, LDA, TAU, WORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, K, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE UNG2R(M, [N], [K], A, [LDA], TAU, [WORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, K, LDA, INFO

SUBROUTINE UNG2R_64(M, [N], [K], A, [LDA], TAU, [WORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, K, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zung2r(int m, int n, int k, doublecomplex *a, int  lda,
```

```
          doublecomplex *tau, int *info);

void zung2r_64(long m, long n,  long  k,  doublecomplex  *a,
          long lda, doublecomplex *tau, long *info);
```

PURPOSE

zung2r R generates an m by n complex matrix Q with orthonormal columns, which is defined as the first n columns of a product of k elementary reflectors of order m

      Q = H(1) H(2) . . . H(k)

as returned by CGEQRF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. M >= N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. N >= K >= 0.

A (input/output)
On entry, the i-th column must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGEQRF in the first k columns of its array argument A. On exit, the m by n matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGEQRF.

WORK (workspace)
dimension(N)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
zungbr - generate one of the complex unitary matrices  Q  or
P**H  determined  by CGEBRD when reducing a complex matrix A
to bidiagonal form
```

SYNOPSIS

```
SUBROUTINE ZUNGBR(VECT, M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

CHARACTER * 1 VECT
DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, K, LDA, LWORK, INFO

SUBROUTINE ZUNGBR_64(VECT, M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

CHARACTER * 1 VECT
DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, K, LDA, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNGBR(VECT, M, [N], K, A, [LDA], TAU, [WORK], [LWORK],
       [INFO])

CHARACTER(LEN=1) :: VECT
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, K, LDA, LWORK, INFO

SUBROUTINE UNGBR_64(VECT, M, [N], K, A, [LDA], TAU, [WORK], [LWORK],
       [INFO])

CHARACTER(LEN=1) :: VECT
COMPLEX(8), DIMENSION(:) :: TAU, WORK
```

```
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, K, LDA, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zungbr(char vect, int m, int n,  int  k,  doublecomplex
          *a, int lda, doublecomplex *tau, int *info);

void zungbr_64(char vect, long m, long n, long k, doublecom-
          plex   *a,  long  lda,  doublecomplex  *tau,  long
          *info);
```

PURPOSE

zungbr generates one of the complex unitary matrices Q or P**H determined by CGEBRD when reducing a complex matrix A to bidiagonal form: A = Q * B * P**H. Q and P**H are defined as products of elementary reflectors H(i) or G(i) respectively.

If VECT = 'Q', A is assumed to have been an M-by-K matrix, and Q is of order M:
if m >= k, Q = H(1) H(2) . . . H(k) and CUNGBR returns the first n columns of Q, where m >= n >= k;
if m < k, Q = H(1) H(2) . . . H(m-1) and CUNGBR returns Q as an M-by-M matrix.

If VECT = 'P', A is assumed to have been a K-by-N matrix, and P**H is of order N:
if k < n, P**H = G(k) . . . G(2) G(1) and CUNGBR returns the first m rows of P**H, where n >= m >= k;
if k >= n, P**H = G(n-1) . . . G(2) G(1) and CUNGBR returns P**H as an N-by-N matrix.

ARGUMENTS

VECT (input)
Specifies whether the matrix Q or the matrix P**H is required, as defined in the transformation applied by CGEBRD:
= 'Q': generate Q;
= 'P': generate P**H.

M (input) The number of rows of the matrix Q or P**H to be returned. M >= 0.

N (input) The number of columns of the matrix Q or P**H to be returned. N >= 0. If VECT = 'Q', M >= N >= min(M,K); if VECT = 'P', N >= M >= min(N,K).

K (input) If VECT = 'Q', the number of columns in the original M-by-K matrix reduced by CGEBRD. If VECT = 'P', the number of rows in the original K-by-N matrix reduced by CGEBRD. K >= 0.

A (input/output)
On entry, the vectors which define the elementary reflectors, as returned by CGEBRD. On exit, the M-by-N matrix Q or P**H.

LDA (input)
The leading dimension of the array A. LDA >= M.

TAU (input)
(min(M,K)) if VECT = 'Q' (min(N,K)) if VECT = 'P' TAU(i) must contain the scalar factor of the elementary reflector H(i) or G(i), which determines Q or P**H, as returned by CGEBRD in its array argument TAUQ or TAUP.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,min(M,N)). For optimum performance LWORK >= min(M,N)*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

zunghr - generate a complex unitary matrix Q which is defined as the product of IHI-ILO elementary reflectors of order N, as returned by CGEHRD

SYNOPSIS

```
SUBROUTINE ZUNGHR(N, ILO, IHI, A, LDA, TAU, WORK, LWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER N, ILO, IHI, LDA, LWORK, INFO

SUBROUTINE ZUNGHR_64(N, ILO, IHI, A, LDA, TAU, WORK, LWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 N, ILO, IHI, LDA, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNGHR([N], ILO, IHI, A, [LDA], TAU, [WORK], [LWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, ILO, IHI, LDA, LWORK, INFO

SUBROUTINE UNGHR_64([N], ILO, IHI, A, [LDA], TAU, [WORK], [LWORK],
       [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, ILO, IHI, LDA, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zunghr(int n, int ilo, int ihi, doublecomplex  *a,  int
          lda, doublecomplex *tau, int *info);

void zunghr_64(long n, long ilo, long ihi, doublecomplex *a,
          long lda, doublecomplex *tau, long *info);
```

PURPOSE

zunghr generates a complex unitary matrix Q which is defined as the product of IHI-ILO elementary reflectors of order N, as returned by CGEHRD:

Q = H(ilo) H(ilo+1) . . . H(ihi-1).

ARGUMENTS

N (input) The order of the matrix Q. N >= 0.

ILO (input)
ILO and IHI must have the same values as in the previous call of CGEHRD. Q is equal to the unit matrix except in the submatrix Q(ilo+1:ihi,ilo+1:ihi). 1 <= ILO <= IHI <= N, if N > 0; ILO=1 and IHI=0, if N=0.

IHI (input)
See the description of IHI.

A (input/output)
On entry, the vectors which define the elementary reflectors, as returned by CGEHRD. On exit, the N-by-N unitary matrix Q.

LDA (input)
The leading dimension of the array A. LDA >= max(1,N).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGEHRD.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)

The dimension of the array WORK. LWORK >= IHI-ILO. For optimum performance LWORK >= (IHI-ILO)*NB, where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zungl2 - generate an m-by-n complex matrix Q with  orthonor-
mal rows,
```

SYNOPSIS

```
SUBROUTINE ZUNGL2(M, N, K, A, LDA, TAU, WORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, K, LDA, INFO

SUBROUTINE ZUNGL2_64(M, N, K, A, LDA, TAU, WORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, K, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE UNGL2([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, K, LDA, INFO

SUBROUTINE UNGL2_64([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, K, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zungl2(int m, int n, int k, doublecomplex *a, int  lda,
```

```
          doublecomplex *tau, int *info);

void zungl2_64(long m, long n,  long  k,  doublecomplex  *a,
          long lda, doublecomplex *tau, long *info);
```

PURPOSE

zungl2 generates an m-by-n complex matrix Q with orthonormal rows, which is defined as the first m rows of a product of k elementary reflectors of order n

```
      Q  =  H(k)' . . . H(2)' H(1)'
```

as returned by CGELQF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. N >= M.

K (input) The number of elementary reflectors whose product defines the matrix Q. M >= K >= 0.

A (input/output)
On entry, the i-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGELQF in the first k rows of its array argument A. On exit, the m by n matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGELQF.

WORK (workspace)
dimension(M)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
zunglq - generate an M-by-N complex matrix Q with  orthonor-
mal rows,
```

SYNOPSIS

```
SUBROUTINE ZUNGLQ(M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, K, LDA, LWORK, INFO

SUBROUTINE ZUNGLQ_64(M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, K, LDA, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNGLQ(M, [N], [K], A, [LDA], TAU, [WORK], [LWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, K, LDA, LWORK, INFO

SUBROUTINE UNGLQ_64(M, [N], [K], A, [LDA], TAU, [WORK], [LWORK],
       [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, K, LDA, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zunglq(int m, int n, int k, doublecomplex *a, int  lda,
          doublecomplex *tau, int *info);

void zunglq_64(long m, long n,  long  k,  doublecomplex  *a,
          long lda, doublecomplex *tau, long *info);
```

PURPOSE

zunglq generates an M-by-N complex matrix Q with orthonormal rows, which is defined as the first M rows of a product of K elementary reflectors of order N

```
      Q  =  H(k)' . . . H(2)' H(1)'
```

as returned by CGELQF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. N >= M.

K (input) The number of elementary reflectors whose product defines the matrix Q. M >= K >= 0.

A (input/output)
On entry, the i-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGELQF in the first k rows of its array argument A. On exit, the M-by-N matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGELQF.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,M). For optimum performance LWORK >= M*NB,

where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit;
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
zungql - generate an M-by-N complex matrix Q with  orthonor-
mal columns,
```

SYNOPSIS

```
SUBROUTINE ZUNGQL(M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, K, LDA, LWORK, INFO

SUBROUTINE ZUNGQL_64(M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, K, LDA, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNGQL(M, [N], [K], A, [LDA], TAU, [WORK], [LWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, K, LDA, LWORK, INFO

SUBROUTINE UNGQL_64(M, [N], [K], A, [LDA], TAU, [WORK], [LWORK],
       [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, K, LDA, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zungql(int m, int n, int k, doublecomplex *a, int  lda,
          doublecomplex *tau, int *info);

void zungql_64(long m, long n,  long  k,  doublecomplex  *a,
          long lda, doublecomplex *tau, long *info);
```

PURPOSE

zungql generates an M-by-N complex matrix Q with orthonormal columns, which is defined as the last N columns of a product of K elementary reflectors of order M

      Q = H(k) . . . H(2) H(1)

as returned by CGEQLF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. M >= N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. N >= K >= 0.

A (input/output)
On entry, the (n-k+i)-th column must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGEQLF in the last k columns of its array argument A. On exit, the M-by-N matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGEQLF.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >=

```
            max(1,N).   For optimum performance LWORK >= N*NB,
            where NB is the optimal blocksize.

            If LWORK = -1, then a workspace query is  assumed;
            the  routine  only  calculates the optimal size of
            the WORK array, returns this value  as  the  first
            entry  of  the  WORK  array,  and no error message
            related to LWORK is issued by XERBLA.

INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument has an ille-
            gal value
```

Contents

NAME

```
zungqr - generate an M-by-N complex matrix Q with  orthonor-
mal columns,
```

SYNOPSIS

```
SUBROUTINE ZUNGQR(M, N, K, A, LDA, TAU, WORKIN, LWORKIN, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORKIN(*)
INTEGER M, N, K, LDA, LWORKIN, INFO

SUBROUTINE ZUNGQR_64(M, N, K, A, LDA, TAU, WORKIN, LWORKIN, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORKIN(*)
INTEGER*8 M, N, K, LDA, LWORKIN, INFO
```

F95 INTERFACE

```
SUBROUTINE UNGQR(M, [N], [K], A, [LDA], TAU, [WORKIN], [LWORKIN],
       [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORKIN
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, K, LDA, LWORKIN, INFO

SUBROUTINE UNGQR_64(M, [N], [K], A, [LDA], TAU, [WORKIN], [LWORKIN],
       [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORKIN
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, K, LDA, LWORKIN, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zungqr(int m, int n, int k, doublecomplex *a, int  lda,
          doublecomplex *tau, int *info);

void zungqr_64(long m, long n,  long  k,  doublecomplex  *a,
          long lda, doublecomplex *tau, long *info);
```

PURPOSE

zungqr generates an M-by-N complex matrix Q with orthonormal columns, which is defined as the first N columns of a product of K elementary reflectors of order M

```
      Q  =  H(1) H(2) . . . H(k)
```

as returned by CGEQRF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. M >= N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. N >= K >= 0.

A (input/output)
On entry, the i-th column must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGEQRF in the first k columns of its array argument A. On exit, the M-by-N matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGEQRF.

WORKIN (workspace)
On exit, if INFO = 0, WORKIN(1) returns the optimal LWORKIN.

LWORKIN (input)
The dimension of the array WORKIN. LWORKIN >=

max(1,N). For optimum performance LWORKIN >= N*NB, where NB is the optimal blocksize.

If LWORKIN = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORKIN array, returns this value as the first entry of the WORKIN array, and no error message related to LWORKIN is issued by XERBLA.

INFO (output)

= 0: successful exit

< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
zungr2 - generate an m by n complex matrix Q with  orthonor-
mal rows,
```

SYNOPSIS

```
SUBROUTINE ZUNGR2(M, N, K, A, LDA, TAU, WORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, K, LDA, INFO

SUBROUTINE ZUNGR2_64(M, N, K, A, LDA, TAU, WORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, K, LDA, INFO
```

F95 INTERFACE

```
SUBROUTINE UNGR2([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, K, LDA, INFO

SUBROUTINE UNGR2_64([M], [N], [K], A, [LDA], TAU, [WORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, K, LDA, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zungr2(int m, int n, int k, doublecomplex *a, int  lda,
```

```
          doublecomplex *tau, int *info);

void zungr2_64(long m, long n,  long  k,  doublecomplex  *a,
          long lda, doublecomplex *tau, long *info);
```

PURPOSE

zungr2 generates an m by n complex matrix Q with orthonormal rows, which is defined as the last m rows of a product of k elementary reflectors of order n

```
      Q  =  H(1)' H(2)' . . . H(k)'
```

as returned by CGERQF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. N >= M.

K (input) The number of elementary reflectors whose product defines the matrix Q. M >= K >= 0.

A (input/output)
On entry, the (m-k+i)-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGERQF in the last k rows of its array argument A. On exit, the m-by-n matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGERQF.

WORK (workspace)
dimension(M)

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument has an illegal value

Contents

NAME

```
zungrq - generate an M-by-N complex matrix Q with  orthonor-
mal rows,
```

SYNOPSIS

```
SUBROUTINE ZUNGRQ(M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER M, N, K, LDA, LWORK, INFO

SUBROUTINE ZUNGRQ_64(M, N, K, A, LDA, TAU, WORK, LWORK, INFO)

DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 M, N, K, LDA, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNGRQ(M, [N], [K], A, [LDA], TAU, [WORK], [LWORK], [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: M, N, K, LDA, LWORK, INFO

SUBROUTINE UNGRQ_64(M, [N], [K], A, [LDA], TAU, [WORK], [LWORK],
       [INFO])

COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: M, N, K, LDA, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zungrq(int m, int n, int k, doublecomplex *a, int  lda,
          doublecomplex *tau, int *info);

void zungrq_64(long m, long n,  long  k,  doublecomplex  *a,
          long lda, doublecomplex *tau, long *info);
```

PURPOSE

zungrq generates an M-by-N complex matrix Q with orthonormal rows, which is defined as the last M rows of a product of K elementary reflectors of order N

Q = H(1)' H(2)' . . . H(k)'

as returned by CGERQF.

ARGUMENTS

M (input) The number of rows of the matrix Q. M >= 0.

N (input) The number of columns of the matrix Q. N >= M.

K (input) The number of elementary reflectors whose product defines the matrix Q. M >= K >= 0.

A (input/output)
On entry, the (m-k+i)-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGERQF in the last k rows of its array argument A. On exit, the M-by-N matrix Q.

LDA (input)
The first dimension of the array A. LDA >= max(1,M).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGERQF.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= max(1,M). For optimum performance LWORK >= M*NB,

```
            where NB is the optimal blocksize.

            If LWORK = -1, then a workspace query is  assumed;
            the  routine  only  calculates the optimal size of
            the WORK array, returns this value  as  the  first
            entry  of  the  WORK  array,  and no error message
            related to LWORK is issued by XERBLA.

INFO (output)
            = 0:  successful exit
            < 0:  if INFO = -i, the i-th argument has an ille-
            gal value
```

Contents

NAME

zungtr - generate a complex unitary matrix Q which is defined as the product of n-1 elementary reflectors of order N, as returned by CHETRD

SYNOPSIS

```
SUBROUTINE ZUNGTR(UPLO, N, A, LDA, TAU, WORK, LWORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER N, LDA, LWORK, INFO

SUBROUTINE ZUNGTR_64(UPLO, N, A, LDA, TAU, WORK, LWORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX A(LDA,*), TAU(*), WORK(*)
INTEGER*8 N, LDA, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNGTR(UPLO, [N], A, [LDA], TAU, [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER :: N, LDA, LWORK, INFO

SUBROUTINE UNGTR_64(UPLO, [N], A, [LDA], TAU, [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A
INTEGER(8) :: N, LDA, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zungtr(char uplo, int n,  doublecomplex  *a,  int  lda,
          doublecomplex *tau, int *info);

void zungtr_64(char uplo, long  n,  doublecomplex  *a,  long
          lda, doublecomplex *tau, long *info);
```

PURPOSE

zungtr generates a complex unitary matrix Q which is defined as the product of n-1 elementary reflectors of order N, as returned by CHETRD:
if UPLO = 'U', Q = H(n-1) . . . H(2) H(1),

if UPLO = 'L', Q = H(1) H(2) . . . H(n-1).

ARGUMENTS

UPLO (input)
= 'U': Upper triangle of A contains elementary reflectors from CHETRD; = 'L': Lower triangle of A contains elementary reflectors from CHETRD.

N (input) The order of the matrix Q. N >= 0.

A (input/output)
On entry, the vectors which define the elementary reflectors, as returned by CHETRD. On exit, the N-by-N unitary matrix Q.

LDA (input)
The leading dimension of the array A. LDA >= N.

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CHETRD.

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. LWORK >= N-1. For optimum performance LWORK >= (N-1)*NB, where

NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zunmbr - VECT = 'Q', CUNMBR overwrites the  general  complex
M-by-N matrix C with  SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE ZUNMBR(VECT, SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 VECT, SIDE, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE ZUNMBR_64(VECT, SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 VECT, SIDE, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNMBR(VECT, SIDE, [TRANS], [M], [N], K, A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: VECT, SIDE, TRANS
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE UNMBR_64(VECT, SIDE, [TRANS], [M], [N], K, A, [LDA], TAU,
       C, [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: VECT, SIDE, TRANS
```

```
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zunmbr(char vect, char side, char trans, int m, int  n,
          int  k,  doublecomplex  *a, int lda, doublecomplex
          *tau, doublecomplex *c, int ldc, int *info);

void zunmbr_64(char vect, char side,  char  trans,  long  m,
          long  n, long k, doublecomplex *a, long lda, doub-
          lecomplex *tau, doublecomplex *c, long  ldc,  long
          *info);
```

PURPOSE

```
zunmbr VECT = 'Q', CUNMBR overwrites the general complex  M-
by-N matrix C with
                SIDE = 'L'      SIDE  =  'R'  TRANS  =  'N':
Q  *  C           C * Q TRANS = 'C':      Q**H * C       C *
Q**H

If VECT = 'P', CUNMBR overwrites the general complex  M-by-N
matrix C with
                SIDE = 'L'     SIDE = 'R'
TRANS = 'N':      P * C          C * P
TRANS = 'C':      P**H * C       C * P**H

Here Q and P**H  are  the  unitary  matrices  determined  by
CGEBRD  when reducing a complex matrix A to bidiagonal form:
A = Q * B * P**H. Q and P**H are defined as products of ele-
mentary reflectors H(i) and G(i) respectively.

Let nq = m if SIDE = 'L' and nq = n if SIDE = 'R'.  Thus  nq
is  the  order  of  the  unitary  matrix  Q  or P**H that is
applied.

If VECT = 'Q', A is assumed to have been an NQ-by-K  matrix:
if nq >= k, Q = H(1) H(2) . . . H(k);
if nq < k, Q = H(1) H(2) . . . H(nq-1).

If VECT = 'P', A is assumed to have been a  K-by-NQ  matrix:
if k < nq, P = G(1) G(2) . . . G(k);
if k >= nq, P = G(1) G(2) . . . G(nq-1).
```

ARGUMENTS

VECT (input)
= 'Q': apply Q or Q**H;
= 'P': apply P or P**H.

SIDE (input)
= 'L': apply Q, Q**H, P or P**H from the Left;
= 'R': apply Q, Q**H, P or P**H from the Right.

TRANS (input)
= 'N': No transpose, apply Q or P;
= 'C': Conjugate transpose, apply Q**H or P**H.

TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.
N (input) The number of columns of the matrix C. N >= 0.

K (input) If VECT = 'Q', the number of columns in the original matrix reduced by CGEBRD. If VECT = 'P', the number of rows in the original matrix reduced by CGEBRD. K >= 0.

A (input) (LDA,min(nq,K)) if VECT = 'Q' (LDA,nq) if VECT = 'P' The vectors which define the elementary reflectors H(i) and G(i), whose products determine the matrices Q and P, as returned by CGEBRD.

LDA (input)
The leading dimension of the array A. If VECT = 'Q', LDA >= max(1,nq); if VECT = 'P', LDA >= max(1,min(nq,K)).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i) or G(i) which determines Q or P, as returned by CGEBRD in the array argument TAUQ or TAUP.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by Q*C or Q**H*C or C*Q**H or C*Q or P*C or P**H*C or C*P or C*P**H.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)

On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If SIDE = 'L', LWORK >= max(1,N); if SIDE = 'R', LWORK >= max(1,M). For optimum performance LWORK >= N*NB if SIDE = 'L', and LWORK >= M*NB if SIDE = 'R', where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zunmhr - overwrite the general complex M-by-N matrix C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE ZUNMHR(SIDE, TRANS, M, N, ILO, IHI, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, ILO, IHI, LDA, LDC, LWORK, INFO

SUBROUTINE ZUNMHR_64(SIDE, TRANS, M, N, ILO, IHI, A, LDA, TAU, C,
      LDC, WORK, LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, ILO, IHI, LDA, LDC, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNMHR(SIDE, [TRANS], [M], [N], ILO, IHI, A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER :: M, N, ILO, IHI, LDA, LDC, LWORK, INFO

SUBROUTINE UNMHR_64(SIDE, [TRANS], [M], [N], ILO, IHI, A, [LDA], TAU,
       C, [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, ILO, IHI, LDA, LDC, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zunmhr(char side, char trans, int m, int  n,  int  ilo,
          int  ihi, doublecomplex *a, int lda, doublecomplex
          *tau, doublecomplex *c, int ldc, int *info);

void zunmhr_64(char side, char trans, long m, long  n,  long
          ilo,  long  ihi, doublecomplex *a, long lda, doub-
          lecomplex *tau, doublecomplex *c, long  ldc,  long
          *info);
```

PURPOSE

zunmhr overwrites the general complex M-by-N matrix C with
TRANS = 'C':   Q**H * C   C * Q**H

where Q is a complex unitary matrix of order nq, with nq = m if SIDE = 'L' and nq = n if SIDE = 'R'. Q is defined as the product of IHI-ILO elementary reflectors, as returned by CGEHRD:

Q = H(ilo) H(ilo+1) . . . H(ihi-1).

ARGUMENTS

SIDE (input)
= 'L': apply Q or Q**H from the Left;
= 'R': apply Q or Q**H from the Right.

TRANS (input)
= 'N': apply Q (No transpose)
= 'C': apply Q**H (Conjugate transpose)

TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

ILO (input)
ILO and IHI must have the same values as in the previous call of CGEHRD. Q is equal to the unit

matrix except in the submatrix Q(ilo+1:ihi,ilo+1:ihi). If SIDE = 'L', then 1 <= ILO <= IHI <= M, if M > 0, and ILO = 1 and IHI = 0, if M = 0; if SIDE = 'R', then 1 <= ILO <= IHI <= N, if N > 0, and ILO = 1 and IHI = 0, if N = 0.

IHI (input)
See the description of ILO.

A (input) (LDA,M) if SIDE = 'L' (LDA,N) if SIDE = 'R' The vectors which define the elementary reflectors, as returned by CGEHRD.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M) if SIDE = 'L'; LDA >= max(1,N) if SIDE = 'R'.

TAU (input)
(M-1) if SIDE = 'L' (N-1) if SIDE = 'R' TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGEHRD.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by Q*C or Q**H*C or C*Q**H or C*Q.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If SIDE = 'L', LWORK >= max(1,N); if SIDE = 'R', LWORK >= max(1,M). For optimum performance LWORK >= N*NB if SIDE = 'L', and LWORK >= M*NB if SIDE = 'R', where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit

< 0: if INFO = -i, the i-th argument had an ille-
gal value

Contents

NAME

```
zunml2 - overwrite the general complex m-by-n matrix C  with
Q  *  C  if SIDE = 'L' and TRANS = 'N', or   Q'* C if SIDE =
'L' and TRANS = 'C', or   C * Q if SIDE = 'R'  and  TRANS  =
'N', or   C * Q' if SIDE = 'R' and TRANS = 'C',
```

SYNOPSIS

```
SUBROUTINE ZUNML2(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      INFO)

CHARACTER * 1 SIDE, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, K, LDA, LDC, INFO

SUBROUTINE ZUNML2_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      INFO)

CHARACTER * 1 SIDE, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, K, LDA, LDC, INFO
```

F95 INTERFACE

```
SUBROUTINE UNML2(SIDE, TRANS, [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER :: M, N, K, LDA, LDC, INFO

SUBROUTINE UNML2_64(SIDE, TRANS, [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, K, LDA, LDC, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zunml2(char side, char trans, int  m,  int  n,  int  k,
          doublecomplex  *a,  int  lda,  doublecomplex *tau,
          doublecomplex *c, int ldc, int *info);

void zunml2_64(char side, char trans, long m, long  n,  long
          k, doublecomplex *a, long lda, doublecomplex *tau,
          doublecomplex *c, long ldc, long *info);
```

PURPOSE

zunml2 overwrites the general complex m-by-n matrix C with

where Q is a complex unitary matrix defined as the product of k elementary reflectors

Q = H(k)' . . . H(2)' H(1)'

as returned by CGELQF. Q is of order m if SIDE = 'L' and of order n if SIDE = 'R'.

ARGUMENTS

SIDE (input)
= 'L': apply Q or Q' from the Left
= 'R': apply Q or Q' from the Right

TRANS (input)
= 'N': apply Q (No transpose)
= 'C': apply Q' (Conjugate transpose)

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. If SIDE = 'L', M >= K >= 0; if SIDE = 'R', N >= K >= 0.

```
A (input) (LDA,M) if SIDE = 'L', (LDA,N) if SIDE =  'R'  The
          i-th row must contain the vector which defines the
          elementary reflector H(i), for i =  1,2,...,k,  as
          returned  by  CGELQF  in  the  first k rows of its
          array argument A.  A is modified  by  the  routine
          but restored on exit.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,K).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i), as returned by CGELQF.

C (input/output)
          On entry, the m-by-n matrix  C.   On  exit,  C  is
          overwritten by Q*C or Q'*C or C*Q' or C*Q.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          (N) if SIDE = 'L', (M) if SIDE = 'R'

INFO (output)
          = 0: successful exit
          < 0: if INFO = -i, the i-th argument had an  ille-
          gal value
```

Contents

NAME

```
zunmlq - overwrite the general complex M-by-N matrix C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE ZUNMLQ(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE ZUNMLQ_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNMLQ(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE UNMLQ_64(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zunmlq(char side, char trans, int  m,  int  n,  int  k,
          doublecomplex  *a,  int  lda,  doublecomplex *tau,
          doublecomplex *c, int ldc, int *info);

void zunmlq_64(char side, char trans, long m, long  n,  long
          k, doublecomplex *a, long lda, doublecomplex *tau,
          doublecomplex *c, long ldc, long *info);
```

PURPOSE

```
zunmlq overwrites the general complex M-by-N matrix  C  with
TRANS = 'C':      Q**H * C       C * Q**H

where Q is a complex unitary matrix defined as  the  product
of k elementary reflectors

      Q = H(k)' . . . H(2)' H(1)'

as returned by CGELQF. Q is of order M if SIDE = 'L' and  of
order N if SIDE = 'R'.
```

ARGUMENTS

```
SIDE (input)
          = 'L': apply Q or Q**H from the Left;
          = 'R': apply Q or Q**H from the Right.

TRANS (input)
          = 'N':  No transpose, apply Q;
          = 'C':  Conjugate transpose, apply Q**H.

          TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose  product
          defines the matrix Q.  If SIDE = 'L', M >= K >= 0;
          if SIDE = 'R', N >= K >= 0.
```

```
A (input) (LDA,M) if SIDE = 'L', (LDA,N) if SIDE =  'R'  The
          i-th row must contain the vector which defines the
          elementary reflector H(i), for i =  1,2,...,k,  as
          returned  by  CGELQF  in  the  first k rows of its
          array argument A.  A is modified  by  the  routine
          but restored on exit.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,K).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i), as returned by CGELQF.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten by Q*C or Q**H*C or C*Q**H or C*Q.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK.  If SIDE  =  'L',
          LWORK  >=  max(1,N);  if  SIDE  =  'R',  LWORK  >=
          max(1,M).  For optimum performance LWORK  >=  N*NB
          if  SIDE  'L',  and  LWORK  >= M*NB if SIDE = 'R',
          where NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
zunmql - overwrite the general complex M-by-N matrix C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE ZUNMQL(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE ZUNMQL_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNMQL(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE UNMQL_64(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zunmql(char side, char trans, int  m,  int  n,  int  k,
          doublecomplex  *a,  int  lda,  doublecomplex *tau,
          doublecomplex *c, int ldc, int *info);

void zunmql_64(char side, char trans, long m, long  n,  long
          k, doublecomplex *a, long lda, doublecomplex *tau,
          doublecomplex *c, long ldc, long *info);
```

PURPOSE

zunmql overwrites the general complex M-by-N matrix C with
TRANS = 'C':   Q**H * C   C * Q**H

where Q is a complex unitary matrix defined as the product of k elementary reflectors

      Q = H(k) . . . H(2) H(1)

as returned by CGEQLF. Q is of order M if SIDE = 'L' and of order N if SIDE = 'R'.

ARGUMENTS

SIDE (input)
  = 'L': apply Q or Q**H from the Left;
  = 'R': apply Q or Q**H from the Right.

TRANS (input)
  = 'N': No transpose, apply Q;
  = 'C': Transpose, apply Q**H.

  TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. If SIDE = 'L', M >= K >= 0; if SIDE = 'R', N >= K >= 0.

```
A (input) The i-th column  must  contain  the  vector  which
          defines  the  elementary  reflector  H(i), for i =
          1,2,...,k, as returned by CGEQLF  in  the  last  k
          columns of its array argument A.  A is modified by
          the routine but restored on exit.

LDA (input)
          The leading dimension of the array A.  If  SIDE  =
          'L',  LDA  >=  max(1,M);  if  SIDE  =  'R', LDA >=
          max(1,N).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i), as returned by CGEQLF.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten by Q*C or Q**H*C or C*Q**H or C*Q.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK.  If SIDE  =  'L',
          LWORK  >=  max(1,N);  if  SIDE  =  'R',  LWORK  >=
          max(1,M).  For optimum performance LWORK  >=  N*NB
          if  SIDE  =  'L', and LWORK >= M*NB if SIDE = 'R',
          where NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
zunmqr - overwrite the general complex M-by-N matrix C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE ZUNMQR(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE ZUNMQR_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNMQR(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE UNMQR_64(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zunmqr(char side, char trans, int  m,  int  n,  int  k,
          doublecomplex  *a,  int  lda,  doublecomplex *tau,
          doublecomplex *c, int ldc, int *info);

void zunmqr_64(char side, char trans, long m, long  n,  long
          k, doublecomplex *a, long lda, doublecomplex *tau,
          doublecomplex *c, long ldc, long *info);
```

PURPOSE

```
zunmqr overwrites the general complex M-by-N matrix  C  with
TRANS = 'C':      Q**H * C       C * Q**H

where Q is a complex unitary matrix defined as  the  product
of k elementary reflectors

      Q = H(1) H(2) . . . H(k)

as returned by CGEQRF. Q is of order M if SIDE = 'L' and  of
order N if SIDE = 'R'.
```

ARGUMENTS

```
SIDE (input)
          = 'L': apply Q or Q**H from the Left;
          = 'R': apply Q or Q**H from the Right.

TRANS (input)
          = 'N':  No transpose, apply Q;
          = 'C':  Conjugate transpose, apply Q**H.

          TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose  product
          defines the matrix Q.  If SIDE = 'L', M >= K >= 0;
          if SIDE = 'R', N >= K >= 0.
```

A (input) The i-th column must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGEQRF in the first k columns of its array argument A. A is modified by the routine but restored on exit.

LDA (input)
The leading dimension of the array A. If SIDE = 'L', LDA >= max(1,M); if SIDE = 'R', LDA >= max(1,N).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGEQRF.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by Q*C or Q**H*C or C*Q**H or C*Q.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If SIDE = 'L', LWORK >= max(1,N); if SIDE = 'R', LWORK >= max(1,M). For optimum performance LWORK >= N*NB if SIDE = 'L', and LWORK >= M*NB if SIDE = 'R', where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zunmr2 - overwrite the general complex m-by-n matrix C  with
Q  *  C  if SIDE = 'L' and TRANS = 'N', or   Q'* C if SIDE =
'L' and TRANS = 'C', or   C * Q if SIDE = 'R'  and  TRANS  =
'N', or   C * Q' if SIDE = 'R' and TRANS = 'C',
```

SYNOPSIS

```
SUBROUTINE ZUNMR2(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      INFO)

CHARACTER * 1 SIDE, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, K, LDA, LDC, INFO

SUBROUTINE ZUNMR2_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      INFO)

CHARACTER * 1 SIDE, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, K, LDA, LDC, INFO
```

F95 INTERFACE

```
SUBROUTINE UNMR2(SIDE, TRANS, [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER :: M, N, K, LDA, LDC, INFO

SUBROUTINE UNMR2_64(SIDE, TRANS, [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [INFO])
```

```
CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, K, LDA, LDC, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zunmr2(char side, char trans, int  m,  int  n,  int  k,
          doublecomplex  *a,  int  lda,  doublecomplex *tau,
          doublecomplex *c, int ldc, int *info);

void zunmr2_64(char side, char trans, long m, long  n,  long
          k, doublecomplex *a, long lda, doublecomplex *tau,
          doublecomplex *c, long ldc, long *info);
```

PURPOSE

zunmr2 overwrites the general complex m-by-n matrix C with

where Q is a complex unitary matrix defined as the product of k elementary reflectors

Q = H(1)' H(2)' . . . H(k)'

as returned by CGERQF. Q is of order m if SIDE = 'L' and of order n if SIDE = 'R'.

ARGUMENTS

SIDE (input)
= 'L': apply Q or Q' from the Left
= 'R': apply Q or Q' from the Right

TRANS (input)
= 'N': apply Q (No transpose)
= 'C': apply Q' (Conjugate transpose)

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. If SIDE = 'L', M >= K >= 0; if SIDE = 'R', N >= K >= 0.

A (input) (LDA,M) if SIDE = 'L', (LDA,N) if SIDE = 'R' The i-th row must contain the vector which defines the elementary reflector H(i), for i = 1,2,...,k, as returned by CGERQF in the last k rows of its array argument A. A is modified by the routine but restored on exit.

LDA (input)
The leading dimension of the array A. LDA >= max(1,K).

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CGERQF.

C (input/output)
On entry, the m-by-n matrix C. On exit, C is overwritten by Q*C or Q'*C or C*Q' or C*Q.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
(N) if SIDE = 'L', (M) if SIDE = 'R'

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zunmrq - overwrite the general complex M-by-N matrix C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE ZUNMRQ(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE ZUNMRQ_64(SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, K, LDA, LDC, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNMRQ(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C, [LDC],
       [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER :: M, N, K, LDA, LDC, LWORK, INFO

SUBROUTINE UNMRQ_64(SIDE, [TRANS], [M], [N], [K], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, TRANS
```

```
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, K, LDA, LDC, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zunmrq(char side, char trans, int  m,  int  n,  int  k,
          doublecomplex  *a,  int  lda,  doublecomplex *tau,
          doublecomplex *c, int ldc, int *info);

void zunmrq_64(char side, char trans, long m, long  n,  long
          k, doublecomplex *a, long lda, doublecomplex *tau,
          doublecomplex *c, long ldc, long *info);
```

PURPOSE

```
zunmrq overwrites the general complex M-by-N matrix  C  with
TRANS = 'C':      Q**H * C       C * Q**H

where Q is a complex unitary matrix defined as  the  product
of k elementary reflectors

      Q = H(1)' H(2)' . . . H(k)'

as returned by CGERQF. Q is of order M if SIDE = 'L' and  of
order N if SIDE = 'R'.
```

ARGUMENTS

```
SIDE (input)
          = 'L': apply Q or Q**H from the Left;
          = 'R': apply Q or Q**H from the Right.

TRANS (input)
          = 'N':  No transpose, apply Q;
          = 'C':  Transpose, apply Q**H.

          TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose  product
          defines the matrix Q.  If SIDE = 'L', M >= K >= 0;
          if SIDE = 'R', N >= K >= 0.
```

```
A (input) (LDA,M) if SIDE = 'L', (LDA,N) if SIDE =  'R'  The
          i-th row must contain the vector which defines the
          elementary reflector H(i), for i =  1,2,...,k,  as
          returned by CGERQF in the last k rows of its array
          argument A.  A is  modified  by  the  routine  but
          restored on exit.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,K).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i), as returned by CGERQF.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten by Q*C or Q**H*C or C*Q**H or C*Q.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK.  If SIDE  =  'L',
          LWORK  >=  max(1,N);  if  SIDE  =  'R',  LWORK  >=
          max(1,M).  For optimum performance LWORK  >=  N*NB
          if  SIDE  =  'L', and LWORK >= M*NB if SIDE = 'R',
          where NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
zunmrz - overwrite the general complex M-by-N matrix C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE ZUNMRZ(SIDE, TRANS, M, N, K, L, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, K, L, LDA, LDC, LWORK, INFO

SUBROUTINE ZUNMRZ_64(SIDE, TRANS, M, N, K, L, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 SIDE, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, K, L, LDA, LDC, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE ZUNMRZ(SIDE, TRANS, M, N, K, L, A, LDA, TAU, C, LDC,
       WORK, LWORK, INFO)

CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER :: M, N, K, L, LDA, LDC, LWORK, INFO

SUBROUTINE ZUNMRZ_64(SIDE, TRANS, M, N, K, L, A, LDA, TAU, C, LDC,
       WORK, LWORK, INFO)
```

```
CHARACTER(LEN=1) :: SIDE, TRANS
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, K, L, LDA, LDC, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zunmrz(char side, char trans, int m, int n, int k,  int
          l,  doublecomplex *a, int lda, doublecomplex *tau,
          doublecomplex *c, int ldc, int *info);

void zunmrz_64(char side, char trans, long m, long  n,  long
          k,  long l, doublecomplex *a, long lda, doublecom-
          plex  *tau,  doublecomplex  *c,  long  ldc,   long
          *info);
```

PURPOSE

zunmrz overwrites the general complex M-by-N matrix C with TRANS = 'C': Q**H * C C * Q**H

where Q is a complex unitary matrix defined as the product of k elementary reflectors

Q = H(1) H(2) . . . H(k)

as returned by CTZRZF. Q is of order M if SIDE = 'L' and of order N if SIDE = 'R'.

ARGUMENTS

SIDE (input)
= 'L': apply Q or Q**H from the Left;
= 'R': apply Q or Q**H from the Right.

TRANS (input)
= 'N': No transpose, apply Q;
= 'C': Conjugate transpose, apply Q**H.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

K (input) The number of elementary reflectors whose product defines the matrix Q. If SIDE = 'L', M >= K >= 0; if SIDE = 'R', N >= K >= 0.

```
L (input) The number of columns of the matrix  A  containing
          the meaningful part of the Householder reflectors.
          If SIDE = 'L', M >= L >= 0, if SIDE = 'R', N >=  L
          >= 0.

A (input) (LDA,M) if SIDE = 'L', (LDA,N) if SIDE =  'R'  The
          i-th row must contain the vector which defines the
          elementary reflector H(i), for i =  1,2,...,k,  as
          returned by CTZRZF in the last k rows of its array
          argument A.  A is  modified  by  the  routine  but
          restored on exit.

LDA (input)
          The leading dimension  of  the  array  A.  LDA  >=
          max(1,K).

TAU (input)
          TAU(i) must contain the scalar factor of the  ele-
          mentary reflector H(i), as returned by CTZRZF.

C (input/output)
          On entry, the M-by-N matrix  C.   On  exit,  C  is
          overwritten by Q*C or Q**H*C or C*Q**H or C*Q.

LDC (input)
          The leading dimension  of  the  array  C.  LDC  >=
          max(1,M).

WORK (workspace)
          On exit, if INFO = 0, WORK(1) returns the  optimal
          LWORK.

LWORK (input)
          The dimension of the array WORK.  If SIDE  =  'L',
          LWORK  >=  max(1,N);  if  SIDE  =  'R',  LWORK  >=
          max(1,M).  For optimum performance LWORK  >=  N*NB
          if  SIDE  =  'L', and LWORK >= M*NB if SIDE = 'R',
          where NB is the optimal blocksize.

          If LWORK = -1, then a workspace query is  assumed;
          the  routine  only  calculates the optimal size of
          the WORK array, returns this value  as  the  first
          entry  of  the  WORK  array,  and no error message
          related to LWORK is issued by XERBLA.

INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

FURTHER DETAILS

```
Based on contributions by
  A. Petitet, Computer Science Dept., Univ. of Tenn.,  Knox-
ville, USA
```

Contents

NAME

```
zunmtr - overwrite the general complex M-by-N matrix C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE ZUNMTR(SIDE, UPLO, TRANS, M, N, A, LDA, TAU, C, LDC, WORK,
      LWORK, INFO)

CHARACTER * 1 SIDE, UPLO, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, LDA, LDC, LWORK, INFO

SUBROUTINE ZUNMTR_64(SIDE, UPLO, TRANS, M, N, A, LDA, TAU, C, LDC,
      WORK, LWORK, INFO)

CHARACTER * 1 SIDE, UPLO, TRANS
DOUBLE COMPLEX A(LDA,*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, LDA, LDC, LWORK, INFO
```

F95 INTERFACE

```
SUBROUTINE UNMTR(SIDE, UPLO, [TRANS], [M], [N], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANS
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER :: M, N, LDA, LDC, LWORK, INFO

SUBROUTINE UNMTR_64(SIDE, UPLO, [TRANS], [M], [N], A, [LDA], TAU, C,
       [LDC], [WORK], [LWORK], [INFO])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANS
```

```
COMPLEX(8), DIMENSION(:) :: TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: A, C
INTEGER(8) :: M, N, LDA, LDC, LWORK, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zunmtr(char side, char uplo, char trans, int m, int  n,
          doublecomplex  *a,  int  lda,  doublecomplex *tau,
          doublecomplex *c, int ldc, int *info);

void zunmtr_64(char side, char uplo,  char  trans,  long  m,
          long  n, doublecomplex *a, long lda, doublecomplex
          *tau, doublecomplex *c, long ldc, long *info);
```

PURPOSE

zunmtr overwrites the general complex M-by-N matrix C with
TRANS = 'C':   Q**H * C   C * Q**H

where Q is a complex unitary matrix of order nq, with nq = m if SIDE = 'L' and nq = n if SIDE = 'R'. Q is defined as the product of nq-1 elementary reflectors, as returned by CHETRD:

if UPLO = 'U', Q = H(nq-1) . . . H(2) H(1);

if UPLO = 'L', Q = H(1) H(2) . . . H(nq-1).

ARGUMENTS

SIDE (input)
= 'L': apply Q or Q**H from the Left;
= 'R': apply Q or Q**H from the Right.

UPLO (input)
= 'U': Upper triangle of A contains elementary reflectors from CHETRD; = 'L': Lower triangle of A contains elementary reflectors from CHETRD.

TRANS (input)
= 'N': No transpose, apply Q;
= 'C': Conjugate transpose, apply Q**H.

TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

A (input) (LDA,M) if SIDE = 'L' (LDA,N) if SIDE = 'R' The vectors which define the elementary reflectors, as returned by CHETRD.

LDA (input)
The leading dimension of the array A. LDA >= max(1,M) if SIDE = 'L'; LDA >= max(1,N) if SIDE = 'R'.

TAU (input)
(M-1) if SIDE = 'L' (N-1) if SIDE = 'R' TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CHETRD.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by Q*C or Q**H*C or C*Q**H or C*Q.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input)
The dimension of the array WORK. If SIDE = 'L', LWORK >= max(1,N); if SIDE = 'R', LWORK >= max(1,M). For optimum performance LWORK >= N*NB if SIDE = 'L', and LWORK >=M*NB if SIDE = 'R', where NB is the optimal blocksize.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

zupgtr - generate a complex unitary matrix Q which is defined as the product of n-1 elementary reflectors H(i) of order n, as returned by CHPTRD using packed storage

SYNOPSIS

```
SUBROUTINE ZUPGTR(UPLO, N, AP, TAU, Q, LDQ, WORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AP(*), TAU(*), Q(LDQ,*), WORK(*)
INTEGER N, LDQ, INFO

SUBROUTINE ZUPGTR_64(UPLO, N, AP, TAU, Q, LDQ, WORK, INFO)

CHARACTER * 1 UPLO
DOUBLE COMPLEX AP(*), TAU(*), Q(LDQ,*), WORK(*)
INTEGER*8 N, LDQ, INFO
```

F95 INTERFACE

```
SUBROUTINE UPGTR(UPLO, [N], AP, TAU, Q, [LDQ], [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: AP, TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: Q
INTEGER :: N, LDQ, INFO

SUBROUTINE UPGTR_64(UPLO, [N], AP, TAU, Q, [LDQ], [WORK], [INFO])

CHARACTER(LEN=1) :: UPLO
COMPLEX(8), DIMENSION(:) :: AP, TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: Q
INTEGER(8) :: N, LDQ, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zupgtr(char uplo, int n, doublecomplex *ap,  doublecom-
          plex *tau, doublecomplex *q, int ldq, int *info);

void zupgtr_64(char uplo, long n, doublecomplex  *ap,  doub-
          lecomplex  *tau,  doublecomplex *q, long ldq, long
          *info);
```

PURPOSE

zupgtr generates a complex unitary matrix Q which is defined as the product of n-1 elementary reflectors H(i) of order n, as returned by CHPTRD using packed storage:

if UPLO = 'U', Q = H(n-1) . . . H(2) H(1),

if UPLO = 'L', Q = H(1) H(2) . . . H(n-1).

ARGUMENTS

UPLO (input)
= 'U': Upper triangular packed storage used in previous call to CHPTRD; = 'L': Lower triangular packed storage used in previous call to CHPTRD.

N (input) The order of the matrix Q. N >= 0.

AP (input)
The vectors which define the elementary reflectors, as returned by CHPTRD.

TAU (input)
TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CHPTRD.

Q (output)
The N-by-N unitary matrix Q.

LDQ (input)
The leading dimension of the array Q. LDQ >= max(1,N).

WORK (workspace)
dimension(N-1)

```
INFO (output)
          = 0:  successful exit
          < 0:  if INFO = -i, the i-th argument had an ille-
          gal value
```

Contents

NAME

```
zupmtr - overwrite the general complex M-by-N matrix C  with
SIDE = 'L' SIDE = 'R' TRANS = 'N'
```

SYNOPSIS

```
SUBROUTINE ZUPMTR(SIDE, UPLO, TRANS, M, N, AP, TAU, C, LDC, WORK,
      INFO)

CHARACTER * 1 SIDE, UPLO, TRANS
DOUBLE COMPLEX AP(*), TAU(*), C(LDC,*), WORK(*)
INTEGER M, N, LDC, INFO

SUBROUTINE ZUPMTR_64(SIDE, UPLO, TRANS, M, N, AP, TAU, C, LDC, WORK,
      INFO)

CHARACTER * 1 SIDE, UPLO, TRANS
DOUBLE COMPLEX AP(*), TAU(*), C(LDC,*), WORK(*)
INTEGER*8 M, N, LDC, INFO
```

F95 INTERFACE

```
SUBROUTINE UPMTR(SIDE, UPLO, [TRANS], [M], [N], AP, TAU, C, [LDC],
       [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANS
COMPLEX(8), DIMENSION(:) :: AP, TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: C
INTEGER :: M, N, LDC, INFO

SUBROUTINE UPMTR_64(SIDE, UPLO, [TRANS], [M], [N], AP, TAU, C, [LDC],
       [WORK], [INFO])

CHARACTER(LEN=1) :: SIDE, UPLO, TRANS
```

```
COMPLEX(8), DIMENSION(:) :: AP, TAU, WORK
COMPLEX(8), DIMENSION(:,:) :: C
INTEGER(8) :: M, N, LDC, INFO
```

C INTERFACE

```
#include <sunperf.h>

void zupmtr(char side, char uplo, char trans, int m, int  n,
          doublecomplex  *ap, doublecomplex *tau, doublecom-
          plex *c, int ldc, int *info);

void zupmtr_64(char side, char uplo,  char  trans,  long  m,
          long  n,  doublecomplex  *ap,  doublecomplex *tau,
          doublecomplex *c, long ldc, long *info);
```

PURPOSE

zupmtr overwrites the general complex M-by-N matrix C with
TRANS = 'C':   Q**H * C   C * Q**H

where Q is a complex unitary matrix of order nq, with nq = m if SIDE = 'L' and nq = n if SIDE = 'R'. Q is defined as the product of nq-1 elementary reflectors, as returned by CHPTRD using packed storage:

if UPLO = 'U', Q = H(nq-1) . . . H(2) H(1);

if UPLO = 'L', Q = H(1) H(2) . . . H(nq-1).

ARGUMENTS

SIDE (input)
= 'L': apply Q or Q**H from the Left;
= 'R': apply Q or Q**H from the Right.

UPLO (input)
= 'U': Upper triangular packed storage used in previous call to CHPTRD; = 'L': Lower triangular packed storage used in previous call to CHPTRD.

TRANS (input)
= 'N': No transpose, apply Q;
= 'C': Conjugate transpose, apply Q**H.

TRANS is defaulted to 'N' for F95 INTERFACE.

M (input) The number of rows of the matrix C. M >= 0.

N (input) The number of columns of the matrix C. N >= 0.

AP (input)
(M*(M+1)/2) if SIDE = 'L' (N*(N+1)/2) if SIDE = 'R' The vectors which define the elementary reflectors, as returned by CHPTRD. AP is modified by the routine but restored on exit.

TAU (input)
or (N-1) if SIDE = 'R' TAU(i) must contain the scalar factor of the elementary reflector H(i), as returned by CHPTRD.

C (input/output)
On entry, the M-by-N matrix C. On exit, C is overwritten by Q*C or Q**H*C or C*Q**H or C*Q.

LDC (input)
The leading dimension of the array C. LDC >= max(1,M).

WORK (workspace)
(N) if SIDE = 'L' (M) if SIDE = 'R'

INFO (output)
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value

Contents

NAME

```
zvbrmm - variable block sparse row format matrix-matrix
multiply
```

SYNOPSIS

```
 SUBROUTINE ZVBRMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, MB, N, KB, DESCRA(5), LDB, LDC, LWORK
 INTEGER    INDX(*), BINDX(*), RPNTR(MB+1), CPNTR(KB+1),
*           BPNTRB(MB), BPNTRE(MB)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(*), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE ZVBRMM_64( TRANSA, MB, N, KB, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, MB, N, KB, DESCRA(5), LDB, LDC, LWORK
 INTEGER*8  INDX(*), BINDX(*), RPNTR(MB+1), CPNTR(KB+1),
*           BPNTRB(MB), BPNTRE(MB)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX VAL(*), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE VBRMM(TRANSA, MB, [N], KB, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, KB
 INTEGER, DIMENSION(:) ::  DESCRA, INDX, BINDX
 INTEGER, DIMENSION(:) ::  RPNTR, CPNTR, BPNTRB, BPNTRE
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL
```

```
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C

 SUBROUTINE VBRMM_64(TRANSA, MB, [N], KB, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, KB
 INTEGER*8, DIMENSION(:) ::  DESCRA, INDX, BINDX
 INTEGER*8, DIMENSION(:) ::  RPNTR, CPNTR, BPNTRB, BPNTRE
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
        C <- alpha op(A) B + beta C
where ALPHA and BETA are scalar, C and B are  matrices,
A is a matrix represented in variable block sparse row  format
and op( A )  is one  of

op( A ) = A   or   op( A ) = A'   or   op( A ) = conjg( A' ).
                                   ( ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if the matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

KB            Number of block columns in matrix A

ALPHA         Scalar parameter

DESCRA()      Descriptor argument.  Five element integer array
              DESCRA(1) matrix structure
                0 : general
                1 : symmetric (A=A')
                2 : Hermitian (A= CONJG(A'))
                3 : Triangular
                4 : Skew(Anti)-Symmetric (A=-A')
                5 : Diagonal
                6 : Skew-Hermitian (A= -CONJG(A'))
```

DESCRA(2) upper/lower triangular indicator
  1 : lower
  2 : upper
DESCRA(3) main diagonal type
  0 : non-unit
  1 : unit
DESCRA(4) Array base (NOT IMPLEMENTED)
  0 : C/C++ compatible
  1 : Fortran compatible
DESCRA(5) repeated indices? (NOT IMPLEMENTED)
  0 : unknown
  1 : no repeated indices

VAL() scalar array of length NNZ consisting of the block entries of A where each block entry is a dense rectangular matrix stored column by column.
NNZ is the total number of point entries in all nonzero block entries of a matrix A.

INDX() integer array of length BNNZ+1 where BNNZ is the number of block entries of a matrix A such that the I-th element of INDX[] points to the location in VAL of the (1,1) element of the I-th block entry.

BINDX() integer array of length BNNZ consisting of the block column indices of the block entries of A where BNNZ is the number block entries of a matrix A.

RPNTR() integer array of length MB+1 such that RPNTR(I)-RPNTR(1)+1 is the row index of the first point row in the I-th block row.
RPNTR(MB+1) is set to M+RPNTR(1) where M is the number of rows in matrix A.
Thus, the number of point rows in the I-th block row is RPNTR(I+1)-RPNTR(I).

CPNTR() integer array of length KB+1 such that CPNTR(J)-CPNTR(1)+1 is the column index of the first point column in the J-th block column. CPNTR(KB+1) is set to K+CPNTR(1) where K is the number of columns in matrix A.
Thus, the number of point columns in the J-th block column is CPNTR(J+1)-CPNTR(J).

BPNTRB() integer array of length MB such that BPNTRB(I)-BPNTRB(1)+1 points to location in BINDX of the first block entry of the I-th block row of A.

BPNTRE() integer array of length MB such that BPNTRE(I)-BPNTRB(1) points to location in BINDX of the last block entry of the I-th block row of A.

B()   rectangular array with first dimension LDB.

LDB   leading dimension of B

BETA   Scalar parameter

C()   rectangular array with first dimension LDC.

LDC   leading dimension of C

WORK()   scratch array of length LWORK. WORK is not referenced in the current version.

LWORK   length of WORK array. LWORK is not referenced in the current version.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:
http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. For a general matrix (*DESCRA*(1)=0), array CPNTR can be different from RPNTR. For all other matrix types, RPNTR must equal CPNTR and a single array can be passed for both arguments.

2.It is known that there exists another representation of the variable block sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of six array instead of the seven used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block row in the array BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of variable block sparse row format the following calling sequence should be used

```
  SUBROUTINE ZVBRMM( TRANSA, MB, N, KB, ALPHA, DESCRA,
 *            VAL, INDX, BINDX, RPNTR, CPNTR, IA, IA(2),
 *            B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

```
zvbrsm - variable block sparse row format triangular solve
```

SYNOPSIS

```
 SUBROUTINE ZVBRSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER    TRANSA, MB, N, UNITD, DESCRA(5), LDB, LDC, LWORK
 INTEGER    INDX(*), BINDX(*), RPNTR(MB+1), CPNTR(MB+1),
*           BPNTRB(MB), BPNTRE(MB)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX DV(*), VAL(*), B(LDB,*), C(LDC,*), WORK(LWORK)

 SUBROUTINE ZVBRSM_64( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, LDB, BETA, C, LDC, WORK, LWORK)
 INTEGER*8  TRANSA, MB, N, UNITD, DESCRA(5), LDB, LDC, LWORK
 INTEGER*8  INDX(*), BINDX(*), RPNTR(MB+1), CPNTR(MB+1),
*           BPNTRB(MB), BPNTRE(MB)
 DOUBLE COMPLEX ALPHA, BETA
 DOUBLE COMPLEX DV(*), VAL(*), B(LDB,*), C(LDC,*), WORK(LWORK)
```

F95 INTERFACE

```
 SUBROUTINE VBRSM(TRANSA, MB, [N], UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER    TRANSA, MB, UNITD
 INTEGER, DIMENSION(:) ::  DESCRA, INDX, BINDX
 INTEGER, DIMENSION(:) ::  RPNTR, CPNTR, BPNTRB, BPNTRE
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL, DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

```
 SUBROUTINE VBRSM_64(TRANSA, MB, [N], UNITD, DV, ALPHA, DESCRA,
*           VAL, INDX, BINDX, RPNTR, CPNTR, BPNTRB, BPNTRE,
*           B, [LDB], BETA, C,[LDC], [WORK], [LWORK])
 INTEGER*8    TRANSA, MB, UNITD
 INTEGER*8, DIMENSION(:) ::  DESCRA, INDX, BINDX
 INTEGER*8, DIMENSION(:) ::  RPNTR, CPNTR, BPNTRB, BPNTRE
 DOUBLE COMPLEX    ALPHA, BETA
 DOUBLE COMPLEX, DIMENSION(:) :: VAL, DV
 DOUBLE COMPLEX, DIMENSION(:, :) ::  B, C
```

DESCRIPTION

```
  C <- ALPHA  op(A) B + BETA C     C <- ALPHA D op(A) B + BETA C
  C <- ALPHA op(A) D B + BETA C
where ALPHA and BETA are scalar, C and B are m by n dense matrices,
D is a block  diagonal matrix,  A is a unit, or non-unit, upper or
lower triangular matrix represented in variable block sparse row
format and  op( A )  is one  of

 op( A ) = inv(A) or  op( A ) = inv(A')  or  op( A ) =inv(conjg( A' ))
 (inv denotes matrix inverse,  ' indicates matrix transpose)
```

ARGUMENTS

```
TRANSA        Indicates how to operate with the sparse matrix
                0 : operate with matrix
                1 : operate with transpose matrix
                2 : operate with the conjugate transpose of matrix.
                    2 is equivalent to 1 if matrix is real.

MB            Number of block rows in matrix A

N             Number of columns in matrix C

UNITD         Type of scaling:
                1 : Identity matrix (argument DV[] is ignored)
                2 : Scale on left (row block scaling)
                3 : Scale on right (column block scaling)

DV()          Array containing the block entries of the block
              diagonal matrix D.  The size of the J-th block is
              RPNTR(J+1)-RPNTR(J) and each block contains matrix
              entries stored column-major.  The total length of
              array DV is given by the formula:

              sum over J from 1 to MB:
```

```
                ((RPNTR(J+1)-RPNTR(J))*(RPNTR(J+1)-RPNTR(J)))

ALPHA          Scalar parameter

DESCRA()       Descriptor argument.  Five element integer array
               DESCRA(1) matrix structure
                 0 : general
                 1 : symmetric (A=A')
                 2 : Hermitian (A= CONJG(A'))
                 3 : Triangular
                 4 : Skew(Anti)-Symmetric (A=-A')
                 5 : Diagonal
                 6 : Skew-Hermitian (A= -CONJG(A'))
               Note: For the routine, DESCRA(1)=3 is only supported.
               DESCRA(2) upper/lower triangular indicator
                 1 : lower
                 2 : upper
               DESCRA(3) main diagonal type
                 0 : non-identity  blocks on the main diagonal
                 1 : identity diagonal blocks
                 2 : diagonal blocks are dense matrices
               DESCRA(4) Array base  (NOT IMPLEMENTED)
                 0 : C/C++ compatible
                 1 : Fortran compatible
               DESCRA(5) repeated indices? (NOT IMPLEMENTED)
                 0 : unknown
                 1 : no repeated indices

VAL()          scalar array of length NNZ consisting of the block entries
               of A where each block entry is a dense rectangular matrix
               stored column by column.
               NNZ is the total number of point entries in all nonzero
               block  entries of a matrix A.

INDX()         integer array of length BNNZ+1 where BNNZ is the number
               block entries of a matrix A such that the I-th element of
               INDX[] points to the location in VAL of the (1,1) element
               of the I-th block entry.

BINDX()        integer array of length BNNZ consisting of the block
               column indices of the block entries of A where BNNZ is
               the number block entries of a matrix A.   Block column
               indices MUST be sorted in increasing order for each block
               row.

RPNTR()        integer array of length MB+1 such that RPNTR(I)-RPNTR(1)+1
               is the row index of the first point row in the I-th block
               row.
               RPNTR(MB+1) is set to M+RPNTR(1) where M is the number
               of rows  in square triangular matrix A.
```

Thus, the number of point rows in the I-th block row is RPNTR(I+1)-RPNTR(I).

NOTE: For the current version CPNTR must equal RPNTR and a single array can be passed for both arguments

CPNTR()	integer array of length MB+1 such that CPNTR(J)-CPNTR(1)+1 is the column index of the first point column in the J-th block column. CPNTR(MB+1) is set to M+CPNTR(1). Thus, the number of point columns in the J-th block column is CPNTR(J+1)-CPNTR(J). NOTE: For the current version CPNTR must equal RPNTR and a single array can be passed for both arguments
BPNTRB()	integer array of length MB such that BPNTRB(I)-BPNTRB(1)+1 points to location in BINDX of the first block entry of the I-th block row of A.
BPNTRE()	integer array of length MB such that BPNTRE(I)-BPNTRB(1) points to location in BINDX of the last block entry of the I-th block row of A.
B()	rectangular array with first dimension LDB.
LDB	leading dimension of B
BETA	Scalar parameter
C()	rectangular array with first dimension LDC.
LDC	leading dimension of C
WORK()	scratch array of length LWORK. On exit, if LWORK= -1, WORK(1) returns the optimum size of LWORK.
LWORK	length of WORK array. LWORK should be at least M = RPNTR(MB+1)-RPNTR(1). For good performance, LWORK should generally be larger. For optimum performance on multiple processors, LWORK >=M*N_CPUS where N_CPUS is the maximum number of processors available to the program. If LWORK=0, the routine is to allocate workspace needed. If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimum size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued

by XERBLA.

SEE ALSO

NIST FORTRAN Sparse Blas User's Guide available at:

http://math.nist.gov/mcsd/Staff/KRemington/fspblas/

"Document for the Basic Linear Algebra Subprograms (BLAS) Standard", University of Tennessee, Knoxville, Tennessee, 1996:

http://www.netlib.org/utk/papers/sparse.ps

NOTES/BUGS

1. No test for singularity or near-singularity is included in this routine. Such tests must be performed before calling this routine.

2. If *DESCRA*(3)=0,the lower or upper triangular part of each diagonal block is used by the routine depending on *DESCRA*(2).

3. If *DESCRA*(3)=1, the unit diagonal blocks might or might not be referenced in the VBR representation of a sparse matrix. They are not used anyway.

4. If *DESCRA*(3)=2, diagonal blocks are considered as dense matrices and the LU factorization with partial pivoting is used by the routine. *WORK*(1)=0 on return if the factorization for all diagonal blocks has been completed successfully, otherwise *WORK*(1) = -i where i is the block number for which the LU factorization could not be computed.

5. The routine can be applied for solving triangular systems when the upper or lower triangle of the general sparse matrix A is used. Howerver *DESCRA*(1) must be equal to 3.

6. It is known that there exists another representation of the variable block sparse row format (see for example Y.Saad, "Iterative Methods for Sparse Linear Systems", WPS, 1996). Its data structure consists of six array instead of the seven used in the current implementation. The main difference is that only one array, IA, containing the pointers to the beginning of each block row in the array BINDX is used instead of two arrays BPNTRB and BPNTRE. To use the routine with this kind of variable block sparse row format the following calling sequence should be used

```
      SUBROUTINE ZVBRSM( TRANSA, MB, N, UNITD, DV, ALPHA, DESCRA,
     *           VAL, INDX, BINDX, RPNTR, CPNTR, IA, IA(2),
     *           B, LDB, BETA, C, LDC, WORK, LWORK )
```

Contents

NAME

```
zvmul - compute the scaled product of complex vectors
```

SYNOPSIS

```
SUBROUTINE ZVMUL(N, ALPHA, X, INCX, Y, INCY, BETA, Z, INCZ)

DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX X(*), Y(*), Z(*)
INTEGER N, INCX, INCY, INCZ

SUBROUTINE ZVMUL_64(N, ALPHA, X, INCX, Y, INCY, BETA, Z, INCZ)

DOUBLE COMPLEX ALPHA, BETA
DOUBLE COMPLEX X(*), Y(*), Z(*)
INTEGER*8 N, INCX, INCY, INCZ
```

F95 INTERFACE

```
SUBROUTINE VMUL([N], ALPHA, X, [INCX], Y, [INCY], BETA, Z, [INCZ])

COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:) :: X, Y, Z
INTEGER :: N, INCX, INCY, INCZ

SUBROUTINE VMUL_64([N], ALPHA, X, [INCX], Y, [INCY], BETA, Z, [INCZ])

COMPLEX(8) :: ALPHA, BETA
COMPLEX(8), DIMENSION(:) :: X, Y, Z
INTEGER(8) :: N, INCX, INCY, INCZ
```

C INTERFACE

```
#include <sunperf.h>
```

```
void zvmul(int n, doublecomplex  *alpha,  doublecomplex  *x,
          int  incx,  doublecomplex *y, int incy, doublecom-
          plex *beta, doublecomplex *z, int incz);

void zvmul_64(long n,  doublecomplex  *alpha,  doublecomplex
          *x,  long incx, doublecomplex *y, long incy, doub-
          lecomplex *beta, doublecomplex *z, long incz);
```

PURPOSE

```
zvmul computes the scaled product of complex vectors:
  z(i) = ALPHA * x(i) * y(i) + BETA * z(i)
for 1 <= i <= N.
```

ARGUMENTS

```
N (input)
          Length of the vectors.  N >= 0.  ZVMUL will return
          immediately if N = 0.

ALPHA (input)
          Scale factor on the multiplicand vectors.

X (input)  dimension(*)
          Multiplicand vector.

INCX (input)
          Stride between elements of the multiplicand vector
          X.  INCX > 0.

Y (input)  dimension(*)
          Multiplicand vector.

INCY (input)
          Stride between elements of the multiplicand vector
          Y.  INCY > 0.

BETA (input)
          Scale factor on the product vector.

Z (input/output)
           dimension(*)
          Product vector.  On exit, z(i) = ALPHA  *  x(i)  *
          y(i) + BETA * z(i).

INCZ (input)
          Stride between elements of Z.  INCZ > 0.
```